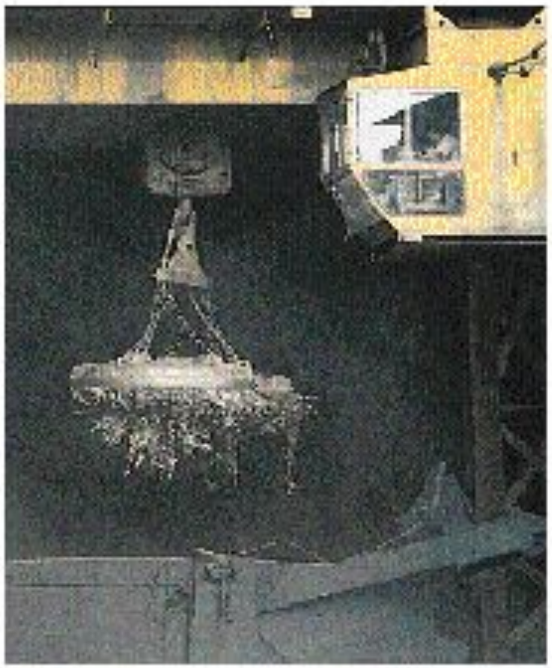
MAGNETIC FIELDS

Magnetic Fields

If you are outside on a dark night in the middle to high latitudes, you might be able to see an aurora, a ghostly "curtain" of light that hangs down from the sky. This curtain is not just local: it may be several hundred kilometers high and several thousand kilometers long, stretching around Earth in an arc. However, it is less than 1 km thick. What produces this huge display, and what makes it so thin?

29-1 THE MAGNETIC FIELD

We have discussed how a charged plastic rod produces a vector field–the electric field **E**–at all points in the space around it. Similarly, a magnet produces a vector field the –**magnetic field B**– at all points in the space around it. You get a hint of that magnetic field whenever you attach a note to a refrigerator door with a small magnet, or accidentally erase a computer disk by bringing it near a magnet. The magnet acts on the door or disk *by means of* its magnetic field.



In a familiar type of magnet, a wire coil is wound around an iron core and a current is sent through the coil; the strength of the magnetic field is determined by the size of the current. In industry, such **electromagnets** are used for sorting scrap iron (Fig. 29-1) among many other things. You are probably more familiar with **permanent magnets**–magnets, like the refrigerator-door type, that do not need current to have a magnetic field.

FIGURE 29-1
Scrap metal collected by an electromagnet at a steel mill.

29-2 THE DEFINITION OF B

We determined the electric field **E** at a point by putting a test particle of charge q at rest at that point and measuring the electric force $\mathbf{F}_E$ acting on the particle. We then defined **E** as

$$\mathbf{E} = \frac{\mathbf{F}_E}{q}. \qquad (29\text{-}1)$$

If a magnetic monopole were available, we could define **B** in a similar way. Because such particles have not been found, we must define **B** in another way, in terms of the magnetic force $\mathbf{F}_B$ exerted on a moving electrically charged test particle.

In principle, we do this by firing a charged particle through the point where **B** is to be defined, using various directions and speeds for the particle and determining the force $\mathbf{F}_B$ that acts on the particle at that point. After many such trials we would find that when the particle's velocity **v** is along a particular axis through the point, force $\mathbf{F}_B$ is zero. For all other directions of **v**, the magnitude of $\mathbf{F}_B$ is always proportional to $v \sin\phi$, where ϕ is the angle between the zero-force axis and the direction of **v**. Furthermore, the direction of $\mathbf{F}_B$ is always perpendicular to the direction of **v**. (These results suggest that a cross product is involved.)

29-2 THE DEFINITION OF B

We can then define a magnetic field **B** to be a vector quantity that is directed along the zero-force axis. We can next measure the magnitude of $\mathbf{F}_B$ when **v** is directed perpendicular to that axis and then define the magnitude of **B** in terms of that force magnitude:

$$B = \frac{F_B}{|q|v},$$

where q is the charge of the particle.

We can summarize all these results with the following vector equation:

$$\mathbf{F} = q\mathbf{v} \times B. \qquad (29\text{-}2)$$

That is, the force $\mathbf{F}_B$ on the particle is equal to the charge q times the cross product of its velocity **v** and the magnetic field **B.** Using Eq. 3-20 to evaluate the cross product, we can write the magnitude of $\mathbf{F}_B$ as

$$F_B = |q|vB \sin \phi, \qquad (29\text{-}3)$$

where ϕ is the angle between the directions of velocity **v** and magnetic field **B.**

Finding the Magnetic Force on a Particle

Equation 29-3 tells us that the magnitude of the force $\mathbf{F}_B$ acting on a particle in a magnetic field is proportional to the charge q and speed v of the particle. Thus, the force is equal to zero if the charge is zero or if the particle is stationary. Equation 29-3 also tells us that the magnitude of the force is zero if $\mathbf{v}$ and $\mathbf{B}$ are either parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$), and the force is at its maximum when $\mathbf{v}$ and $\mathbf{B}$ are perpendicular to each other.

Equation 29-2 tells us all this plus the direction of $\mathbf{F}_B$. From Section 3-7, we know that the cross product $\mathbf{v} \times \mathbf{B}$ in Eq. 29-2 is a vector that is perpendicular to the two vectors $\mathbf{v}$ and $\mathbf{B}$. The right-hand rule (Fig. 29-2*a*) tells us that the thumb of the right hand points in the direction of $\mathbf{v} \times \mathbf{B}$ when the fingers sweep $\mathbf{v}$ into $\mathbf{B}$. If q is positive, then (by Eq. 29-2) the force $\mathbf{F}_B$ has the same sign as $\mathbf{v} \times \mathbf{B}$ and thus must be in the same direction. That is, for positive q, $\mathbf{F}_B$ points along the thumb as in Figs. 29-2*b*. If q is negative, then the force $\mathbf{F}$ and the cross product $\mathbf{v} \times \mathbf{B}$ have opposite signs and thus must be in opposite directions. So,for negative q, $\mathbf{F}_B$ points opposite the thumb as in Fig. 29-2*c*.

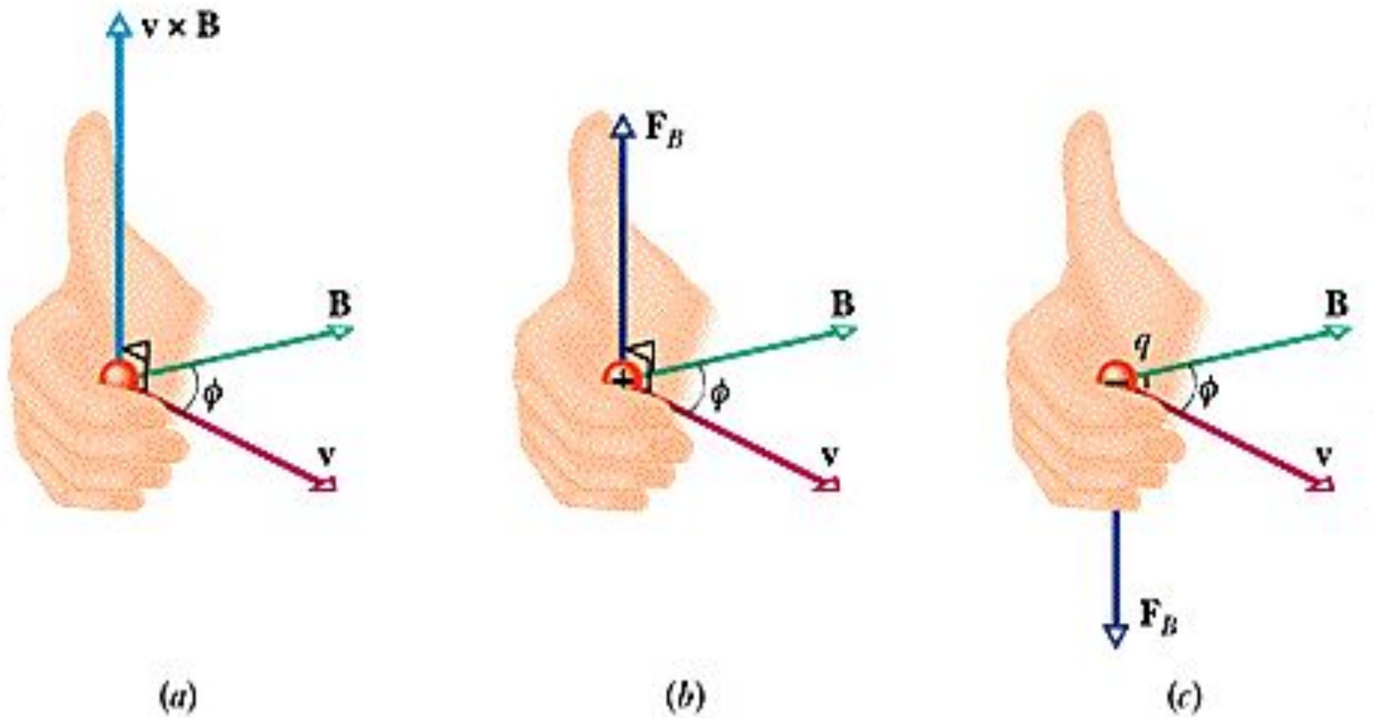


FIGURE 29-2 (*a*) The right-hand rule (in which **v** is swept into **B** through the smaller angle ϕ between them) gives the direction of $\mathbf{v} \times \mathbf{B}$ as the direction of the thumb. (*b*) If q is positive, then the direction of $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ is in the direction of $\mathbf{v} \times \mathbf{B}$. (*c*) If q is negative, then the direction of $\mathbf{F}_B$ is opposite that of $\mathbf{v} \times \mathbf{B}$.

29-2 THE DEFINITION OF B

Regardless of the sign of the charge, however,

> The force $\mathbf{F}_B$ acting on a charged particle moving with velocity $\mathbf{v}$ through a magnetic field $\mathbf{B}$ is *always* perpendicular to $\mathbf{v}$ and $\mathbf{B}$.

Thus $\mathbf{F}_B$ *never* has a component parallel to $\mathbf{v}$. This means that $\mathbf{F}_B$ cannot change the particle's speed v (and thus it cannot change the particle's kinetic energy). The force can change only the direction of $\mathbf{v}$ (and thus the direction of travel); only in this sense does $\mathbf{F}_B$ accelerate the particle.

To develop a feeling for Eq. 29-2, consider Fig. 29-3, which shows some tracks left by charged particles moving rapidly through a *bubble chamber* at the Lawrence Berkeley Laboratory. The chamber, which is filled with liquid hydrogen, is immersed in a strong uniform magnetic field that points out of the plane of the figure. At the left in Fig. 29-3 an incoming gamma ray–which leaves no track because it is uncharged–transforms into an electron (spiral track marked e^-) and a positron (track marked e^+) while it knocks an electron out of a hydrogen atom (long track marked e^-). Check with Eq. 29-2 and Fig. 29-2 that the three tracks made by these two negative particles and one positive particle curve in the proper directions.

29-2 THE DEFINITION OF B

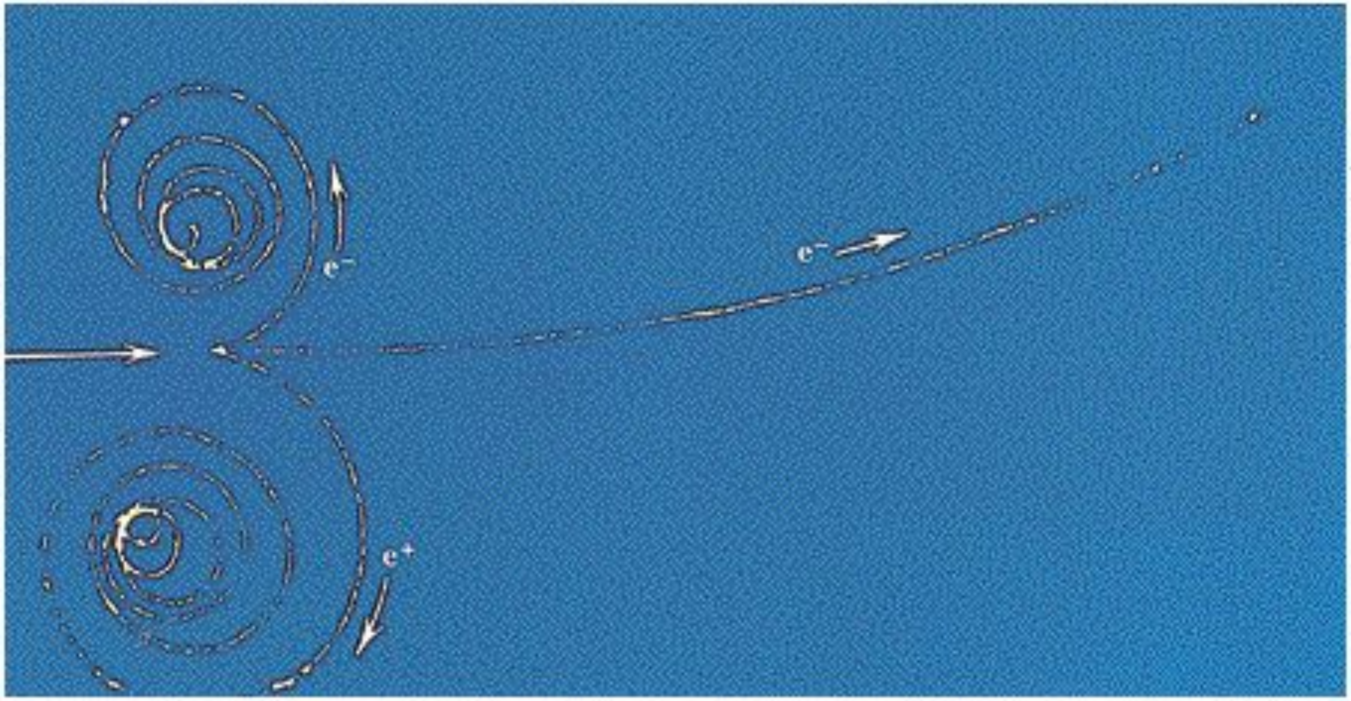


FIGURE 29-3 The tracks of two electrons (e^-) and a positron (e^+) in a bubble chamber that is immersed in a uniform magnetic field that points out of the plane of the page.

29-2 THE DEFINITION OF B

The SI unit for **B** that follows from Eqs. 29-2 and 29-3 is the newton per coulomb-meter per second. For convenience, this is called the **tesla** (T):

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}.$$

Recalling that a coulomb per second is an ampere, we have

$$1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})}$$

$$= 1 \frac{\text{N}}{\text{A} \cdot \text{m}}. \qquad (29\text{-}4)$$

An earlier (non-SI) unit for **B**, still in common use, is the *gauss* (G), and

$$1 \text{ tesla} = 10^4 \text{ gauss}. \qquad (29\text{-}5)$$

Table 29-1 lists the magnetic fields that occur in a few situations. Note that Earth's magnetic field near the planet's surface is about 10^{-4} T (= 100 μT or 1 gauss).

Answers

CHECKPOINT 1:

The figure shows three situations in which a charged particle with velocity **v** travels through a uniform magnetic field **B**. In each situation, what is the direction of the magnetic force $\mathbf{F}_B$ on the particle?

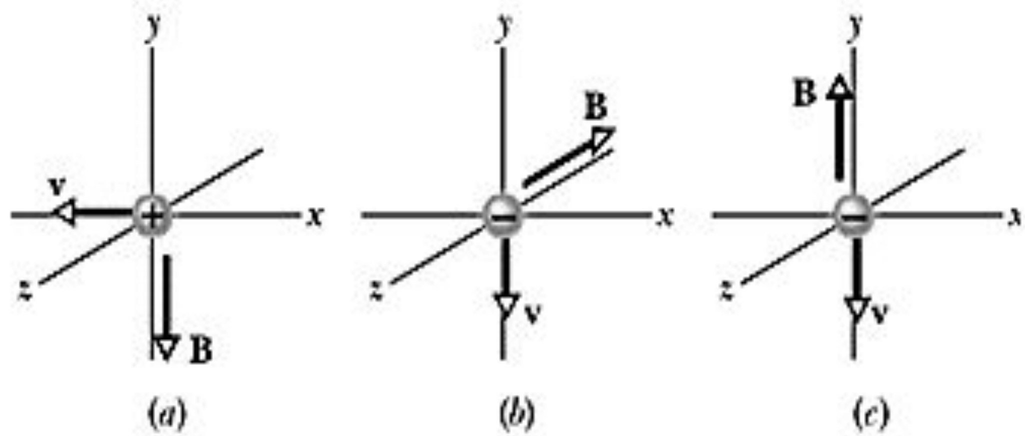


TABLE 29-1 SOME APPROXIMATE MAGNETIC FIELDS

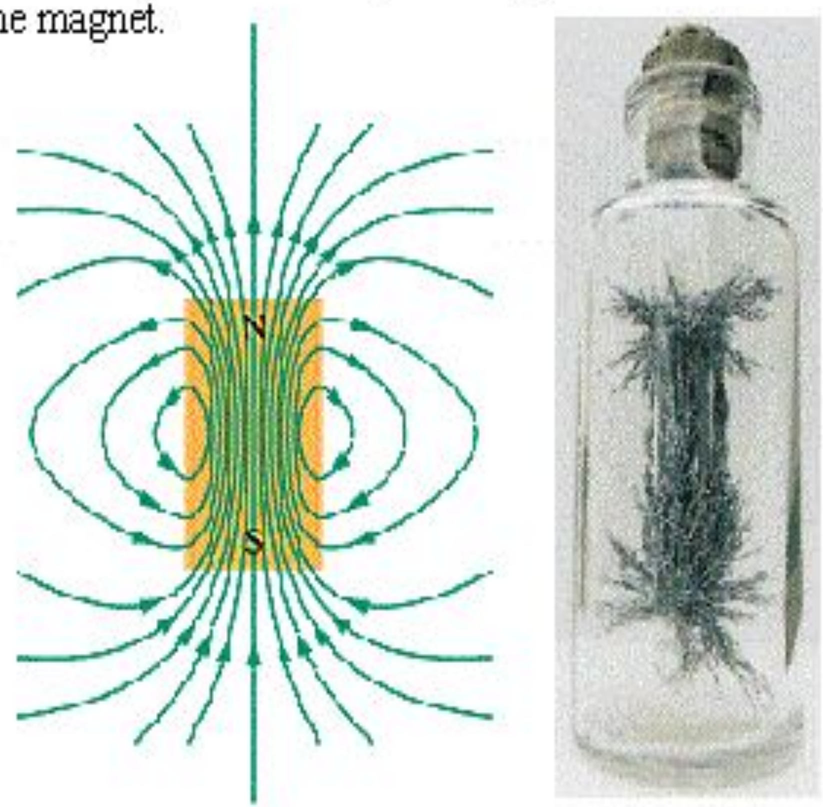
At the surface of a neutron star	10^{8} T
Near a big electromagnet	1.5 T
Near a small bar magnet	10^{-4} T
At Earth's surface	10^{-10} T
In interstellar space	10^{-14} T
Smallest value in a magnetically shielded room	10 T

Magnetic Field Lines

We can represent magnetic fields with field lines, just as we did for electric fields. Similar rules apply. That is, (1) the direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point, and (2) the spacing of the lines represents the magnitude of **B**–the magnetic field is stronger where the lines are closer together, and conversely.

29-2 THE DEFINITION OF B

Figure 29-4*a* shows how the magnetic field near a *bar magnet* (a permanent magnet in the shape of a bar) can be represented by magnetic field lines. The lines all pass through the magnet, and they form closed loops (even those that are not shown closed in the figure). The external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced. Thus the magnetic field of the bar magnet in Fig. 29-4*b* collects the iron filings near the two ends of the magnet.



(a) (b)

FIGURE 29-4
(*a*) The magnetic field lines for a bar magnet.(*b*) A "cow magnet": a bar magnet that is intended to be slipped down into the rumen of a cow to prevent accidentally ingested bits of scrap iron from reaching the cow's intestines.

Because a magnetic field has direction, the (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole*. The magnets we use to fix notes on refrigerators are short bar magnets. Figure 29-5 shows two other common shapes for magnets: a *horseshoe magnet* and a magnet that has been bent around into the shape of a **C** so that the *pole faces* are facing each other. (The magnetic field between the pole faces can then be approximately uniform.) Regardless of the shape of the magnets, if we place two of them near each other we find:

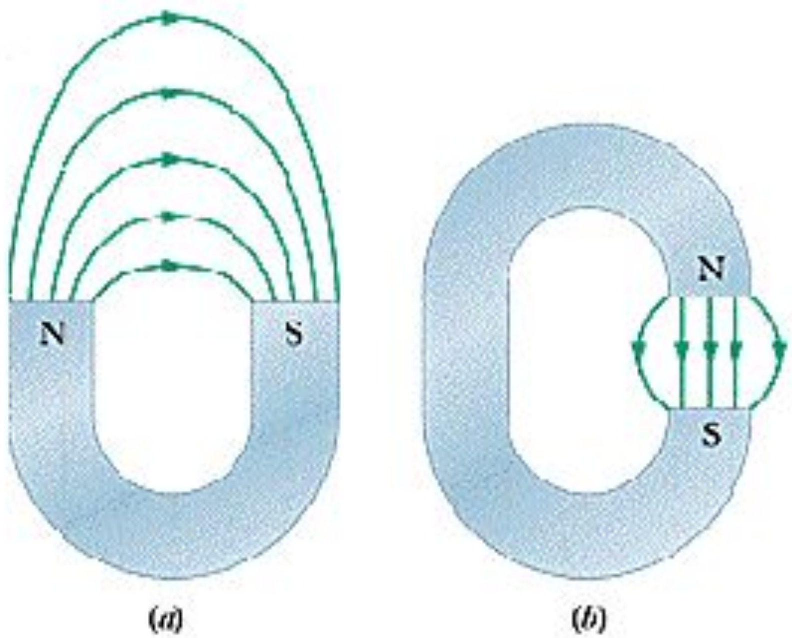


FIGURE 29-5 (*a*) A horseshoe magnet and (*b*) a **C**-shaped magnet. (Only some of the external field lines are shown.)

SAMPLE PROBLEM 29-1

A uniform magnetic field **B**, with magnitude 1.2 mT, points vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg.

SOLUTION: The magnetic deflecting force depends on the speed of the proton, which we can find from $K = \frac{1}{2}mv^2$. Solving for v, we find

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\ \text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV})}{1.67 \times 10^{-27}\ \text{kg}}}$$

$$= 3.2 \times 10^7\ \text{m/s}.$$

Equation 29-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin\phi \\ &= (1.60 \times 10^{-19}\ \text{C})(3.2 \times 10^7\ \text{m/s}) \\ &\quad \times (1.2 \times 10^{-3}\ \text{T})(\sin 90^\circ) \\ &= 6.1 \times 10^{-15}\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration, namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15}\ \mathrm{N}}{1.67 \times 10^{-27}\ \mathrm{kg}} = 3.7 \times 10^{12}\ \mathrm{m/s^2}.$$

It remains to find the direction of $\mathbf{F}_B$. We know that $\mathbf{v}$ points horizontally from south to north and $\mathbf{B}$ points vertically up. The right-hand rule (see Fig. 29-2*b*) shows us that the deflecting force $\mathbf{F}_B$ must point horizontally from west to east, as Fig. 29-6 shows. (The array of dots in the figure represents a magnetic field pointing directly out of the plane of the figure. An array of Xs would have represented a magnetic field pointing directly into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would point in the opposite direction, that is, horizontally from east to west. This is predicted automatically by Eq. 29-2, if we substitute $-e$ for q.

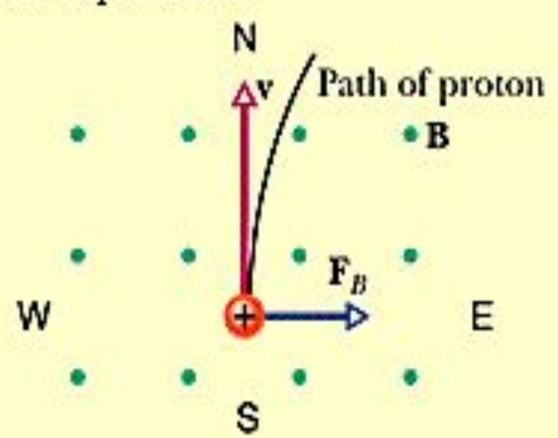


FIGURE 29-6 Sample Problem 29-1. An overhead view of a proton moving from south to north with velocity $\mathbf{v}$ in a chamber. A magnetic field points vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

PROBLEM SOLVING TACTICS

TACTIC 1: *Classical and Relativistic Formulas for Kinetic Energy*

In Sample Problem 29-1, we used the (approximate) classical expression ($K = \frac{1}{2}mv^2$) for the kinetic energy of the proton rather than the (exact) relativistic expression (see Eq. 7-51). The criterion for when the classical expression may safely be used is that $K \ll mc^2$, where mc^2 is the rest energy of the particle. In this case, $K = 5.3$ MeV and the rest energy of a proton is 938 MeV. This proton passes the test and we were justified in treating it as "slow," that is, in using the classical $K = \frac{1}{2}mv^2$ formula for the kinetic energy. That is not always the case in dealing with energetic particles.

29-3 CROSSED FIELDS: DISCOVERY OF THE ELECTRON

Both an electric field **E** and a magnetic field **B** can produce a force on a charged particle. When the two fields are perpendicular to each other, they are said to be *crossed fields.* Here we shall examine what happens to charged particles, namely, electrons, as they move through crossed fields. We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University.

Figure 29-7 shows a modern, simplified version of Thomson's experimental apparatus–a *cathode ray tube* (which is like the "picture tube" in a standard television set). Charged particles (which we now know as electrons) are emitted by a hot filament at the rear of the evacuated tube and are accelerated by an applied potential difference V. After they pass through a slit in screen C, they form a narrow beam. They then pass through a region of crossed **E** and **B** fields, headed toward a fluorescent screen S, where they will produce a spot of light (on a television screen the spot would be part of the picture). The forces on the charged particles in the crossed-fields region can deflect them from the center of the screen. By controlling the magnitudes and directions of the fields, Thomson could thus control where the spot of light appeared on the screen. For the particular field arrangement of Fig. 29-7, electrons are forced up the page by the electric field **E** and down the page by the magnetic field **B**–that is, the forces are *in opposition.* Thomson's procedure was equivalent to the following series of steps.

29-3 CROSSED FIELDS: DISCOVERY OF THE ELECTRON

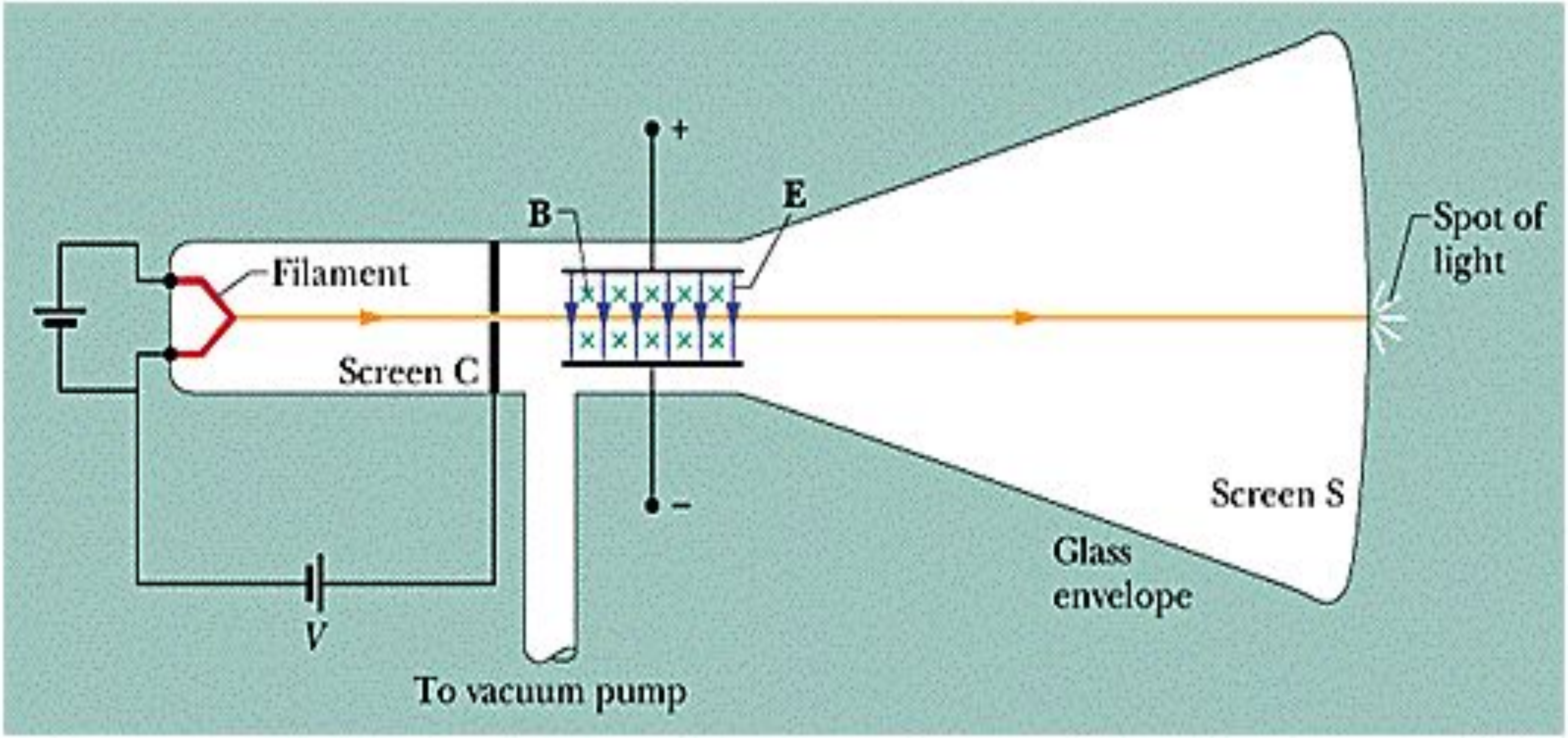


FIGURE 29-7 A modern version of J. J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. The electric field **E** is established by connecting a battery across the deflecting plate terminals. The magnetic field **B** is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of **X**s (which resemble the feathered ends of arrows).

1. Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam.
2. Turn on **E** and measure the resulting beam deflection.

3. Maintaining **E**, now turn on **B** and adjust its value until the beam returns to the undeflected position. (With the forces in opposition, they can be made to cancel.)

We discussed the deflection of a charged particle moving through an electric field **E** between two plates (step 2 here) in Sample Problem 23-8. We found that the deflection of the particle at the far end of the plates is

$$y = \frac{qEL^2}{2mv^2}, \qquad (29\text{-}6)$$

where v is the particle's speed, m its mass, and q its charge, and L is the length of the plates. We can apply this same equation to the beam of electrons in Fig. 29-7 by measuring the deflection of the beam on screen S and then working back to calculate the deflection y at the end of the plates. (Because the direction of the deflection is set by the sign of the particle's charge, Thomson was able to show that the particles that were lighting up his screen were negatively charged.)

29-3 CROSSED FIELDS: DISCOVERY OF THE ELECTRON

When the two fields in Fig. 29-7 are adjusted so that the two deflecting forces cancel (step 3), we have from Eqs. 29-1 and 29-3

$$|q|E = |q|vB\sin(90°) = |q|vB,$$

or

$$v = \frac{E}{B}. \tag{29-7}$$

Thus the crossed fields allow us to measure the speed of the charged particles passing through them. Substituting Eq. 29-7 for v in Eq. 29-6 and rearranging yield

$$\frac{m}{q} = \frac{B^2L^2}{2yE}, \tag{29-8}$$

in which all quantities on the right can be measured. Thus, the crossed fields allow us to measure the ratio m/q of the particles moving through Thomson's apparatus.

Thomson claimed that these particles are found in all matter. He also claimed that they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. (The exact ratio proved later to be 1836.15.) His m/q measurement, coupled with the boldness of his two claims, is considered to be the "discovery of the electron."

29-4 CROSSED FIELDS: THE HALL EFFECT

As we just discussed, a beam of electrons in a vacuum can be deflected by a magnetic field. Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field? In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can. This **Hall effect** allows us to find out whether the charge carriers in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor.

Figure 29-8*a* shows a copper strip of width d, carrying a current i whose conventional direction is from the top of the figure to the bottom. The charge carriers are electrons and, as we know, they drift (with drift speed v_d) in the opposite direction, from bottom to top. At the instant shown in Fig. 29-8*a*, an external magnetic field $\mathbf{B}$, pointing into the plane of the figure, has just been turned on. From Eq. 29-2 we see that a magnetic deflecting force $\mathbf{F}_B$ will act on each drifting electron, pushing it toward the right edge of the strip.

As time goes on, electrons will move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge. The separation of positive and negative charges produces an electric field $\mathbf{E}$ within the strip, pointing from left to right in Fig. 29-8*b*. This field will exert an electric force $\mathbf{F}_E$ on each electron, tending to push it to the left.

29-4 CROSSED FIELDS: THE HALL EFFECT

An equilibrium quickly develops in which the electric force on each electron builds up until it just cancels the magnetic force. When this happens, as Fig. 29-8*b* shows, the force due to **B** and the force due to **E** are in balance. The drifting electrons then move along the strip toward the top of the page with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field **E**.

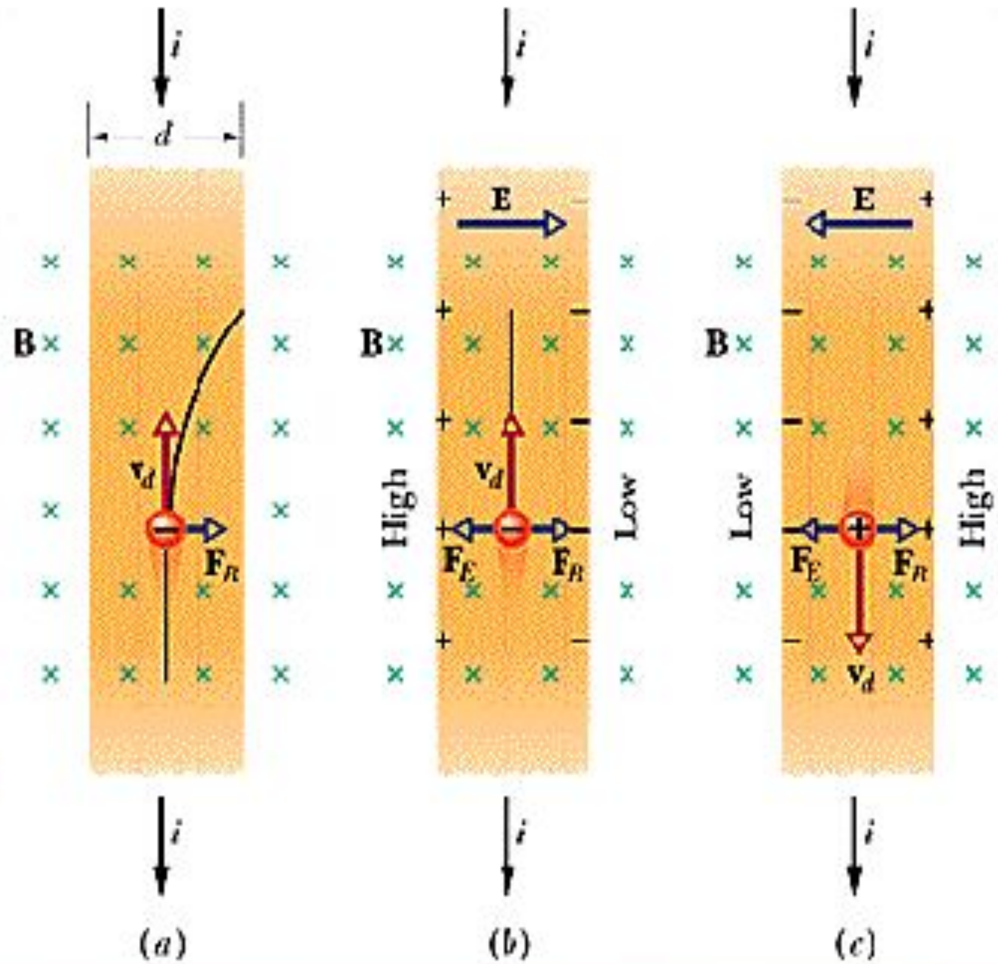


FIGURE 29-8 A strip of copper carrying a current *i* is immersed in a magnetic field **B**. (*a*) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by an electron is shown. (*b*) The situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus the left side is at a higher potential than the right side. (*c*) For the same current direction, if the charge carriers were positively charged, *they* would pile up on the right side, and the right side would be at the higher potential.

29-4 CROSSED FIELDS: THE HALL EFFECT

A *Hall potential difference* V is associated with the electric field across strip width d. From Eq. 25-42, the magnitude of that potential difference is

$$V = Ed. \tag{29-9}$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip. Moreover, the voltmeter can tell us which edge is at higher potential. For the situation of Fig. 29-8*a*, we would find that the left edge is at higher potential, which is consistent with our assumption that the charge carriers are negatively charged.

For a moment, let us make the opposite assumption, that the charge carriers in current i are positively charged (Fig. 29-8*c*). Convince yourself that as these charge carriers moved from top to bottom in the strip, they would be pushed to the right edge by $\mathbf{F}_B$ and thus that the *right* edge would be at higher potential. Because that last statement is contradicted by our voltmeter reading, the charge carriers must be negatively charged.

Now for the quantitative part. When the electric and magnetic forces are in balance (Fig. 29-8*b*), Eqs. 29-1 and 29-3 give us

$$eE = ev_dB. \tag{29-10}$$

29-4 CROSSED FIELDS: THE HALL EFFECT

From Eq. 27-7, the drift speed v_d is

$$v_d = \frac{J}{ne} = \frac{i}{neA}, \qquad (29\text{-}11)$$

in which J (= i/A) is the current density in the strip, A is the cross-sectional area of the strip, and n is the *number density* of charge carriers (their number per unit volume).

In Eq. 29-10, substituting for E with Eq. 29-9 and substituting for v_d with Eq. 29-11, we obtain

$$n = \frac{Bi}{Vle}, \qquad (29\text{-}12)$$

in which l (= A/d) is the thickness of the strip. Thus we can find n in terms of quantities that we can measure.

It is also possible to use the Hall effect to measure directly the drift speed v_d of the charge carriers, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers. The speed of the moving strip is then adjusted until the Hall potential difference vanishes. At this condition, with no Hall effect, the velocity of the charge carriers *with respect to the magnetic field* must be zero. So the velocity of the strip must be equal in magnitude but opposite in direction to the velocity of the negative charge carriers.

SAMPLE PROBLEM 29-2

Figure 29-9 shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity **v** of magnitude 4.0 m/s. The cube moves through a uniform magnetic field **B** of magnitude 0.050 T and pointing in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

SOLUTION: When the cube first began to move through the magnetic field, the conduction electrons within the cube also began to move through the field. Because of their motion, they experienced a force $\mathbf{F}_B$ given by Eq. 29-2. In Fig. 29-9, $\mathbf{F}_B$ acts in the negative direction of the x axis. This means that some of the electrons were deflected by $\mathbf{F}_B$ to the (hidden) left cube face, making that face negatively charged and the right face positively charged. This charge separation produces an electric field **E** directed from the right face toward the left face.

29-4 CROSSED FIELDS:THE HALL EFFECT

Thus, the left face is at lower potential and the right face is at higher potential.

(b) What is the potential difference V between the faces of higher and lower electric potential?

SOLUTION: The electric field **E** that is produced by the charge separation causes a force $\mathbf{F}_E$ to act on the electrons; $\mathbf{F}_E$ is directed toward the right cube face, in the direction opposite that of force $\mathbf{F}_B$. Equilibrium, in which $F_E = F_B$, is reached quickly after the cube begins to move through the magnetic field. From Eqs. 29-1 and 29-3, we then have

$$eE = evB.$$

Substituting for E with Eq. 29-9 ($V = Ed$) then yields

$$V = dvB. \qquad (29\text{-}13)$$

Substituting the given data, we now find

$$V = (0.015\text{ m})(4.0\text{ m/s})(0.050\text{ T})$$
$$= 0.0030\text{ V} = 3.0\text{ mV}. \qquad \text{(Answer)}$$

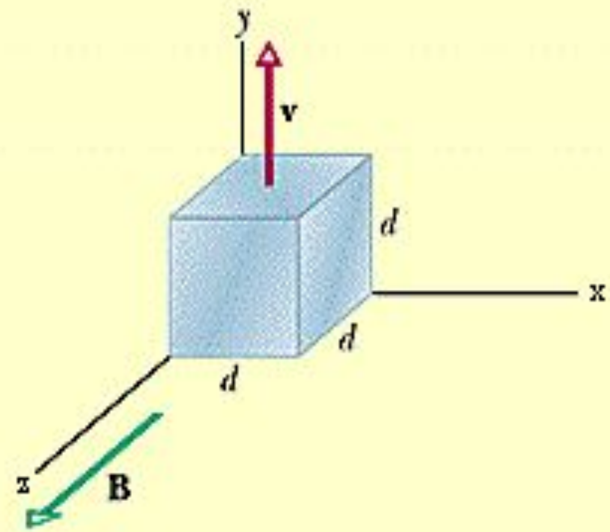
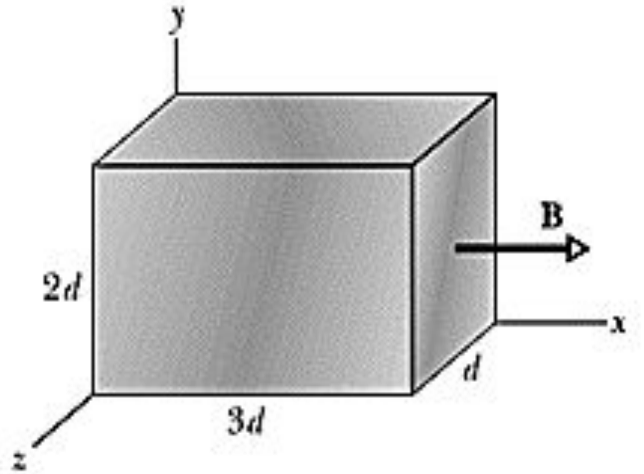


FIGURE 29-9 Sample Problem 29-2. A solid metal cube of edge length d moves at constant velocity **v** through a uniform magnetic field **B.**

Answers

CHECKPOINT 3:

The figure shows a metallic, rectangular solid that is to move at a certain speed v through the uniform magnetic field **B.** Its dimensions are multiples of d, as shown. You have six choices for the direction of the velocity of the solid: it can be parallel to x, y, or z, in either the positive or negative direction. (a) Rank the six choices according to the potential set up across the solid, greatest first. (b) For which choice is the front face at lower potential?



29-5 A CIRCULATING CHARGED PARTICLE

If a particle moves in a circle at constant speed, we can be sure that the net force acting on the particle is constant in magnitude and points toward the center of the circle, always perpendicular to the particle's velocity. Think of a stone tied to a string and whirled in a circle on a smooth horizontal surface, or of a satellite moving in a circular orbit around Earth. In the first case, the tension in the string provides the necessary force and centripetal acceleration. In the second case, Earth's gravitational attraction provides the force and acceleration.

Figure 29-10 shows another example: a beam of electrons is projected into a chamber by an *electron gun* G. The electrons enter in the plane of the page with velocity **v** and move in a region of uniform magnetic field **B** directed out of the plane of the figure. As a result, a magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ continually deflects the electrons, and because **v** and **B** are perpendicular to each other, this deflection causes the electrons to follow a circular path. The path is visible in the photo because atoms of gas in the chamber emit light when some of the circulating electrons collide with them.

29-5 A CIRCULATING CHARGED PARTICLE

We would like to determine the parameters that characterize the circular motion of these electrons, or of any particle of charge magnitude q and mass m moving perpendicular to a uniform magnetic field **B** at speed v. From Eq. 29-3, the force acting on the particle has a magnitude of qvB. So from Newton's second law applied to uniform circular motion (Eq.6-20),

$$F = ma = \frac{mv^2}{r}, \qquad (29\text{-}14)$$

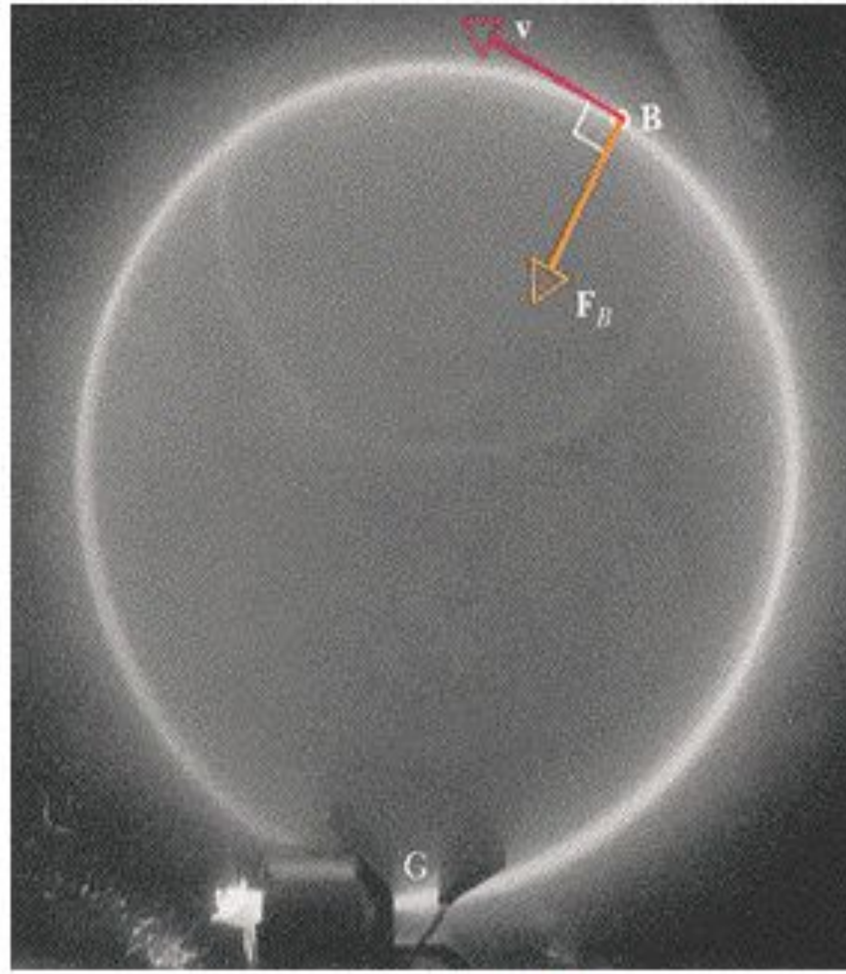


Figure 29-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field **B**, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\mathbf{F}_B$: for circular motion to occur, $\mathbf{F}_B$ *must* point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ gives $\mathbf{F}_B$ the proper direction.

29-5 A CIRCULATING CHARGED PARTICLE

we have

$$\boxed{qvB = \frac{mv^2}{r}.} \qquad (29\text{-}15)$$

Solving for r, we find the radius of the circular path as

$$r = \frac{mv}{qB} \qquad \text{(radius)} \qquad (29\text{-}16)$$

The period T (the time for one full revolution) is equal to the circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} \quad \text{(period).} \qquad (29\text{-}17)$$

The frequency f is

$$f = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \text{(frequency)} \qquad (29\text{-}18)$$

The angular frequency ω of the motion is then

$$\omega = 2\pi f = \frac{qB}{m} \qquad \text{(angular frequency).} \quad (29\text{-}19)$$

The quantities T, f, and ω do not depend on the speed of the particle (provided that speed is much less than the speed of light). Fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio q/m take the same time T (the period) to complete one round trip. Using Eq. 29-2, you can show that if you are looking in the direction of **B**, the direction of rotation for a positive particle is always counterclockwise; that for a negative particle is always clockwise.

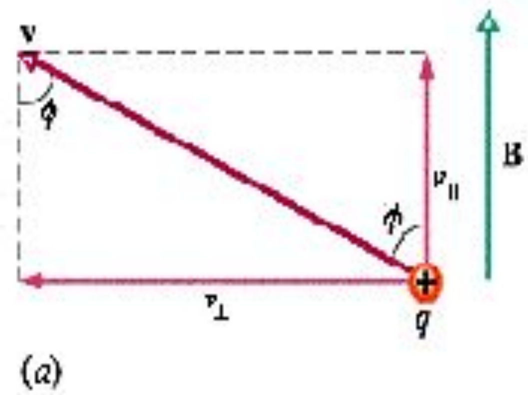
Helical Paths

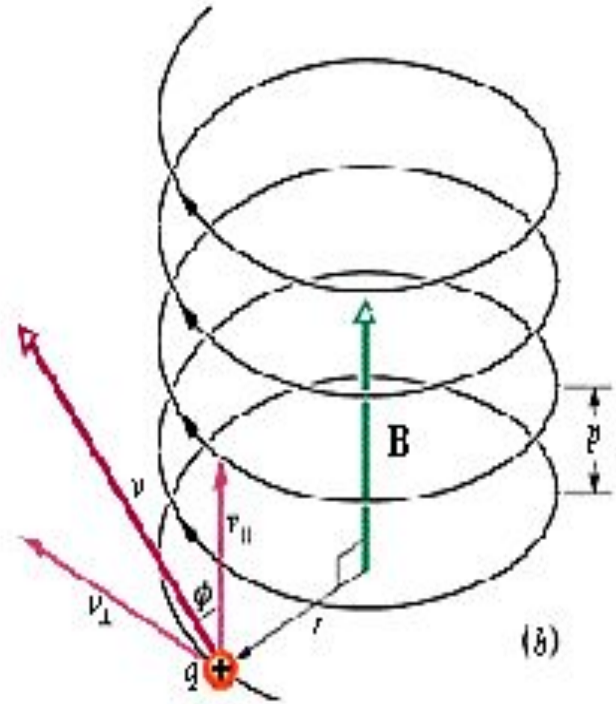
If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector. Figure 29-11*a*, for example, shows the velocity vector **v** of such a particle resolved into two components, one parallel to **B** and one perpendicular to it:

$$v_{\parallel} = v \cos \phi \quad \text{and} \quad v_{\perp} = v \sin \phi. \qquad (29\text{-}20)$$

The parallel component determines the *pitch* p of the helix, that is, the distance between adjacent turns (Fig. 29-11*b*). The perpendicular component determines the radius of the helix and is the quantity to be substituted for v in Eq. 29-16.

29-5 A CIRCULATING CHARGED PARTICLE





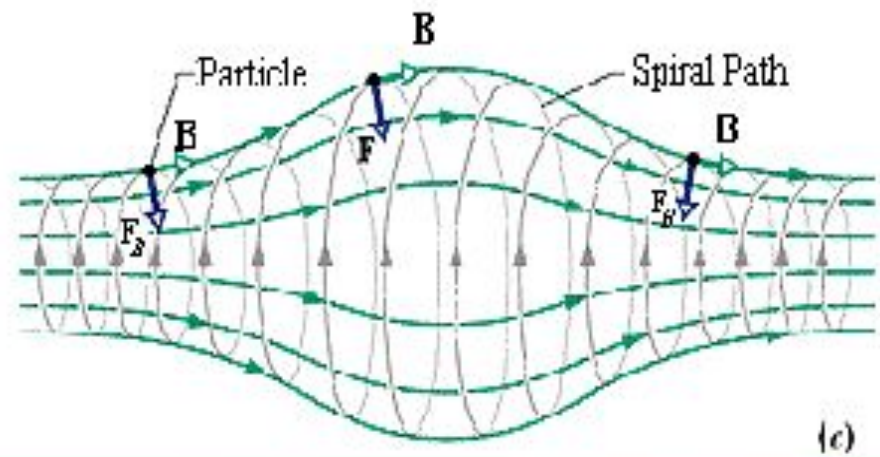


FIGURE 29-11 (*a*) A charged particle moves in a magnetic field, its velocity making an angle ϕ with the field direction. (*b*) The particle follows a helical path, of radius r and pitch p. (*c*) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.

29-5 A CIRCULATING CHARGED PARTICLE

Figure 29-11*c* shows a charged particle spiraling in a nonuniform magnetic field. The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle "reflects" from that end. If the particle reflects from both ends, it is said to be trapped in a *magnetic bottle.*

Electrons and protons are trapped in this way by the terrestrial magnetic field, forming the *Van Allen radiation belts,* which loop well above Earth's atmosphere, between Earth's north and south geomagnetic poles. The trapped particles bounce back and forth, from end to end of the magnetic bottle, within a few seconds.

When a large solar flare shoots additional energetic electrons and protons into the radiation belts, an electric field is produced in the region where electrons normally reflect. This field eliminates the reflection and drives electrons down into the atmosphere, where they collide with atoms and molecules of air, causing that air to emit light. This light forms the aurora–a curtain of light that hangs down to an altitude of about 100 km. Green light is emitted by oxygen atoms, and pink light is emitted by nitrogen molecules, but often the light is so dim that we perceive only white light.

An auroral display extends in an arc above Earth in a region called the *auroral oval* (Figs. 29-12 and 29-13). Although the display is long, it is less than 1 km thick (north to south) because the paths of the electrons producing it converge as the electrons spiral down the converging magnetic field lines (Fig. 29-12).

29-5 A CIRCULATING CHARGED PARTICLE

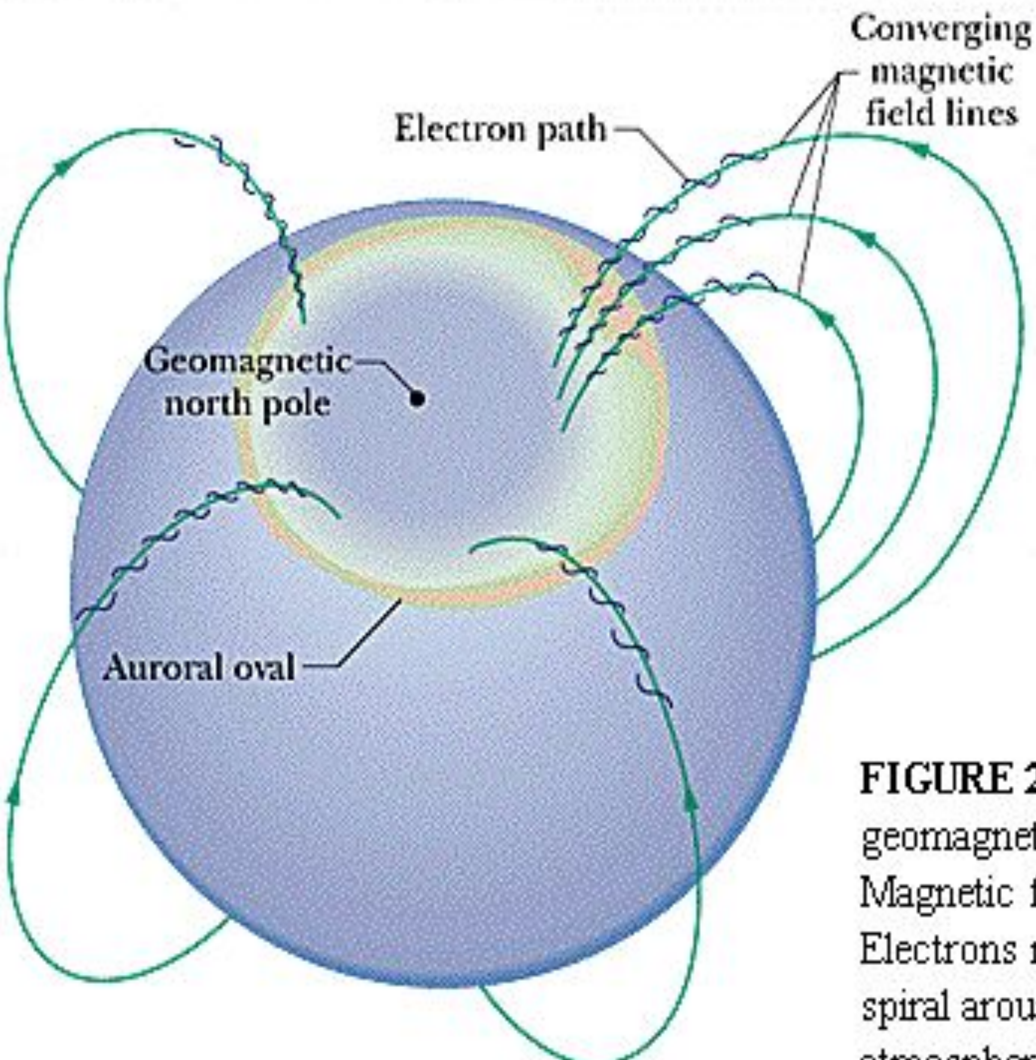


FIGURE 29-12 The auroral oval surrounding Earth's geomagnetic north pole (in northwestern Greenland) Magnetic field lines converg toward that pole. Electrons moving toward Earth are "caught by" and spiral around these field lines, entering the terrestrail atmosphere at high latitudes and producing aurora within the oval.

FIGURE 29-13 A false-color image of aurora inside the north auroral oval, recorded by the satellite *Dynamic Explorer,* using ultraviolet light emitted by oxygen atoms excited in the aurora. The sun-lit portion of Earth is the crescent at the left.

SAMPLE PROBLEM 29-3

Figure 29-14 shows the essentials of a *mass spectrometer*, which can be used to measure the mass of an ion: an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference V. The ion leaves S and enters a separator chamber in which a uniform magnetic field **B** is perpendicular to the path of the ion. The magnetic field causes the ion to move in a semicircle, striking (and thus altering) a photographic plate at distance x from the entry slit. Suppose that in a certain trial $B = 80.000$ mT and $V = 1000.0$ V and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the plate at $x = 1.6254$ m. What is the mass m of the individual ions, in unified atomic mass units ($1\ \text{u} = 1.6605 \times 10^{-27}$ kg)?

SOLUTION: We need to relate the ion mass m to the measured distance x in Fig. 29-14. To do so, we first note that $x = 2r$, where r is the radius of the semicircular path taken by the ion. Then we note that r is related to mass m by $r = mv/qB$ (Eq. 29-16), where v is the speed of the ion upon entering and then moving through the magnetic field.

29-5 A CIRCULATING CHARGED PARTICLE

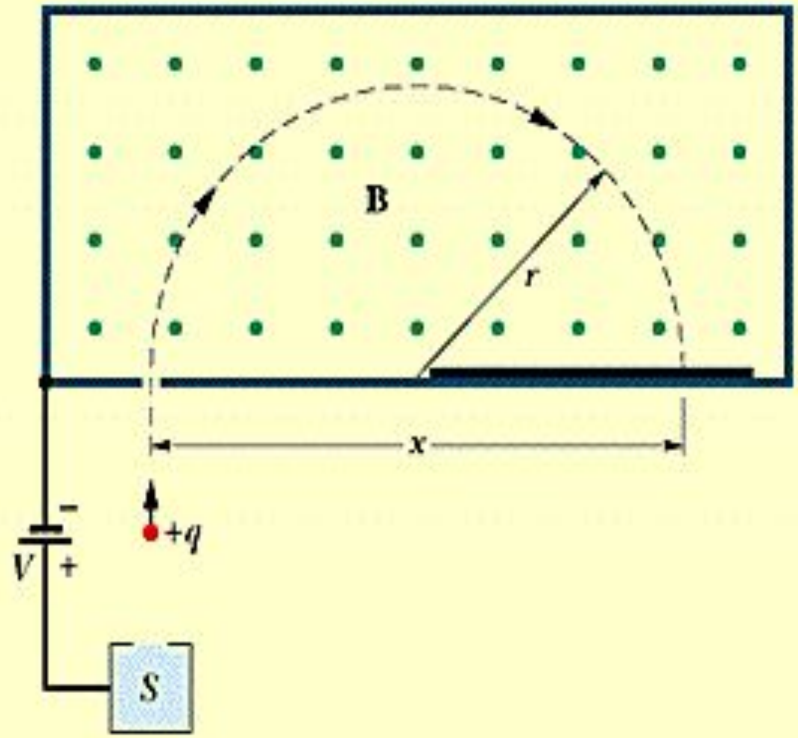


FIGURE 29-14 Sample Problem 29-3. Essentials of a mass spectrometer. A positive ion, after being accelerated from its source S by potential difference V, enters a chamber of uniform magnetic field **B.** There it travels through a semicircle of radius r and strikes a photographic plate at a distance x from where it entered the chamber.

29-5 A CIRCULATING CHARGED PARTICLE

We can relate the speed v to the accelerating potential V by applying the law of conservation of energy to the ion: its kinetic energy $\frac{1}{2}mv^2$ at the end of the accleration is equal to its potential energy qV at the start of the acceleration. Thus

$$\tfrac{1}{2}mv^2 = qV$$

and

$$v = \sqrt{\frac{2qV}{m}}. \qquad (29\text{-}21)$$

Substituting this into Eq. 29-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving this for m and substituting the given data yield

$$m = \frac{B^2qx^2}{8V}$$

$$= \frac{(0.080000\ \mathrm{T})^2(1.6022\times 10^{-19}\ \mathrm{C})(1.6254\ \mathrm{m})^2}{8(1000.0\ \mathrm{V})}$$

$$= 3.3863\times 10^{-25}\ \mathrm{kg} = 203.93\ \mathrm{u}. \qquad \text{(Answer)}$$

SAMPLE PROBLEM 29-4

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field **B** of magnitude 4.55×10^{-4} T. The angle between the directions of **B** and the electron's velocity **v** is 65.5°. What is the pitch of the helical path taken by the electron?

SOLUTION: The pitch p is the distance the electron travels parallel to the magnetic field **B** during one period T of revolution. That distance is $v_{\parallel} T$, where $v_{\parallel}$ is the electron's speed parallel to **B.** Using Eqs. 29-20 and 29-17, we find that

$$p = v_{\parallel} T = (v \cos \phi) \frac{2\pi m}{qB}. \quad (29\text{-}22)$$

We can calculate the electron's speed v from its kinetic energy as we did for the proton in Sample Problem 29-1. (The kinetic energy of 22.5 eV is much less than the electron's rest energy of 5.11×10^5 eV, so we need not use the relativistic formula for the kinetic energy.) We find that $v = 2.81 \times 10^6$ m/s. Substituting this and known data in Eq. 29-22 gives us

$$\begin{aligned} p &= (2.81 \times 10^6 \text{ m/s})(\cos 65.5°) \\ &\quad \times \frac{2\pi (9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(4.55 \times 10^{-4} \text{ T})} \\ &= 9.16 \text{ cm.} \qquad \text{(Answer)} \end{aligned}$$

29-6 CYCLOTRONS AND SYNCHROTRONS

What is the structure of matter on the smallest scale? This question has always intrigued physicists. One way of getting at the answer is to allow an energetic charged particle (a proton, for example) to slam into a solid target. Better yet, allow two such energetic protons to collide head-on. Then analyze the debris from many such collisions to learn the nature of the subatomic particles of matter. The Nobel Prizes in physics for 1976 and 1984 were awarded for just such studies.

How can we give a proton enough kinetic energy for such an experiment? The direct approach is to allow the proton to "fall" through a potential difference V, thereby increasing its kinetic energy by eV. As we want higher and higher energies, however, it becomes more and more difficult to establish the necessary potential difference.

A better way is to arrange for the proton to circulate in a magnetic field, and to give it a modest electrical "kick" once per revolution. For example, if a proton circulates 100 times in a magnetic field and receives an energy boost of 100 keV every time it completes an orbit, it will end up with a kinetic energy of (100)(100 keV) or 10 MeV. Two very useful devices are based on this principle.

29-6 CYCLOTRONSAND SYNCHROTRONS

The *proton synchrotron* is designed to meet these two difficulties. The magnetic field B and the oscillator frequency f_{osc} , instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle. When this is done properly, (1) the frequency of the circulating protons remains in step with the oscillator at all times, and (2) the protons follow a circular–not a spiral–path. Thus the magnet need extend only along that circular path, not over some 4×10^6 m^2. The circular path, however, still must be large if high energies are to be achieved. The proton synchrotron at the Fermi National Accelerator Laboratory (Fermilab) in Illinois (Fig. 29-16) has a circumference of 6.3 kmand can produce protons with energies of about 1 TeV ($= 10^{12}$ eV).

FIGURE 29-16
An aerial view of Fermilab.

29-7 MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

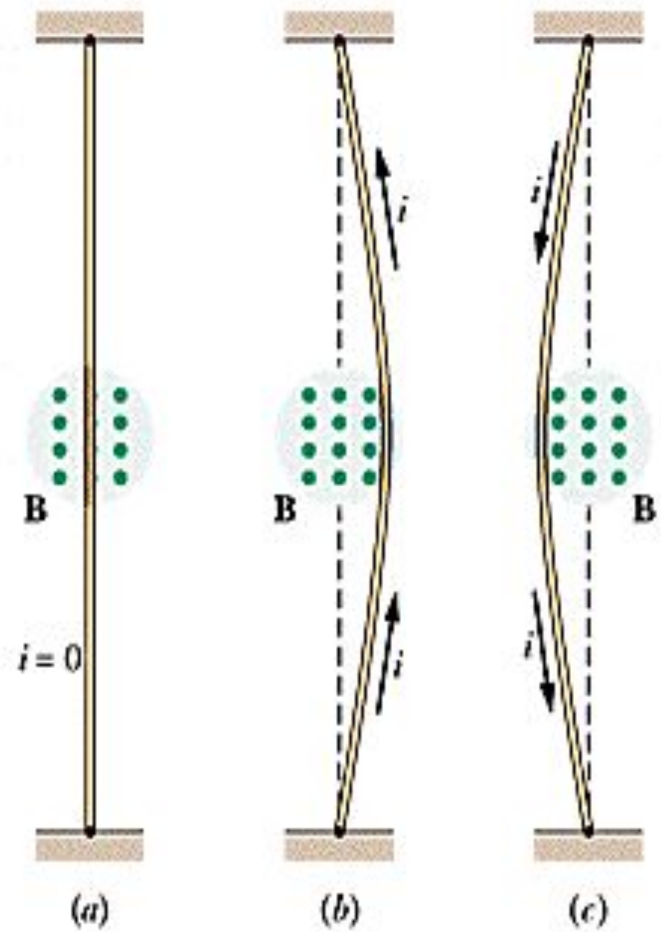
We have already seen (in connection with the Hall effect) that a magnetic field exerts a sideways force on moving electrons in a wire. This force must be transmitted to the wire itself, because the conduction electrons cannot escape sideways out of the wire.

In Fig. 29-17*a*, a vertical wire, carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet. The magnetic field between the faces points outward from the page. In Fig. 29-17*b*, a current is sent upward through the wire; the wire deflects to the right. In Fig. 29-17*c*, we reverse the direction of the current and the wire deflects to the left.

Figure 29-18 shows what happens inside the wire of Fig. 29-17. We see one of the conduction electrons, drifting downward with an assumed drift speed v_d. Equation 29-3, in which we must put $\phi = 90°$, tells us that a force $\mathbf{F}_B$ of magnitude $ev_d B$ must act on each such electron. From Eq. 29-2 we see that this force must point to the right. We expect then that the wire as a whole will experience a force to the right, in agreement with Fig. 29-17*b*.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

FIGURE 29-17 A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.



29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

If, in Fig. 29-18, we were to reverse *either* the direction of the magnetic field *or* the direction of the current, the force on the wire would reverse, pointing now to the left. Note too that it does not matter whether we consider negative charges drifting downward in the wire (the actual case) or positive charges drifting upward. The direction of the deflecting force on the wire is the same. We are safe then in dealing with the conventional direction of current, which assumes positive charge carriers.

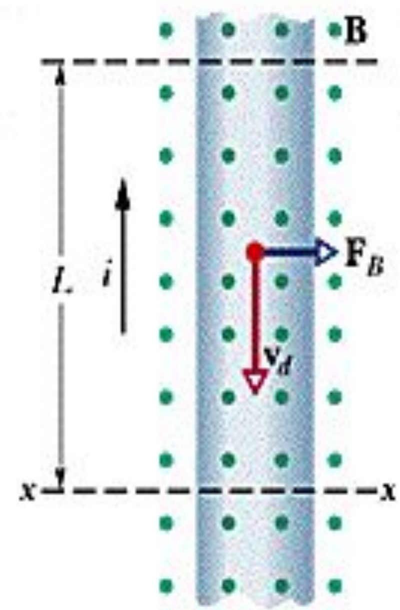


FIGURE 29-18
A close-up view of a section of the wire of Fig. 29-17*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

Consider a length L of the wire in Fig. 29-18. The conduction electrons in this section of wire will drift past plane xx in Fig. 29-18 in a time $t = L/v_d$. Thus in that time a charge given by

$$q = it = i\frac{L}{v_d}$$

will pass through that plane. Substituting this into Eq. 29-3 yields

$$\begin{aligned} F_B &= qv_d B \sin\phi \\ &= \frac{iL}{v_d} v_d B \sin 90^\circ \end{aligned}$$

or

$$F_B = iLB. \qquad (29\text{-}25)$$

This equation gives the force that acts on a segment of a straight wire of length L, carrying a current i and immersed in a magnetic field $\mathbf{B}$ that is perpendicular to the wire.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

If the magnetic field is *not* perpendicular to the wire, as in Fig. 29-19, the magnetic force is given by a generalization of Eq. 29-25:

$$\mathbf{F}_B = i\mathbf{L} \times \mathbf{B} \qquad \text{(force on a current).(29-26)}$$

Here **L** is a *length vector* that points along the wire segment in the direction of the (conventional) current.

Equation 29-26 is equivalent to Eq. 29-2 in that either can be taken as the defining equation for **B**. In practice, we define **B** from Eq. 29-26. It is much easier to measure the magnetic force acting on a wire than that on a single moving charge.

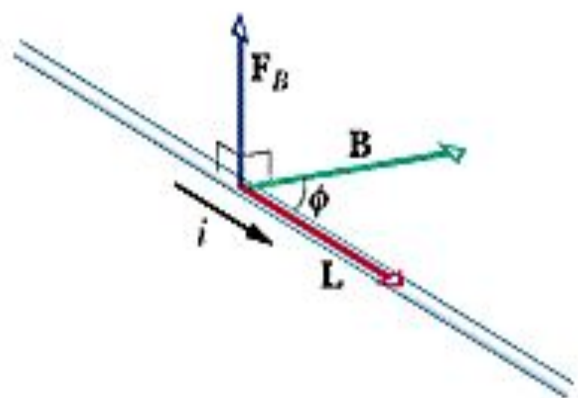


FIGURE 29-19 A wire carrying current i makes an angle ϕ with magnetic field **B**. The wire has length L in the field and length vector **L** (in the direction of the current). A magnetic force $\mathbf{F}_B = i\mathbf{L} \times \mathbf{B}$ acts on the wire.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

If a wire is not straight, we can imagine it broken up into small straight segments and apply Eq. 29-26 to each segment. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

$$d\mathbf{F}_B = i\, d\mathbf{L} \times \mathbf{B}, \quad (29\text{-}27)$$

and we can find the resultant force on any given arrangement of currents by integrating Eq. 29-27 over that arrangement.

In using Eq. 29-27, bear in mind that there is no such thing as an isolated current-carrying wire segment of length dL. There must always be a way to introduce the current into the segment at one end and take it out at the other end.

SAMPLE PROBLEM 29-7

Figure 29-21 shows a length of wire with a central semicircular arc, placed in a uniform magnetic field **B** that points out of the plane of the figure. If the wire carries a current i, what resultant magnetic force **F** acts on it?

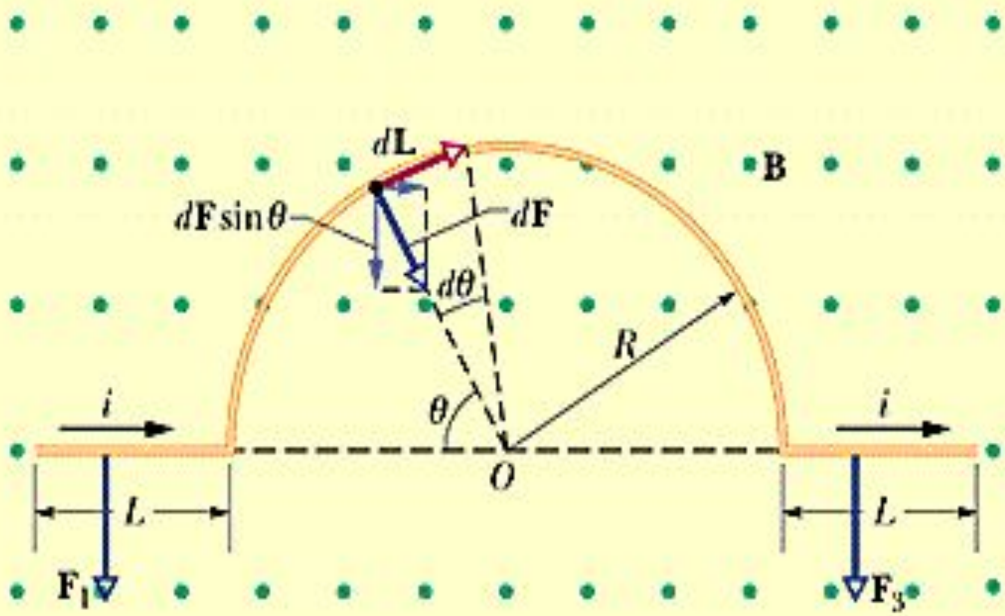


FIGURE 29-21 Sample Problem 29-7. A wire segment carrying a current i is immersed in a magnetic field. The resultant force on the wire is directed downward.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

SOLUTION: The force that acts on each straight section has the magnitude, from Eq. 29-25,

$$F_1 = F_3 = iLB$$

and points down, as shown by $\mathbf{F}_1$ and $\mathbf{F}_3$ in Figure 29-21.

A segment of the central arc of length dL has a force $d\boldsymbol{F}$ acting on it, whose magnitude is given by

$$dF = iB\,dL = iB(R\,d\theta)$$

and whose direction is radially toward point O, the center of the arc. Only the downward component $dF \sin\theta$ of this force element is effective. The horizontal component is canceled by an oppositely directed horizontal component associated with a symmetrically located segment on the opposite side of the arc.

29-7 MAGNETIC FORCEON A CURRENT-CARRYING WIRE

Thus the total force on the central arc points down and is given by

$$F_2 = \int_0^\pi dF \sin\,\theta = \int_0^\pi (iBR\,d\theta)\sin\,\theta$$
$$= iBR\int_0^\pi \sin\,\theta\,d\theta = 2iBR.$$

The resultant force on the entire wire is then

$$F = F_1 + F_2 + F_3 = iLB + 2iBR + iLB$$
$$= 2iB(L + R). \qquad \text{(Answer)}$$

Note that this force is equal to the force that would act on a straight wire of length $2(L + R)$. This would be true no matter what the shape of the central segment.

SAMPLE PROBLEM 29-8

Figure 29-22*a* shows a wire carrying a current $i = 6.0$ A in the positive direction of the x axis and lying in a *nonuniform* magnetic field given by $\mathbf{B} = (2.0\ \text{T/m})x\mathbf{i} + (2.0\ \text{T/m})x\mathbf{j}$, with **B** in teslas and x in meters. What is the net magnetic force $\mathbf{F}_B$ on the section of the wire between $x = 0$ and $x = 2.0$ m?

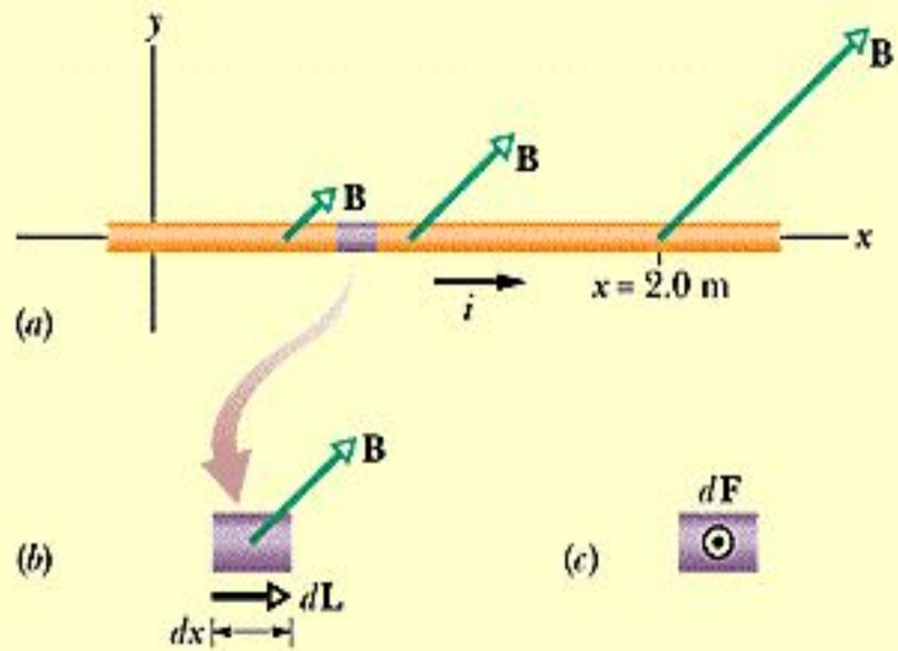


FIGURE 29-22 Sample Problem 29-8. (*a*) A wire with current *i* lies in a nonuniform magnetic field **B.** (*b*) An element of the wire, with differential length vector $d\boldsymbol{L}$ and length dx. (*c*) The differential force $d\boldsymbol{F}$ acting on the element of (*b*) due to the magnetic field; the force is directed out of the page.

SOLUTION: Because the field varies along the section of wire, we cannot just substitute the data into Eq. 29-26, which holds only for a uniform magnetic field **B.** Instead, we must mentally divide the wire into differential lengths and then use Eq. 29-26 to find the differential force $d\mathbf{F}_B$ on each length. Then we can sum these differential forces to find the net magnetic force $\mathbf{F}_B$ on the full section of wire.

Figure 29-22*b* shows a differential length vector $d\mathbf{L}$ along the wire in the direction of the current; the vector has length dx and points in the positive direction of the x axis. Thus we can write this vector $d\mathbf{L}$ as

$$d\mathbf{L} = dx\,\mathbf{i}. \tag{29-28}$$

(Be careful not to confuse the unit vector **i** with the current i.) Now, by Eq. 29-26, the differential force $d\mathbf{F}_B$ on the length dx of the wire is

$$\begin{aligned} d\mathbf{F}_B &= i\,d\mathbf{L} \times \mathbf{B} \\ &= i(dx\,\mathbf{i}) \times (2.0x\mathbf{i} + 2.0x\mathbf{j}) \\ &= i\,dx[2.0x(\mathbf{i} \times \mathbf{i}) + 2.0x(\mathbf{i} \times \mathbf{j})] \\ &= i\,dx[0 + 2.0x\mathbf{k}] = 2.0ix\,dx\,\mathbf{k}, \end{aligned} \tag{29-29}$$

where the constant 2.0 has the unit teslas per meter. From this result we see that the magnetic force does not depend on the x component of **B** (because that component is along the direction of the current). We also see that the magnetic force $d\boldsymbol{F}_B$ on length dx of the wire is in the positive direction of the z axis (out of the page in Fig. 29-22c) and has magnitude $dF_B = (2.0\ \mathrm{T/m})ix\ dx$.

Because the direction of the force $d\boldsymbol{F}$ is the same for all the differential lengths dx of the wire, we can find the magnitude of the total force by summing all the differential force magnitudes dF_B. To do so, we integrate dF_B from $x = 0$ to $x = 2.0$ m and then substitute the given data. We get

$$F_B = \int dF_B = \int_0^{2.0\,\mathrm{m}} (2.0\ \mathrm{T/m})ix\,dx$$

$$= (2.0\ \mathrm{T/m})i\left[\tfrac{1}{2}x^2\right]_0^{2.0\,\mathrm{m}} = (2.0\ \mathrm{T/m})(6.0\ \mathrm{A})(\tfrac{1}{2})(2.0\ \mathrm{m})^2$$

$$= 24\ (\mathrm{T\cdot A\cdot m}) = 24\ \mathrm{N}. \qquad \text{(Answer)}$$

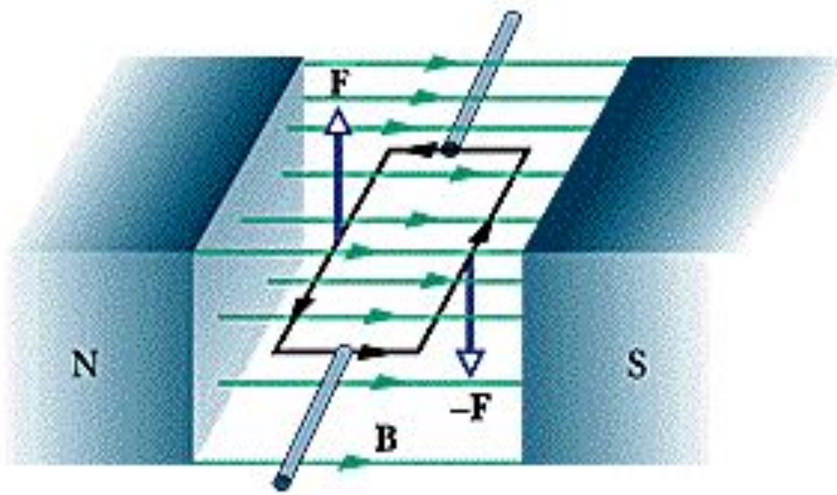
This force is directed along the positive direction of the z axis.

29-8 TORQUE ON A CURRENT LOOP

Much of the world's work is done by electric motors. The forces behind this work are the magnetic forces that we studied in the preceding section, that is, the forces that a magnetic field exerts on a wire that carries a current.

Figure 29-23 shows a simple motor, consisting of a single current-carrying loop immersed in a magnetic field **B**. The two magnetic forces **F** and –**F** combine to exert a torque on the loop, tending to rotate it about its central axis. Although many essential details have been omitted, the figure does suggest how the action of a magnetic field, exerting a torque on a current loop, produces the rotary motion of the electric motor. Let us analyze the action.

FIGURE 29-23 The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. A commutator (not shown) reverses the direction of the current every half-revolution so that the magnetic torque always acts in the same direction.



29-8 TORQUE ON A CURRENT LOOP

Figure 29-24*a* shows a rectangular loop of sides *a* and *b*, carrying a current *i* and immersed in a uniform magnetic field **B**. We place it in the field so that its long sides, labeled 1 and 3, are perpendicular to the field direction (which is into the page), but its short sides, labeled 2 and 4, are not. Wires to lead the current into and out of the loop are needed but, for simplicity, they are not shown.

To define the orientation of the loop in the magnetic field, we use a normal vector **n** that is perpendicular to the plane of the loop. Figure 29-24*b* shows a right-hand rule for finding the direction of **n**. Point or curl the fingers of your right hand in the direction of the current at any point on the loop. Your extended thumb then points in the direction of the normal vector **n.**

The normal vector of the loop is at an angle θ to the direction of the magnetic field **B**, as shown in Fig. 29-24*c*. We wish to find the net force and net torque acting on the loop in this orientation.

The net force is the vector sum of the forces acting on each of the four sides of the loop. For side 2 the vector **L** in Eq. 29-26 points in the direction of the current and has magnitude *b*. The angle between **L** and **B** for side 2 (see Fig. 29-24*c*) is $90° - \theta$. Thus the magnitude of the force acting on this side is

$$F_2 = ibB \sin(90° - \theta) = ibB \cos \theta. \quad (29\text{-}30)$$

29-8 TORQUE ON A CURRENT LOOP

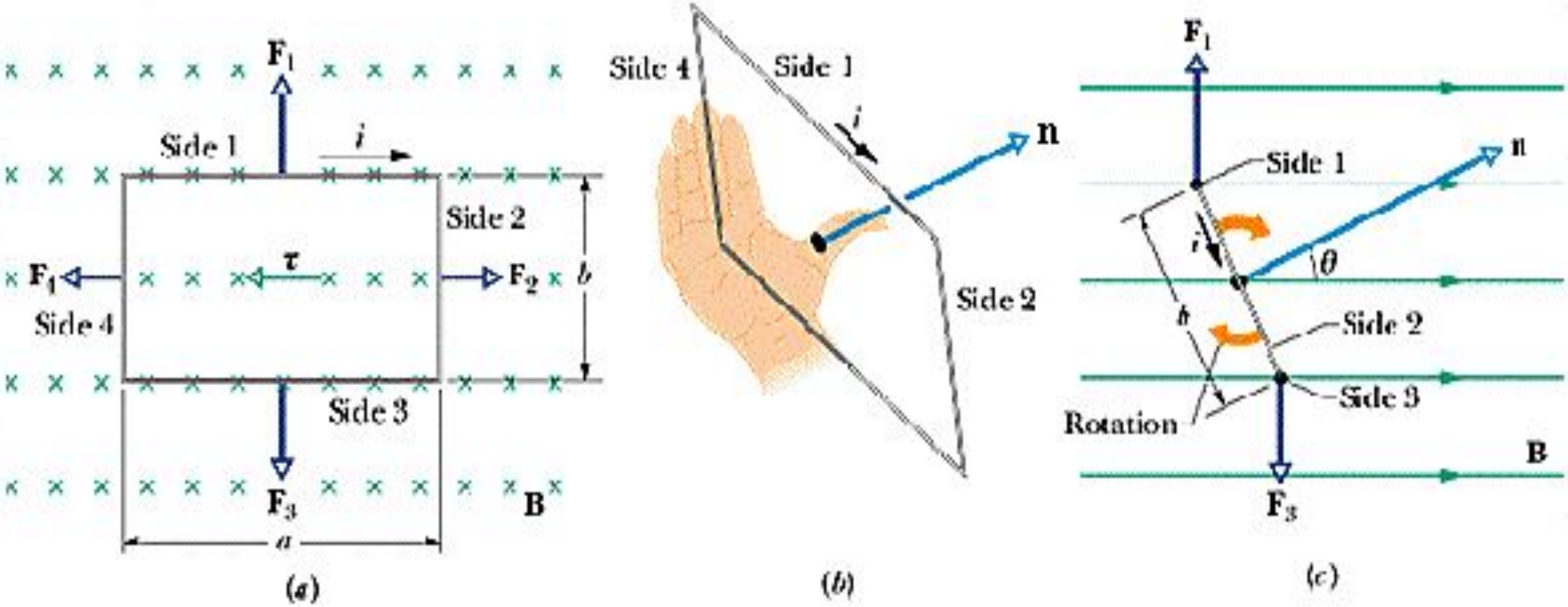


FIGURE 29-24 A rectangular loop, of length a and width b and carrying a current i, is placed in a uniform magnetic field. A torque τ acts to align the normal vector **n** with the direction of the field. (*a*) The loop as seen by looking in the direction of the magnetic field. (*b*) A perspective of the loop showing how a right-hand rule gives the direction of **n**, which is perpendicular to the plane of the loop. (*c*) A side view of the loop, from side 2. The loop rotates as indicated.

29-8 TORQUE ON A CURRENT LOOP

You can show that the force $\mathbf{F}_4$ acting on side 4 has the same magnitude as $\mathbf{F}_2$ but points in the opposite direction. Thus $\mathbf{F}_2$ and $\mathbf{F}_4$ cancel out exactly. Their net force is zero and, because their common line of action is through the center of the loop, their net torque is also zero.

The situation is different for sides 1 and 3. Here the common magnitude of $\mathbf{F}_1$ and $\mathbf{F}_3$ is iaB, and the two forces point in opposite directions so that they do not tend to move the loop up or down. However, as Fig. 29-24*c* shows, these two forces do *not* share the same line of action so they *do* produce a net torque. The torque tends to rotate the loop so as to align its normal vector $\mathbf{n}$ with the direction of the magnetic field $\mathbf{B}$. That torque has moment arm $(b/2)\sin\theta$ about the center of the loop. The magnitude τ' of the torque due to forces $\mathbf{F}_1$ and $\mathbf{F}_3$ is (see Fig. 29-24*c*)

$$\begin{aligned}\tau' &= \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right)\\ &= iabB\sin\theta.\end{aligned}$$

REVIEW & SUMMARY

***Magnetic Field* B**

A magnetic field B is defined in terms of the force $\mathbf{F}_B$ acting on a test particle with charge q moving through the field with velocity $\mathbf{v}$:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}. \qquad (29\text{-}2)$$

The SI unit for **B** is the **tesla** (T): 1 T = 1 N/(A·m) = 10^4 gauss.

The Hall Effect

When a conducting strip of thickness l carrying a current i is placed in a magnetic field **B**, some charge carriers (with charge e) build up on the sides of the conductor, creating a potential difference V across the strip. The polarity of V gives the sign of the charge carriers; the number density n of charge carriers can be calculated with

$$n = \frac{Bi}{Vle}, \qquad (29\text{-}12)$$

A Charged Particle Circulating in a Magnetic Field

A charged particle with mass m and charge magnitude q moving with velocity $\mathbf{v}$ perpendicular to a magnetic field $\mathbf{B}$ will travel in a circle. Applying Newton's second law to the circular motion yields

$$qvB = \frac{mv^2}{r}, \qquad (29\text{-}15)$$

from which we find the radius r of the circle to be

$$r = \frac{mv}{qB} \qquad (29\text{-}16)$$

The frequency of revolution f, the angular frequency ω, and the period of the motion T are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{qB}{2\pi m} \qquad (29\text{-}19,\ 29\text{-}18,\ 29\text{-}17)$$

Cyclotrons and Synchrotrons

A cyclotron is a particle accelerator that uses a magnetic field to hold a charged particle in a circular orbit of increasing radius so that a modest accelerating potential may act on the particle repeatedly, providing it with high energy. Because the moving particle gets out of step with the oscillator as its speed approaches that of light, there is an upper limit to the energy attainable with the cyclotron. A synchrotron avoids this difficulty. Here both B and the oscillator frequency f_{osc} are programmed to change cyclically so that the particle not only can go to high energies but can do so at a constant orbital radius.

Magnetic Force on a Current-Carrying Wire

A straight wire carrying a current i in a uniform magnetic field experiences a sideways force

$$\mathbf{F}_B = i\mathbf{L} \times \mathbf{B} \qquad (29\text{-}26)$$

The force acting on a current element $i\, d\boldsymbol{L}$ in a magnetic field is

$$d\mathbf{F}_B = i\, d\mathbf{L} \times \mathbf{B}, \qquad (29\text{-}27)$$

The direction of the length vector $\mathbf{L}$ or $d\boldsymbol{L}$ is that of the current i.

Torque on a Current-Carrying Coil

A coil (of area A and carrying current i, with N turns) in a uniform magnetic field **B** will experience a torque τ given by

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}, \qquad (29\text{-}35)$$

Here $\boldsymbol{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude $\mu = NiA$ and direction given by a right-hand rule.

Orientation Energy of a Magnetic Dipole

The **magnetic potential energy** of a magnetic dipole in a magnetic field is

$$U(\theta) = -\boldsymbol{\mu} \cdot \mathbf{B}. \qquad (29\text{-}36)$$

Question Menu E & P Menu

31-1 Calculating the Magnetic Field Due to a Current

The **Biot-Savart law** is a prescription for calculating the magnetic field of a current. To calculate the magnetic field produced at any point P by a wire carrying a current i, first choose an infinitesimal element d**s** of the wire, in the same direction as the current. Draw the displacement vector **r** from the selected current element to P. The field produced at P by the current element is given by

$$dB = \left(\frac{\mu_0}{4\pi}\right) \frac{i\,ds \times r}{r^3}.$$

The field of the element is perpendicular to both d**s** and **r**. To find the total field at P, sum (integrate) the contributions from all elements of the wire.

The constant μ_0 is called the **permeability constant** and its value is exactly $4\pi \times 10^{-7}$ T·m/A. Do not confuse the symbol with that for the magnitude of a magnetic dipole moment (μ without a subscript)

In Section 30-1 of the text, the Biot-Savart law is used to find an expression for the magnitude of the magnetic field produced by a long straight wire carrying current i. Each infinitesimal element of the wire produces a field in the same direction so the magnitude of the total field is the sum of the magnitudes of the fields produced by all the elements. Go over the calculation carefully.

The result is $B_0 = \mu\, i \,/\, 2\pi R$ where R is the perpendicular distance from the wire to the field point.

Main Text Study Guide Menu

30-2 Two Parallel Currents

The expression for the field of a long straight wire can be combined with the expression for the magnetic force on a wire, developed in the last chapter, to find an expression for the force exerted by two parallel wires on each other. The magnetic field produced by one wire at a point on the other wire is perpendicular to the second wire. If the wires are separated by a distance d and carry currents i_a and i_b, then the magnitude of the force per unit length of one on the other is given by $F / L = \mu_0 \, i_a \, i_b / 2\pi d$ If the currents are in the same direction, the wires attract each other; if the currents are in opposite directions, they repel each other. The forces of the wires on each other obey Newton's third law: they are equal in magnitude and opposite in direction.

Main Text Study Guide Menu

30-3 Ampere's Law

Ampere's law tells us that the integral of the tangential component of the magnetic field around any closed path is equal to the product of μ_0 and the net current that pierces the loop. Mathematically, for any closed path, $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$,

where d**s** is an infinitesimal displacement vector, tangent to the path. The integral on the left side is a path integral around a *closed* path, called an Amperian path. The current i on the right side is the net current through a surface that is bounded by the path. For example, if the path is formed by the edges of a page, then i is the net current through the page. The Amperian path need not be the boundary of any physical surface and, in fact, may be purely imaginary. The surface need not be a plane.

To apply Ampere's law, choose a direction (clockwise or counterclockwise) to be used in evaluating the integral on the left side. The choice is immaterial but it must be made since it determines the direction of d**s**. If the tangential component of the field is in the direction of d**s**, then the integral is positive; if it is in the opposite direction, the integral is negative. Now, curl the fingers of your right hand around the loop in the direction chosen. Your thumb will point in the direction of positive current. Examine each current through the surface and algebraically sum them. If a current arrow is in the direction your thumb pointed, it enters the sum with a positive sign; if it is in the opposite direction it enters with a negative sign.

Main Text Study Guide Menu

30-4 Solenoids and Toroids

Ampere's law can be applied to a **solenoid**, a cylinder tightly wrapped with a thin wire. For an ideal solenoid (long and tightly wrapped), the magnetic field outside is negligible and the field inside is uniform and is parallel to the axis.

As the Amperian path, take a rectangle with one side, of length h, inside the solenoid and parallel to its axis, and the opposite side outside the solenoid. The first of these sides contributes Bh to the left side of the Ampere's law equation and the other three sides of the rectangle contribute zero. Thus, $\oint \mathbf{B} \cdot d\mathbf{s} = Bh$. If the solenoid has n turns of wire per unit length, then the number of turns that pass through the surface bounded by the Amperian path is nh. Each carries current i so the right side of the Ampere's law equation is $\mu_0 nhi$. $Bh = \mu_0 nhi$, so $B = \mu_0 ni$.

Ampere's law can also be applied to a toroid, with a core shaped like a doughnut and wrapped with a wire, like a solenoid bent so its ends join. The magnetic field is confined to the interior of the core and the field lines are concentric circles centered at the center of the hole. The Amperian path is a circle of radius r, inside the core and centered at the center of the hole. The integral $\oint \mathbf{B} \cdot d\mathbf{s}$ $i 2\pi r B$ and if there are N turns of wire, each carrying current i total current through the surface bounded by the path is Ni. Thus, the Ampere's law equation is $2\pi r B = \mu_0 Ni$ and $B = \mu_0 Ni / 2\pi r$. The field is zero inside the hole and outside the toroid.

Main Text Study Guide Menu

30-5 A Current-Carrying Coil as a Magnetic Dipole

The Biot-Savart law can be used to show that the magnetic field produced by a circular loop of wire of radius R and carrying current i, at a point on its axis of symmetry a distance z from its center, is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}} .$$

For points far away from the loop ($z \gg R$), the expression becomes $B(z) = \mu_0 i R^2/2z^3$, which can be written in terms of the dipole moment of the loop: $B(z) = \mu_0 \mu/2\pi z^3$. **B** is in the direction of $\boldsymbol{\mu}$ for points above the loop (positive z) and in the opposite direction for points below the loop (z negative). This expression is valid for the magnetic field of *any* plane loop, regardless of its shape, for points far away along the axis defined by the direction of the dipole moment.

Main Text Study Guide Menu

31-1 Two Symmetric Situations

When a current-carrying loop of wire is placed in a magnetic field, the field exerts a torque on the loop and the loop rotates. The reverse also happens. If the loop initially has no current but is then rotated in a magnetic field, current is induced in it.

31-2 Two Experiments

- If a magnet is moved toward a closed loop of wire, current is induced in the wire. When the magnet stops moving, the current stops. As the magnet is withdrawn, current is again induced.

  The direction of the current is reversed when the direction of motion of the magnet is reversed.

- If two loops of wire are close to each other and a changing current is produced in one, then current is induced in the second loop.

- In each of these cases, a changing magnetic field through a closed loop induces an emf around the loop and this emf generates a current. The emf is induced only when the magnetic field through the loop is changing or else when the loop is moving in a magnetic field.

Main Text Study Guide Menu

31-3 Faraday's Law of Induction

The **magnetic flux** through a surface is defined by the integral

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

over the surface. Here d**A** is an infinitesimal vector area, normal to the surface. If the field is uniform over the surface, then $\Phi_B = BA\cos\theta$, where θ is the angle between the field and the normal to the surface.

The magnetic flux through any area is proportional to the number of magnetic field lines through that area. Recall that the number of field lines through a small area perpendicular to the field is proportional to the magnitude of the field and that the number of lines through *any* small area is proportional to the component B_n along a normal to the area.

Main Text Study Guide Menu

31-4 Lenz's Law

Lenz's law provides another way to determine the direction of an induced emf. Imagine that the boundary of an area is a conducting wire and that an externally produced magnetic field penetrates the area. When the field changes, a current is induced in the wire and the current also produces a magnetic field. According to Lenz's law, the sign of the flux of the induced current is the same as that of the externally produced flux if that flux is decreasing and is opposite that of the externally produced flux if that flux is increasing. In the first case, the magnetic field of the induced current is roughly in the same direction as the externally produced field; in the second case, it is roughly in the opposite direction.

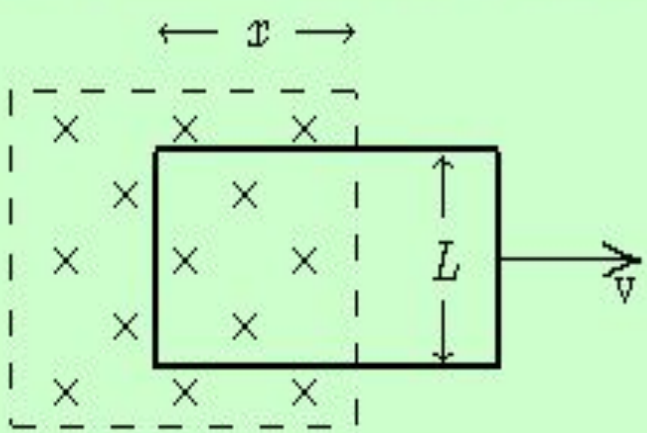
Once you have determined the direction of the magnetic field produced by the induced current, you can determine the direction of the current itself and, hence, that of the emf. Very near any segment of the loop, the field is quite similar to the field of a long straight wire; the lines are nearly circles around the segment. Use the right-hand rule explained in the last chapter: curl your fingers around the segment so they point in the direction of the field in the interior of the loop. That is, they should curl upward through the loop if the field is upward through the loop and they should curl downward if the field is downward. Your thumb will then point along the segment in the direction of the current.
This is also the direction of the induced emf.

Main Text Study Guide Menu

31-5 Induction and Energy Transfers

- An emf is also generated if all or part of the loop moves in a manner that changes the flux through it. If a loop is moved through a uniform field, the flux through it does not change and no emf is generated. If, however, the field is not uniform, then the flux changes as the loop moves and an emf is generated. Find an expression for the flux as a function of the position of the loop, then differentiate it with respect to time to obtain $d\Phi_B/dt$. The emf clearly depends on the velocity of the loop. Either Lenz's law or the sign convention associated with Faraday's law can be used to find the direction of the emf.

- Suppose a closed loop is pulled with constant speed out of the region of a uniform magnetic field, perpendicular to the loop, as shown below. At any instant, x is the length of the loop within the region of the field and the flux through the loop is $\Phi = BLx$. The emf generated around it is $\varepsilon = -BL\, dx/dt = -BLv$, where v is the speed of the loop. The emf is proportional to the speed.



Main Text
Study Guide Menu

31-6 Induced Electric Fields

An electric field is always associated with a changing magnetic field and this electric field is responsible for the induced emf. The relationship between the electric field **E** and the emf around a closed loop is given by the integral

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s},$$

where d**s** is an infinitesimal displacement vector. The integral is zero for a conservative field, such as the electrostatic field produced by charges at rest. The electric field induced by a changing magnetic field, however, is non-conservative and the integral is not zero. For a changing magnetic field, Faraday's law becomes

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi}{dt}.$$

Suppose a cylindrical region of space contains a uniform magnetic field, directed along the axis of the cylinder, and that the field is zero outside the region. If the magnetic field changes with time, the lines of the electric field it induces are circles, concentric with the cylinder.

Main Text Study Guide Menu

31-7 Inductors and Inductance

Current in a circuit produces a magnetic field and magnetic flux through the circuit. If the circuit consists of N turns and the flux is the same through all of them, then its **inductance** L is defined by

$$L = \frac{N\Phi_B}{i},$$

where i is the current that produces flux Φ_B through each turn. Since Φ_B is proportional to i, L does not depend on the current or flux. It does depend on the geometry of the circuit.

The SI unit of inductance is called the henry (abbreviated H). The quantity $N\Phi$ is called the **flux linkage**.

For an ideal solenoid of length l and cross-sectional area A, with n turns per unit length and carrying current i, the magnetic field in the interior is given by $B = \mu_0 in$, the flux through each turn is given by $\Phi_B = BA = \mu_0 inA$, the flux linkage is $N\Phi_B = \mu_0 in^2 Al$, and the inductance

is

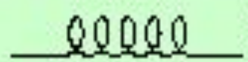
$$\mathrm{L} = nl\,\Phi_B / i = \mu_0 n^2 Al.$$

Main Text Study Guide Menu

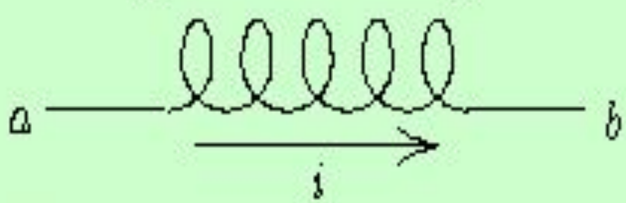
31-8 Self-Induction

- When the current in a circuit changes, the flux changes and an emf is induced in the circuit. If the circuit has inductance L, the induced emf is

$$\mathcal{E}_L = -L\,\frac{di}{dt}\,.$$

- Every circuit has an inductance, usually small, but there are electrical devices, called **inductors**, that are used expressly to add inductance to a circuit. They usually consist of a coil of wire, like a solenoid. The symbol for an inductor is  .

- The diagram below shows an inductor carrying current i, directed from *a* to *b*. The rest of the circuit is not shown. If the current is increasing, then the emf induced in the inductor is from *b* toward *a* (*a* is the positive terminal); if it is decreasing, then the emf is from *a* toward *b* (*b* is the positive terminal).



In either case, the potential V_b at point b is given by
$V_b = V_a - L\, di/dt$, where V_a is potential at point *a* and L is the inductance.

Main Text
HISTORY

31-9 *RL* Circuits

Consider a circuit consisting of an inductor L, a resistor R, and an emf ε. Take the current to be positive in the direction of the emf. The loop rule then gives $\varepsilon - iR - L\,di/dt = 0$. Assume the source of emf ε is connected to the circuit at time $t = 0$, at which time the current is 0. The solution to the loop equation is then

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right),$$

where τ = L/R is the **inductive time constant**. This expression predicts $i = 0$ at t=0, but di/dt is not zero then. It also predicts that long after the emf is connected, the current is $i = \varepsilon/R$, as if the inductor were not present. The rate at which the current increases to its final value is controlled by the inductive time constant. The larger the time constant, the slower the rate.

The potential difference across the inductor is given by

$$V_L(t) = L\frac{di}{dt} = \mathcal{E}\,e^{-t/\tau_L}$$

and the potential difference across the resistor is given by

$$V_R(t) = iR = \mathcal{E}\left(1 - e^{-t/\tau_L}\right).$$

Main Text Study Guide Menu

31-10 Energy Stored in a Magnetic Field

- Energy must be supplied to build up the magnetic field in an inductor, perhaps by a source of emf. The energy may be considered to be stored in the magnetic field and can be retrieved when the current and field decrease. If current i is in an inductor with inductance L, the energy stored is given by $U_B = \frac{1}{2} L i^2$.

- Multiply the loop equation for a series LR circuit by the current to obtain $\varepsilon = i^2 R + Li\, di/dt$. The quantity on the left is the rate with which the emf device is supplying energy to the circuit.

  The first term on the right is the rate with which energy is dissipated in the resistor. The second term on the right is the rate with which energy is being stored in the magnetic field of the inductor.

Main Text

Study Guide Menu

31-11 Energy Density of a Magnetic Field

The inductance of a solenoid with cross-sectional area A, length l, and n turns per unit length is $L = \mu_0 n^2 Al$ and the energy stored is $U_B = \frac{1}{2} Li^2 = \frac{1}{2}\mu_0 n^2 Al\, i^2$, where i is the current. Since the magnetic field in the solenoid is $B = \mu_0 ni$, this can be written $U_B = B^2 Al/2\mu_0$. Al is the volume of the solenoid, so the energy density (energy per unit volume) is

$$u_B = \frac{B^2}{2\mu_0}.$$

This expression gives the energy density stored in *any* magnetic field, not just the field of a solenoid. The total energy stored in a field is the volume integral of the energy density.

Main Text
Study Guide Menu
HISTORY

31-12 Mutual Induction

- The current in one circuit influences the current in another, although the two are not physically connected. The current in the first circuit produces a magnetic field at all points in space and thus is responsible for a magnetic flux through the second circuit. When the current in the first circuit changes, the flux through the second circuit changes and an emf is induced in that circuit.

- If the second circuit has N_2 turns and Φ_{21} is the flux produced through it by the current i_1 in the first circuit, then

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$

  is the **mutual inductance** of circuit 2 with respect to circuit 1. Similarly, the mutual inductance of circuit 1 with respect to circuit 2 is $M_{12} = N_1 \Phi_{12} / i_2$, where N_1 is the number of turns in circuit 1, i_2 is the current in circuit 2, and Φ_{12} is the flux through circuit 1 produced by the current in circuit 2. The two mutual inductances M_{12} and M_{21} are always equal and the subscripts are not needed.

- When the current i^1 in circuit 1 changes at the rate di^1/dt, the emf induced in circuit 2 is $\varepsilon^2 = -M\, di^1 / dt$ and when the current i^2 in circuit 2 changes at the rate di^2 / dt, the emf induced in circuit 1 is $E^1 = -M\, di^2 / dt$.

Main Text
Study Guide Menu

32-1 Magnets

- The magnetic dipole is the simplest known magnetic structure. Magnetism arises from the dipole moments of electrons in materials, associated with their spins and orbital motions.

- Magnetic field lines exit a bar magnet from the end called the north pole and enter the end called the south pole. The lines continue through the interior of the magnet to form closed loops.

- The magnetic field in the exterior of a permanent bar magnet can be closely approximated by the field that would be produced by a positive monopole (a north pole) at one end and a negative monopole (a south pole) at the other but the field does not actually arise from single monopoles but rather from magnetic dipoles associated with electron motion. As proof that the field is not due to monopoles, the magnet can be cut in half with the result that each half produces a field that is closely approximated as a dipole field. This process can be continued to the atomic level with the same result.

Main Text Study Guide Menu

32-2 Gauss' Law for Magnetic Fields

- If the normal component of the magnetic field is integrated over a *closed* surface (one that completely surrounds a volume), the result is zero no matter what closed surface is chosen. That is,

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0.$$

for every closed surface. The direction of the infinitesimal element of area d**A** is normal to the surface. This is Gauss' law for magnetism.

- The law does not necessarily mean that the magnetic field is zero at any point on the surface, only that the total magnetic flux through any closed surface is zero. Usually the field is essentially outward over some portions and essentially inward over others. Every magnetic field line that enters any region also leaves that region: no field lines start or stop anywhere; they are closed curves.

Main Text
Study Guide Menu

32-4 Magnetism and Electrons

Every electron has an intrinsic angular momentum, often called its spin angular momentum or simply its spin. Only one component, usually taken to be the z component, can be measured. It is

$$S_z = \pm\frac{h}{4\pi} = \pm 5.2729 \times 10^{-35}\,\mathrm{J \cdot s},$$

where h is the Planck constant. A magnetic dipole moment is associated with spin and its z component is

$$\mu_{S,z} = -\frac{e}{m}S_z = \mp\frac{eh}{4\pi m} = \mp 9.27 \times 10^{-24}\,\mathrm{J/T},$$

where m is the electron mass. Since an electron is negatively charged, its dipole moment and spin angular momentum are in opposite directions.

Particle and atomic magnetic moments are often measured in units of the **Bohr magneton** μ_B : $\mu_B = eh/4\pi m$ This is the magnitude of the electron spin dipole moment.

Main Text

Study Guide Menu

32-5 Magnetic Materials

- Three types of magnetic materials (diamagnetic, paramagnetic, and ferromagnetic) are discussed in the following sections. All atoms are diamagnetic. A dipole moment is induced by an external field. Some atoms are paramagnetic. They have permanent dipole moments but these are randomly oriented in the absence of an external field. When an external field is turned on they tend to align with the field. Ferromagnetic atoms also have permanent dipole moments, but they align with each other even in the absence of an external field.

- Paramagnetism and ferromagnetism, when they occur, are much stronger than diamagnetism.

32-6 Diamagnetism

- In the absence of an applied field, the atoms of a diamagnetic substance have no magnetic dipole moments, but moments are induced when a field is applied. As an external field is turned on, the orbits of the electrons change so the electrons produce an opposing field, in accordance with Faraday's law. The direction of the magnetization (and the induced dipole moments, on average) is opposite that of the local magnetic field, so the total magnetic field in a diamagnetic material is less in magnitude than the applied field. For most diamagnetic materials, the dipole field is extremely weak.

- The effect occurs for all materials, but if the atoms have permanent dipole moments, the effect of their alignment with the field dominates and the material is paramagnetic rather than diamagnetic.

- If the external field is not uniform, a diamagnetic material is repelled from a region of greater field toward a region of lesser field.

Main Text Study Guide Menu

32-7 Paramagnetism

- The **magnetization** of a uniformly magnetized object is its magnetic moment per unit volume. The SI unit of magnetization is **A**/m. If the substance is not uniformly magnetized, the magnetization at a point in the object is the limiting value of the dipole moment per unit volume as the volume shrinks to the point.

- Atoms of paramagnetic materials have permanent dipole moments: the dipole moments of the electrons in one of these atoms do not sum to zero. When no external magnetic field is applied, however, the magnetization of the material is zero because the atomic dipole moments are randomly oriented. An external field aligns the atomic moments, and the substance becomes magnetized. When the applied field is removed, the magnetization is quickly reduced to zero by atomic oscillations, which randomize the moments.

- When an external magnetic field is applied, the direction of the field produced by dipoles of the material is in the same direction as the applied field, so the total field in the material is greater in magnitude than the applied field.

- According to **Curie's law**, the magnetization at any point in a paramagnetic substance is directly proportional to the magnetic field B at that point and inversely proportional to the absolute temperature T:

$$M = C\left(\frac{B}{T}\right),$$

Main Text Study Guide Menu

32-8 Ferromagnetism

- Atoms of ferromagnetic materials have permanent dipole moments but unlike the atomic moments of paramagnetic materials, they *spontaneously* align with each other. Iron is the best known ferromagnetic material.

- The spontaneous alignment of dipoles in a ferromagnet is *not* due to the magnetic torque exerted by one magnetic dipole on another. These torques are not sufficiently strong to overcome thermal agitation that tends to randomize the dipole orientations. The source of dipole alignment is in the quantum mechanics of electrons in solids.

- For temperatures above its **Curie temperature**, a ferromagnetic substance is paramagnetic. For iron, this temperature is 1043 K.

- Ferromagnetic materials exhibit **hysteresis**. This may be demonstrated by plotting the magnetic field B_M, due to the atomic dipoles, as a function of the applied field B_0. See Fig.32-14. If the substance starts in the unmagnetized state and the external field is increased from zero, B is nearly linear in B_M at first but then at higher applied fields, the slope becomes less. Eventually the magnetization becomes saturated and B_M is constant. When the applied field is reduced, B does not follow the same curve downward and, in fact, when $B_0 = 0$, B_M is not zero. A residual magnetism remains: the material is magnetized even though there is no external field.

Main Text
Study Guide Menu

32-9 Induced Magnetic Fields

A changing electric field produces a magnetic field. The relationship is

$$\oint \mathrm{B} \cdot \mathrm{d}\mathrm{s} = \mu_0 \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}.$$

The left side is a path integral around a closed path. Φ_E is the electric flux through the area bounded by the path. In this form, the equation is called the **Maxwell law of induction**. A sign convention is associated with the Maxwell induction law. First, pick the direction of d**A** to be used to compute the electric flux. It is usually chosen to make the flux positive. Then, point the thumb of your right hand in the direction of d**A**; your fingers then curl around the boundary in the direction of d**s**.

Main Text Study Guide Menu

32-10 Displacement Current

- According to the Ampere-Maxwell law, a changing electric field in a region produces a magnetic field around the boundary of the region, just as if a current passed through the region. In fact, the quantity

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

is called a **displacement current**. A displacement current is emphatically NOT a true current, which consists of moving charges.

- For a cylindrical region of radius R containing a uniform changing electric field along its axis, the displacement current through a circular loop of radius r is $i_d = \varepsilon_0 \pi r^2 dE / dt$ if $r<R$ and is $i_d = \varepsilon_0 \pi R^2 dE / dt$ is $r>R$. The direction of the displacement current is the same as the direction of the field if E is increasing and opposite the direction of the field if E is decreasing.

- The equations for the magnetic field associated with the cylindrical region considered above are just like the equations for the magnetic field of a long straight wire, developed in Chapter 30, but the true current i is replaced by the displacement current i_d. That is, the field inside the cylinder is $B = \mu_0 i_d (r^2/R^2)/2\pi r$ and the field outside the cylinder is $B = \mu_0 i_d d / 2\pi r$, where i_d is the total displacement current in the cylinder.

Main Text Study Guide Menu

32-11 Maxwell's Equations

Here is list of Maxwell's equations, including the displacement current term in the Ampere-Maxwell law:

Gauss' law for electricity: $$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q}{\epsilon_0}.$$

Gauss' law for magnetism: $$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0.$$

Faraday's law of induction: $$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}.$$

Ampere-Maxwell law: $$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}.$$