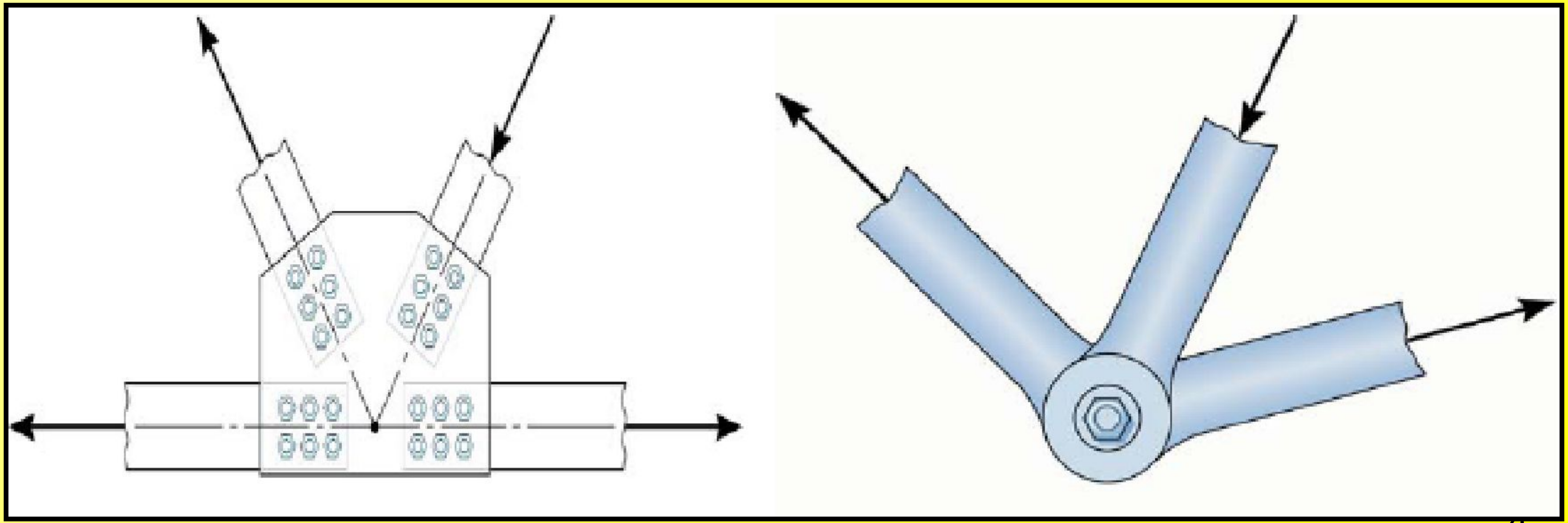
# Structural Analysis of Trusses – Method of Joints

# Simple Trusses

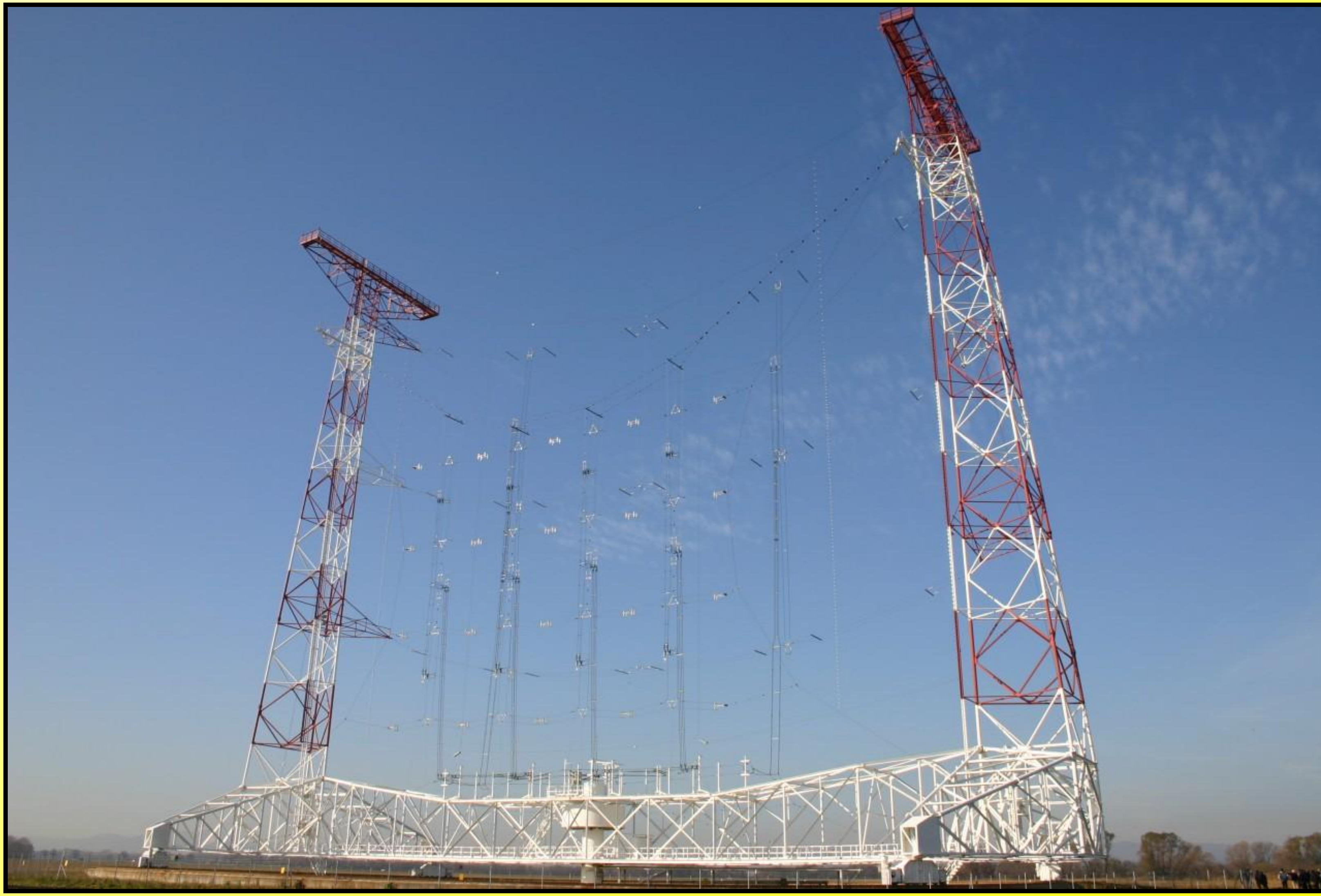
# Truss

- Definition: A truss is a structure composed of slender members joined together at their end points. The members are usually made of wood or metal

- Steel trusses: Joints are usually formed by bolting or welding the members to a common plate, called a gusset plate, or simply passing a large bolt through each member.

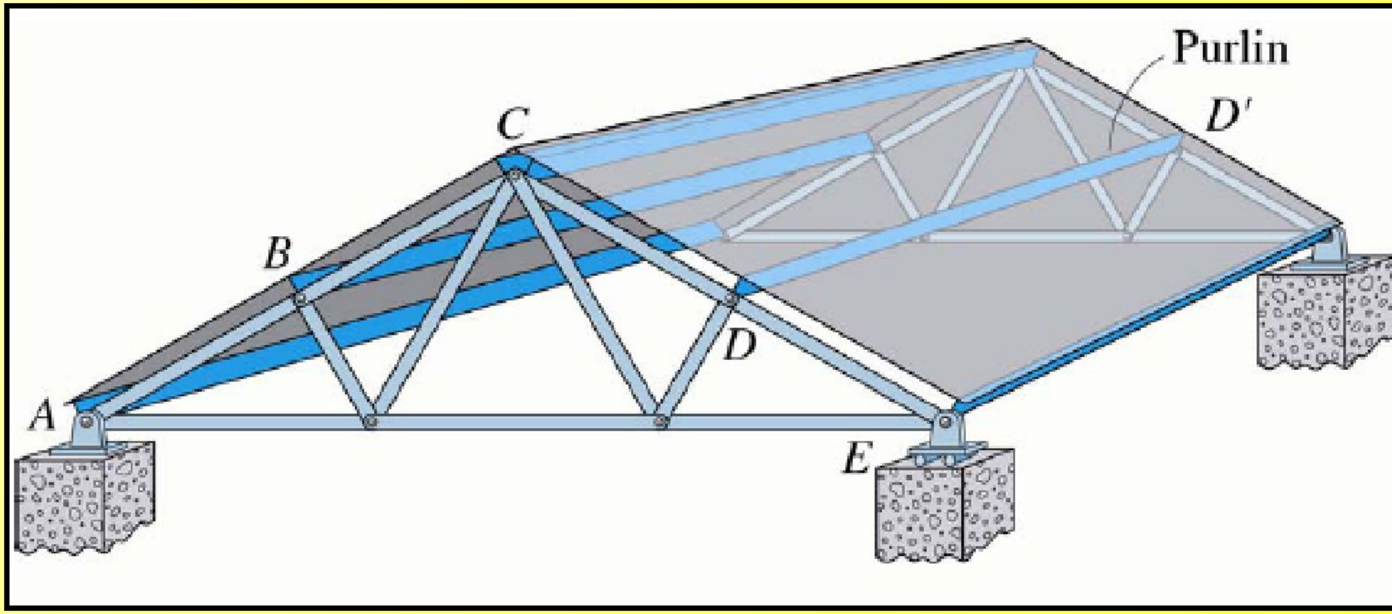


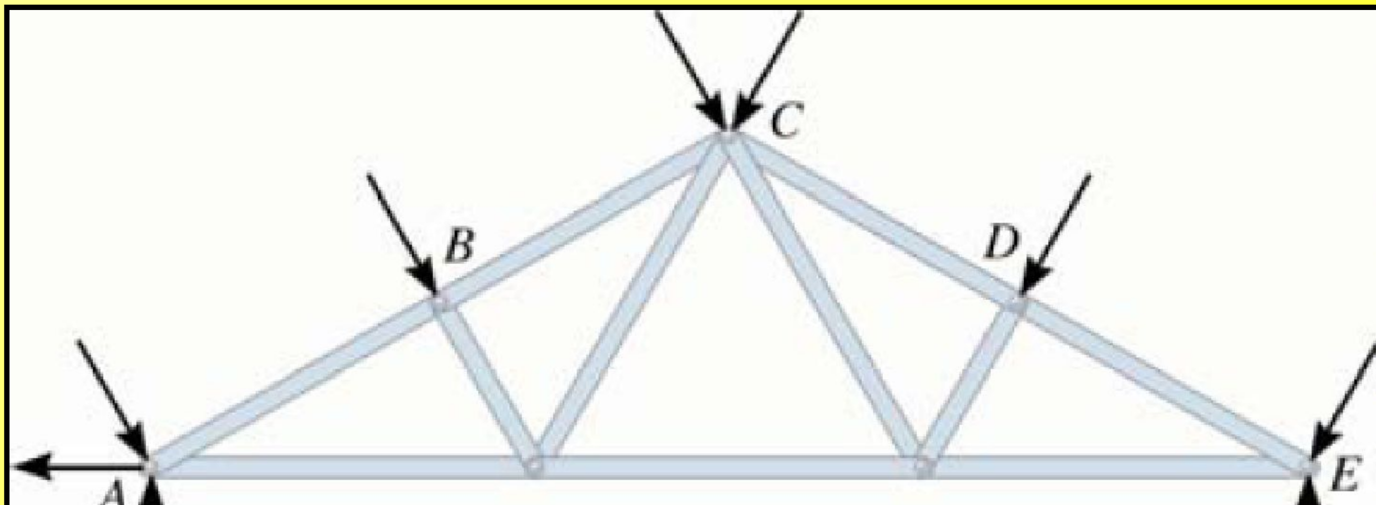

NO CAMPING



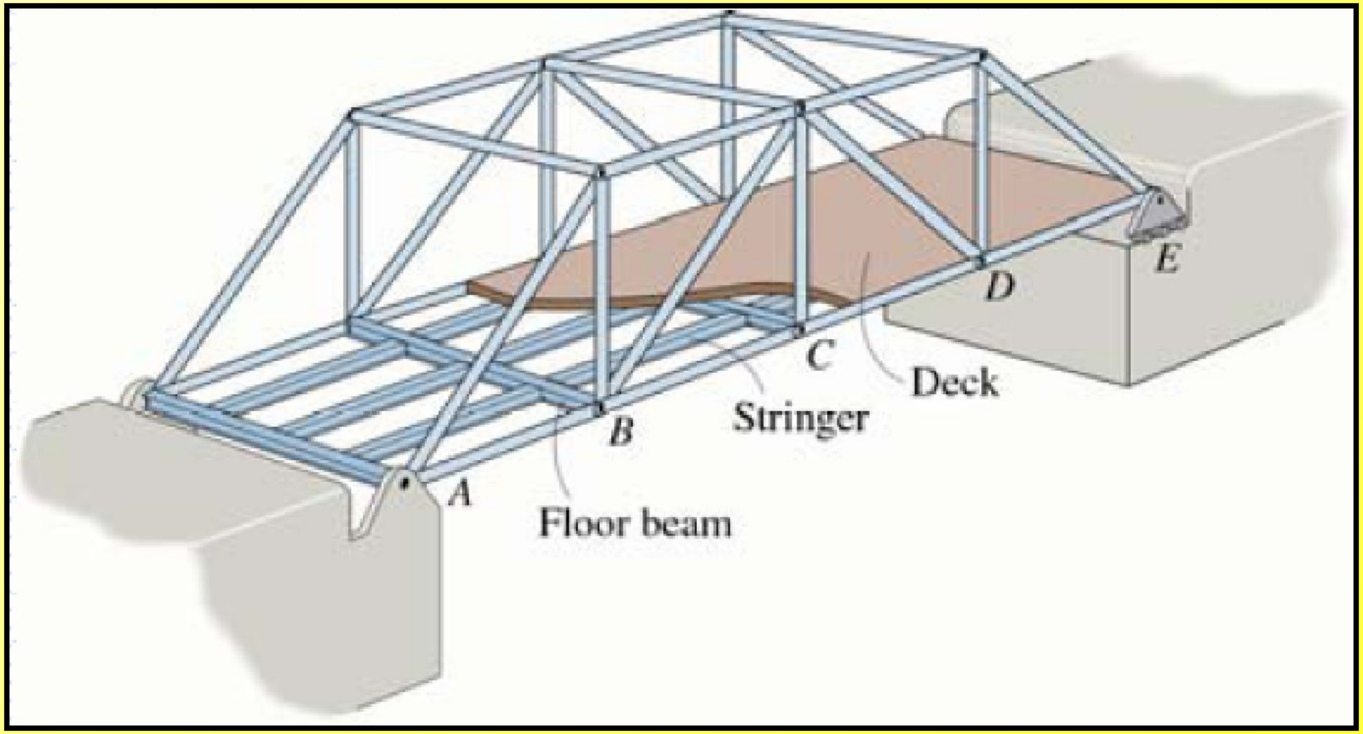
# Planar Trusses

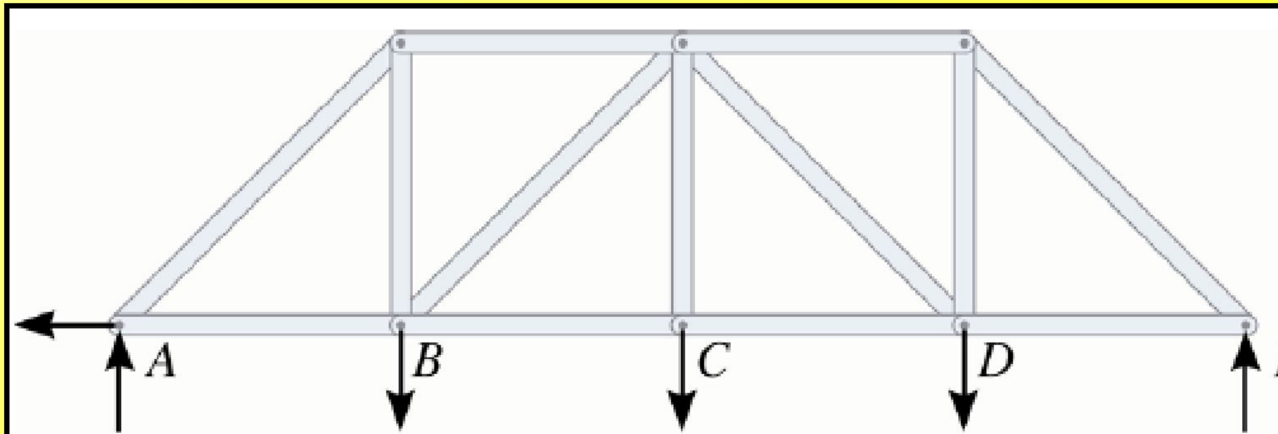
- What we will encounter most frequently is planar truss structure.
- Planar truss structures are structures which can be assumed to lie in a single plane and are often used to support roof and bridges.



Roof truss




Bridge truss

# Assumptions for Design

- To design truss structure and its connections, it is first necessary to determine force in each member.
- Newton's First and Third Law will be used to solve the equilibrium equations.

- General assumptions concerning truss structures are:
    1. All loadings are applied at the joints
    2. The weights of the members are neglected.
    3. All members are two force members
    4. The members are joined together by smooth pins.

- Note:
    1. To prevent collapse, the framework of a truss must be rigid
    2. The simplest framework which is rigid and stable is a triangle.
    3. Therefore, a triangle is the building block of all truss structures.

# The Method of Joints

- It is important to know that for equilibrium of forces, the sufficient condition is

  *External Force = Internal Force*

- External force: reaction forces, applied forces
- Internal force: tension/compression force set up in the member

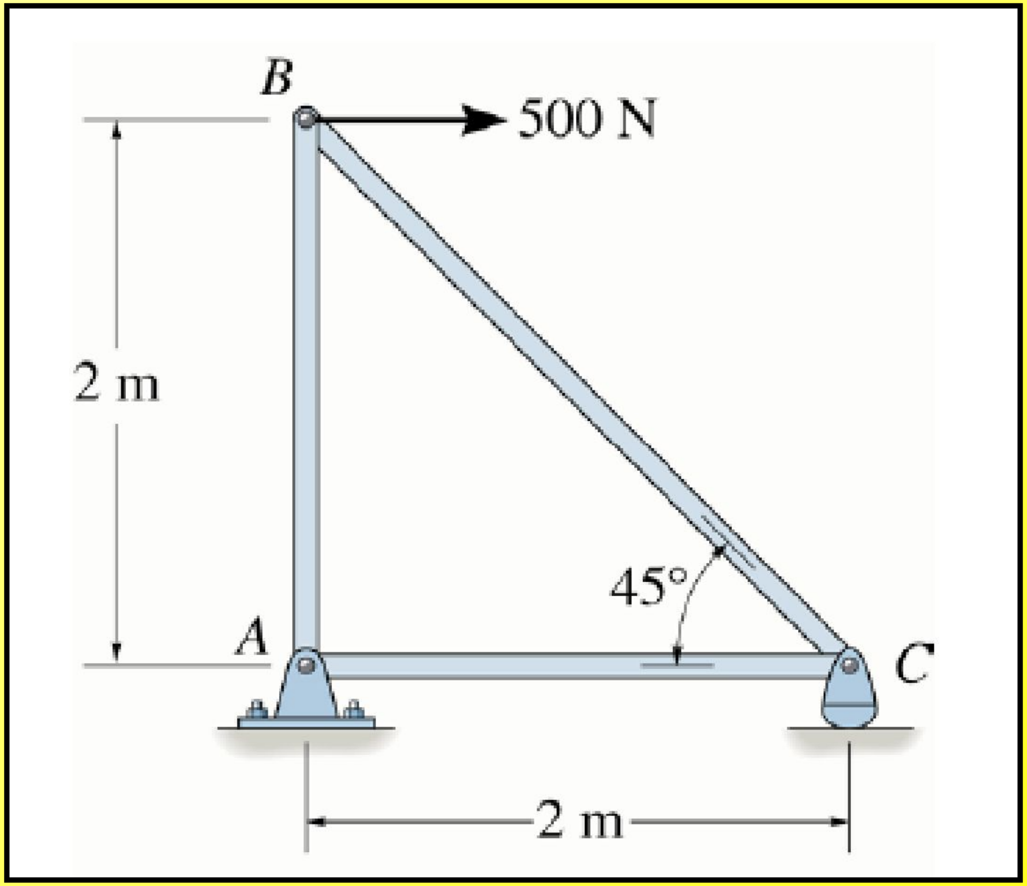
- To analyze a truss structure using method of joint, these steps are useful:
  1. Draw the free-body diagram of a joint having at least one known force and at most two unknown forces.
  2. Orient the *x*- and *y*-axes such that the force can be resolved into their *x* and *y* components
  3. Apply force equilibrium equations
  4. Continue to analyze each of the other joints by repeating procedure (i) to (iii)
  5. A member in compression (C) "pushes" on the joint and a member in tension (T) "pulls" on the joints. Normally, take compression as –ve and tension as +ve.

**Note:**

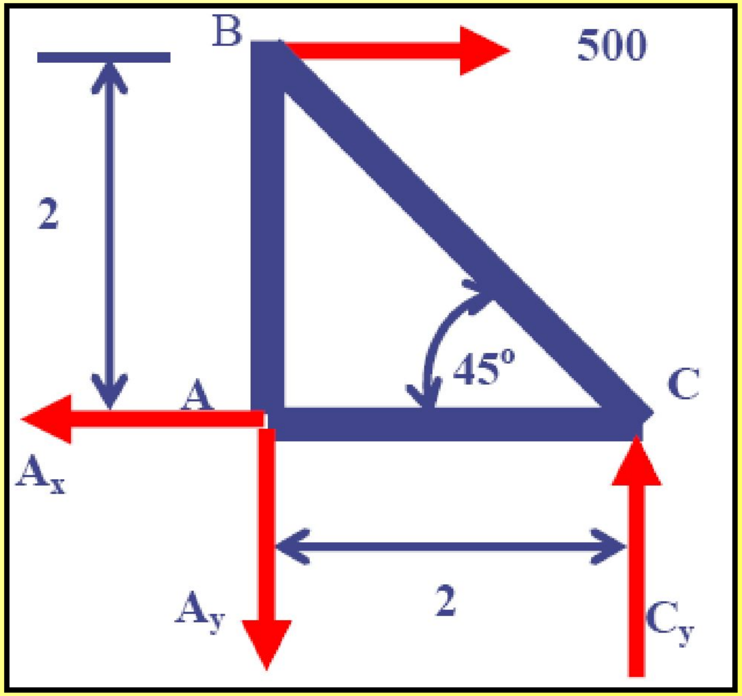
- In the beginning of a calculation, ignore all the compression/tension sign.
- Start off by simply labeling each member consistently according to the applied force (a little bit of common sense is required here).
- The final calculated numerical values will tell us if we have labeled the members correctly.

Example:

Determine force in each member of the truss shown. Indicate if the members are in tension/compression



***Step 1***: Draw the free-body diagram & determine the support reactions



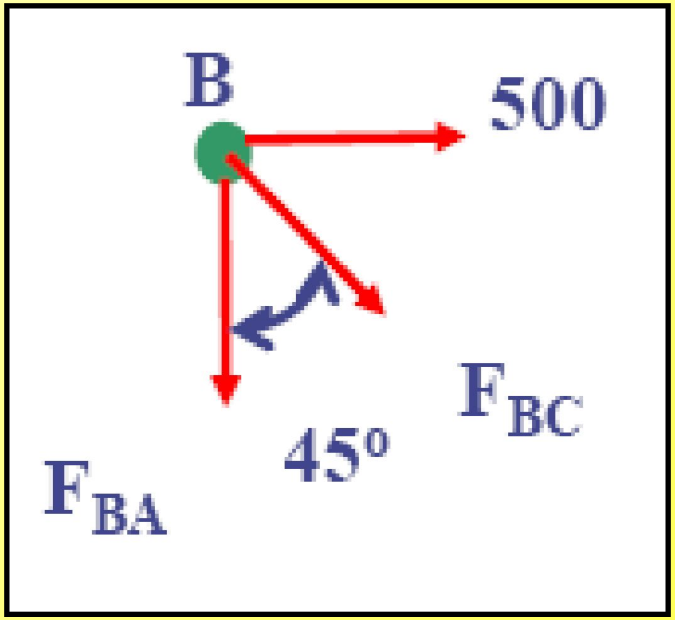

$$\sum F_x = 0 = 500 - A_x \qquad \therefore A_x = 500N$$

$$\sum M_A = 0 = -500(2) + 2C_y \qquad \therefore C_y = 500N$$

$$\sum F_y = 0 = C_y - A_y \qquad \therefore A_y = 500N$$

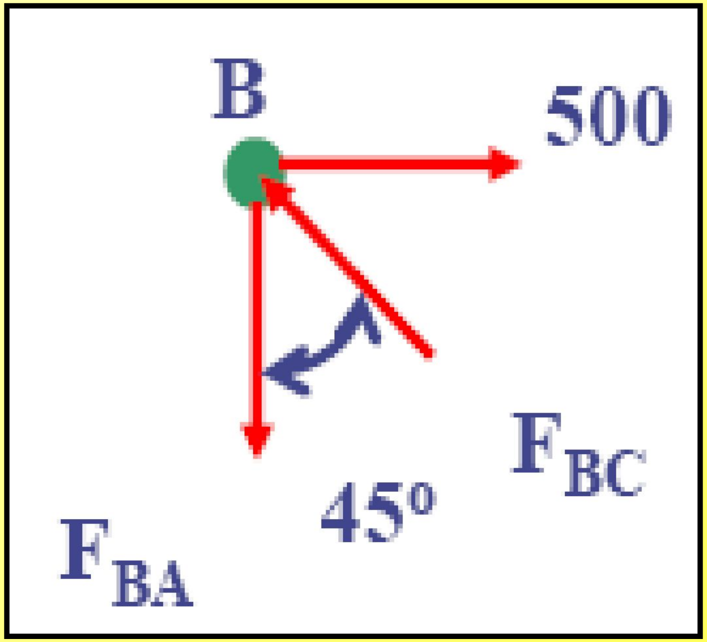
***Step 2*:** Resolved forces in members into their *x* and *y* components. Start with the joints having at least one known force and at most two unknown forces. Assume the force in each member.

## Joint B




Alternative 1: Assume $F_{BC}$ is in tension

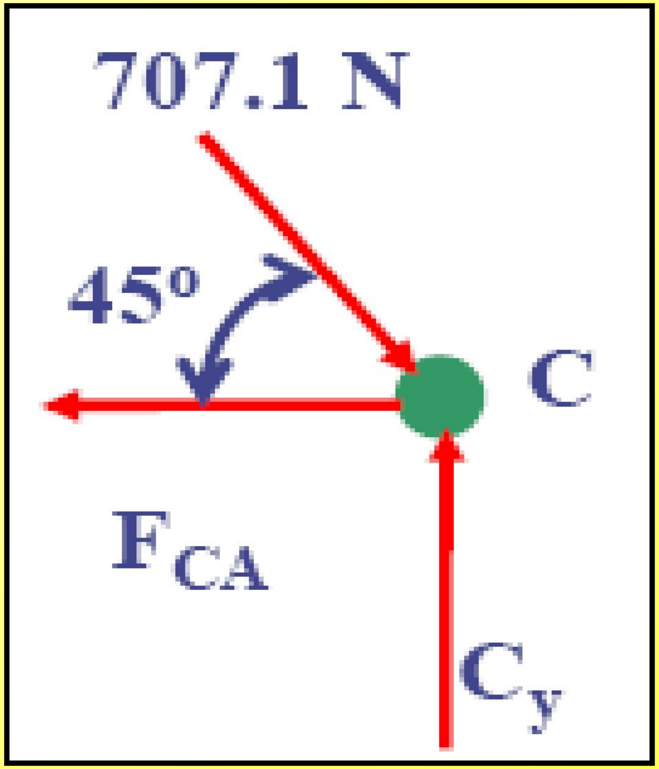
$$\sum F_x = 0$$
$$500 + F_{BC} \sin 45^\circ = 0$$
$$F_{BC} = -707.1 \text{ N}$$
$$F_{BC} = 707.1 \text{ N (C)}$$
$$\sum F_y = 0$$
$$-F_{BC} \cos 45^\circ - F_{BA} = 0$$
$$F_{BA} = 500 \text{ N (T)}$$




Alternative 2: Assume
$F_{BC}$ is in compression

$$\sum F_x = 0$$
$$500 - F_{BC} \sin 45^\circ = 0$$
$$F_{BC} = 707.1 \text{ N } (C)$$
$$\sum F_y = 0$$
$$F_{BC} \cos 45^\circ - F_{BA} = 0$$
$$F_{BA} = 500 \text{ N } (T)$$

## Joint C




$$\sum F_x = 0$$

$$-F_{CA} + 707.1 \cos 45^0 = 0$$

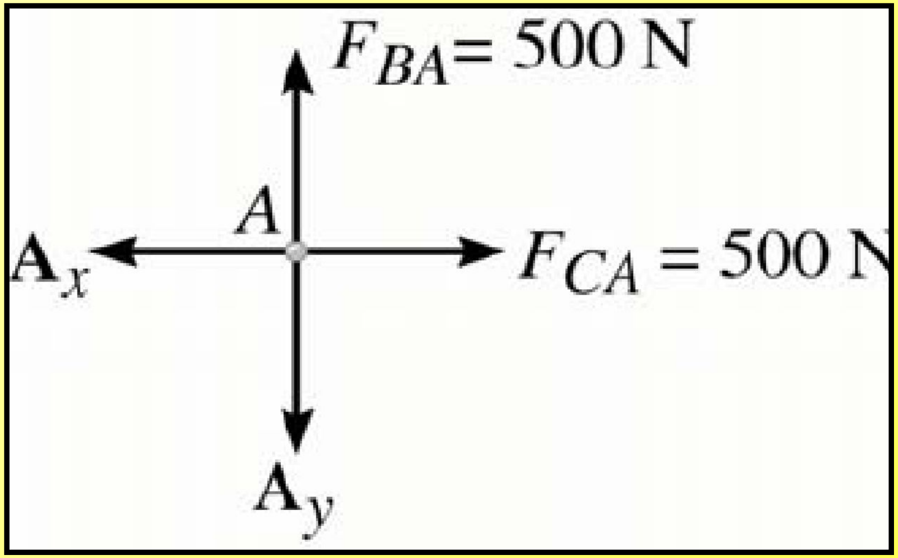
$$F_{CA} = 500 \text{ N} \quad (T)$$

$$\sum F_y = 0$$

$$C_y - 707.1 \sin 45^0 = 0$$

$$C_y = 500 \text{ N}$$

**Joint A**




$$\sum F_x = 0$$

$$-A_x + 500 = 0$$

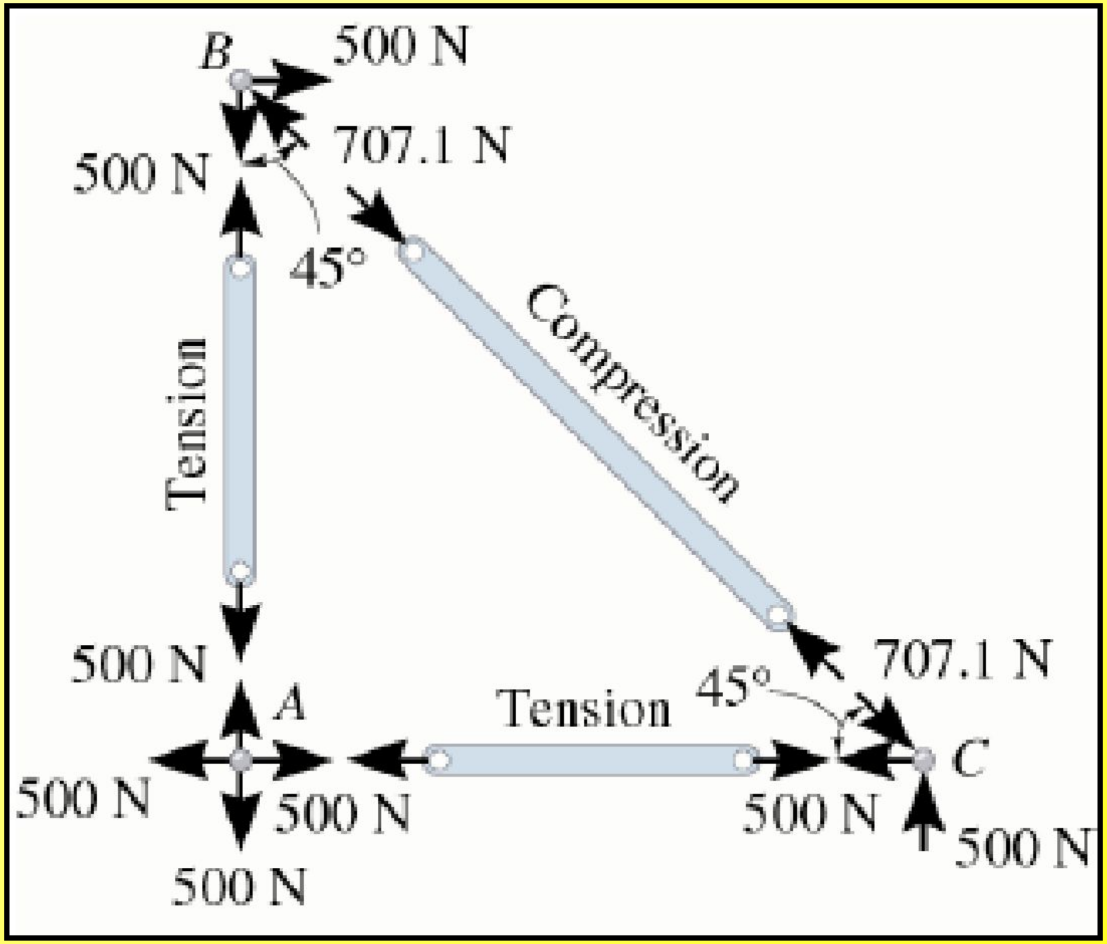
$$A_x = 500 \text{ N}$$

$$\sum F_y = 0$$

$$-A_y + 500 = 0$$

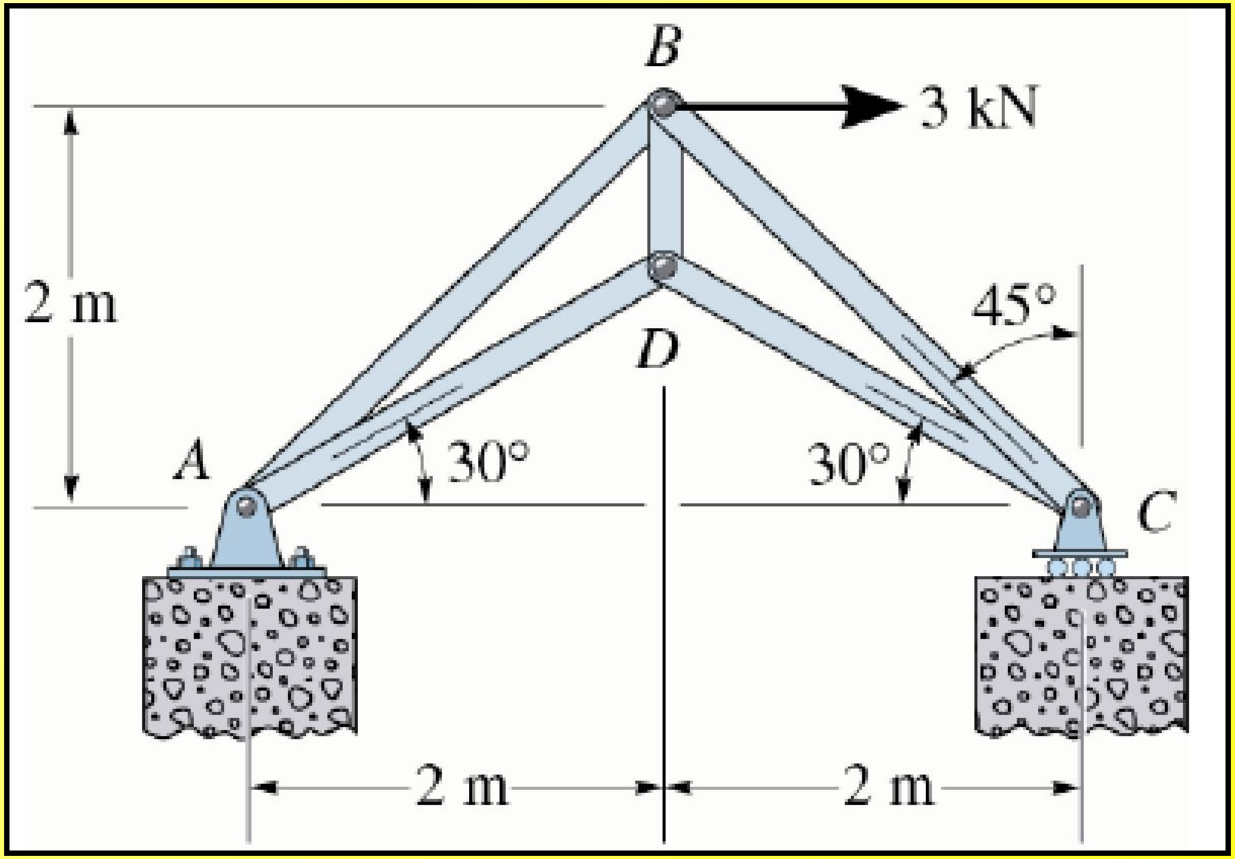
$$A_y = 500 \text{ N}$$

***Step 3*:** Complete the force distribution diagram

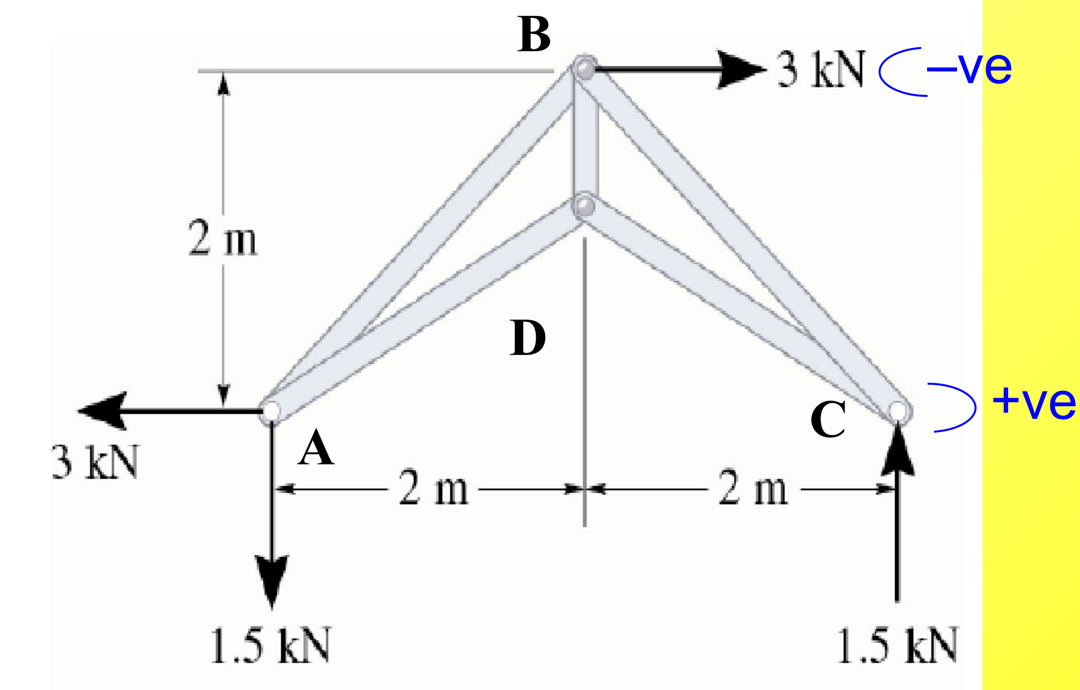


**Example:**

Determine force in each member of the truss shown. Indicate if the members are in tension/compression.



***Step 1***: Draw the free-body diagram & determine the support reactions




$$\sum F_x = 0 \qquad \sum M_A = 0$$

$$-A_x + 3\ \text{kN} = 0 \qquad C_y(4) - 3\ \text{kN}(2) = 0$$

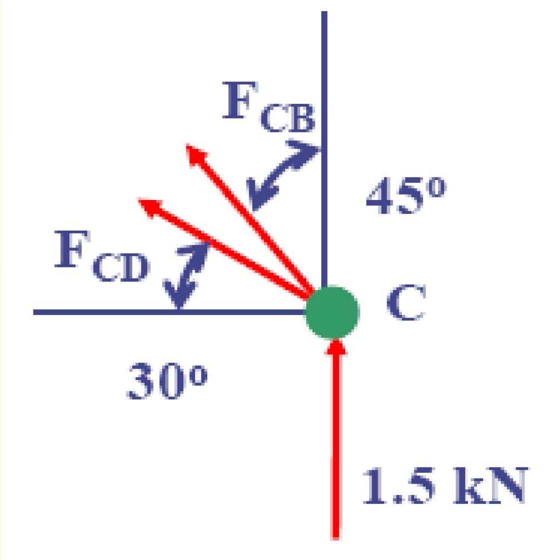
$$A_x = 3\ \text{kN} \qquad C_y = 1.5\ \text{kN}$$

$$\sum F_y = 0$$

$$C_y - Ay = 0$$

$$A_y = 1.5\ \text{kN}$$

## *Step 2*: Calculate forces at the joints




$$\sum F_x = 0$$

$$-F_{CD}\cos 30^{\circ} - F_{CB}\sin 45^{\circ} = 0$$

$$\sum F_y = 0$$

$$F_{CD}\sin 30^{\circ} + F_{CB}\cos 45^{\circ} + 1.5 = 0$$

$$-0.866\ F_{CD} = 0.7071\ F_{CB}$$

$$F_{CD} = -\frac{0.7071}{0.866} F_{CB} = -0.817\ F_{CB}$$

$$-0.817\ F_{CB}(0.5) + 0.7071 F_{CB} + 1.5 = 0$$

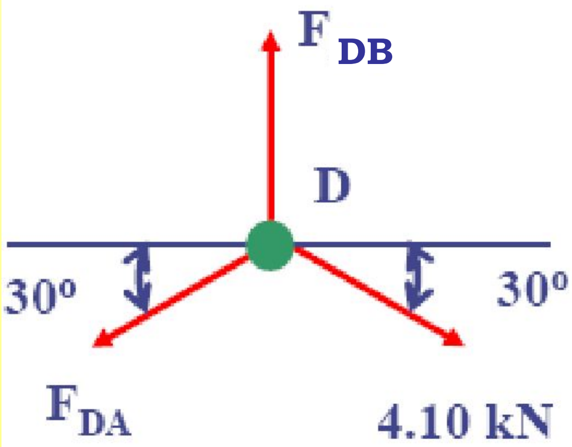
$$0.299\ F_{CB} = -1.5$$

$$F_{CB} = -5.02\ \text{kN}$$

$$F_{CD} = -0.817\ F_{CB} = -0.817(-5.02\ \text{kN}) = 4.10\ \text{kN}$$

$$F_{CB} = 5.02\ \text{kN} \quad (\text{C})$$

$$F_{CD} = 4.10\ \text{kN} \quad (\text{T})$$




$$\sum F_x = 0$$

$$-F_{DA}\cos 30^\circ + 4.10\cos 30^\circ = 0$$

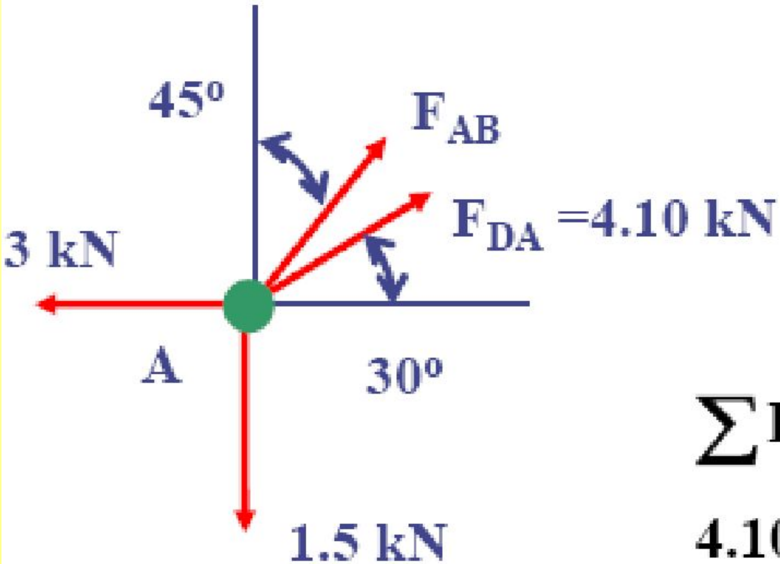
$$F_{DA} = 4.10\ \text{kN} \quad (T)$$

$$\sum F_y = 0$$

$$-F_{DA}\sin 30^\circ - F_{DC}\sin 30^\circ + F_{DB} = 0$$

$$-4.10(0.5) - 4.10(0.5) + F_{DB} = 0$$

$$F_{DB} = 4.10\ \text{kN} \quad (T)$$



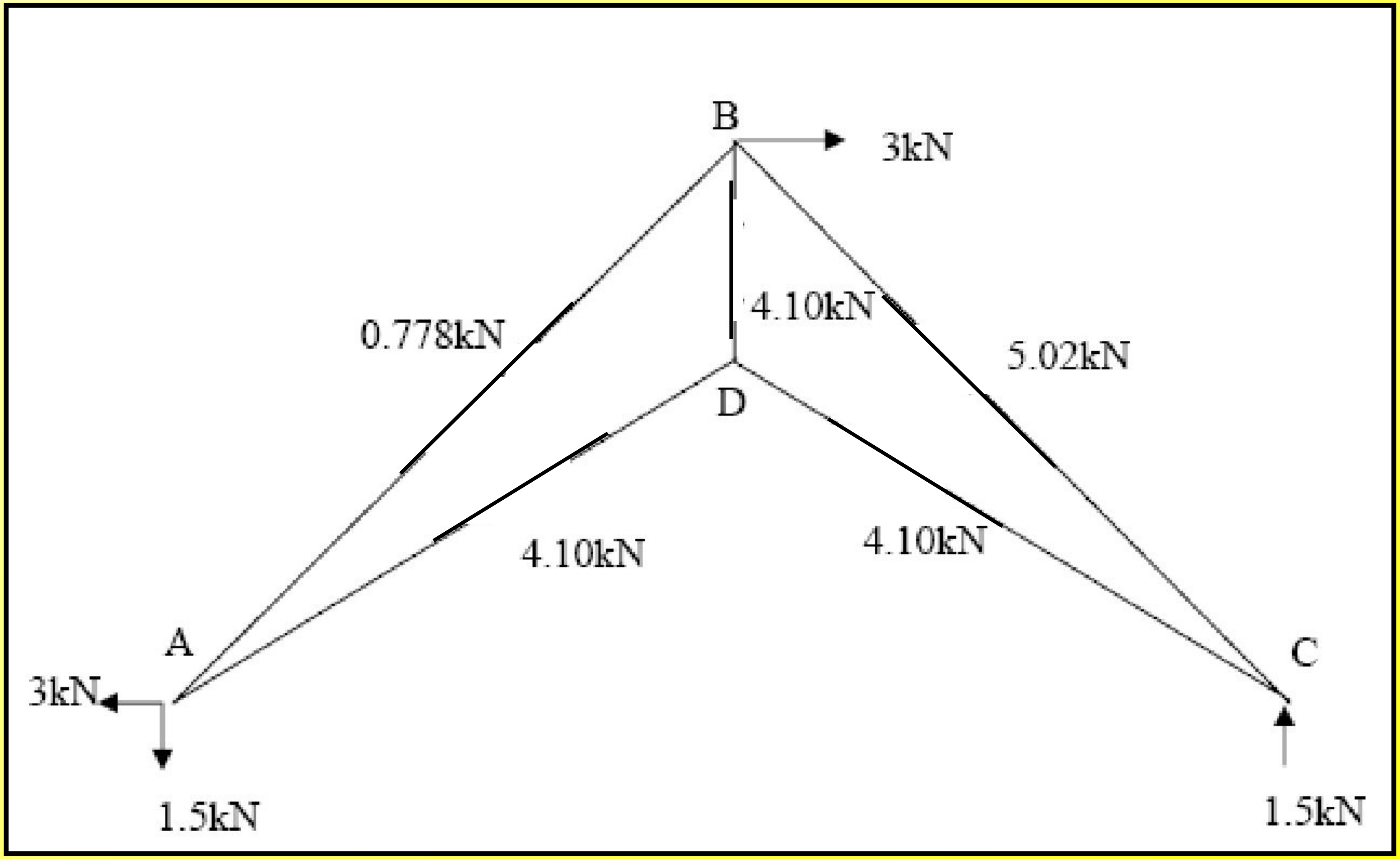

$$\sum F_x = 0$$

$$4.10\cos 30^{o} + F_{AB}\sin 45^{o} - 3 = 0$$

$$F_{AB} = -0.776\ \text{kN}$$

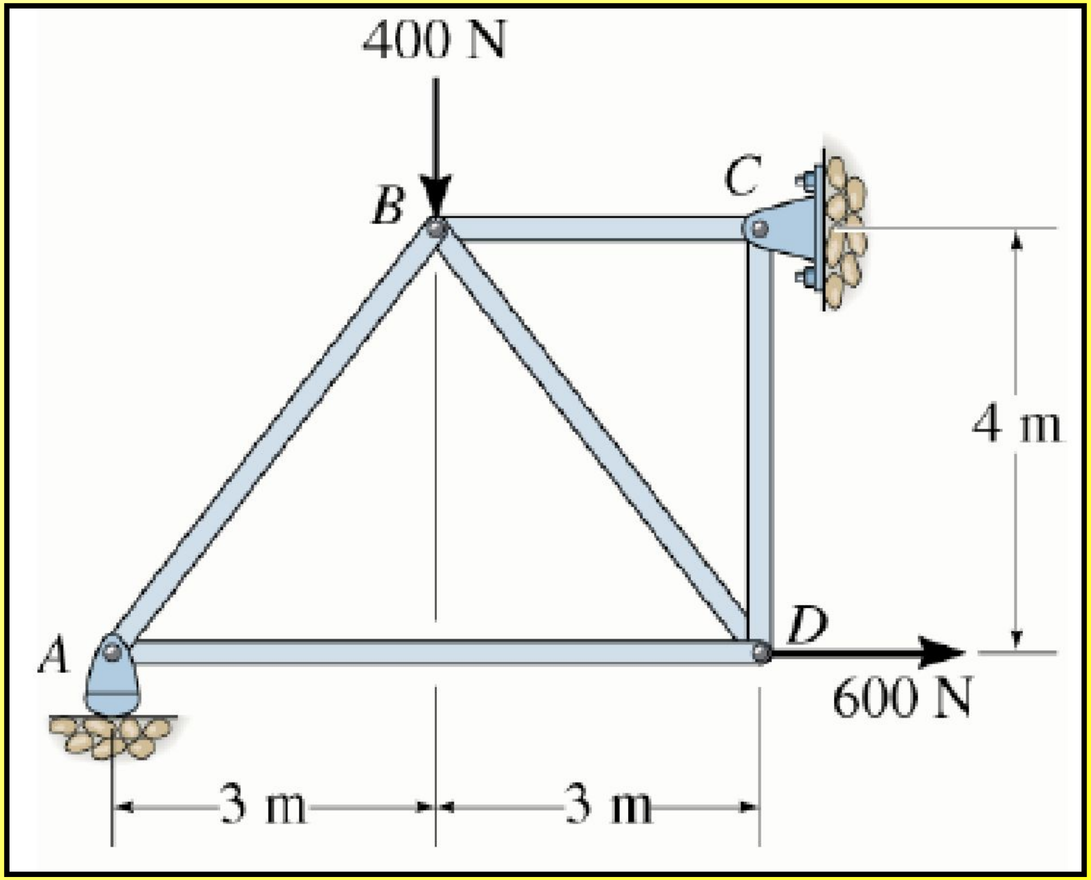
$$F_{AB} = 0.776\ \text{kN} \quad (C)$$

## ***Step 3***: Complete the force distribution diagram

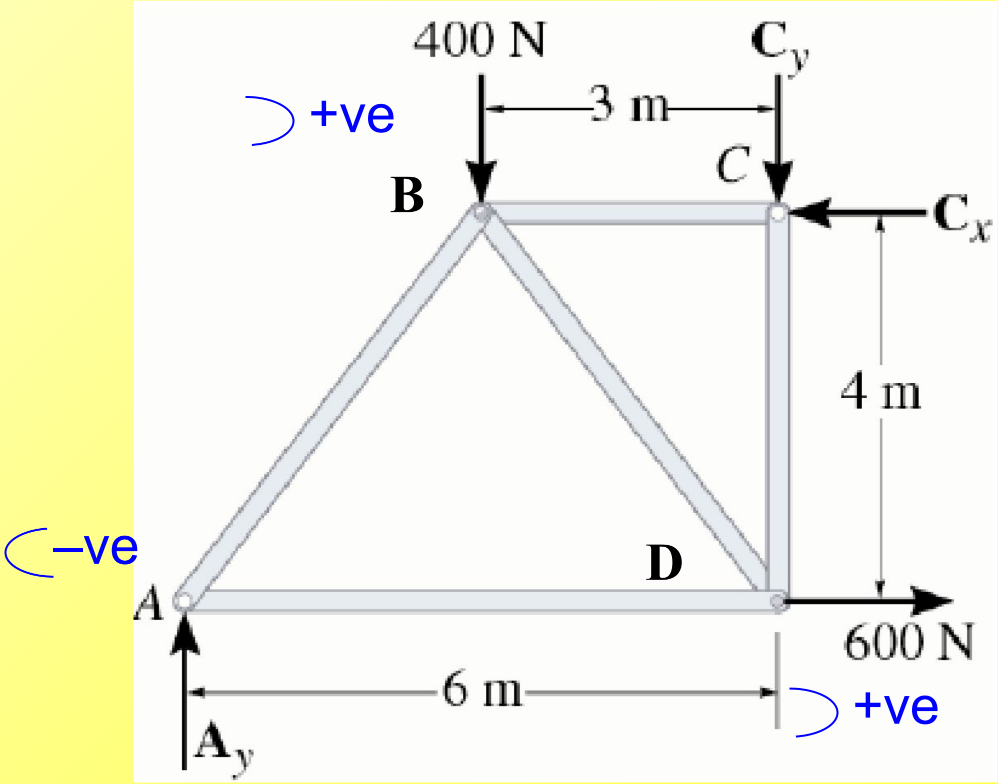


Example:

Determine force in each member. Indicate whether the members are in tension or compression.



***Step 1***: Draw the free-body diagram & determine the support reactions




$$\sum F_x = 0$$

$$600 - C_x = 0$$

$$C_x = 600 \ \text{N}$$

$$\sum M_C = 0 \ (+\text{ccw})$$

$$-A_y(6) + 400(3) + 600(4) = 0$$

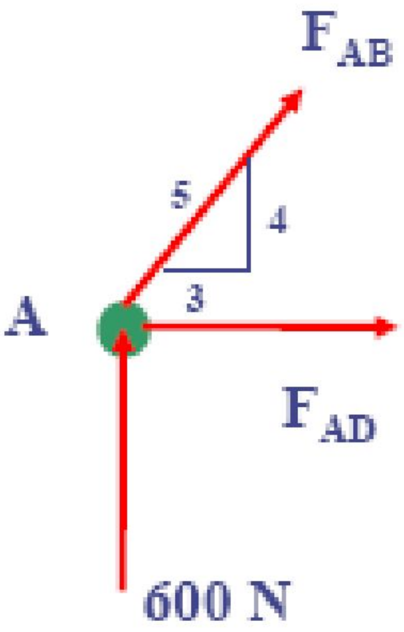
$$A_y = 600 \ \text{N}$$

$$\sum F_y = 0$$

$$A_y - 400 - C_y = 0$$

$$C_y = 200 \ \text{N}$$

***Step 2***: Calculate forces at the joints




$$\sum F_x = 0$$

$$F_{AD} + F_{AB}\left(\frac{3}{5}\right) = 0$$

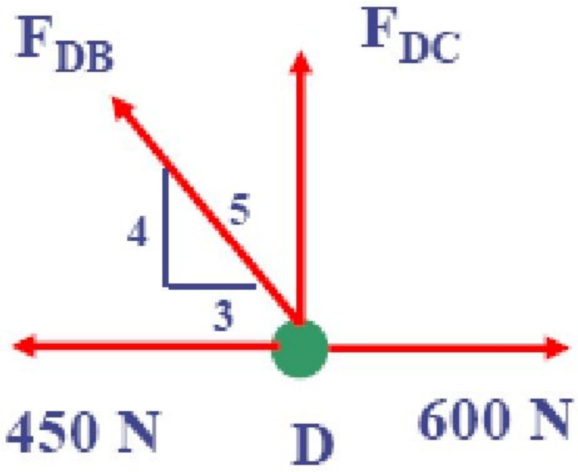
$$\sum F_y = 0$$

$$600 + F_{AB}\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = -750\ \text{N}$$

$$F_{AB} = 750\ \text{N} \quad (\text{C})$$

$$F_{AD} = 450\ \text{N} \quad (\text{T})$$




$$\sum F_x = 0$$

$$600 - 450 - F_{DB}\left(\frac{3}{5}\right) = 0$$

$$F_{DB} = +250 \text{ N}$$

$$F_{DB} = 250 \text{ N} \quad (T)$$

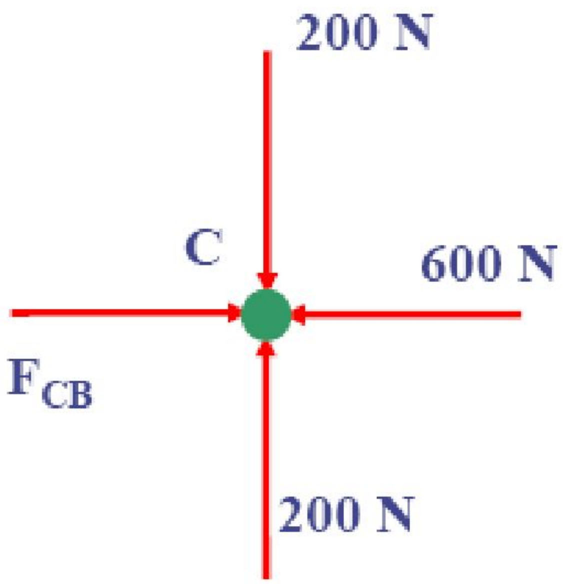
$$\sum F_y = 0$$

$$F_{DC} + F_{DB}\left(\frac{4}{5}\right) = 0$$

$$F_{DC} + 250\left(\frac{4}{5}\right) = 0$$

$$F_{DC} = -200 \text{ N}$$

$$F_{DB} = 250 \text{ N} \quad (T)$$



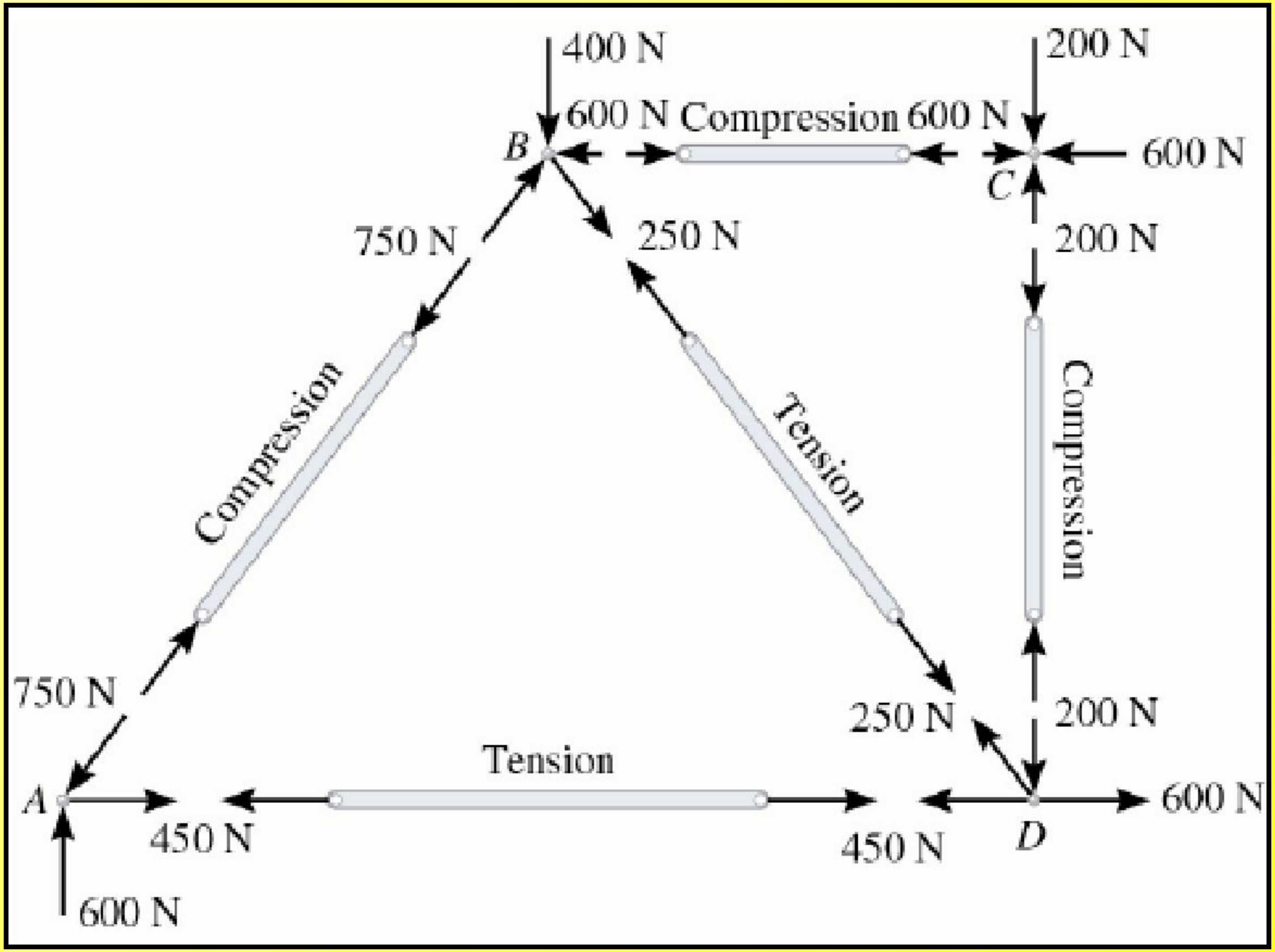

$$\sum F_x = 0$$

$$-600 + F_{CB} = 0$$

$$F_{DB} = 600\ \mathrm{N}$$

$$F_{DB} = 600\ \mathrm{N} \quad (C)$$

***Step 3***: Complete the force distribution diagram



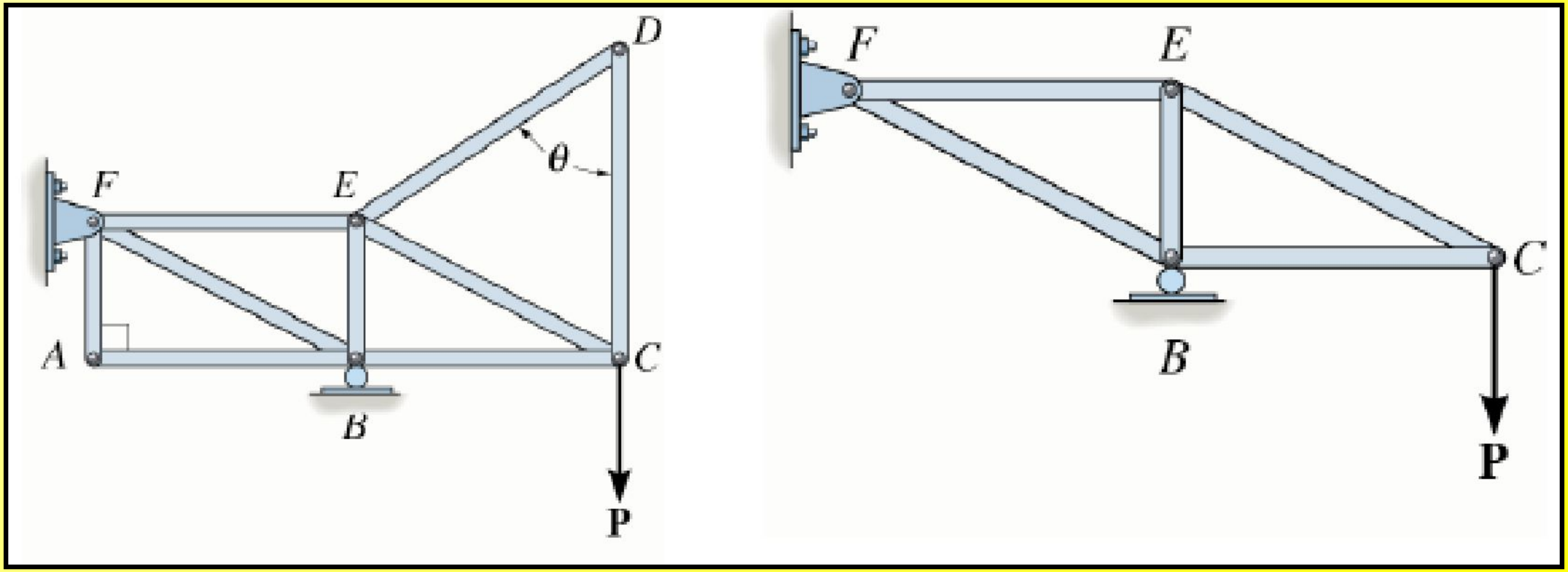
# Structural Analysis of Trusses – Method of Sections

# Zero-Force Members

- Our analysis can be greatly simplified if one can identify those members that support no loads. We call these zero-force members.
- These members can used to increase the stability of the truss during construction and to provide support if the applied loading is change.

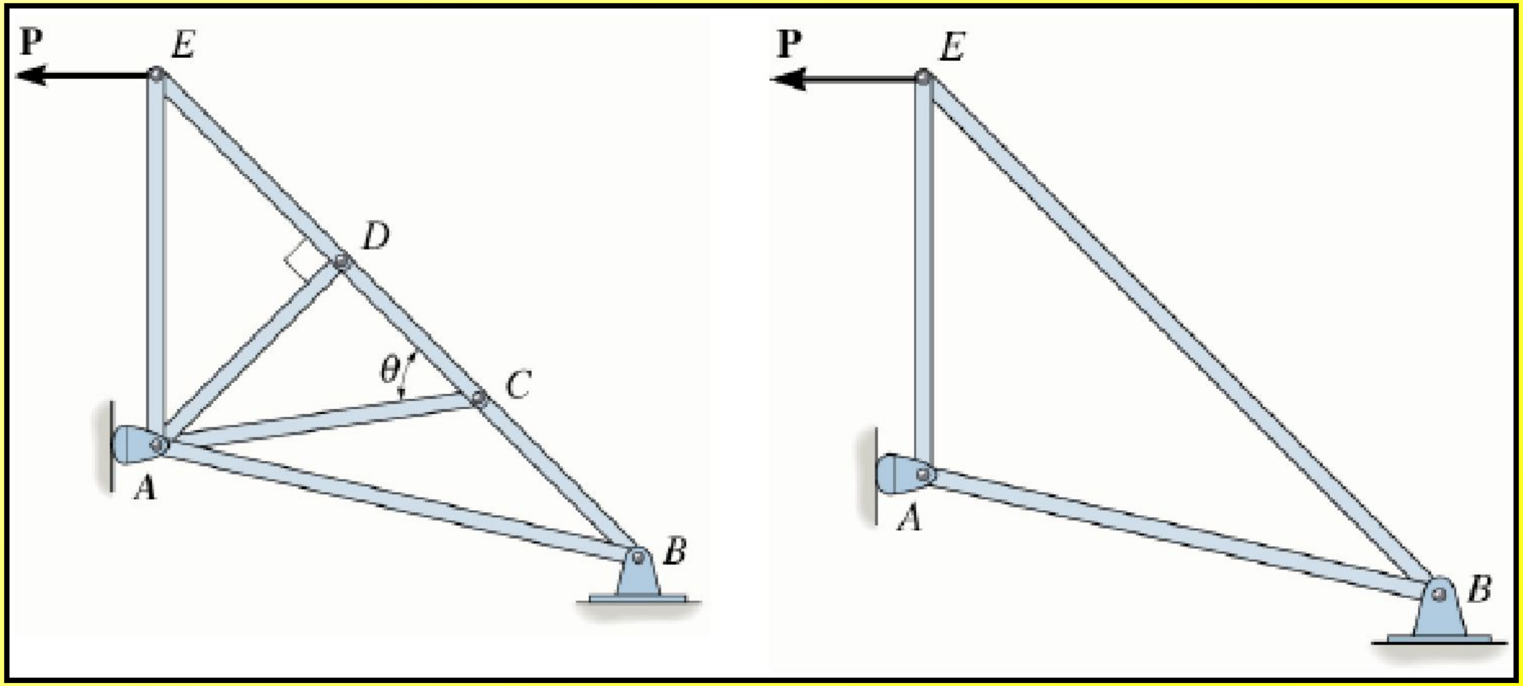
- ***Rule 1***:

  If only two members form a truss joint and no external load or support reaction is applied to the joint, the members must be a *zero-force* member.



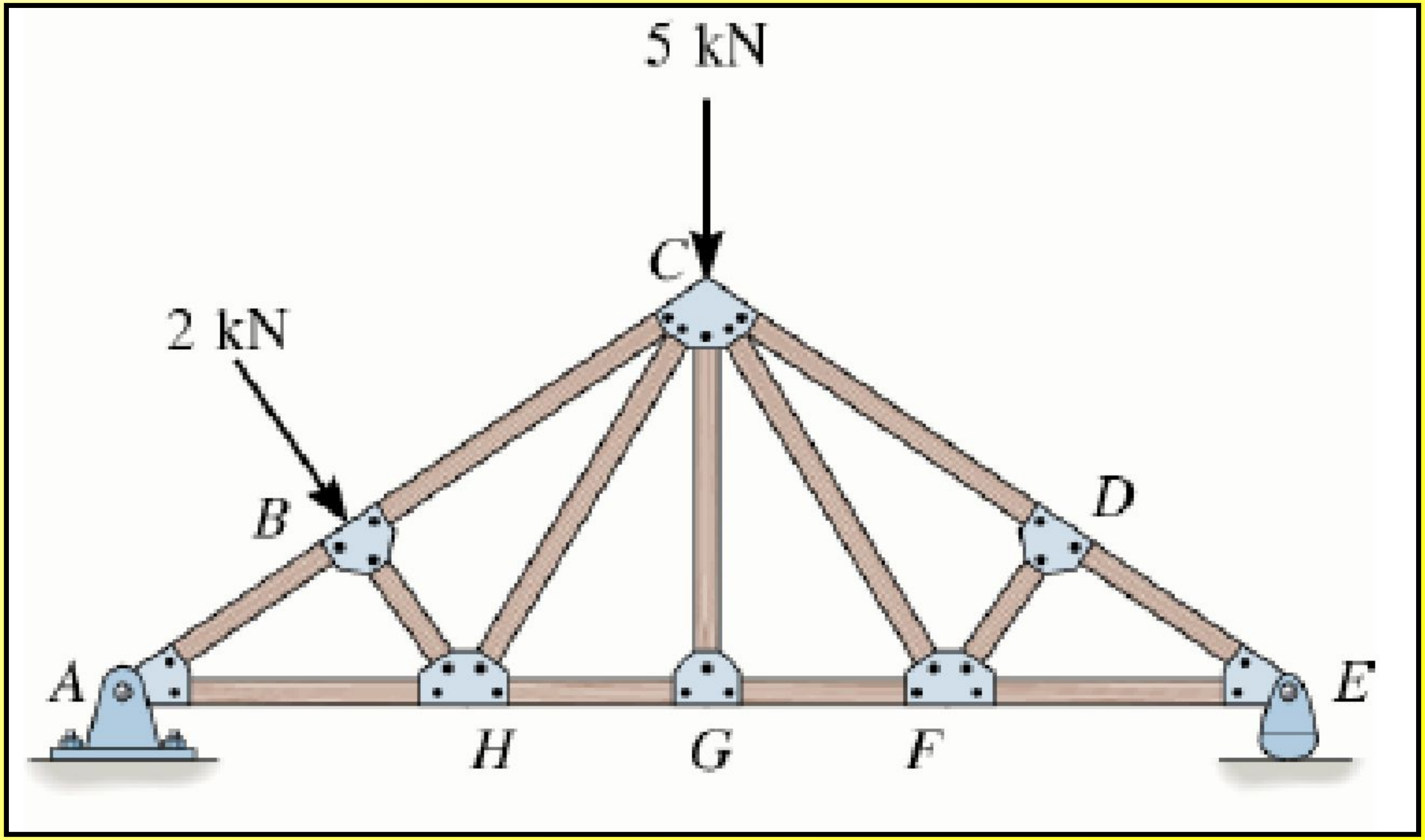
- ***Rule 2***:

  If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.

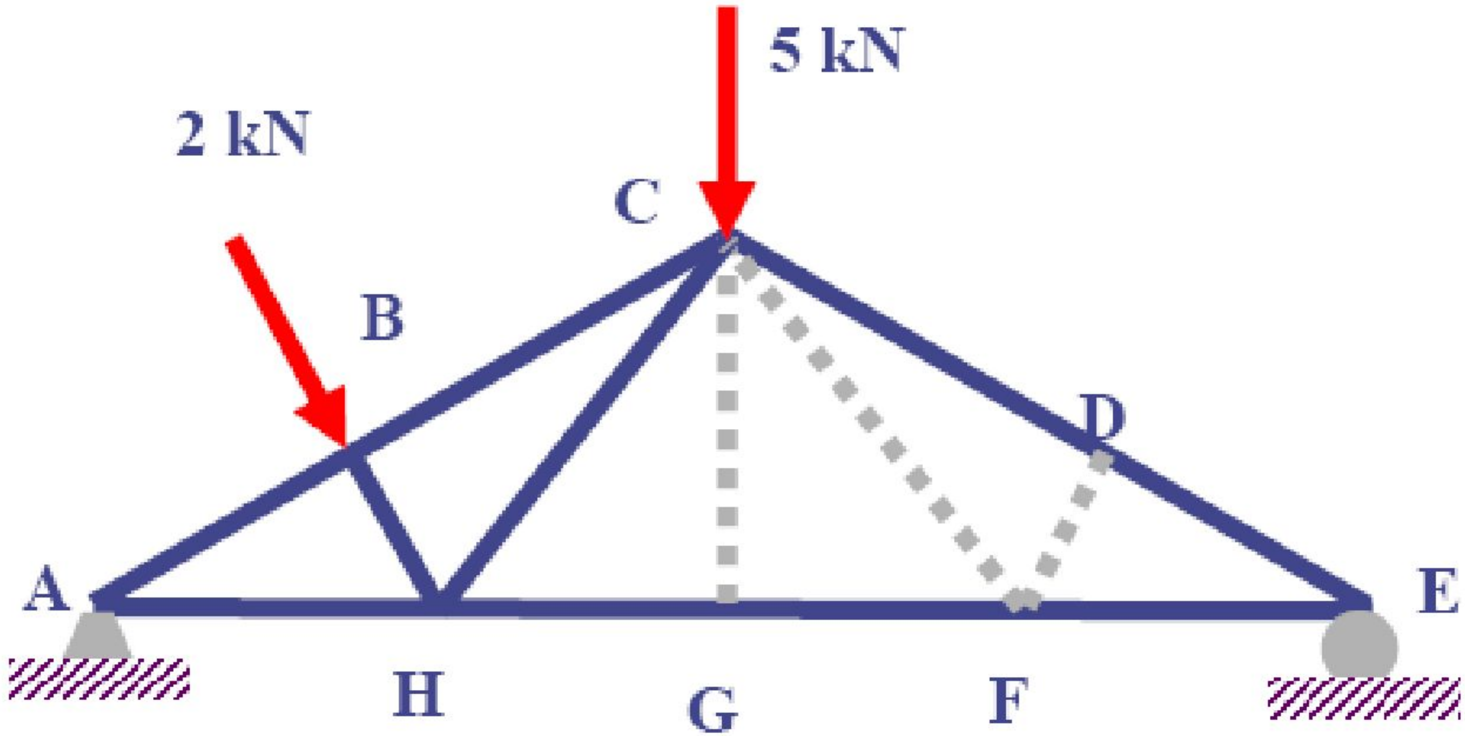



**$F_{DA}$ & $F_{CA}$ are zero-force members**

Which are the zero-force members?

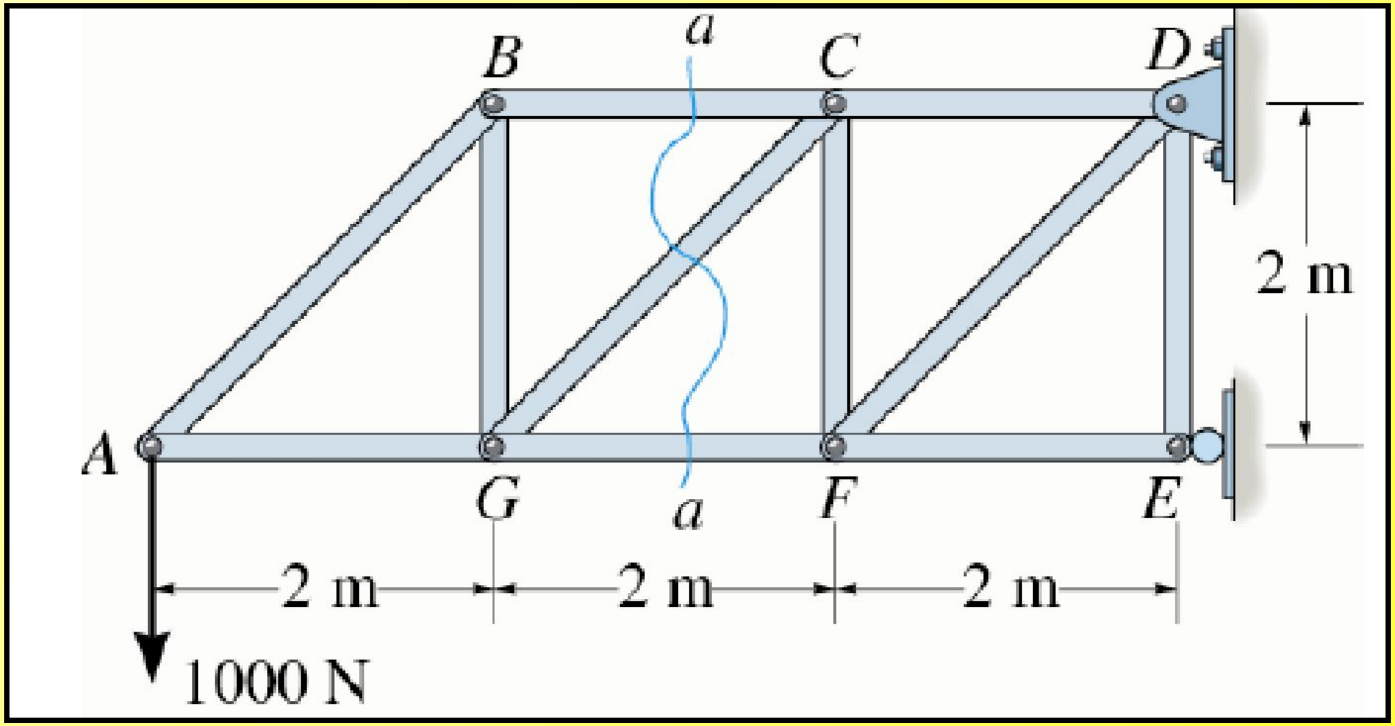


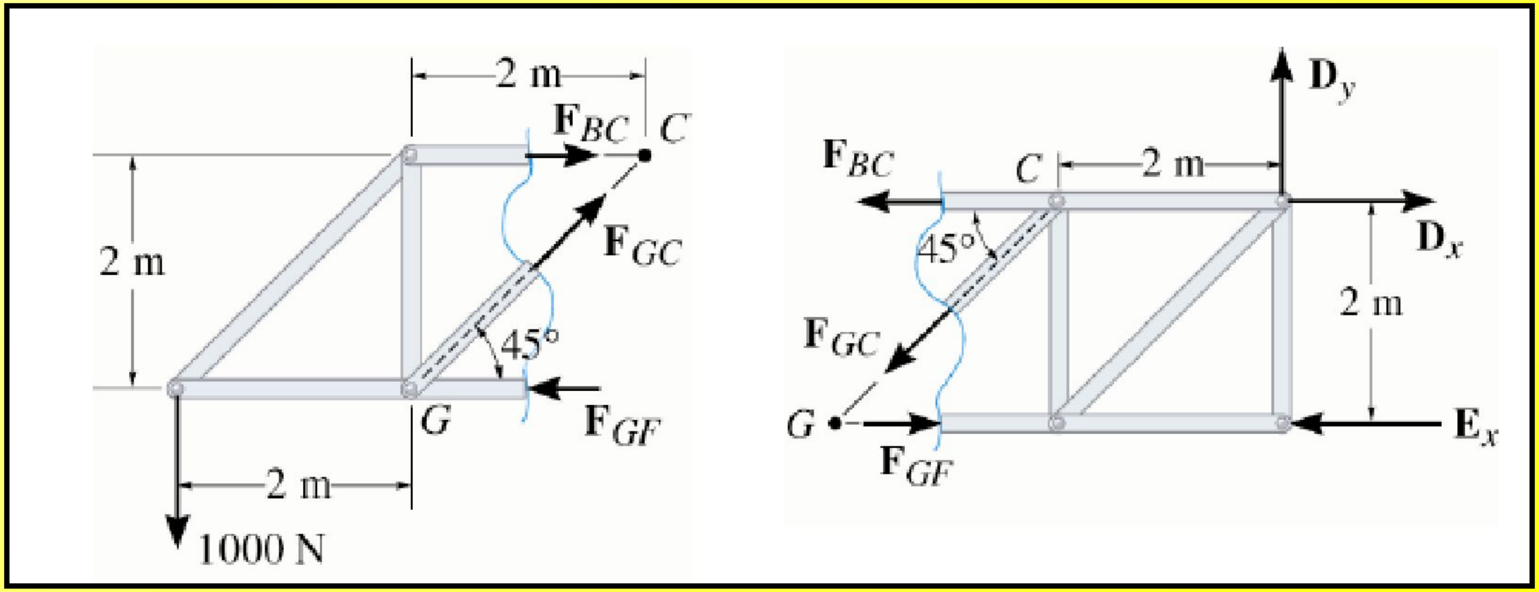
Answer:



# The Method of Sections

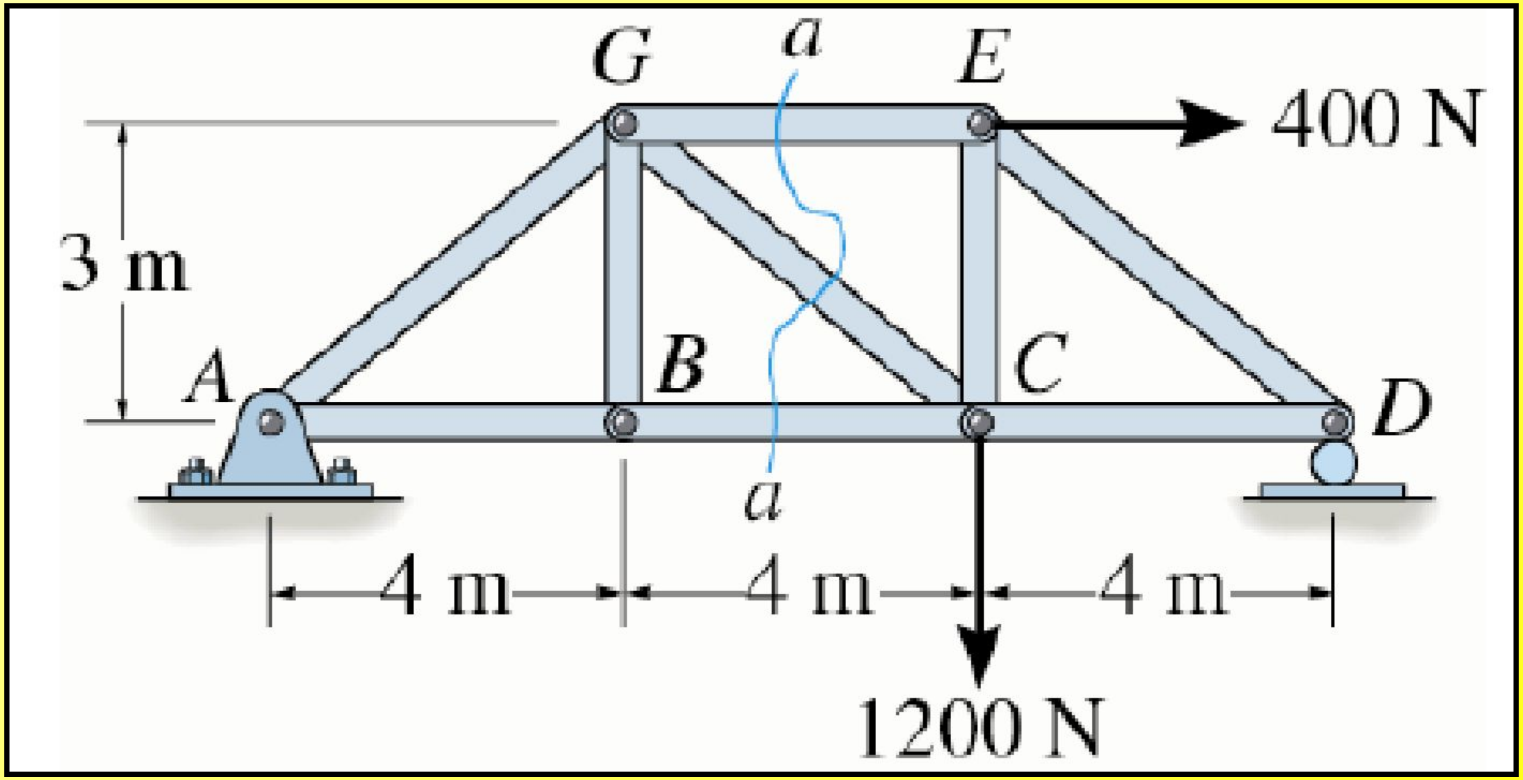
- Based on principle that if a body is in equilibrium then any (all) parts of the body must be in equilibrium.
- If a body is in equilibrium, then any parts of the body is also in equilibrium. We can thus ‘cut’ the body and analyze the section in isolation.
- Generally, the ‘cut’ should not pass more than three members in which the forces are unknown


a
B
C
D
2 m
A
G
a
F
E
2 m
2 m
2 m
1000 N

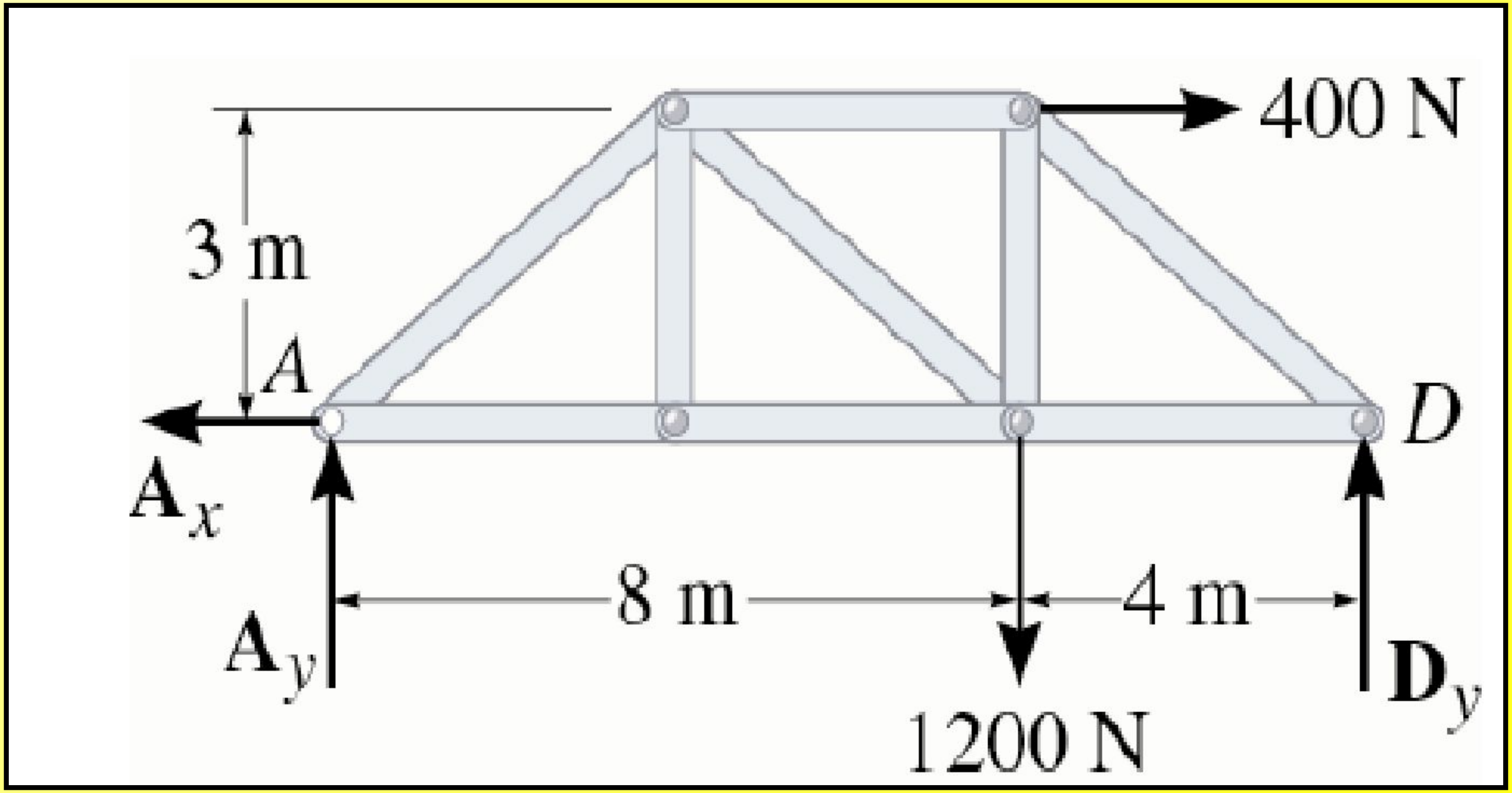

2 m
$F_{BC}$
C
$F_{GC}$
2 m
45°
G
$F_{GF}$
2 m
1000 N
$D_y$
$F_{BC}$
C
2 m
$D_x$
45°
2 m
$F_{GC}$
G
$E_x$
$F_{GF}$

**Example:**

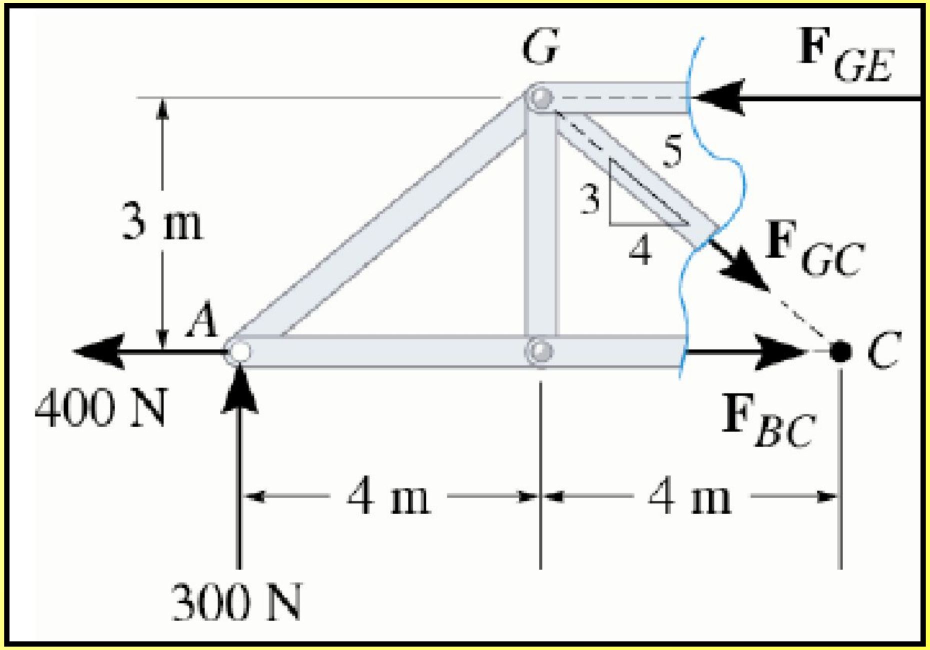
Determine the force in members GE, GC and BC. Indicate whether the members are in tension/compression.



***Step 1***: Calculate the reaction force



***Step 2****:* Section the structure




Taking moment about G:

$$\sum M_G = 0 = -300(4) - 400(3) + 3F_{BC}$$

$$\therefore F_{BC} = 800N(T)$$

Taking moment about C:

$$\sum M_C = 0 = -300(8) + 3F_{GE}$$

$$\therefore F_{GE} = 800N(C)$$

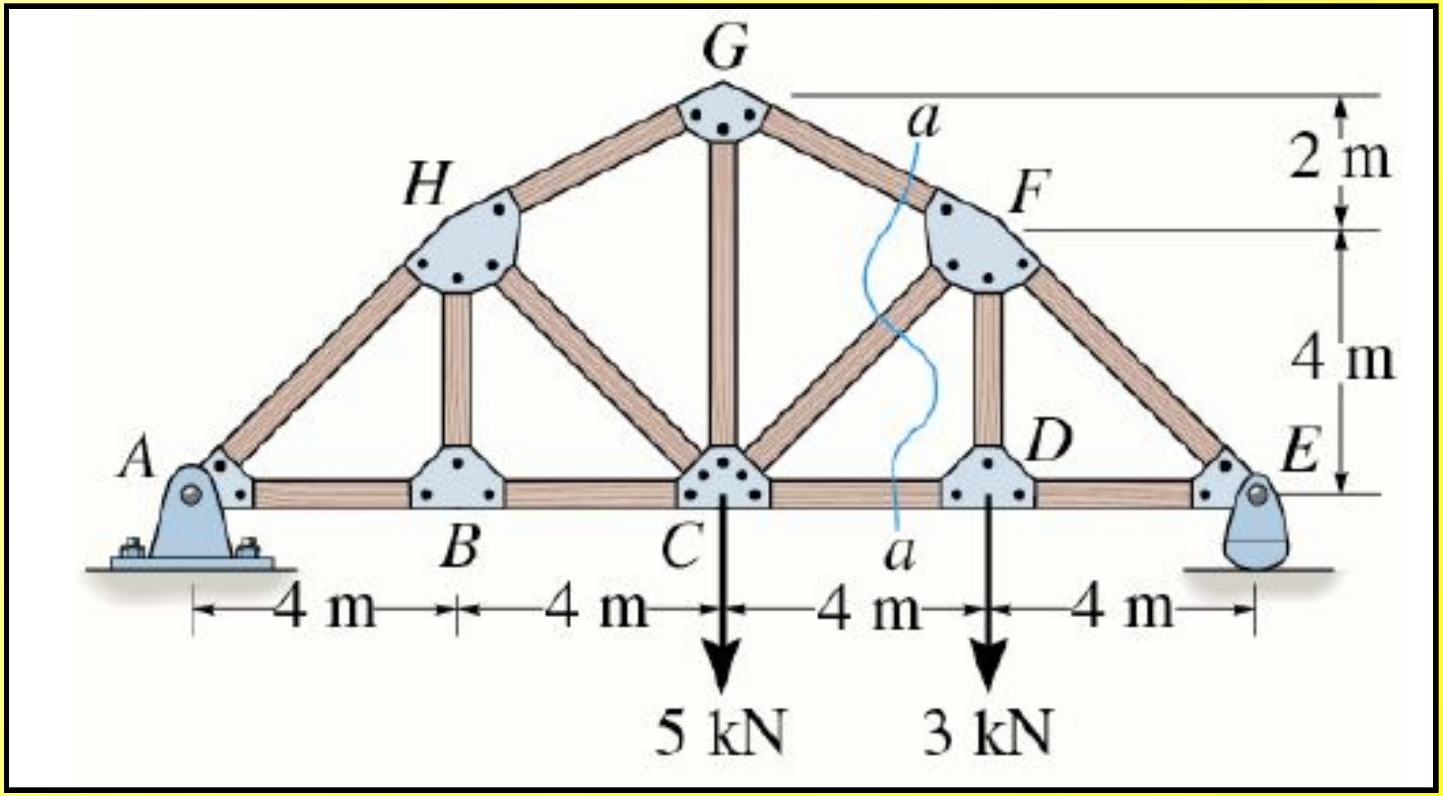
To obtain $F_{GC}$, sum force vertically gives

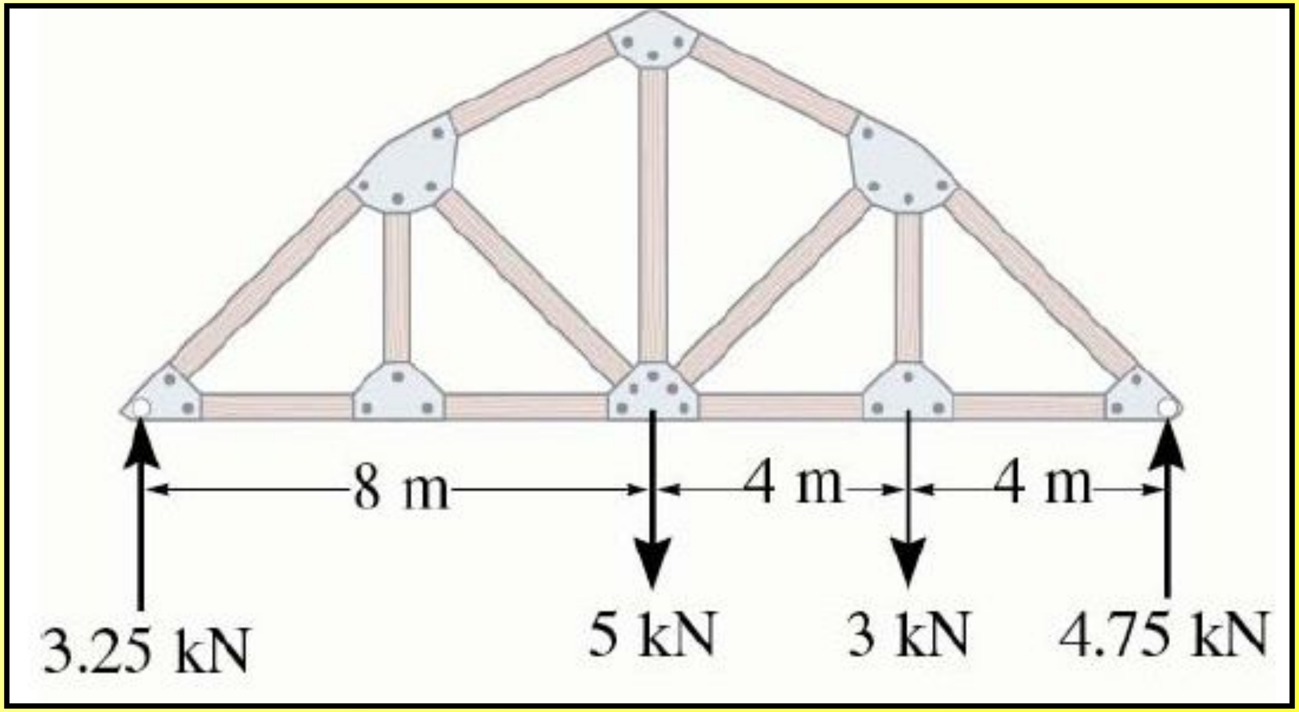
$$\sum F_y = 0 = 300 - \tfrac{3}{5}F_{GC}$$

$$\therefore F_{GC} = 500N(T)$$

Example:

Determine the force in member CF.





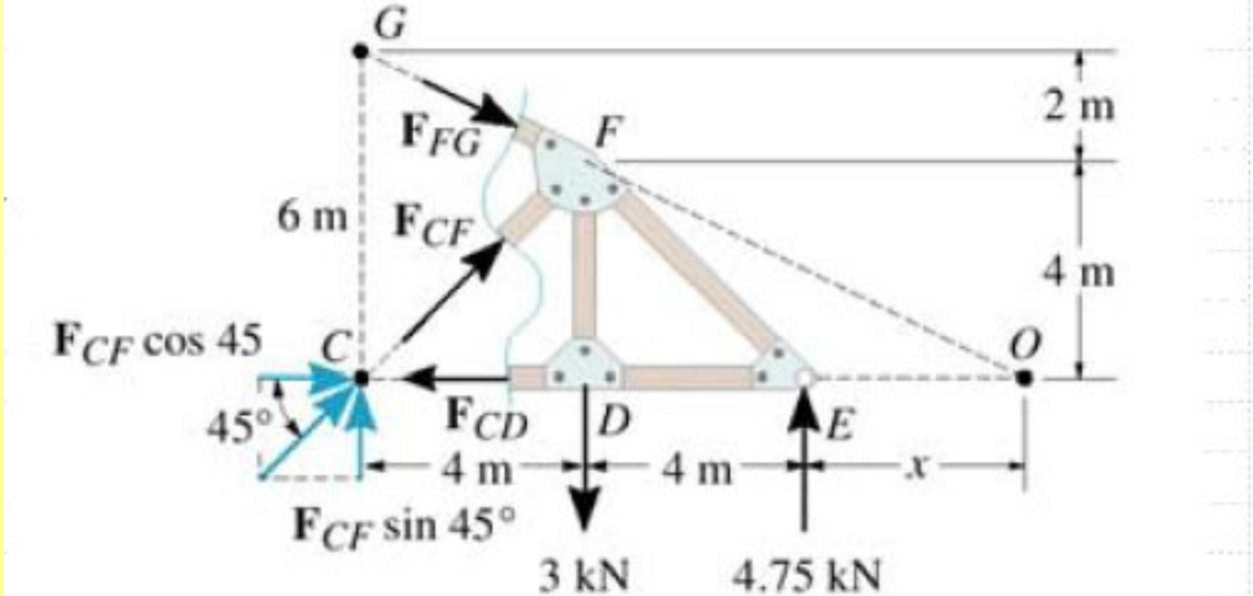

***Step 1*:** Calculate the reaction force

Taking moment about A,

$$\sum M_A = 0 = E_y(16) - 5(8) - 3(12)$$
$$E_y = 4.75\text{ kN}$$

$$\sum F_y = 0 = A_y + 4.75 - 5 - 3$$
$$A_y = 3.25\text{ kN}$$

***Step 2***: Section the structure




$$\sum M_O = 0$$

$$-F_{CF}\sin 45^\circ(12\,\text{m}) + (3\,\text{kN})(8\,\text{m}) - (4.75\,\text{kN})(4\,\text{m}) = 0$$

$$F_{CF} = 0.589\,\text{kN}\,(\text{C})$$

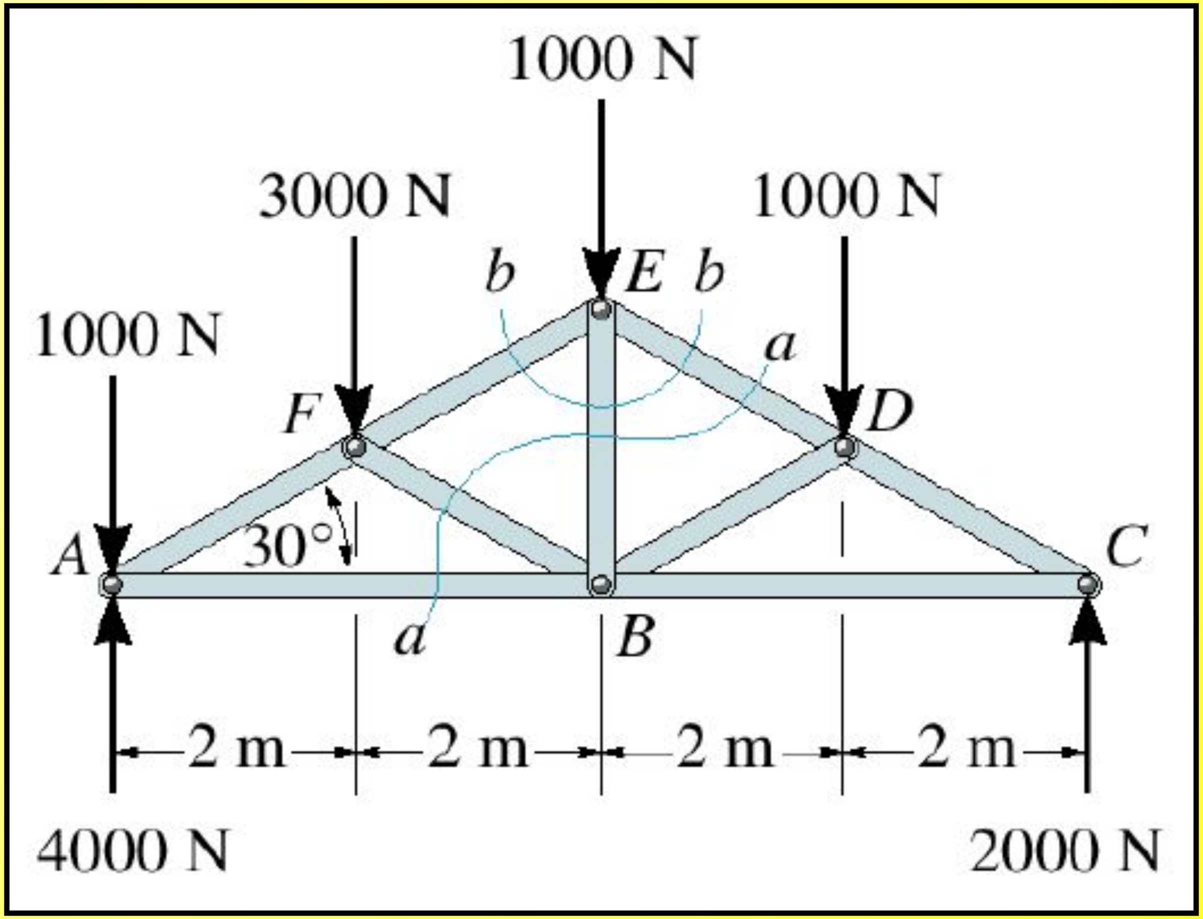
From similar triangles, $x$ can be obtained:

$$\frac{4}{4+x} = \frac{6}{8+x}$$

$$x = 4\text{ m}$$

Example:

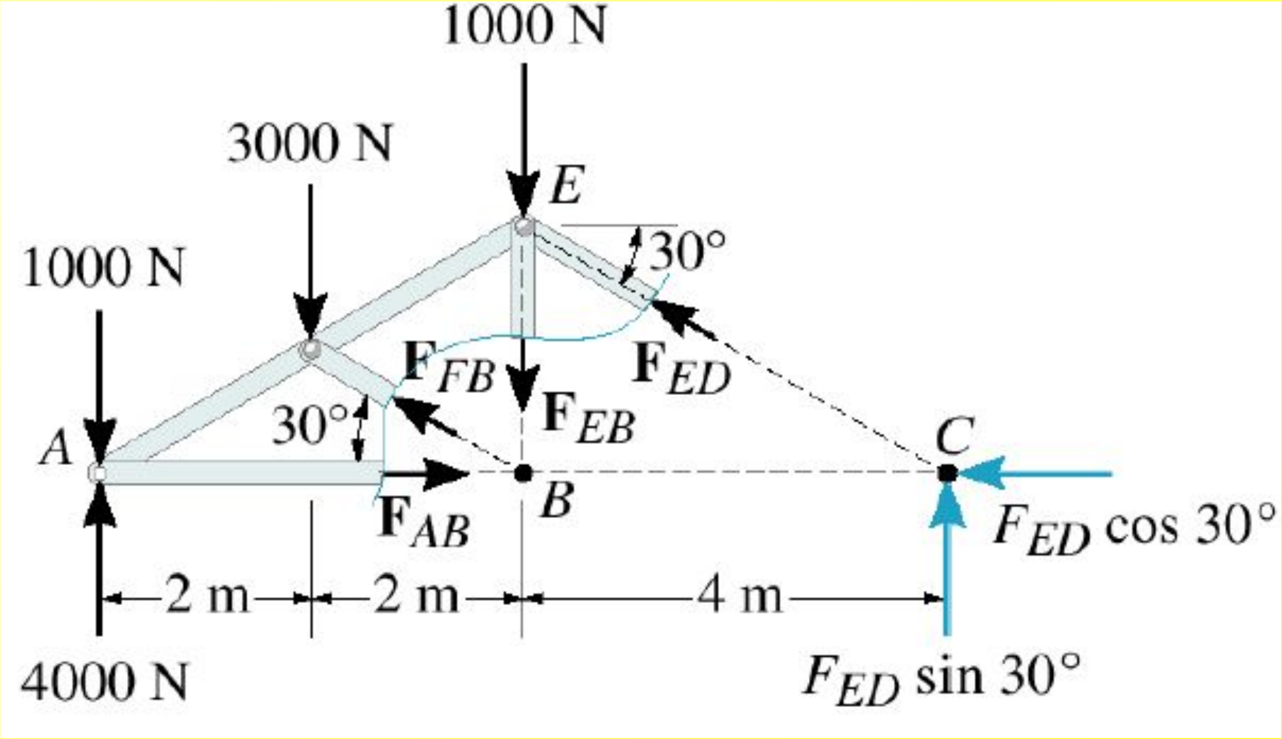
Determine the force in member EB




***Step 1***: Calculate the reaction forces

***Step 2*: Section the structure**

Notice that we cannot cut through section *b-b*. To reduce the section *b-b* into 2 unknowns, we have to cut through section *a-a* first.




Taking moments about B,

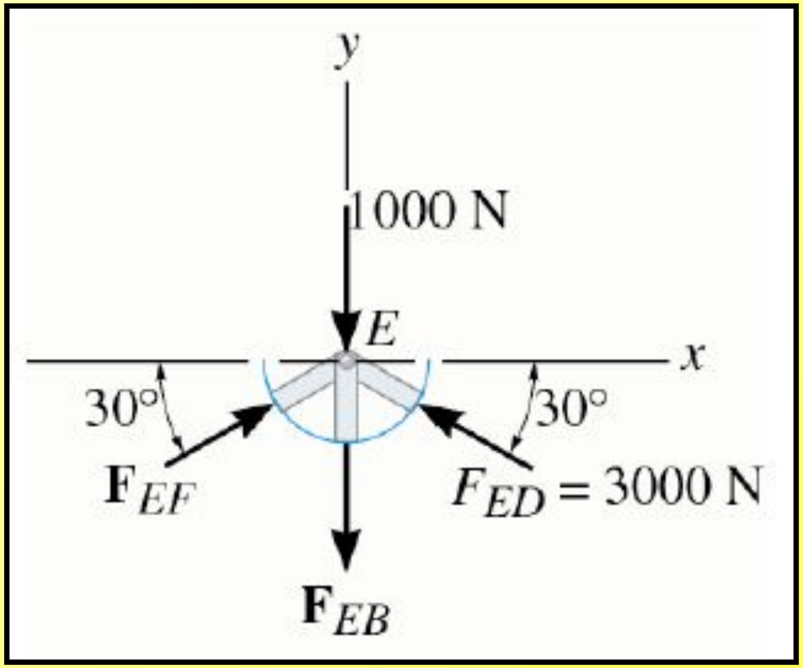
$$\sum M_B = 0$$

$$1000(4) + 3000(2) - 4000(4) - F_{ED} \sin 30° \,(4) = 0$$

$$F_{ED} = -\,3000 \text{ N}$$

$$F_{ED} = 3000 \text{ N (C)}$$

Consider now the free-body diagram of section *b-b*:




$$\sum F_x = 0$$

$$-F_{EF}\cos 30^\circ - 3000\cos 30^\circ = 0$$

$$F_{EF} = -3000 \ \ \text{N}$$

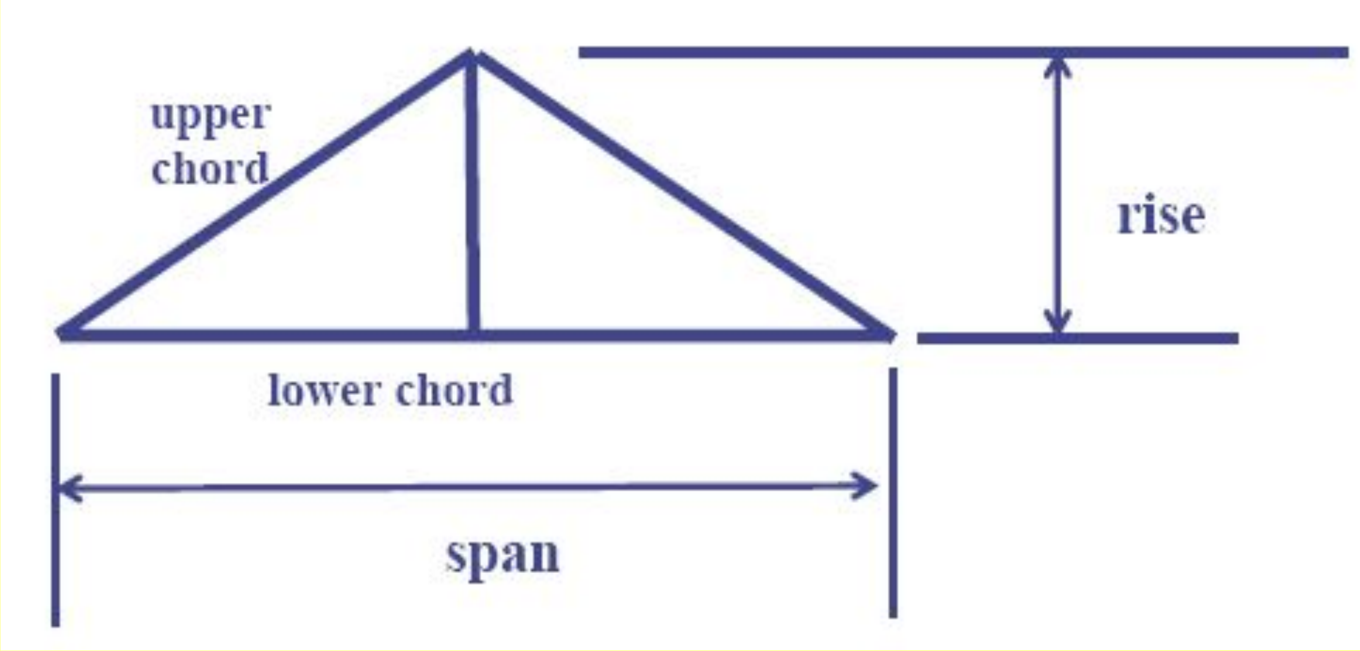
$$F_{EF} = 3000 \ \ \text{N} \ \ \text{(C)}$$

$$\sum F_y = 0$$

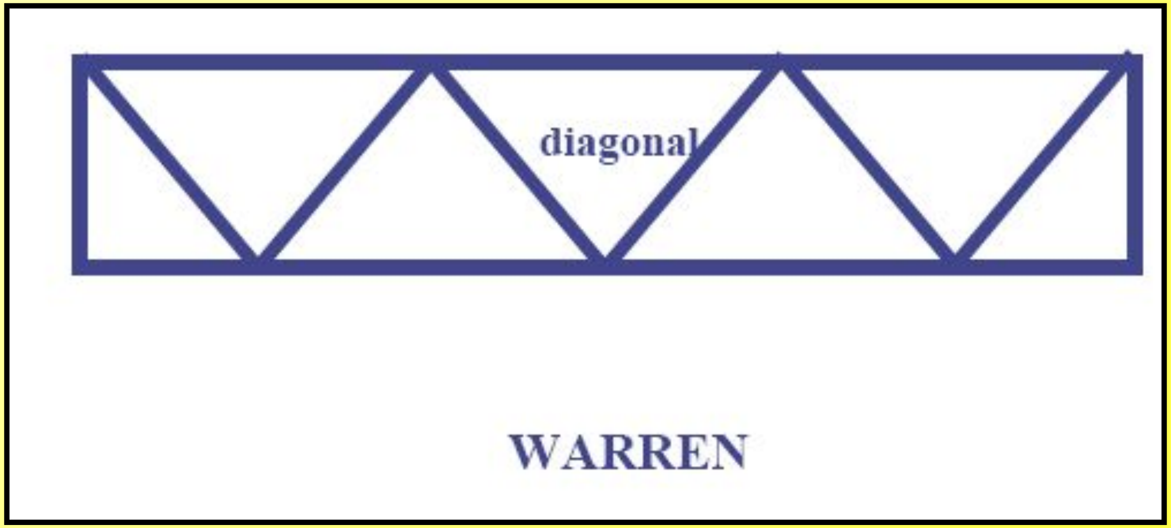
$$-F_{EF}\sin 30^\circ - 3000\sin 30^\circ - 1000 - F_{EB} = 0$$

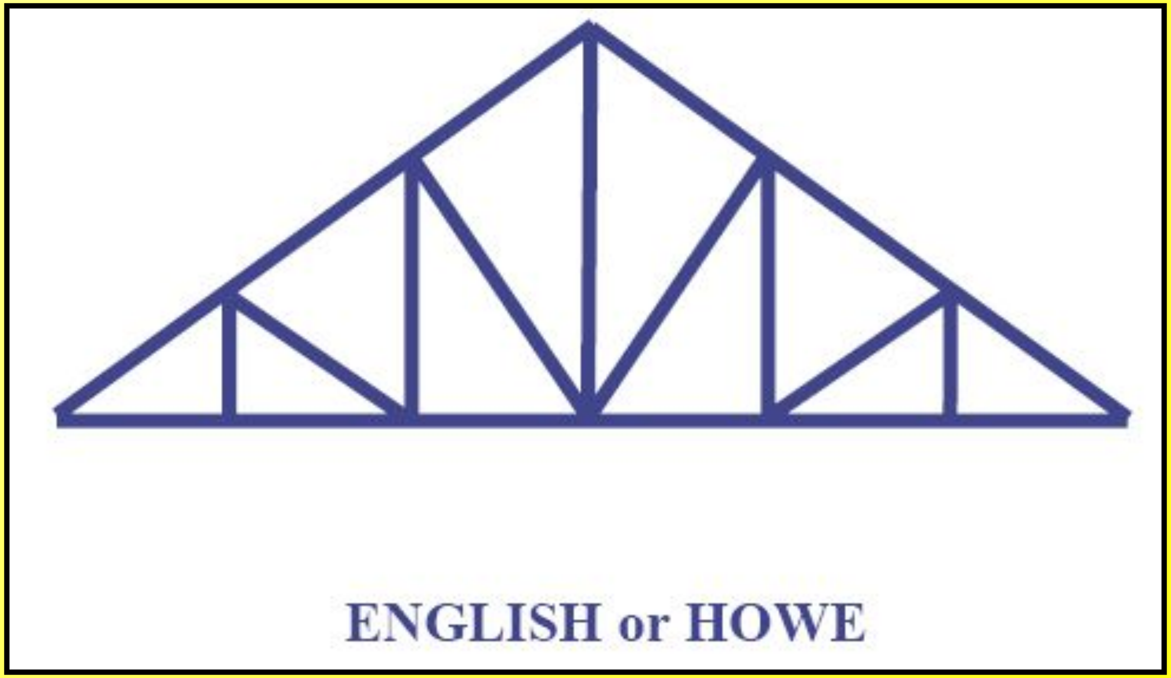
$$F_{EB} = 2000 \ \ \text{N} \ \ \text{(T)}$$

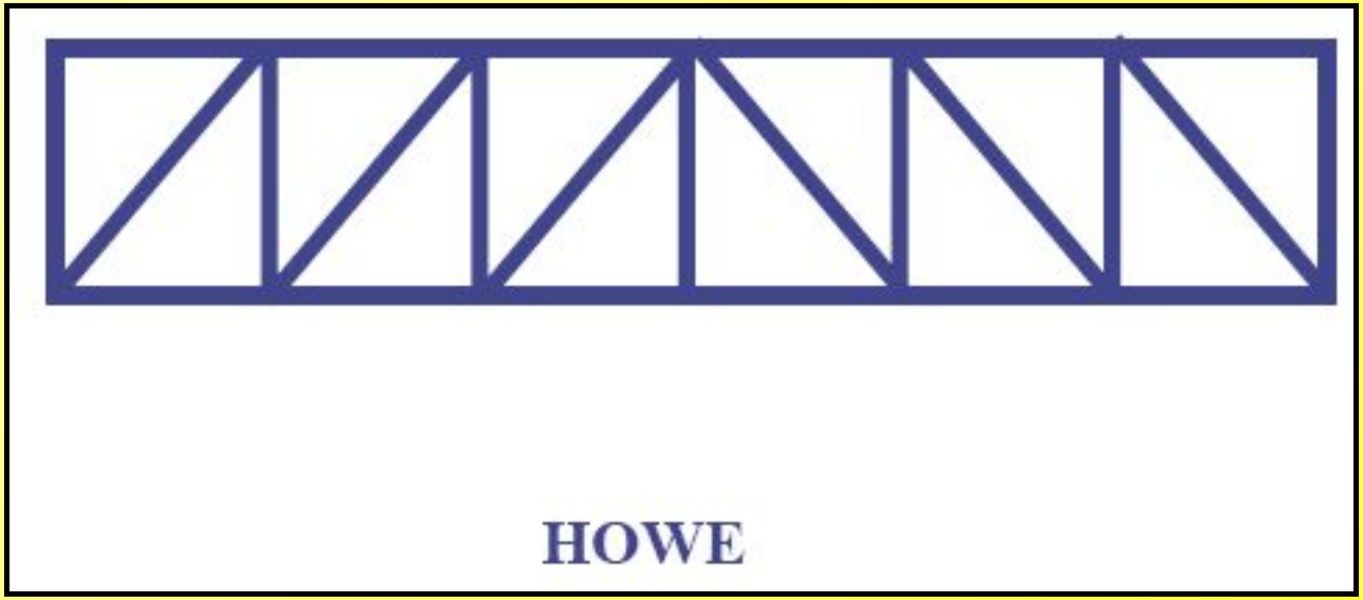
# Different Types of Roof Trusses




**KING POST**


diagonal
WARREN


ENGLISH or HOWE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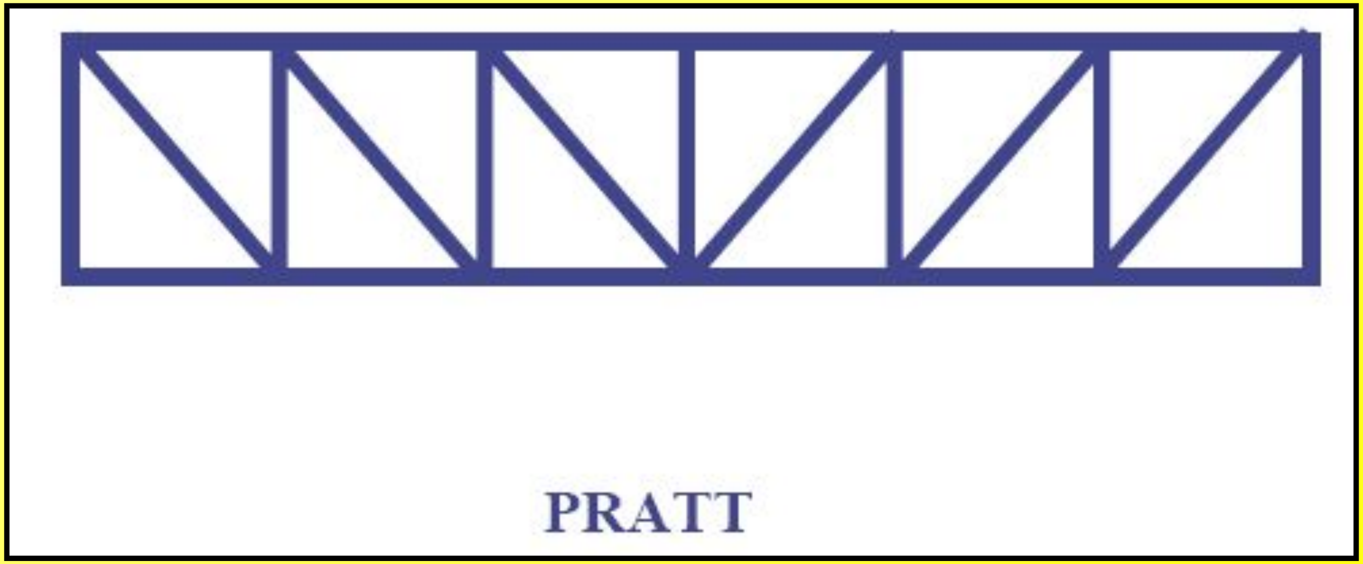
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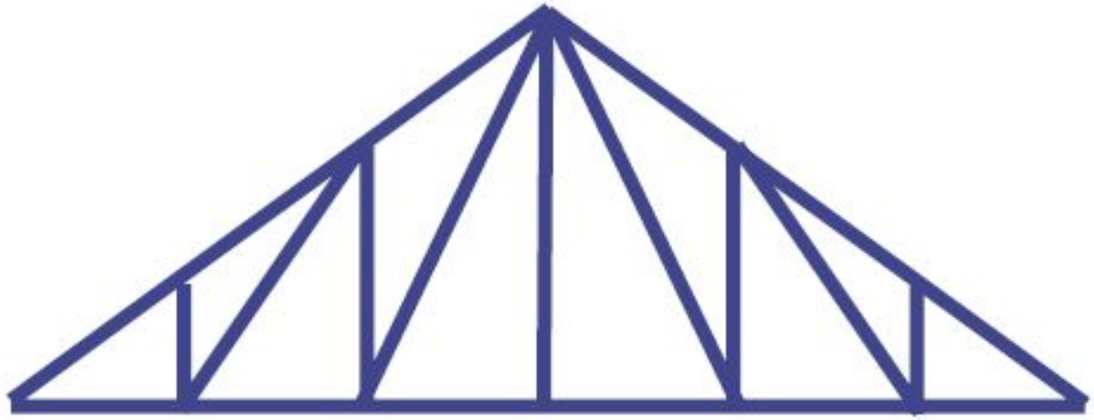
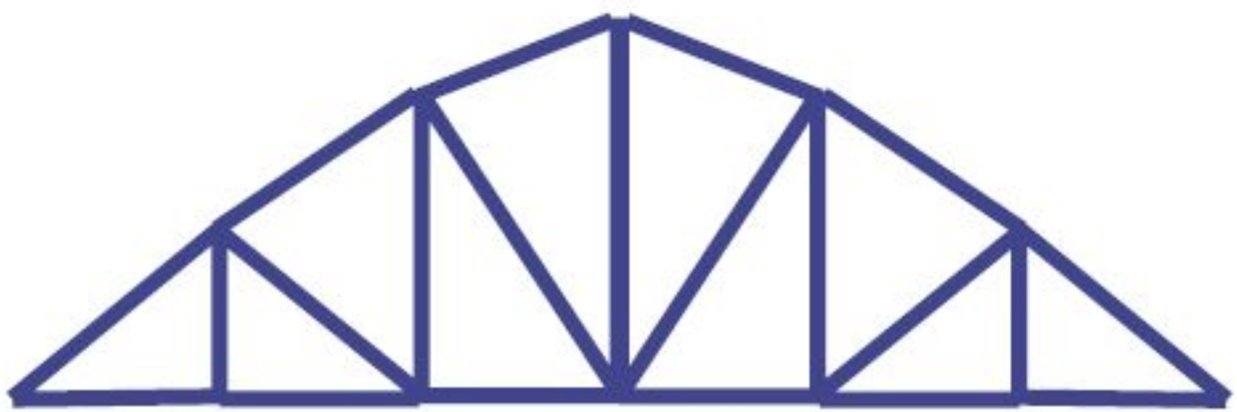
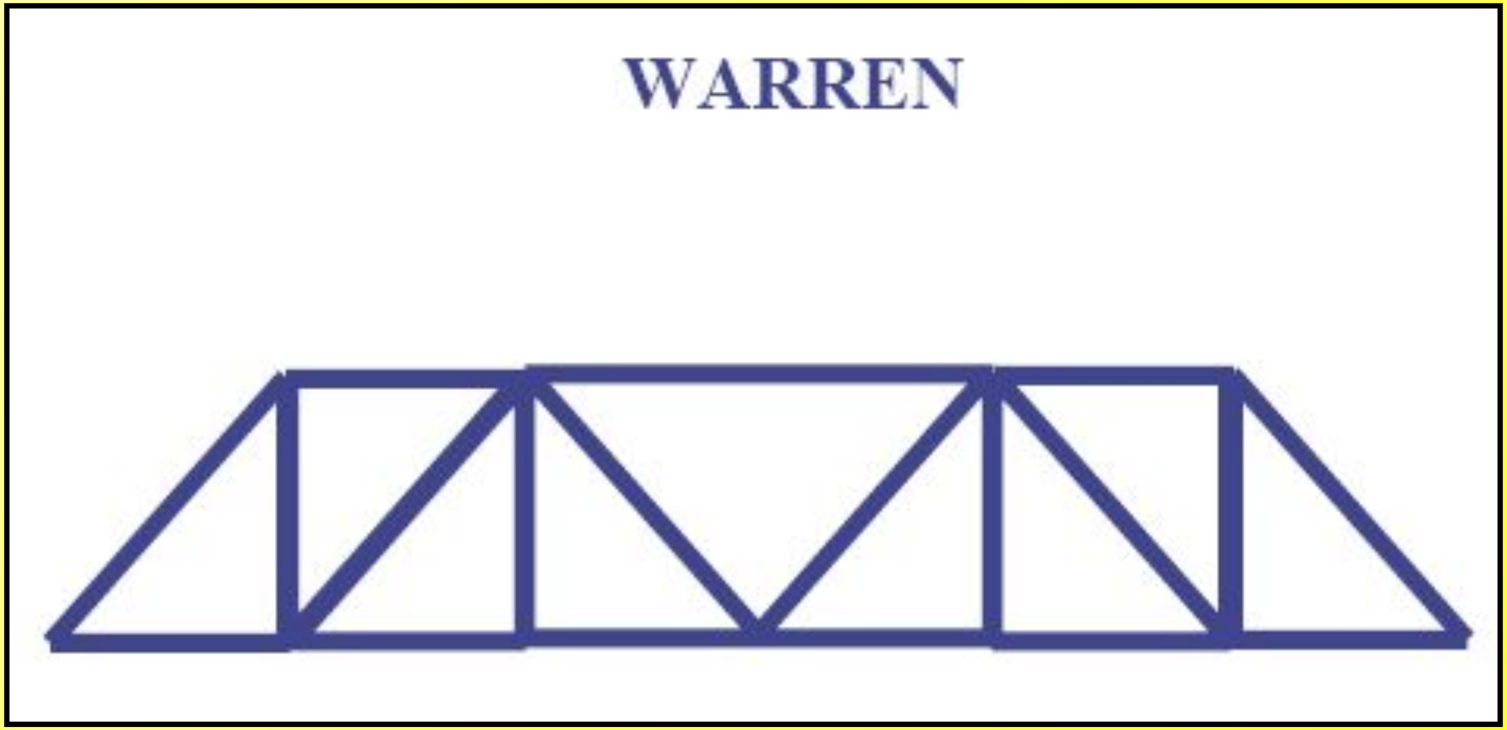


PRATT

BOWSTRING



# Types of bridge trusses



HOWE

PRATT