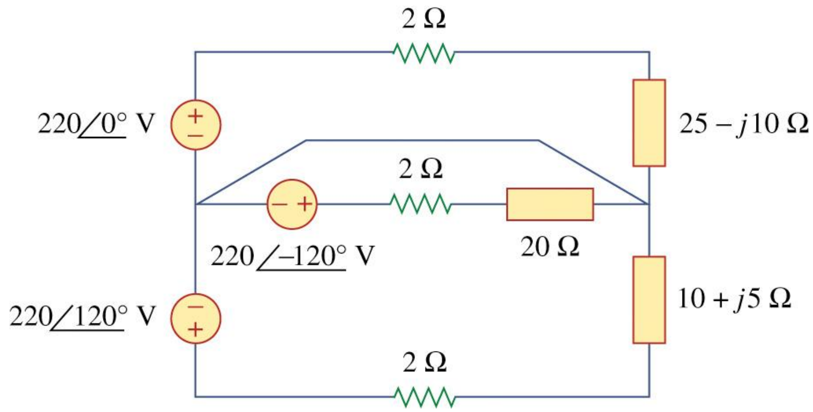
Problem 12.10 Determine the current in the neutral line.



UNBALANCED LOAD

NEUTRAL CURRENT IS NOT ZERO

Chapter 12, Solution 10.

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a, $\mathbf{I}_a = \dfrac{\mathbf{V}_{an}}{\mathbf{Z}_A + 2} = \dfrac{220\angle 0^\circ}{27 - j10} = \dfrac{220}{28.79\angle -20.32^\circ} = 7.642\angle 20.32^\circ$

For phase b, $\mathbf{I}_b = \dfrac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \dfrac{220\angle -120^\circ}{22} = 10\angle -120^\circ$

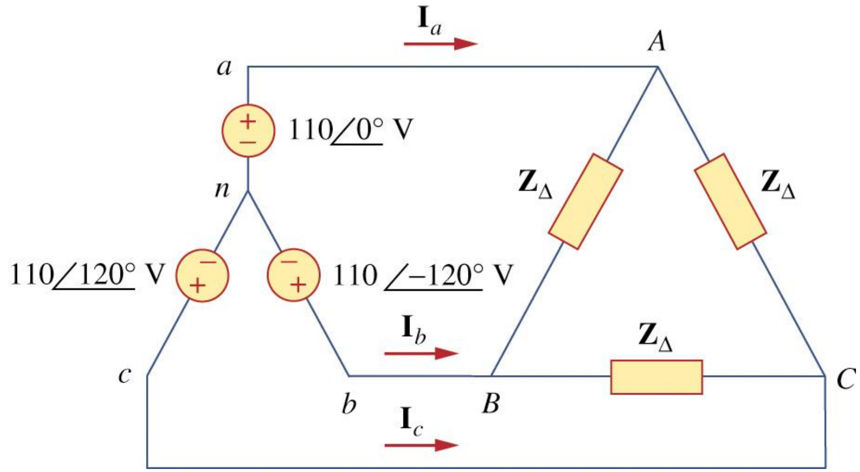
For phase c, $\mathbf{I}_c = \dfrac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \dfrac{220\angle 120^\circ}{12 + j5} = \dfrac{220\angle 120^\circ}{13\angle 22.62^\circ} = 16.923\angle 97.38^\circ$

The current in the neutral line is $\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$ or $-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$

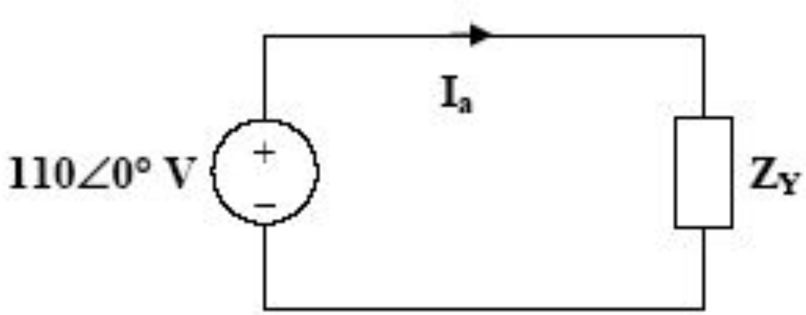
$-\mathbf{I}_n = (7.166 + j2.654) + (-5 - j8.667) + (-2.173 + j16.783)$

$\mathbf{I}_n = 0.007 - j10.77 = \underline{\mathbf{10.77\angle 90^\circ A}}$

Problem 12.12 **Solve for the line currents in the Y-Δ circuit. Take $Z_\Delta = 60\angle 45°\Omega$.**



Convert the delta-load to a Y-load and apply per-phase analysis.



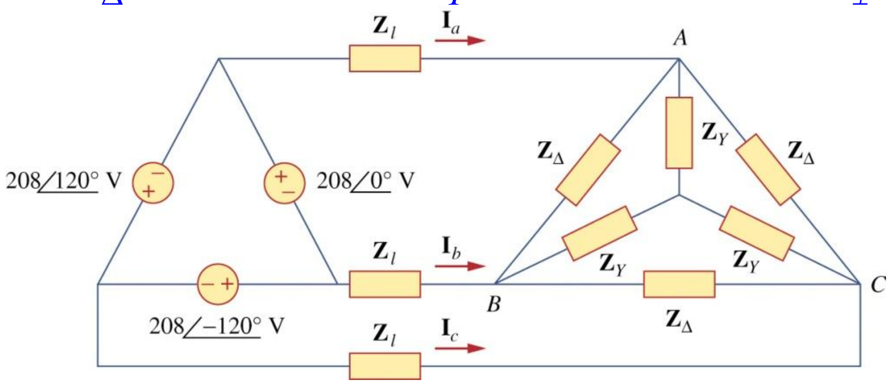
SINGLE PHASE EQUIVALENT CIRCUIT

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \underline{5.5\angle -45°\ \mathrm{A}} \qquad \mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{5.5\angle -165°\ \mathrm{A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{5.5\angle 75°\ \mathrm{A}}$$

Problem 12.22 Find the line currents I_a, I_b, and I_c in the three-phase network below. Take $Z_\Delta = 12 - j15\Omega$, $Z_Y = 4 + j6\ \Omega$, and $Z_l = 2\ \Omega$.

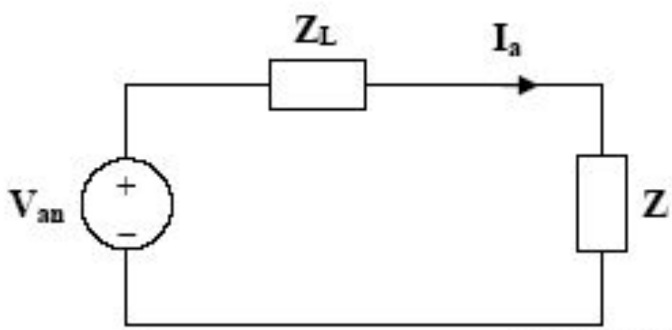


- ONE DELTA AND ONE Y CONNECTED LOAD IS CONNECTED
- TWO Loads are parallel if they are converted to same type.
- Delta connected load is converted to Y connection.

Convert the Δ-connected source to a Y-connected

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ = \frac{208}{\sqrt{3}}\angle -30^\circ = 120\angle -30^\circ$$

Convert the Δ-connected load to a Y-connected load.

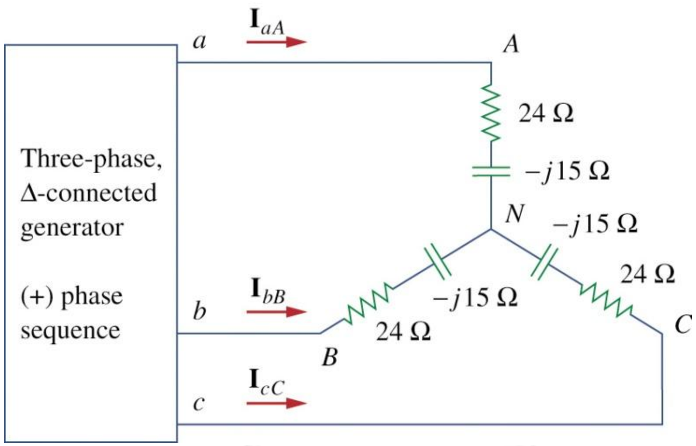


$$\mathbf{Z} = \mathbf{Z}_Y \,||\, \frac{\mathbf{Z}_\Delta}{3} = (4+j6)\,||\,(4-j5) = \frac{(4+j6)(4-j5)}{8+j} = 5.723 - j0.2153$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{120\angle -30^\circ}{7.723 - j0.2153} = \underline{\mathbf{15.53\angle -28.4^\circ\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = \underline{\mathbf{15.53\angle -148.4^\circ\ A}} \quad \mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = \underline{\mathbf{15.53\angle 91.6^\circ\ A}}$$

Problem 12.26 For the balanced circuit below, $V_{ab} = 125\angle 0°$ V. Find the line currents I_{aA}, I_{bB}, and I_{cC}.



BALANCED Y CONNECTED LOAD.

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{\mathbf{V}_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Source voltage given is line to line, obtain the line to neutral voltage.

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \underline{\mathbf{2.55\angle 2°\ A}}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \underline{\mathbf{2.55\angle -118°\ A}}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \underline{\mathbf{2.55\angle 122°\ A}}$$

Problem 12.47 The following three parallel-connected three-phase loads are fed by a balanced three-phase source.

Load 1: 250 kVA, 0.8 pf lagging **Load 2: 300 kVA, 0.95 pf leading** **Load 3: 450 kVA, unity pf**

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

$$\text{pf} = 0.8 \quad \text{(lagging)} \longrightarrow \theta = \cos^{-1}(0.8) = 36.87^\circ$$

$$S_1 = 250\angle 36.87^\circ = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \quad \text{(leading)} \longrightarrow \theta = \cos^{-1}(0.95) = -18.19^\circ$$

$$S_2 = 300\angle -18.19^\circ = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta = \cos^{-1}(1) = 0^\circ$$

$$S_3 = 450 \text{ kVA} \quad S_T = S_1 + S_2 + S_3 = 935 + j56.35 = 936.7\angle 3.45^\circ \text{ kVA}$$

$$|S_T| = \sqrt{3}V_L I_L \qquad I_L = \frac{936.7\times 10^3}{\sqrt{3}(13.8\times 10^3)} = \underline{\mathbf{39.19 \text{ A rms}}}$$

$$\text{pf} = \cos\theta = \cos(3.45^\circ) = \underline{\mathbf{0.9982 \quad (lagging)}}$$

Problem 12.81 A professional center is supplied by a balanced three-phase source. The center has four plants, each a balanced three-phase load as follows:

Load 1: 150 kVA at 0.8 pf leading Load 2: 100 kW at unity pf

Load 3: 200 kVA at 0.6 pf lagging Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is 0.02 + j0.05 Ω per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

$$\text{pf} = 0.8 \quad (\text{leading}) \longrightarrow \theta_1 = -36.87° \quad \mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta_2 = 0° \quad \mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad (\text{lagging}) \longrightarrow \theta_3 = 53.13° \quad \mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA} \qquad \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$

$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L \qquad I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

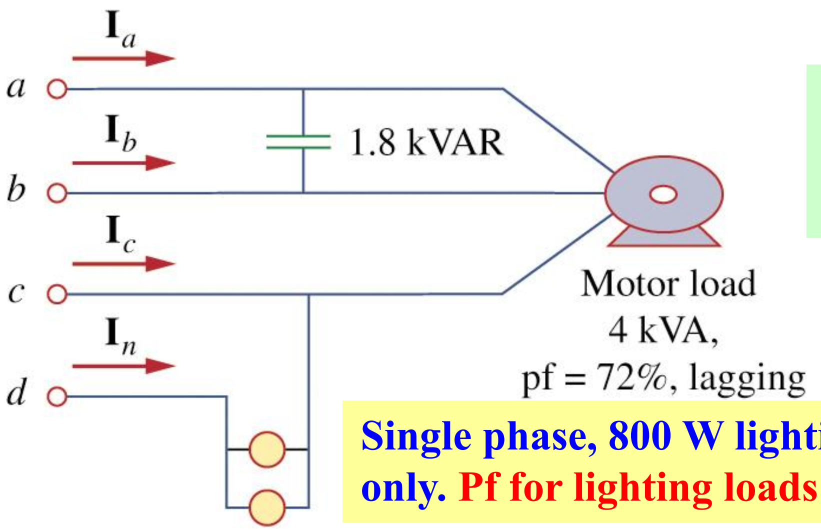
$$\mathbf{S}_L = 3 I_L^2\, \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05) \qquad \mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2 \qquad \mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \underline{\mathbf{516 \text{ V}}}$$

Problem 12.84 The Figure displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines *a* and *b*, while a 800-W lighting load is connected between line *c* and neutral. Assuming the *abc* sequence and taking $V_{an} = V_p \angle 0°$, **find the magnitude and phase angle of currents I_a, I_b, I_c, and I_n.**



Total load is UNBALANCED. LINE CURRENTS I_a, I_b, I_c ARE NOT ERQUAL

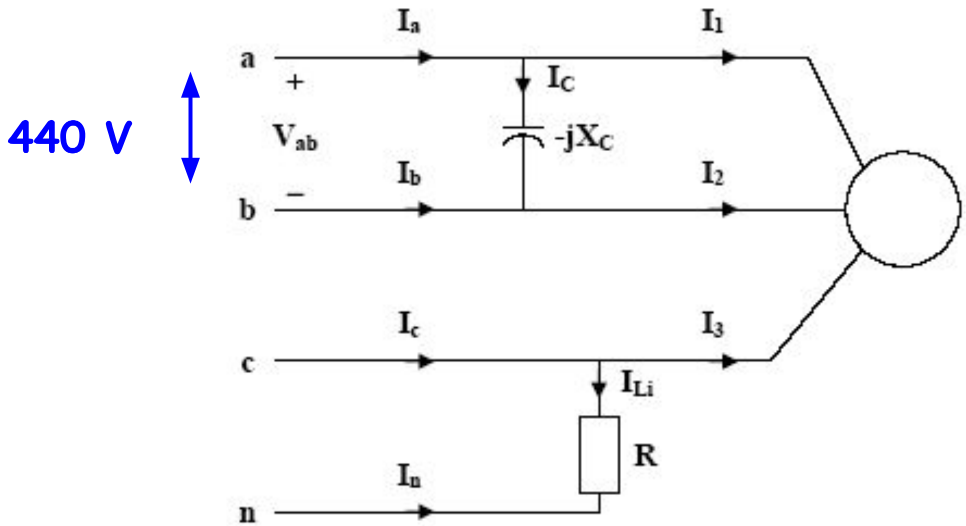
Single phase, 800 W lighting load connected to phase C only. Pf for lighting loads is unity.

We first find the magnitude of the various currents.

For the motor, $I_L = |I_1| = \dfrac{S}{\sqrt{3}V_L} = \dfrac{4000}{440\sqrt{3}} = 5.248 \text{ A}$

For the capacitor, $I_C = \dfrac{Q_c}{V_L} = \dfrac{1800}{440} = 4.091 \text{ A}$

For the lighting, $V_p = \dfrac{440}{\sqrt{3}} = 254 \text{ V}$ $\quad I_{Li} = \dfrac{P_{Li}}{V_p} = \dfrac{800}{254} = 3.15 \text{ A}$



If $\mathbf{V}_{an} = V_p\angle 0°$, $\quad \mathbf{V}_{ab} = \sqrt{3}\, V_p\angle 30°$ $\quad \mathbf{V}_{cn} = V_p\angle 120°$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120° \qquad I_1 = \frac{S}{\sqrt{3}\,V_L} = 5.249\angle(\theta + 30°) \quad \text{here } \theta = \cos^{-1}(0.72) = 43.95°$$

$$\mathbf{I}_1 = 5.249\angle 73.95° \qquad \mathbf{I}_2 = 5.249\angle -46.05° \qquad \mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120° \quad \text{Thus,}$$

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120° \qquad \mathbf{I}_a = \underline{\mathbf{8.608\angle 93.96°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120° \qquad \mathbf{I}_b = \underline{\mathbf{9.271\angle -52.16°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120° \qquad \mathbf{I}_c = \underline{\mathbf{6.827\angle 167.6°\ A}}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \underline{\mathbf{3.15\angle -60°\ A}}$$