

# Point Group For The students

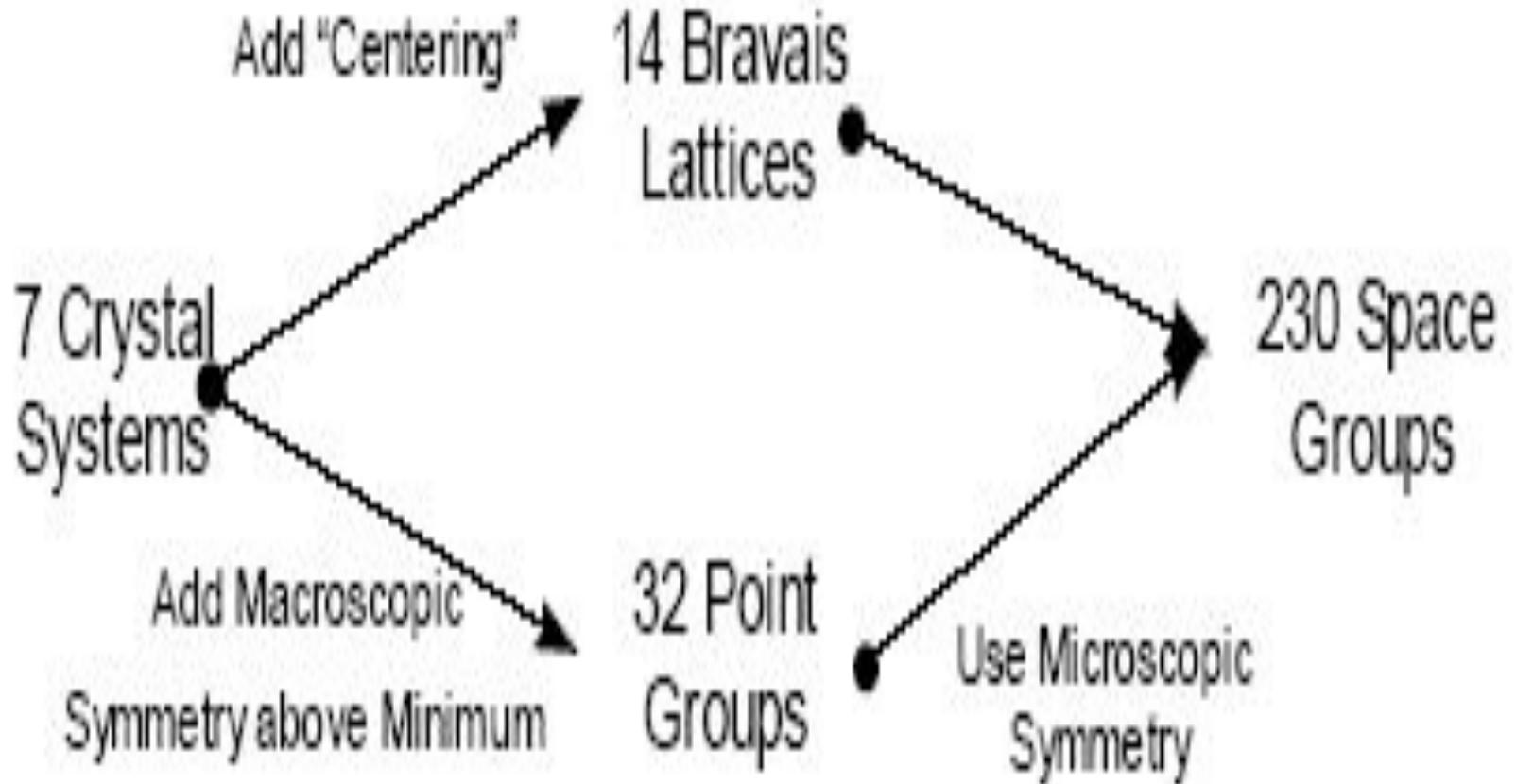
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# Symmetry Elements

- We may have the following combinations:
  - $X$  rotation axis alone
  - $\bar{X}$  inversion axis alone
  - $X/m$  rotation axis normal to a plane of symmetry
  - $Xm$  rotation axis with a vertical plane of symmetry
  - $\bar{X}m$  inversion axis with a vertical plane of symmetry
  - $X2$  rotation axis with a diad axis normal to it
  - $X/mmm$  rotation axis with both kinds of plane symmetry

# Space Group



# SCREW AXIS



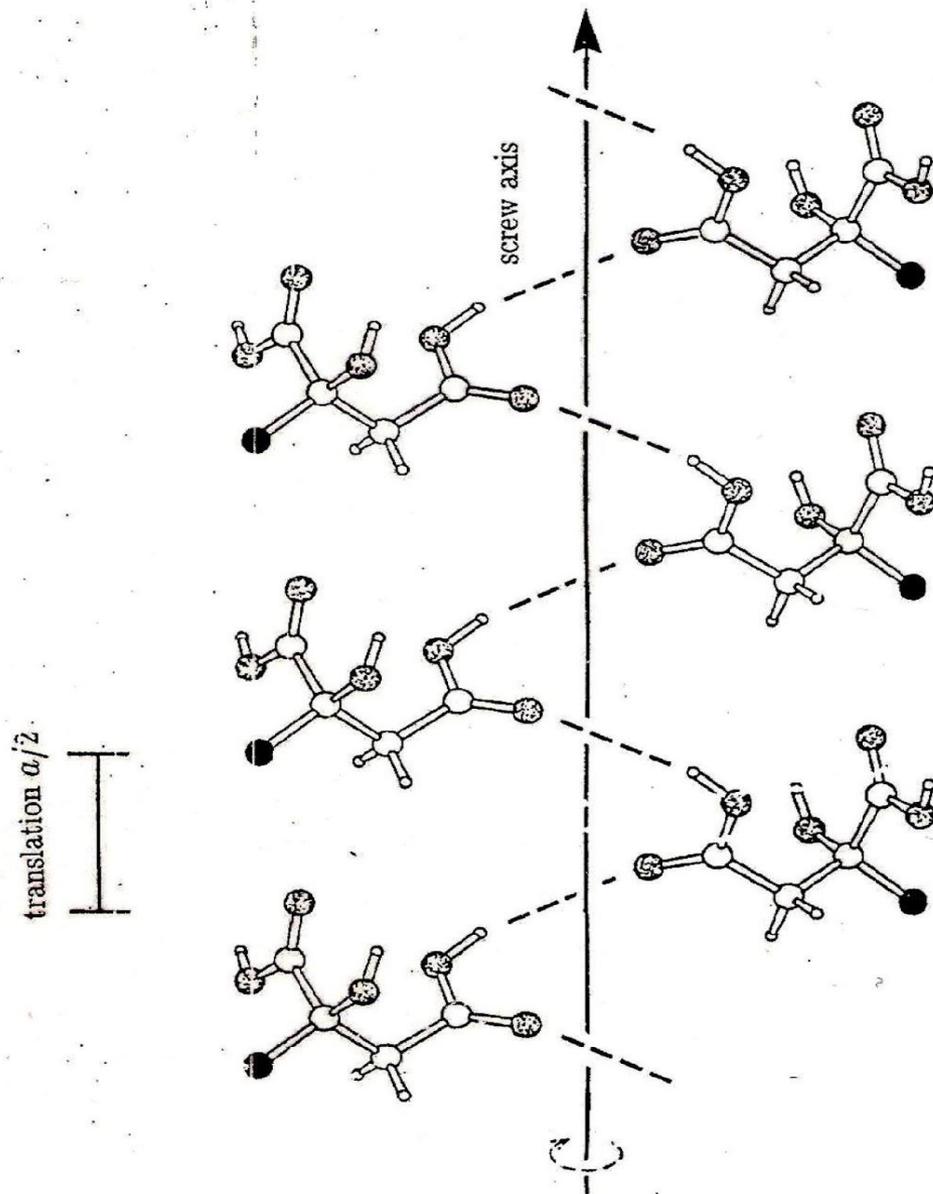
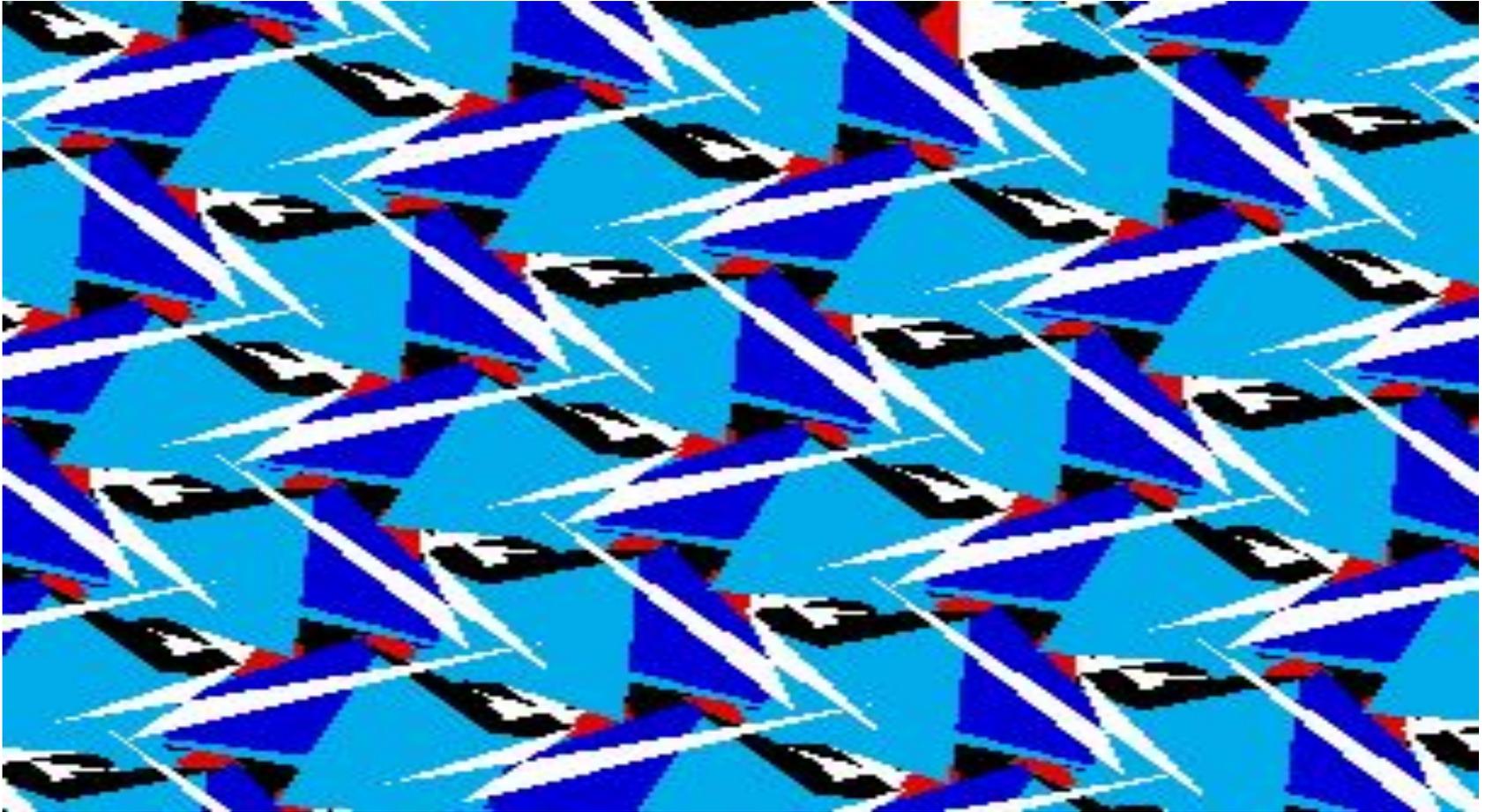


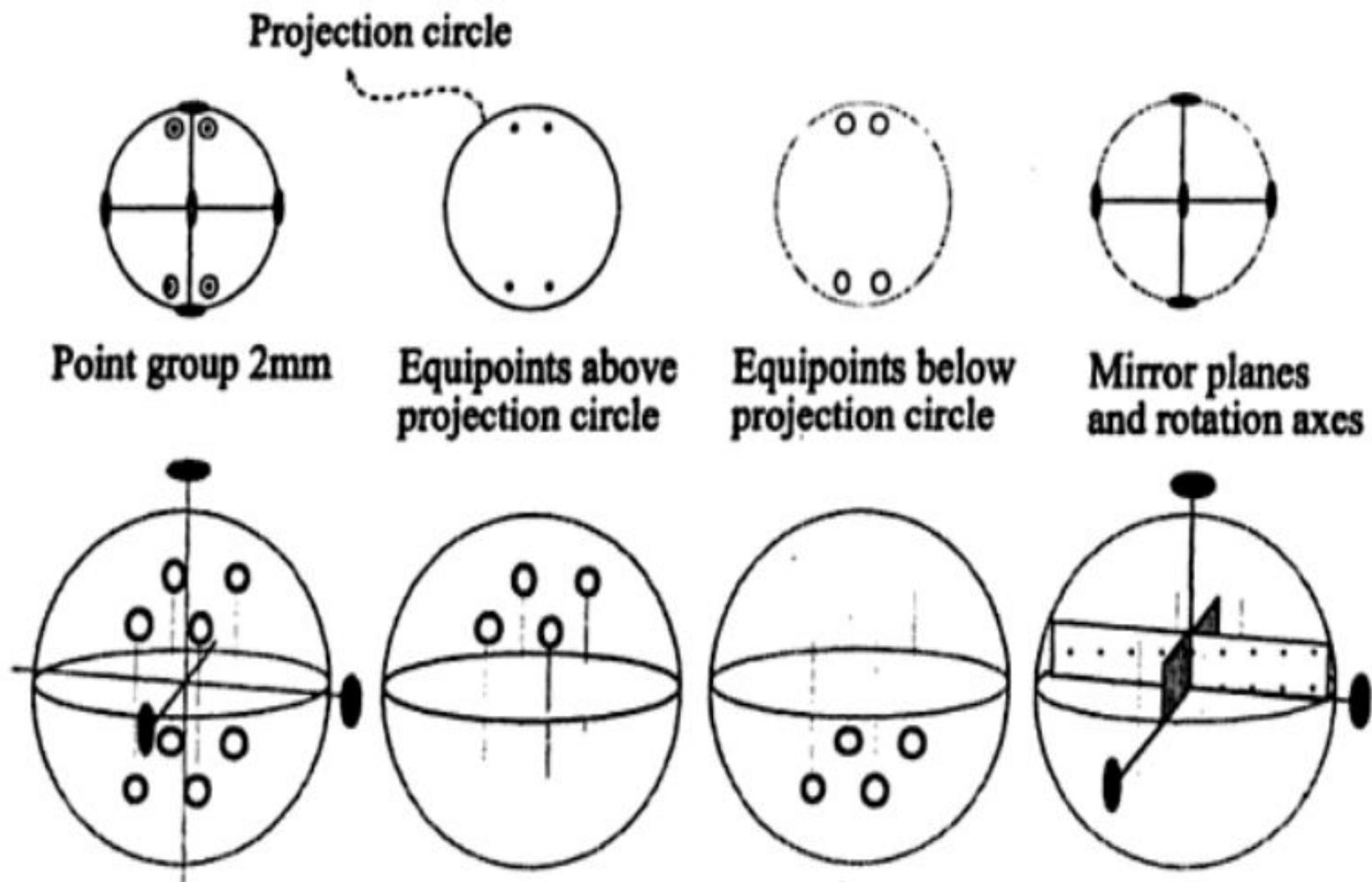
FIGURE 4.11. A twofold screw axis. One molecule is translated half a unit cell ( $a$  in this example) along the screw axis direction, and then a twofold rotation is applied to it. Two such operations give the original molecule translated to the next unit cell ( $a$  away). All molecules have the same handedness.

# Glide Plane



# The point and monoclinic space group

Crystal System	Point Group		Space Group	
	Full	Short	Full	Short
Monoclinic	m	m	Pm Pc Cm Cc	Pm Pc Cm Cc
	2	2	P2 P2 <sub>1</sub> C2	P2 P2 <sub>1</sub> C2
	$\frac{2}{m}$	$\frac{2}{m}$	P $\frac{2}{m}$ P $\frac{2_1}{m}$ P $\frac{2}{c}$ P $\frac{2_1}{c}$ C $\frac{2}{m}$ C $\frac{2}{c}$	P $\frac{2}{m}$ P $\frac{2_1}{m}$ P $\frac{2}{c}$ P $\frac{2_1}{c}$ C $\frac{2}{m}$ C $\frac{2}{c}$



**Figure 2-44** Illustration of the representation of the point group, 2mm

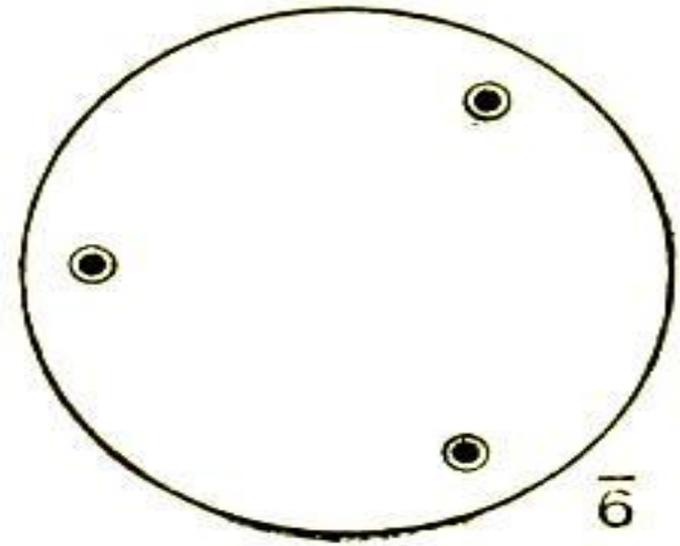
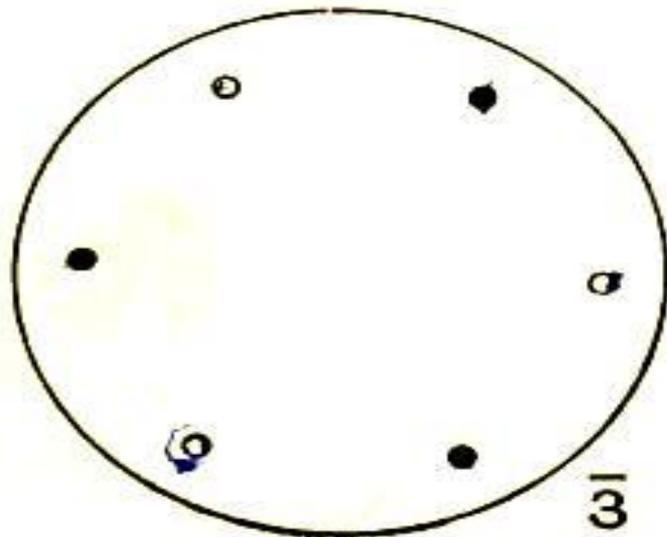
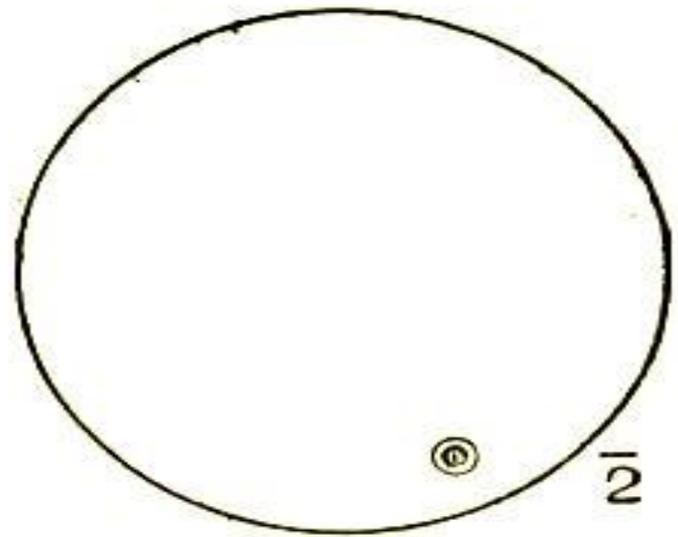
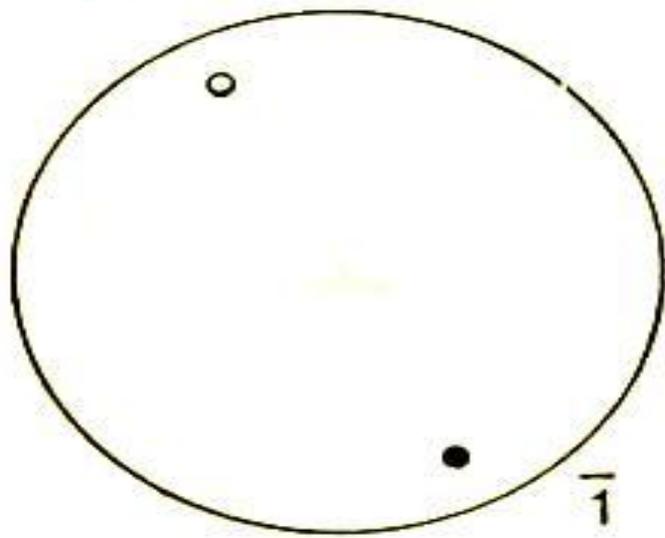


FIG. 171. Stereograms to show the operation of inversion axes  $\bar{1}$ ,  $\bar{2}$ ,  $\bar{3}$  and  $\bar{6}$ .

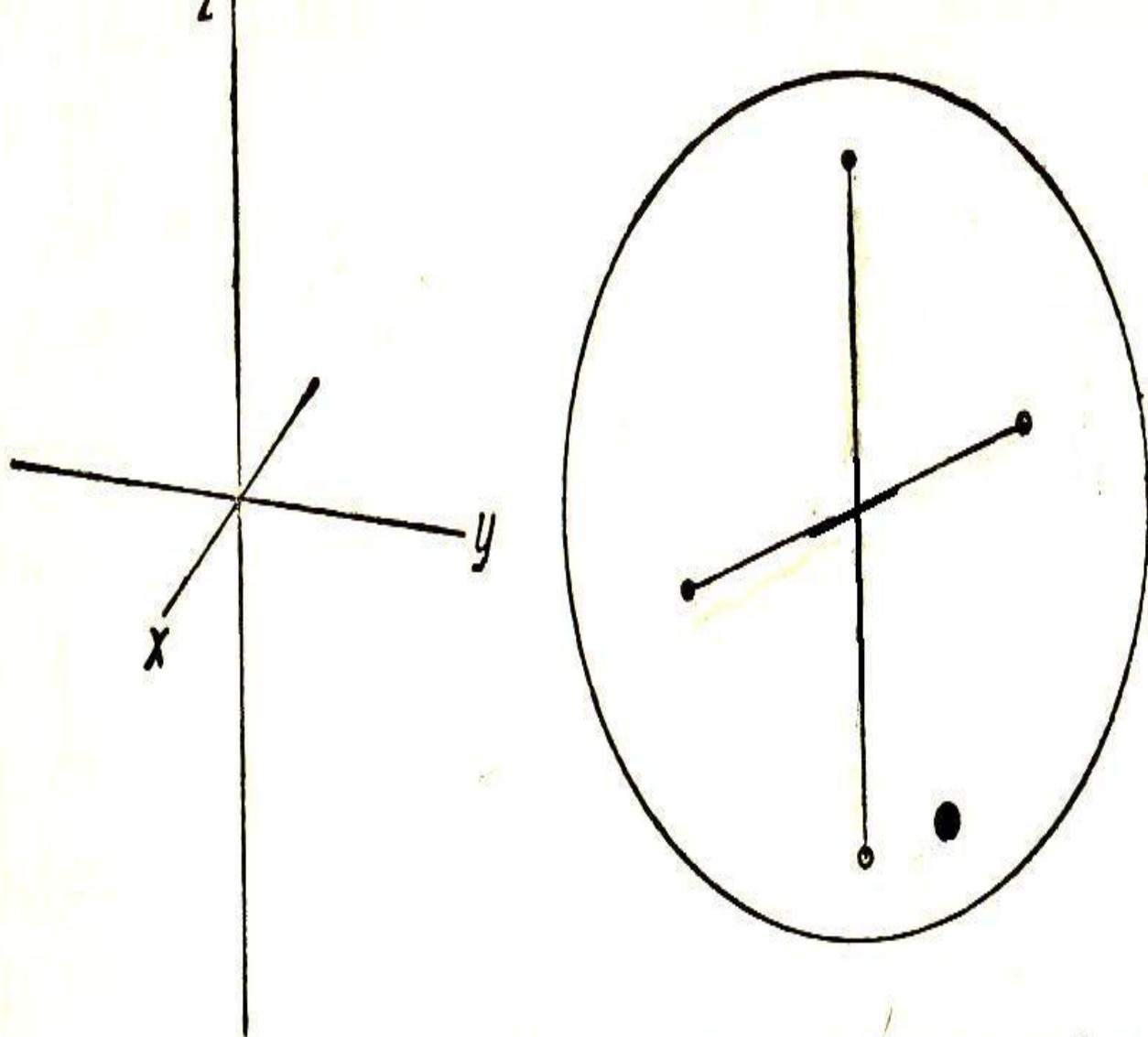


FIG. 172. Class 1; the crystallographic axes and a stereogram of the general form. (See the note to the legend of Fig. 127.)

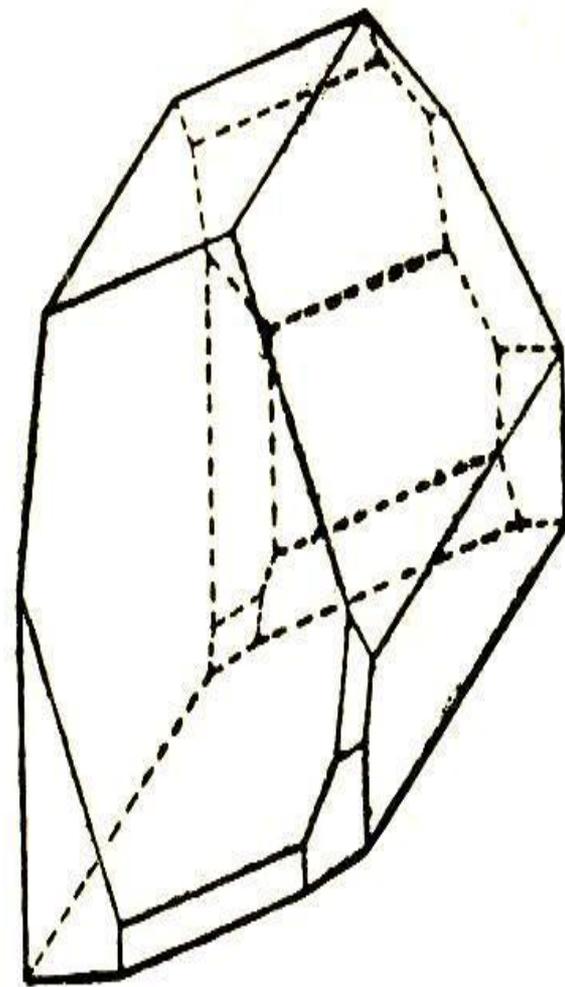
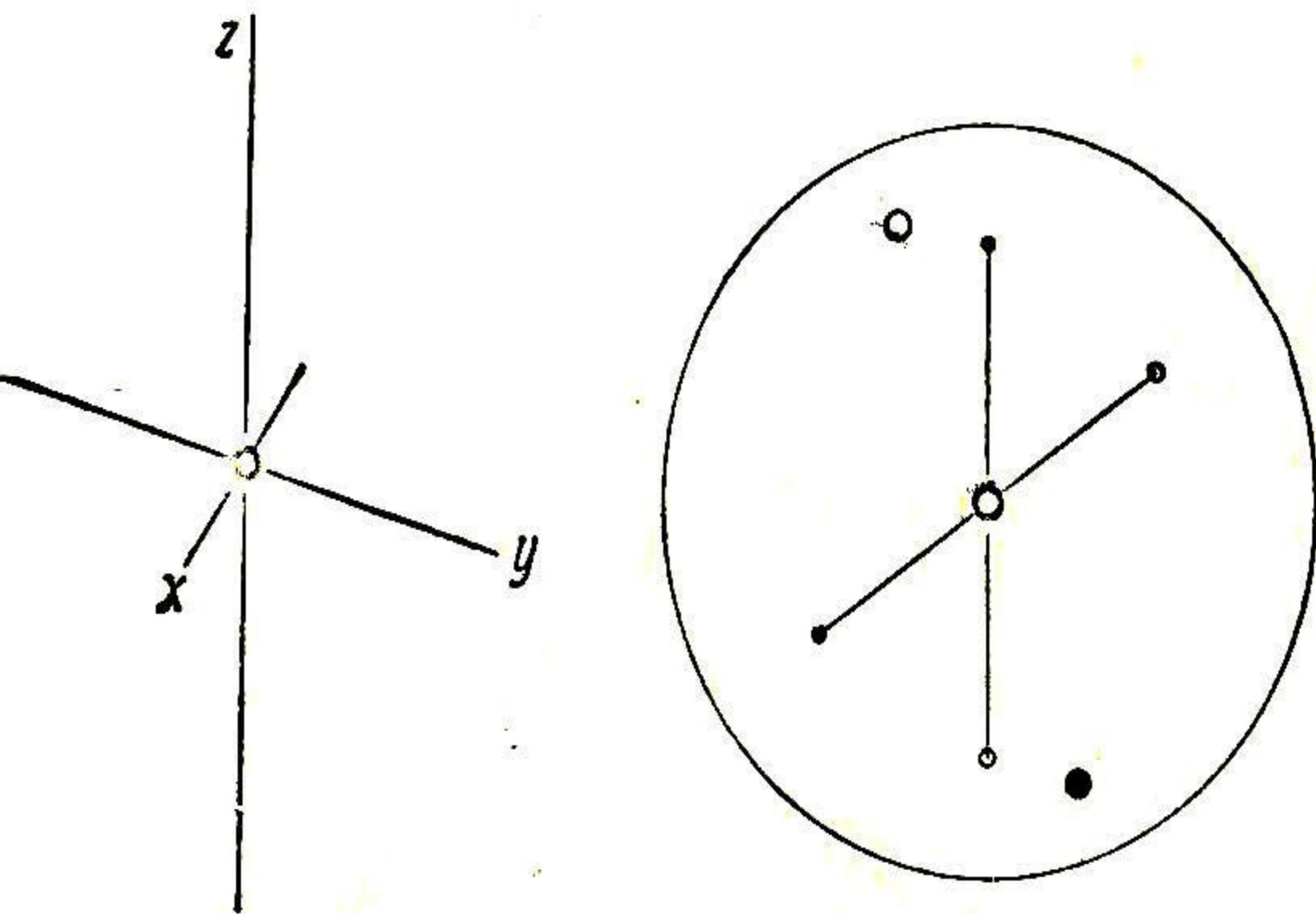
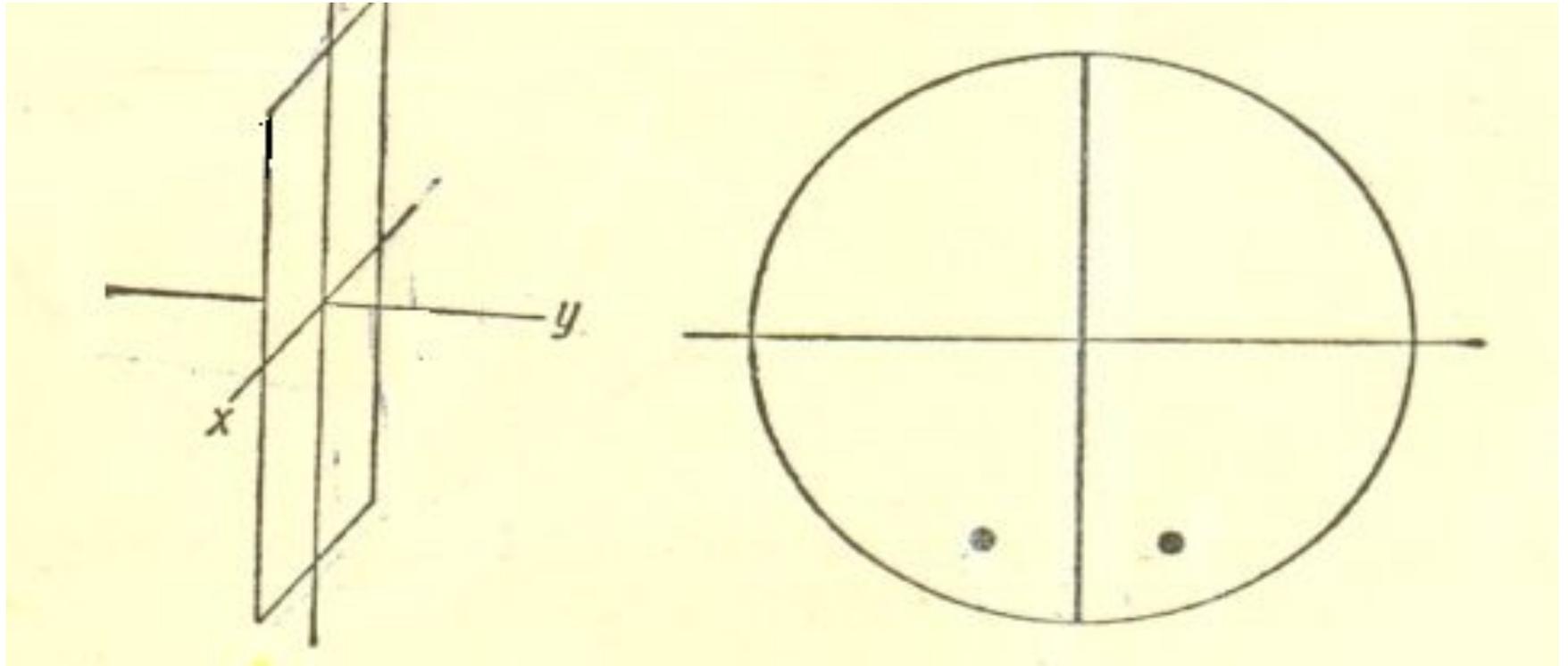


FIG. 173. A crystal of calcium thiosulphate.



Class  $\bar{1}$ ; the crystallographic axes and a stereogram of the general form. The centre of symmetry is denoted by a thick ring.



Class  $m$ ; the crystallographic axes, showing their relationship to the plane of symmetry, and a stereogram of the general form

## MONOCLINIC SYSTEM

**CLASS 2.** (Monoclinic hemimorphic, monoclinic sphenoidal.) One diad axis (always chosen as the  $y$  crystallographic axis, Fig. 182).

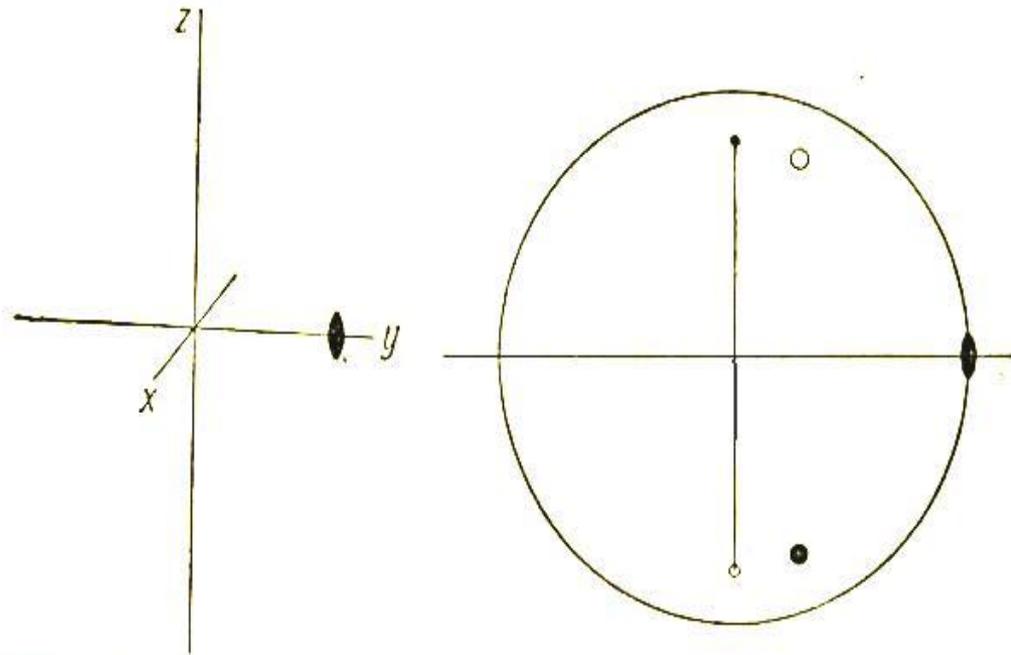


FIG. 182. Class 2; the crystallographic axes and a stereogram of the general form.

**Special forms.** Pedions  $\{010\}$ ,  $\{0\bar{1}0\}$ .

Pinacoids  $\{h\ 0\ l\}$ .

**General forms.** Sphenoids  $\{h\ k\ l\}$ .

# Orthorhombic Class $mm$

**CLASS  $mm$ .** (Orthorhombic hemimorphic, orthorhombic pyramidal.) Two planes of symmetry at right-angles, intersecting in a diad axis (always chosen as the  $z$  crystallographic axis, Fig. 191).

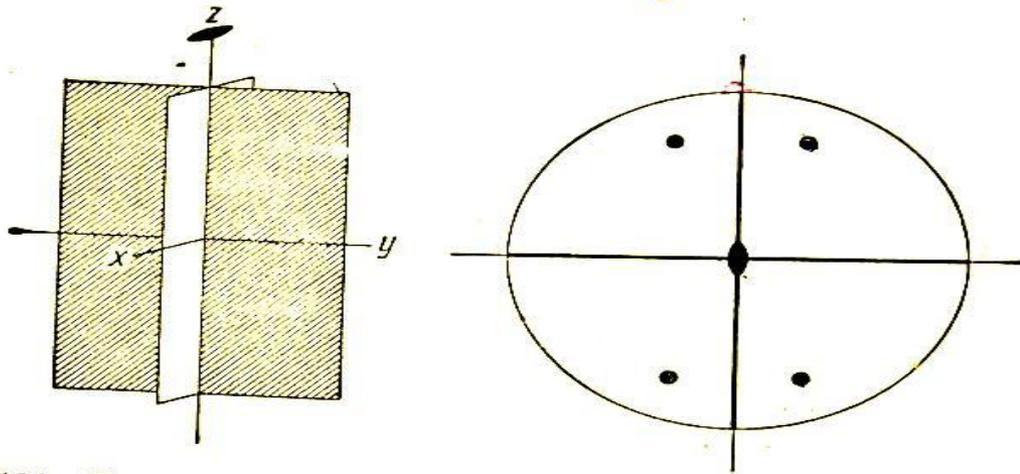
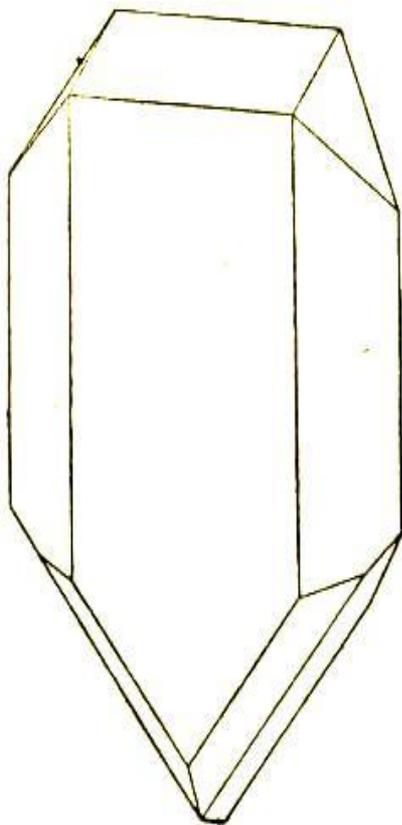
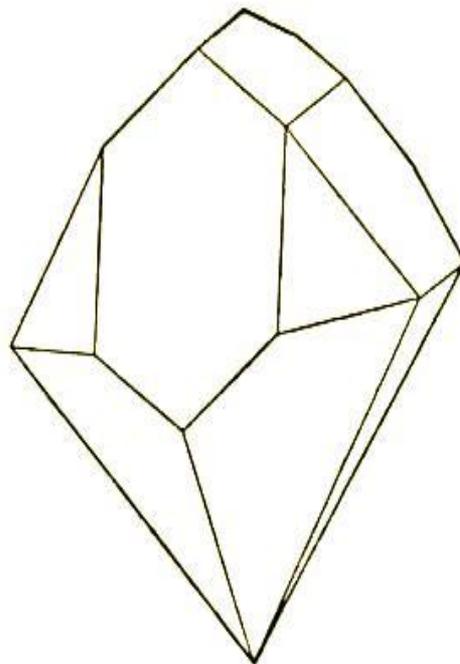


FIG. 191. Class  $mm$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form. The planes of symmetry, parallel to different pinacoids, are differently shaded.

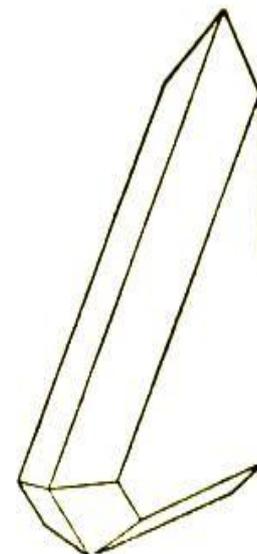
- Special forms.**
- Pedions  $\{001\}$ ,  $\{00\bar{1}\}$ .
  - Pinacoids  $\{100\}$ ,  $\{010\}$ .
  - Prisms  $\{hk0\}$ .
  - Domes  $\{h0l\}$ ,  $\{0kl\}$ .
- General forms.** Pyramids  $\{hkl\}$ .



**A crystal of  
potassium tetrathionate**



**A crystal of  
 $\text{Na}_2\text{SiO}_2 \cdot 5\text{H}_2\text{O}$**



**A crystal of  
hilgardite**

**CLASS 222.** (Orthorhombic sphenoidal.) Three mutually perpendicular diad axes (Fig. 195), always chosen as the directions of the crystallographic axes  $x$ ,  $y$ ,  $z$ .

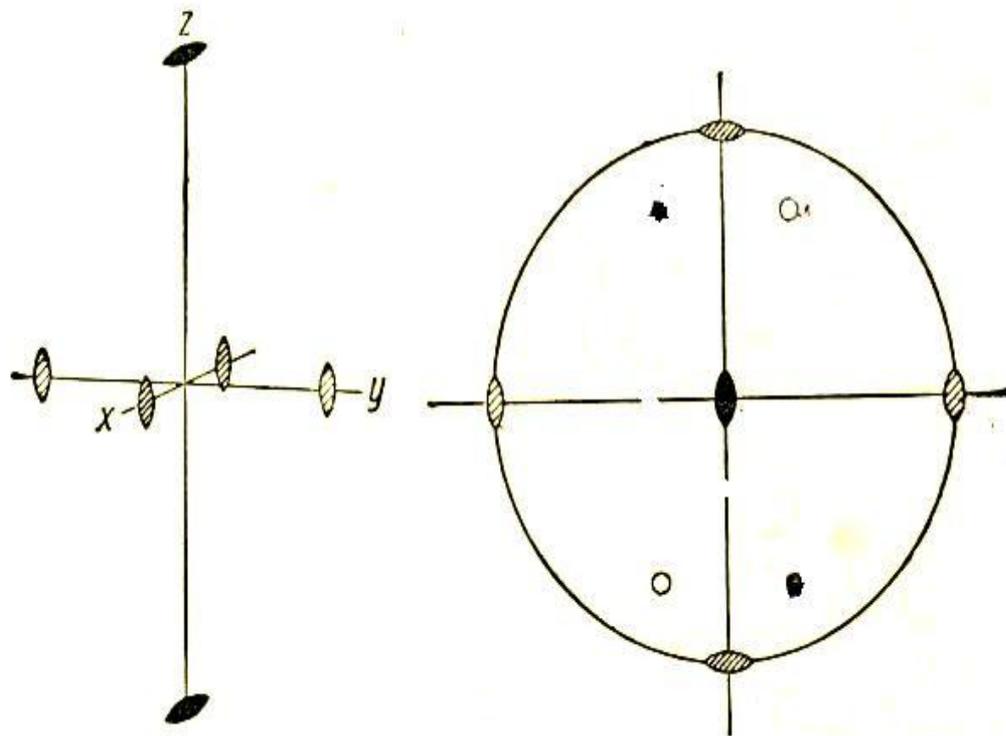


FIG. 195. Class 222; the elements of symmetry, crystallographic axes and a stereogram of the general form. The flags of the three diad axes, normal to different pinacoids, are differently shaded.

- Special forms.** Pinacoids  $\{100\}$ ,  $\{010\}$ ,  $\{001\}$ .  
Prisms  $\{hk0\}$ ,  $\{0kl\}$ ,  $\{h0l\}$ .
- General forms.** Sphenoids  $\{hkl\}$ .

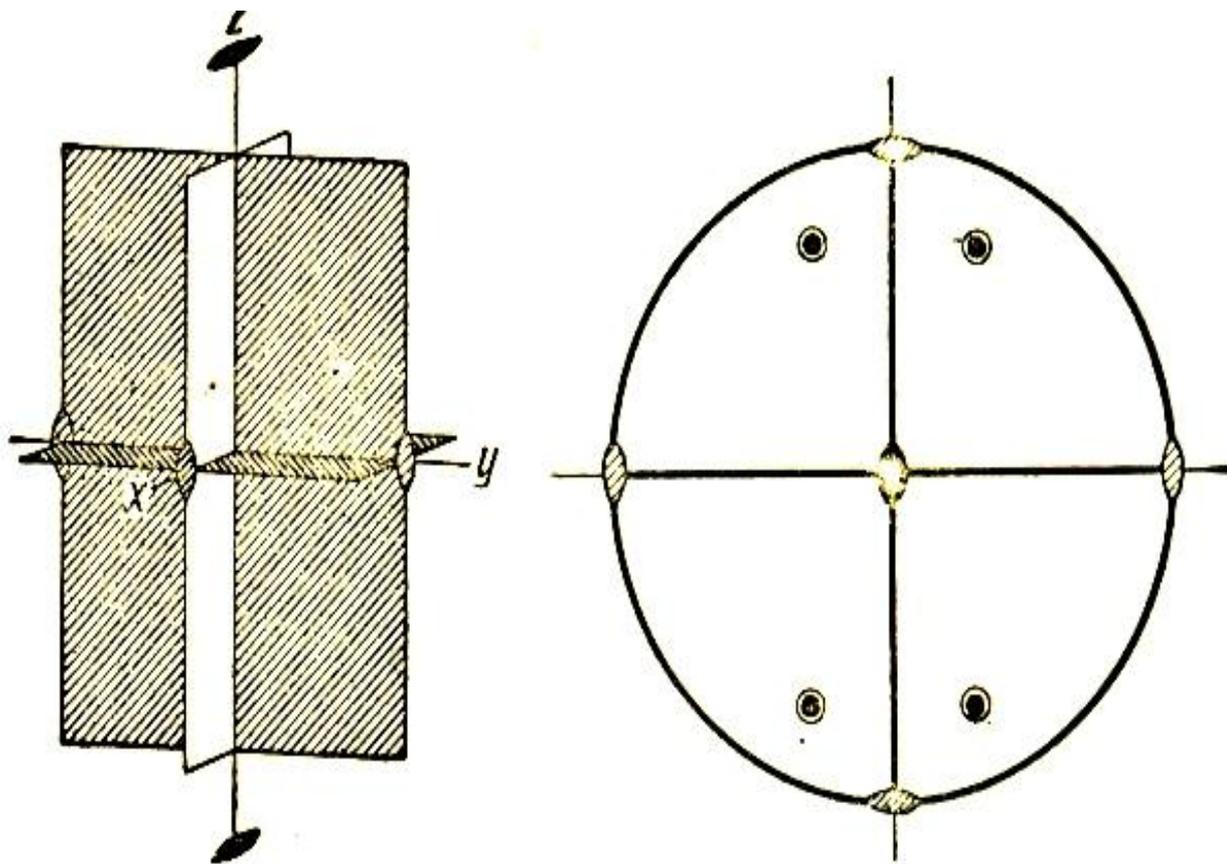


FIG. 202. Class  $mmm$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

- Special forms.** Pinacoids  $\{100\}$ ,  $\{010\}$ ,  $\{001\}$ .  
 Prisms  $\{hk0\}$ ,  $\{h0l\}$ ,  $\{0kl\}$ .
- General forms.** Bipyramids  $\{hkl\}$ .

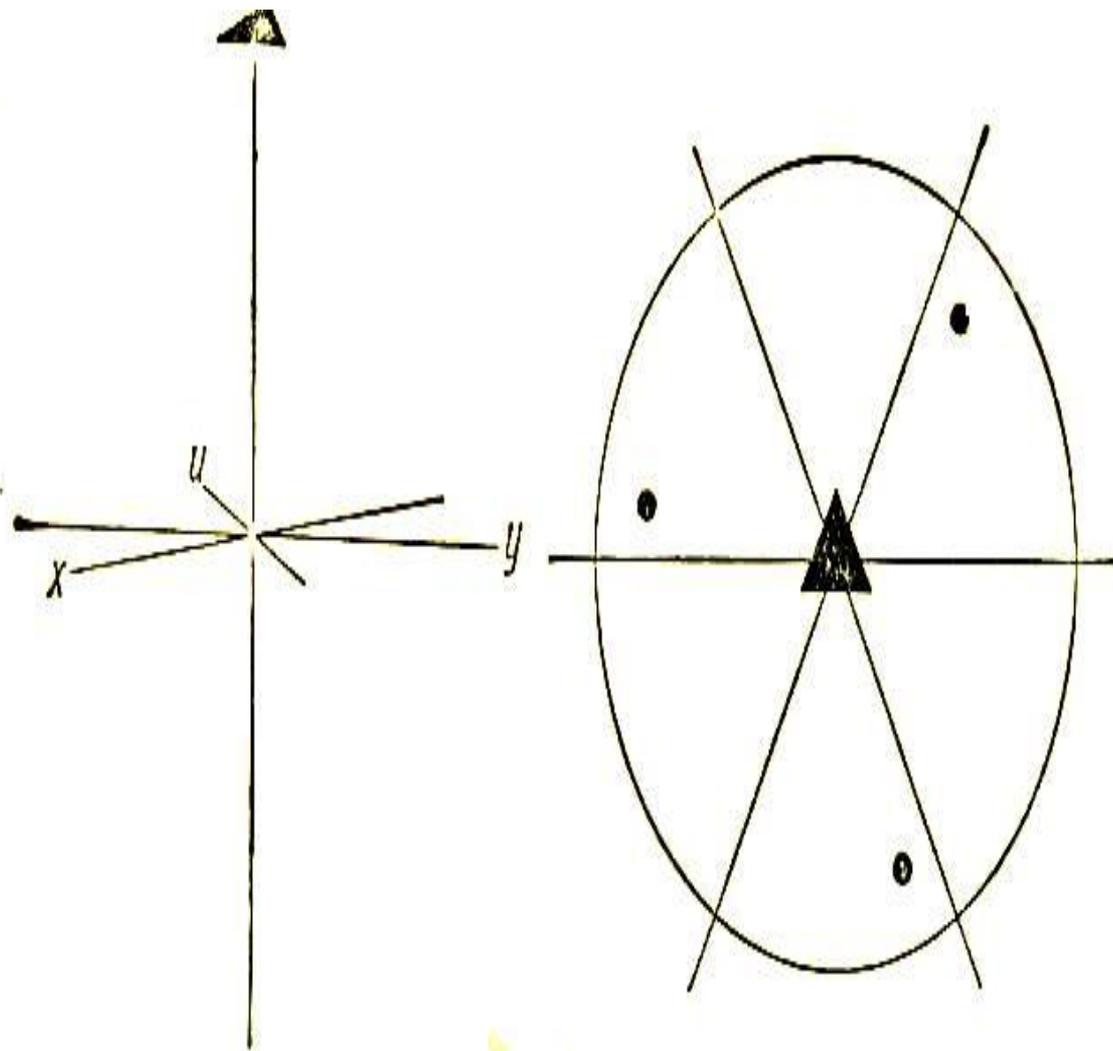


FIG. 203. Class 3; the crystallographic axes, showing their relationship to the triad axis, and a stereogram of the general form.

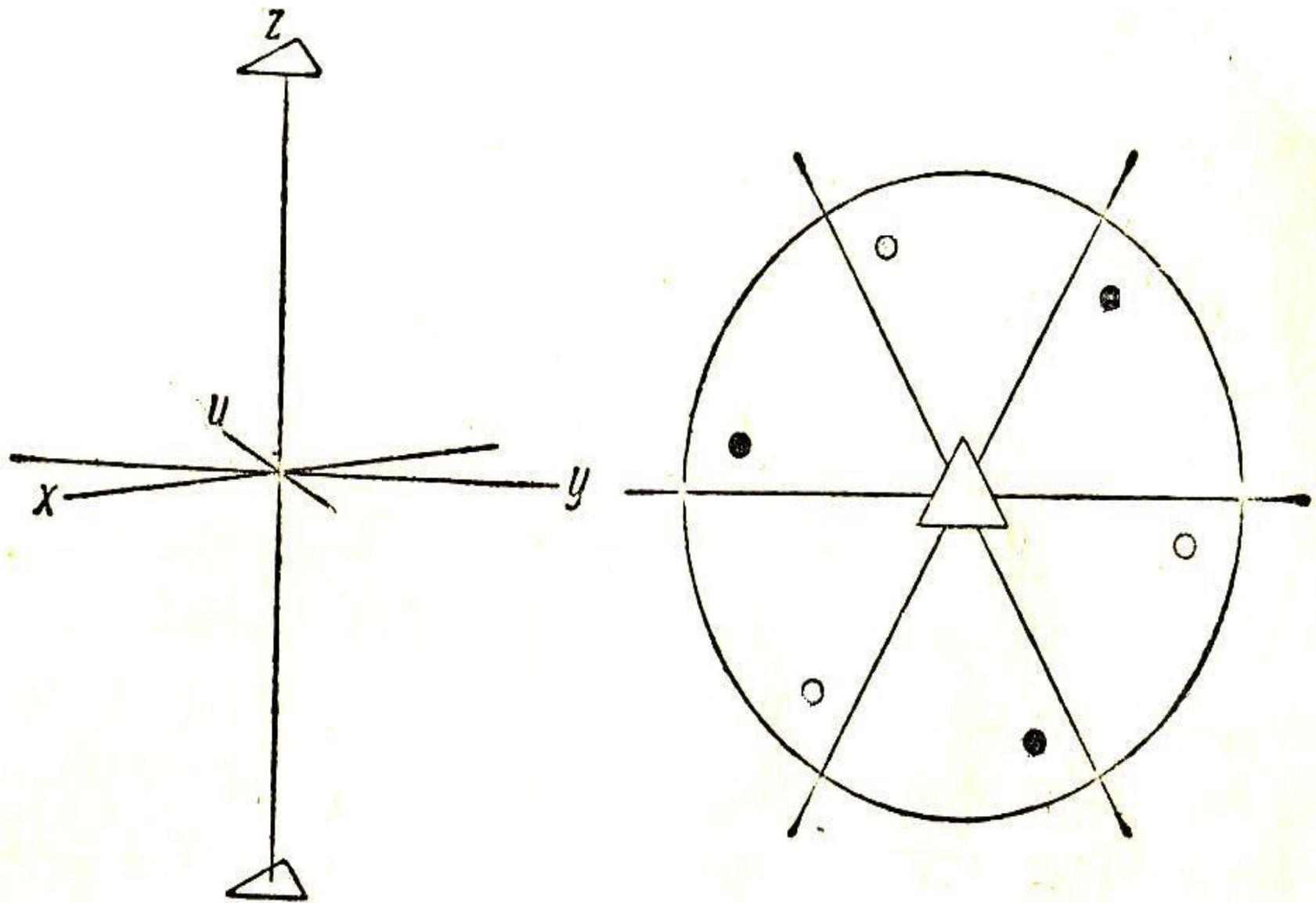


FIG. 206. Class  $\bar{3}$ ; the inversion axis, flagged with open triangles, crystallographic axes and a stereogram of the general form.

substance illustrates a combination of eight different general forms.

**CLASS 3m.** (Ditrigonal hemimorphic, ditrigonal pyramidal.) Three vertical planes of symmetry intersecting in a triad axis (Fig. 210).

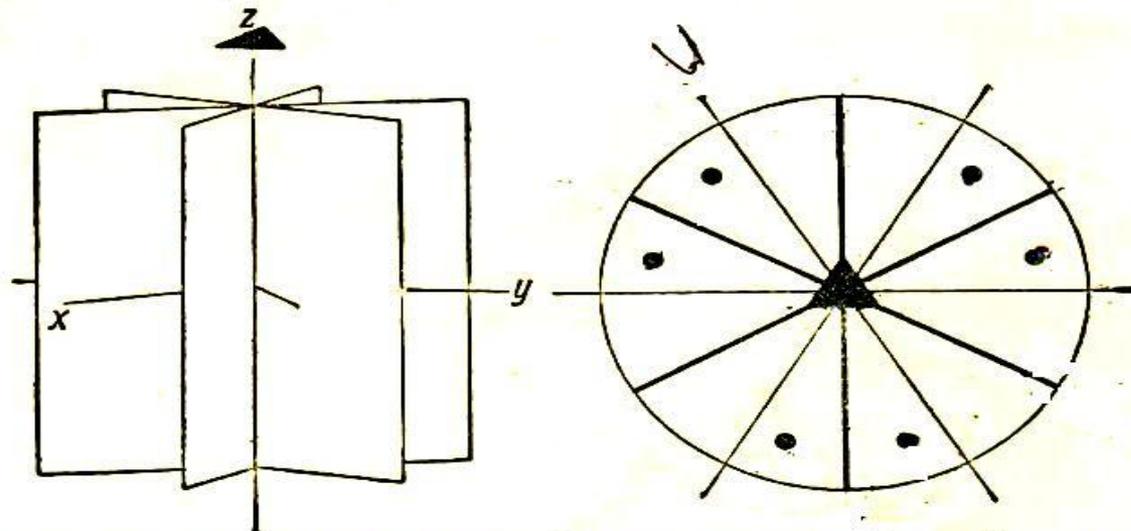


FIG. 210. Class 3m; the elements of symmetry, crystallographic axes and a stereogram of the general form.

		<i>Miller-Bravais</i>	<i>Miller</i>
<b>Special forms.</b>	Pedions	$\{0001\}, \{000\bar{1}\}.$	$\{111\}, \{\bar{1}\bar{1}\bar{1}\}.$
	Trigonal prisms	$\{10\bar{1}0\}, \{01\bar{1}0\}.$	$\{2\bar{1}\bar{1}\}, \{11\bar{2}\}.$
	Hexagonal prism	$\{11\bar{2}0\}.$	$\{10\bar{1}\}.$
	Ditrigonal prisms	$\{h k i 0\}.$	$\{p q -p + q\}.$
	Trigonal pyramids	$\{h 0 \bar{h} l\},$ $\{0 k \bar{k} l\}.$	$\{p q q\},$ $\{p p q\}.$
<b>General forms.</b>	Hexagonal pyramids	$\{h h 2\bar{h} l\}.$	$\{p q 2q - p\}.$
	Ditrigonal pyramids	$\{h k i l\}.$	$\{p q r\}.$

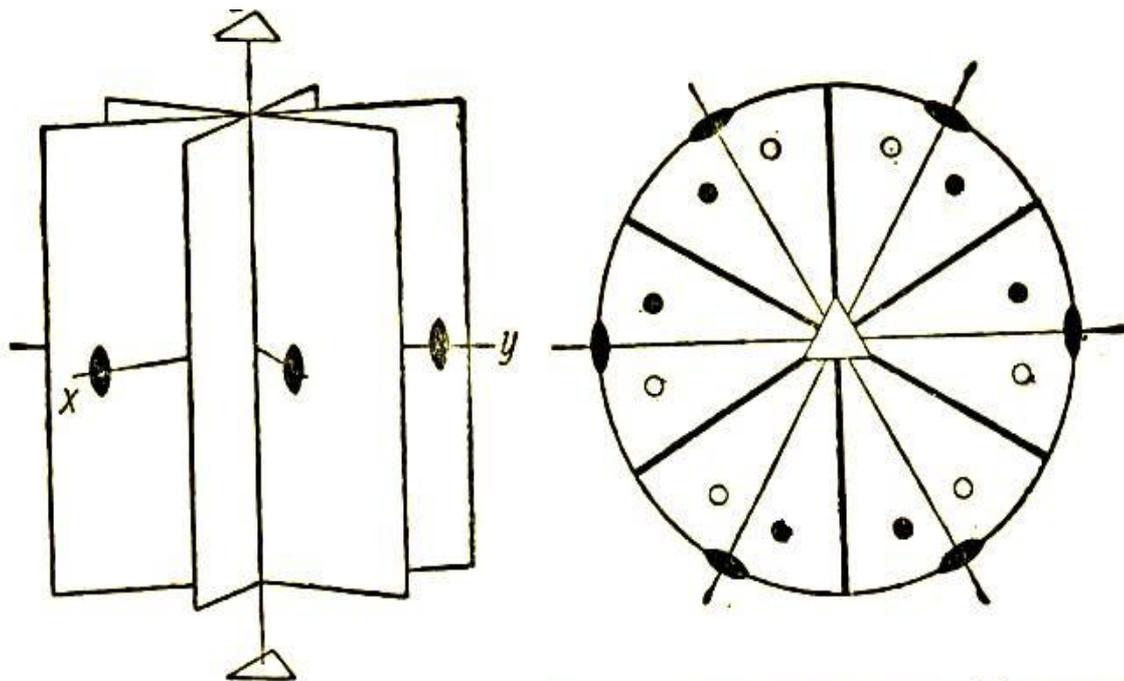


FIG. 214. Class  $\bar{3}m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

**Special forms.**

Pinacoid

Hexagonal prisms

Dihexagonal prisms

Rhombohedra

Hexagonal bipyramids

*Miller-Bravais*

$\{0001\}$ .

$\{10\bar{1}0\}$ ,  $\{11\bar{2}0\}$ .

$\{h k i 0\}$ .

$\{h 0 \bar{h} l\}$ ,  $\{0 k \bar{k} l\}$ .

$\{h h \bar{2}h l\}$ .

*Miller*

$\{111\}$ .

$\{2\bar{1}\bar{1}\}$ ,  $\{10\bar{1}\}$ .

$\{p q -p + q\}$ .

$\{p q q\}$ ,  $\{p p q\}$ .

$\{p q 2q - p\}$ .

**General Forms.**

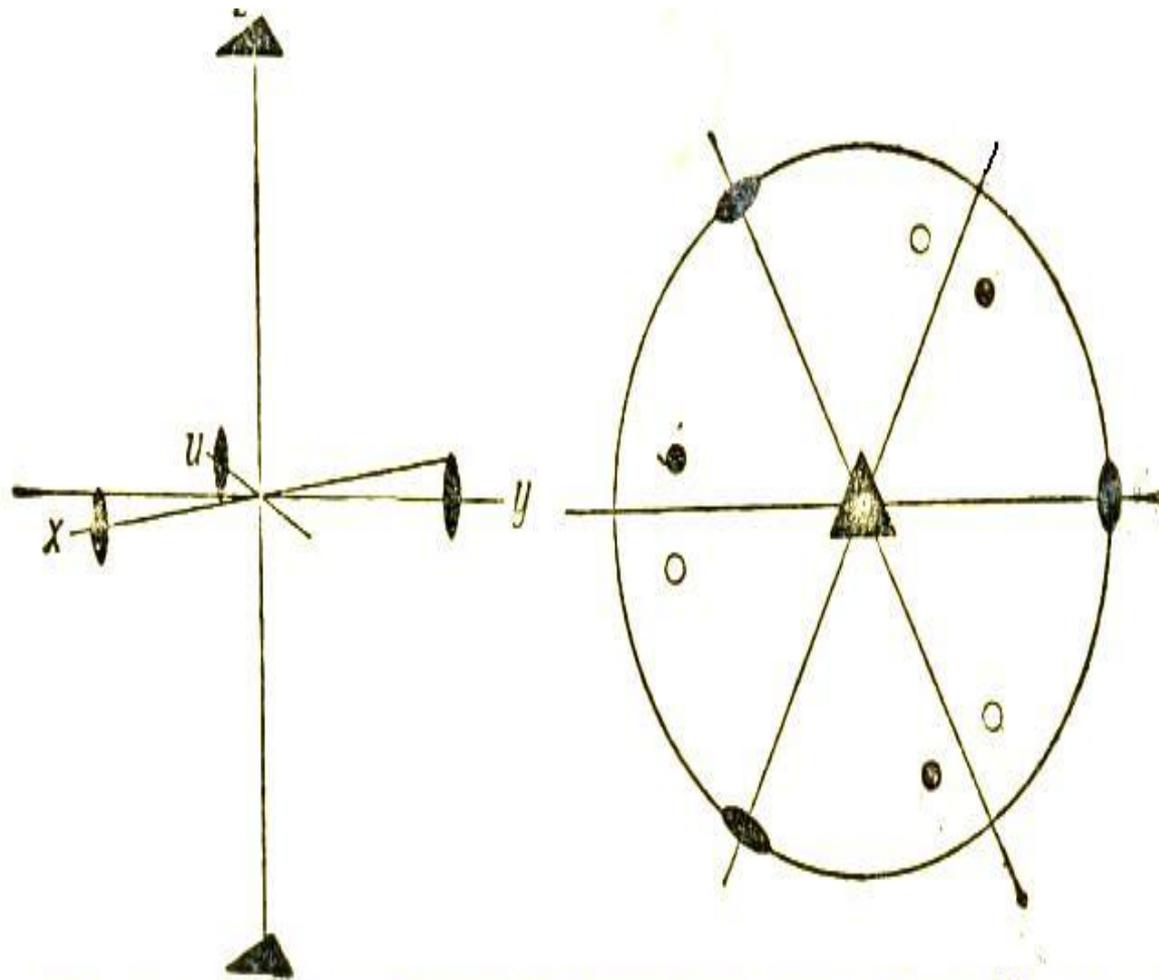
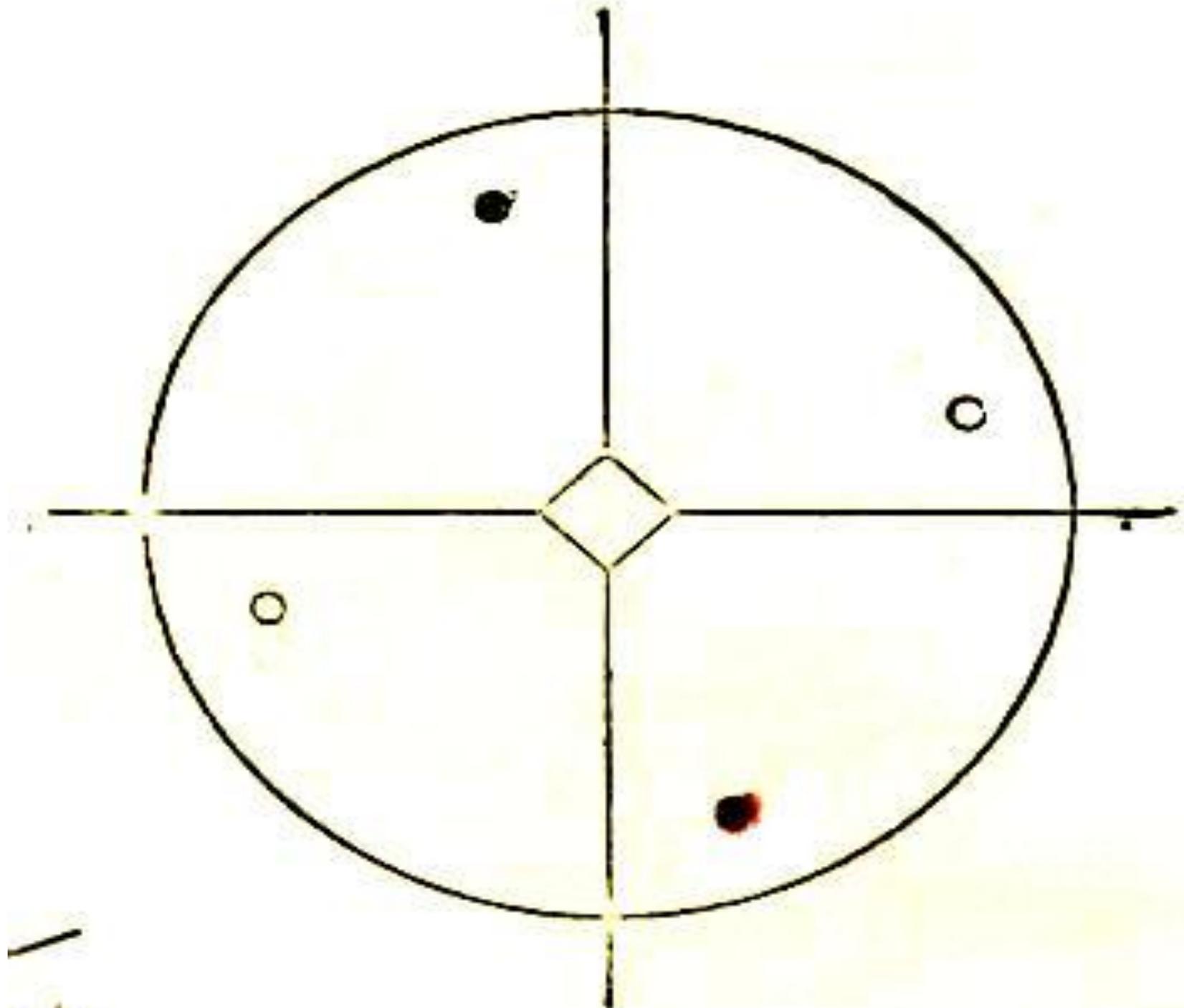


FIG. 215. Class 32; the elements of symmetry, crystallographic axes and a stereogram of the general form.



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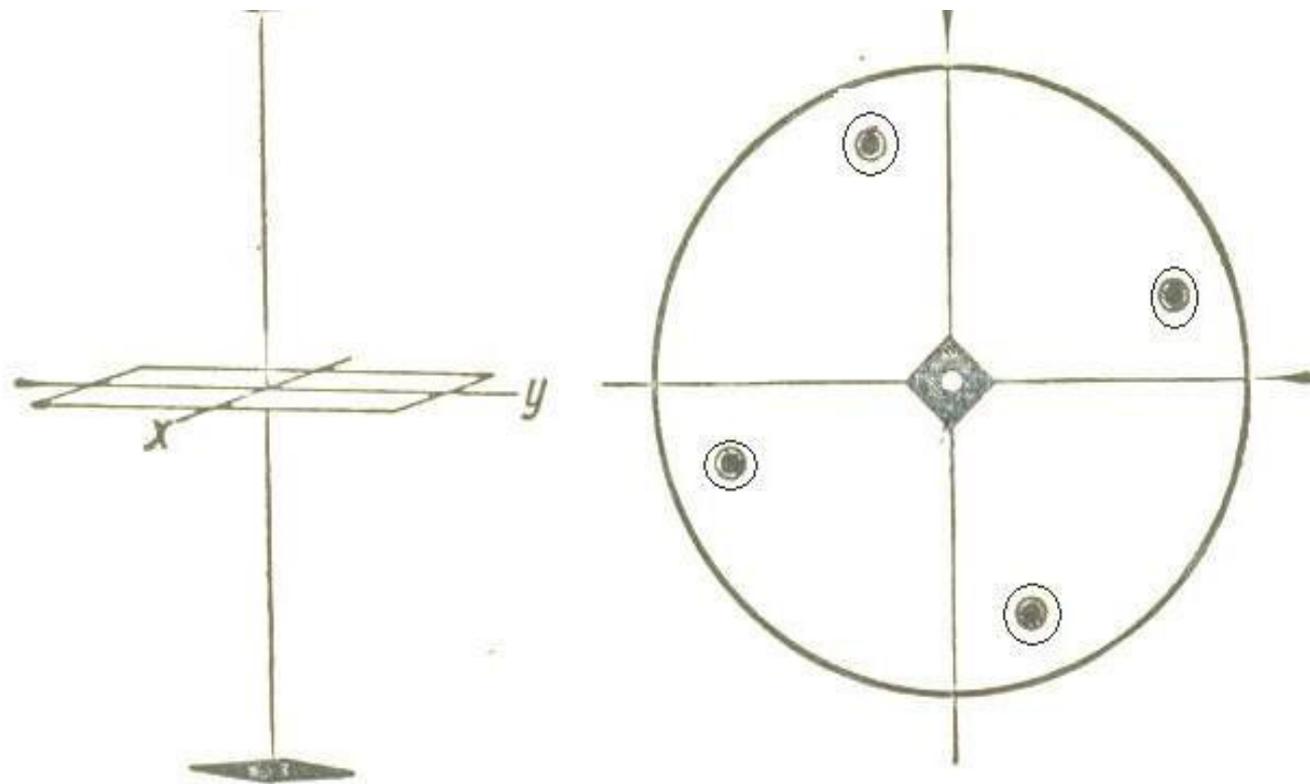


FIG. 229. Class  $4/m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

- Special forms.** Pinacoid  $\{001\}$ .  
 Tetragonal prisms  $\{hk0\}$ .
- General forms.** Tetragonal bipyramids  $\{hkl\}$ .

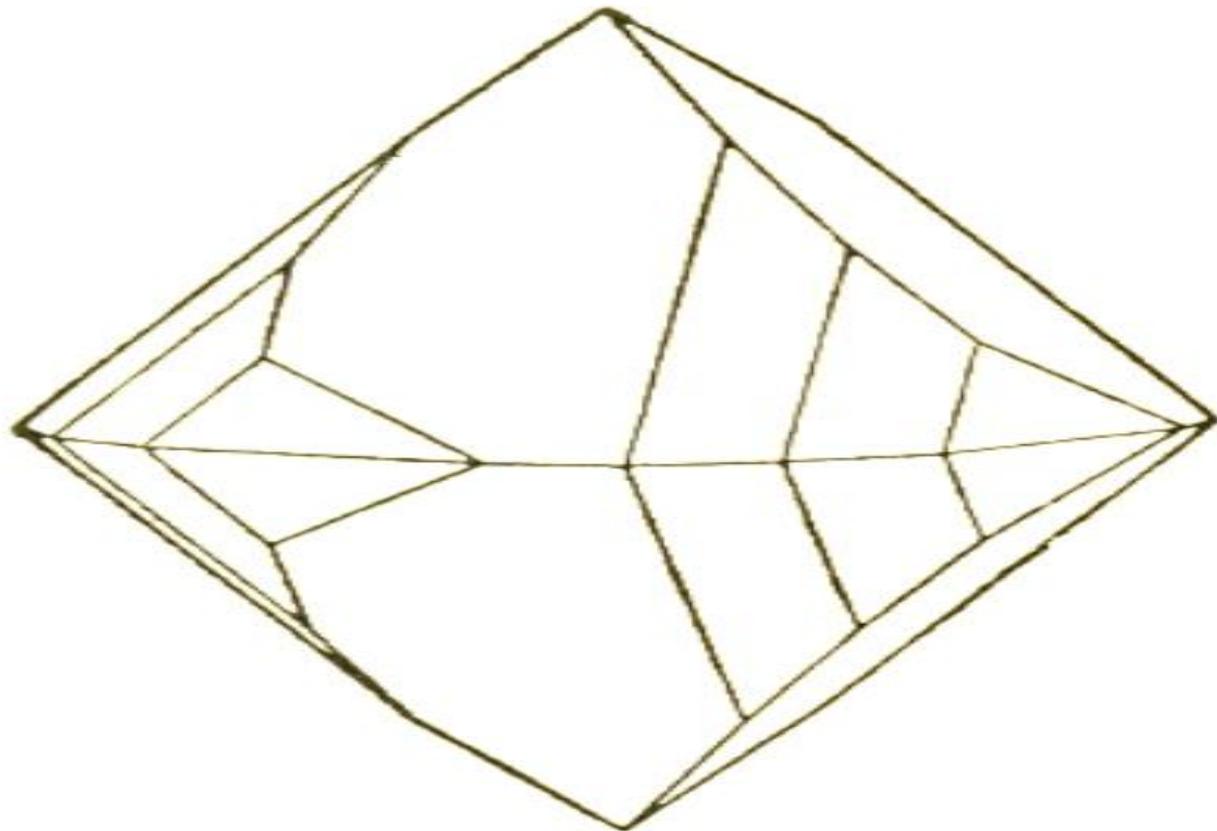


FIG. 230. A crystal of scheelite.

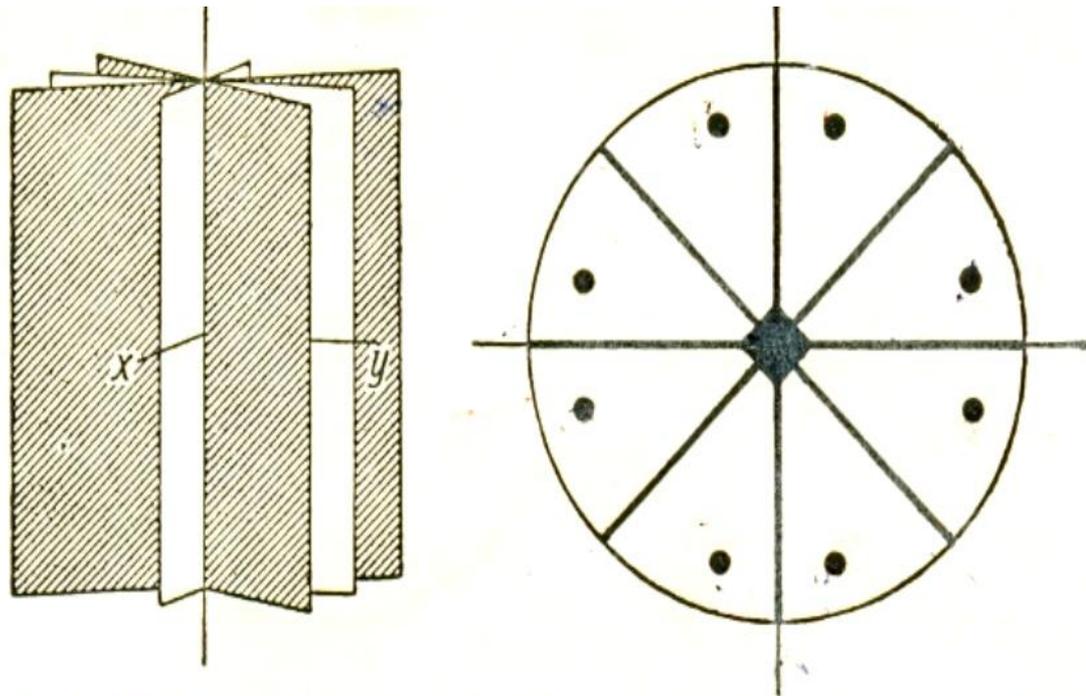


FIG. 232. Class  $4mm$ ; the elements of symmetry (with the two pairs of planes of symmetry differently shaded), crystallographic axes and a stereogram of the general form.

- Special forms.** Pedions  $\{001\}$ ,  $\{00\bar{1}\}$ .  
 Tetragonal prisms  $\{100\}$ ,  $\{110\}$ .  
 Ditetragonal prisms  $\{hk0\}$ .  
 Tetragonal pyramids  $\{h0l\}$ ,  $\{hh\bar{l}\}$ .
- General forms.** Ditetragonal pyramids  $\{hkl\}$ .

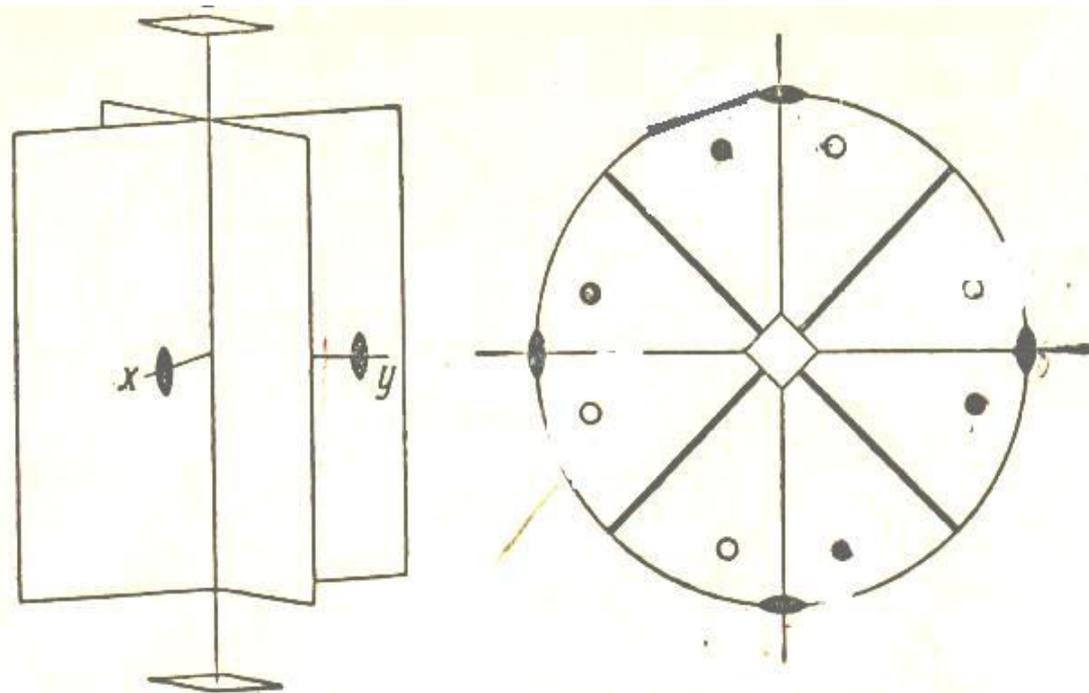


FIG. 234. Class  $\bar{4}2m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

- Special forms.** Pinacoid  $\{001\}$ .  
 Tetragonal prisms  $\{100\}$ ,  $\{110\}$ .  
 Ditetragonal prisms  $\{hk0\}$ .  
 Tetragonal bipyramids  $\{h0l\}$ .  
 Sphenoids  $\{hhl\}$ ,  $\{h\bar{h}l\}$ .
- General forms.** Bisphenoids  $\{hkl\}$ .

**CLASS 42.** (Tetragonal trapezohedral.) A tetrad axis normal to two pairs of diad axes (Fig. 242).

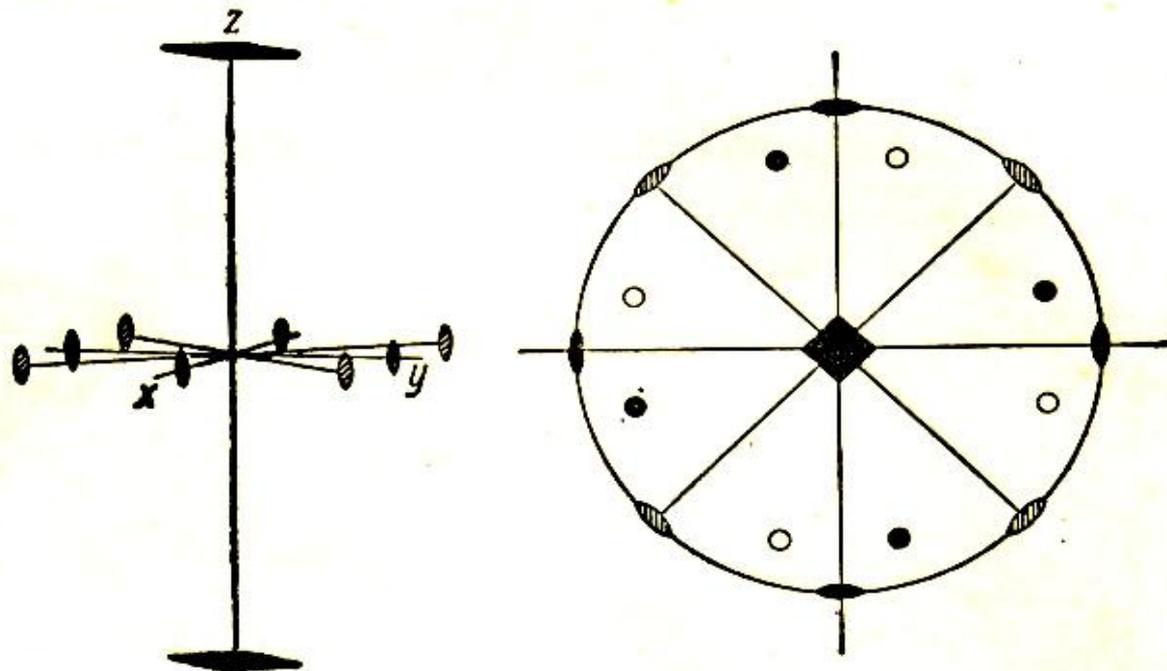


FIG. 242. Class 42; the elements of symmetry (the two pairs of diad axes have differently shaded flags), crystallographic axes and a stereogram of the general form.

**Special forms.**

Pinacoid  $\{001\}$ .

Tetragonal prisms  $\{100\}$ ,  $\{110\}$ .

Ditetragonal prisms  $\{hk0\}$ .

Tetragonal bipyramids  $\{h0l\}$ ,  $\{hh l\}$ .

**General forms.**

Tetragonal trapezohedra  $\{hkl\}$ .

12

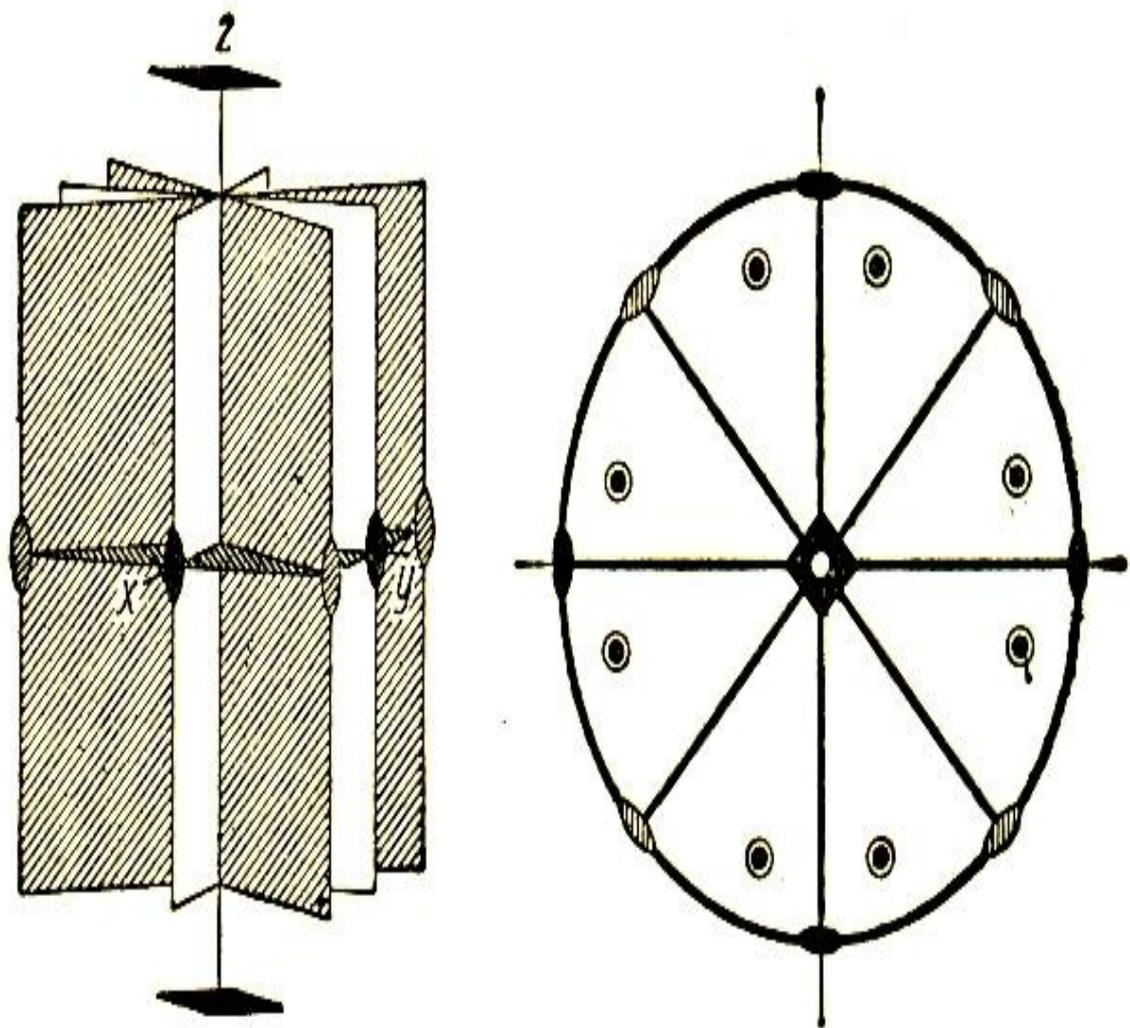


FIG. 247. Class  $4/m\bar{3}m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

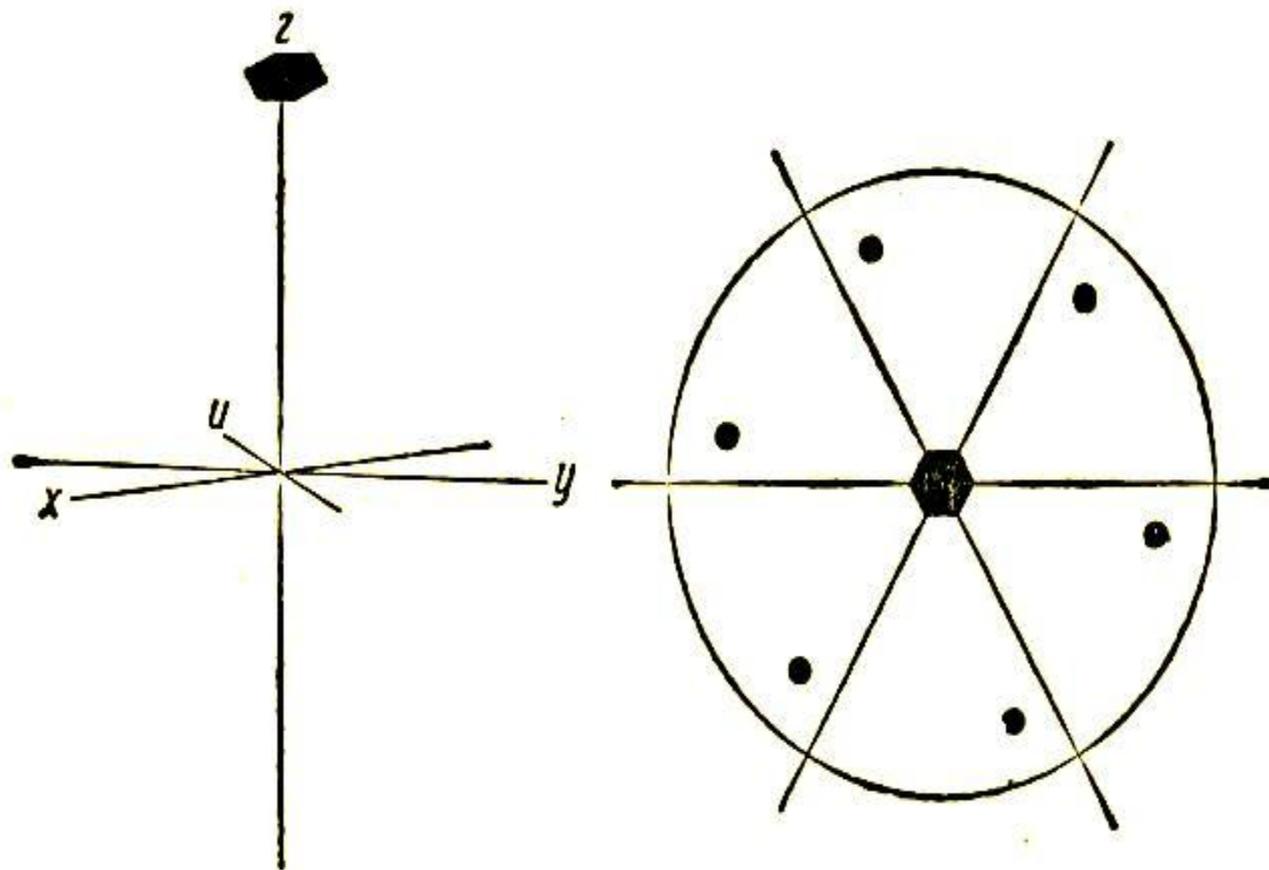


FIG. 248. Class 6; the crystallographic axes and a stereogram of the general form.

**Special forms.** Pedions  $\{0001\}$ ,  $\{000\bar{1}\}$ .

Hexagonal prisms  $\{hk i 0\}$ .

**General forms.** Hexagonal pyramids  $\{hk i l\}$ .

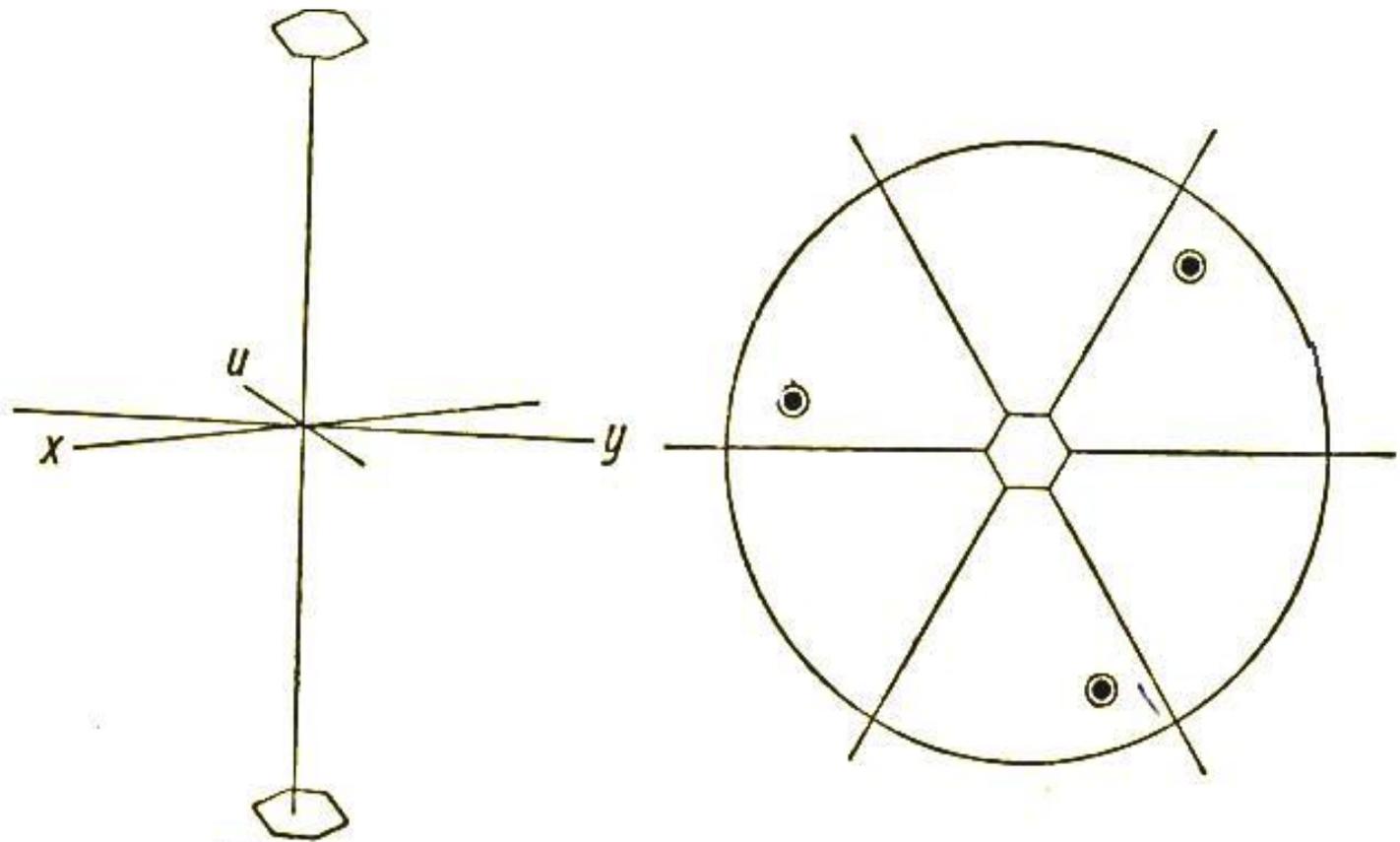


FIG. 252. Class  $\bar{6}$ ; the inversion axis (flagged with open hexagons), crystallographic axis and a stereogram of the general form.

**Special forms.** Pinacoid  $\{0001\}$ .

Trigonal prisms  $\{hk i 0\}$ .

**General forms.** Trigonal bipyramids  $\{hk i l\}$ .

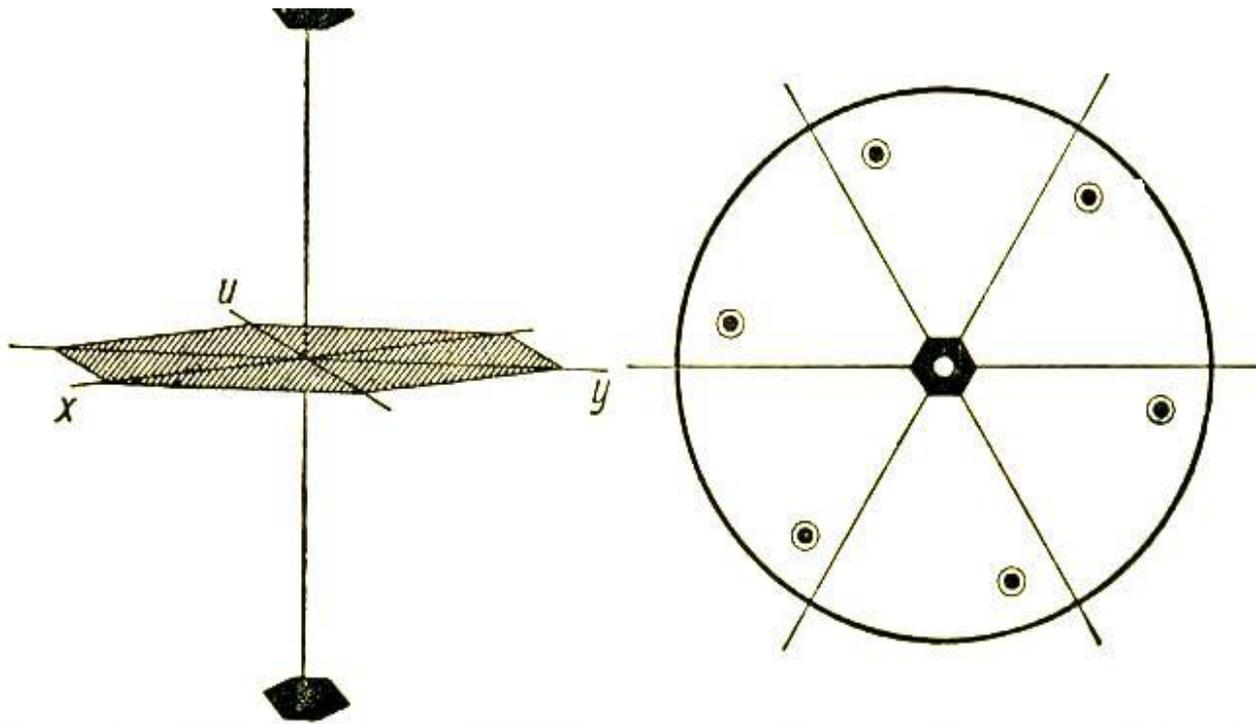


FIG. 253. Class  $6/m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

**CLASS  $6/m$ .** (Hexagonal bipyramidal.) A hexad axis normal to a plane of symmetry, and a centre (Fig. 253).

**Special forms.** Pinacoid  $\{0001\}$ .  
Hexagonal prisms  $\{hkil\}$ .

**General forms.** Hexagonal bipyramids  $\{hkil\}$ .

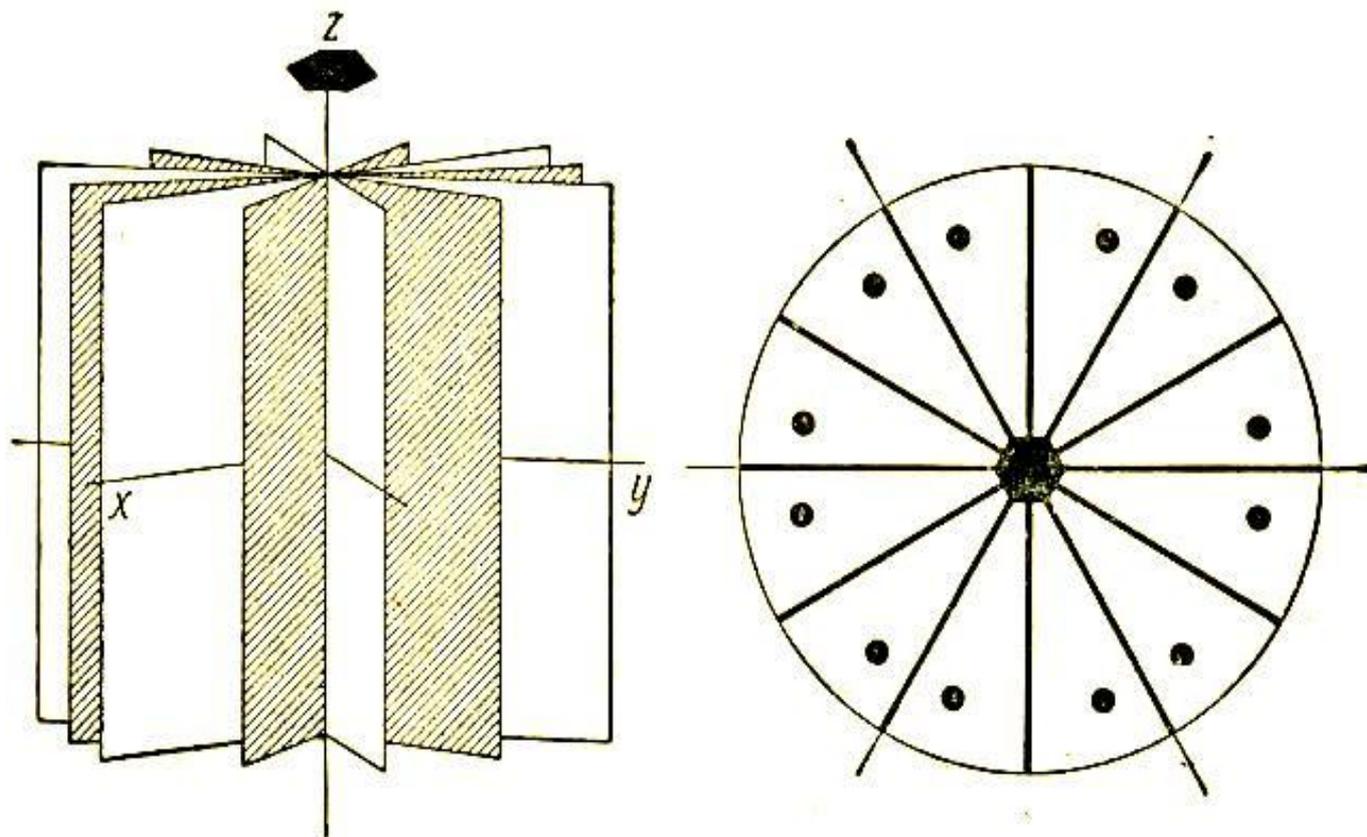


FIG. 255. Class  $6mm$ ; the elements of symmetry (the two families of planes of symmetry are differently shaded), crystallographic axes and a stereogram of the general form.

- Special forms.** Pedions  $\{0001\}$ ,  $\{000\bar{1}\}$ .  
 Hexagonal  $\{10\bar{1}0\}$ ,  $\{11\bar{2}0\}$ .  
 Dihexagonal prisms  $\{hk i 0\}$ .  
 Hexagonal pyramids  $\{h 0 \bar{h} l\}$ ,  $\{h h \bar{2}h l\}$ .
- General forms.** Dihexagonal pyramids  $\{h k i l\}$ .

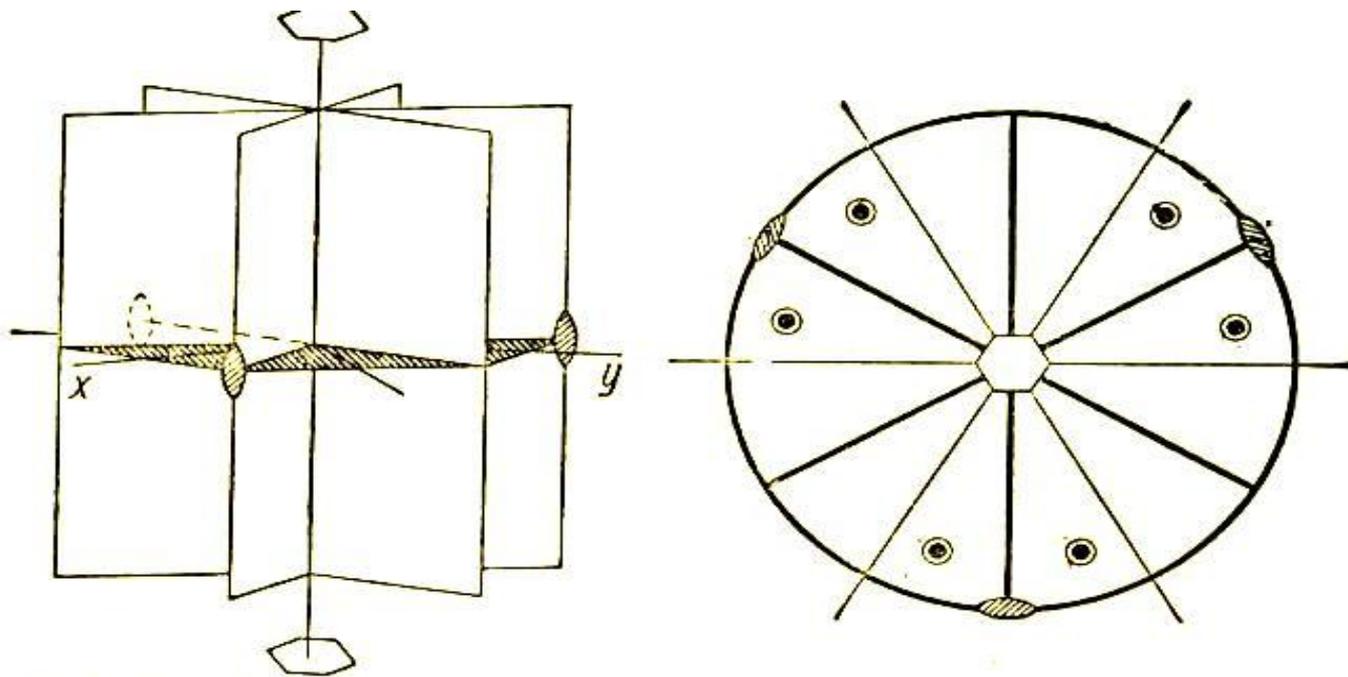


FIG. 259. Class  $\bar{6}m2$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

**Special forms.**

Pinacoid  $\{0001\}$

Trigonal prisms  $\{10\bar{1}0\}$ ,  $\{01\bar{1}0\}$ .

Hexagonal prism  $\{11\bar{2}0\}$ .

Ditrigonal prisms  $\{hk i 0\}$ .

Trigonal bipyramids  $\{h 0 \bar{h} l\}$ ,  $\{0 k \bar{k} l\}$ .

Hexagonal bipyramids  $\{h h \bar{2}h l\}$ .

**General forms.**

Ditrigonal bipyramids  $\{h k i l\}$ .

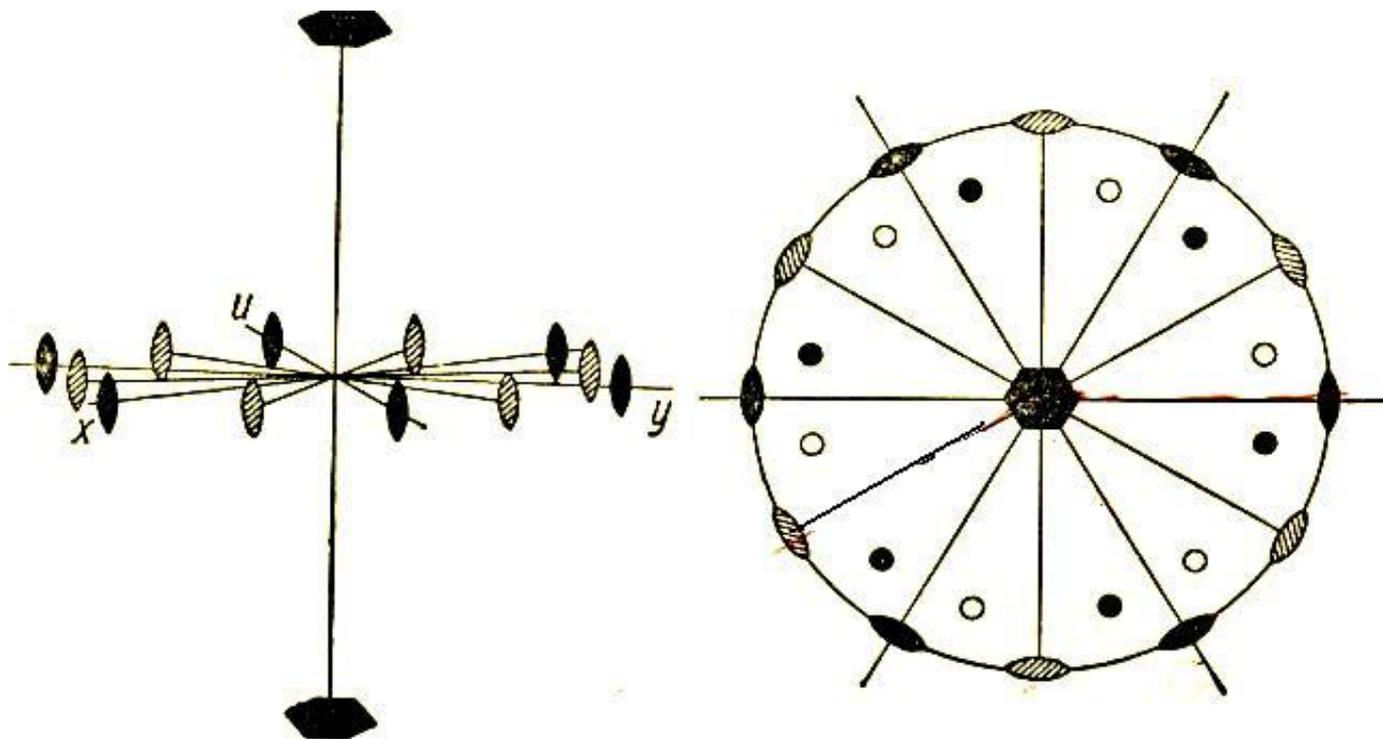


FIG. 262. Class 62; the elements of symmetry, crystallographic axes and stereogram of the general form.

**Special forms.**

Pinacoid  $\{0001\}$

Hexagonal prisms  $\{10\bar{1}0\}$ ,  $\{11\bar{2}0\}$ .

Dihexagonal prisms  $\{hkil\}$ .

Hexagonal bipyramids  $\{h0\bar{h}l\}$ ,  $\{hh\bar{2}hl\}$ .

**General forms.**

Hexagonal trapezohedra  $\{hkil\}$ .

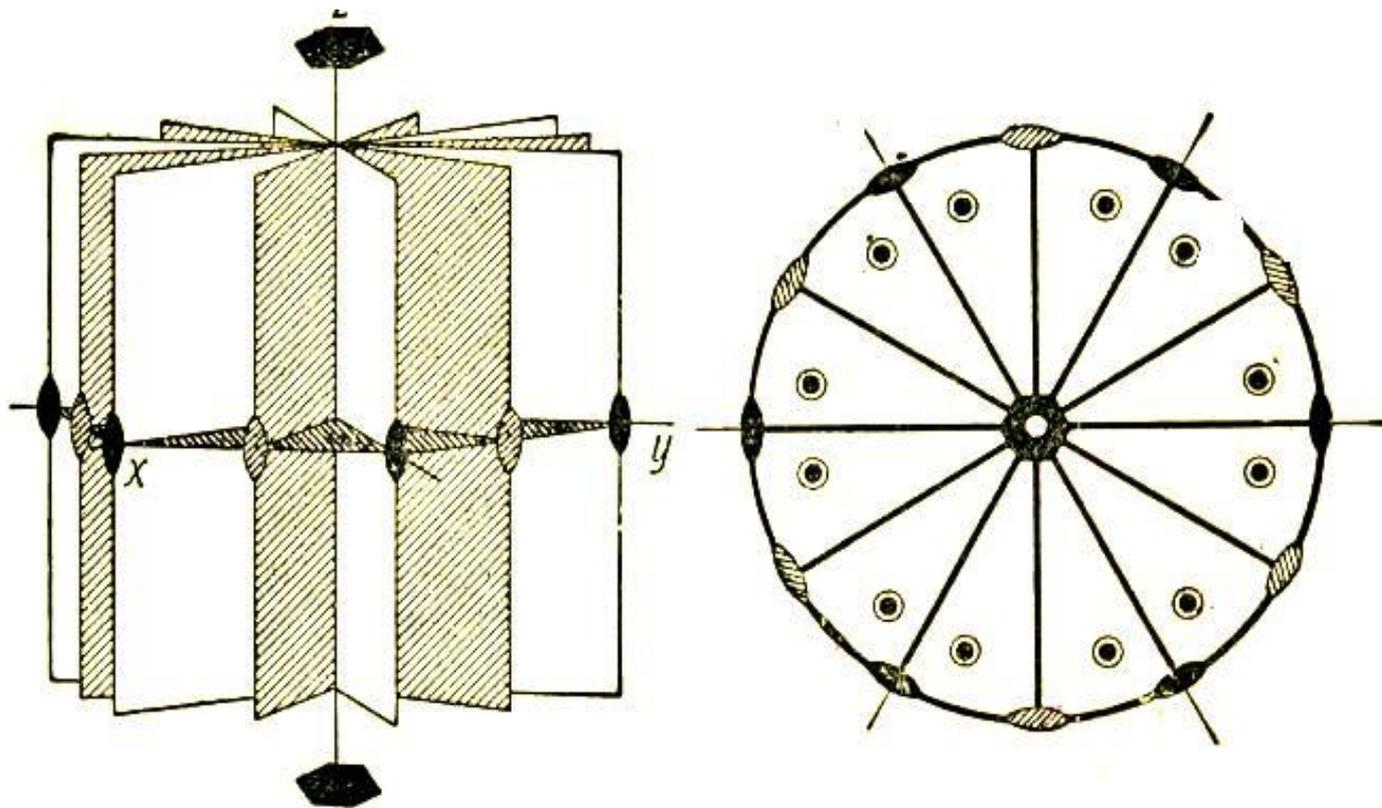


FIG. 204. Class  $6/mmm$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

**Special forms.**

Pinacoid  $\{0001\}$ .

Hexagonal prisms  $\{10\bar{1}0\}$ ,  $\{11\bar{2}0\}$ .

Dihexagonal prisms  $\{hk i 0\}$ .

Hexagonal bipyramids  $\{h 0 \bar{h} l\}$ ,  $\{h h \bar{2}h l\}$ .

**General forms.**

Dihexagonal bipyramids  $\{hk i l\}$ .

# Different combination of The Cubic Point group

in the first column below represent all the different combinations possible:

<i>Full symbol</i>	<i>Abbreviated symbol</i>
23	23
432	432
2m3	m3
$\bar{4}3m$	$\bar{4}3m$
4m3m2	m3m

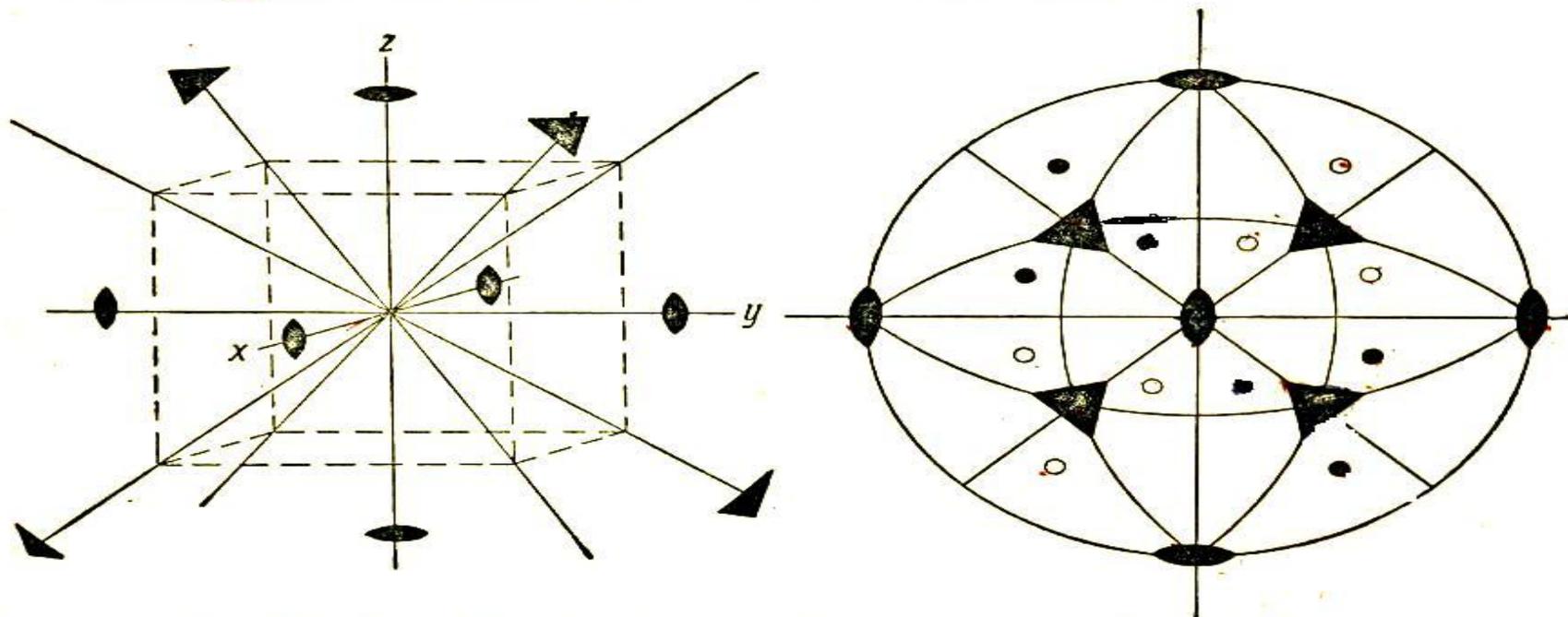


FIG. 266. Class 23; the elements of symmetry, crystallographic axes and a stereogram of the general form. The outline of the cube has been added to the drawing of the elements of symmetry to help in visualising their arrangement in space.

- Special forms.**      Cube  $\{100\}$ .  
                             Rhombic dodecahedron  $\{110\}$ .  
                             Pentagonal dodecahedra  $\{hk0\}$ ,  $\{kh0\}$ .  
                             Tetrahedra  $\{111\}$ ,  $\{1\bar{1}1\}$ .  
                             Tristetrahedra  $\{hll\}$ ,  $\{h\bar{l}l\}$ .  
                             Deltoid dodecahedra  $\{hhl\}$ ,  $\{h\bar{h}l\}$ .
- General forms.**      Tetrahedral pentagonal dodecahedra  $\{hkl\}$ .

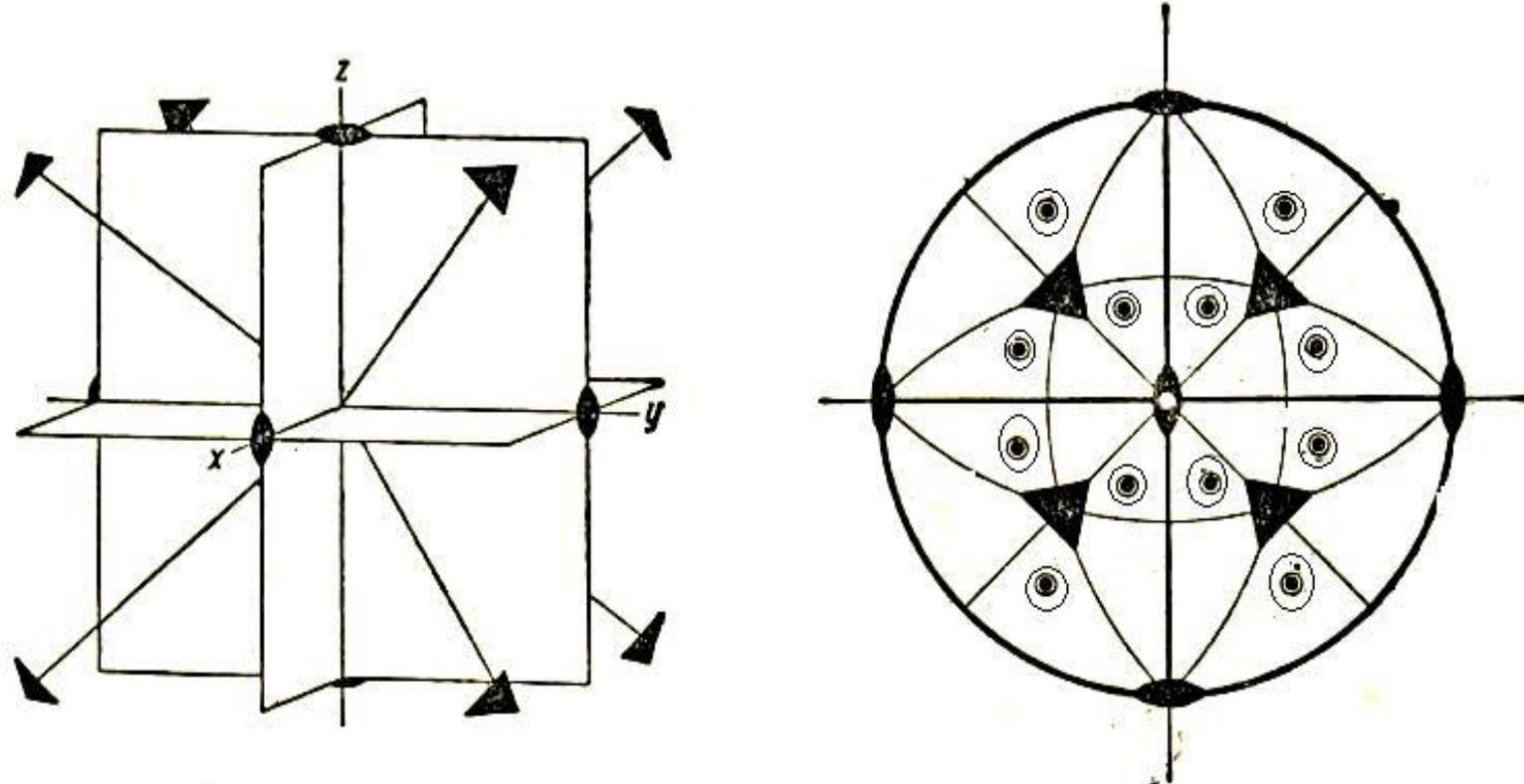
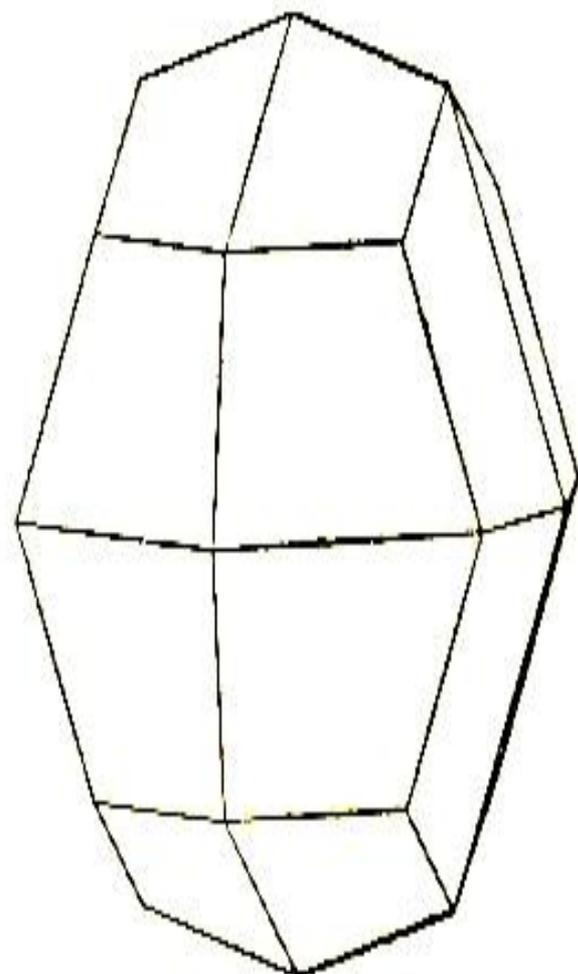
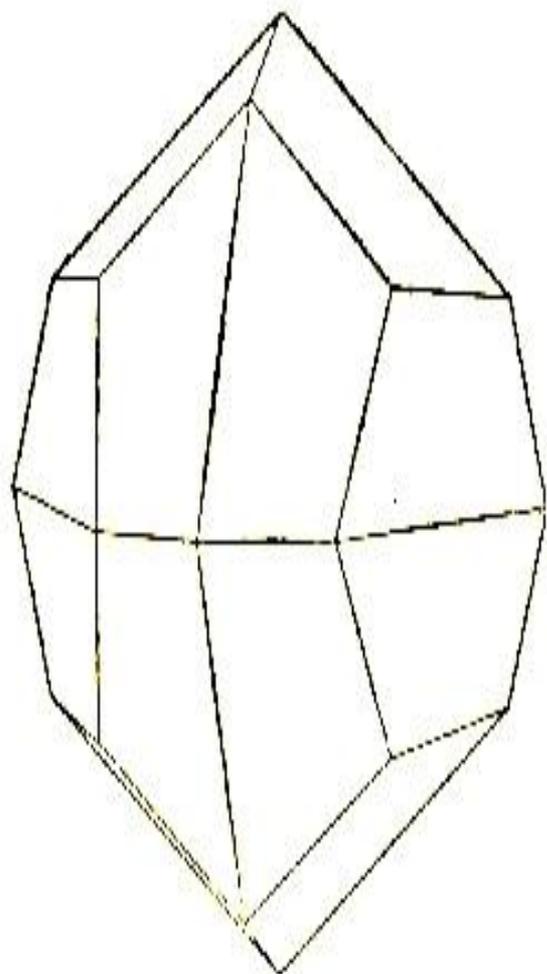
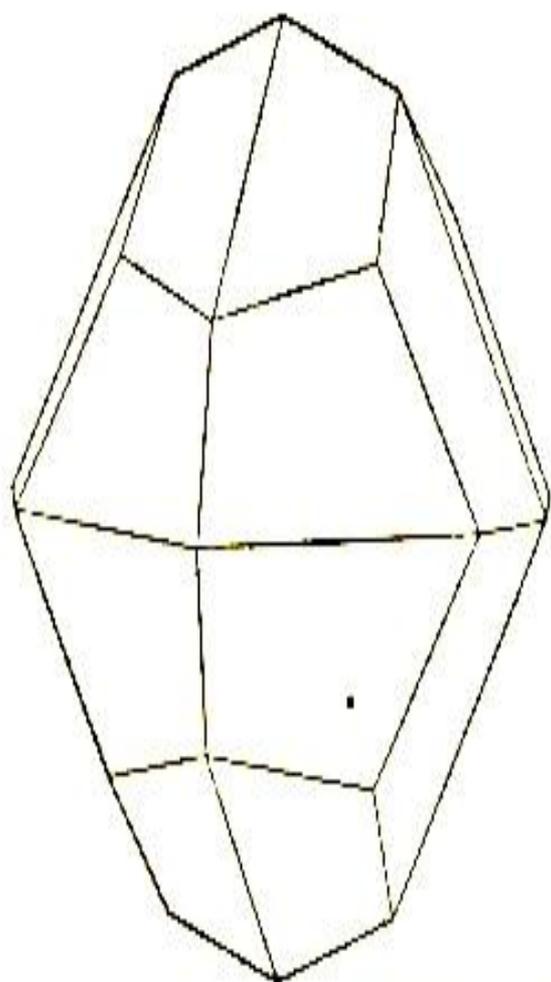


FIG. 279. Class  $m\bar{3}$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.

- Special forms.**
- Cube  $\{100\}$ .
  - Rhombic dodecahedron  $\{110\}$ .
  - Pentagonal dodecahedra  $\{hk0\}$ ,  $\{kh0\}$ .
  - Octahedron  $\{111\}$ .
  - Icositetrahedra  $\{hll\}$ .
  - Trisoctahedra  $\{hhl\}$ .
- General forms.**
- Di(akis)dodecahedra  $\{hkl\}$ .



Figs. 280-282. Didodecahedra {321} {312} {421}.

P.C.

K

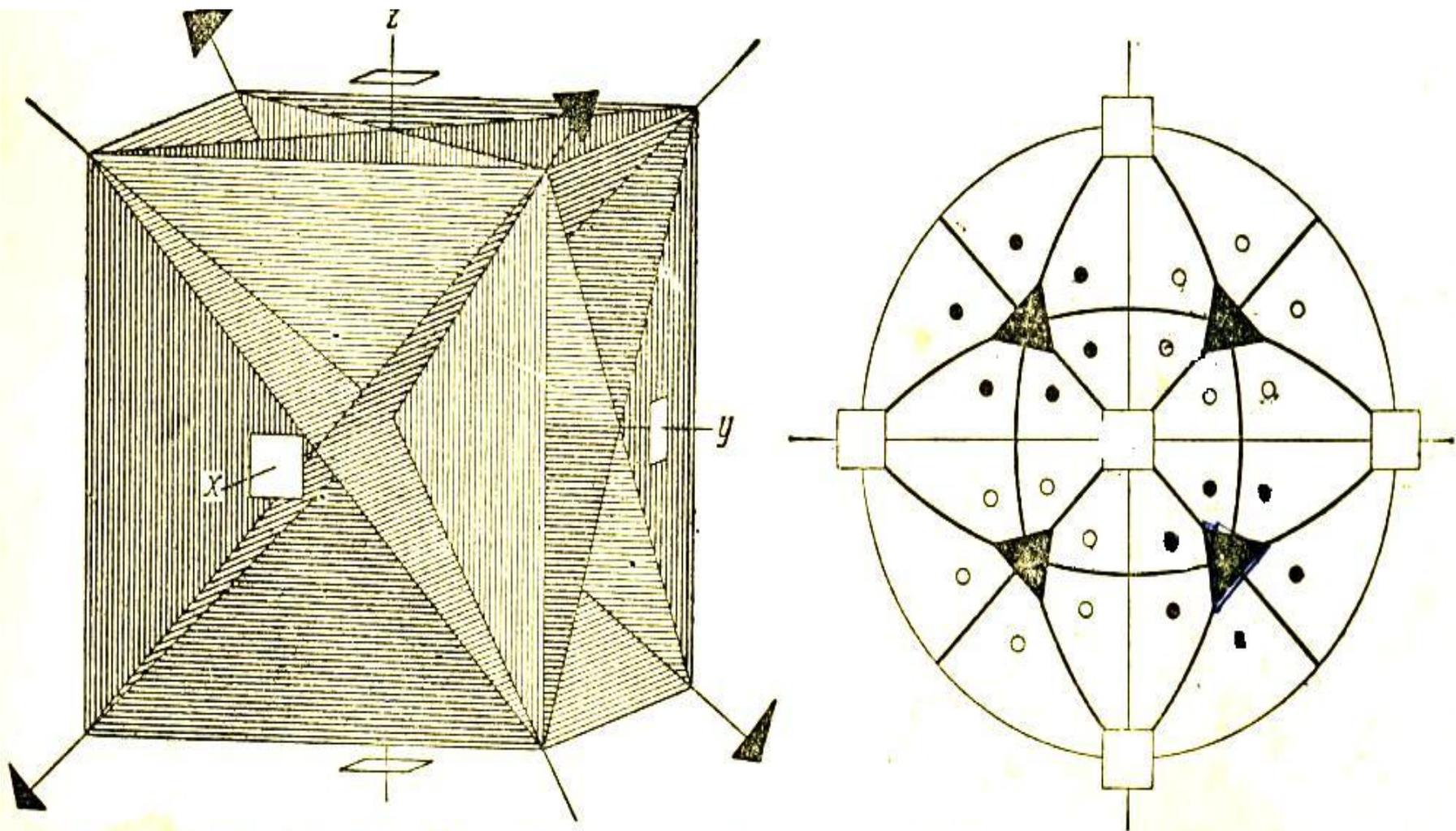
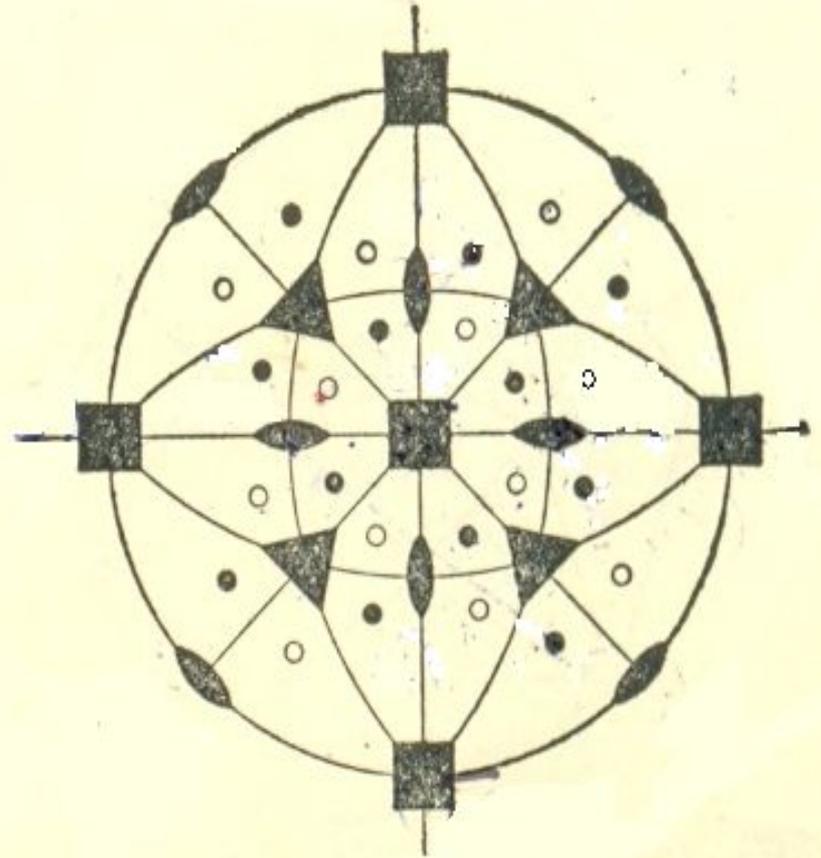
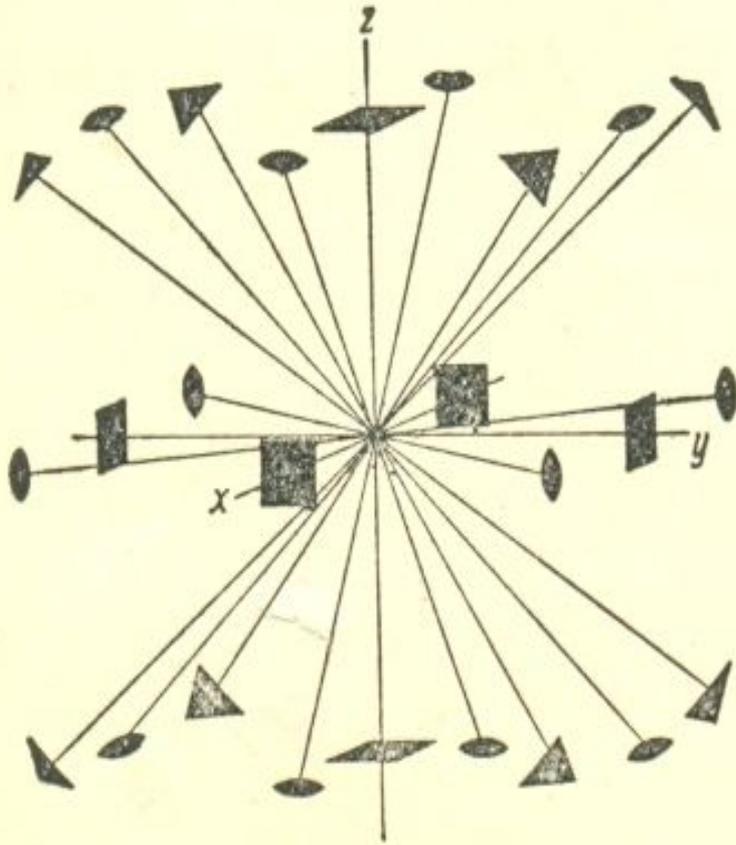
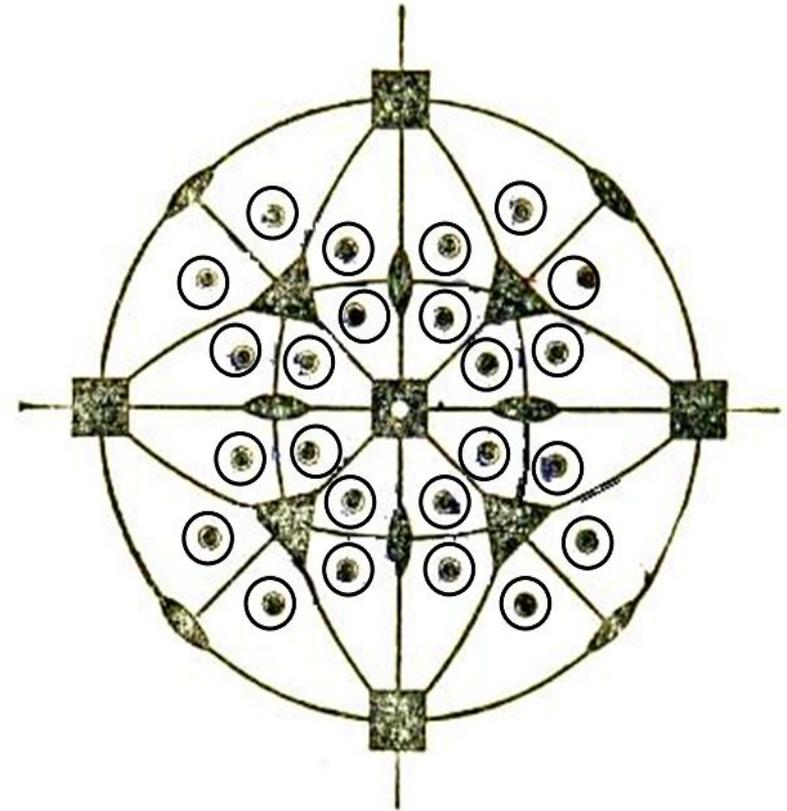
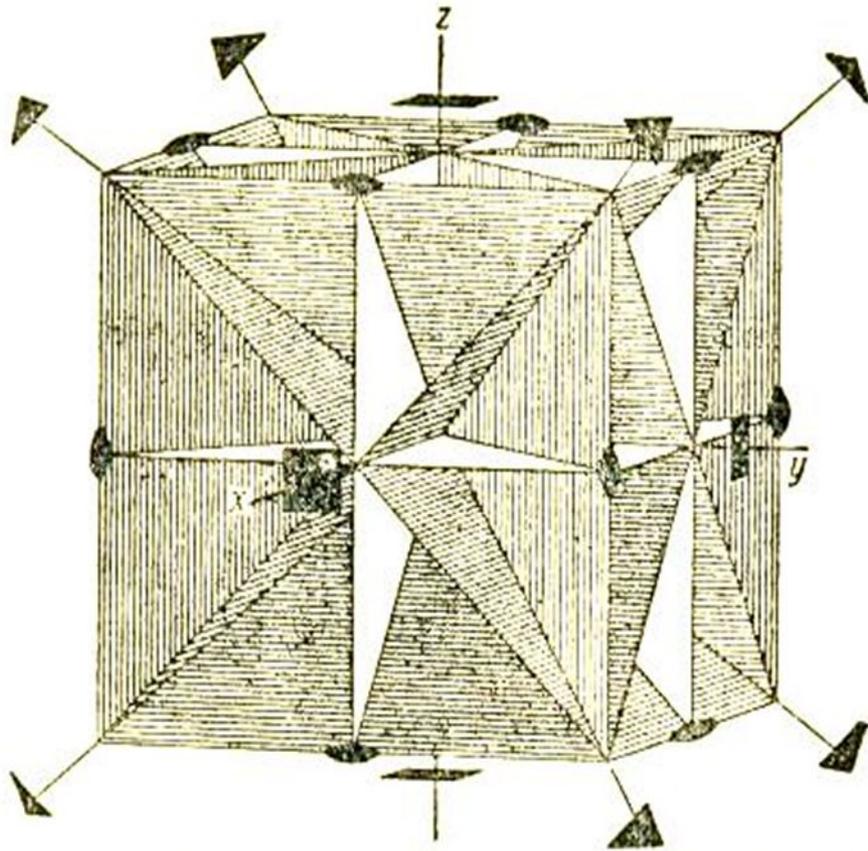


FIG. 286. Class  $\bar{4}3m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form.



Class 432 the elements of symmetry, crystallographic axes and a stereogram of the general form



**Class  $m\bar{3}m$ ; the elements of symmetry, crystallographic axes and a stereogram of the general form**

