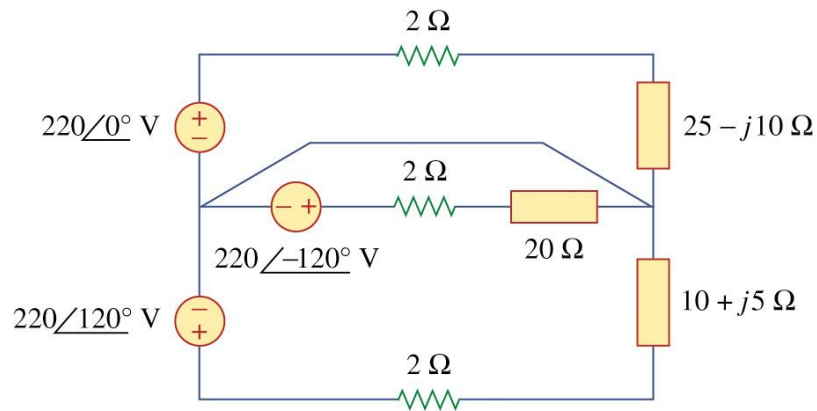


Problem 12.10 Determine the current in the neutral line.



**UNBALANCED LOAD
NEUTRAL CURRENT IS NOT
ZERO**

Chapter 12, Solution 10.

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\text{For phase a, } \mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_A + 2} = \frac{220\angle 0^\circ}{27 - j10} = \frac{220}{28.79\angle -20.32^\circ} = 7.642\angle 20.32^\circ$$

$$\text{For phase b, } \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{220\angle -120^\circ}{22} = 10\angle -120^\circ$$

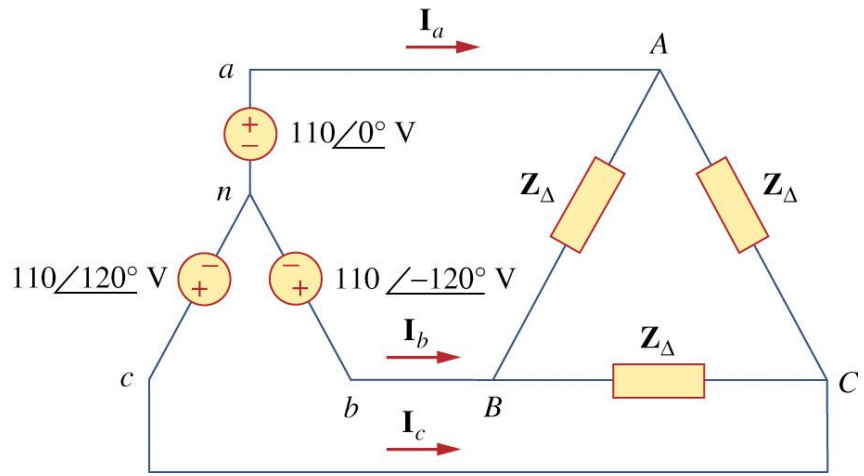
$$\text{For phase c, } \mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{220\angle 120^\circ}{12 + j5} = \frac{220\angle 120^\circ}{13\angle 22.62^\circ} = 16.923\angle 97.38^\circ$$

The current in the neutral line is $\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$ or $-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$

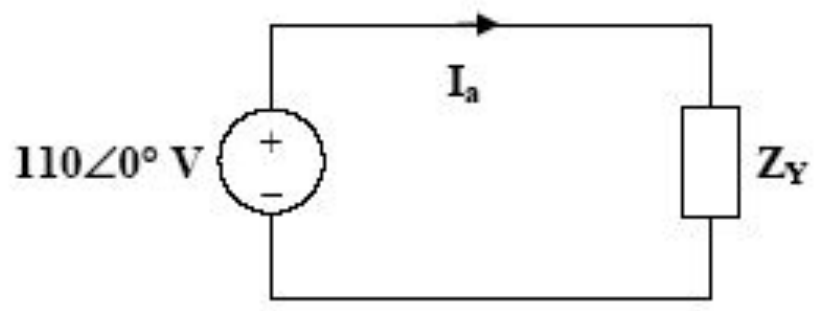
$$-\mathbf{I}_n = (7.166 + j2.654) + (-5 - j8.667) + (-2.173 + j16.783)$$

$$\mathbf{I}_n = 0.007 - j10.77 = \underline{\underline{10.77\angle 90^\circ \text{ A}}}$$

Problem 12.12 Solve for the line currents in the Y-Δ circuit. Take $Z_{\Delta} = 60 \angle 45^{\circ} \Omega$.



Convert the delta-load to a Y-load and apply per-phase analysis.



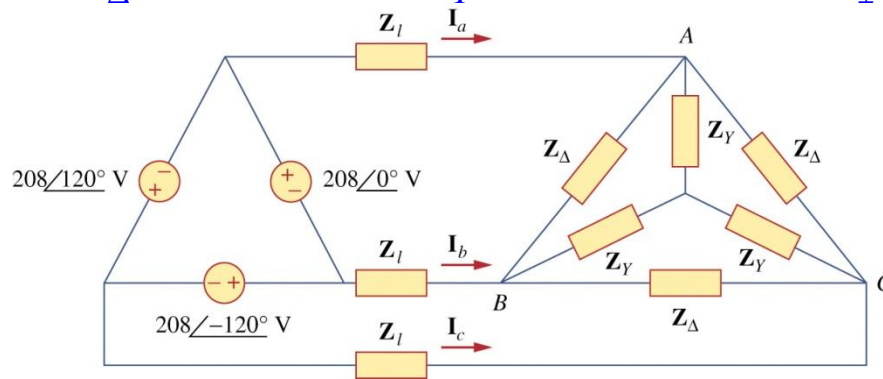
SINGLE PHASE EQUIVALENT
CIRCUIT

$$Z_Y = \frac{Z_{\Delta}}{3} = 20 \angle 45^{\circ} \Omega$$

$$I_a = \frac{110 \angle 0^{\circ}}{20 \angle 45^{\circ}} = \underline{5.5 \angle -45^{\circ} \text{ A}} \quad I_b = I_a \angle -120^{\circ} = \underline{5.5 \angle -165^{\circ} \text{ A}}$$

$$I_c = I_a \angle 120^{\circ} = \underline{5.5 \angle 75^{\circ} \text{ A}}$$

Problem 12.22 Find the line currents I_a , I_b , and I_c in the three-phase network below. Take $Z_\Delta = 12 - j15\Omega$, $Z_Y = 4 + j6\Omega$, and $Z_L = 2\Omega$.



□ ONE DELTA AND ONE Y CONNECTED LOAD IS CONNECTED

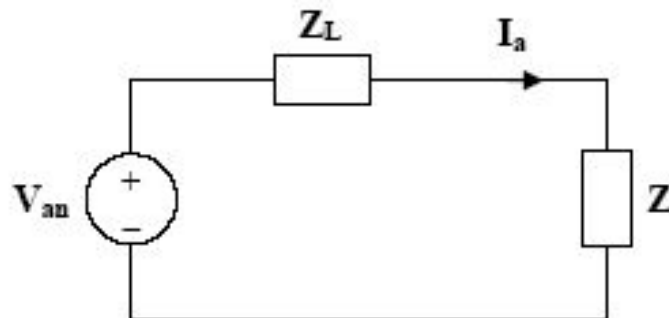
□ TWO Loads are parallel if they are converted to same type.

□ Delta connected load is converted to Y connection.

Convert the Δ -connected source to a Y-connected

$$V_{an} = \frac{V_p}{\sqrt{3}} \angle -30^\circ = \frac{208}{\sqrt{3}} \angle -30^\circ = 120 \angle -30^\circ$$

Convert the Δ -connected load to a Y-connected load.

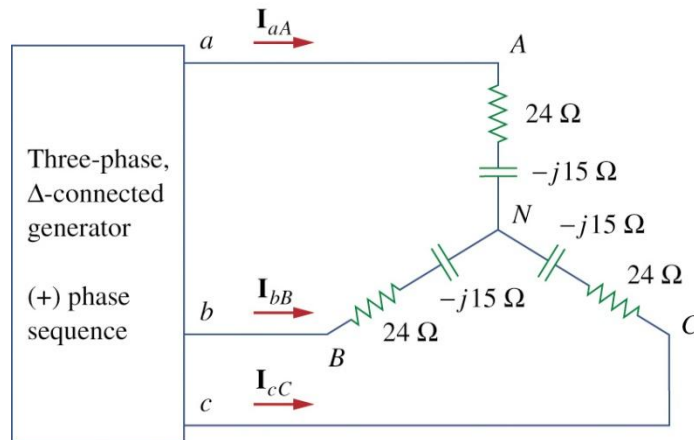


$$Z = Z_Y \parallel \frac{Z_\Delta}{3} = (4 + j6) \parallel (4 - j5) = \frac{(4 + j6)(4 - j5)}{8 + j} = 5.723 - j0.2153$$

$$I_a = \frac{V_{an}}{Z_L + Z} = \frac{120 \angle -30^\circ}{7.723 - j0.2153} = \underline{15.53 \angle -28.4^\circ \text{ A}}$$

$$I_b = I_a \angle -120^\circ = \underline{15.53 \angle -148.4^\circ \text{ A}} \quad I_c = I_a \angle 120^\circ = \underline{15.53 \angle 91.6^\circ \text{ A}}$$

Problem 12.26 For the balanced circuit below, $V_{ab} = 125 \angle 0^\circ$ V. Find the line currents I_{aA} , I_{bB} , and I_{cC} .



BALANCED Y CONNECTED LOAD.

Transform the source to its wye equivalent.

$$V_{an} = \frac{V_p}{\sqrt{3}} \angle -30^\circ = 72.17 \angle -30^\circ$$

Source voltage given is line to line, obtain the line to neutral voltage.

Now, use the per-phase equivalent circuit.

$$I_{aA} = \frac{V_{an}}{Z}, \quad Z = 24 - j15 = 28.3 \angle -32^\circ$$

$$I_{aA} = \frac{72.17 \angle -30^\circ}{28.3 \angle -32^\circ} = \underline{\underline{2.55 \angle 2^\circ \text{ A}}}$$

$$I_{bB} = I_{aA} \angle -120^\circ = \underline{\underline{2.55 \angle -118^\circ \text{ A}}}$$

$$I_{cC} = I_{aA} \angle 120^\circ = \underline{\underline{2.55 \angle 122^\circ \text{ A}}}$$

Problem 12.47 The following three parallel-connected three-phase loads are fed by a balanced three-phase source.

Load 1: 250 kVA, 0.8 pf lagging Load 2: 300 kVA, 0.95 pf leading Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

$$\text{pf} = 0.8 \text{ (lagging)} \longrightarrow \theta = \cos^{-1}(0.8) = 36.87^\circ$$

$$S_1 = 250 \angle 36.87^\circ = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \text{ (leading)} \longrightarrow \theta = \cos^{-1}(0.95) = -18.19^\circ$$

$$S_2 = 300 \angle -18.19^\circ = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta = \cos^{-1}(1) = 0^\circ$$

$$S_3 = 450 \text{ kVA} \quad S_T = S_1 + S_2 + S_3 = 935 + j56.35 = 936.7 \angle 3.45^\circ \text{ kVA}$$

$$|S_T| = \sqrt{3} V_L I_L \quad I_L = \frac{936.7 \times 10^3}{\sqrt{3} (13.8 \times 10^3)} = \underline{\underline{39.19 \text{ A rms}}}$$

$$\text{pf} = \cos \theta = \cos(3.45^\circ) = \underline{\underline{0.9982 \text{ (lagging)}}}$$

Problem 12.81 A professional center is supplied by a balanced three-phase source. The center has four plants, each a balanced three-phase load as follows:

Load 1: 150 kVA at 0.8 pf leading Load 2: 100 kW at unity pf

Load 3: 200 kVA at 0.6 pf lagging Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is $0.02 + j0.05 \Omega$ per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

$$\text{pf} = 0.8 \text{ (leading)} \longrightarrow \theta_1 = -36.87^\circ \quad S_1 = 150 \angle -36.87^\circ \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta_2 = 0^\circ \quad S_2 = 100 \angle 0^\circ \text{ kVA}$$

$$\text{pf} = 0.6 \text{ (lagging)} \longrightarrow \theta_3 = 53.13^\circ \quad S_3 = 200 \angle 53.13^\circ \text{ kVA}$$

$$S_4 = 80 + j95 \text{ kVA} \quad S = S_1 + S_2 + S_3 + S_4$$

$$S = 420 + j165 = 451.2 \angle 21.45^\circ \text{ kVA}$$

$$S = \sqrt{3} V_L I_L \quad I_L = \frac{S}{\sqrt{3} V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

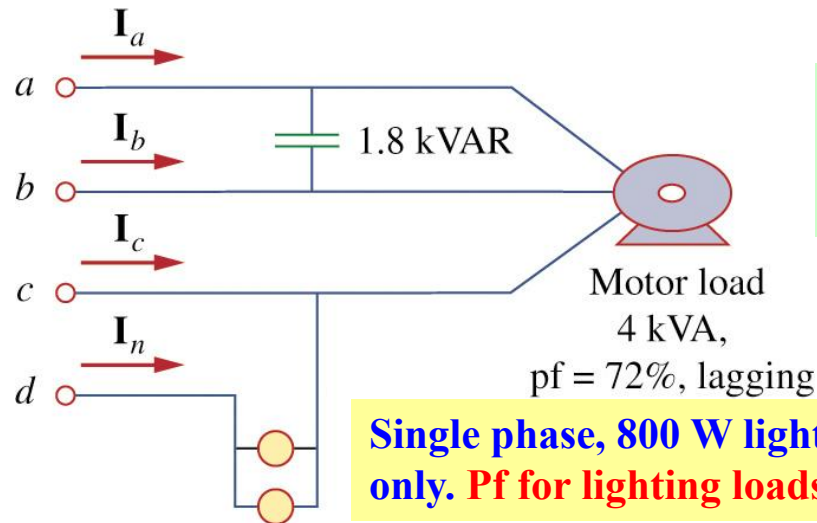
$$S_L = 3 I_L^2 Z_L = (3)(542.7)^2 (0.02 + j0.05) \quad S_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$S_T = S + S_L = 437.7 + j209.2 \quad S_T = 485.1 \angle 25.55^\circ \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3} I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \underline{\underline{516 \text{ V}}}$$

Problem 12.84 The Figure displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines *a* and *b*, while a 800-W lighting load is connected between line *c* and neutral. Assuming the *abc* sequence and taking $V_{an} = V_p \angle 0^\circ$, **find the magnitude and phase angle of currents I_a , I_b , I_c , and I_n .**



Total load is UNBALANCED. LINE CURRENTS I_a , I_b , I_c ARE NOT EQUAL

Single phase, 800 W lighting load connected to phase C only. Pf for lighting loads is unity.

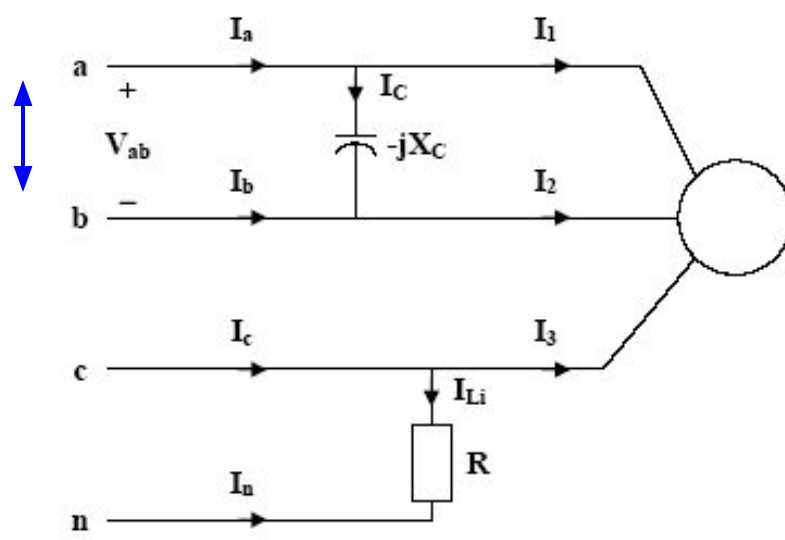
We first find the magnitude of the various currents.

$$\text{For the motor, } I_L = |I_L| = \frac{S}{\sqrt{3}V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

$$\text{For the capacitor, } I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

$$\text{For the lighting, } V_p = \frac{440}{\sqrt{3}} = 254 \text{ V} \quad I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15 \text{ A}$$

440 V



$$\text{If } V_{an} = V_p \angle 0^\circ, \quad V_{ab} = \sqrt{3} V_p \angle 30^\circ \quad V_{cn} = V_p \angle 120^\circ$$

$$I_c = \frac{V_{ab}}{-jX_c} = 4.091 \angle 120^\circ \quad I_1 = \frac{V_{ab}}{\sqrt{3} V_L} = 5.249 \angle (\theta + 30^\circ) \quad \text{here } \theta = \cos^{-1}(0.72) = 43.95^\circ$$

$$I_1 = 5.249 \angle 73.95^\circ \quad I_2 = 5.249 \angle -46.05^\circ \quad I_3 = 5.249 \angle 193.95^\circ$$

$$I_{Li} = \frac{V_{cn}}{R} = 3.15 \angle 120^\circ \quad \text{Thus,}$$

$$I_a = I_1 + I_c = 5.249 \angle 73.95^\circ + 4.091 \angle 120^\circ \quad I_a = \underline{\underline{8.608 \angle 93.96^\circ \text{ A}}}$$

$$I_b = I_2 - I_c = 5.249 \angle -46.05^\circ - 4.091 \angle 120^\circ \quad I_b = \underline{\underline{9.271 \angle -52.16^\circ \text{ A}}}$$

$$I_c = I_3 + I_{Li} = 5.249 \angle 193.95^\circ + 3.15 \angle 120^\circ \quad I_c = \underline{\underline{6.827 \angle 167.6^\circ \text{ A}}}$$

$$I_n = -I_{Li} = \underline{\underline{3.15 \angle -60^\circ \text{ A}}}$$