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Scale of notation, or number system notation

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Positional, or radix notation

$$383 = 3*100+8*10+3*1 = 3*10^2+8*10^1+3*10^0$$

Position of hundreds

Position of tens

Position of units

So, in decimal positional system we use 10 Arabic digits and write the number using powers of the number 10 which is called "radix" of the system.

General positional scheme

Theorem. For any integer **p>=2**, any positive integer number **N** may be represented in the form

$$N = \beta_k p^k + \beta_{k-1} p^{k-1} + \ldots + \beta_2 p^2 + \beta_1 p^1 + \beta_0 p^0 \qquad (1)$$

where coefficients $\boldsymbol{\beta_i}$ are integer and satisfy the inequalities $\mathbf{0 <= \beta_i <= p-1}$

Let's reduce the notation (1) and write the number **N** as the sequence of coefficients $\boldsymbol{\beta_i}$:

$$\mathbf{N = \beta_k \beta_{k-1} \ldots \beta_2 \beta_1 \beta_0} \qquad (2)$$

The reduced form (2) is called positional representation of the number **N** in the number system notation with radix **p**.

General positional scheme

If $p<=10$ all the coefficients $\boldsymbol{\beta}_i$ not greater than 9 and it is possible to use decimal digits from 0 to p-1 as digits in the p-radix system.

In this case we have got a simple rule to transform a number written in p-radix system to decimal one. We should just rewrite the number using the form (1) and calculate this expression.

Example.

Transform the number 423_5 to the decimal system.

$423_5 = 4*5^2 + 2*5^1 +3*5^0 = 100 + 10 + 3 = 113_{10}$

Overdecimal systems

However, if $p>10$ we may get an ambiguity in the notation (2).
Let's consider $p=16$ (hexadecimal system) and take the number 30. We can write this number as

$30 = 16+14 = 1*16^1 + 14*16^0$

and with accordance of the rule (1) $30 = 114_{16}$ .Let's fulfill the reverse transformation: from hexadecimal system to decimal.

$114_{16} = 1*16^2 + 1*16^1 + 4* 16^0 = 276$!!!

Where is the contradiction?

Only one digit may be placed in one position but we tried to put to the left position the number 14 of two digits. As a result the digit 1 which must be at the second position (on the left) has arisen at the third one and as a consequence have changed its value!

Overdecimal systems

How to overcome this contradiction?

The decision is very simple. Let's add new signs to our decimal numerals so that signs will denote numbers 10, 11, 12 and so on. As these signs the uppercase Roman letters are used, i.e.

A=10, B=11, C=12, D=13, E=14, F=15. As in the informatics hexadecimal system is the greatest one it is not necessary to continue.

So, the number $30_{10} = 1E_{16}$

Example.

$123_{12} = ?_{10}$ $AAA_{16} = ?_{10}$

Remainder method

The method of transforming a decimal number to another system with radix being different from 10 can be derived from the expression (1): it is sufficient to find coefficients of powers. However, there is a simple method for sequential computation these coefficients.

Example. $113_{10} = ?_5$

```
 113 |_5___
 10__  22 |_5__
  13   20   4
  10__  2
   3
```

Thank you for attention

- Questions,
  - Remarks,
    - Comments