Point Group For The students

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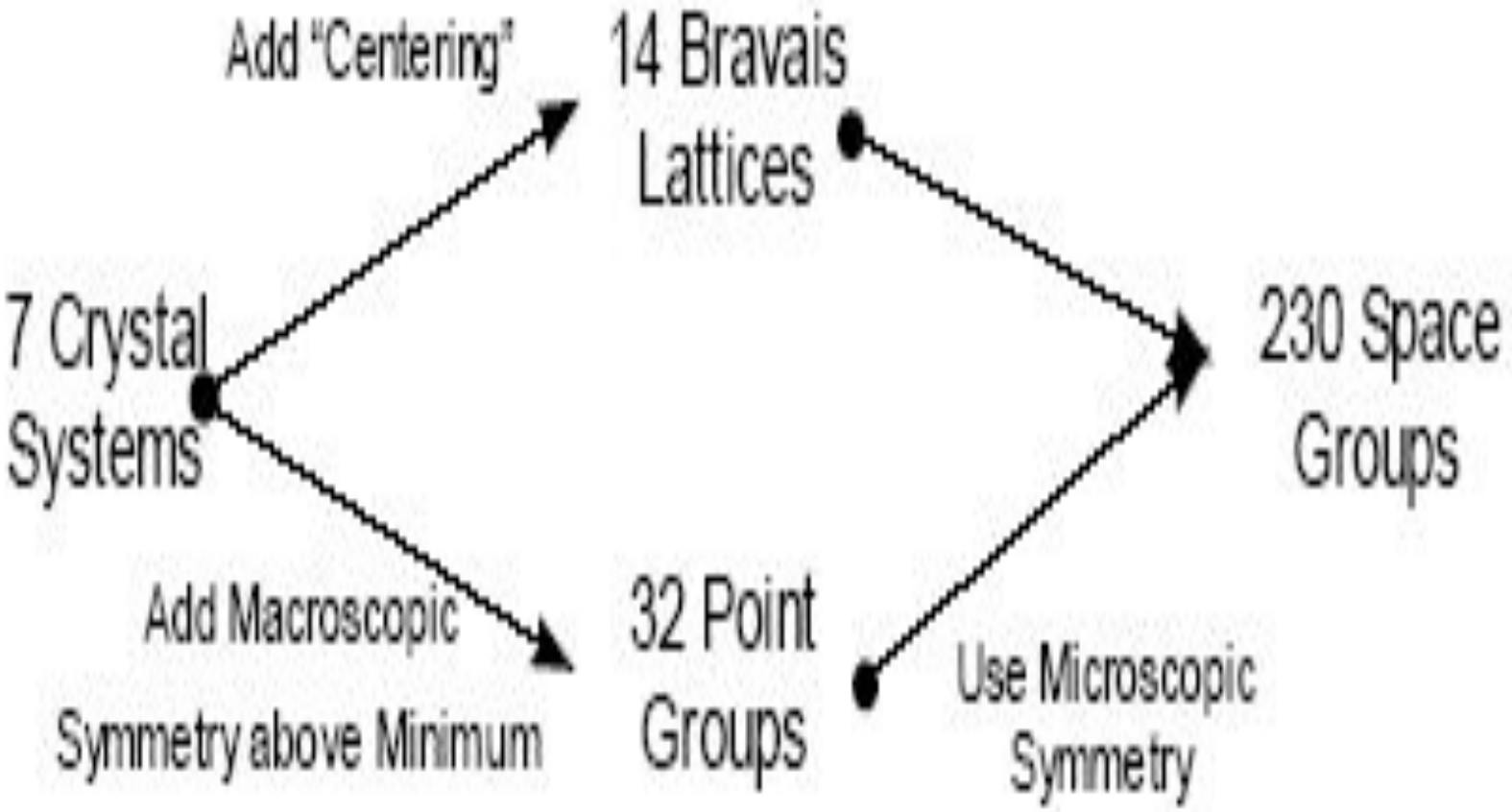
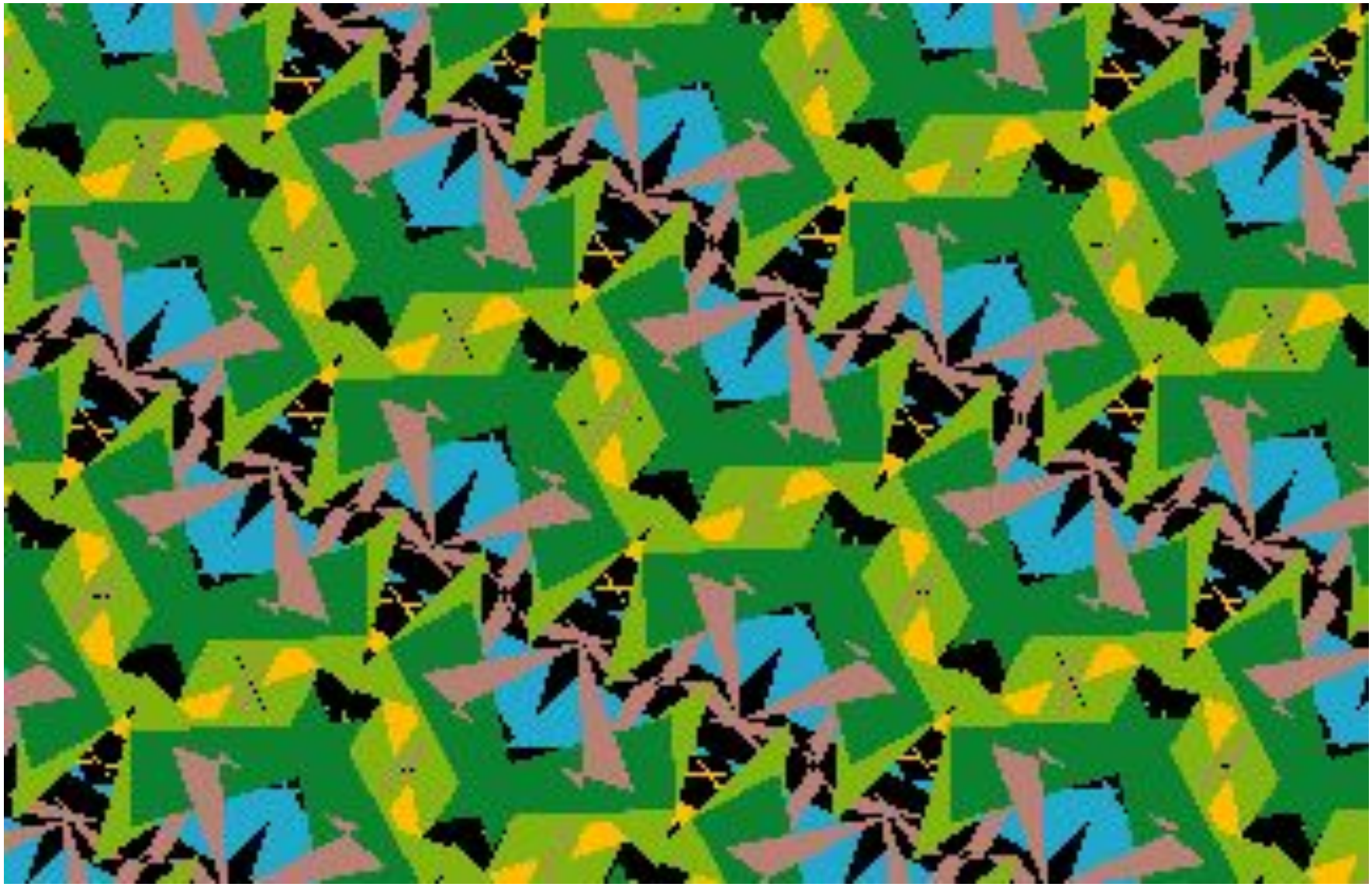
Symmetry Elements

- We may have the following combinations:
  - X rotation axis alone
  - $\bar{X}$ inversion axis alone
  - X/m rotation axis normal to a plane of symmetry
  - Xm rotation axis with a vertical plane of symmetry
  - $\bar{X}m$ inversion axis with a vertical plane of symmetry
  - $X2$ rotation axis with a diad axis normal to it
  - X/mm rotation axis with both kinds of plane symmetry

Space Group


Add "Centering"
14 Bravais Lattices
7 Crystal Systems
230 Space Groups
Add Macroscopic Symmetry above Minimum
32 Point Groups
Use Microscopic Symmetry

SCREW AXIS



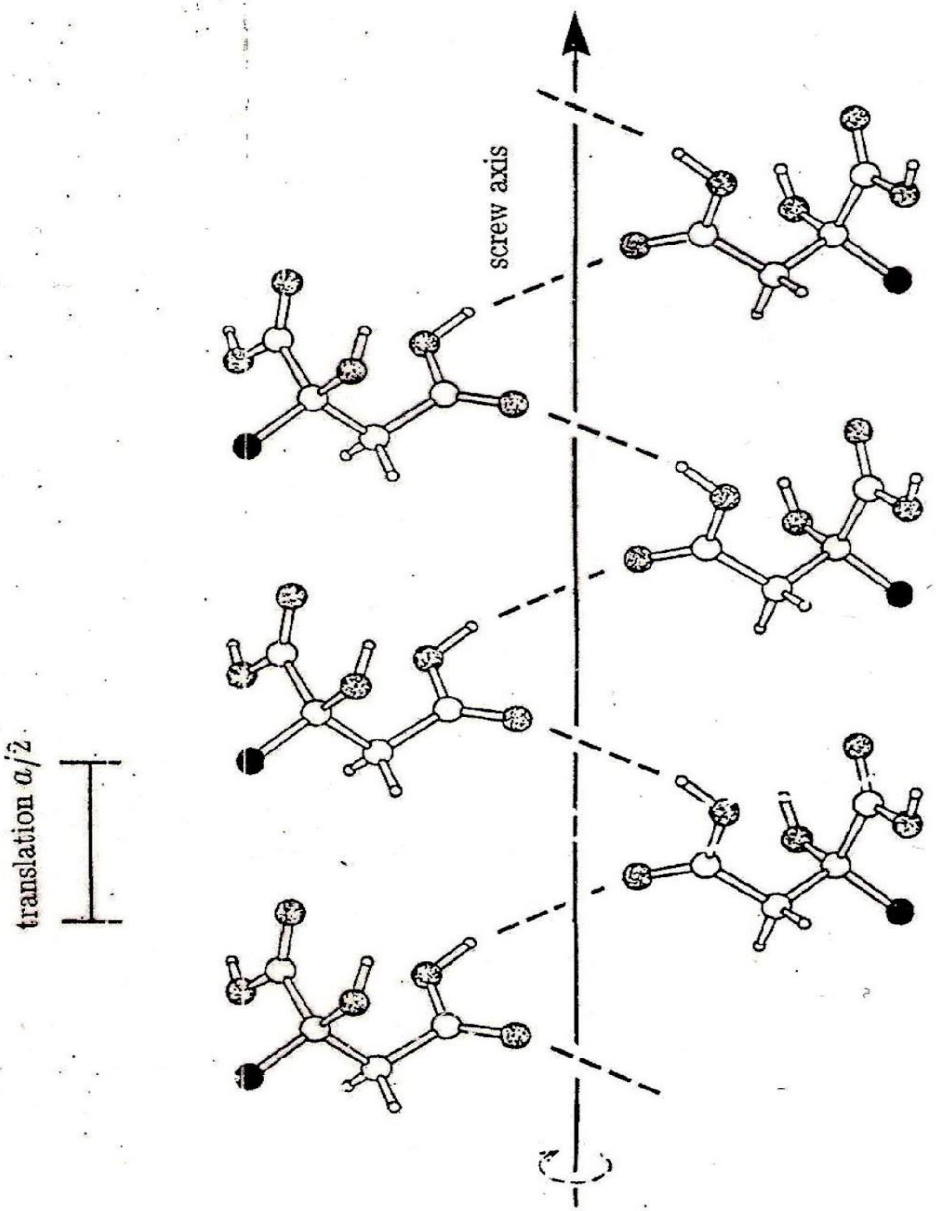
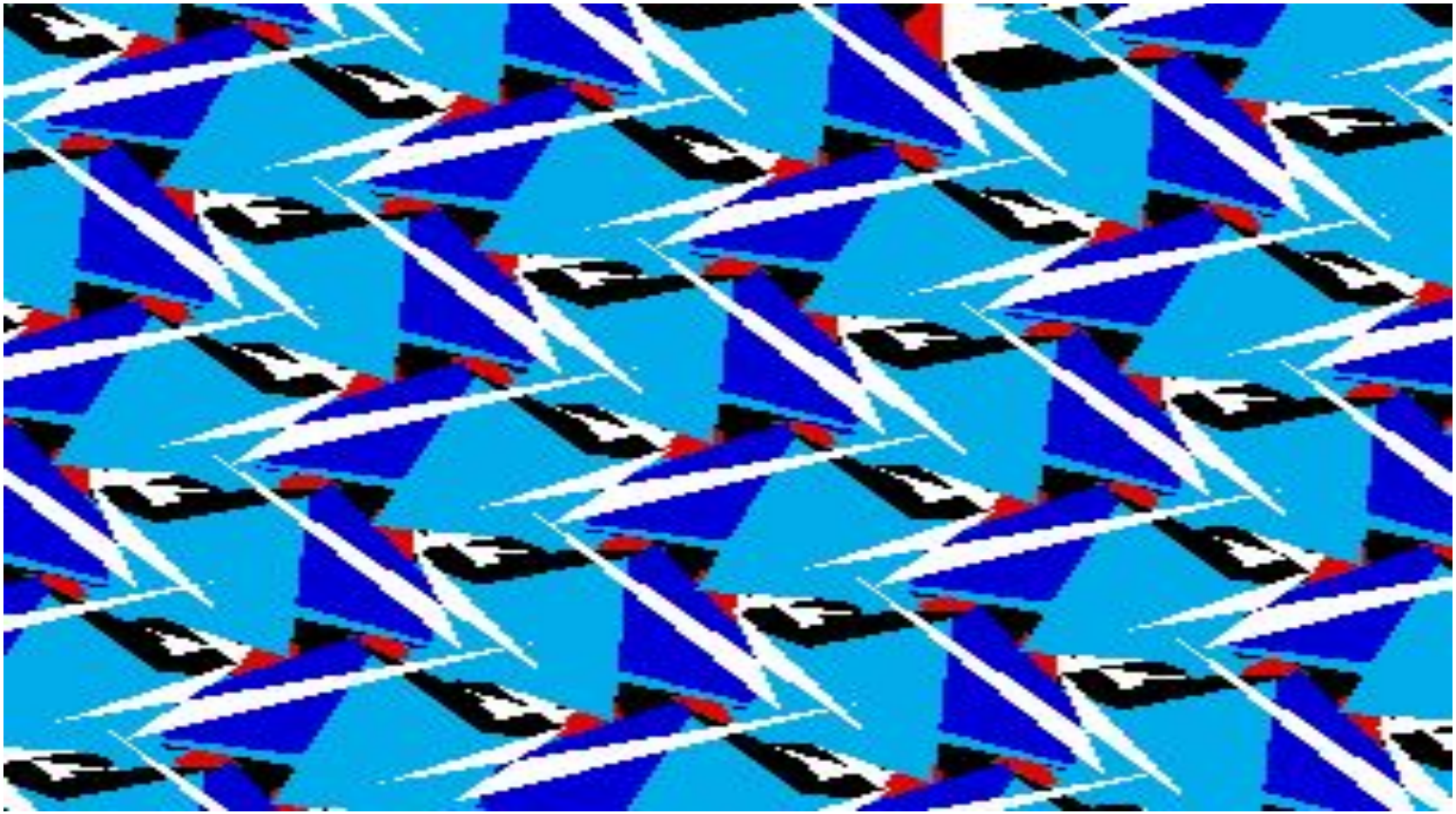


FIGURE 4.11. A twofold screw axis. One molecule is translated half a unit cell (a in this example) along the screw axis direction, and then a twofold rotation is applied to it. Two such operations give the original molecule translated to the next unit cell (a away). All molecules have the same handedness.

Glide Plane



The point and monoclinic space group

Crystal System	Point Group		Space Group	
	Full	Short	Full	Short
Monoclinic	m	m	Pm Pc Cm Cc	Pm Pc Cm Cc
	2	2	P2 $P2_1$ C2	P2 $P2_1$ C2
	$2/m$	$2/m$	$P2/m$ $P2_1/m$ $P2/c$ $P2_1/c$ $C2/m$ $C2/c$	$P2/m$ $P2_1/m$ $P2/c$ $P2_1/c$ $C2/m$ $C2/c$

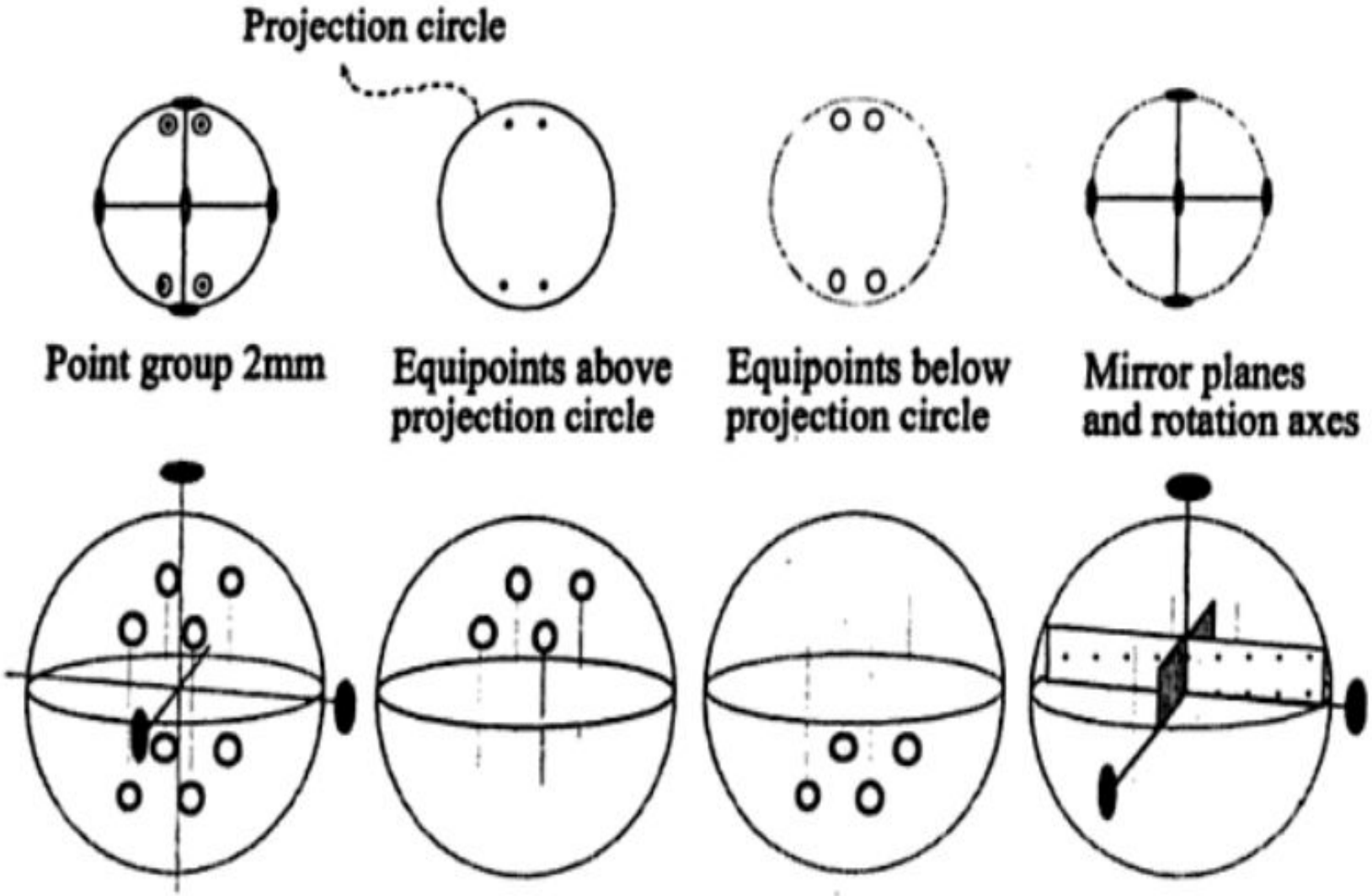


Figure 2-44 Illustration of the representation of the point group, 2mm

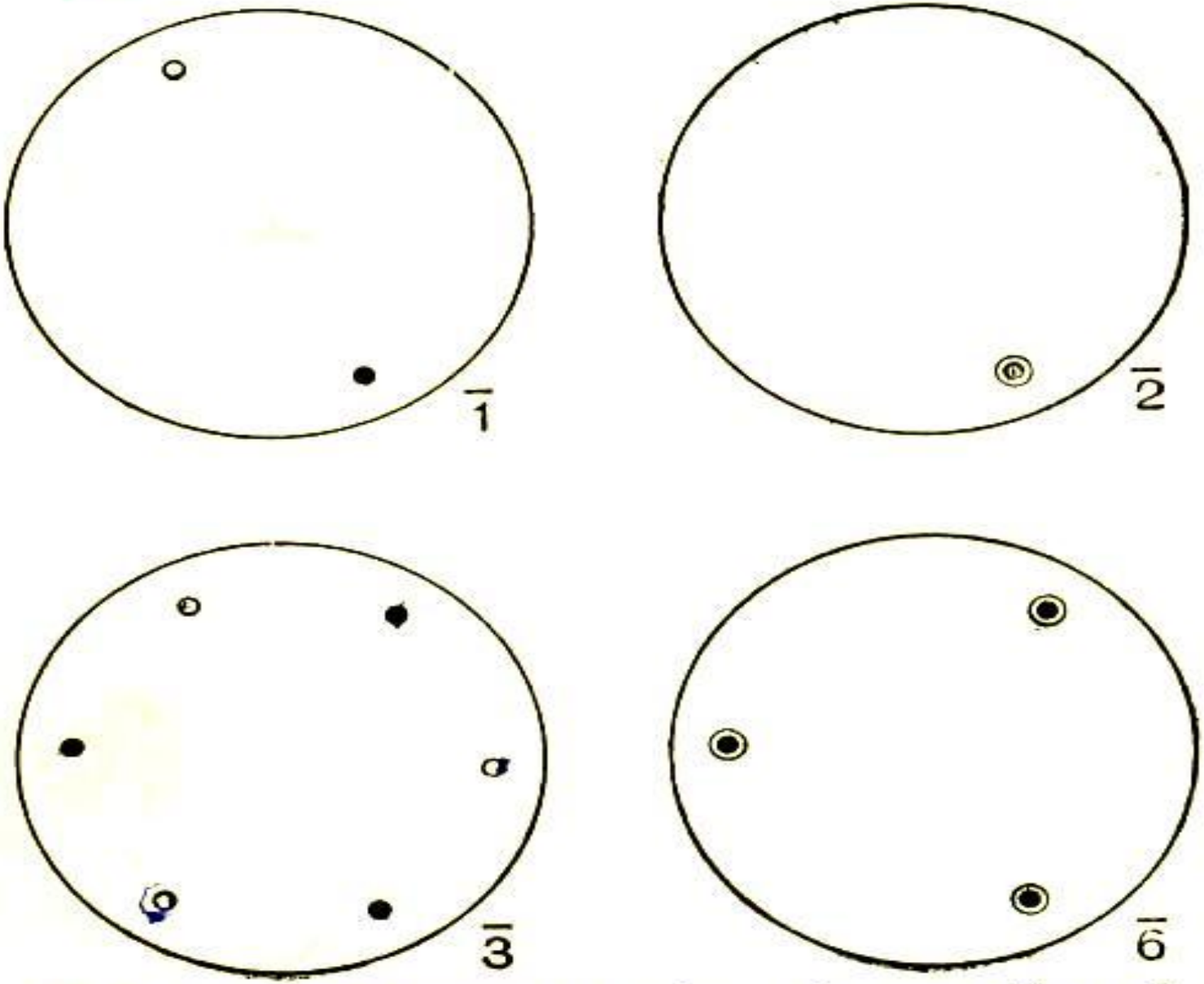
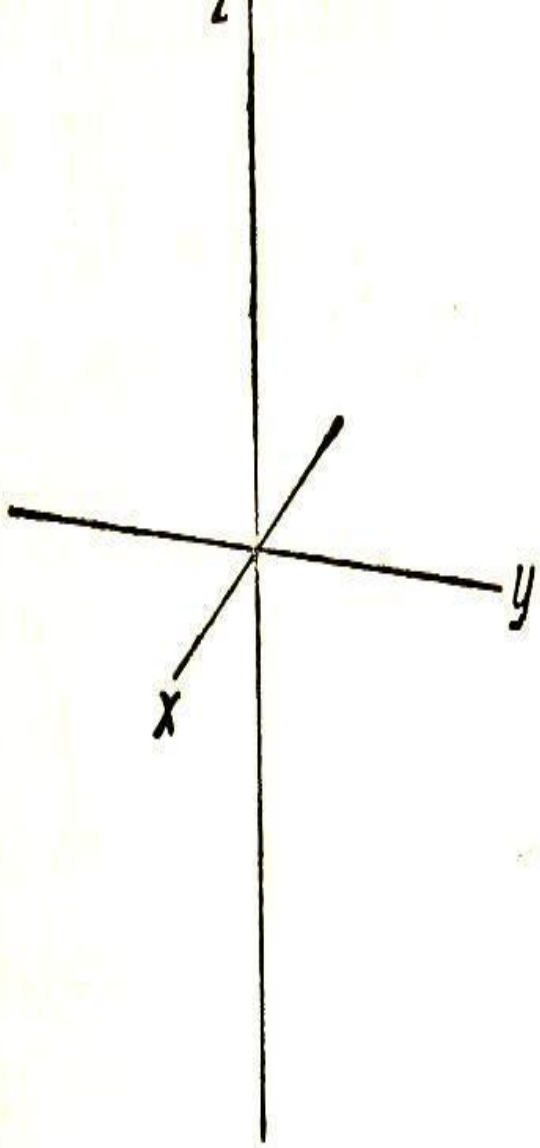


FIG. 171. Stereograms to show the operation of inversion axes $\bar{1}$, $\bar{2}$, $\bar{3}$ and $\bar{6}$.



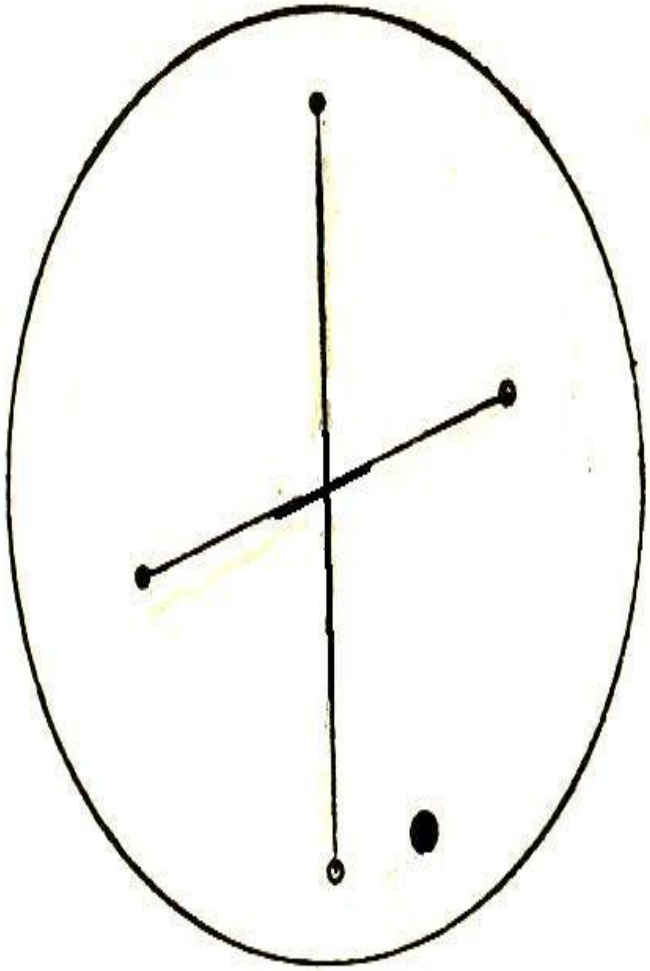


FIG. 172. Class 1; the crystallographic axes and a stereogram of the general form. (See the note to the legend of Fig. 127.)

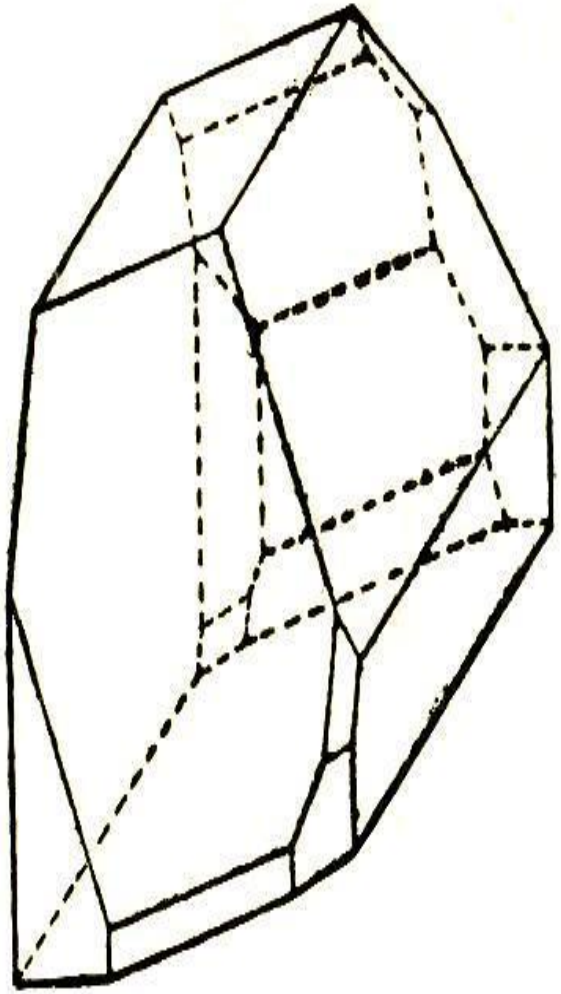
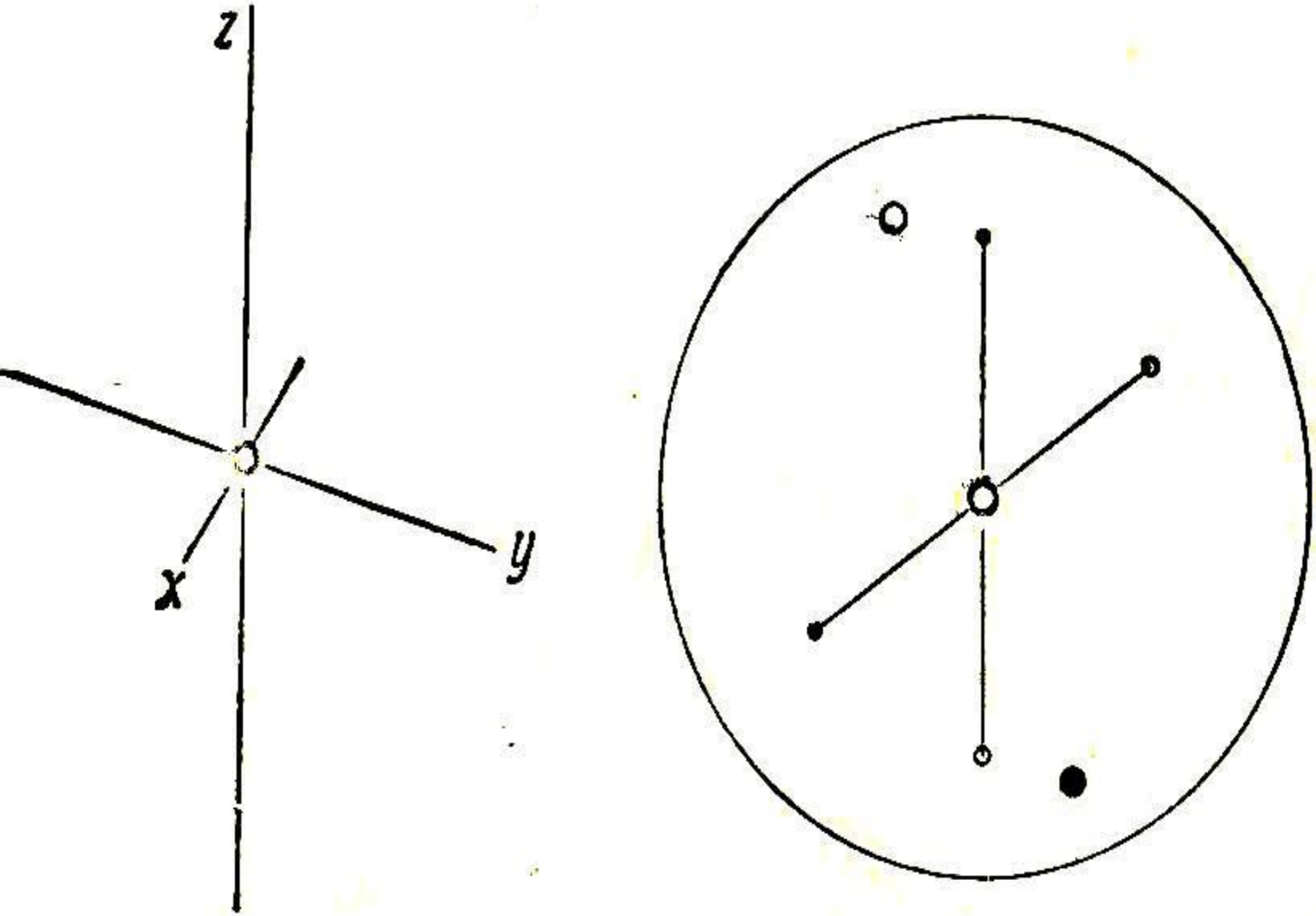
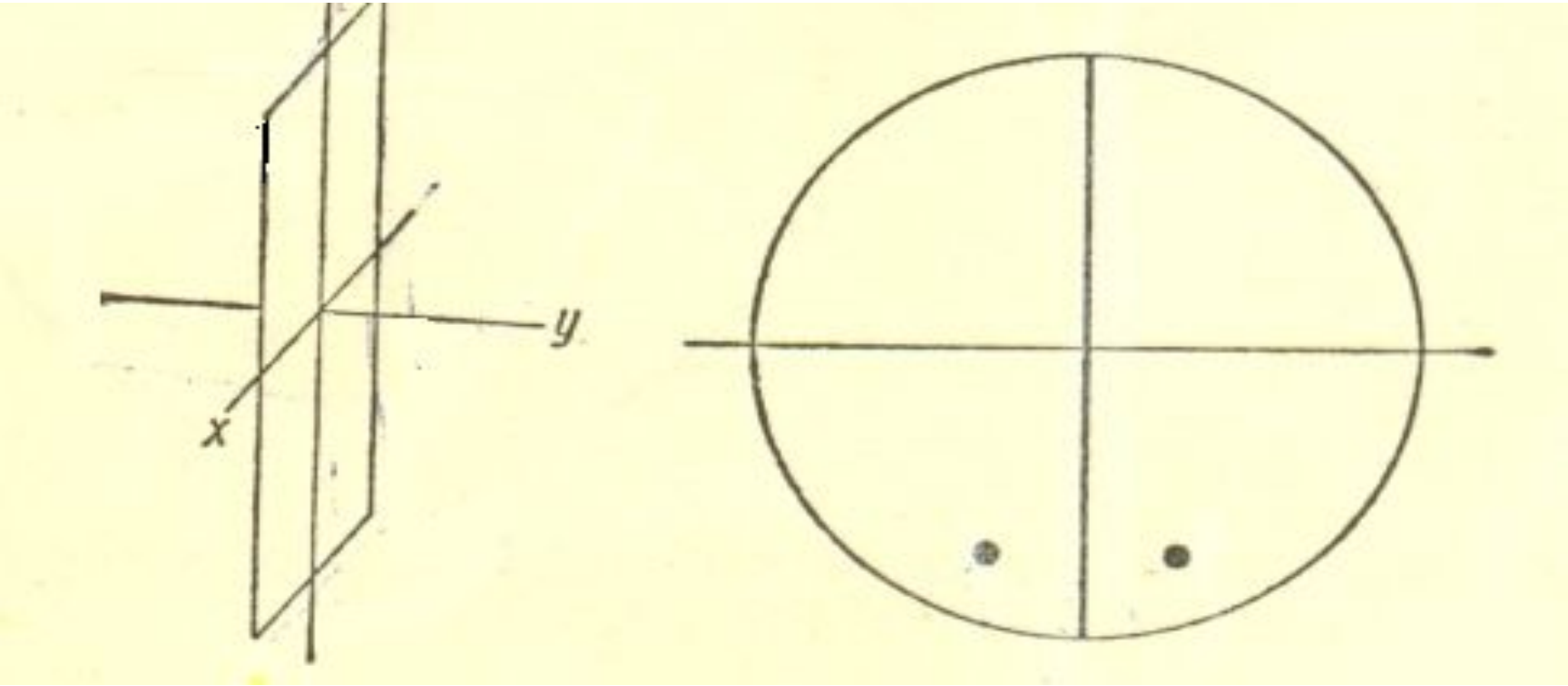


FIG. 173. A crystal of calcium thiosulphate.



Class $\bar{1}$; the crystallographic axes and a stereogram of the general form. The centre of symmetry is denoted by a thick ring.



Class m; the crystallographic axes, showing their relationship to the plane of symmetry, and a stereogram of the general form

MONOCLINIC SYSTEM

CLASS 2. (Monoclinic hemimorphic, monoclinic sphenoidal.) One diad axis (always chosen as the *y* crystallographic axis, Fig. 182).

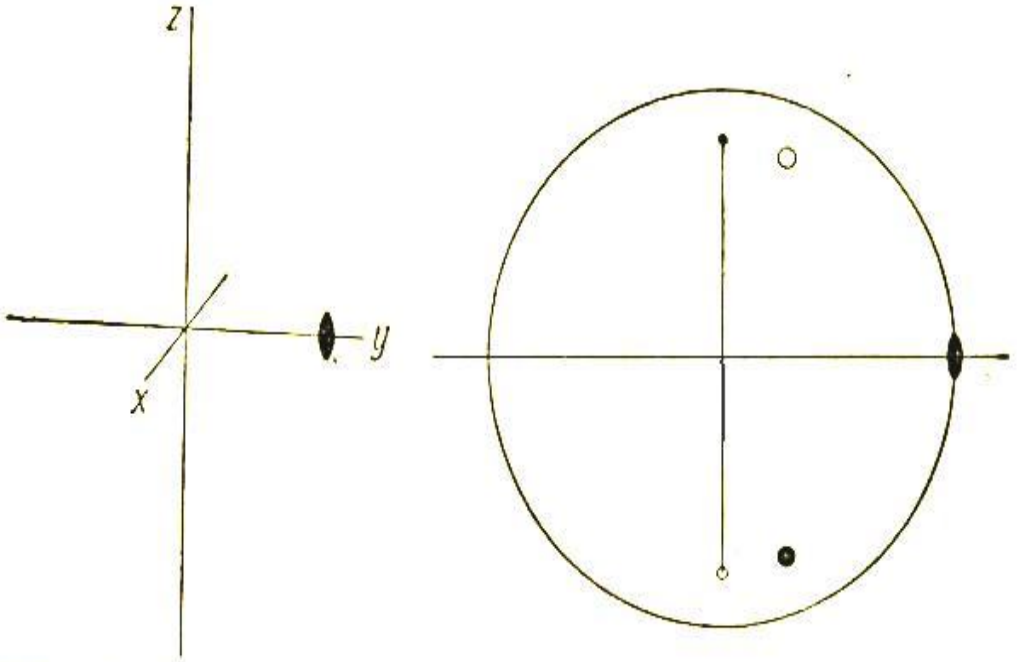


FIG. 182. Class 2; the crystallographic axes and a stereogram of the general form.

Special forms. Pedions {010}, {0$\bar{1}$0}.
Pinacoids {*h* 0 *l*}.

General forms. Sphenoids {*h k l*}.

Orthorhombic Class mm

CLASS *mm*. (Orthorhombic hemimorphic, orthorhombic pyramidal.) Two planes of symmetry at right-angles, intersecting in a diad axis (always chosen as the *z* crystallographic axis, Fig. 191).

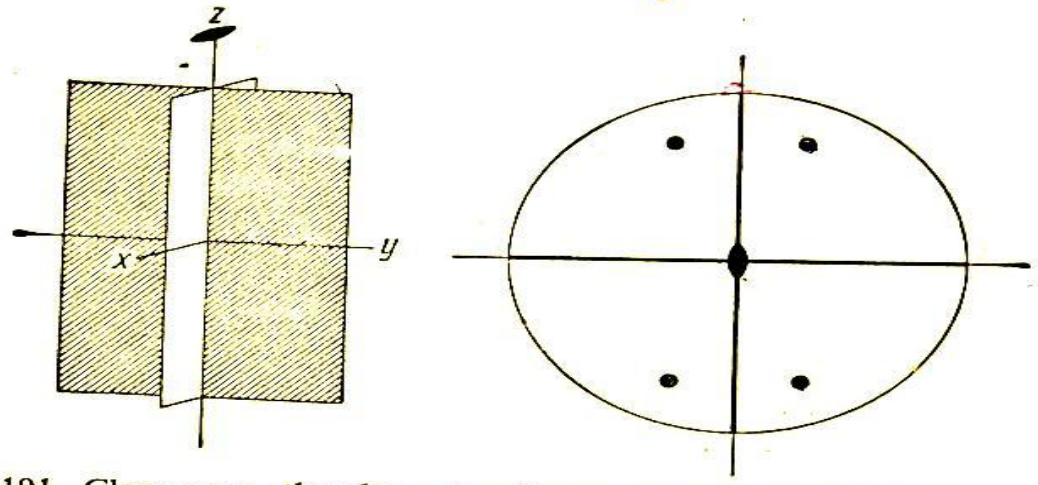
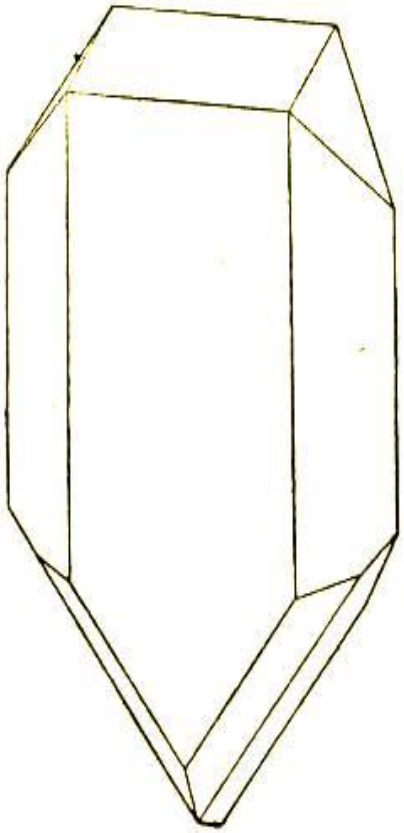


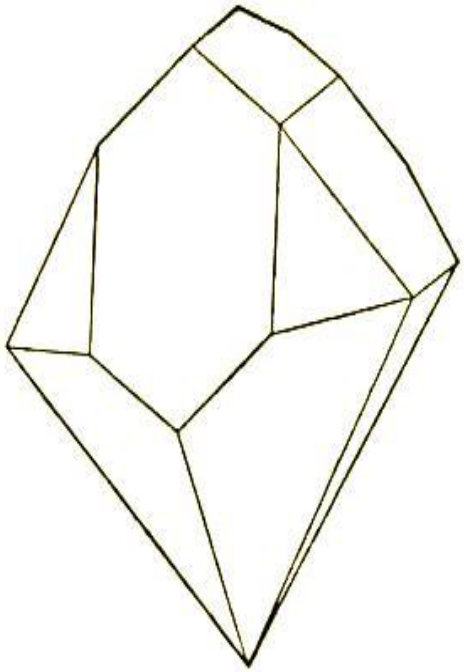
FIG. 191. Class *mm*; the elements of symmetry, crystallographic axes and a stereogram of the general form. The planes of symmetry, parallel to different pinacoids, are differently shaded.

Special forms. Pedions {001}, {00$\bar{1}$}.
Pinacoids {100}, {010}.
Prisms {*h k* 0}.
Domes {*h* 0 *l*}, {0 *k l*}.

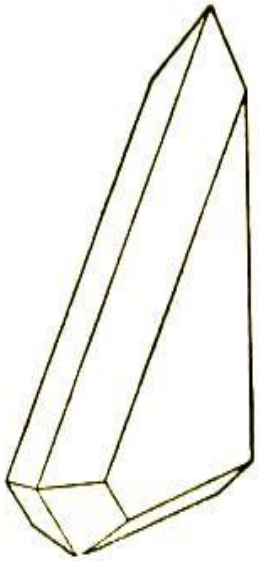
General forms. Pyramids {*h k l*}.



A crystal of potassium tetrathionate



A crystal of $Na_2SiO_2.5H_2O$



A crystal of hilgardite

CLASS 222. (Orthorhombic sphenoidal.) Three mutually perpendicular diad axes (Fig. 195), always chosen as the directions of the crystallographic axes x, y, z.

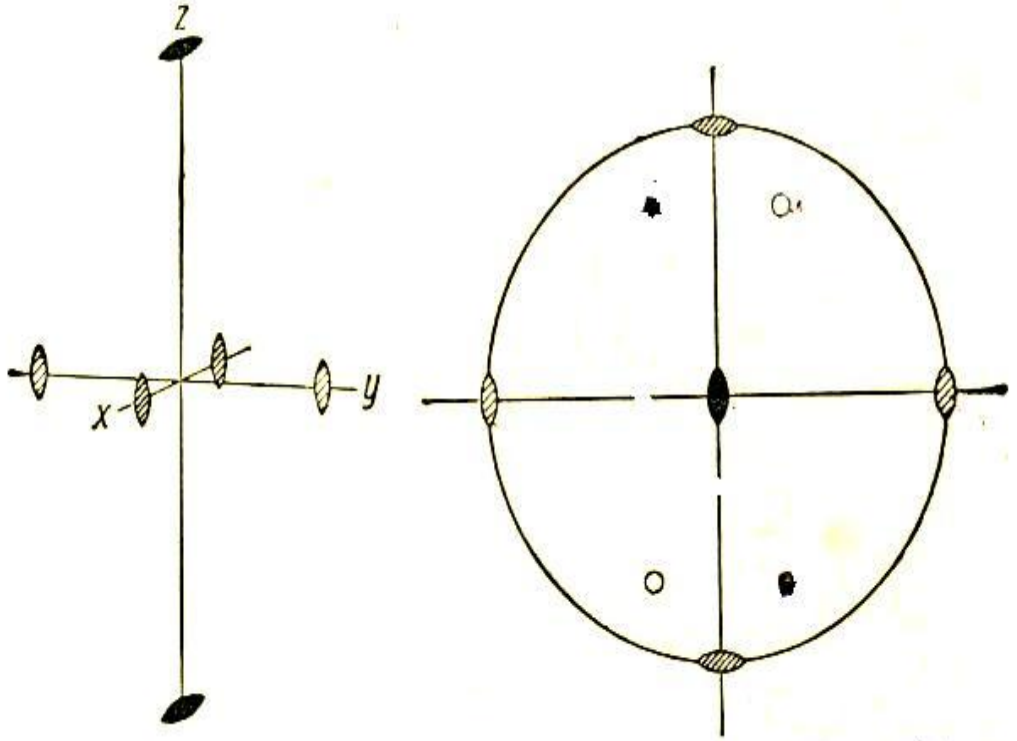


FIG. 195. Class 222; the elements of symmetry, crystallographic axes and a stereogram of the general form. The flags of the three diad axes, normal to different pinacoids, are differently shaded.

Special forms. Pinacoids {100}, {010}, {001}.
Prisms $\{h\,k\,0\}$, $\{0\,k\,l\}$, $\{h\,0\,l\}$.

General forms. Sphenoids $\{h\,k\,l\}$.

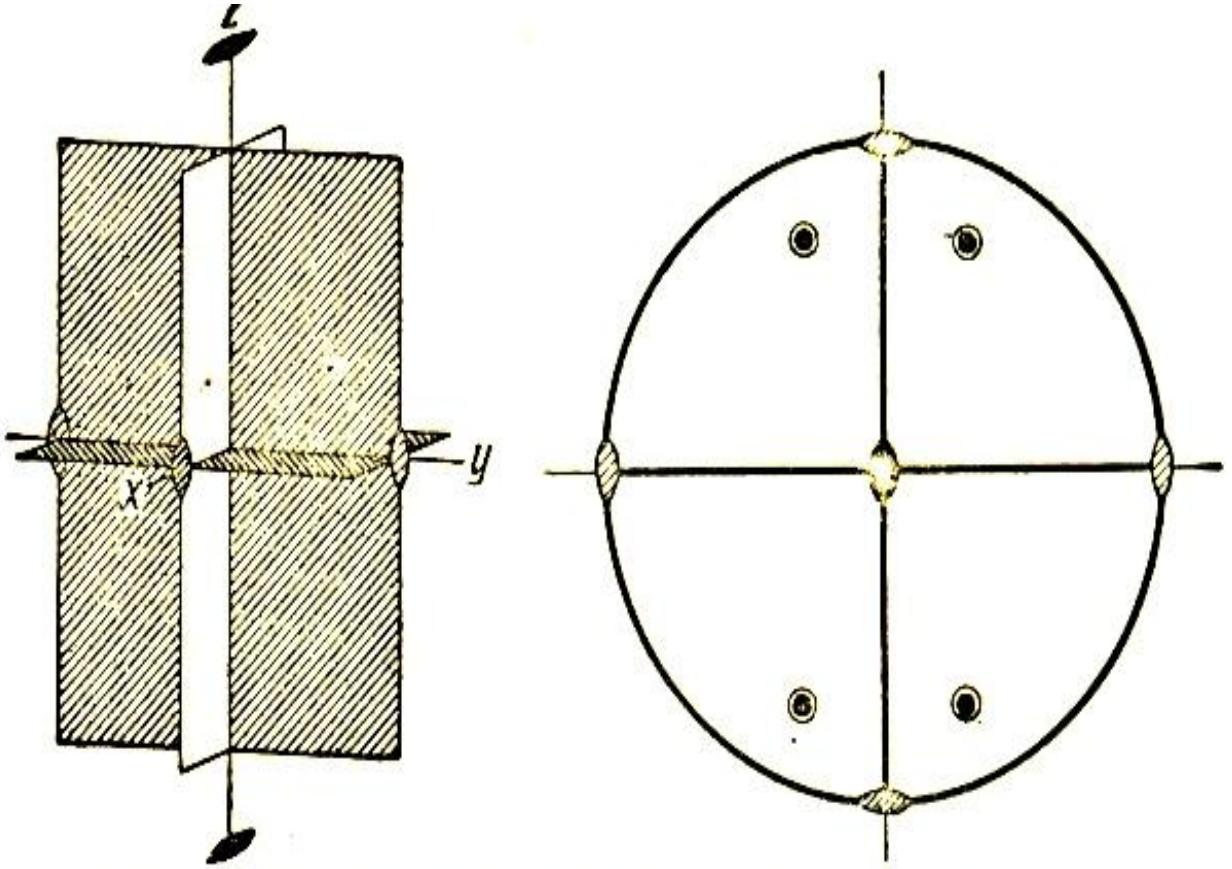


FIG. 202. Class *mmm*; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Pinacoids {100}, {010}, {001}.
Prisms {*h k* 0}, {*h* 0 *l*}, {0 *k l*}.

General forms. Bipyramids {*h k l*}.

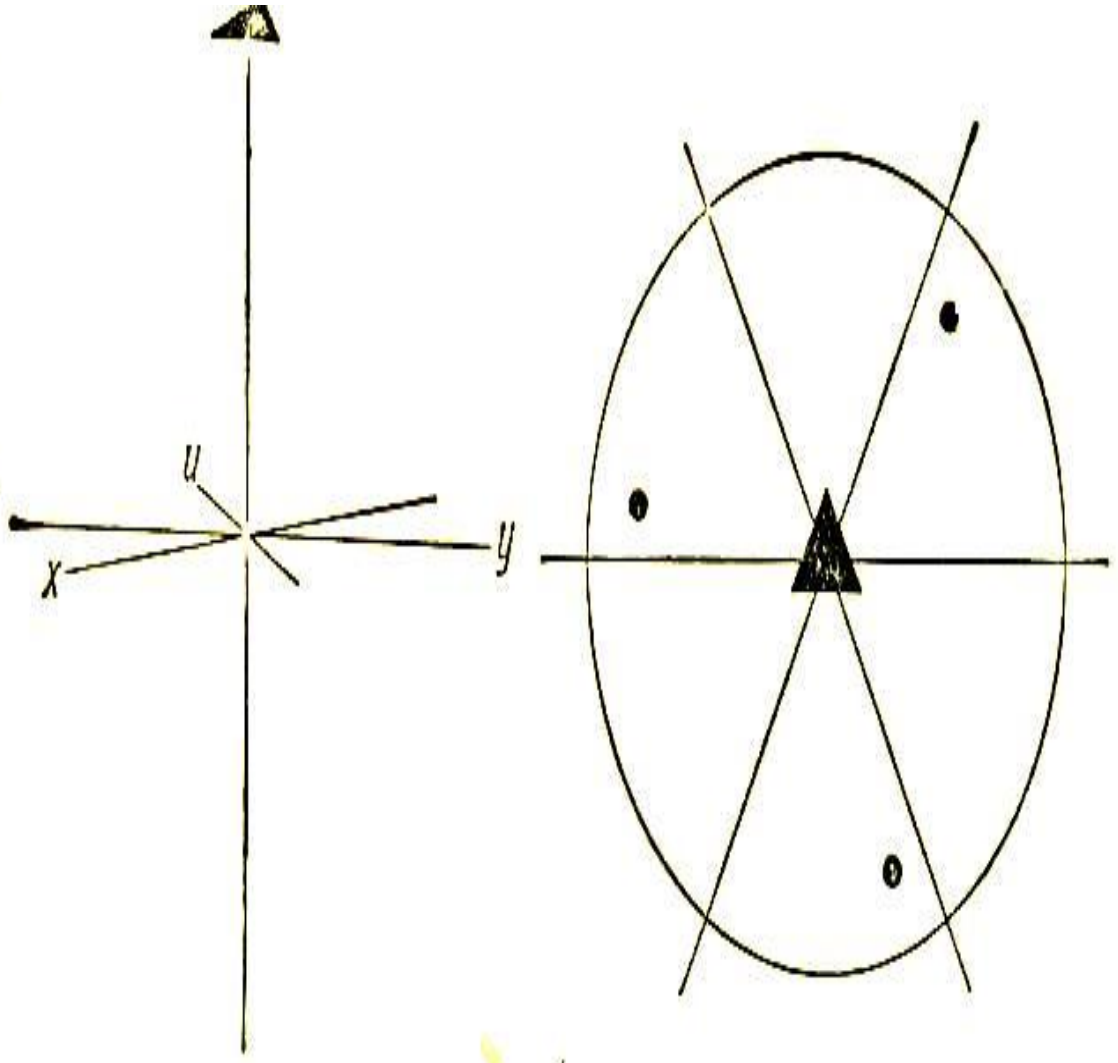


FIG. 203. Class 3; the crystallographic axes, showing their relationship to the triad axis, and a stereogram of the general form.

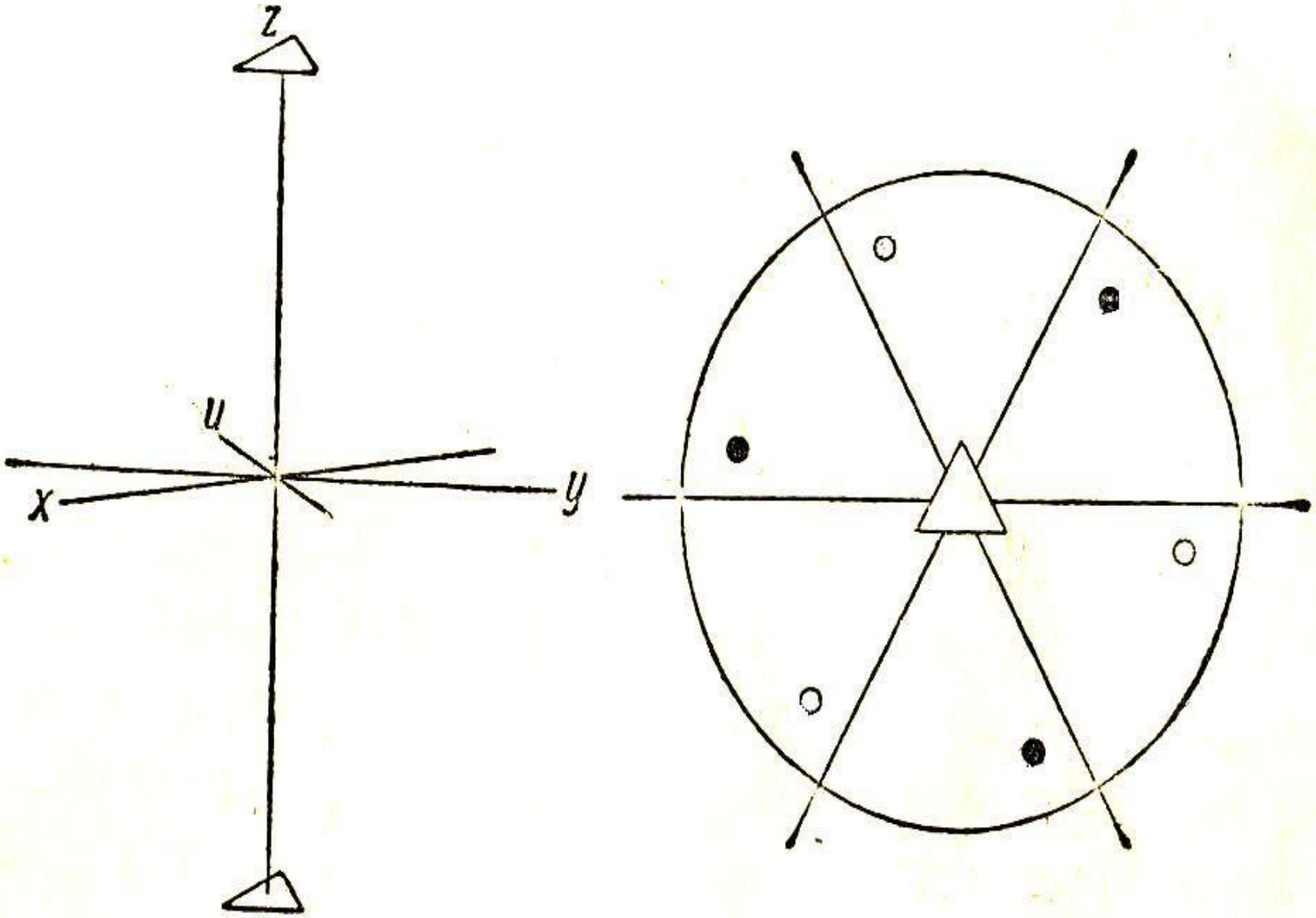


FIG. 206. Class $\bar{3}$; the inversion axis, flagged with open triangles, crystallographic axes and a stereogram of the general form.

substance illustrates a combination of eight different general forms.

CLASS 3*m*. (Ditrigonal hemimorphic, ditrigonal pyramidal.) Three vertical planes of symmetry intersecting in a triad axis (Fig. 210).

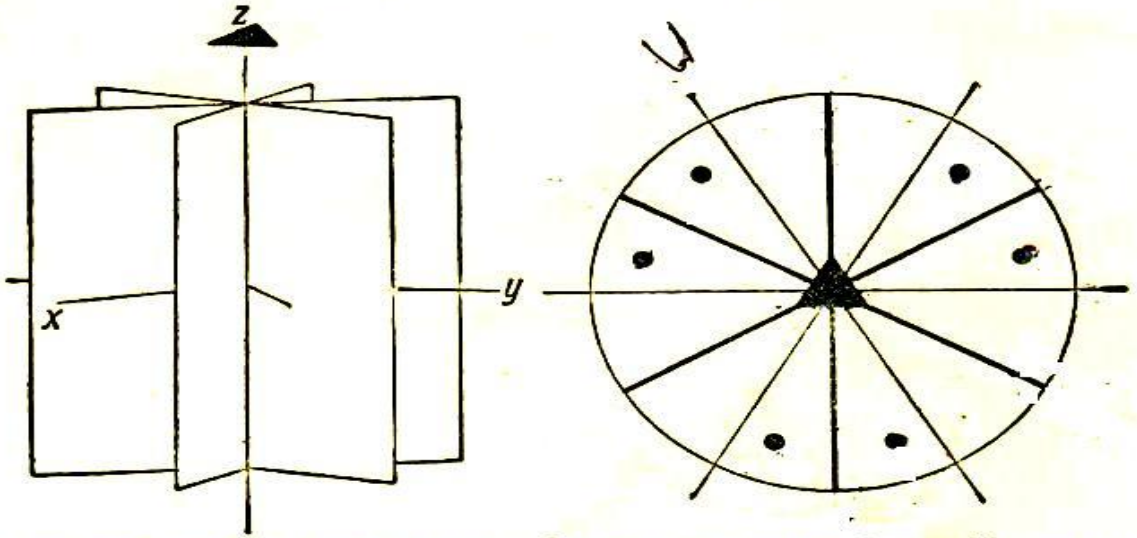


FIG. 210. Class 3*m*; the elements of symmetry, crystallographic axes and a stereogram of the general form.

		Miller-Bravais	*Miller*
Special forms.	Pedions	$\{0001\}$, $\{000\bar{1}\}$.	$\{111\}$, $\{\bar{1}\bar{1}\bar{1}\}$.
	Trigonal prisms	$\{10\bar{1}0\}$, $\{01\bar{1}0\}$.	$\{2\bar{1}\bar{1}\}$, $\{11\bar{2}\}$.
	Hexagonal prism	$\{11\bar{2}0\}$.	$\{10\bar{1}\}$.
	Ditrigonal prisms	$\{h\ k\ i\ 0\}$.	$\{p\ q\ -\overline{p+q}\}$.
	Trigonal pyramids	$\{h\ 0\ \bar{h}\ l\}$, $\{0\ k\ \bar{k}\ l\}$.	$\{p\ q\ q\}$, $\{p\ p\ q\}$.
	Hexagonal pyramids	$\{h\ h\ \overline{2h}\ l\}$.	$\{p\ q\ \overline{2q-p}\}$.
General forms.	Ditrigonal pyramids	$\{h\ k\ i\ l\}$.	$\{p\ q\ r\}$.

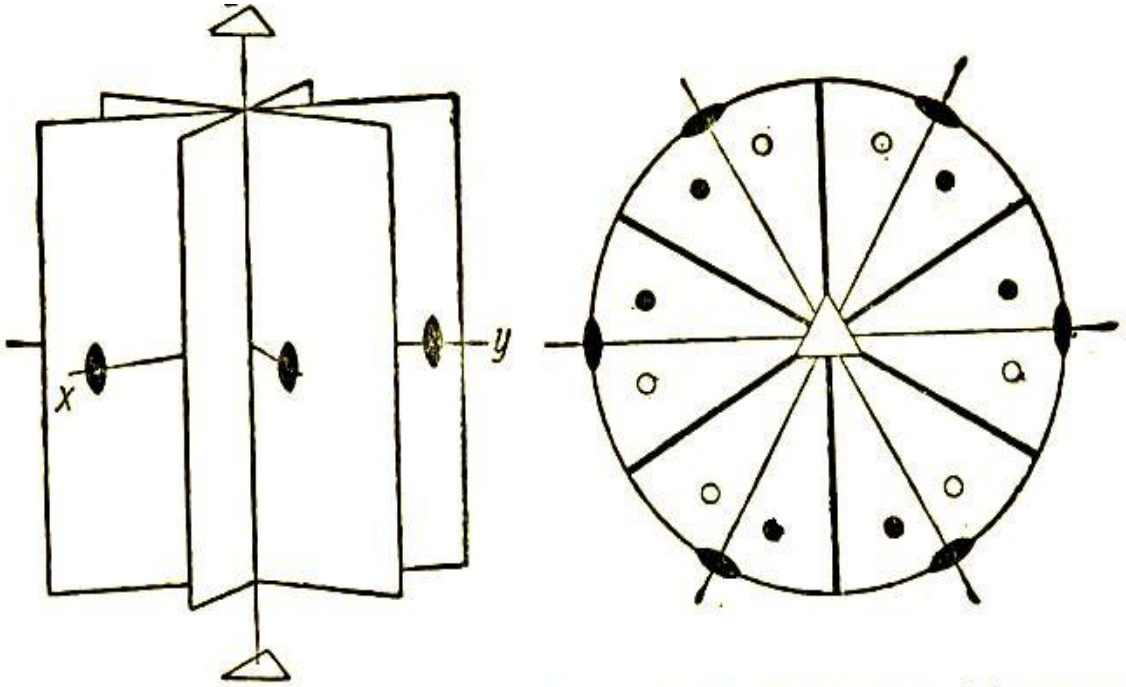


FIG. 214. Class $\bar{3}m$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

	Miller-Bravais	*Miller*
Special forms.		
Pinacoid	$\{0001\}$.	$\{111\}$.
Hexagonal prisms	$\{10\bar{1}0\}$, $\{11\bar{2}0\}$.	$\{2\bar{1}\bar{1}\}$, $\{10\bar{1}\}$.
Dihexagonal prisms	$\{h\,k\,i\,0\}$.	$\{p\,q\,\overline{-p+q}\}$.
Rhombohedra	$\{h\,0\,\bar{h}\,l\}$, $\{0\,k\,\bar{k}\,l\}$.	$\{p\,q\,q\}$, $\{p\,p\,q\}$.
Hexagonal bipyramids	$\{h\,h\,\overline{2h}\,l\}$.	$\{p\,q\,\overline{2q-p}\}$.
General Forms.		

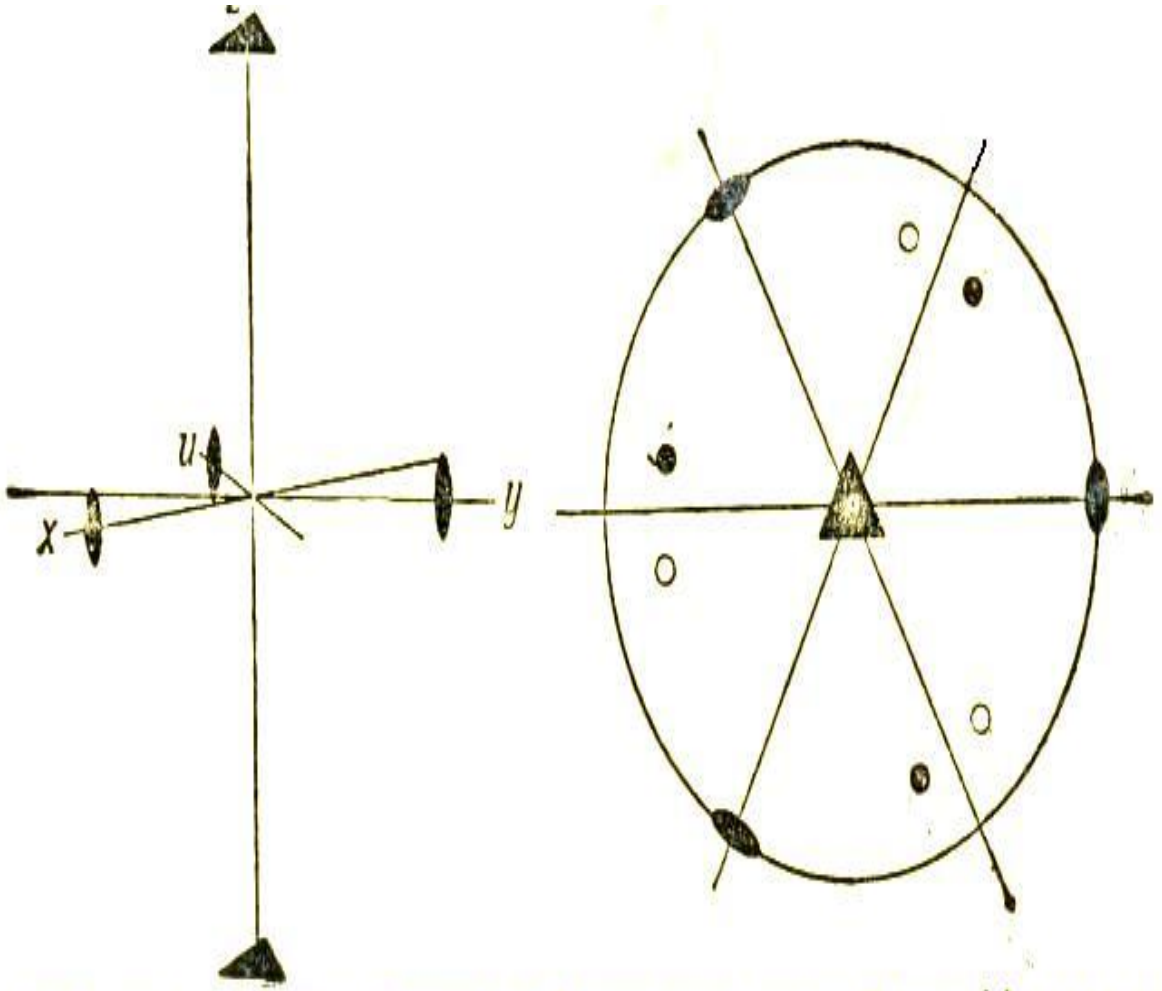
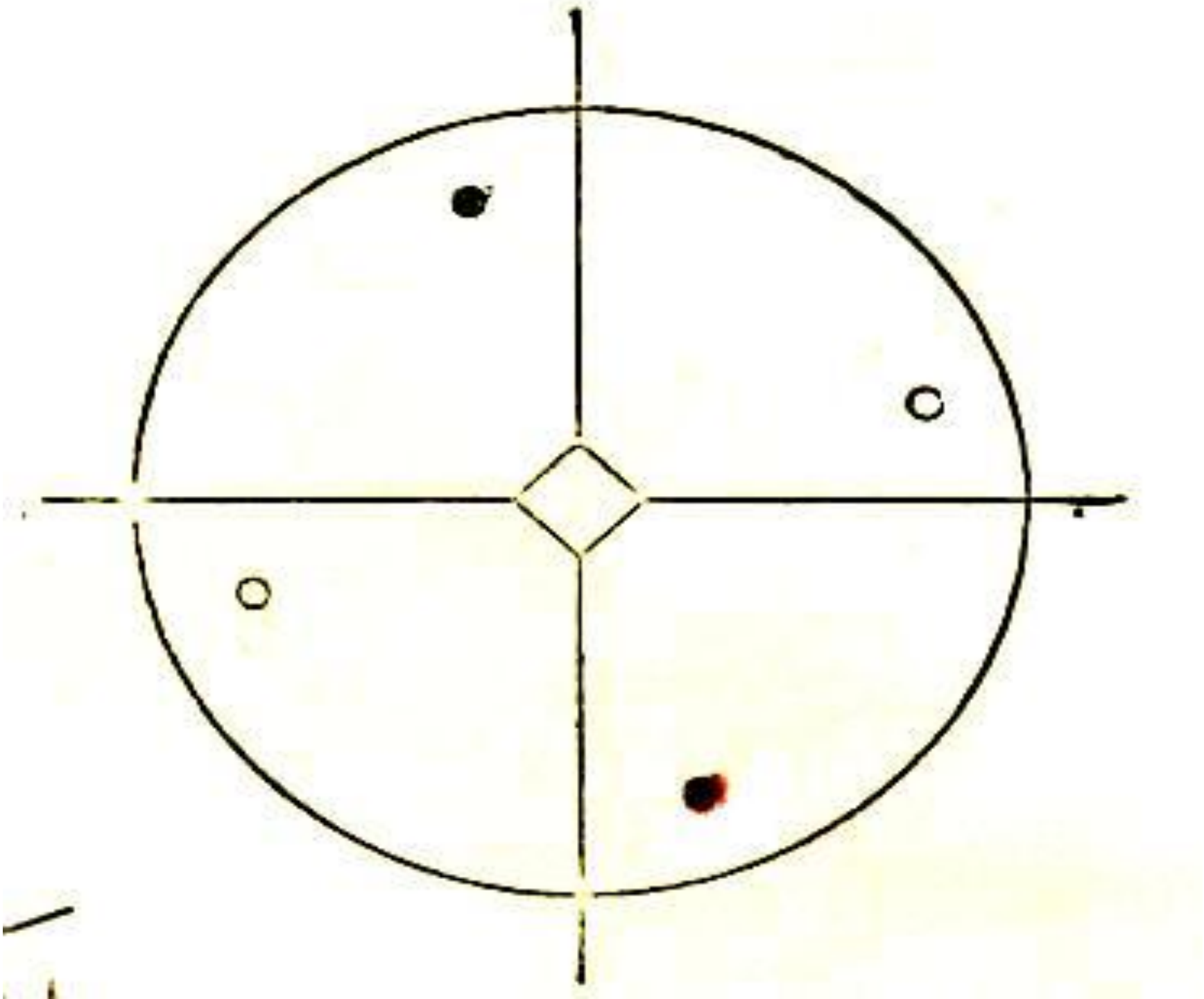


FIG. 215. Class 32; the elements of symmetry, crystallographic axes and a stereogram of the general form.



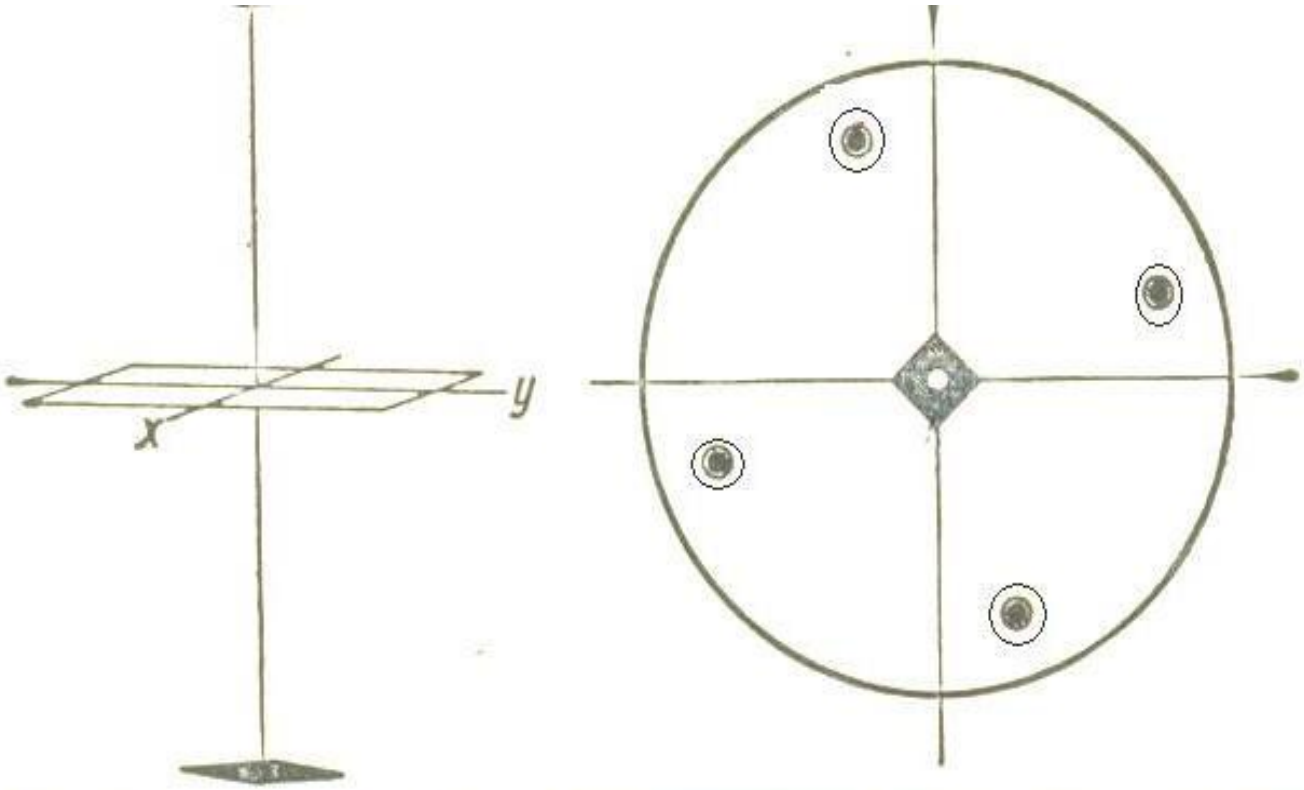


FIG. 229. Class $4/m$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Pinacoid {001}.
Tetragonal prisms $\{h\,k\,0\}$.

General forms. Tetragonal bipyramids $\{h\,k\,l\}$.

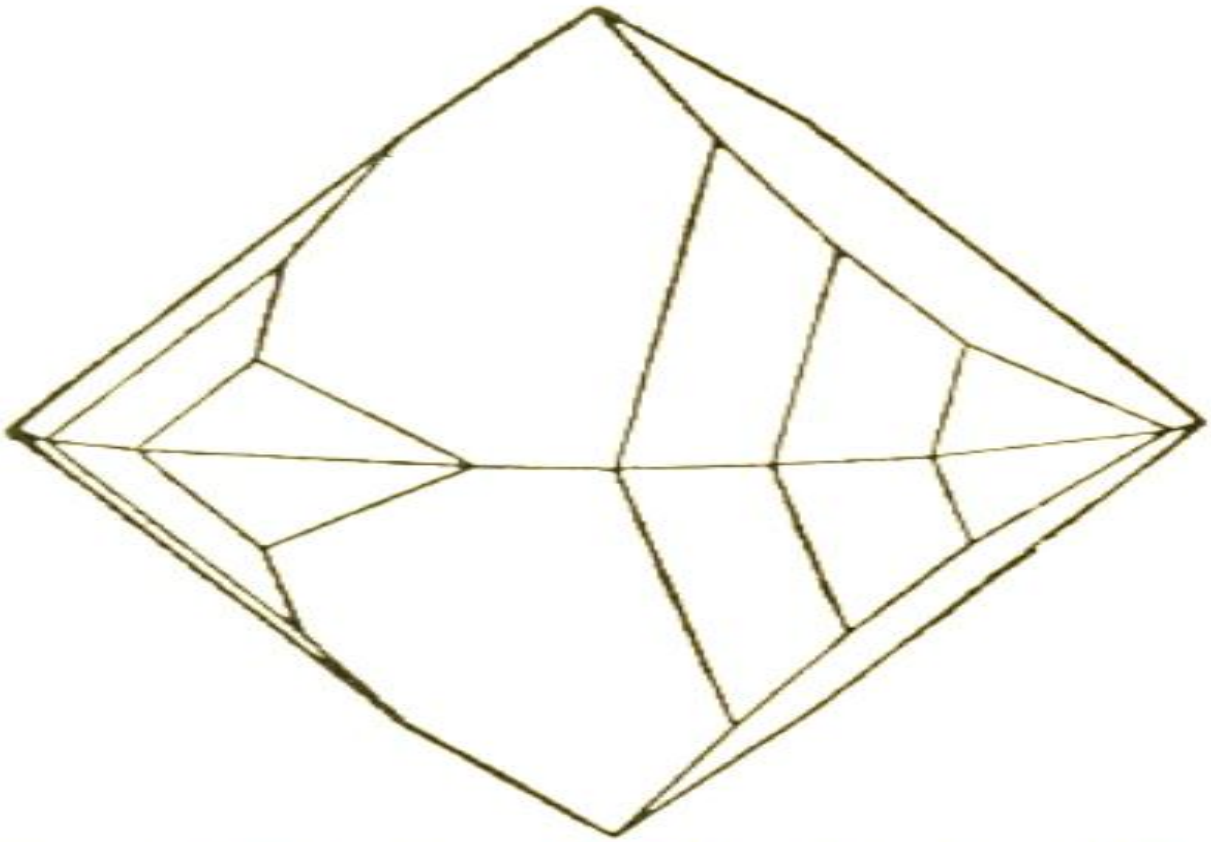


FIG. 230. A crystal of scheelite.

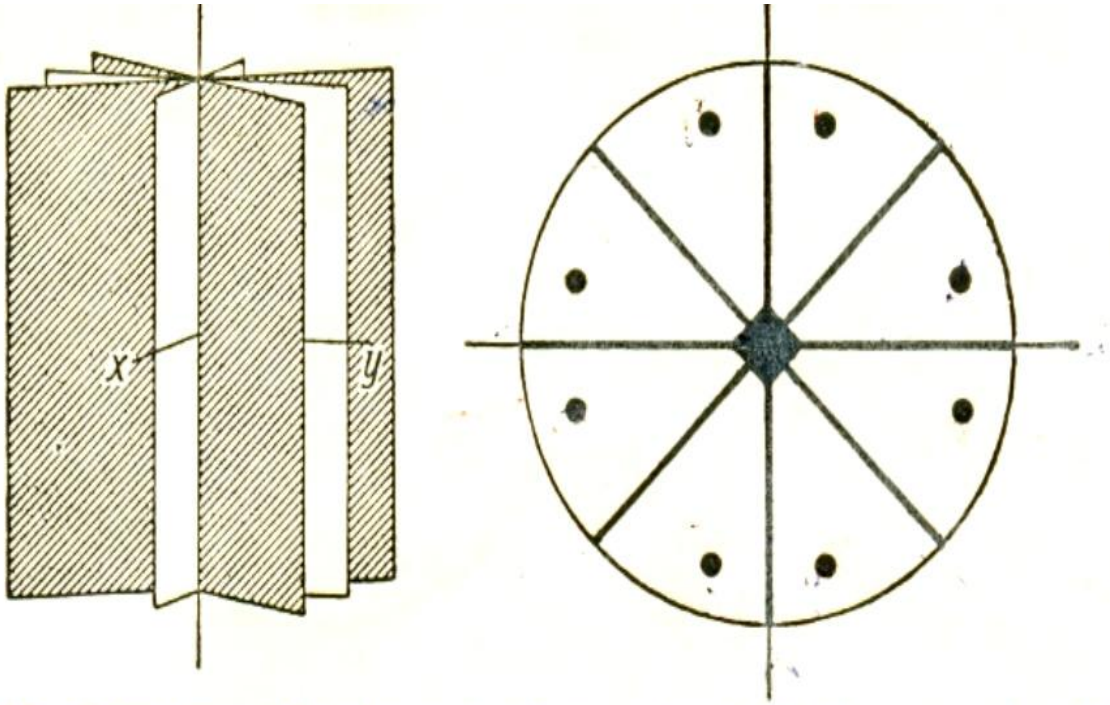


FIG. 232. Class 4*mm*; the elements of symmetry (with the two pairs of planes of symmetry differently shaded), crystallographic axes and a stereogram of the general form.

Special forms. Pedions {001}, {00$\bar{1}$}.
Tetragonal prisms {100}, {110}.
Ditetragonal prisms {*h k* 0}.
Tetragonal pyramids {*h* 0 *l*}, {*h h l*}.

General forms. Ditetragonal pyramids {*h k l*}.

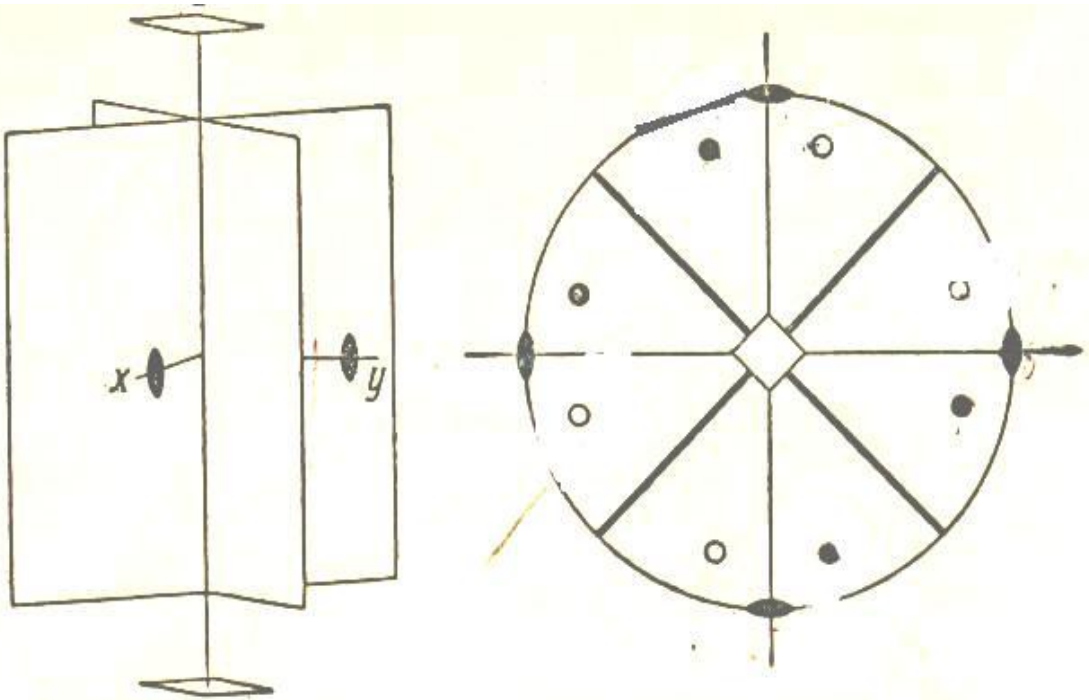


FIG. 234. Class $\bar{4}2m$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Pinacoid {001}.
Tetragonal prisms {100}, {110}.
Ditetragonal prisms $\{h\,k\,0\}$.
Tetragonal bipyramids $\{h\,0\,l\}$.
Sphenoids $\{h\,h\,l\}$, $\{h\,\bar{h}\,l\}$.

General forms. Bisphenoids $\{h\,k\,l\}$.

CLASS 42. (Tetragonal trapezohedral.) A tetrad axis normal to two pairs of diad axes (Fig. 242).

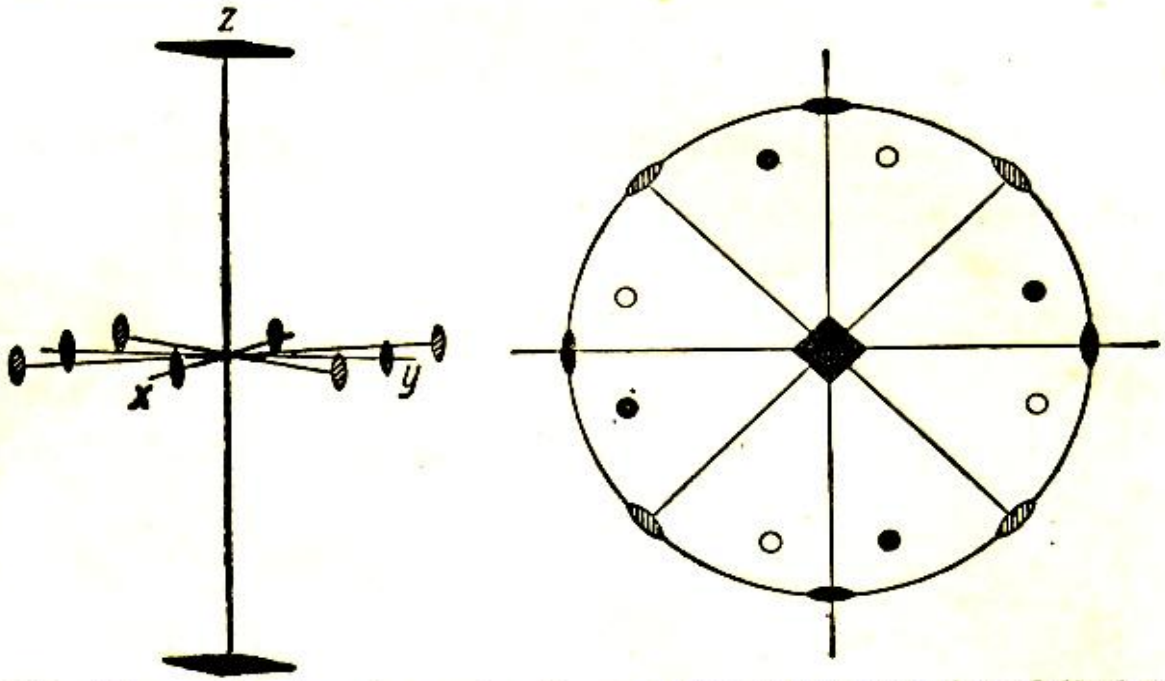


FIG. 242. Class 42; the elements of symmetry (the two pairs of diad axes have differently shaded flags), crystallographic axes and a stereogram of the general form.

Special forms. Pinacoid {001}.
Tetragonal prisms {100}, {110}.
Ditetragonal prisms {*h k* 0}.
Tetragonal bipyramids {*h* 0 *l*}, {*h h l*}.

General forms. Tetragonal trapezohedra {*h k l*}.

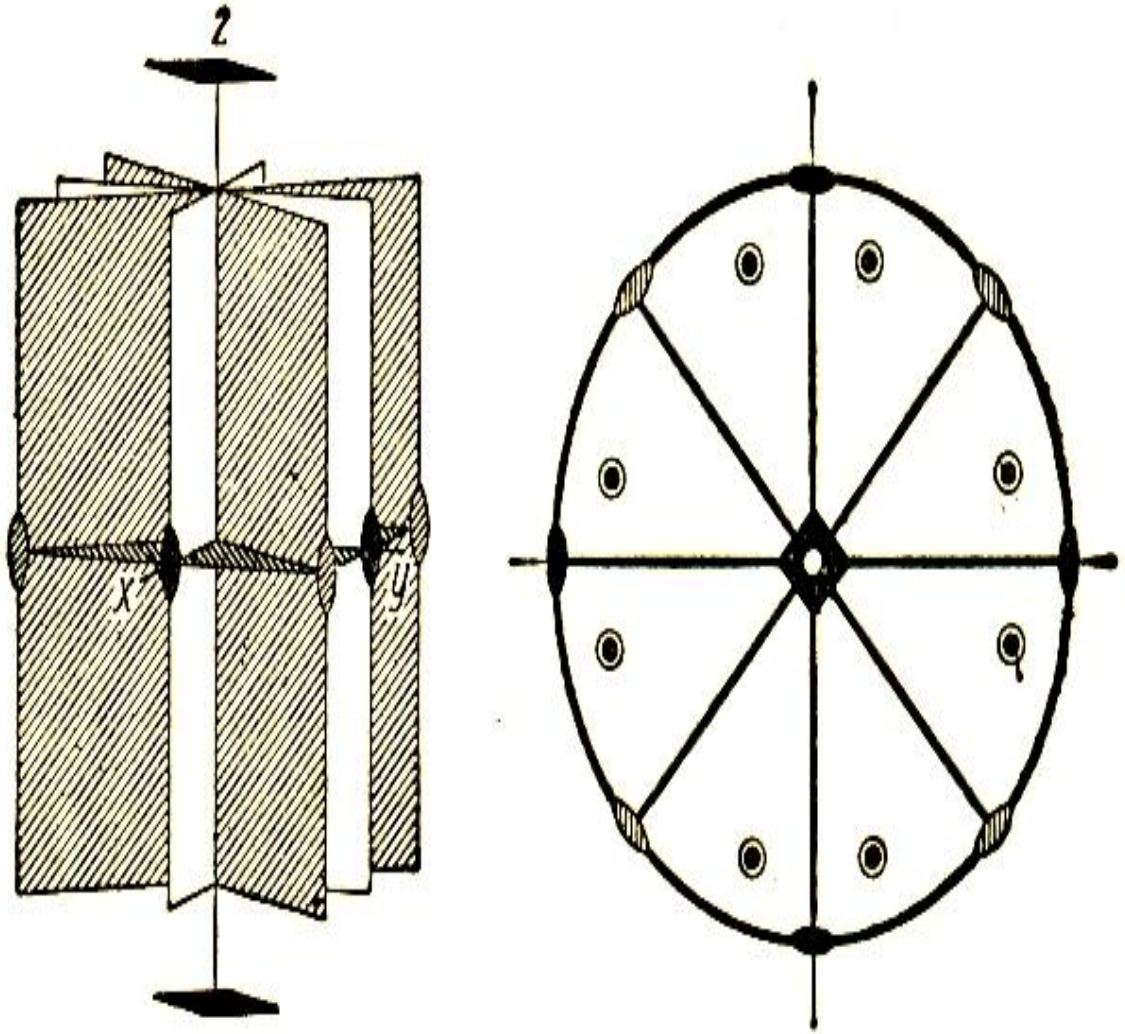


FIG. 247. Class 4/*mmm*; the elements of symmetry, crystallographic axes and a stereogram of the general form.

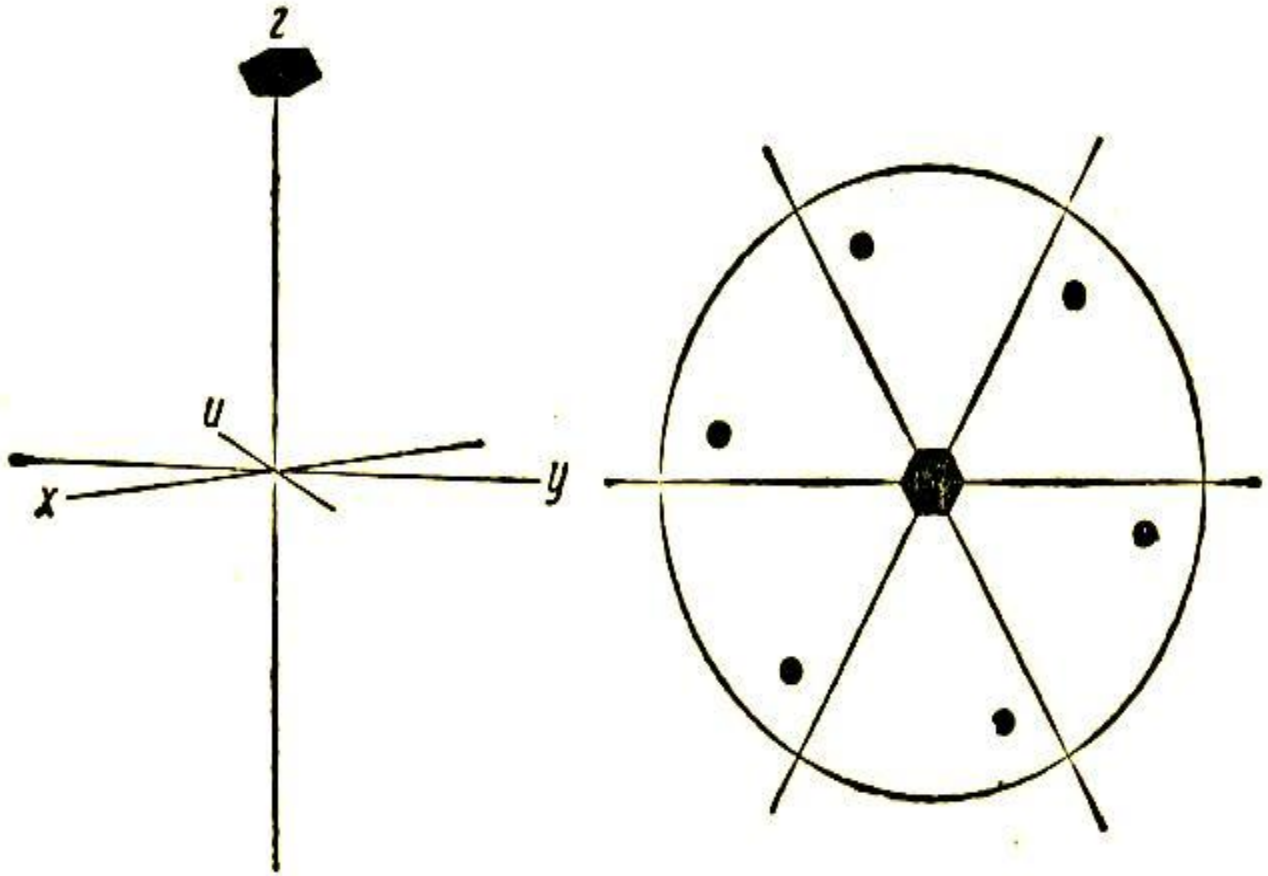


FIG. 248. Class 6; the crystallographic axes and a stereogram of the general form.

Special forms. Pedions $\{0001\}$, $\{000\bar{1}\}$.
Hexagonal prisms $\{h\,k\,i\,0\}$.

General forms. Hexagonal pyramids $\{h\,k\,i\,l\}$.

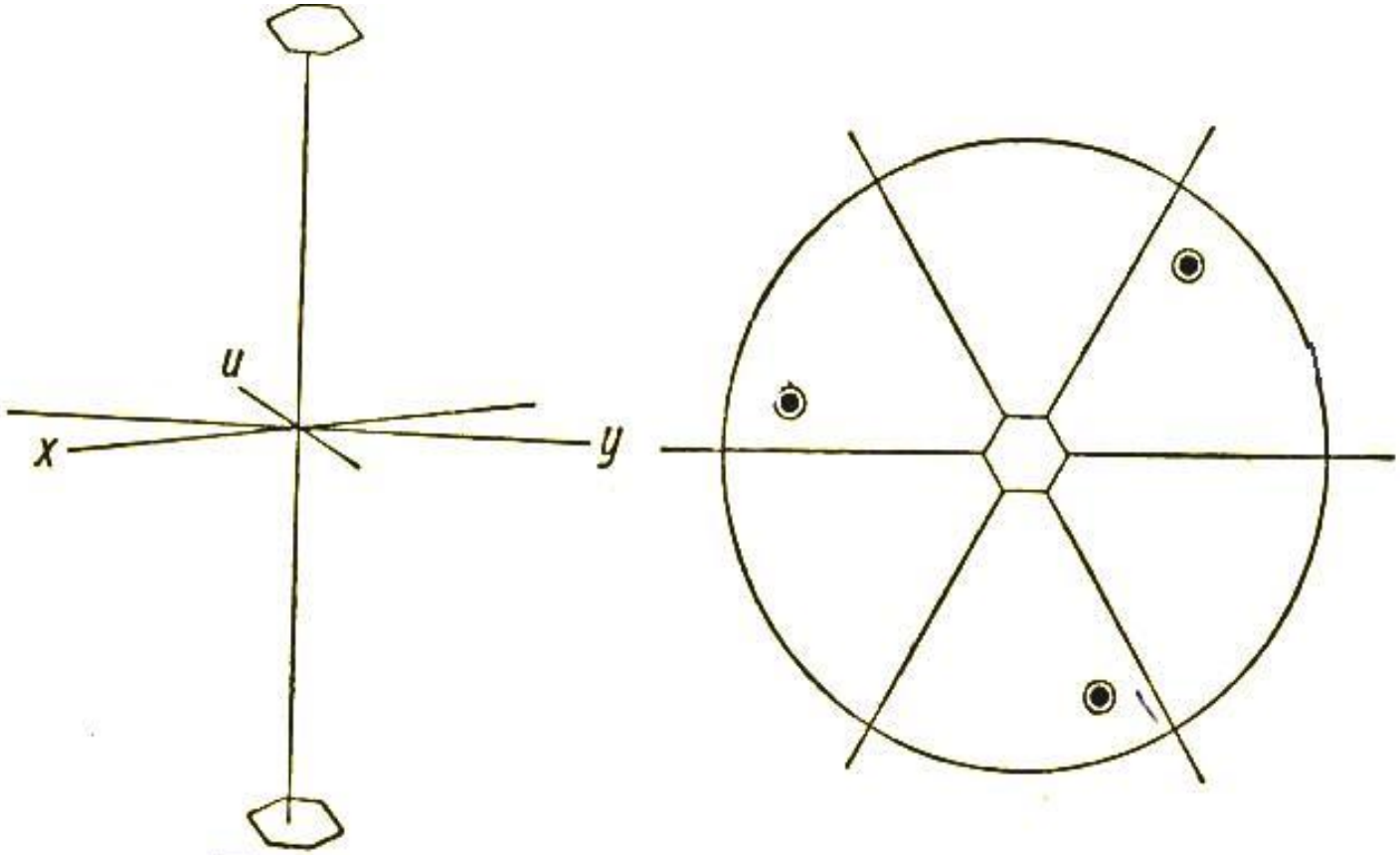


FIG. 252. Class $\bar{6}$; the inversion axis (flagged with open hexagons), crystallographic axis and a stereogram of the general form.

Special forms. Pinacoid {0001}.
Trigonal prisms $\{h\ k\ i\ 0\}$.

General forms. Trigonal bipyramids $\{h\ k\ i\ l\}$.

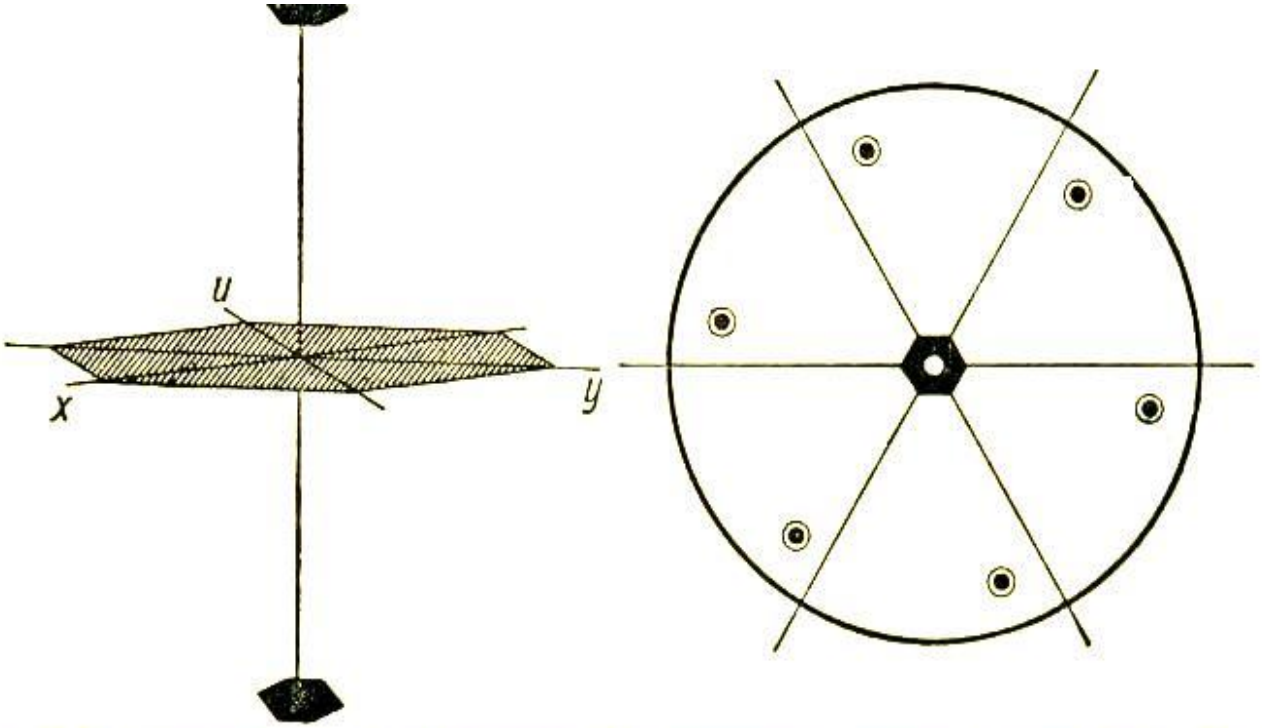


FIG. 253. Class $6/m$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

CLASS $6/m$. (Hexagonal bipyramidal.) A hexad axis normal to a plane of symmetry, and a centre (Fig. 253).

Special forms. Pinacoid {0001}.
Hexagonal prisms $\{h\ k\ i\ 0\}$.

General forms. Hexagonal bipyramids $\{h\ k\ i\ l\}$.

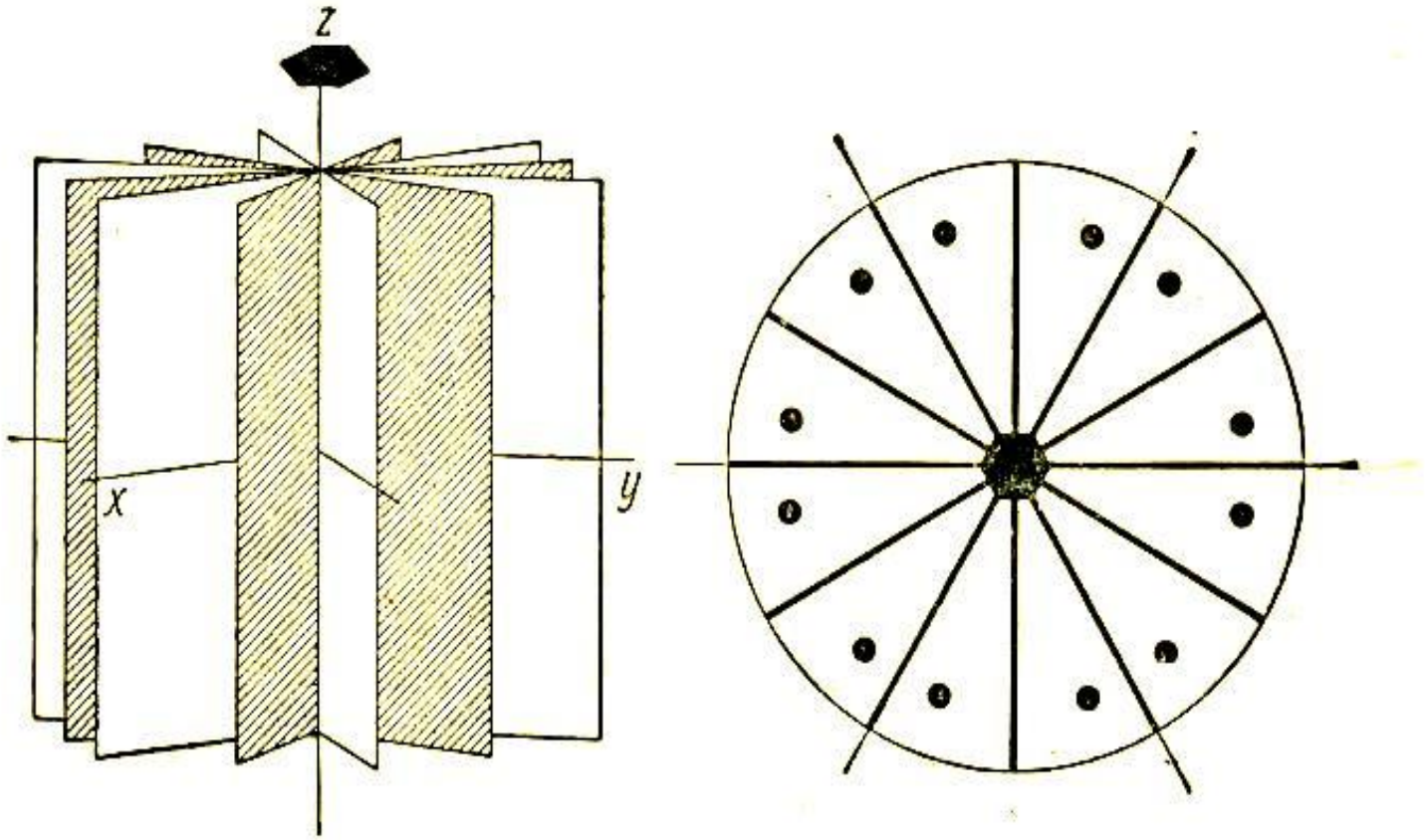


FIG. 255. Class 6*mm*; the elements of symmetry (the two families of planes of symmetry are differently shaded), crystallographic axes and a stereogram of the general form.

Special forms. Pedions $\{0001\}$, $\{000\bar{1}\}$.
Hexagonal $\{10\bar{1}0\}$, $\{11\bar{2}0\}$.
Dihexagonal prisms $\{h\,k\,i\,0\}$.
Hexagonal pyramids $\{h\,0\,\bar{h}\,l\}$, $\{h\,h\,\overline{2h}\,l\}$.

General forms. Dihexagonal pyramids $\{h\,k\,i\,l\}$.

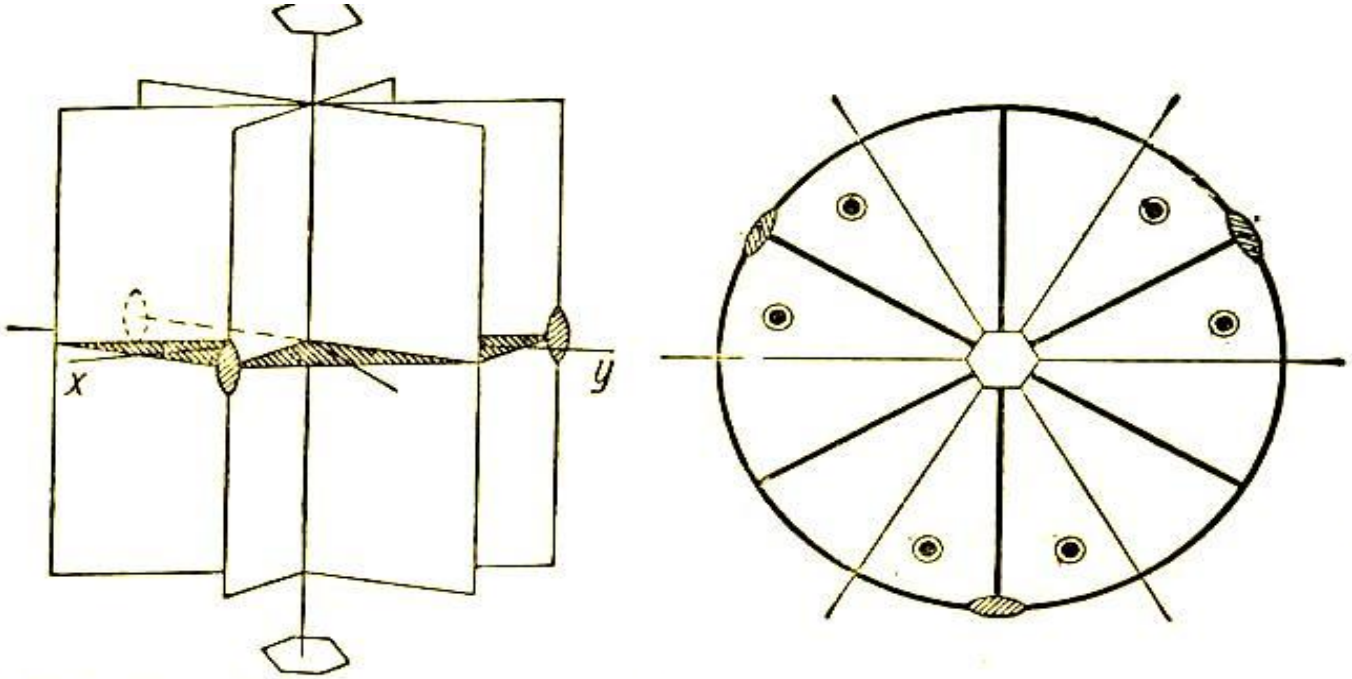


FIG. 259. Class $\bar{6}m2$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Pinacoid {0001}
Trigonal prisms $\{10\bar{1}0\}$, $\{01\bar{1}0\}$.
Hexagonal prism $\{11\bar{2}0\}$.
Ditrigonal prisms $\{h\ k\ i\ 0\}$.
Trigonal bipyramids $\{h\ 0\ \bar{h}\ l\}$, $\{0\ k\ \bar{k}\ l\}$.
Hexagonal bipyramids $\{h\ h\ \overline{2h}\ l\}$.

General forms. Ditrigonal bipyramids $\{h\ k\ i\ l\}$.

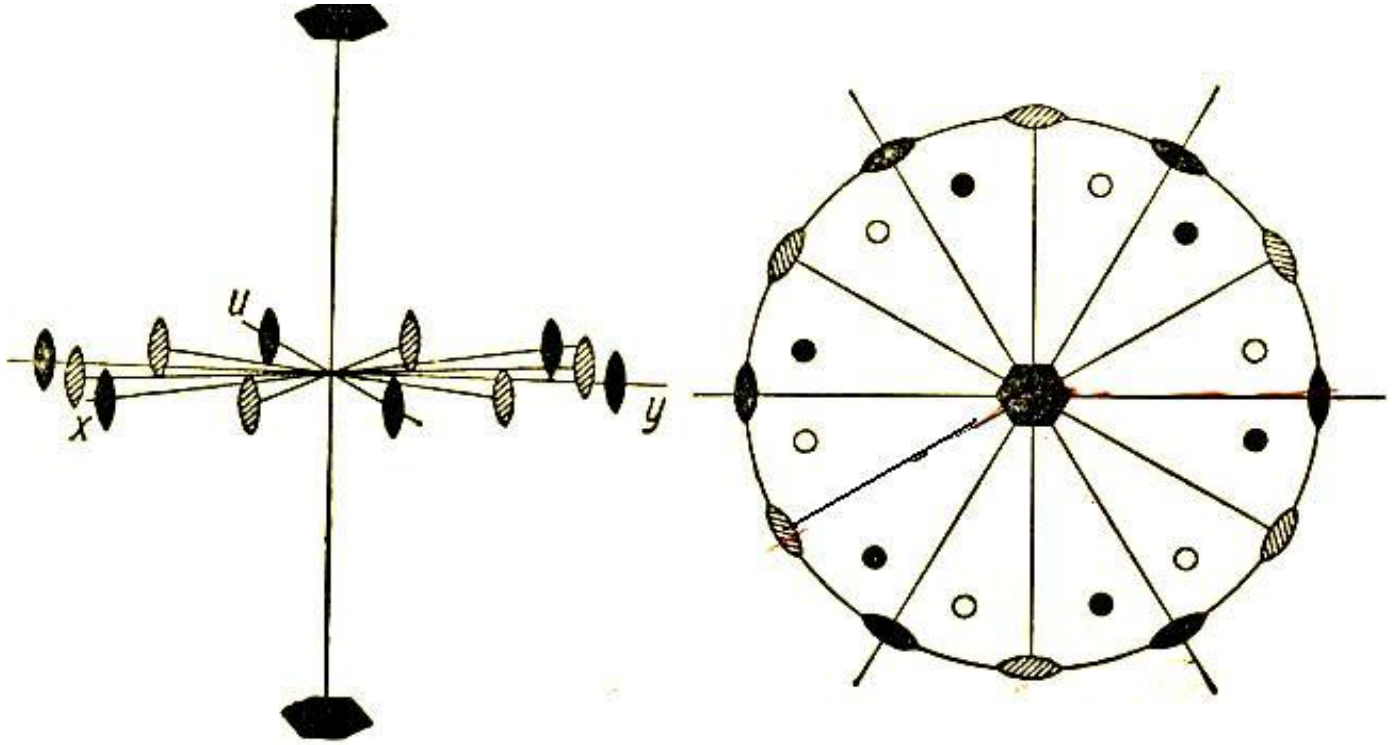


FIG. 262. Class 62; the elements of symmetry, crystallographic axes and stereogram of the general form.

Special forms. Pinacoid {0001}
Hexagonal prisms $\{10\bar{1}0\}$, $\{11\bar{2}0\}$.
Dihexagonal prisms $\{h\ k\ i\ 0\}$.
Hexagonal bipyramids $\{h\ 0\ \bar{h}\ l\}$, $\{hh\ \overline{2h}\ l\}$.

General forms. Hexagonal trapezohedra $\{h\ k\ i\ l\}$.

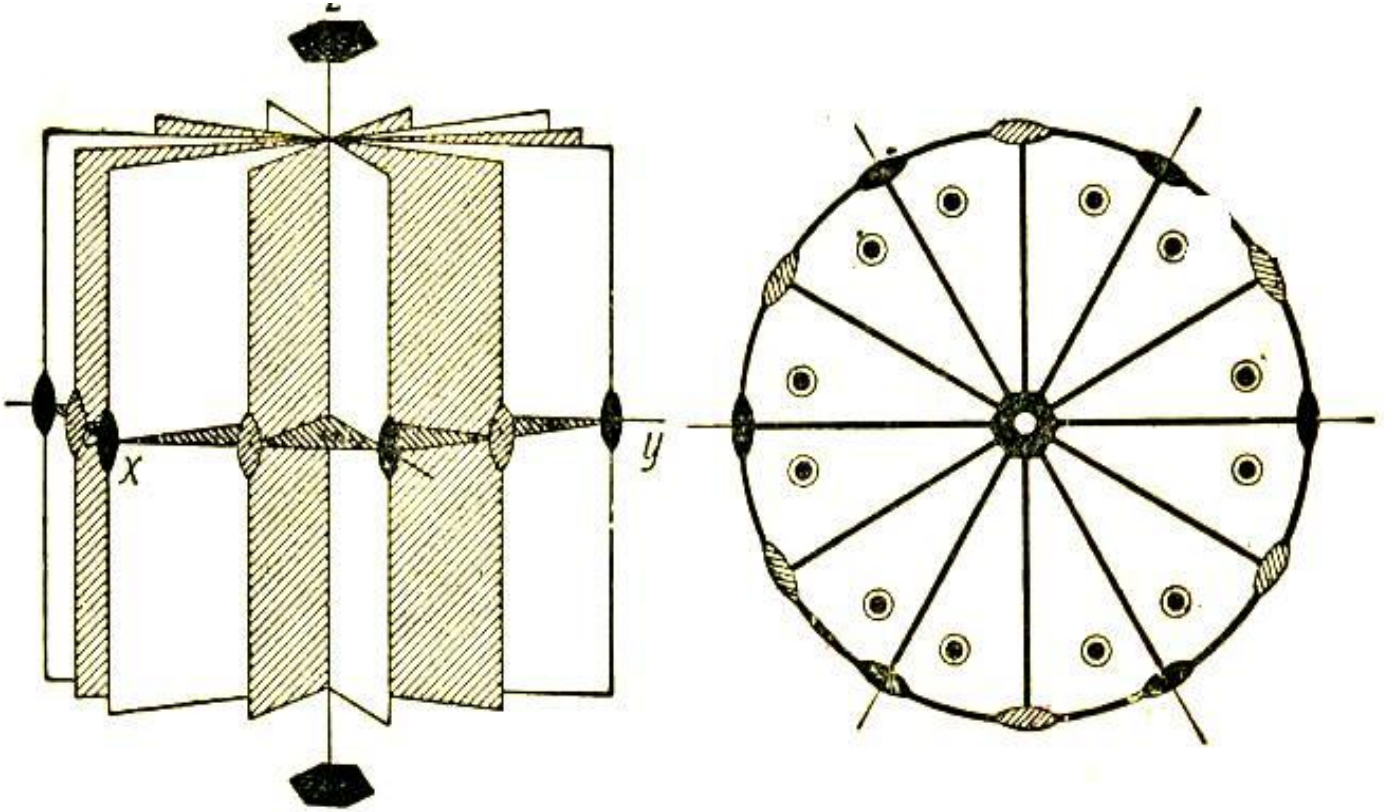


FIG. 204. Class 6/*mmm*; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Pinacoid {0001}.
Hexagonal prisms $\{10\bar{1}0\}$, $\{11\bar{2}0\}$.
Dihexagonal prisms $\{h\ k\ i\ 0\}$.
Hexagonal bipyramids $\{h\ 0\ \bar{h}\ l\}$, $\{h\ h\ \overline{2h}\ l\}$.

General forms. Dihexagonal bipyramids $\{h\ k\ i\ l\}$.

Different combination of The Cubic Point group

in the first column below represent all the different combinations possible:

Full symbol	*Abbreviated symbol*
23	23
432	43
2*m*3	*m*3
$\bar{4}$3*m*	$\bar{4}$3*m*
4*m*3*m*2	*m*3*m*

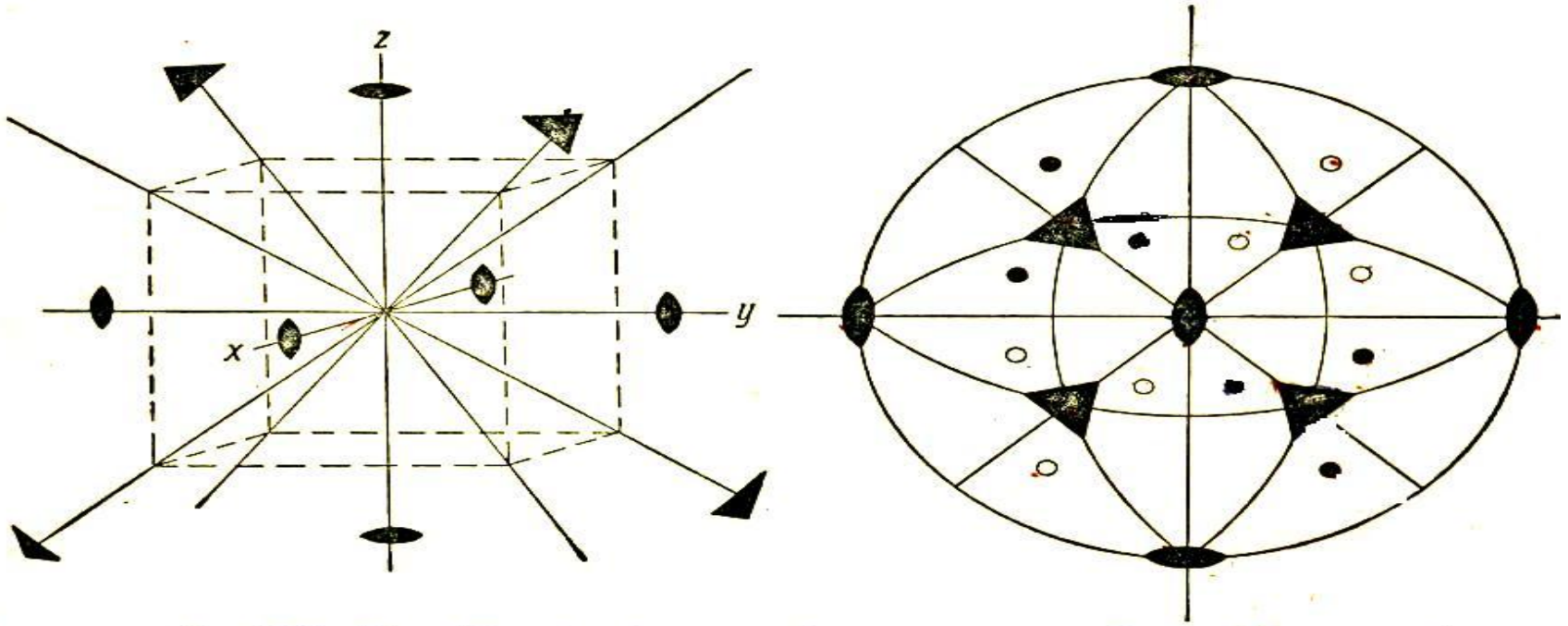


FIG. 266. Class 23; the elements of symmetry, crystallographic axes and a stereogram of the general form. The outline of the cube has been added to the drawing of the elements of symmetry to help in visualising their arrangement in space.

Special forms. Cube {100}.
Rhombic dodecahedron {110}.
Pentagonal dodecahedra {*h k* 0}, {*k h* 0}.
Tetrahedra {111}, {1$\bar{1}$1}.
Tristetrahedra {*h l l*}, {*h* $\bar{l}$ *l*}.
Deltoid dodecahedra {*h h l*}, {*h* $\bar{h}$ *l*}.

General forms. Tetrahedral pentagonal dodecahedra {*h k l*}.

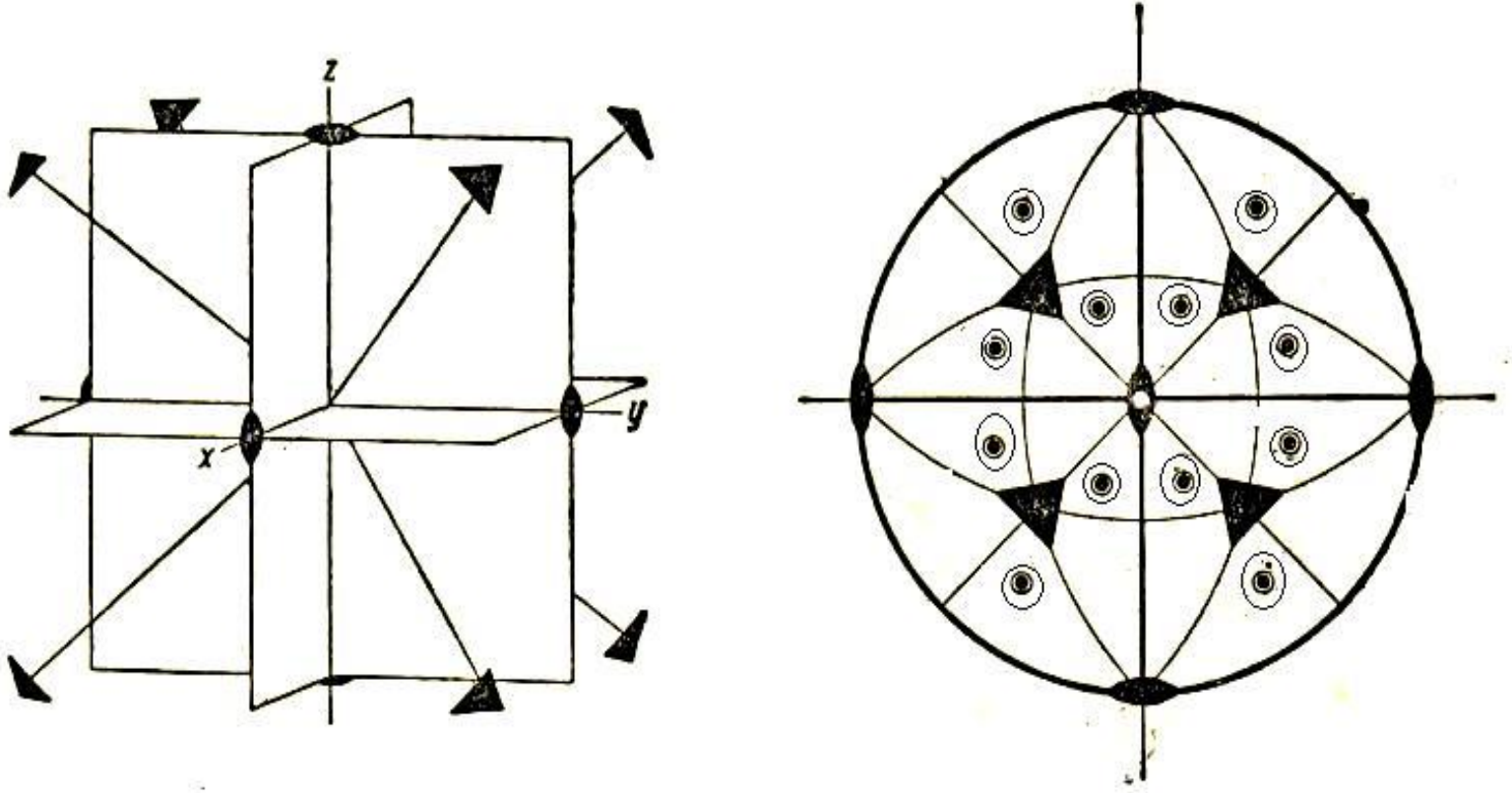
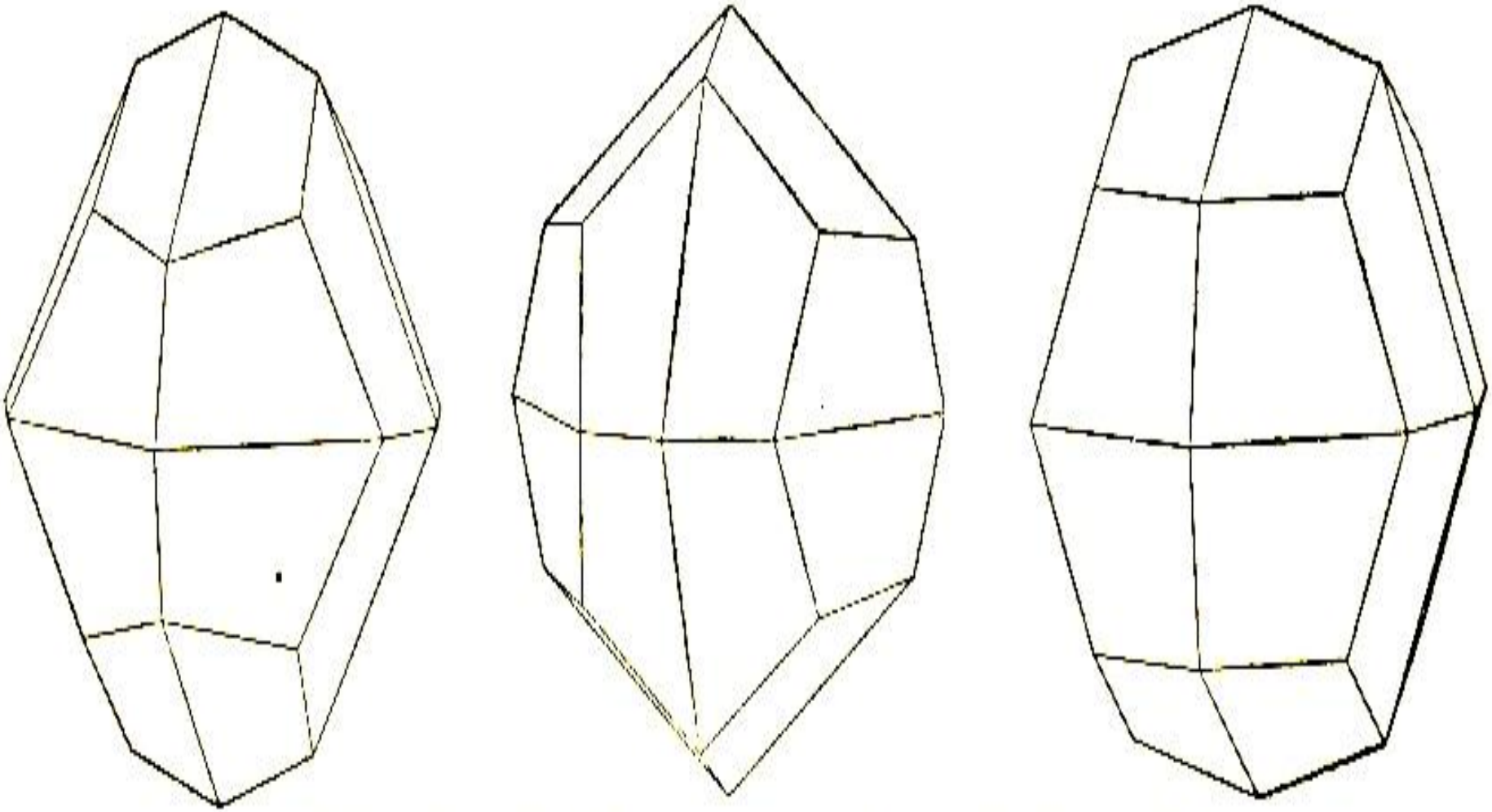


FIG. 279. Class $m3$; the elements of symmetry, crystallographic axes and a stereogram of the general form.

Special forms. Cube {100}.
Rhombic dodecahedron {110}.
Pentagonal dodecahedra {$h\,k\,0$}, {$k\,h\,0$}.
Octahedron {111}.
Icositetrahedra {$h\,l\,l$}.
Trisoctahedra {$h\,h\,l$}.

General forms. Di(akis)dodecahedra {$h\,k\,l$}.



FIGS. 280-282. Didodecahedra {321} {312} {421}.

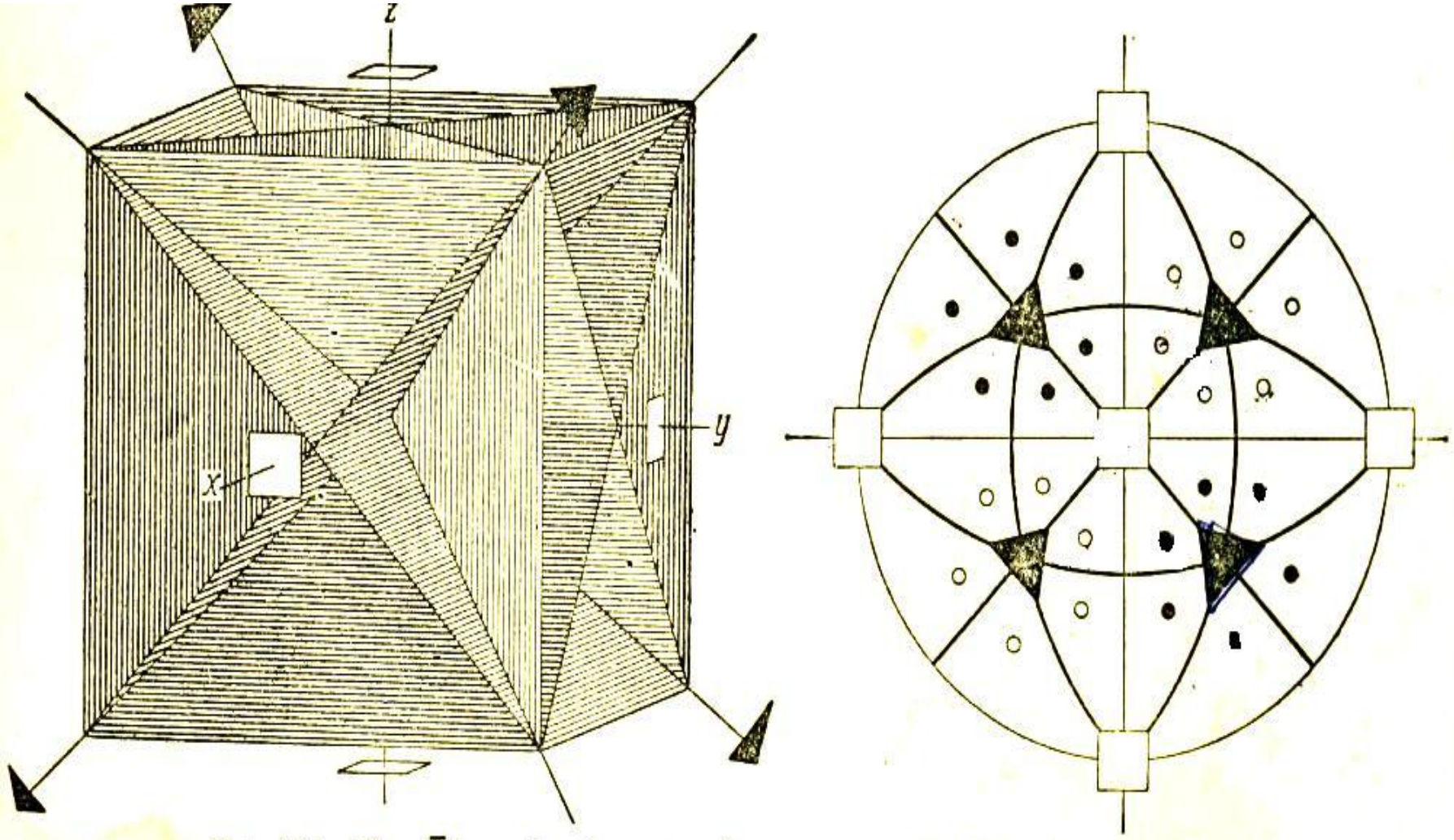
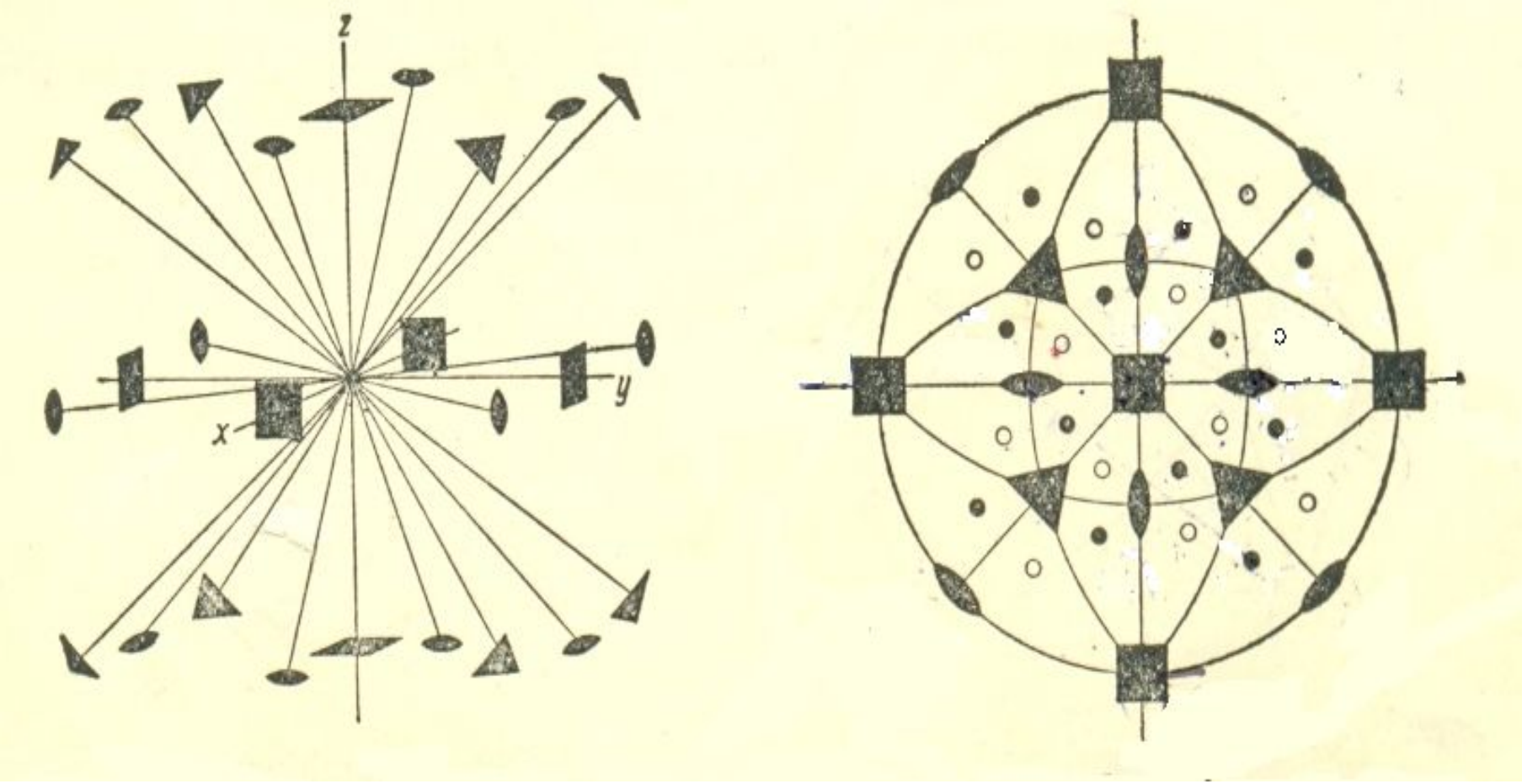
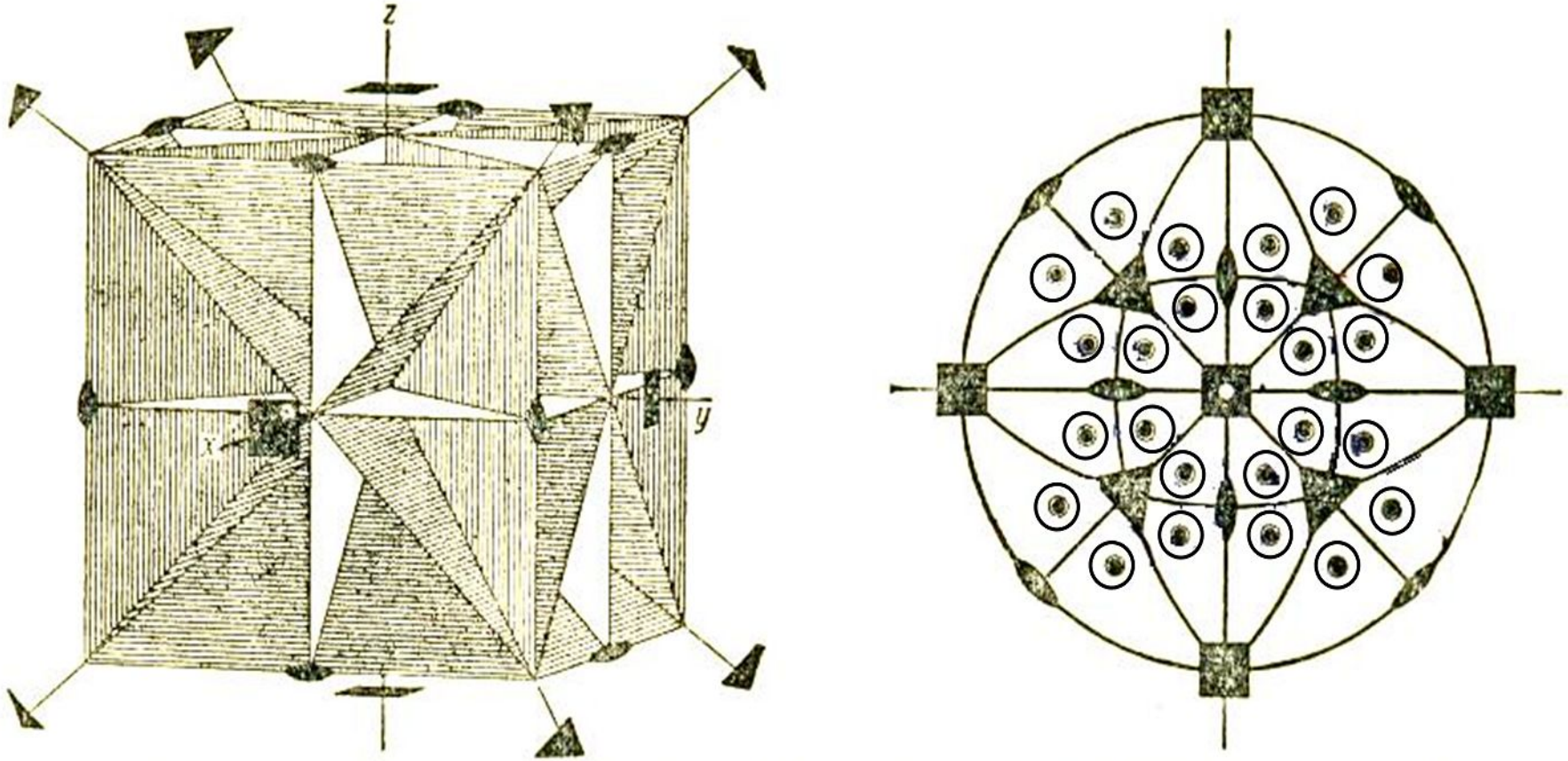


FIG. 286. Class $\bar{4}3m$; the elements of symmetry, crystallographic axes and a stereogram of the general form.



Class 432 the elements of symmetry, crystallographic axes and a stereogram of the general form



Class *m*3*m*; the elements of symmetry, crystallographic axes and a stereogram of the general form