## Class 5

# Optimization models <br> Demand function equation. Revenue maximization. Profit maximization. BEPs. 

Study materials: Slides

## Demand function

What shall we do with our selling Price, if:
$P_{1}=\$ 1,000$, then $Q_{1}=400$ units, and $R_{1}=\$ 400,000$
$P_{2}=\$ 1,750$, then $Q_{2}=250$ units, and $R_{2}=\$ 437,500$ To do:
(a) increase the price, or
(b) decrease the price, or
(c) keep the price at $\$ 1,750$ ?

SOLUTION: The price that MAX the revenue shall be: \$2,250, \$2,000, \$1,750, \$1,500, \$1,250?

## Demand function

## Correct answer:

The "best" price to MAX the revenue would be: $\$ 1,500$
$P_{\text {opt }}=\$ 1,500$, then Qopt $=300$ units, and $\operatorname{Rmax}=\$ 450,000$ To do:
(a) increase the price
(b) decrease the price
(c) keep the price at $\$ 1,750$

This can be solved through (1) finding the demand function equation, and (2) solving a revenue maximization problem.

## Demand function

Can be found using the approaches:

Sales tests:

$$
\begin{aligned}
& \text { P1, Q1 } \\
& \text { P2, Q2 }
\end{aligned}
$$

| Quantity (u) | Price (\$) |
| ---: | ---: |
| 250 | 1750 |
| 350 | 1250 |

NB: Demand function is not always linear.
$P(M A X)$ and $Q(M A X)$ are indicative.
Sales test not always linear.
Need to offset the effect of seasonality.

## Demand Function Equation

$Y=a+b * X$, basic linear equation
$P=a+b^{*} Q$, demand function equation
where:
$\mathrm{a}=\mathrm{P}(\mathrm{MAX}$ in the market $)=3,000$
$b=$ slope of the demand function line
$=$ delta $\mathrm{Y} /$ delta $\mathrm{X}=-5$
$Q(M A X)=-a / b=600$ (units)

NB: Mind the negative value of the variable coefficient of the linear equation"b".

## Task: Revenue maximization

$$
Q^{*}(\text { Revenue MAX })=-a / 2 b=300(u)
$$

Substitute Q* into the Demand function equation, will find $\mathrm{P}^{*}$ (= the price at $\mathrm{Q}^{*}$ point)

$$
\begin{aligned}
& \mathrm{P}^{*}=3,000+(-5)^{*} 300=\$ 1,500 \\
& \mathrm{R}^{*}=\mathrm{P}^{*} \times \mathrm{Q}^{*}=450,000
\end{aligned}
$$

NB: $\mathrm{R}^{*}$ is highest revenue possible at the current demand.

## Profit maximization

## Q** (Profit MAX) = - (a-VC(u)) / 2b

$P^{* *}$ shall correspond to the value of $Q^{* *}$
Data needed: fixed and variable costs FC $=\$ 100,000$ $\mathrm{VC}(\mathrm{u})=\$ 500$
$Q^{* *}=250(u)$, then
$P^{* *}=1,750$, then
$R^{* *}=437,500$, and
$\mathrm{Pr}^{* *}=\mathrm{R}^{* *}-\mathrm{FC}-\mathrm{VC}(\mathrm{u}) \mathrm{Q}^{* *}=\$ 212,500$
Pr** is highest operating profit possible at the current demand and total costs

## Summary

|  | Quantity (u) | Price (\$) | Revenue (\$) | Op Profit (\$) |
| :---: | :---: | :---: | :---: | :---: |
| Demand function | $250$ | $1,750$ | $437,500$ | $212,500$ |
|  |  |  | $437,500$ |  |
| Fixed Costs | 100,000 |  |  |  |
| VC(u) | 500 |  |  |  |
| b | -5 |  |  |  |
| a | 3,000 |  |  |  |
|  | Quantity (u) | Price (\$) | Revenue (\$) | Op Profit (\$) |
| Qmax | 600 | 0 | 0 | -400,000 |
| Q*(Revenue) | 300 | 1,500 | 450,000 | 200,000 |
| Q** (Profit) | 250 | 1,750 | 437,500 | 212,500 |
|  |  |  |  |  |
| Discriminant | 4,250,000 | SqRoot | 2,062 |  |
|  | Quantity (u) | Price (\$) | Revenue (\$) | Op Profit (\$) |
| Q1 | 43.84 | 2,781 | 121,922 | 0 |
| Q2 | 456.16 | 719 | 328,078 | 0 |
| Nota bene: |  |  |  |  |
| input cells |  |  |  |  |
| results ( $\mathrm{u}, \$$ ) |  |  |  |  |

