Metal-Insulator-Semiconductor and Metal-Insulator-Metal Structures. Part VII. Space Charge Effects

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Lecture 14

Fixed space-charge distributions

In the following chapters we will only use one relevant space coordinate between these charges. The relationship between space charge and field is then given by the one-dimensional Poisson equation

$$\boxed{\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\varrho}{\varepsilon\varepsilon_0}.} \tag{1.17}$$

Such a field distribution determines the electrostatic potential distribution for electrons via

$$\boxed{\frac{\mathrm{d}\psi(x)}{\mathrm{d}x} = -\left[F(x) - F(x=0)\right] = -\int_0^x \frac{\varrho(\xi)}{\varepsilon\varepsilon_0}\,\mathrm{d}\xi,} \tag{1.18}$$

with

$$\boxed{\psi(x) = \int_{d_1}^x F(\xi)\mathrm{d}\xi.} \tag{1.19}$$

and for $\xi = d_1$, the corresponding $\psi(d_1)$ serves as reference point for the electrostatic potential.

Fixed space-charge distributions

In summary, we have shown that space-charge regions result in field inhomogeneities. The importance of such field inhomogeneities lies in their ability to influence the current through a semiconductor. With the ability to change space charges by changing a bias, as we will see later, they provide the basis for designing semiconducting devices.

Since a wide variety of space-charge distributions are found in semiconductors, many of which are of technical interest, we will first enumerate some of the basic types of these distributions and start with a catalogue of the interrelationships of various given $\varrho(x)$, resulting in corresponding distributions of electric field $F(x)$ and electrostatic potential $\psi(x)$.

Because of the common practice to plot the distribution of the band edges for devices, we will follow this habit throughout the following sections. The band edge follows the electron potential $\psi_n(x)$ and this relates to the electrostatic potential as

$$E_c(x) = e\psi_n(x) + c = -e\psi(x) + \text{const.} \qquad (1.20)$$

Fixed space-charge distributions

In the examples given in this section, the *space-charge profiles* are arbitrarily introduced as fixed, explicit functions of the independent coordinate (x). The space charge can be kept constant in an insulator that does not contain free carriers. Here all charges are assumed to be trapped in now charged lattice defects.

1.2.1 Sinusoidal Continuous Space-Charge Distribution

A simple sinusoidal space-charge double layer can be described by

$$\varrho(x) = \begin{cases} ea \sin[2\pi x/d] & \text{for} \quad -d/2 \leq x \leq d/2 \\ 0 & \text{elsewhere} \end{cases} \tag{1.21}$$

with $d = d_1 + d_2$ the width of the space charge layer; d_1 and d_2 are the widths of the negative and positive regions of the space charge double layer (here, $d_1 = d_2$). The space charge profile is shown in Fig. 1.1a.

Fixed space-charge distributions

The corresponding field distribution is obtained by integration of (1.21), and assuming $F(x = \pm\infty) = 0$ as boundary conditions:

$$F(x) = \begin{cases} -(ead)\cos[2\pi x/d] & \text{for} \quad -d/2 \leq x \leq d/2 \\ 0 & \text{elsewhere}; \end{cases} \tag{1.22}$$

it is shown in Fig. 1.1b, and presents a negative field with a symmetrical peak; its maximum value lies at the position where the space charge changes its sign. The maximum field increases with increasing space-charge density ea and width d.

The corresponding electron energy (band edge) distribution is obtained by a second integration of 1.21, yielding with an assumed $E_c(\infty) = 0$ as boundary condition:

$$E_c(x) = \begin{cases} e^2ad^2/(4\varepsilon\varepsilon_0) & \text{for} \quad x < -d/2 \\ -e^2ad^2\sin[2\pi x/d]/(4\varepsilon\varepsilon_0) & \text{for} \quad -d/2 \leq x \leq d/2 \\ 0 & \text{for} \quad x > d/2, \end{cases} \tag{1.23}$$

that is, a band edge step down of height $ead^2/(4\varepsilon\varepsilon_0)$, as shown in Fig. 1.1c.

Fixed space-charge distributions

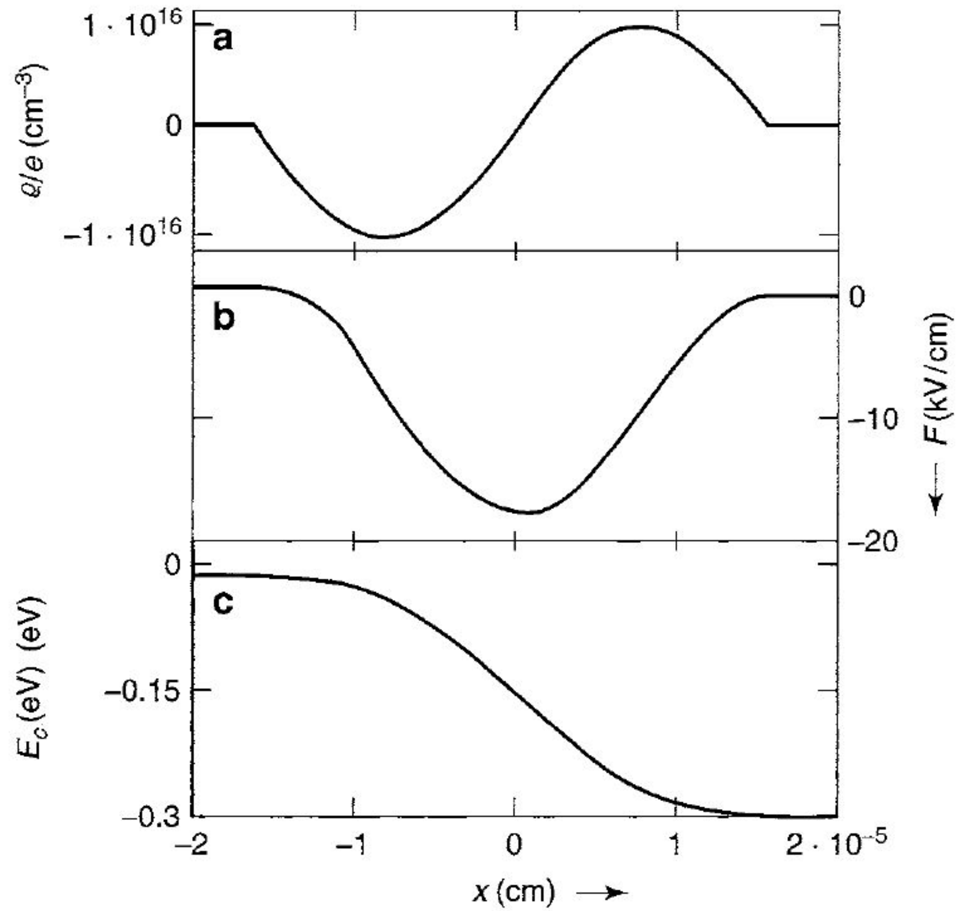

Fig. 1.1. Sinusoidal space charge, and resulting electric field and electron energy distributions. Computed for a maximum charge density, $a = 10^{16}\,\mathrm{cm}^{-3}$, a width $d = 3 \cdot 10^{-5}$ cm, and for a relative dielectric constant, $\varepsilon = 10$

Creation of space-charge regions in solids and corresponding currents

Space-charge regions, which were arbitrarily introduced in Sect. 1.2, occur normally in solids as a consequence of inhomogeneous doping or of the boundary conditions at the contact.

Here, carriers can *leak out* from a region of higher carrier density into a region of lower carrier density. Since the average electron density in thermal equilibrium is equal to the density of ionized uncompensated donors in a homogeneous n-type solid, the leaking out of mobile electrons into an adjacent region of a lesser donor density must create a positive space charge

$$\varrho = e(N_{d2} - n) \tag{2.1}$$

in the more highly doped region where some of the charge-compensating electrons are now missing and a negative space charge

$$\varrho = e(N_{d1} - n), \tag{2.2}$$

in the adjacent, lower doped region, caused by the excess electrons, with an abrupt flip[1] of sign of the space charge at the doping boundary between the two regions,[2] as shown in Figs. 2.1a, b.

Creation of space-charge regions in solids and corresponding currents

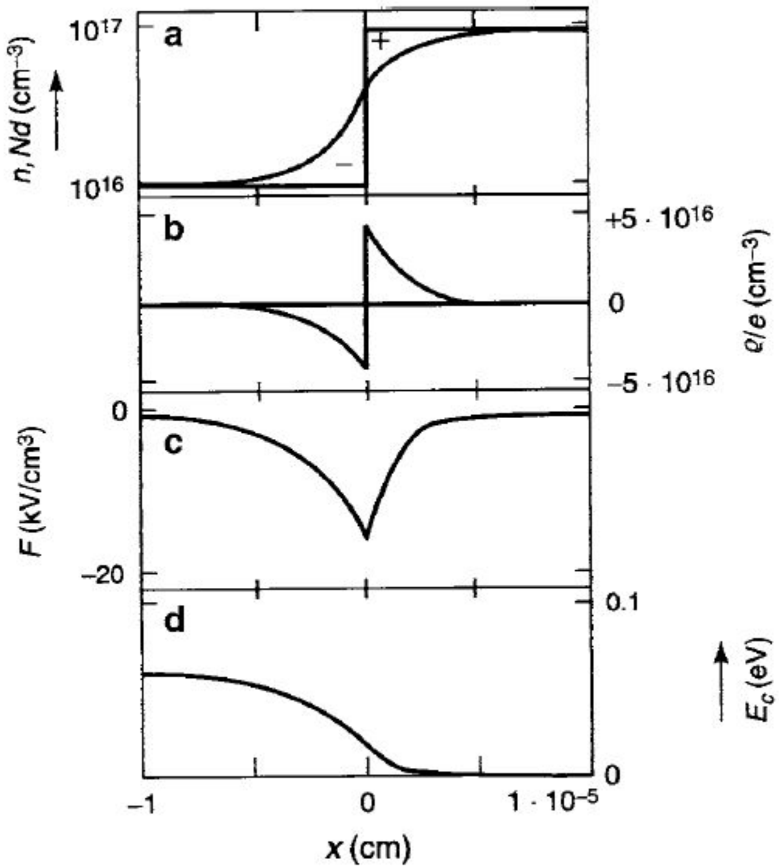

Fig. 2.1. Step-like doping distribution in an nn^+-junction with higher doping density at right, and resulting carrier density (a), space charge (b), field (c), and band edge (d) distributions shown below

Creation of space-charge regions in solids and corresponding currents

The exact shape of these distributions, however, is given by the distribution of the mobile carriers which is caused by carrier diffusion out of the highly doped region ($x > 0$). In equilibrium, such diffusion is counterbalanced by carrier drift in the opposite direction, due to the field induced by the space charge, which was created by the initial carrier diffusion. Hence, in addition to the Poisson equation discussed in the previous section, one must now consider the **carrier transport equation**[3] including drift and diffusion currents, here for electrons:

$$\boxed{j_n = e\mu_n n F + \mu_n kT \frac{\mathrm{d}n}{\mathrm{d}x},} \tag{2.3}$$

where n is the electron density and μ_n, the electron mobility. Equations (1.17), (1.18), and (2.3) are now the **governing set of differential equations** that

for convenience we will repeat here in their basic formulation:

$$j_n = e\mu_n n F + \mu_n kT \frac{\mathrm{d}n}{\mathrm{d}x} \tag{2.4}$$

$$\frac{\mathrm{d}F}{\mathrm{d}x} = e\frac{\varrho}{\epsilon\epsilon_o} \tag{2.5}$$

$$\frac{\mathrm{d}\psi_n}{\mathrm{d}x} = F. \tag{2.6}$$

Creation of space-charge regions in solids and corresponding currents

These determine quantitatively the space-charge distribution and consequently the entire electrical behavior of any junction in which only one carrier is mobile. In this example, it is a simple nn^+-junction.[4]

We will now analyze in more detail the behavior of such an idealized nn^+-junction, which illustrates the main behavior that in a modified form is the basis for the carrier transport in all other barriers or junctions.

Such junctions have technical relevance as high–low junctions, or nn^+-junctions in many devices. The notation n^+ is used to identify a highly doped, or often degenerate region in which the Fermi-level is close to, or inside the conduction band.

In rewriting (2.4)–(2.6) one obtains a set of four **simultaneous nonlinear differential equations**:[5]

$$\frac{\mathrm{d}n}{\mathrm{d}x} = \frac{j_n - e\mu_n nF}{\mu_n kT} \tag{2.7}$$

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{e(N_{d1} - n)}{\varepsilon\varepsilon_0} \quad \text{for} \quad x < 0 \tag{2.8}$$

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{e(N_{d2} - n)}{\varepsilon\varepsilon_0} \quad \text{for} \quad x \geq 0 \tag{2.9}$$

$$\frac{\mathrm{d}\psi_n}{\mathrm{d}x} = F. \tag{2.10}$$

Equations (2.7)–(2.10) cannot be integrated in closed form and certain approximations, that are used in Sect. 3.1, are not sufficiently accurate for the problem presented here. Therefore, the solution curves of (2.7)–(2.10) are obtained by numerical integration.

Space-charge-limited current

With the tools given in the previous sections we are now able to analyze the behavior of some semiconductors that are conventionally described as space charge limited currents. Such behavior is observed in certain nn^+-junctions in sufficient forward bias.

Under such conditions, Poisson and transport equations (1.17) and (2.3) can be integrated in closed form. We will discuss such important example below.

If, in an nn^+-junction device with sufficient forward bias, the electron density in the entire lowly doped region can become much larger than the donor density in this region; then the current through the device becomes controlled by the surplus carriers originating from the adjacent highly doped region. This current behaves much like the current in a vacuum diode [23] in which electrons are injected from the cathode and carried to the anode following the electric field, although limited by the space charge near the injecting cathode. This current is, therefore, often referred to as an *injected current*, or as a *space-charge limited current*[24](Mott and Gurney, 1940; Lampert, 1956; and Rose, 1978).

For spectroscopy of local states using space charge limited currents see also Nespurek and Sworakowski (1990). For the theory of space-charge limited currents in materials with an exponential distribution of capture coefficients see Gildenblat et al. (1989). The temperature dependence of space-charge limited currents in amorphous and disordered semiconductors is discussed by Schauer et al. (1996)

Figure 2.9a shows that $n \gg N_{d1}$ in the entire region 1 with sufficient forward bias; therefore, the space charge in the lower conducting region may be approximated as

$$\varrho = e(N_{d1} - n) \simeq -en. \tag{2.36}$$

Consequently, the Poisson equation becomes independent of the doping in this region:

$$\frac{\mathrm{d}F}{\mathrm{d}x} = -\frac{en(x)}{\varepsilon\varepsilon_0}. \tag{2.37}$$

Space-charge-limited current

In addition, the drift current becomes much larger than the diffusion current in the lowly doped region with large enough forward bias, as one can see from a comparison of Figs. 2.6a, b. This permits, with sufficient forward bias, an approximation of the total current by the drift current alone:

$$j_n = e\mu_n n(x)F(x). \tag{2.38}$$

After replacing $n(x)$ in (2.38) with the Poisson equation (2.37) one obtains

$$j_n = -\varepsilon\varepsilon_0\mu_n F(x)\frac{\mathrm{d}F}{\mathrm{d}x} \tag{2.39}$$

which can be integrated after separating variables, yielding

$$(x_0 - x)j_n = \varepsilon\varepsilon_0\mu_n\frac{[F(x) - F_0]^2}{2}. \tag{2.40}$$

Whenever $F_0 \ll F(-d_1)$, one can evaluate (2.40) at $x = -d_1$ for sufficient forward bias and directly obtain with[25] $F(d_1) \simeq V/d_1$ an analytical expression for the current–voltage characteristic:

$$j_n \simeq \varepsilon\varepsilon_0\frac{\mu_n V^2}{2d_1^3}; \tag{2.41}$$

that is, the current increases proportionally to the square of the applied *voltage* and decreases with the third power of the width of the low conductivity region.

Space-charge-limited current

From the assumption used, it is evident that space-charge-limited currents occur with sufficient forward bias in devices that have a *thin enough* region 1 to have the entire low-conducting region swamped with electrons, and have a density of carriers at the injecting boundary which lies sufficiently above the bulk carrier density in region 1 of the device. Such a device may alternatively consist of a homogeneous semiconductor of length L with an *injecting contact* (see Sect. 3.2.1.1); its current follows the same, well-known **space-charge-limited current equation**:

$$j_n = \varepsilon\varepsilon_0 \frac{\mu_n V^2}{2L^3}. \tag{2.42}$$

From the relation $n \gg N_{d1}$ throughout the device, that is used to evaluate the space charge (2.36) and the characteristics given in Fig. 2.11, one sees that the space-charge-limited current equation holds only for "thin devices" in which the entire low-doped region can be swept over by electrons from the n^+- region. The injected currents then become rather large in such thin

devices even in the mV bias range as shown in Fig. 2.11 and, in the given approximation do not depend on the doping density or the step size beyond a minimum range.

Space-charge-limited current

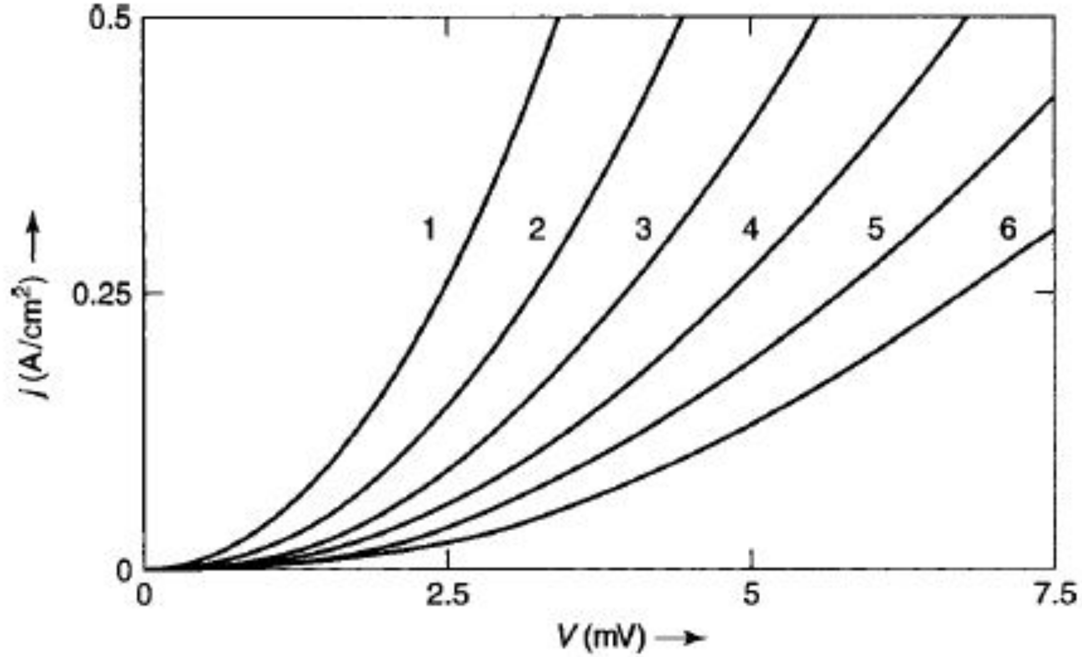

Fig. 2.11. Space-charge-limited currents calculated from (2.42) with μ_n = 100 cm^2 Vs^{-1}, $\varepsilon = 10$, and the device thickness as family parameter with $L = 1$, 1.2, 1.4, 1.6, 1.8, and $2 \cdot 10^{-5}$ cm for curves 1–6, respectively (the thinner the device, the steeper is the increase of the current with bias, the more electrons are swept through the entire device)