



Chapter Six

Demand



Properties of Demand Functions

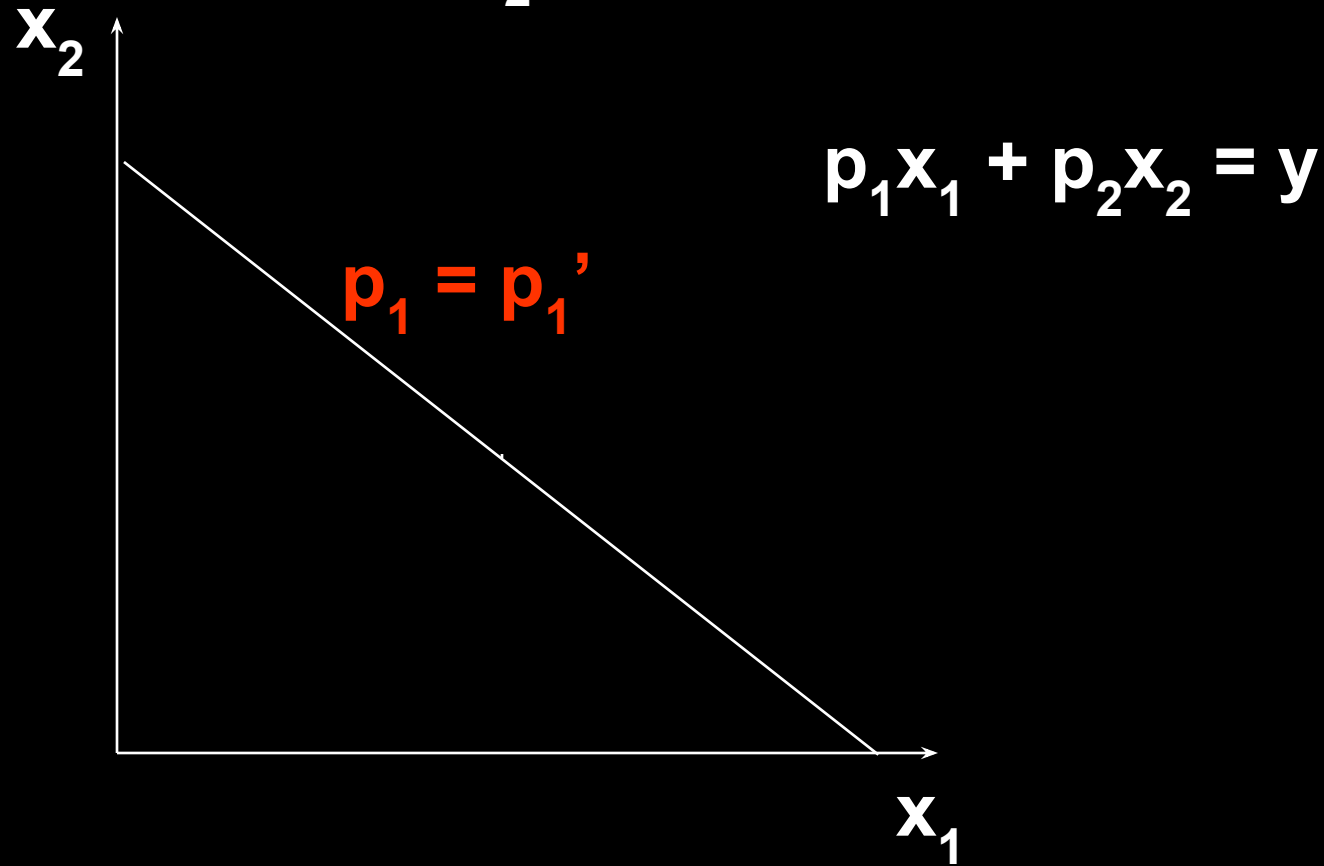
- **Comparative statics analysis** of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1, p_2, y)$ and $x_2^*(p_1, p_2, y)$ change as prices p_1 , p_2 and income y change.

Own-Price Changes

- How does $x_1^*(p_1, p_2, y)$ change as p_1 changes, holding p_2 and y constant?
- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1''' .

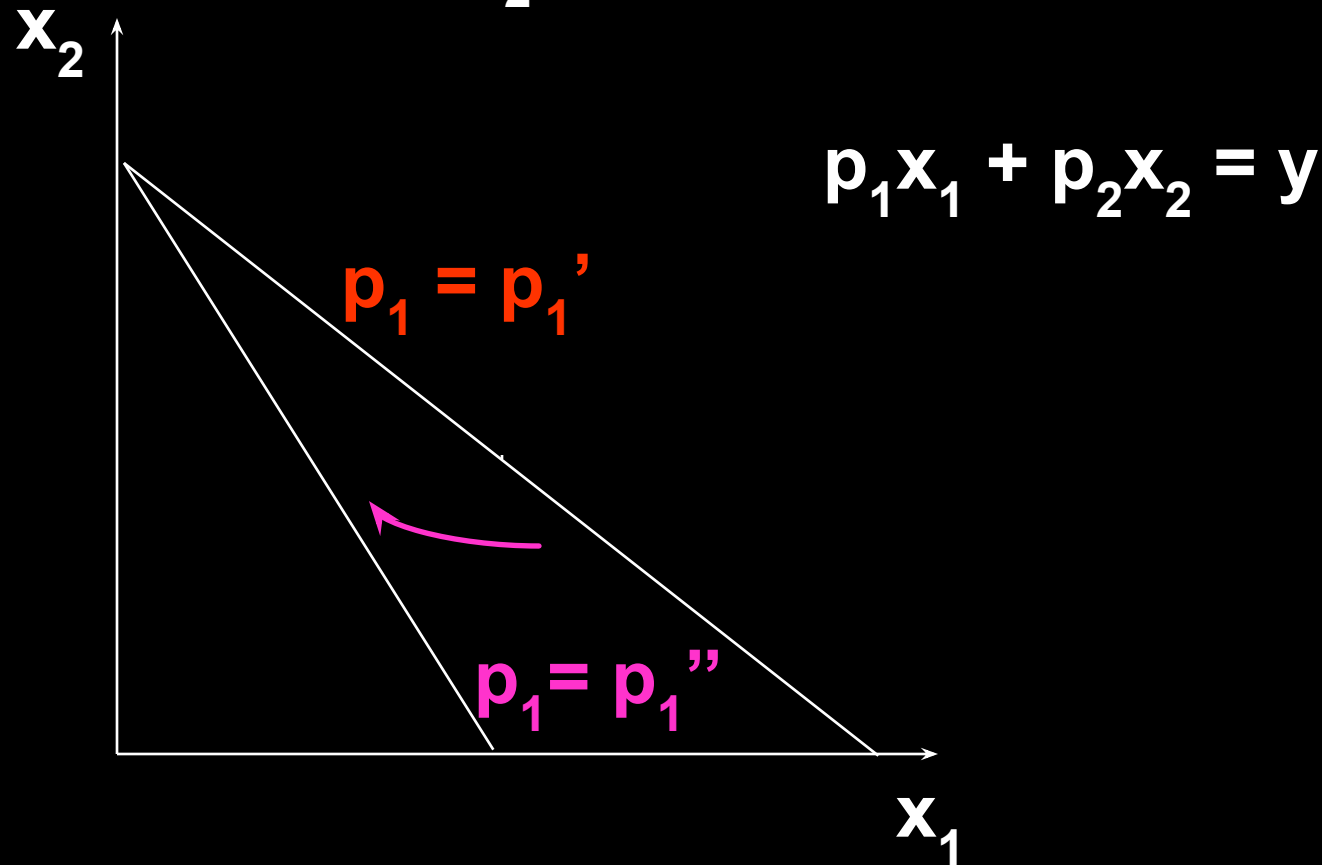
Own-Price Changes

Fixed p_2 and y .



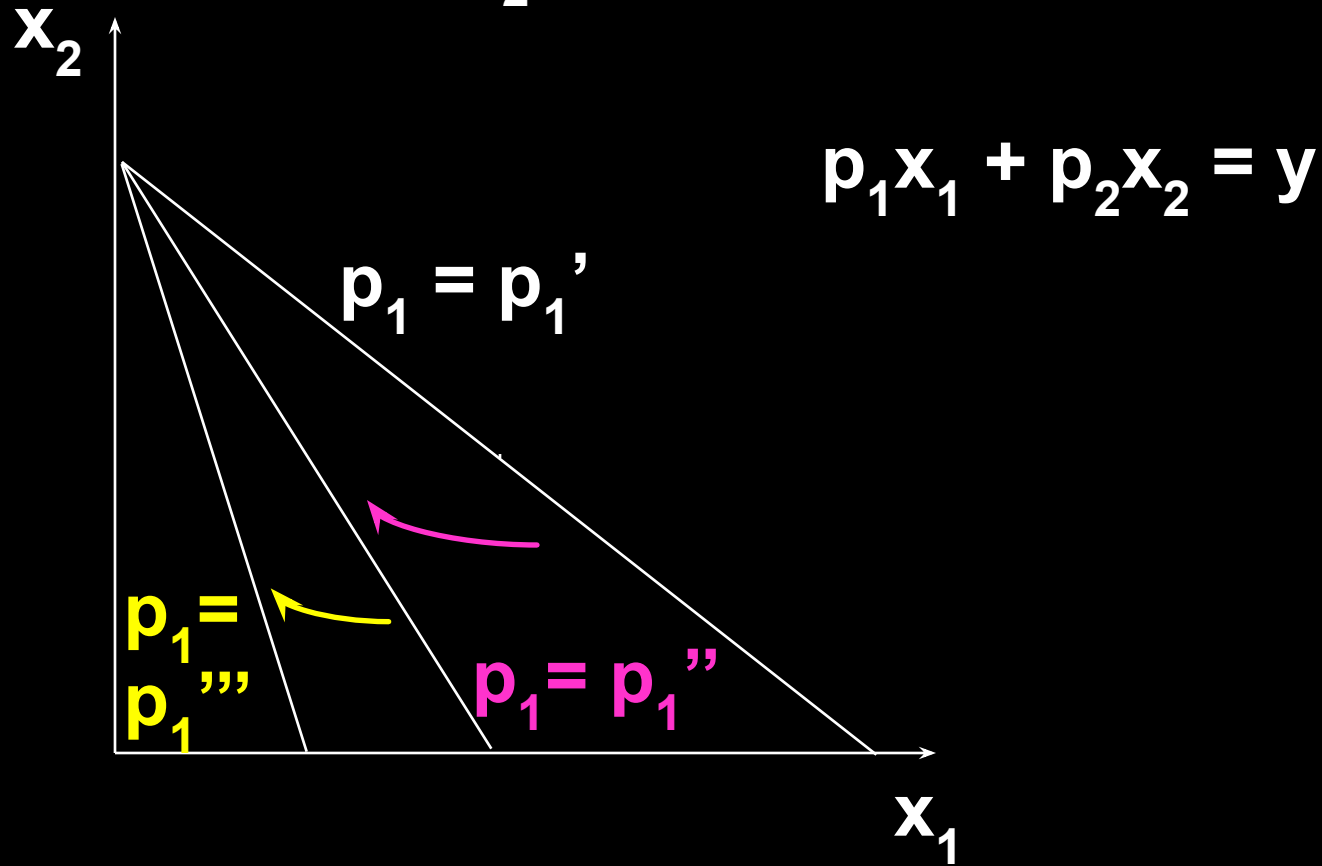
Own-Price Changes

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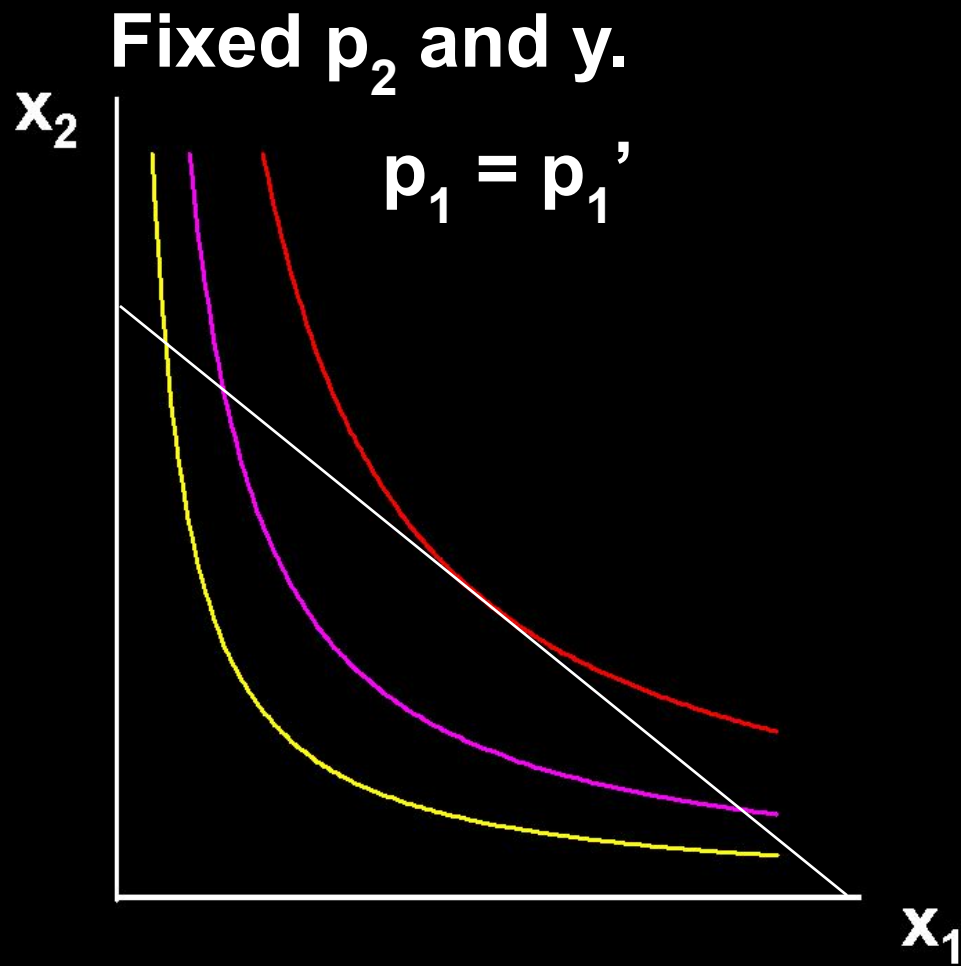


Own-Price Changes

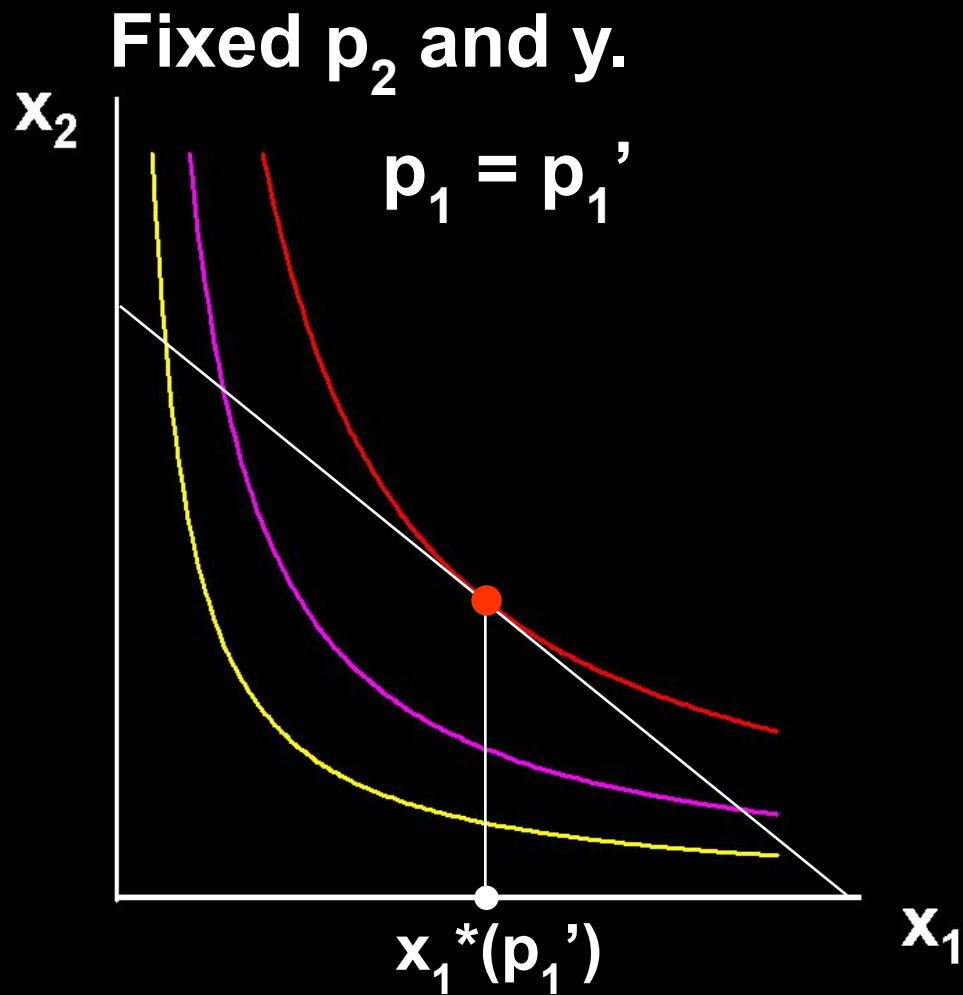
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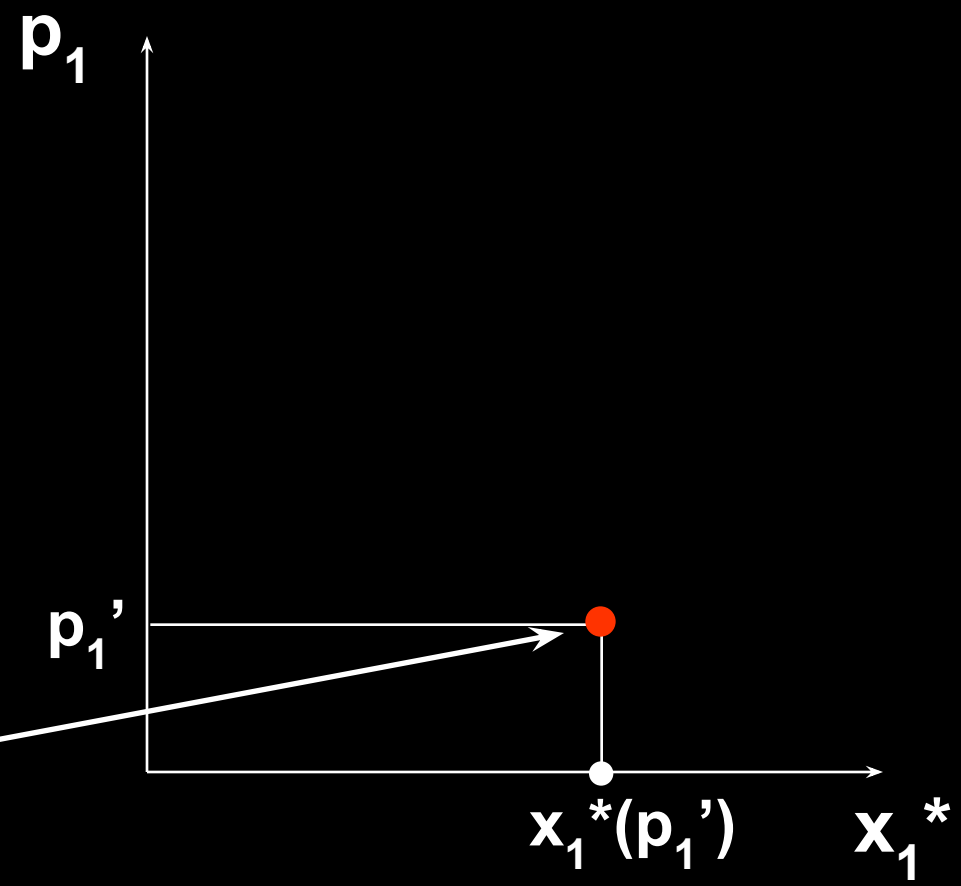
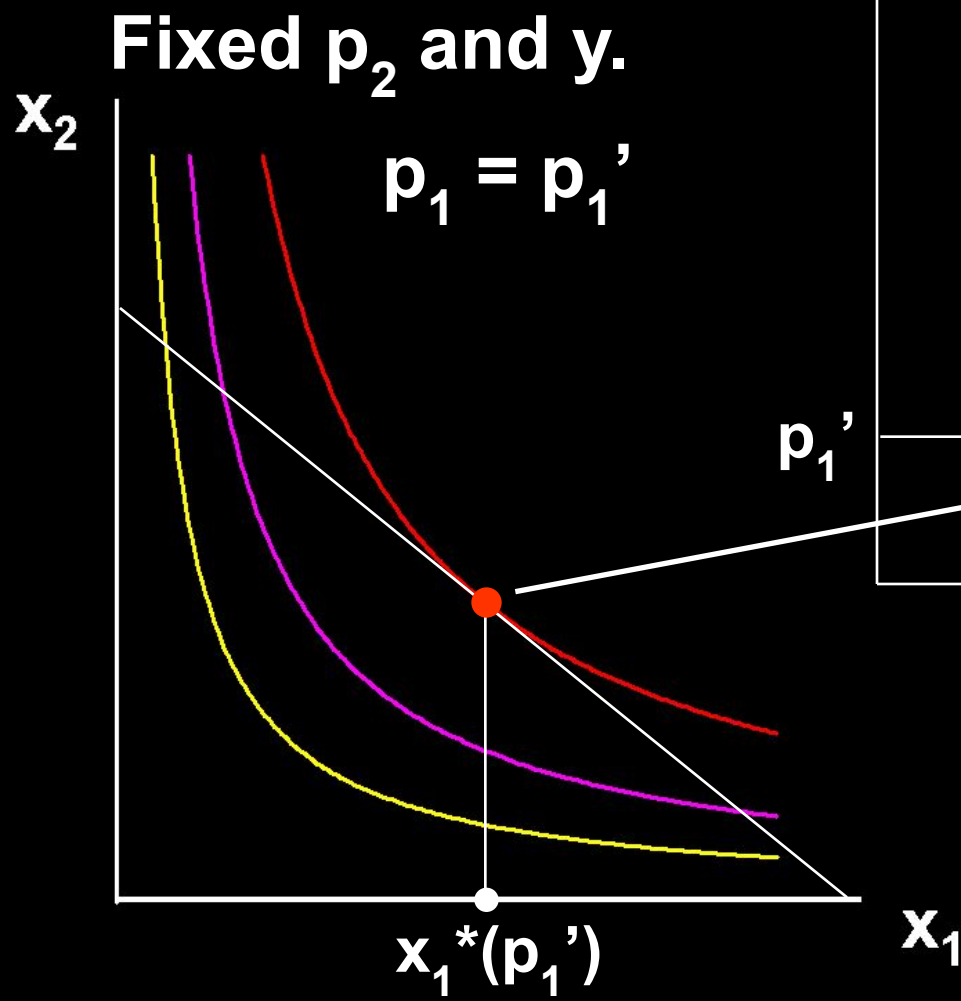
Own-Price Changes



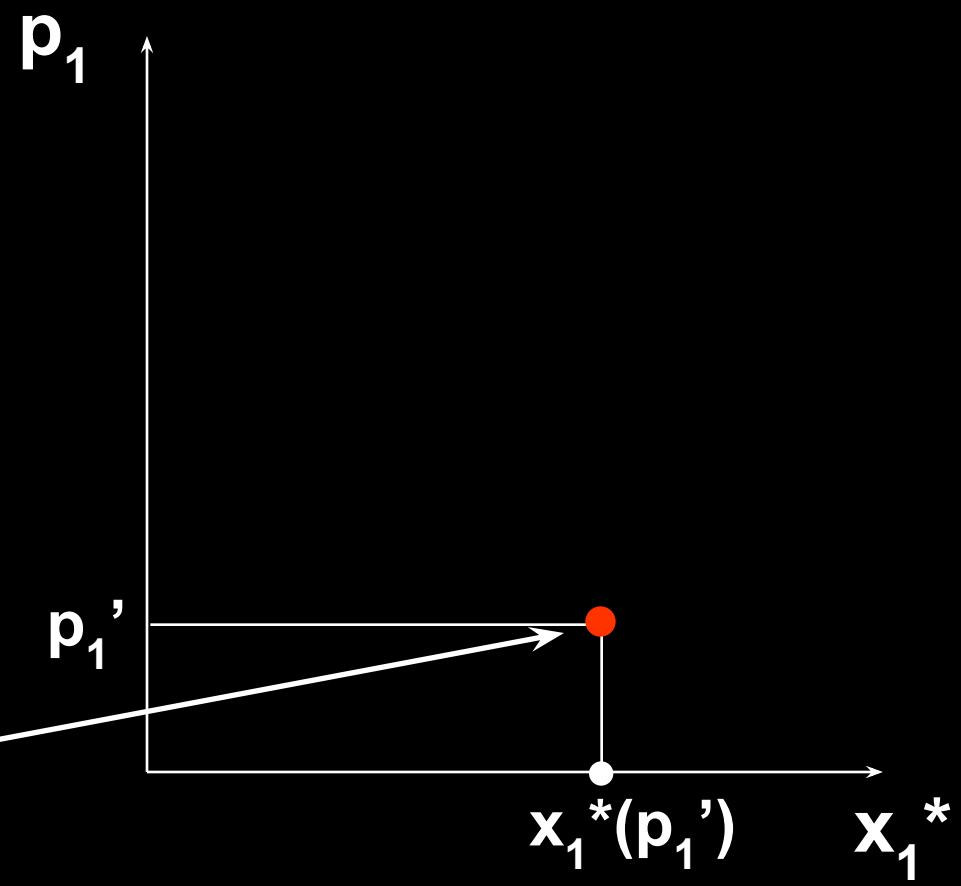
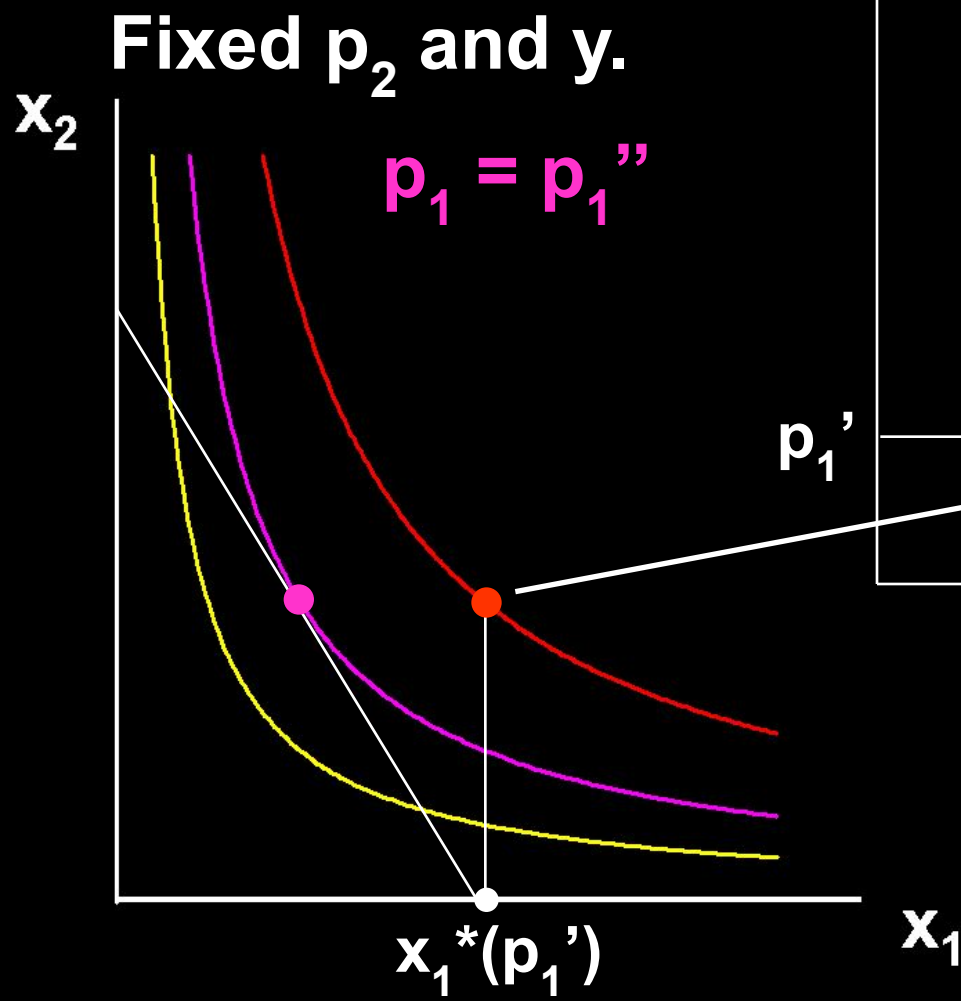
Own-Price Changes



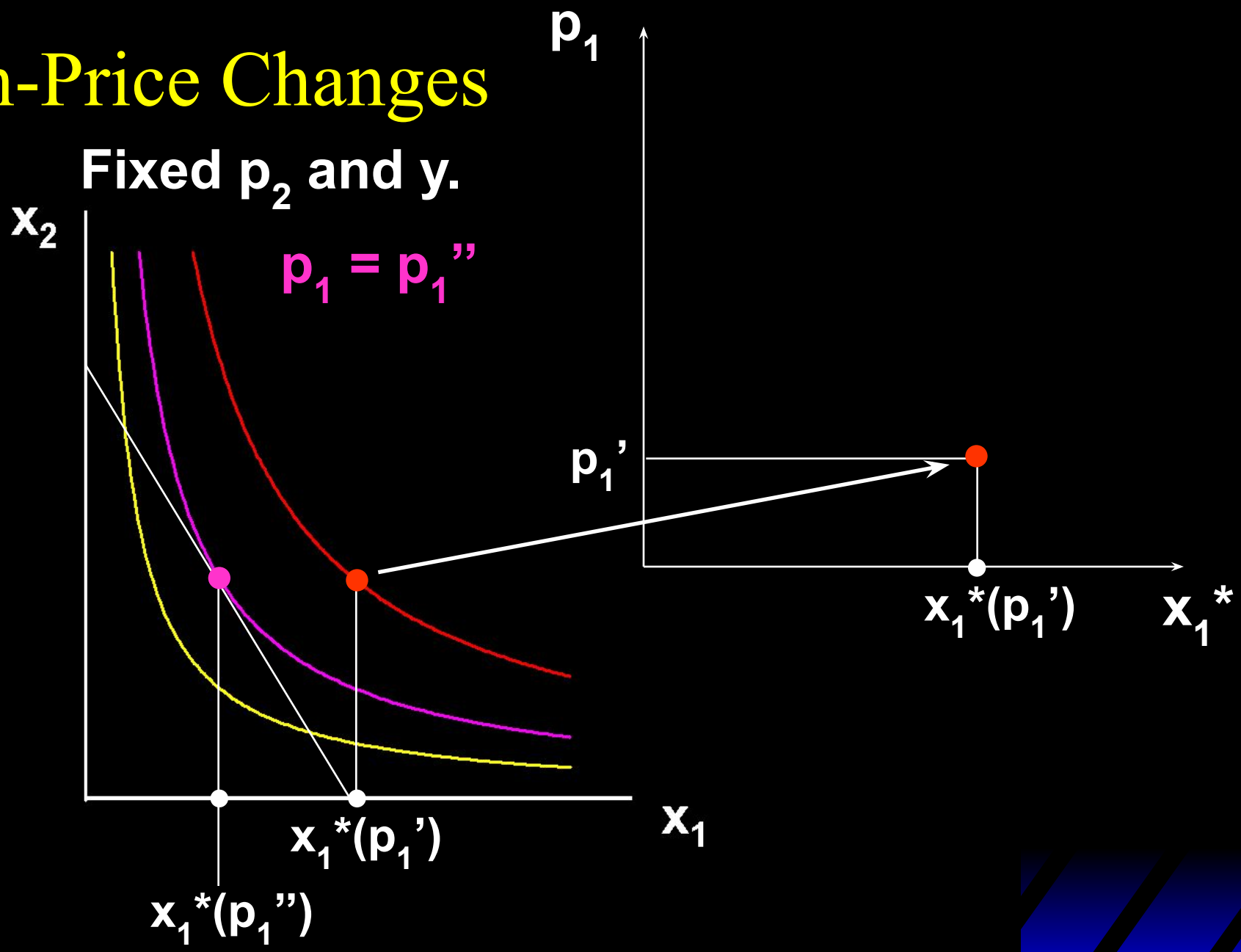
Own-Price Changes



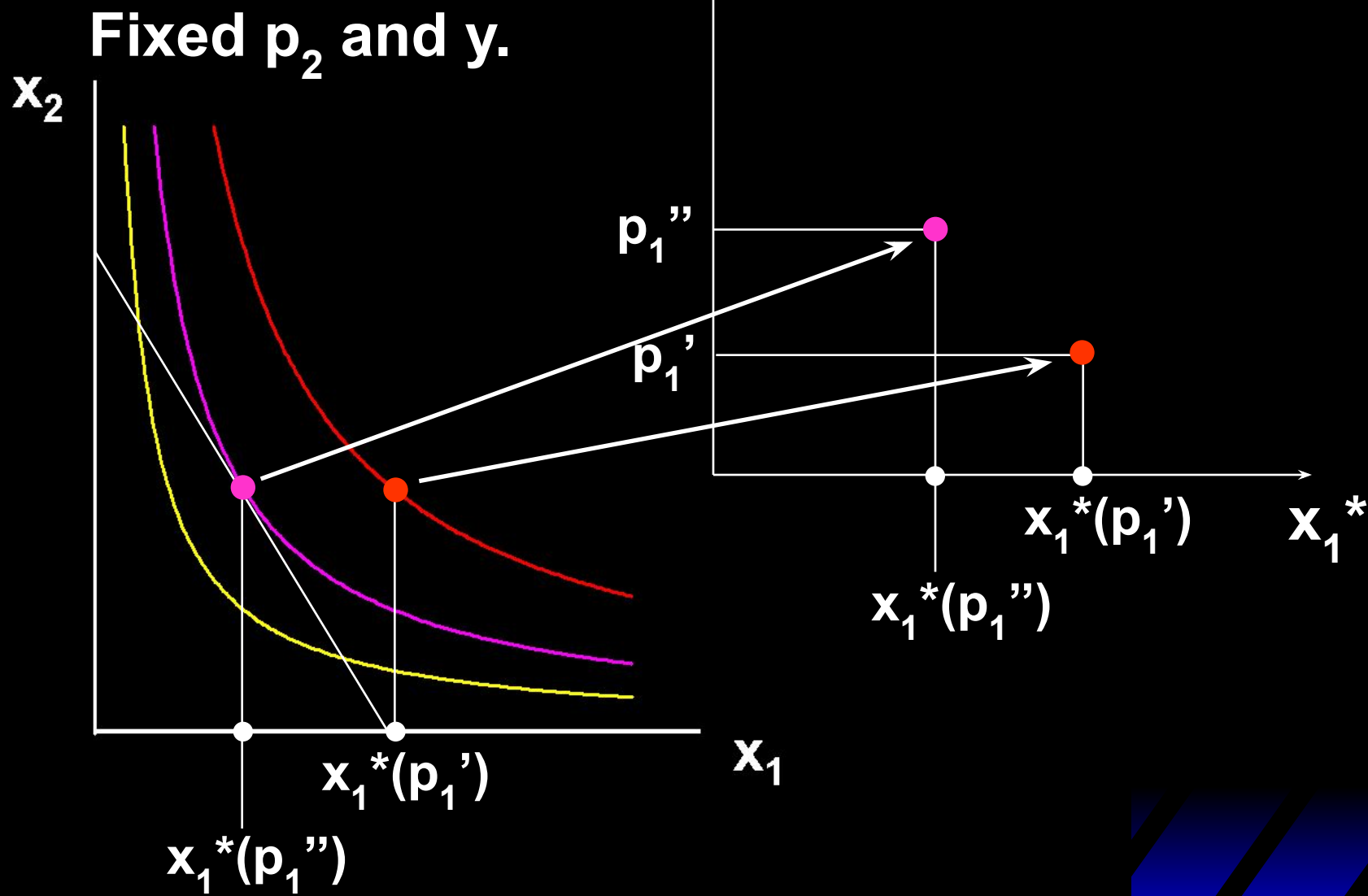
Own-Price Changes



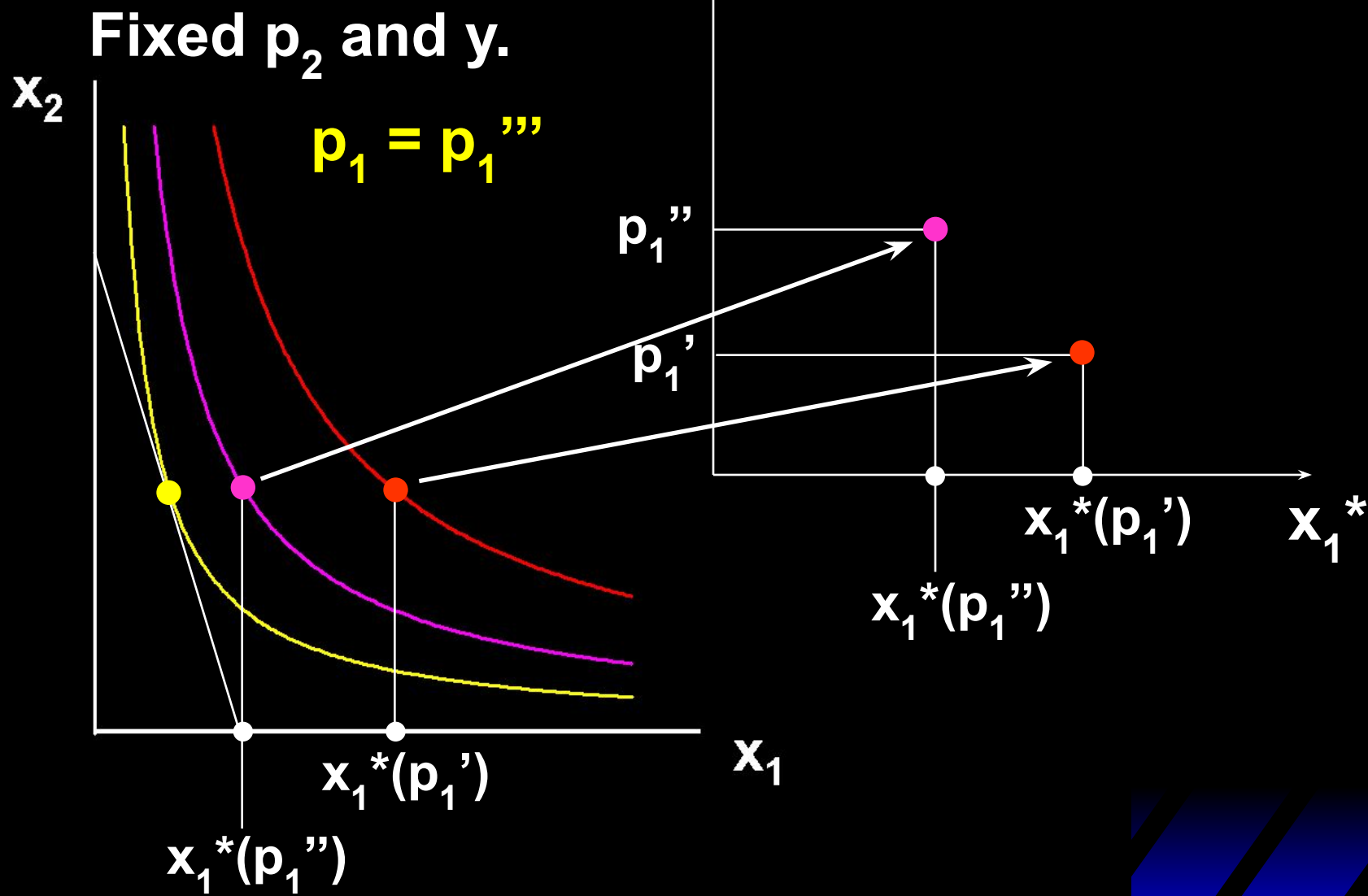
Own-Price Changes



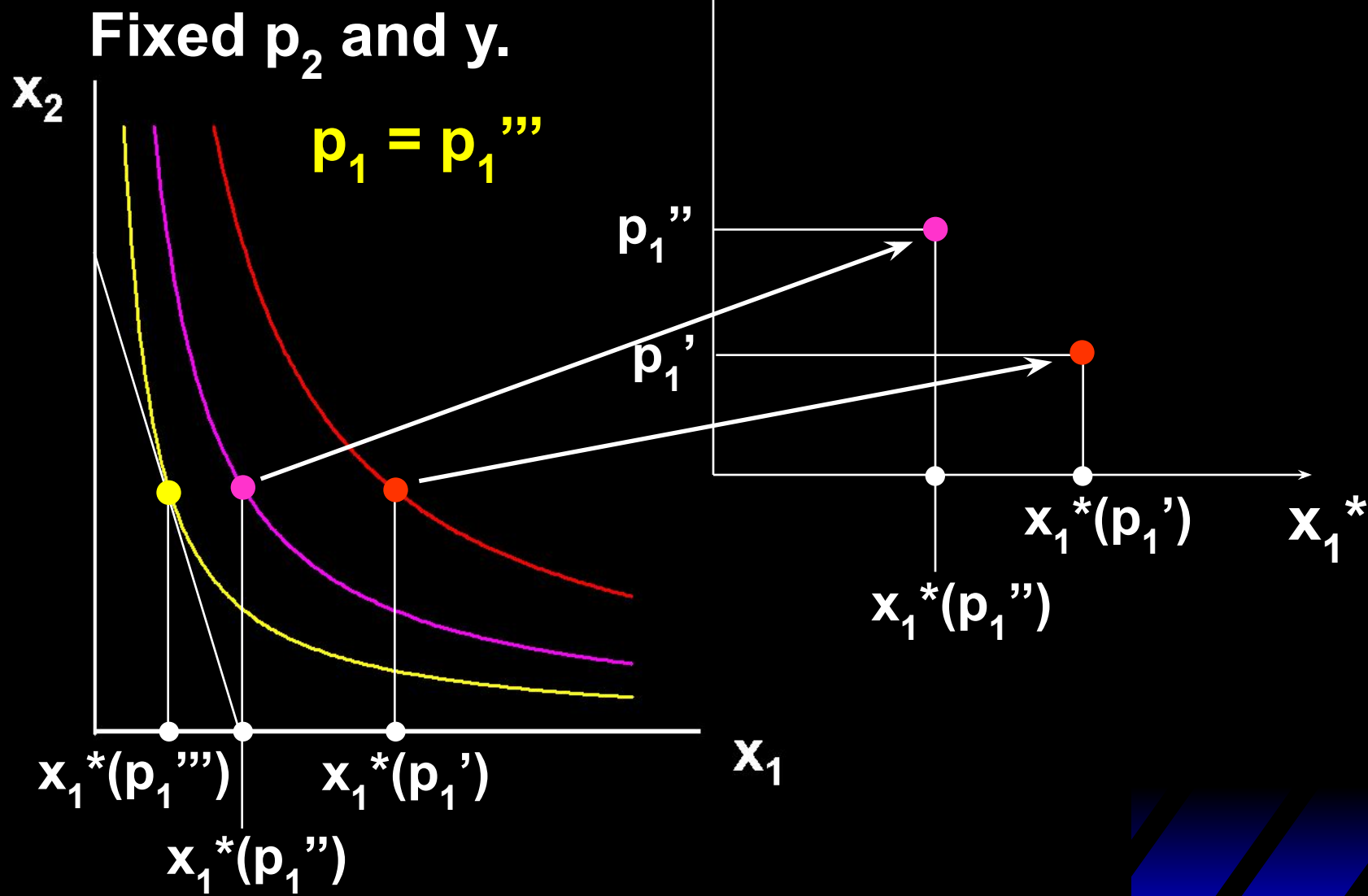
Own-Price Changes



Own-Price Changes

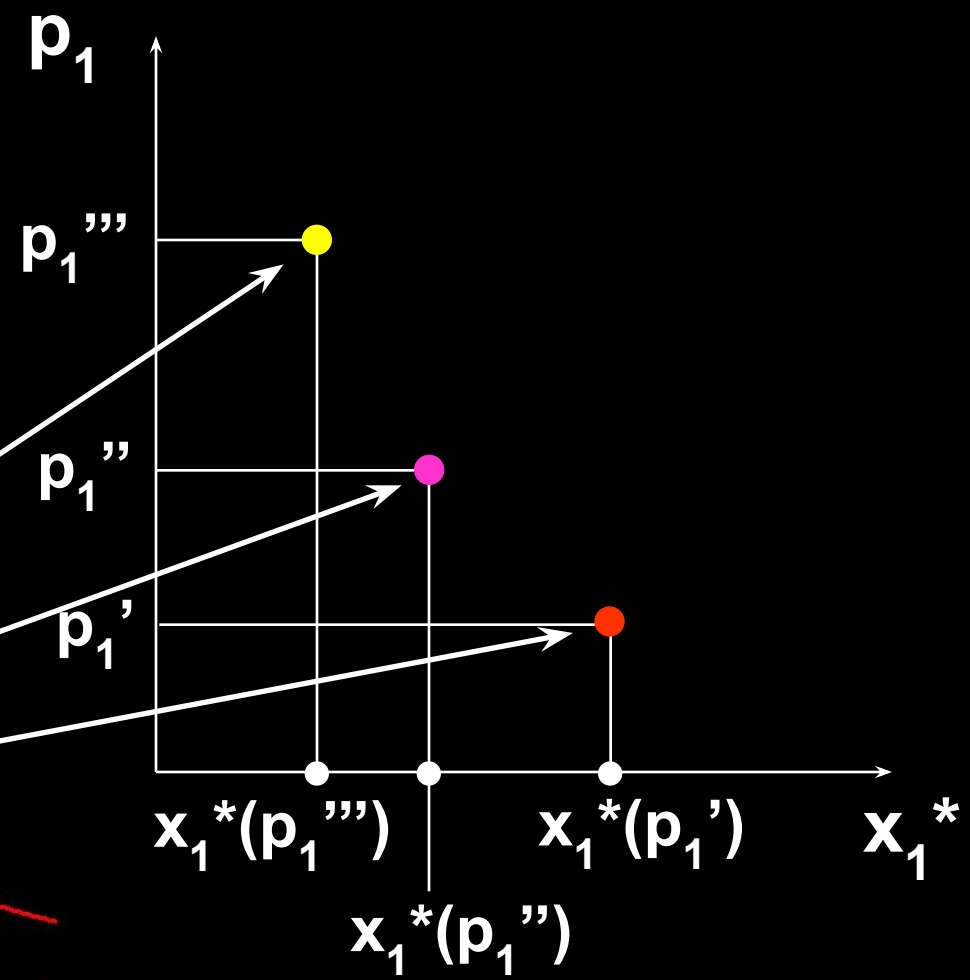
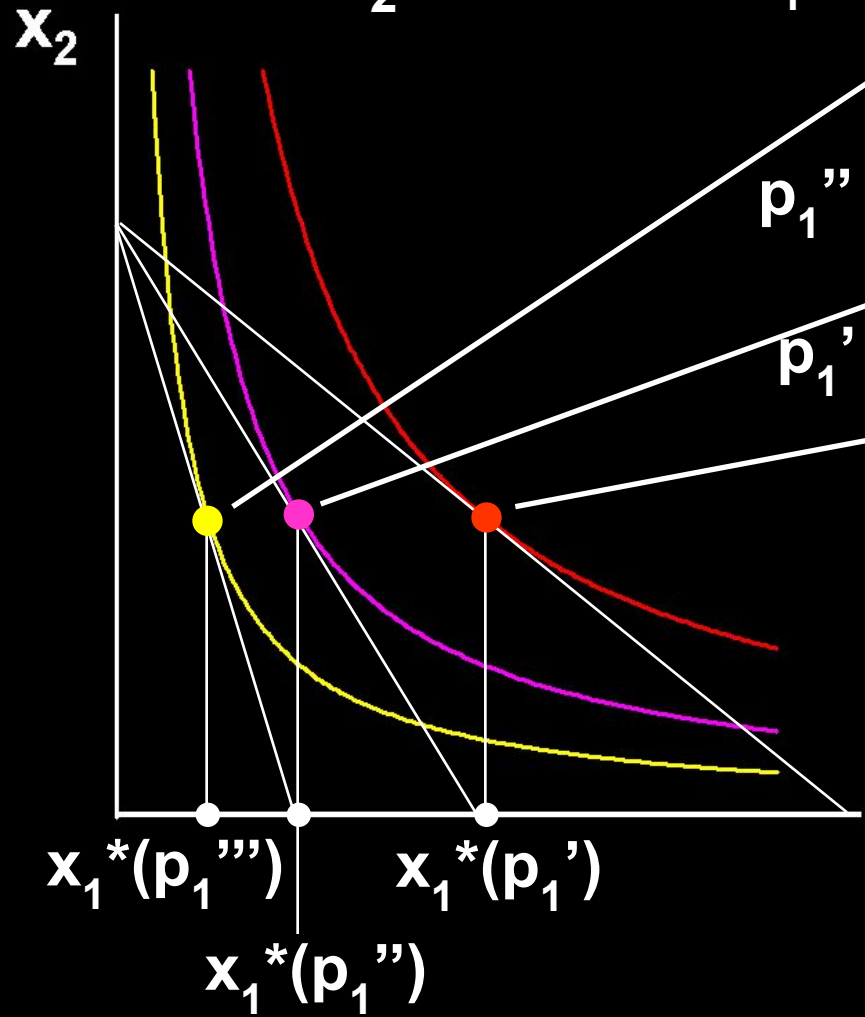


Own-Price Changes



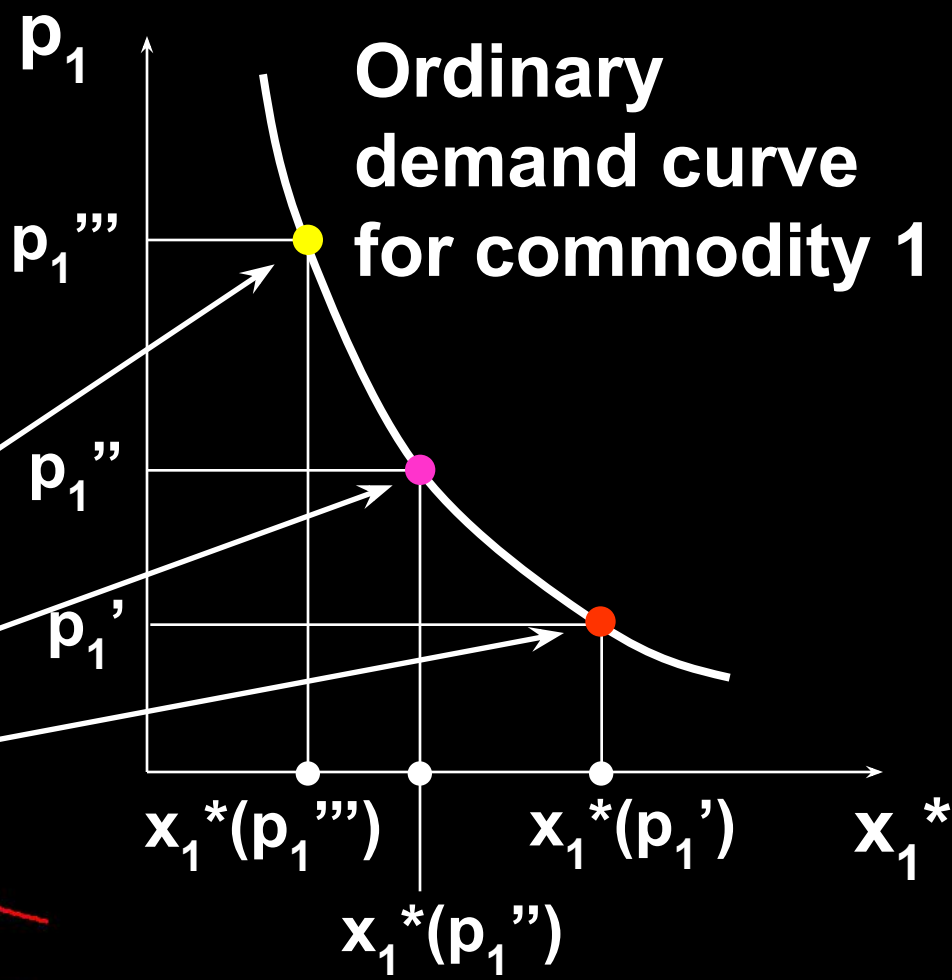
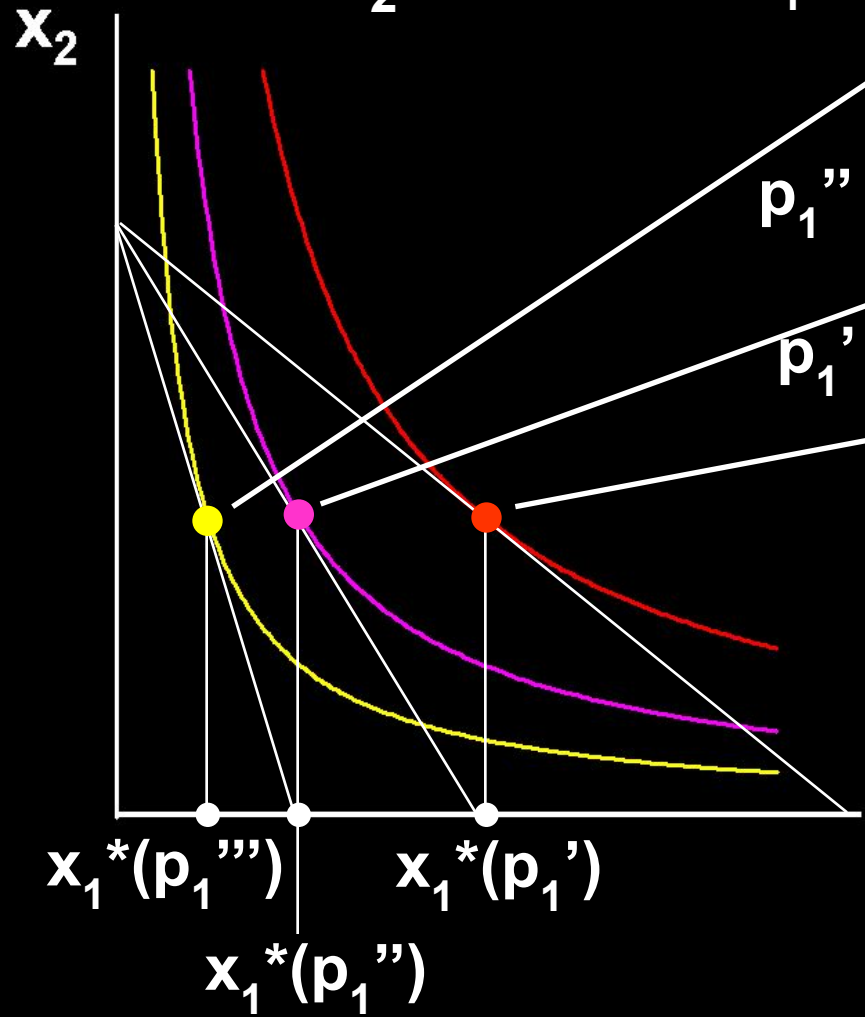
Own-Price Changes

Fixed p_2 and y .



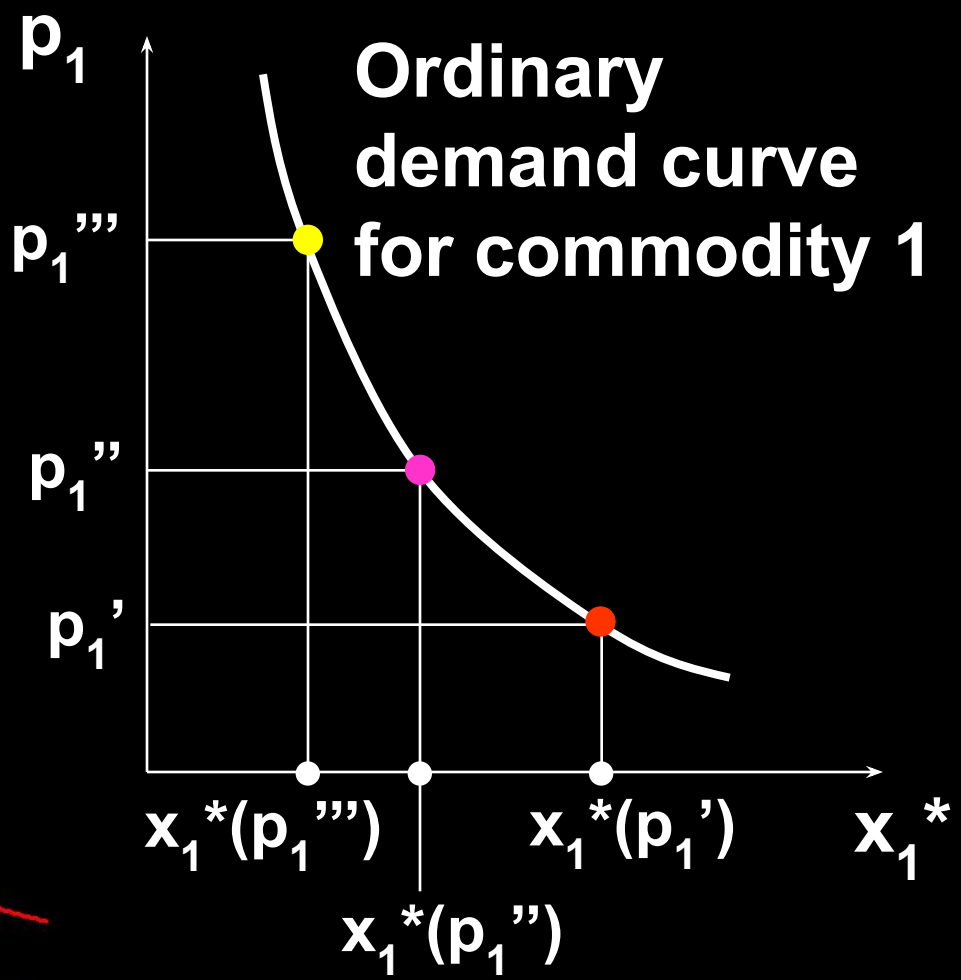
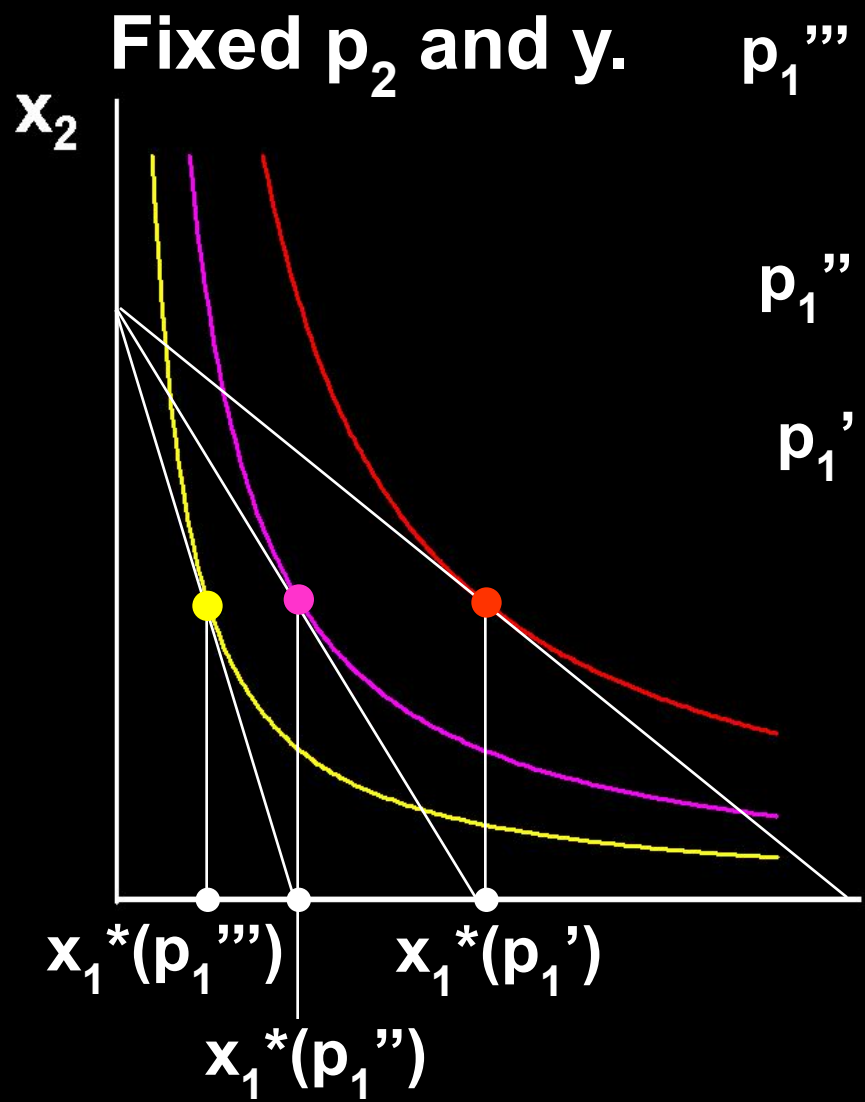
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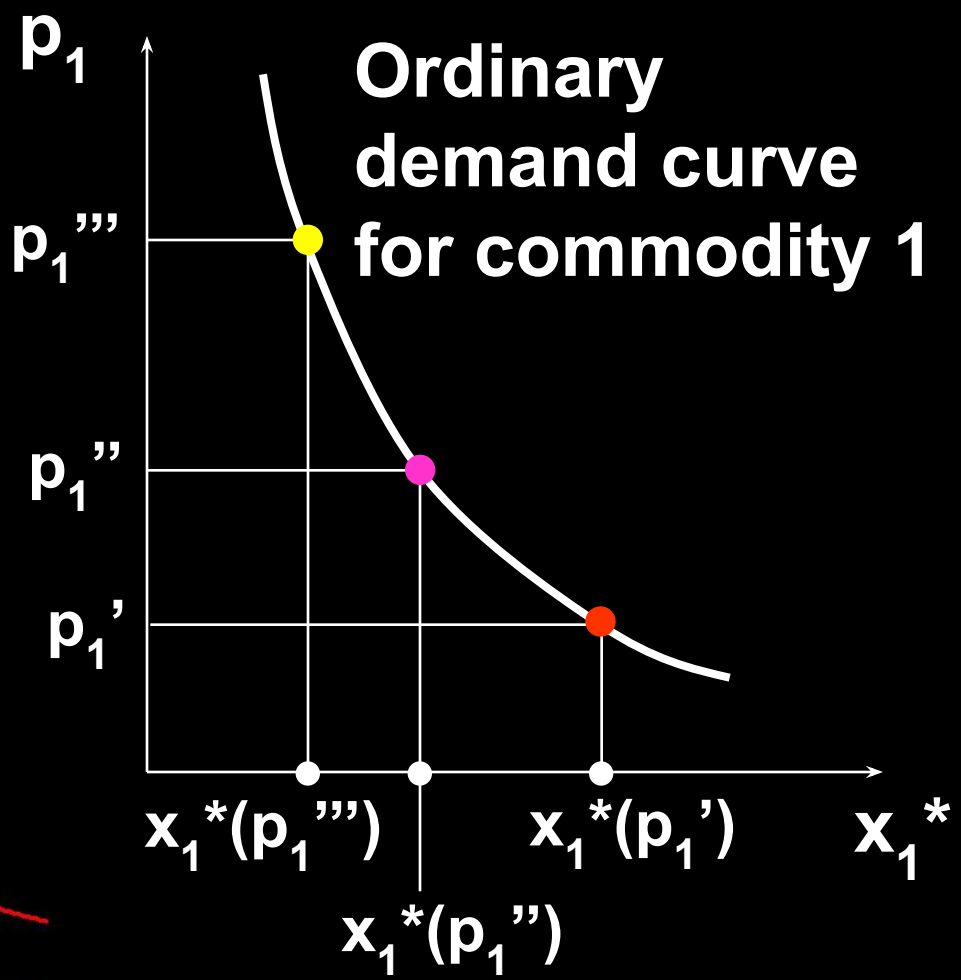
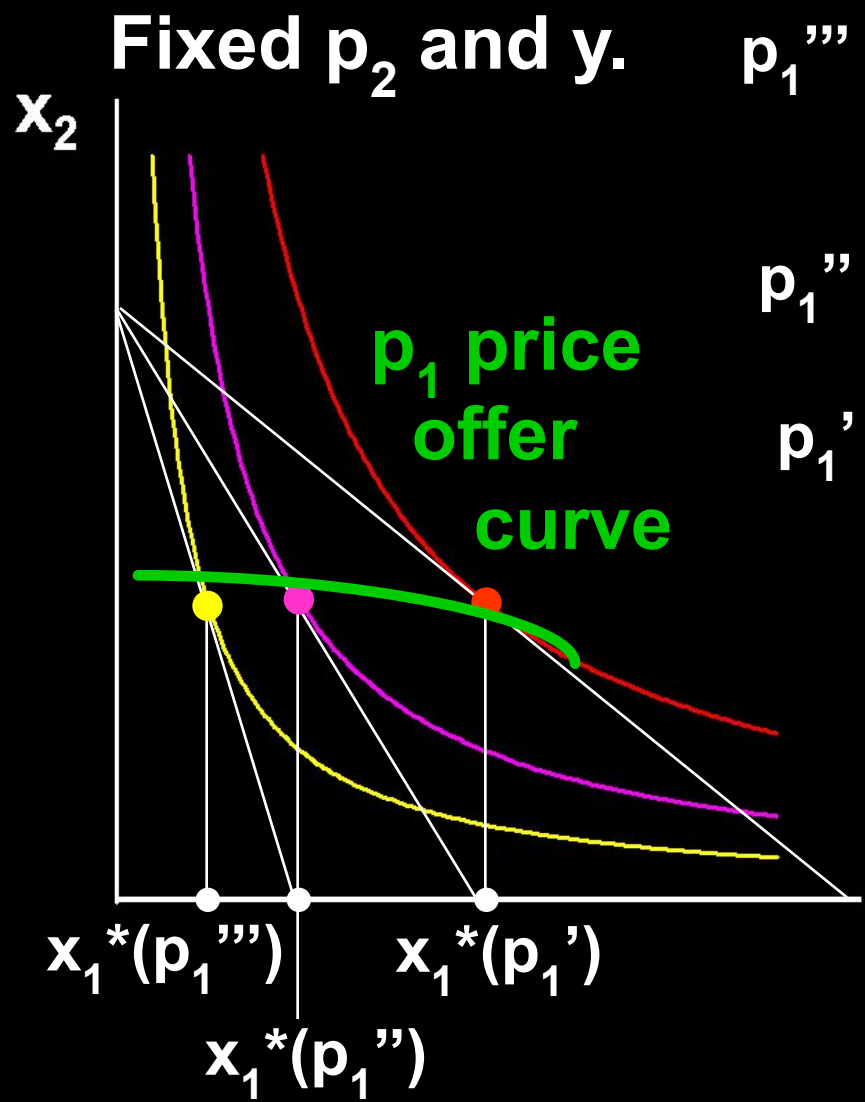


Ordinary demand curve for commodity 1

Own-Price Changes



Own-Price Changes



Own-Price Changes

- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the **p_1 -price offer curve**.
- The plot of the x_1 -coordinate of the p_1 -price offer curve against p_1 is the **ordinary** demand curve for commodity 1.

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?
- Take

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat**

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a

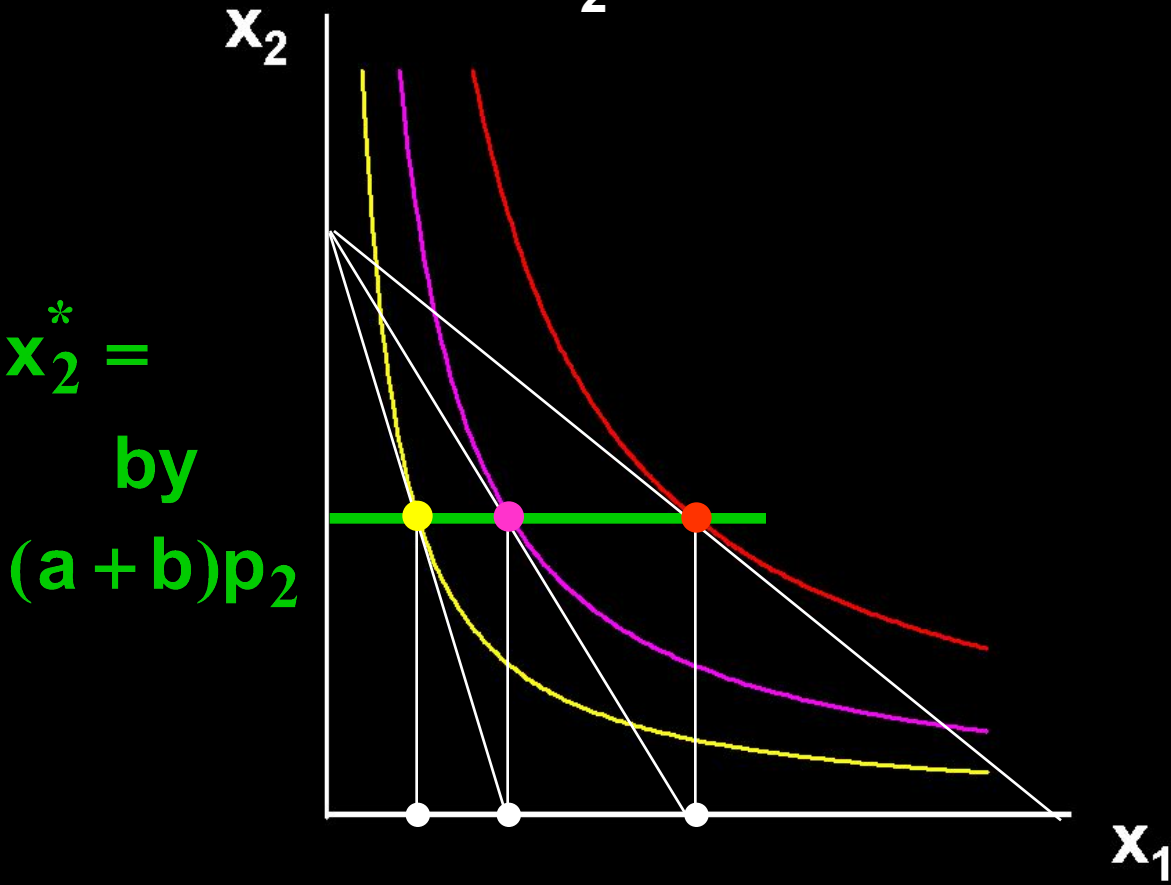
Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a **rectangular hyperbola**.

Own-Price Changes

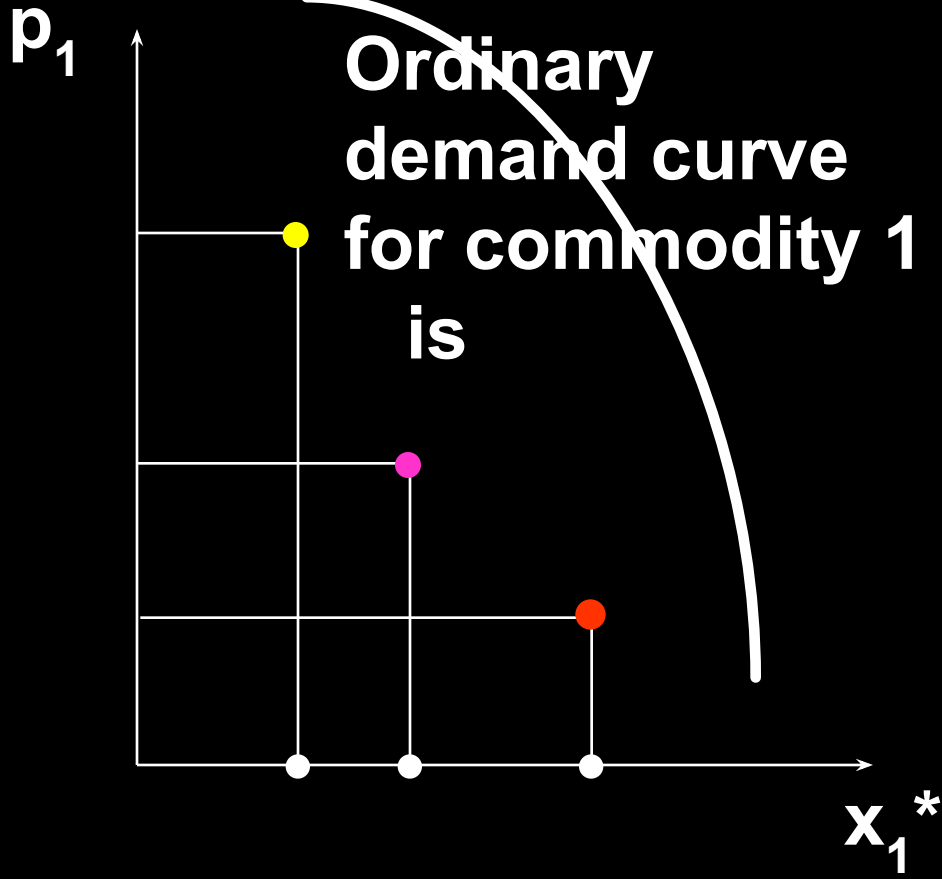
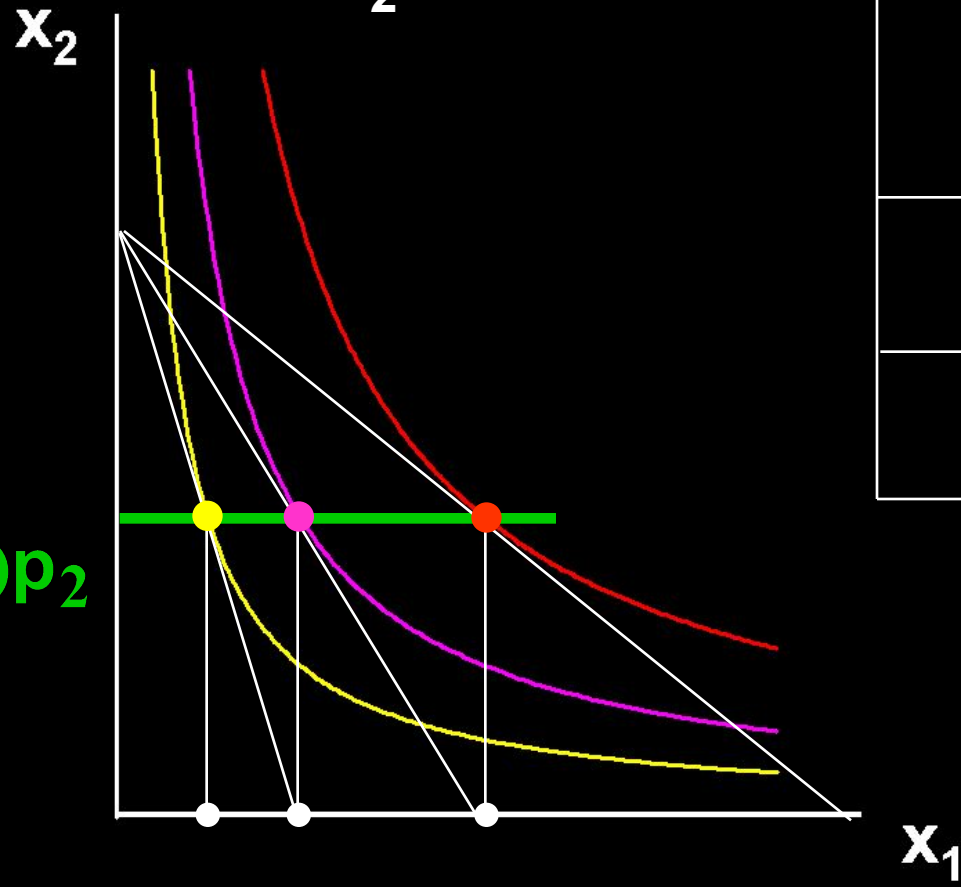
Fixed p_2 and y .



Own-Price Changes

Fixed p_2 and y .

$x_2^* =$
by
 $(a + b)p_2$



Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

Own-Price Changes

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Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes



Own-Price Changes

With p_2 and y fixed, higher p_1 causes smaller x_1^* and x_2^* .

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As

Own-Price Changes

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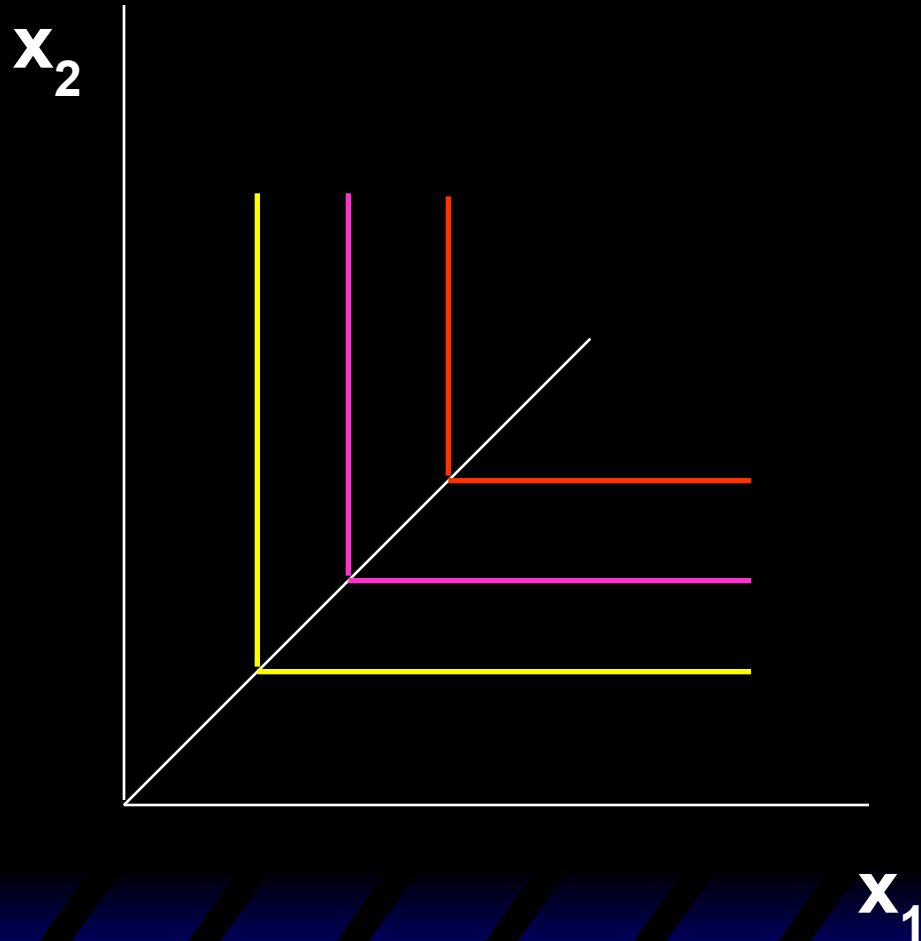
As

As



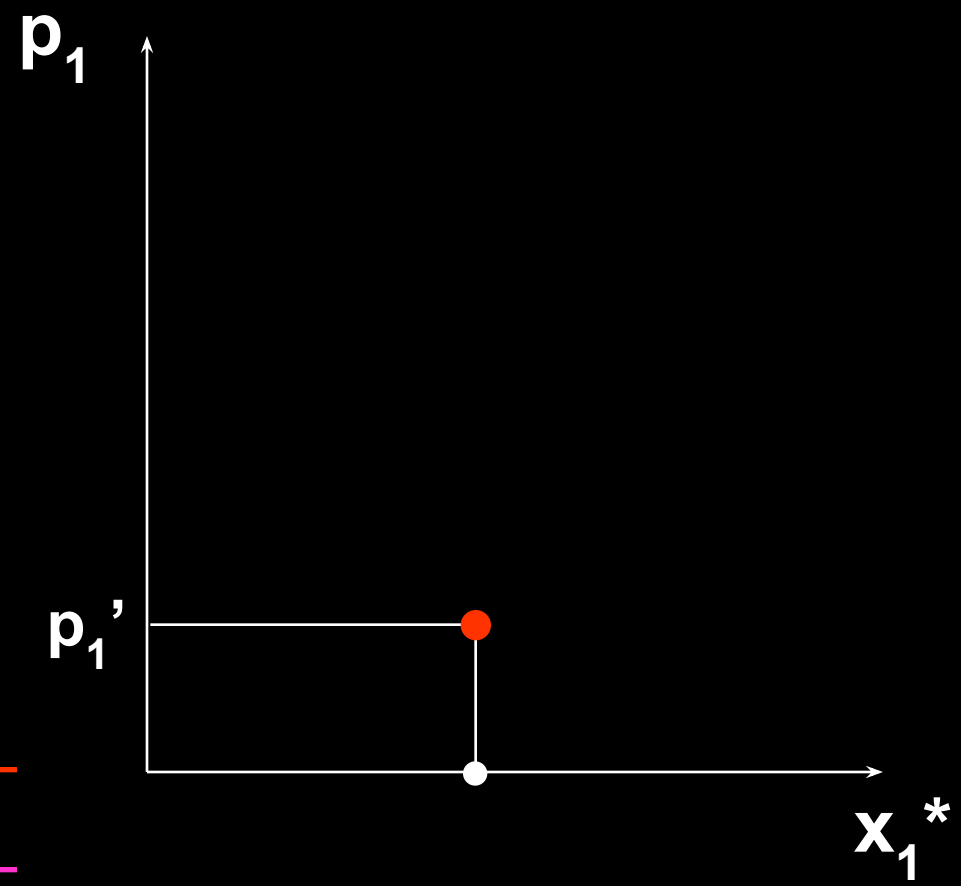
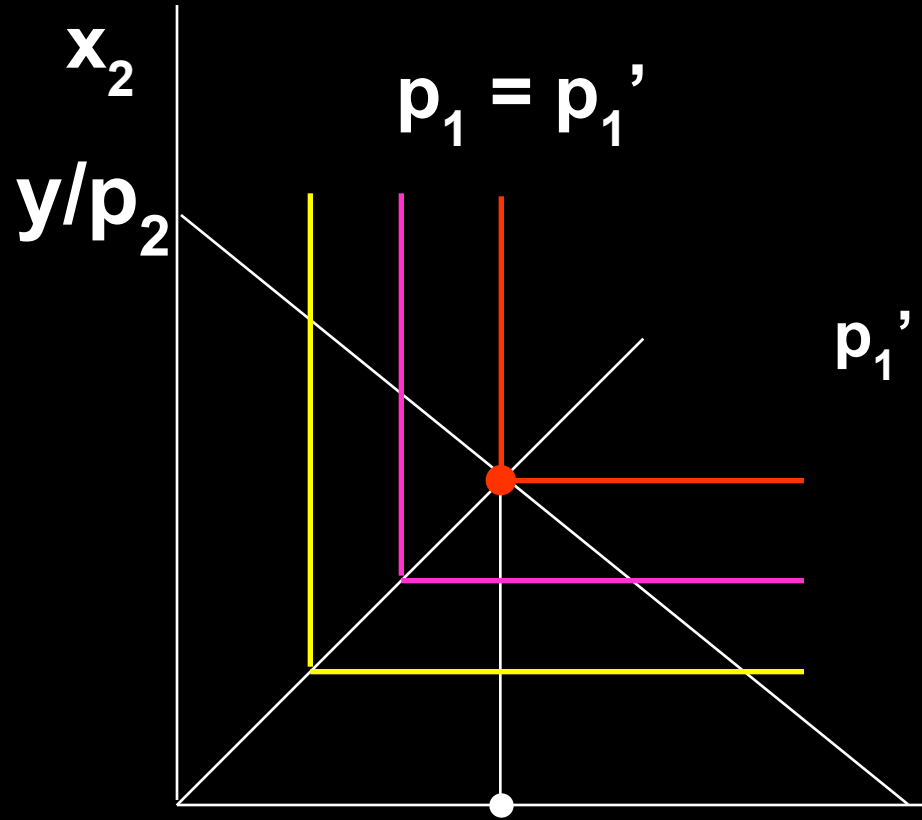
Own-Price Changes

Fixed p_2 and y .



Own-Price Changes

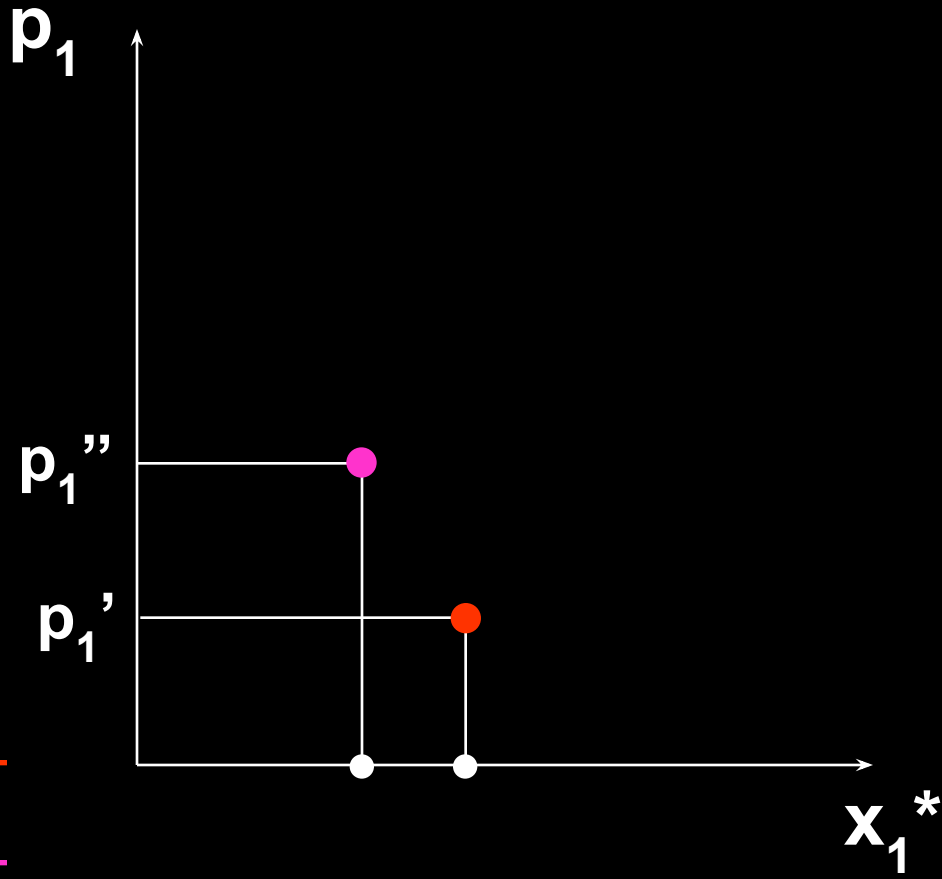
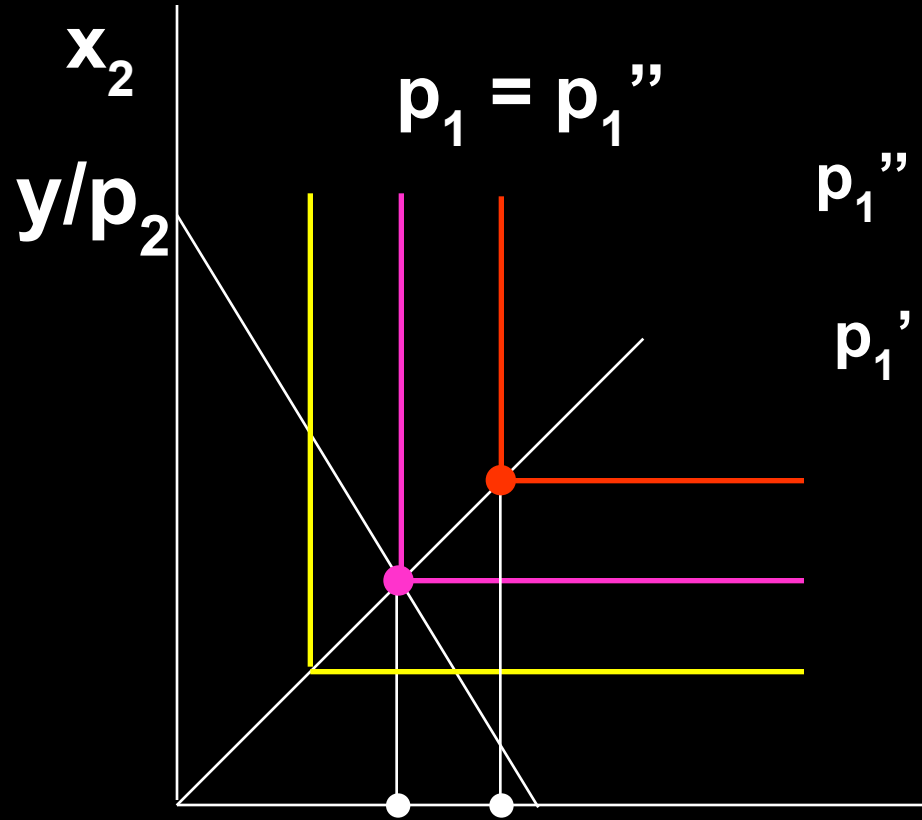
Fixed p_2 and y .



$$x_1 = \frac{y}{p_1 + p_2}$$

Own-Price Changes

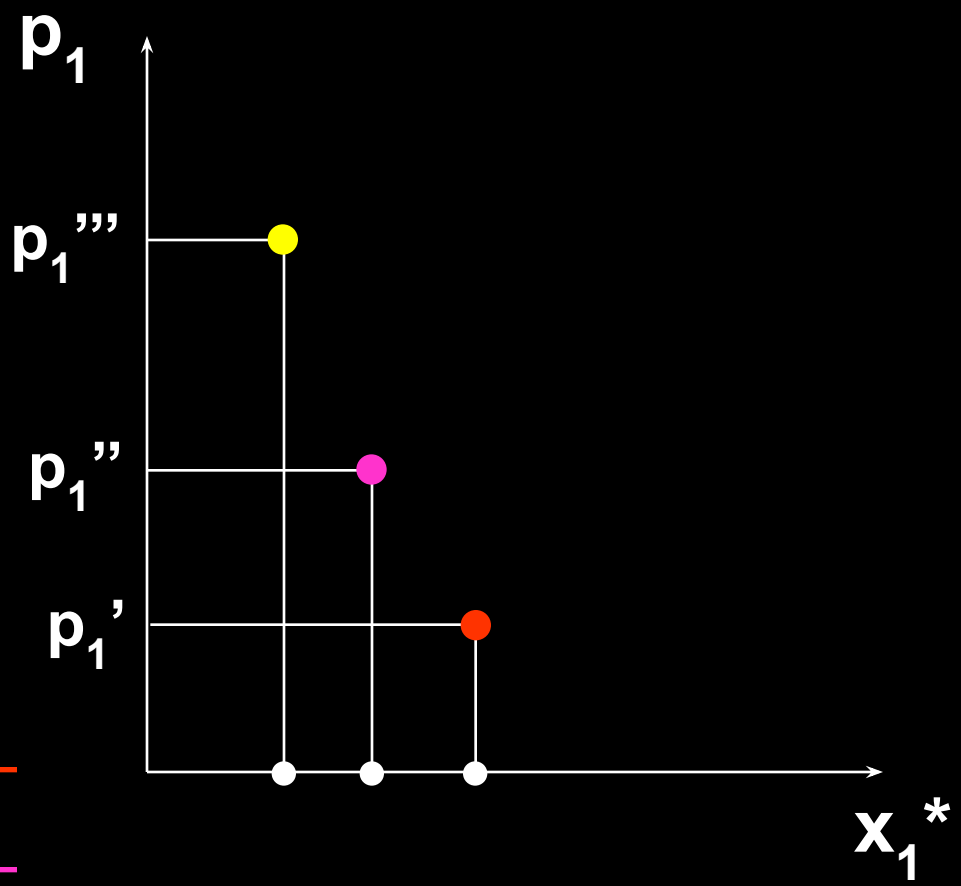
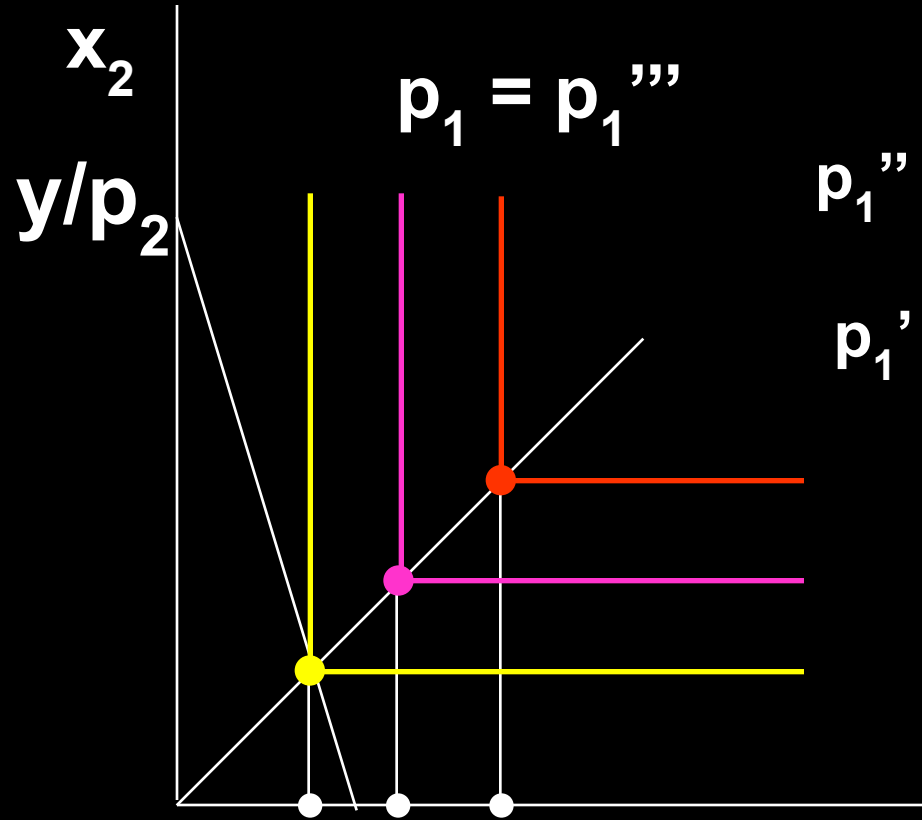
Fixed p_2 and y .



$$x_1 = \frac{y}{p_1'' + p_2}$$

Own-Price Changes

Fixed p_2 and y .



'''

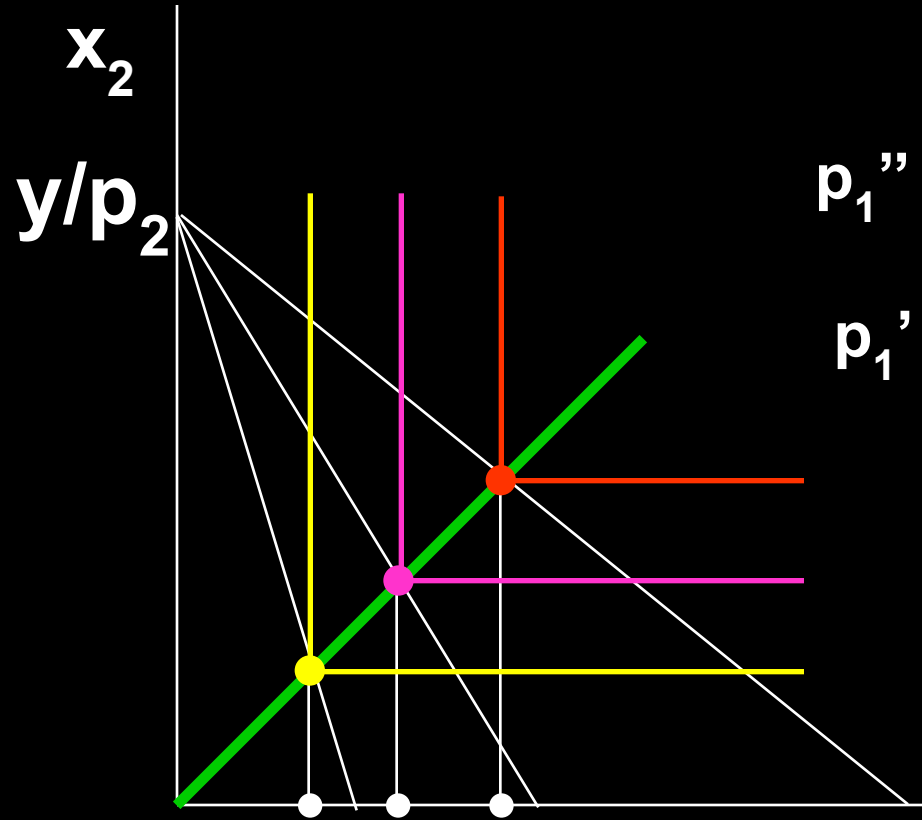
'''

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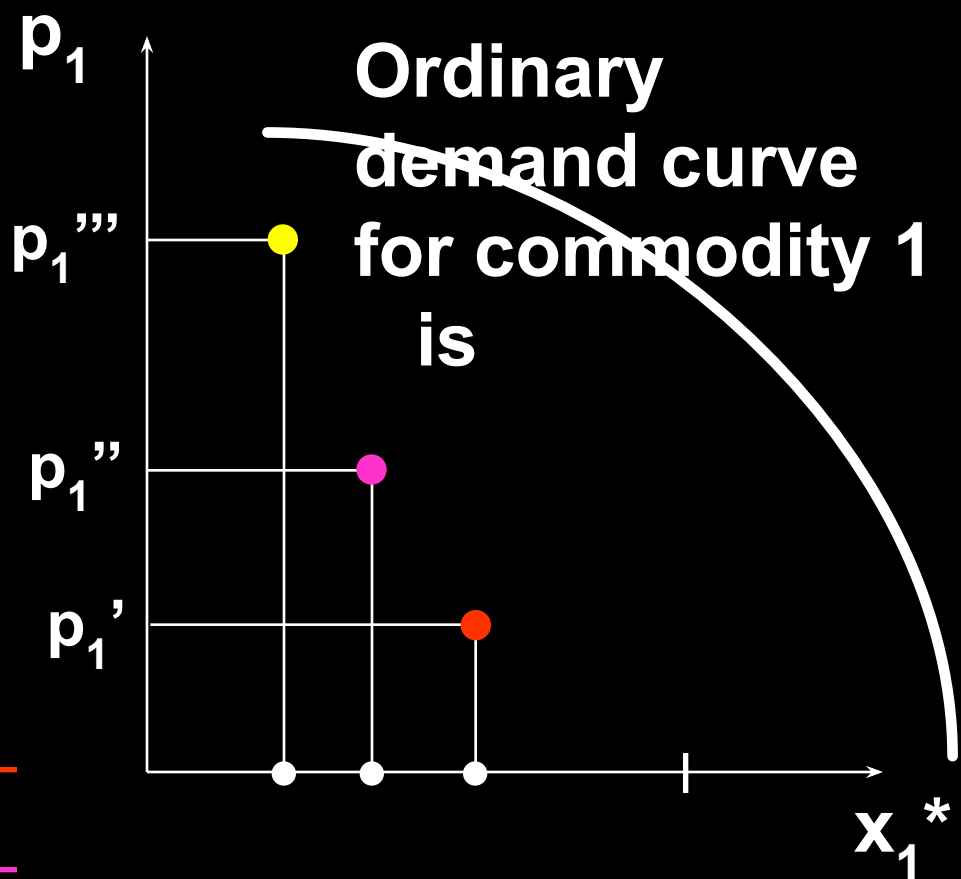
x_1

Own-Price Changes

Fixed p_2 and y .



$$x_1 = \frac{y}{p_1 + p_2}$$



Ordinary demand curve for commodity 1 is

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

Then the ordinary demand functions for commodities 1 and 2 are

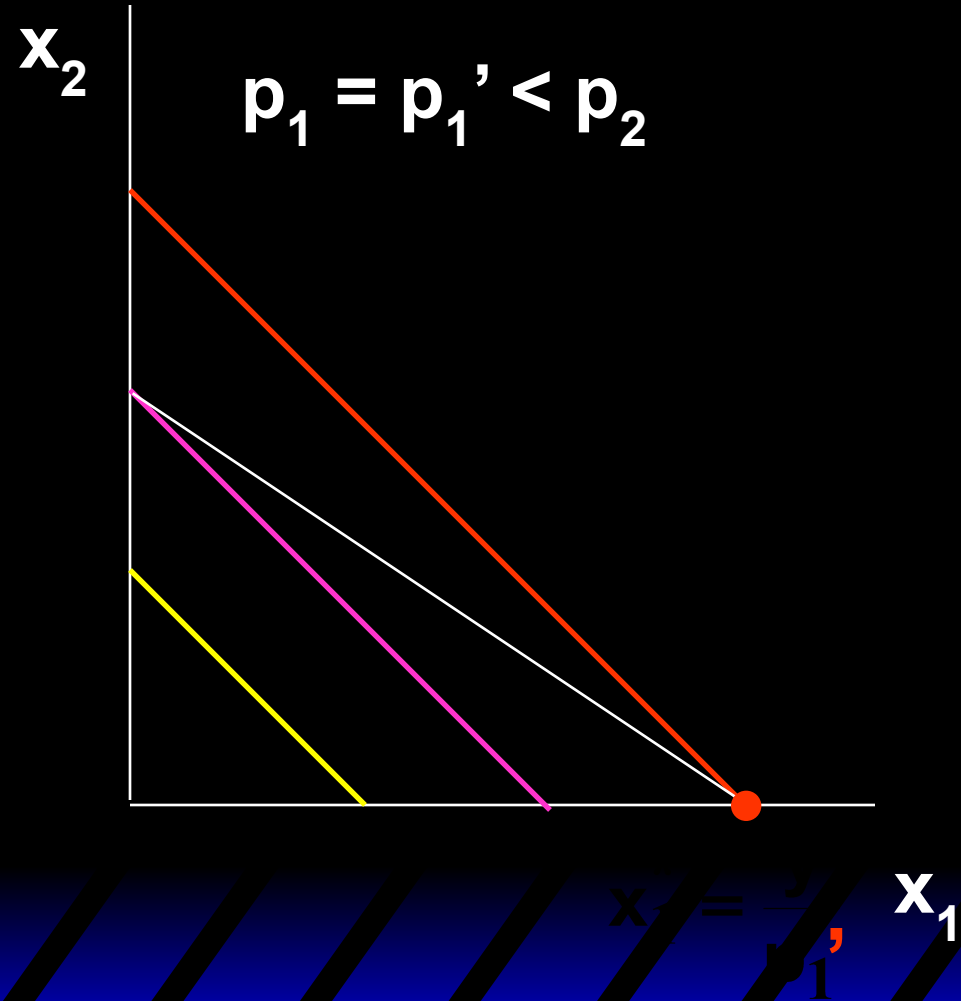
Own-Price Changes

and



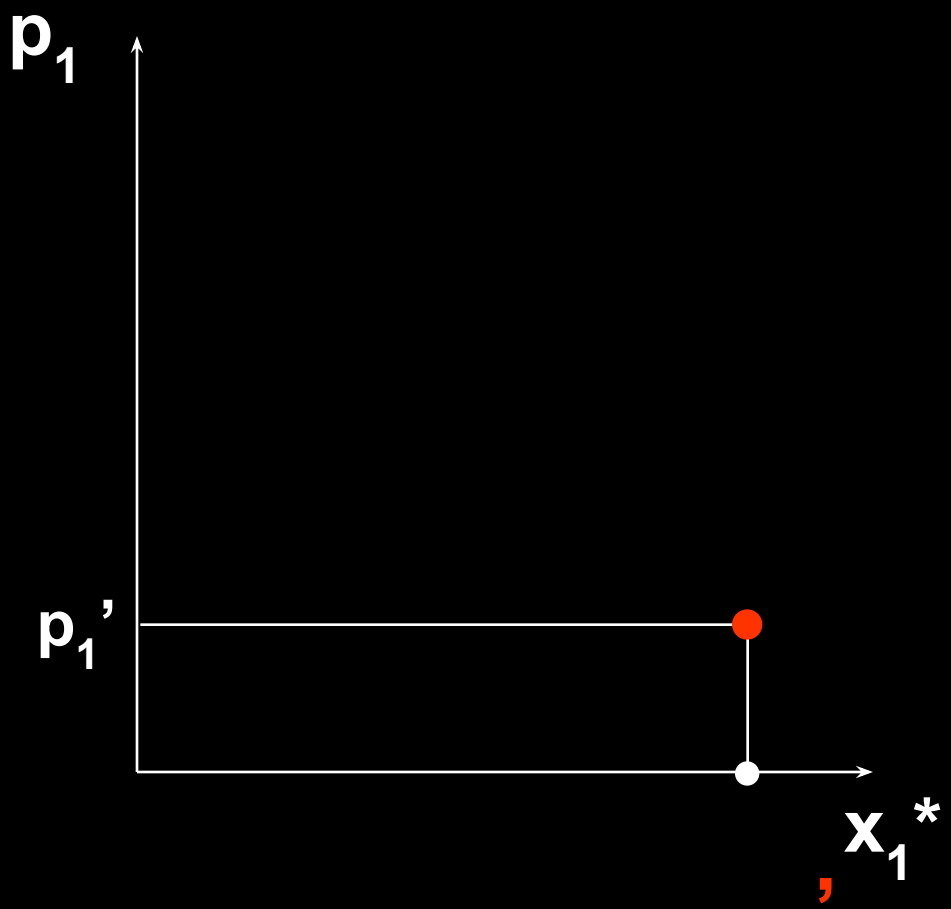
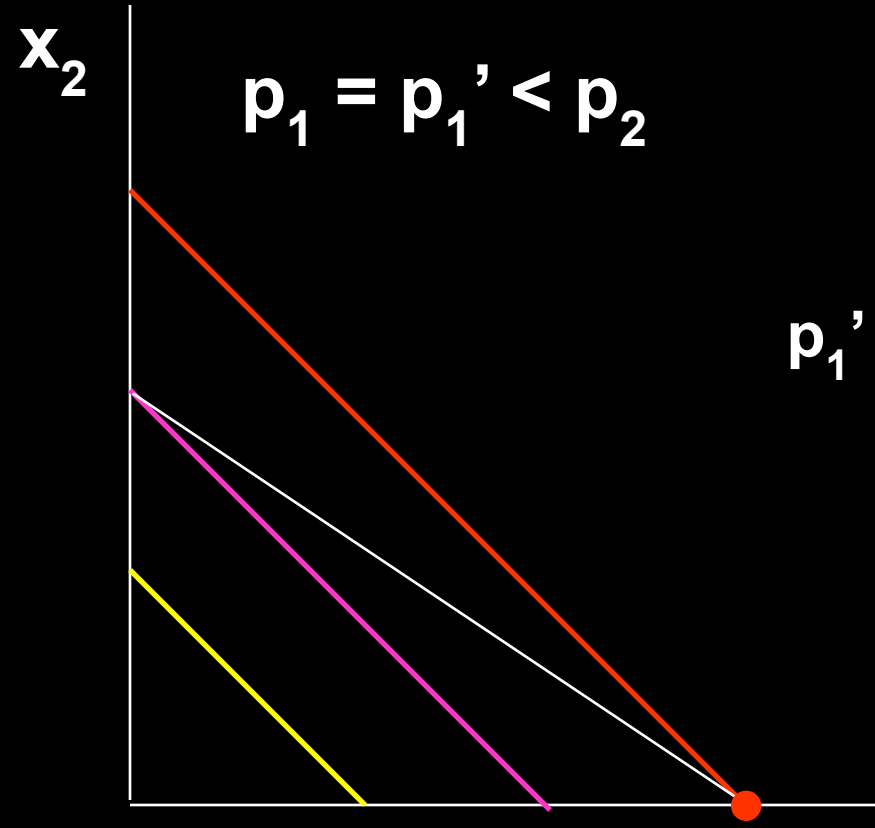
Own-Price Changes

Fixed p_2 and y .



Own-Price Changes

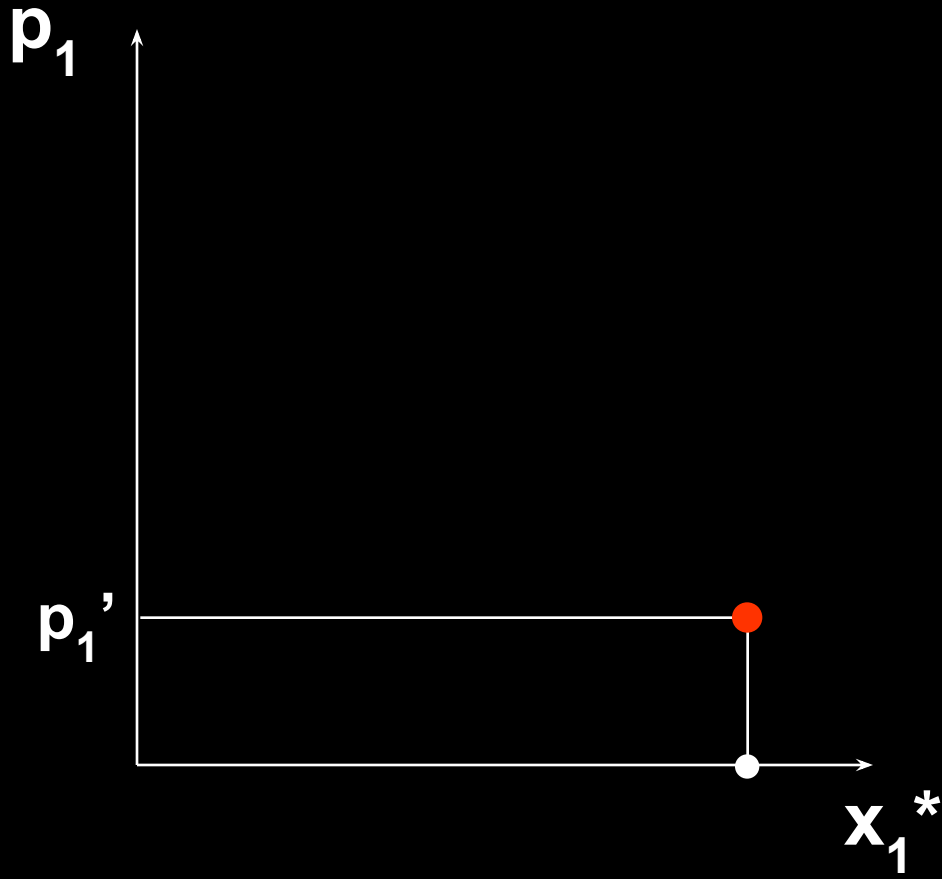
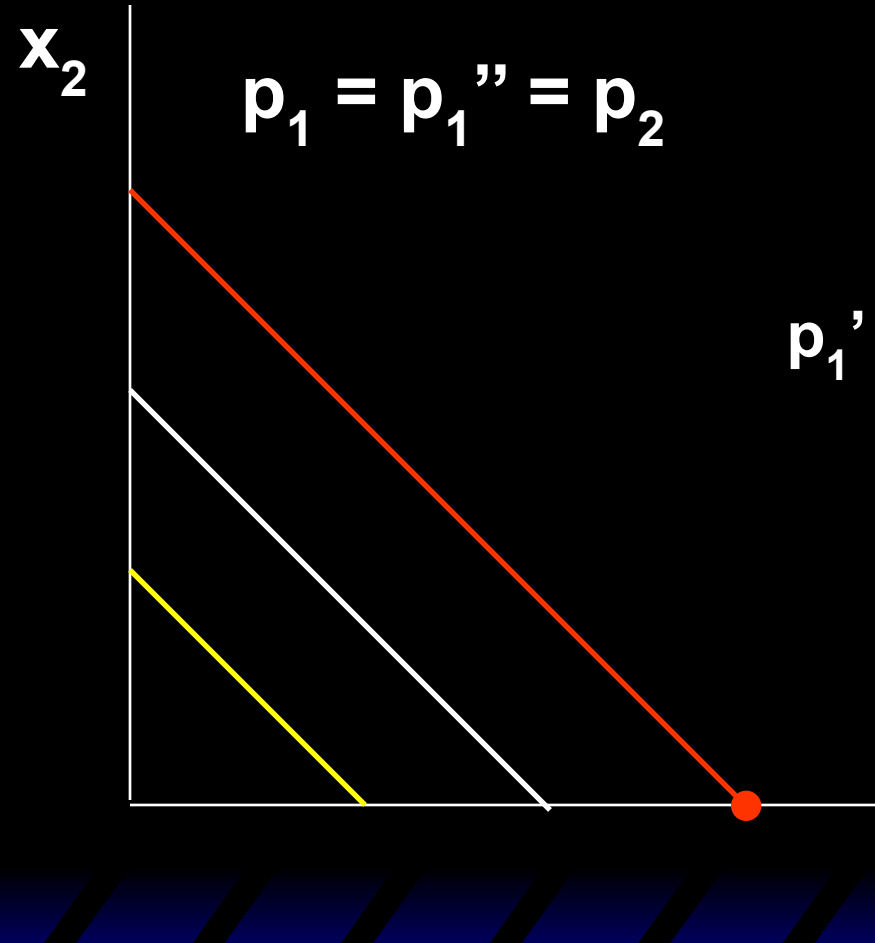
Fixed p_2 and y .



$$x_1 = \frac{y}{p_1}$$

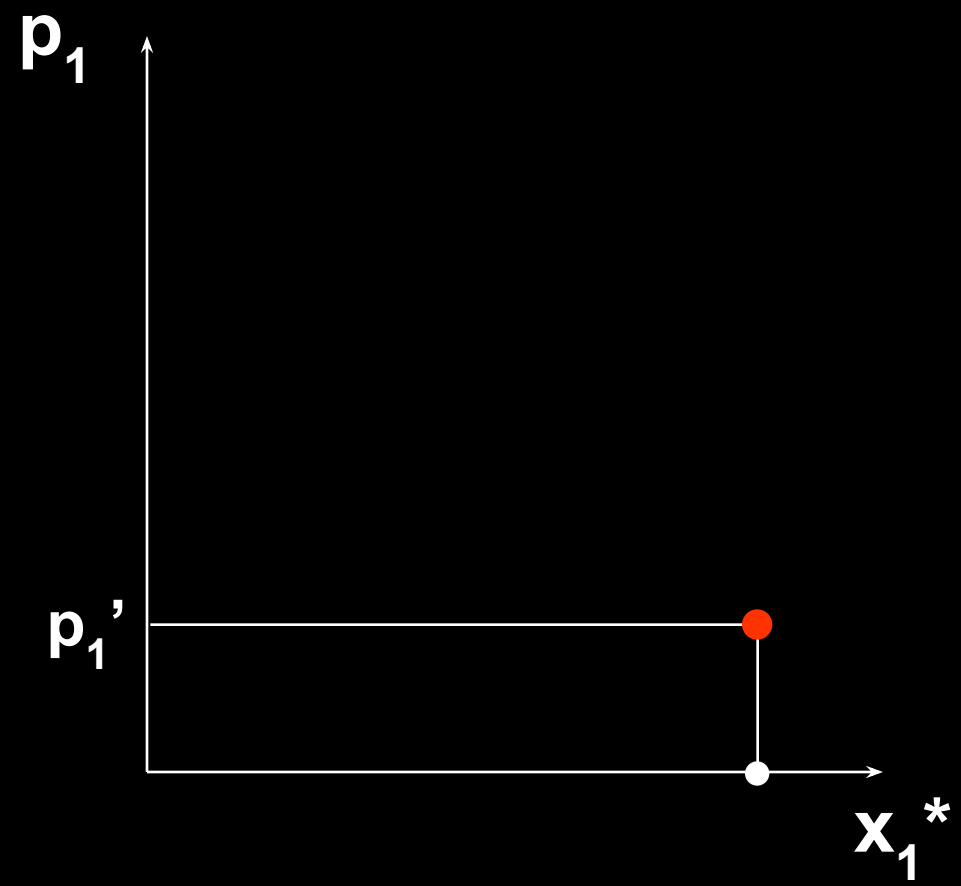
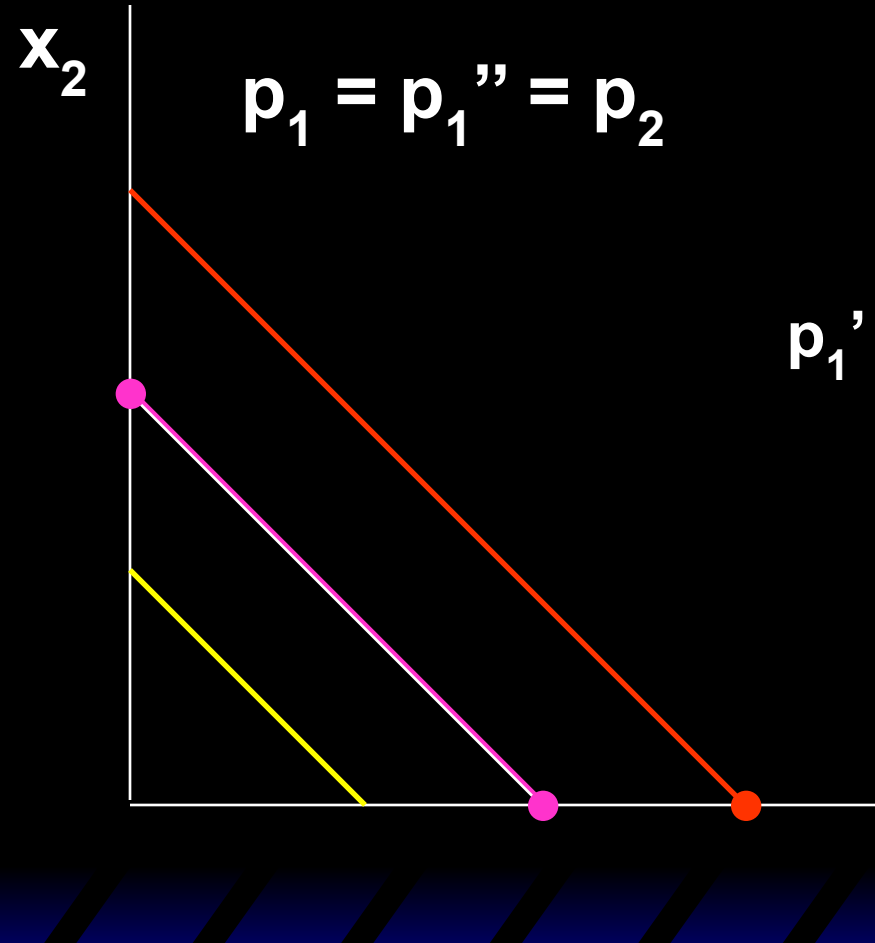
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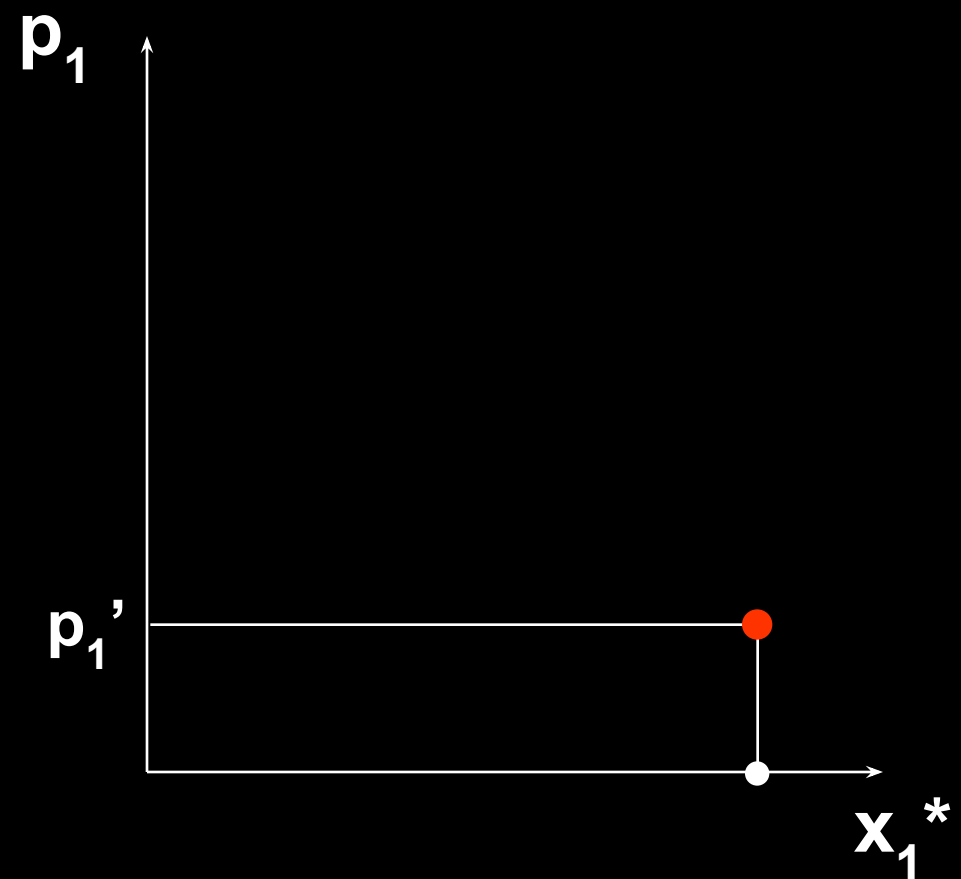
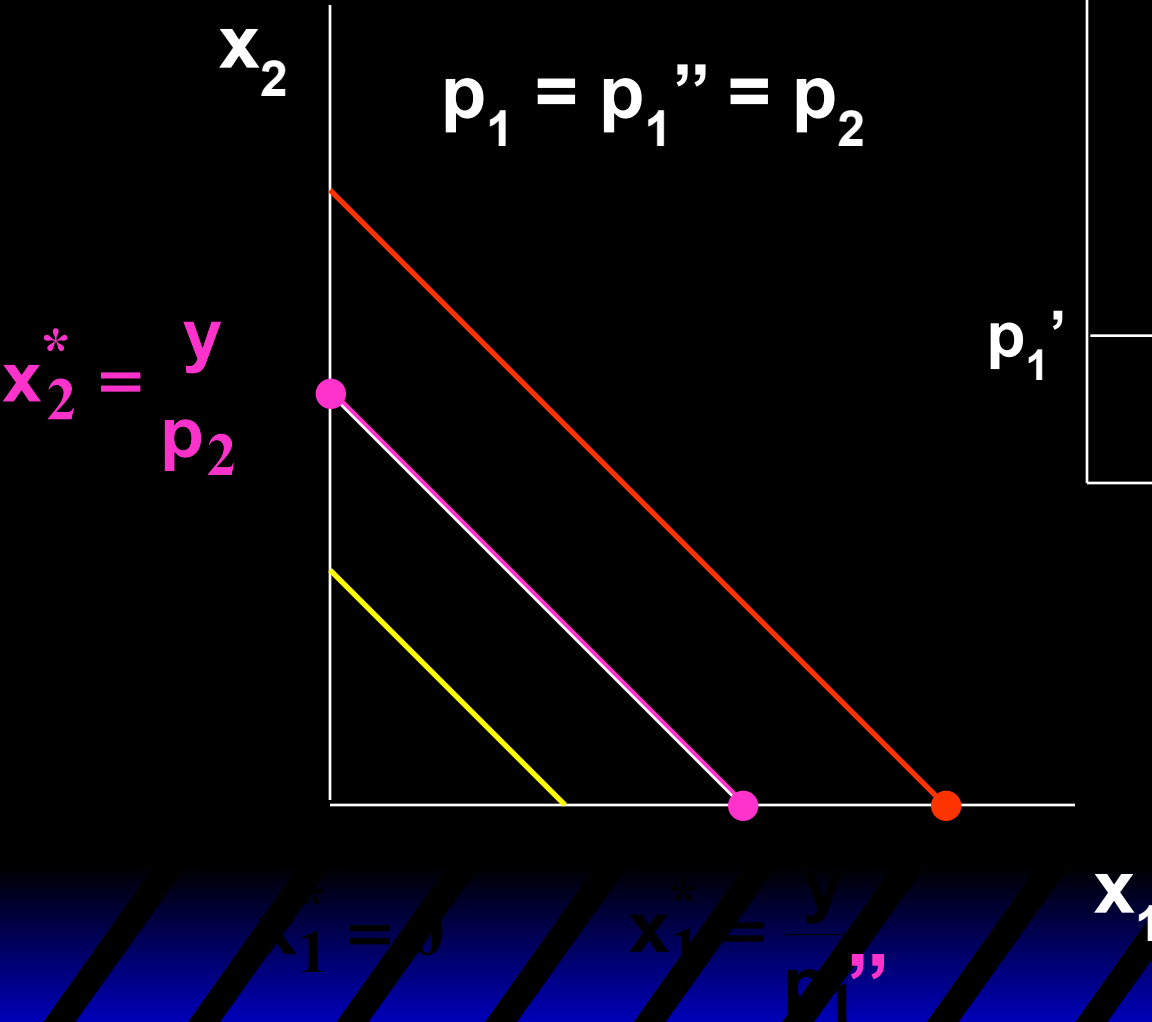
Own-Price Changes

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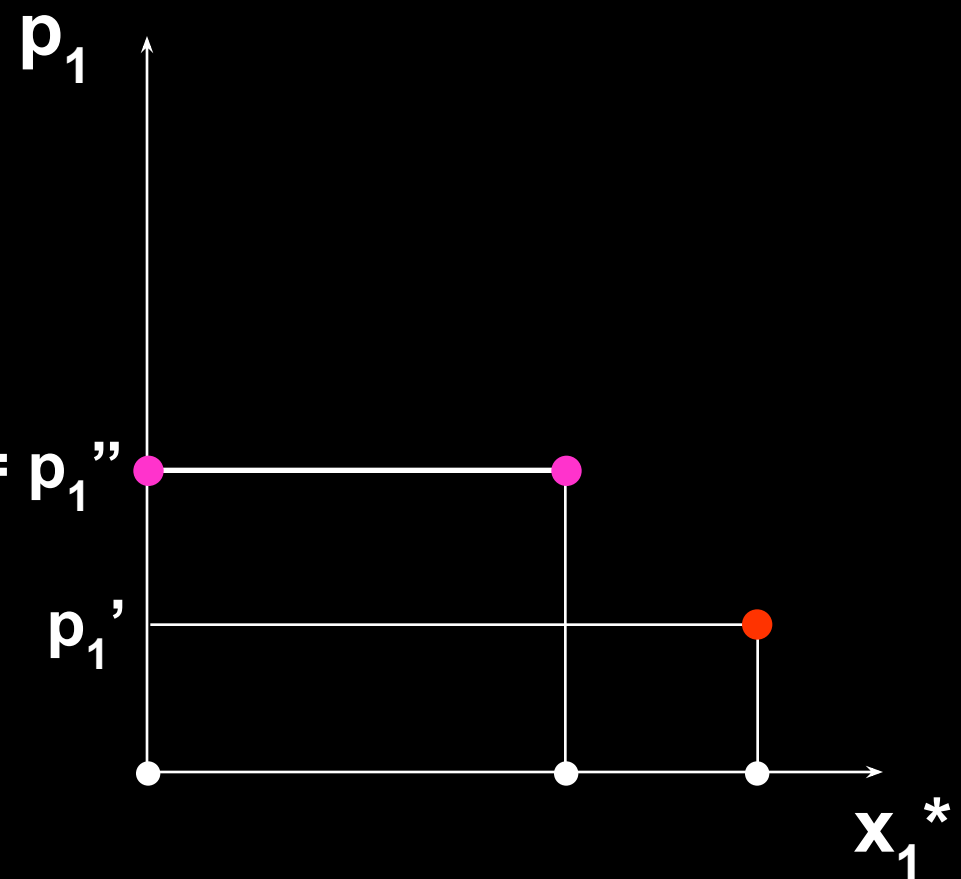
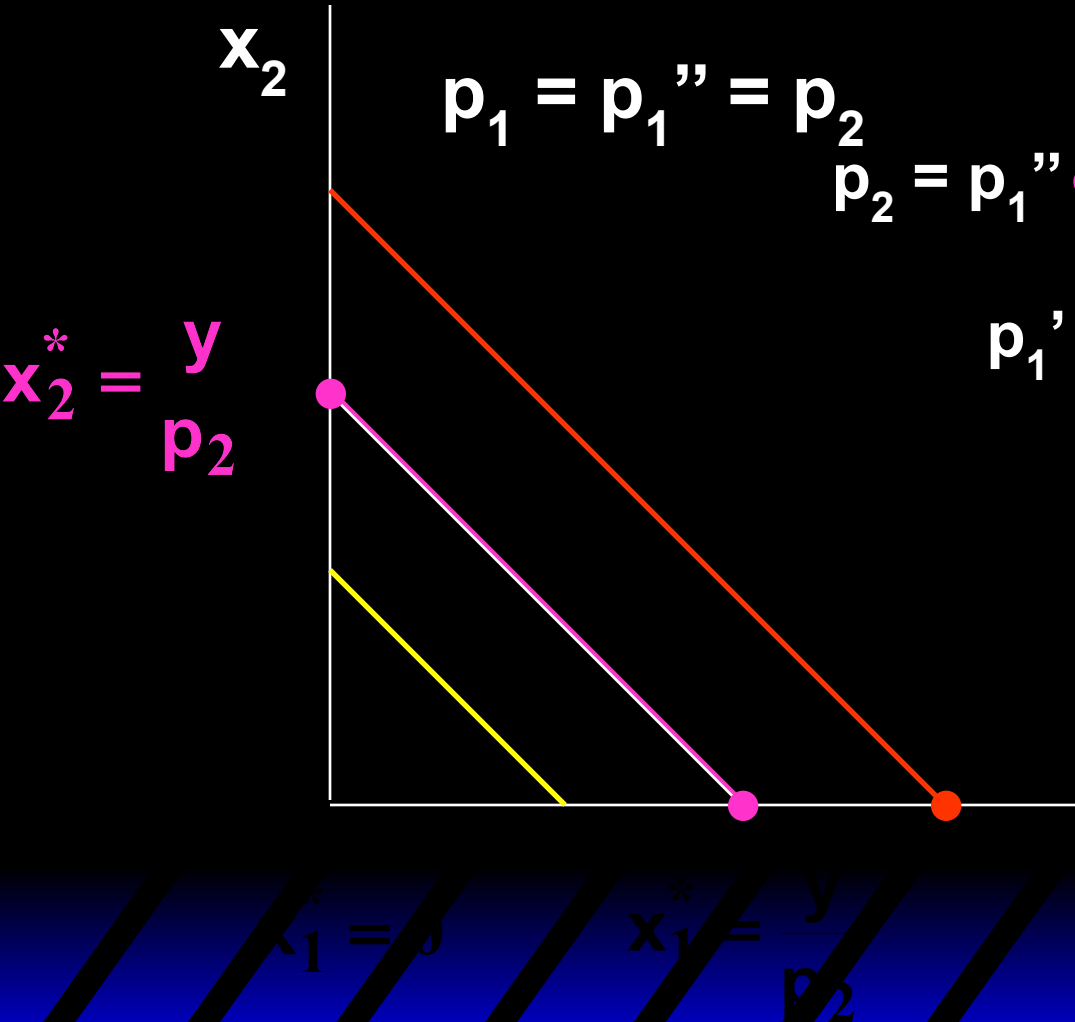
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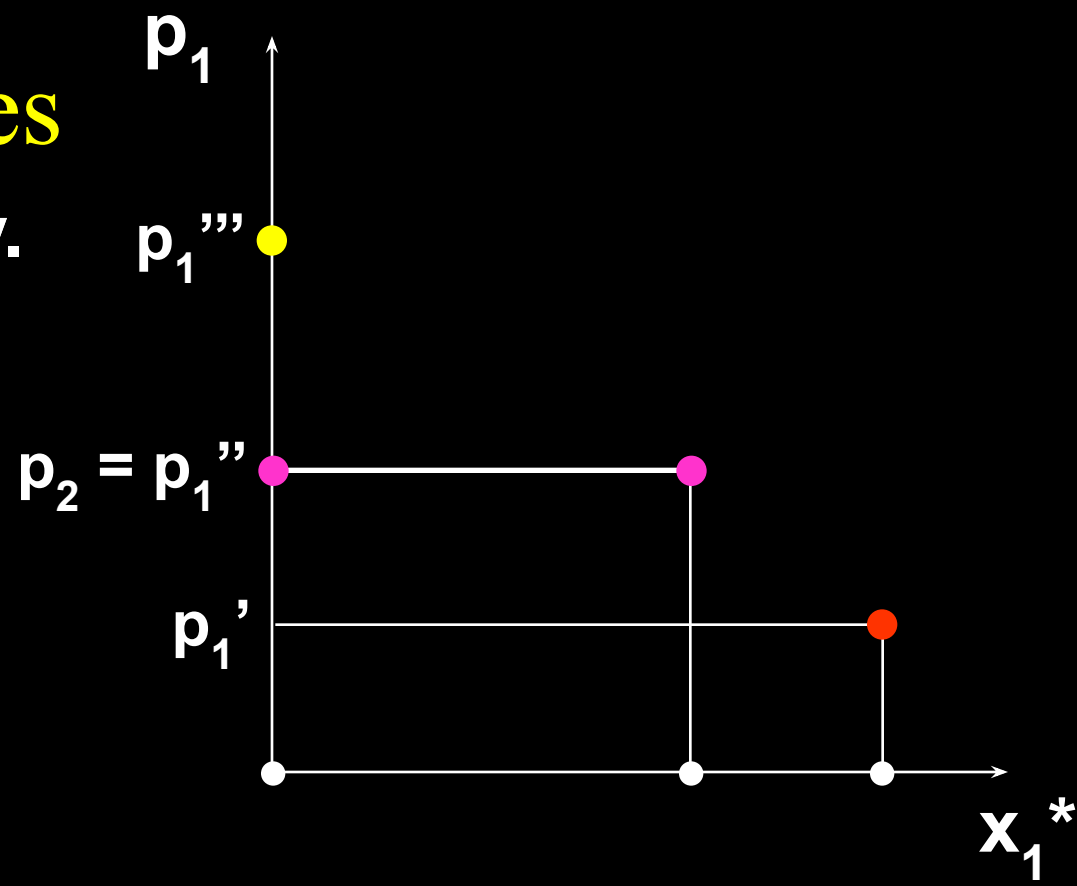
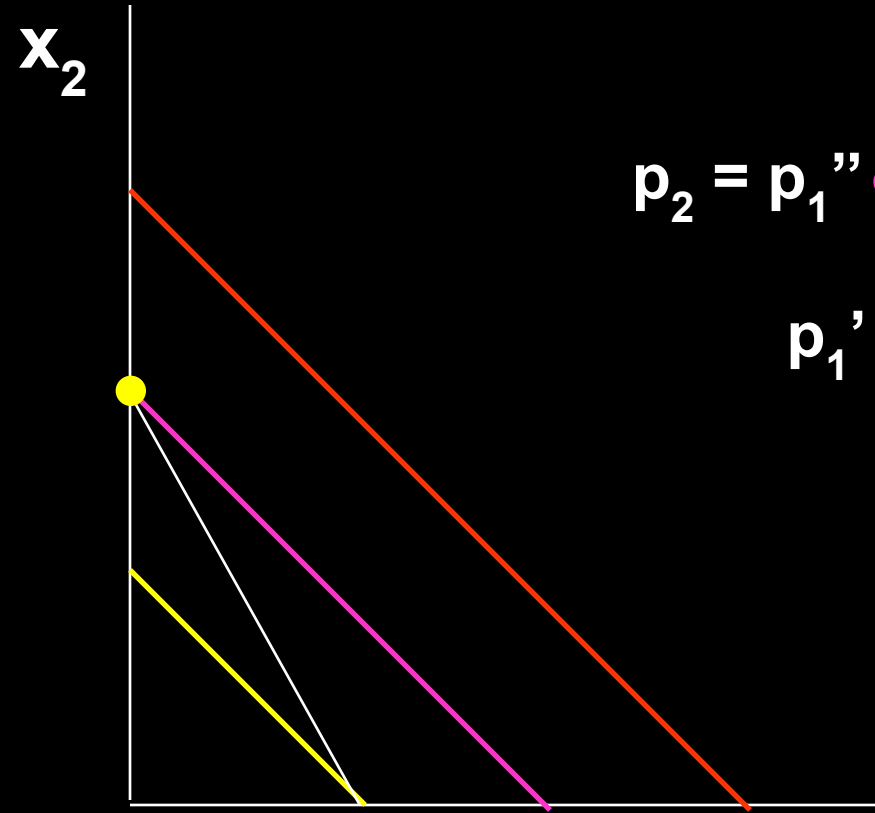
Own-Price Changes

Fixed p_2 and y .



Own-Price Changes

Fixed p_2 and y .

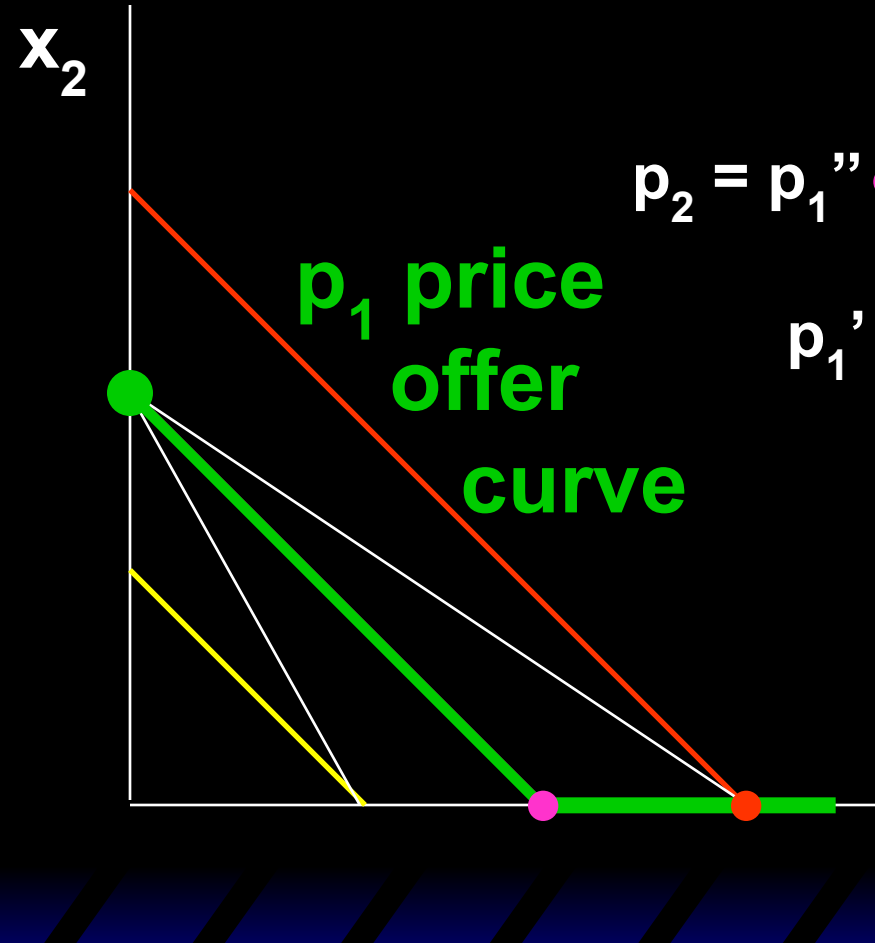


$x_1 = 0$

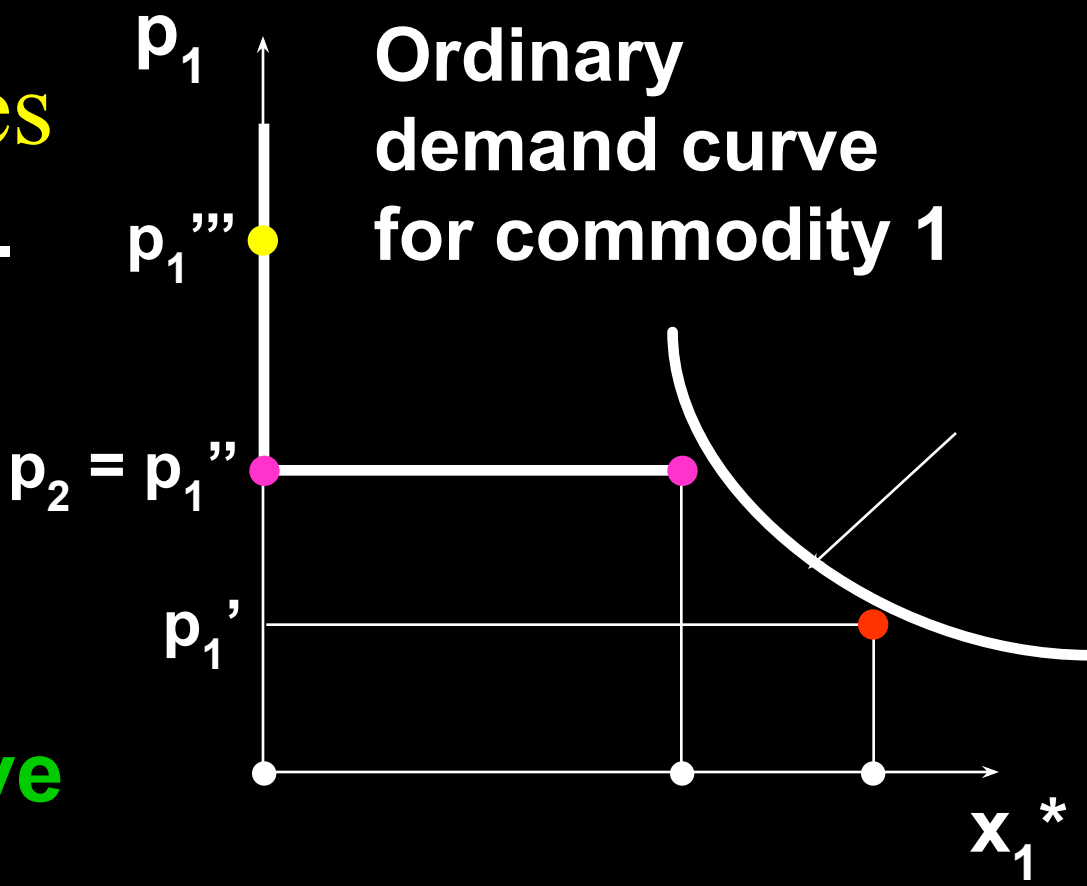
x_1

Own-Price Changes

Fixed p_2 and y .



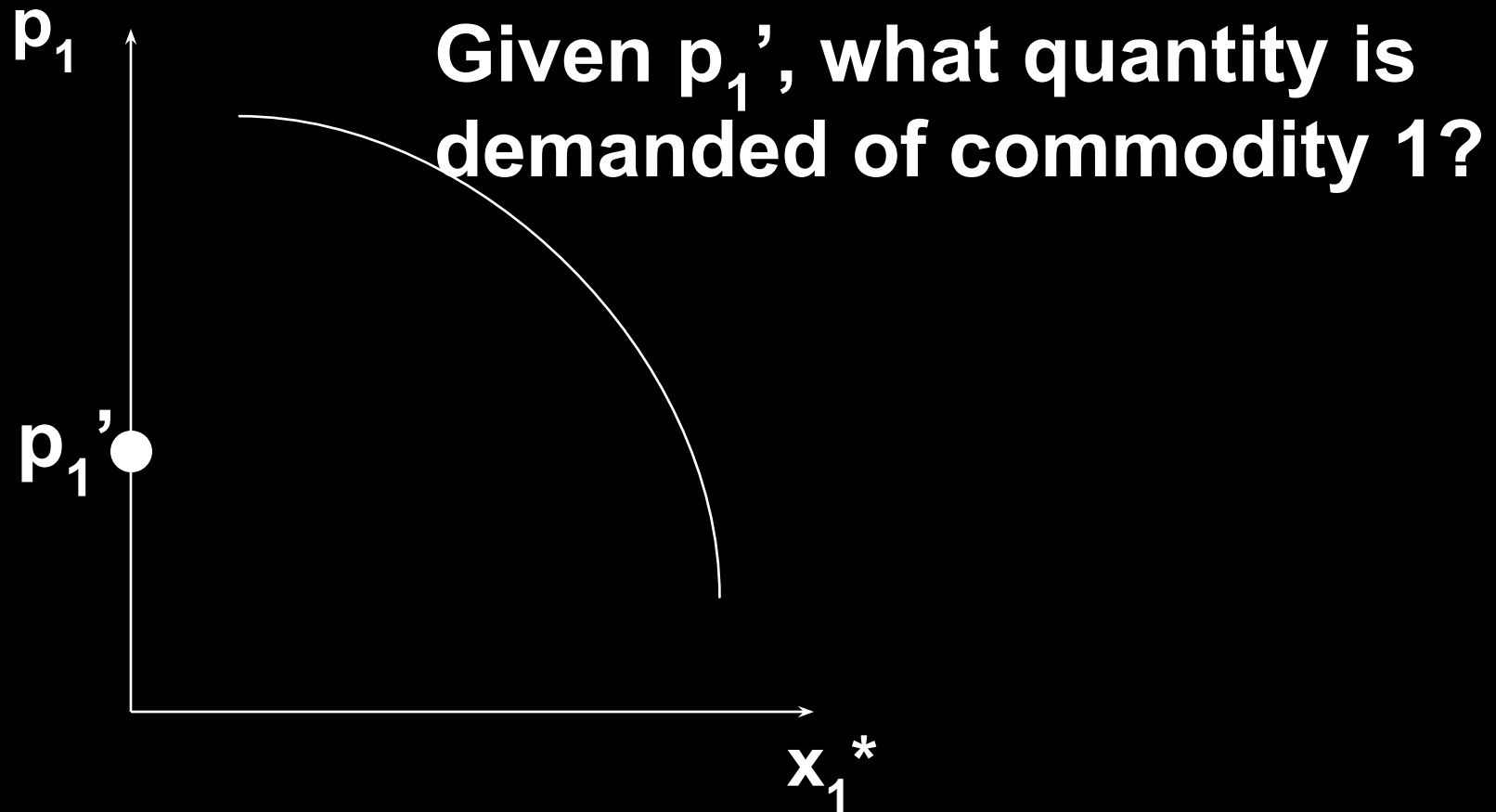
Ordinary demand curve for commodity 1



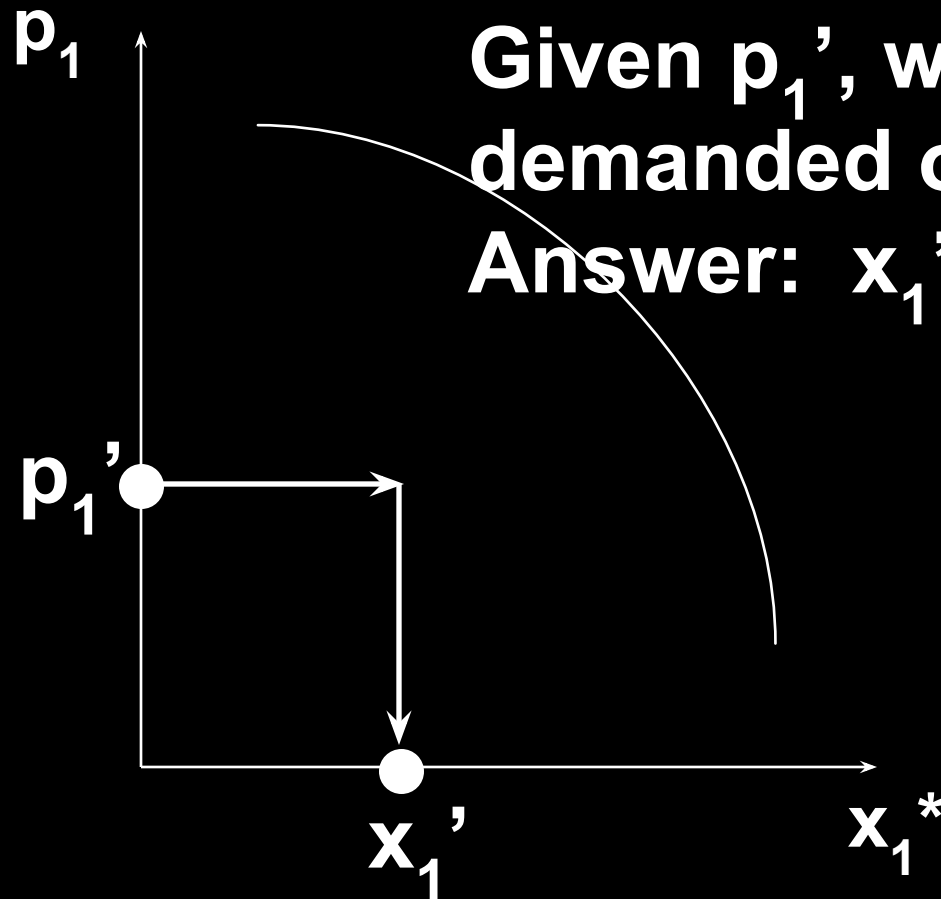
Own-Price Changes

- Usually we ask “Given the price for commodity 1 what is the quantity demanded of commodity 1?”
- But we could also ask the **inverse** question “At what price for commodity 1 would a given quantity of commodity 1 be demanded?”

Own-Price Changes

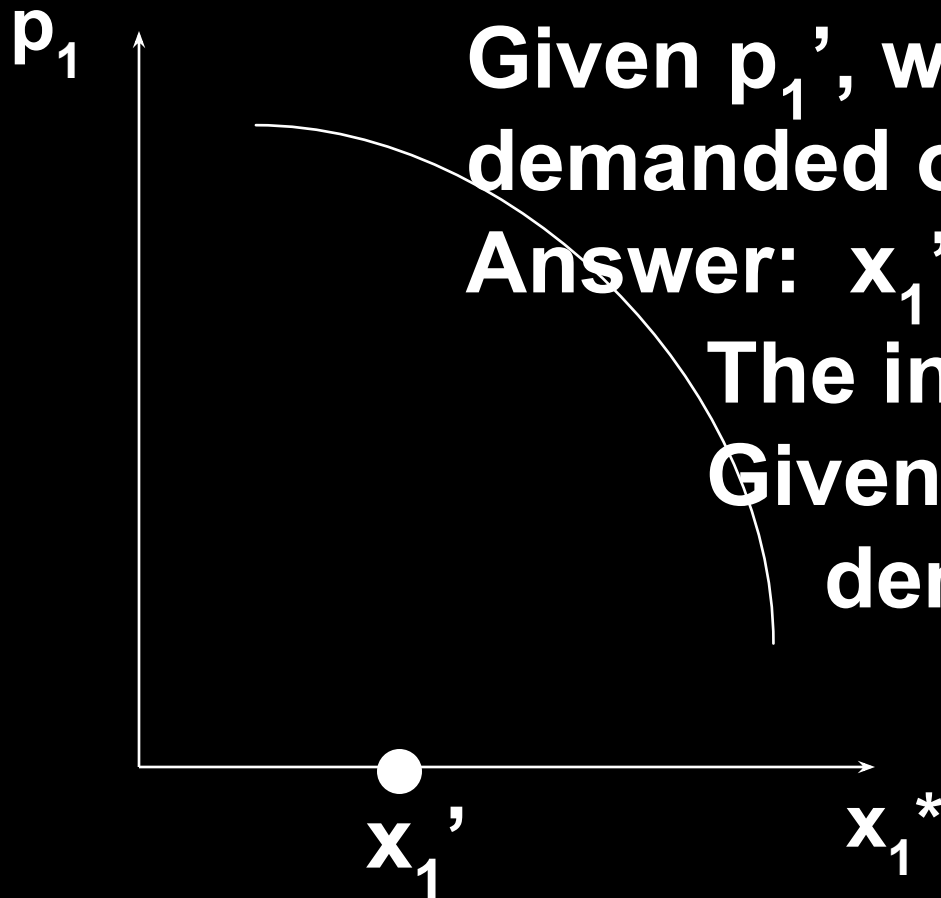


Own-Price Changes



**Given p_1' , what quantity is demanded of commodity 1?
Answer: x_1' units.**

Own-Price Changes



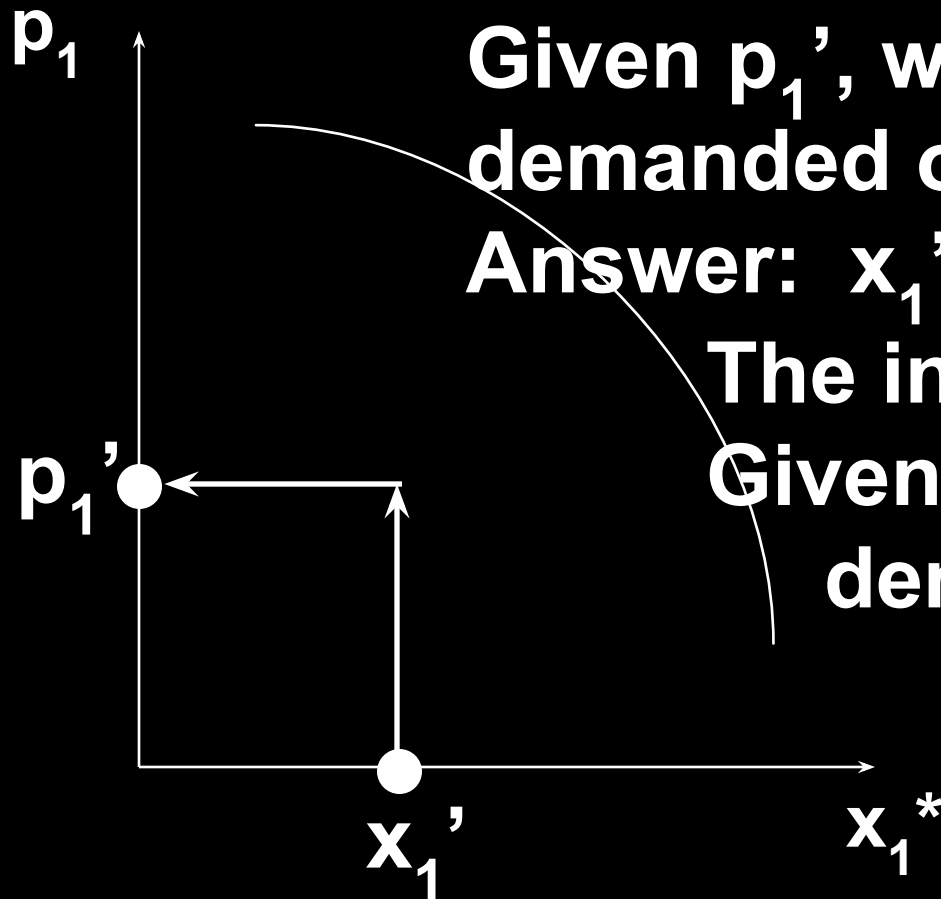
Given p_1' , what quantity is demanded of commodity 1?

Answer: x_1' units.

The inverse question is:

Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



**Given p_1' , what quantity is demanded of commodity 1?
Answer: x_1' units.**

**The inverse question is:
Given x_1' units are demanded, what is the price of commodity 1?
Answer: p_1'**

Own-Price Changes

- Taking quantity demanded as given and then asking what must be price describes the **inverse demand function** of a commodity.

Own-Price Changes

- **Inverse demand function**

At optimal choice

$$|MRS| = p_1/p_2$$

Therefore:

$$p_1 = p_2 |MRS|$$

If $p_2 = 1$, then inverse demand function simply measures MRS, i.e. how much of a good 2 consumer would want to have to compensate for a small reduction in amount of good 1.

Own-Price Changes

- **Inverse demand function**

If good 2 is money, then MRS (and inverse demand function) measure marginal willingness to pay.

Own-Price Changes

A Cobb-Douglas example:

is the ordinary demand function and

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

is the ordinary demand function and

is the inverse demand function.

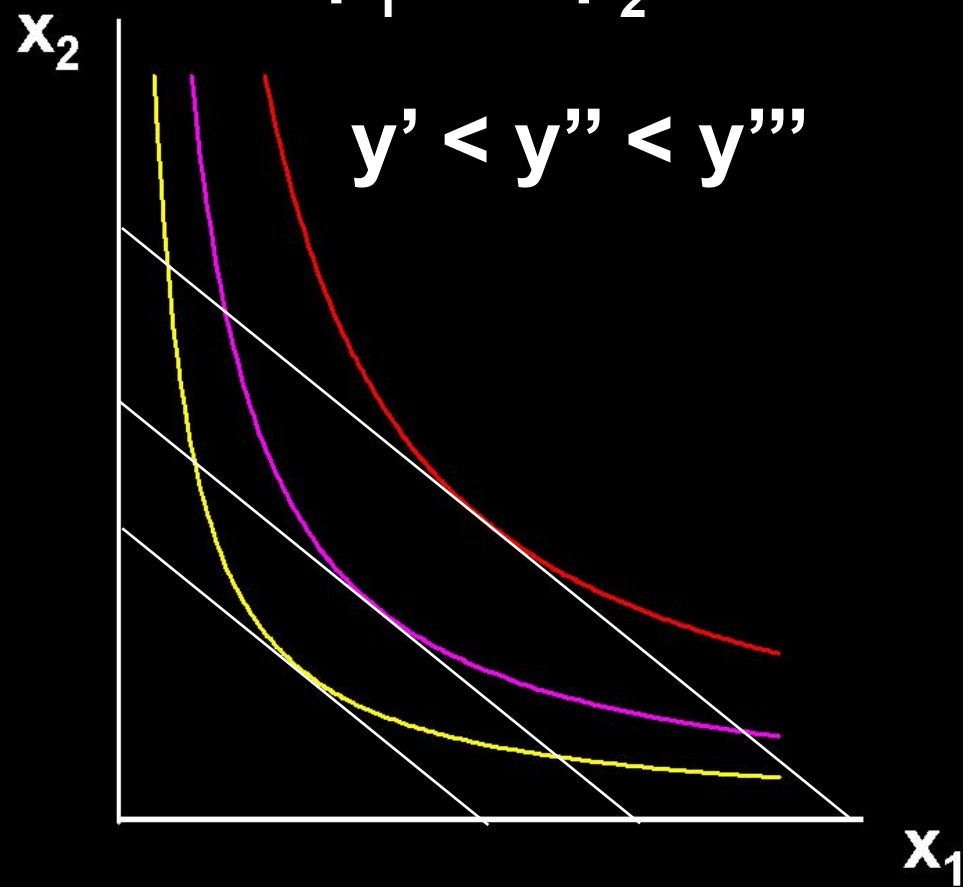


Income Changes

- How does the value of $x_1^*(p_1, p_2, y)$ change as y changes, holding both p_1 and p_2 constant?

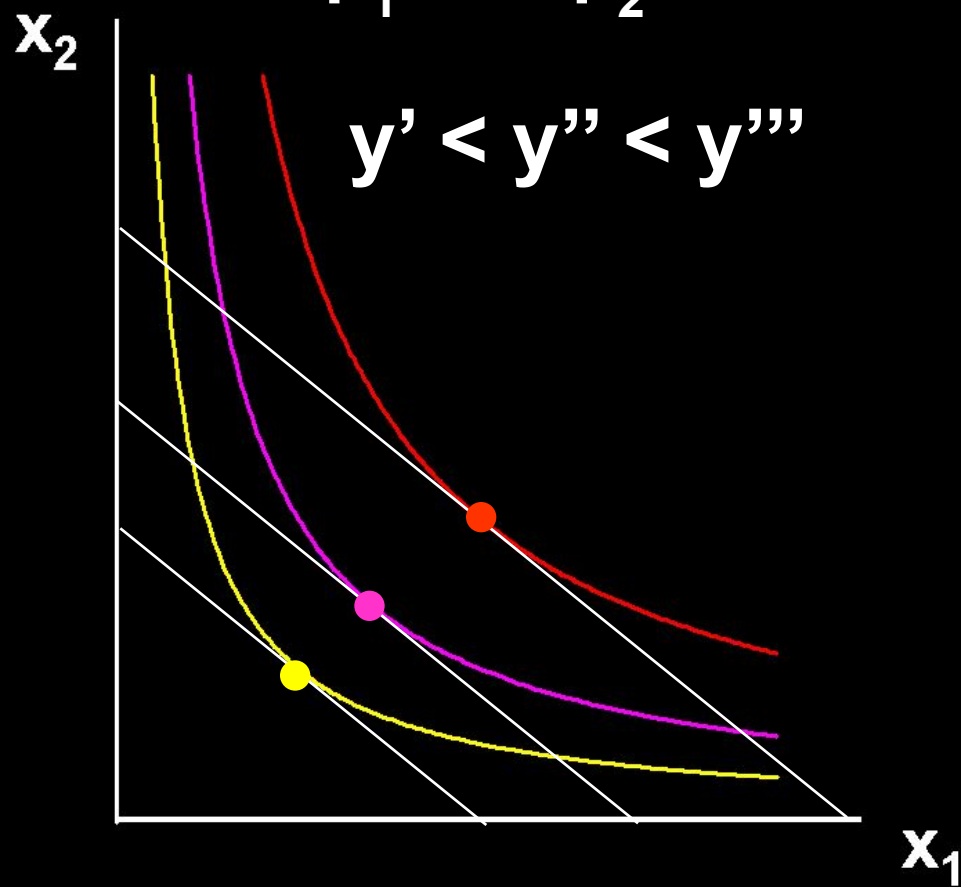
Income Changes

Fixed p_1 and p_2 .



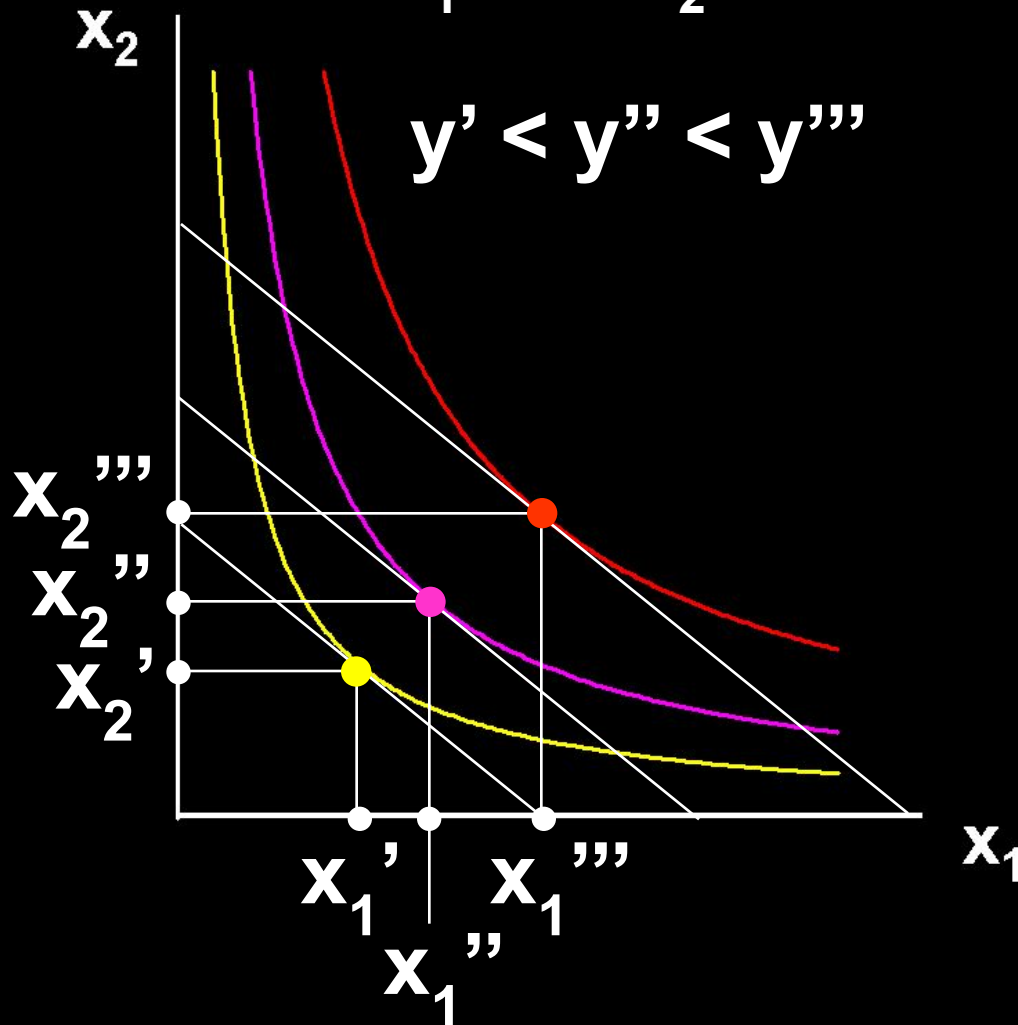
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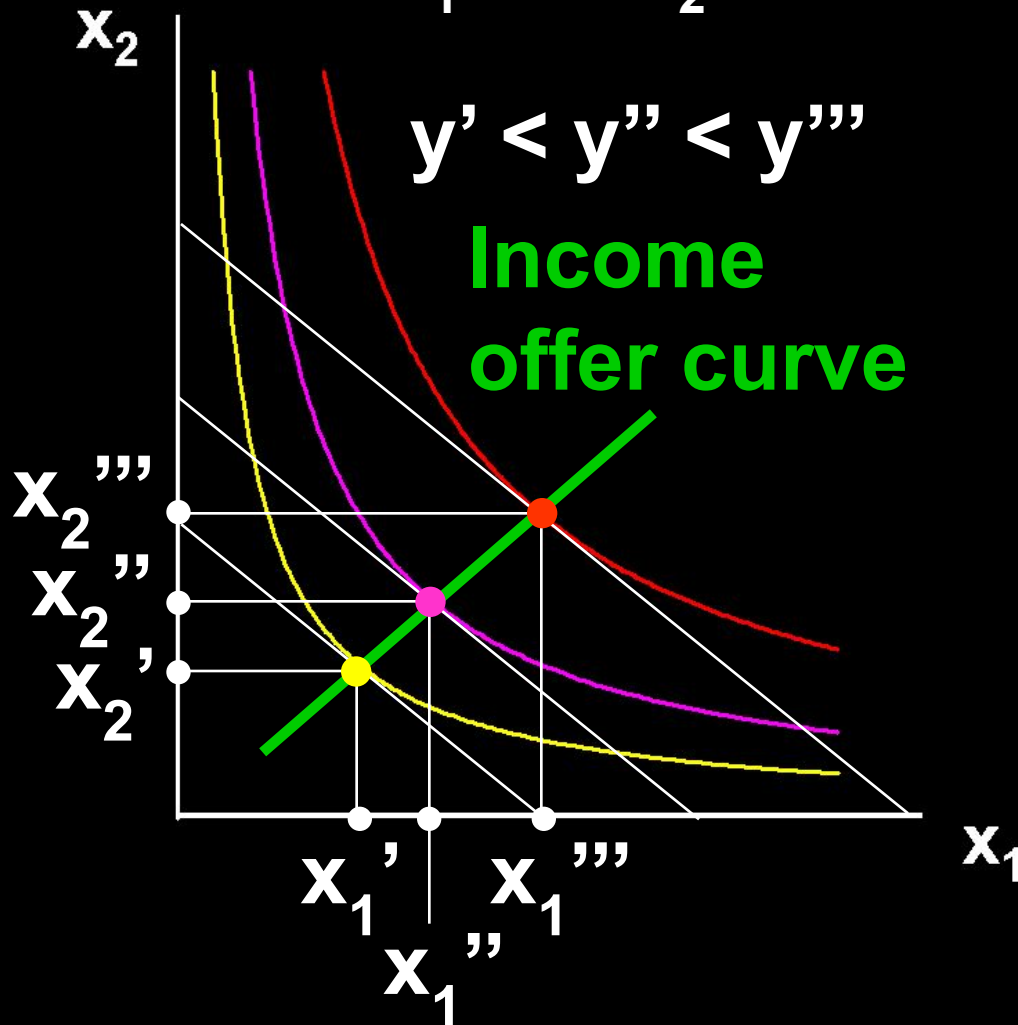
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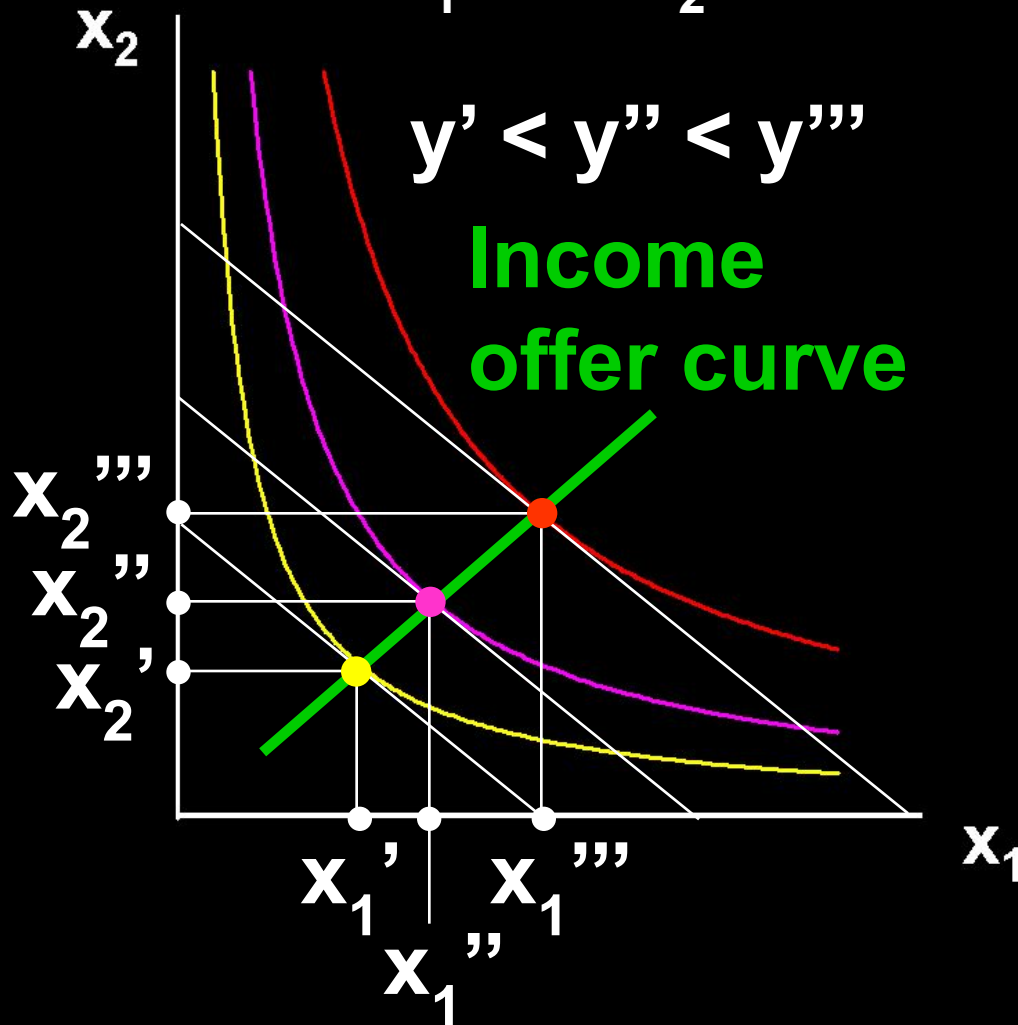


Income Changes

- A plot of quantity demanded against income is called an **Engel curve**.

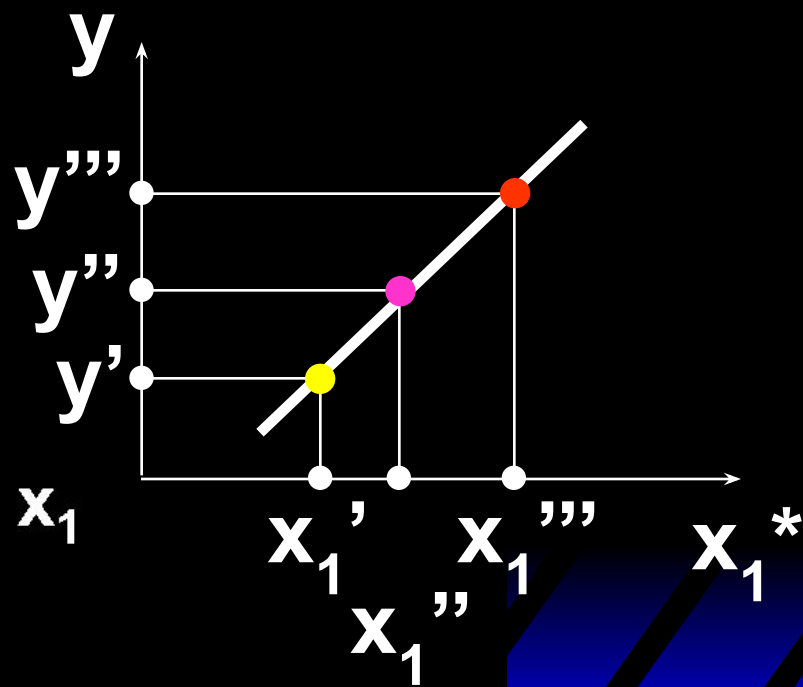
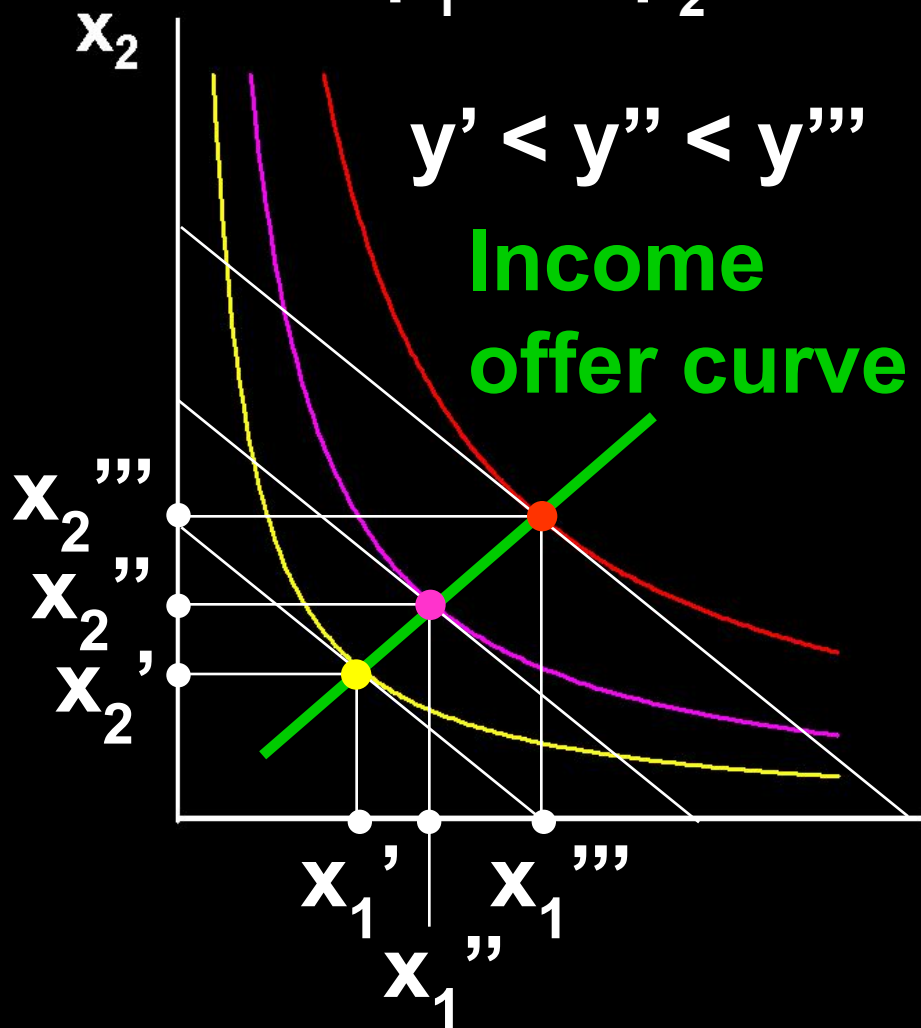
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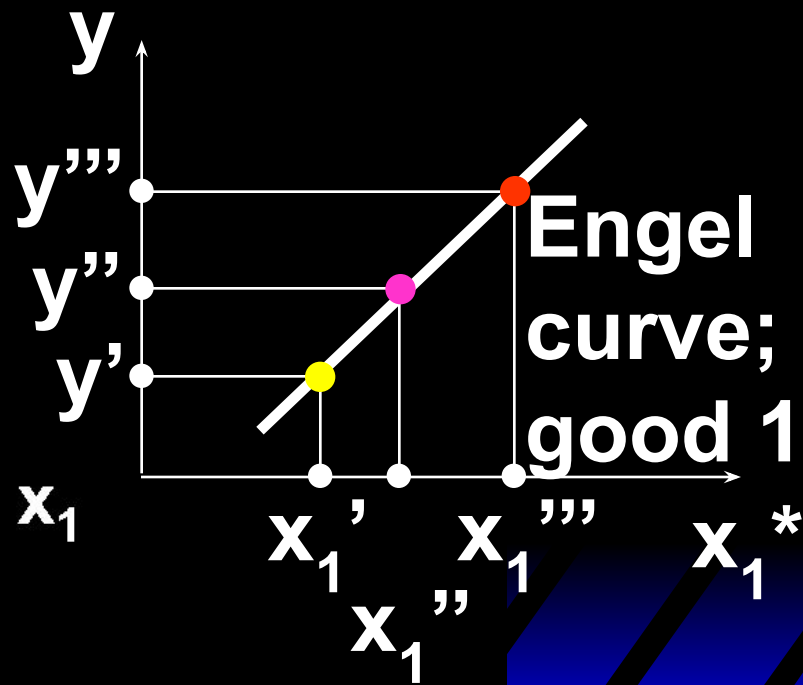
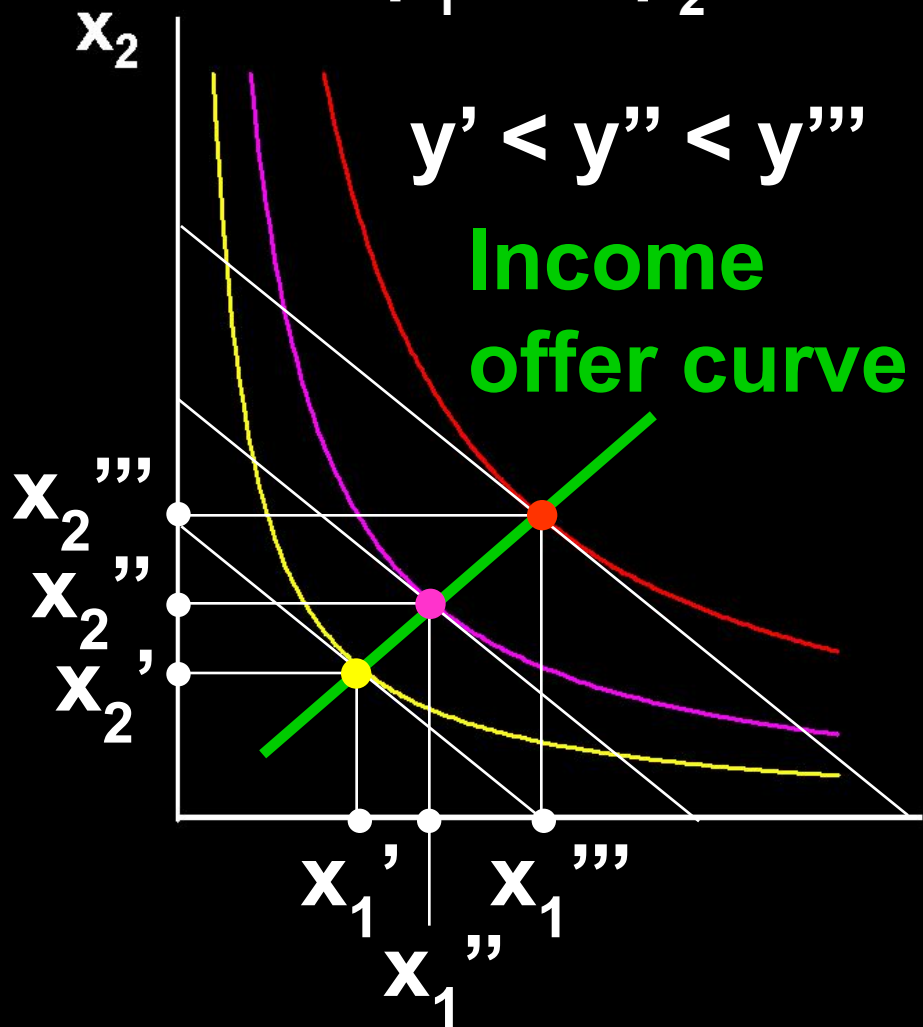
Income Changes

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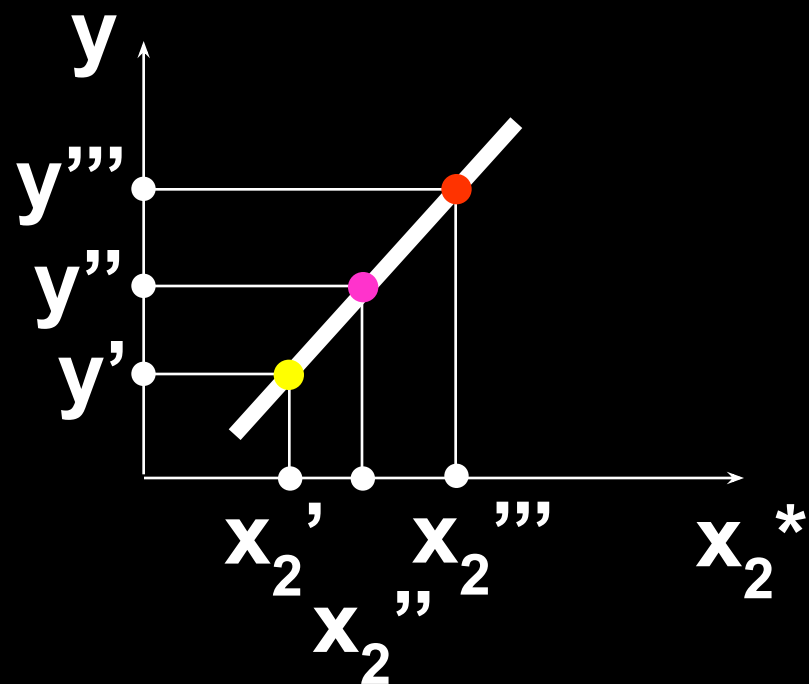
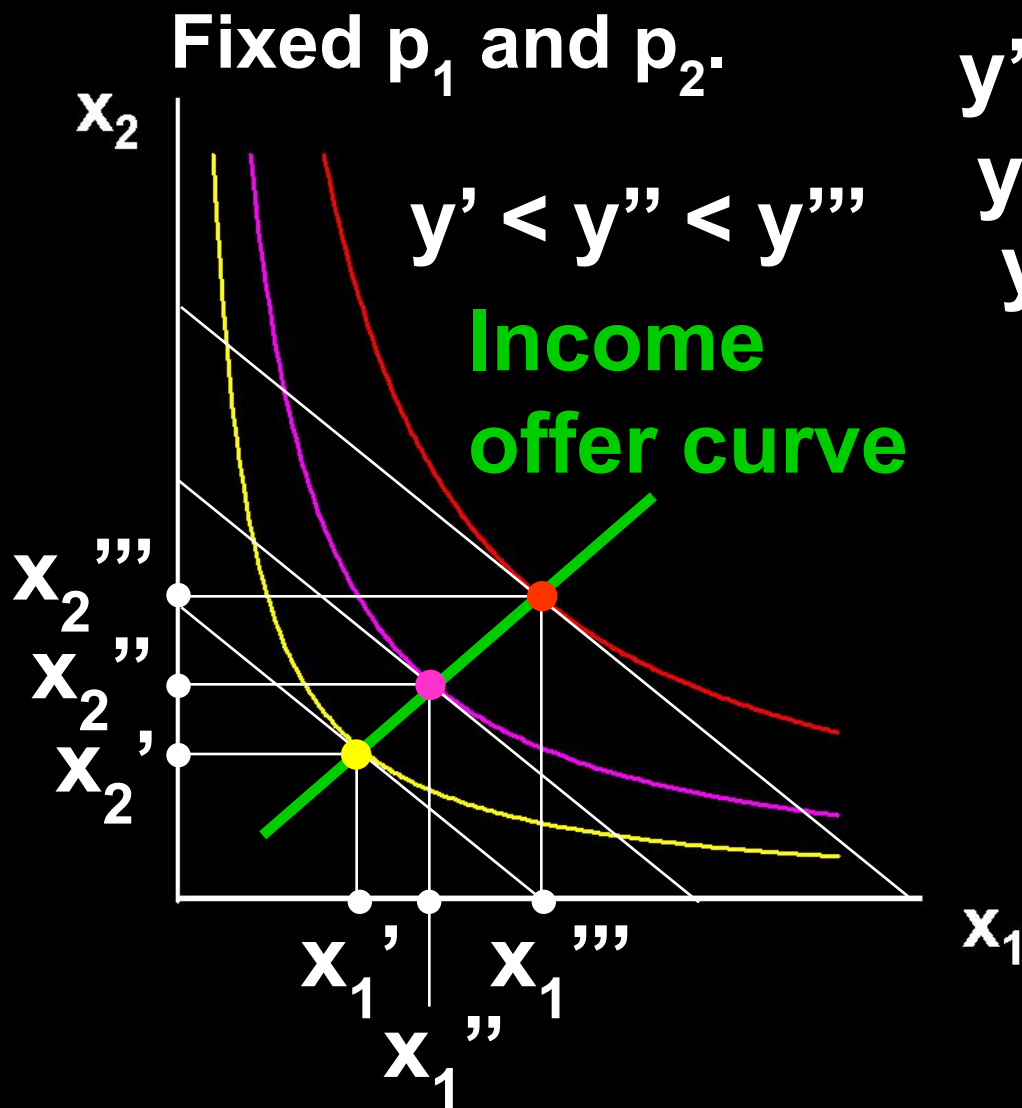


Income Changes

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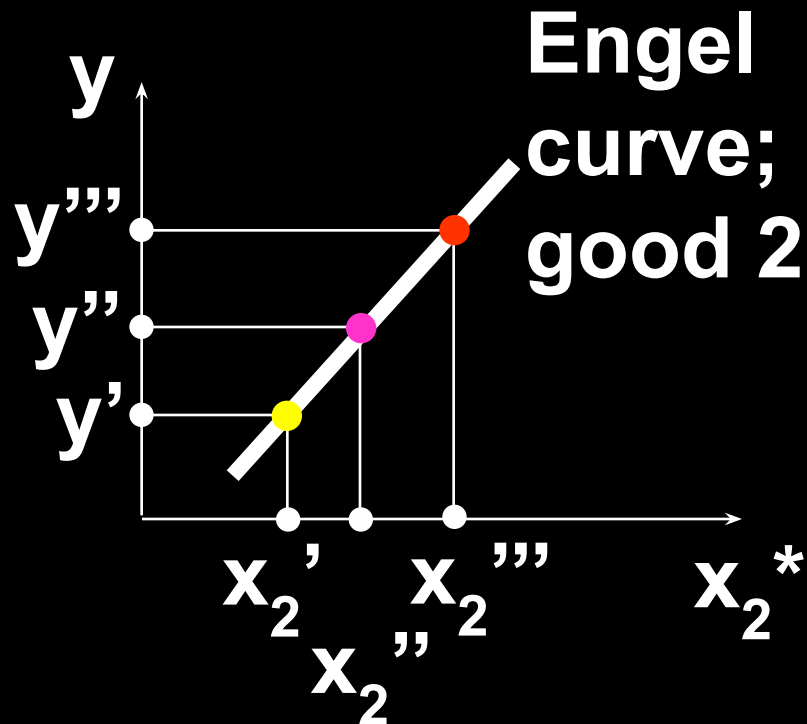
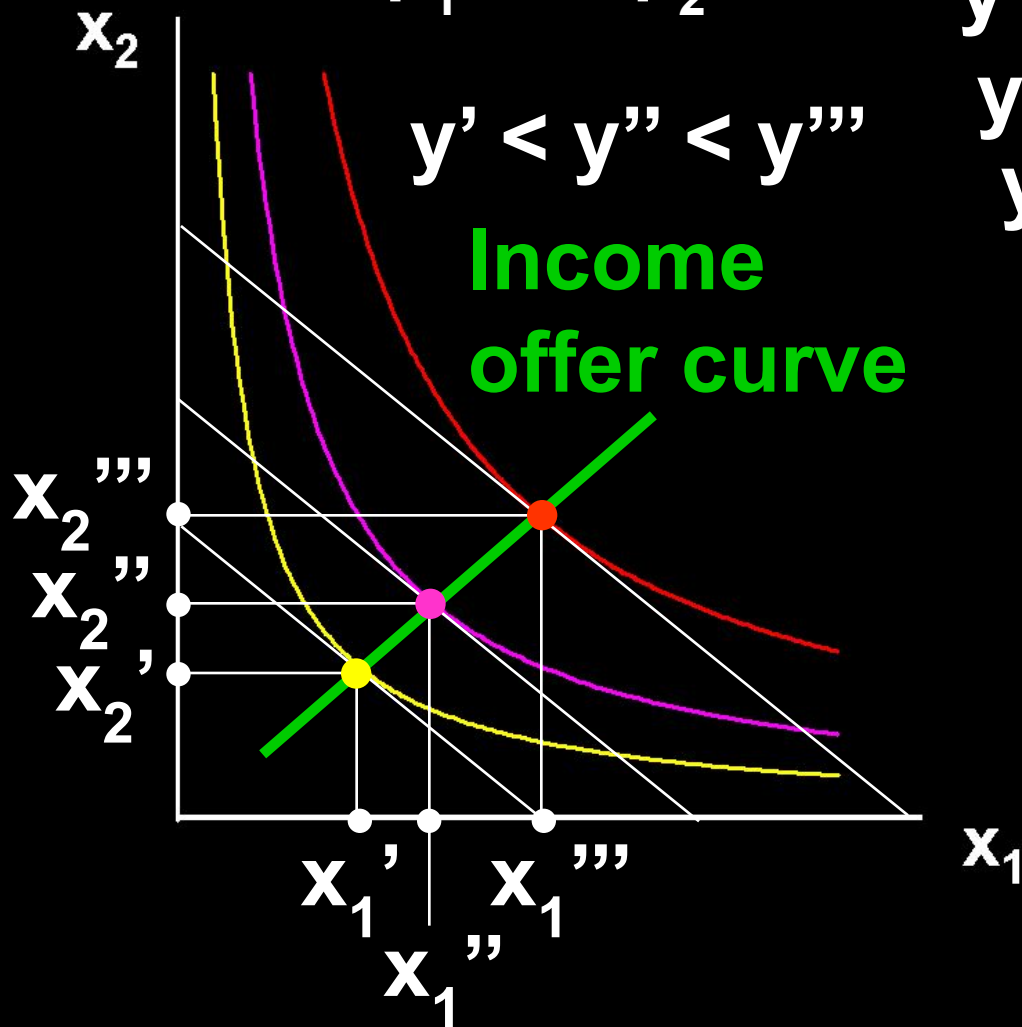


Income Changes



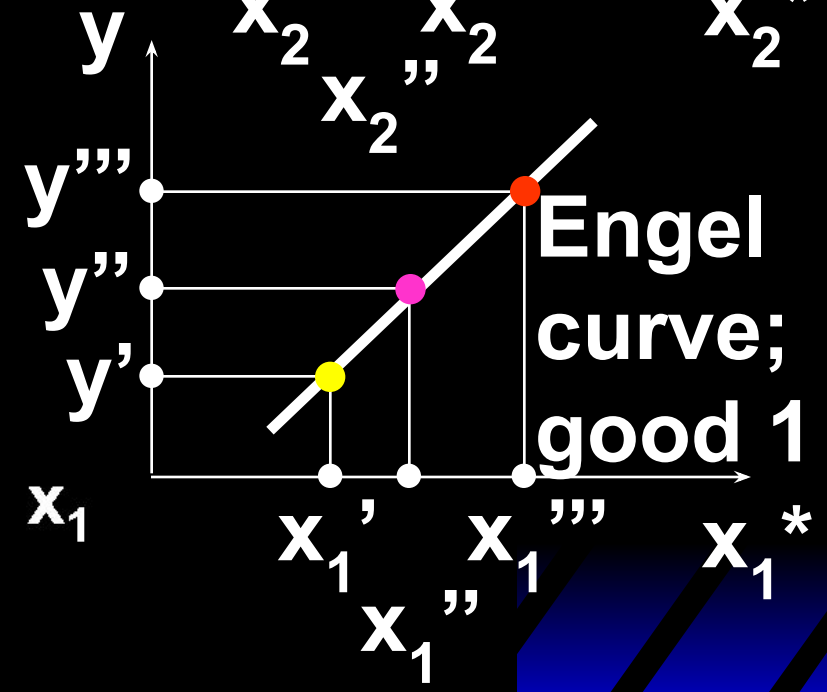
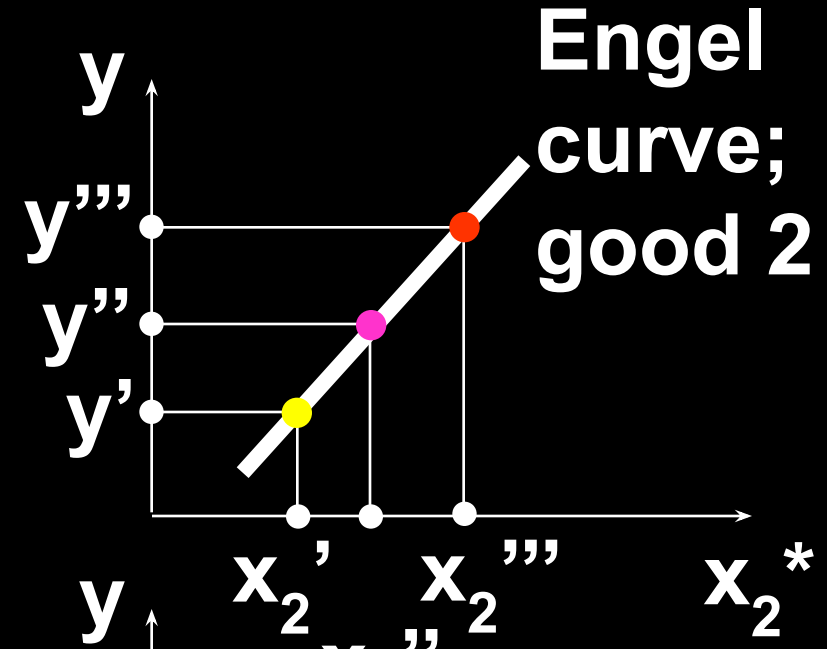
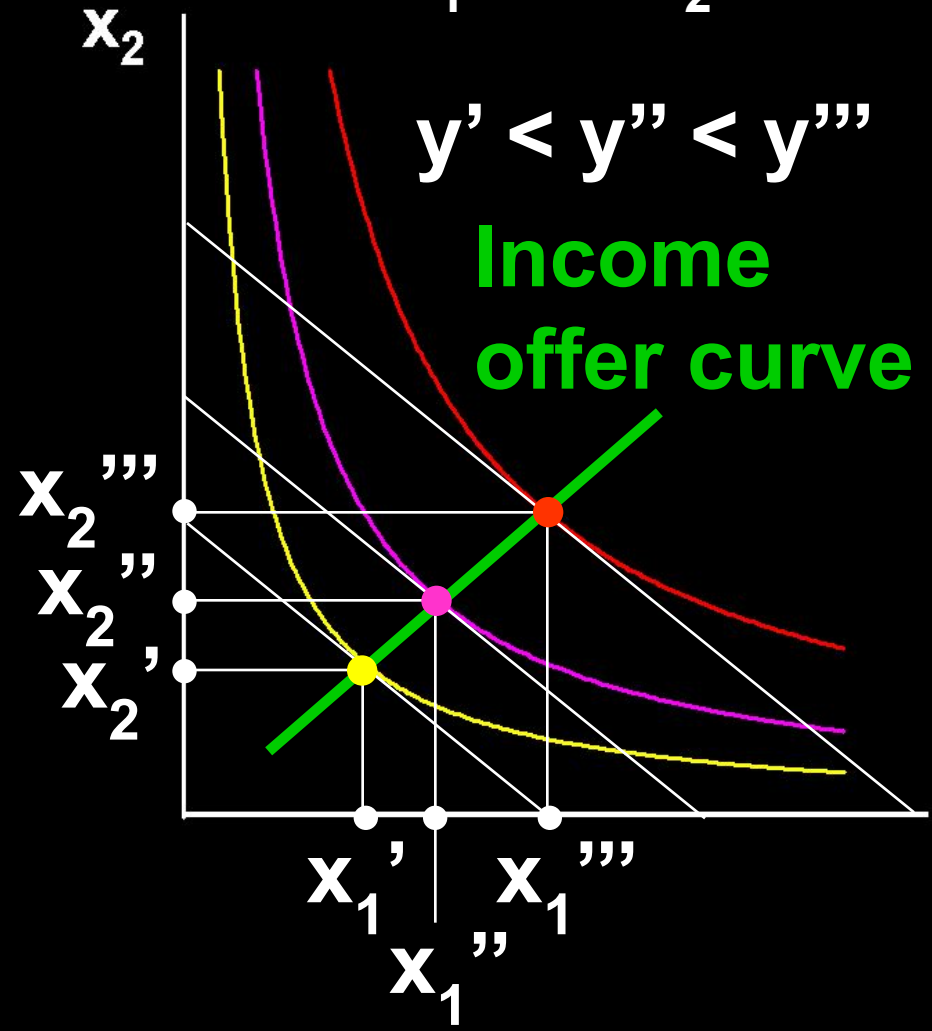
Income Changes

Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .



Income Changes and Cobb-Douglas Preferences

- **An example of computing the equations of Engel curves; the Cobb-Douglas case.**
- **The ordinary demand equations are**

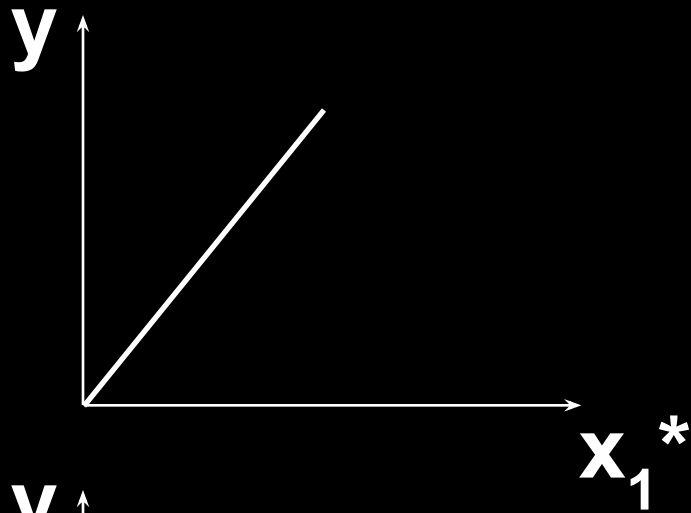
Income Changes and Cobb-Douglas Preferences

Rearranged to isolate y , these are:

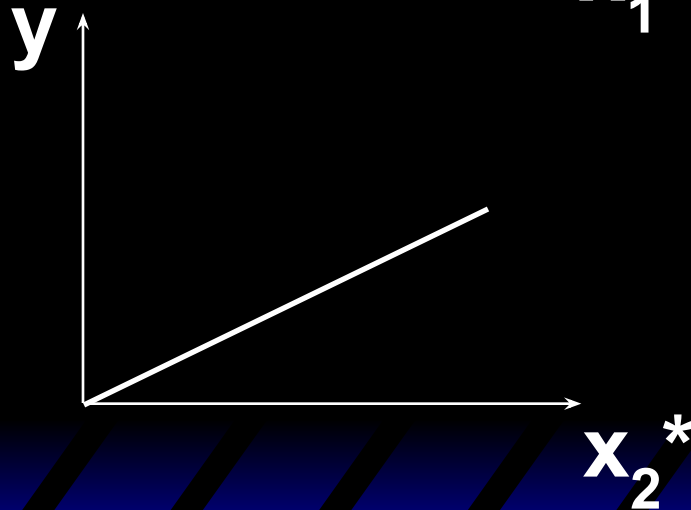
Engel curve for good 1

Engel curve for good 2

Income Changes and Cobb-Douglas Preferences



Engel curve
for good 1



Engel curve
for good 2

Income Changes and Perfectly-Complementary Preferences

- **Another example of computing the equations of Engel curves; the perfectly-complementary case.**
- **The ordinary demand equations are**

Income Changes and Perfectly-Complementary Preferences

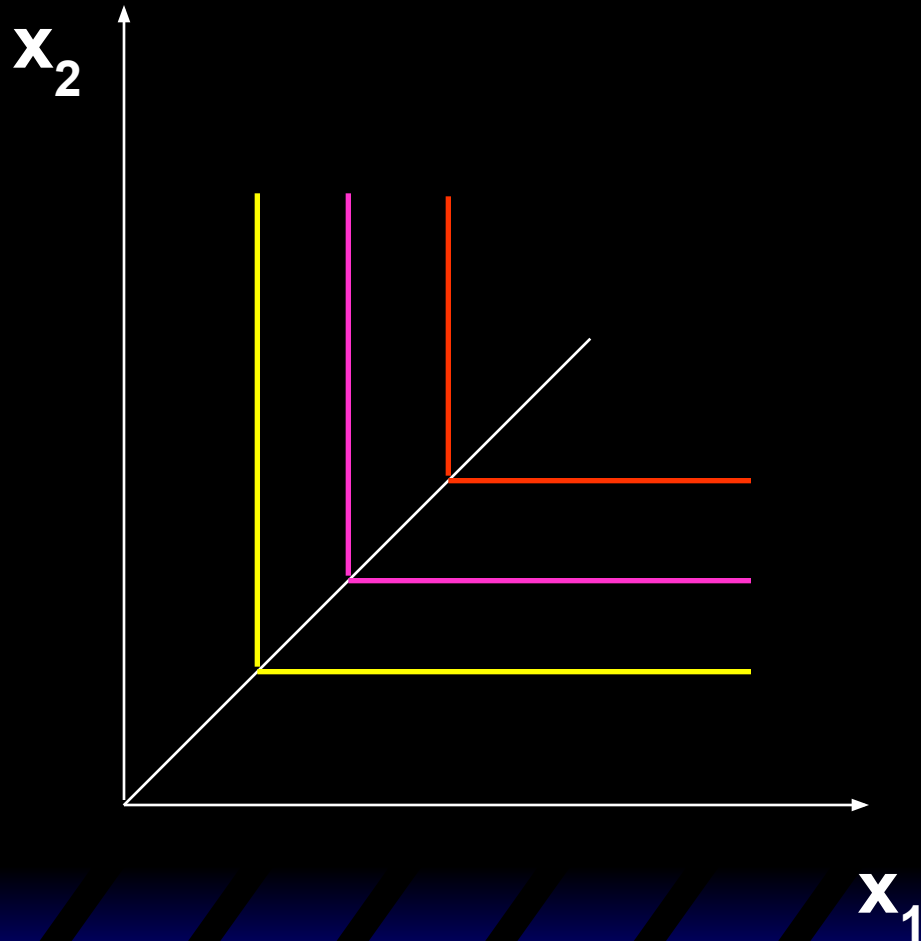
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Engel curve for good 1

Engel curve for good 2

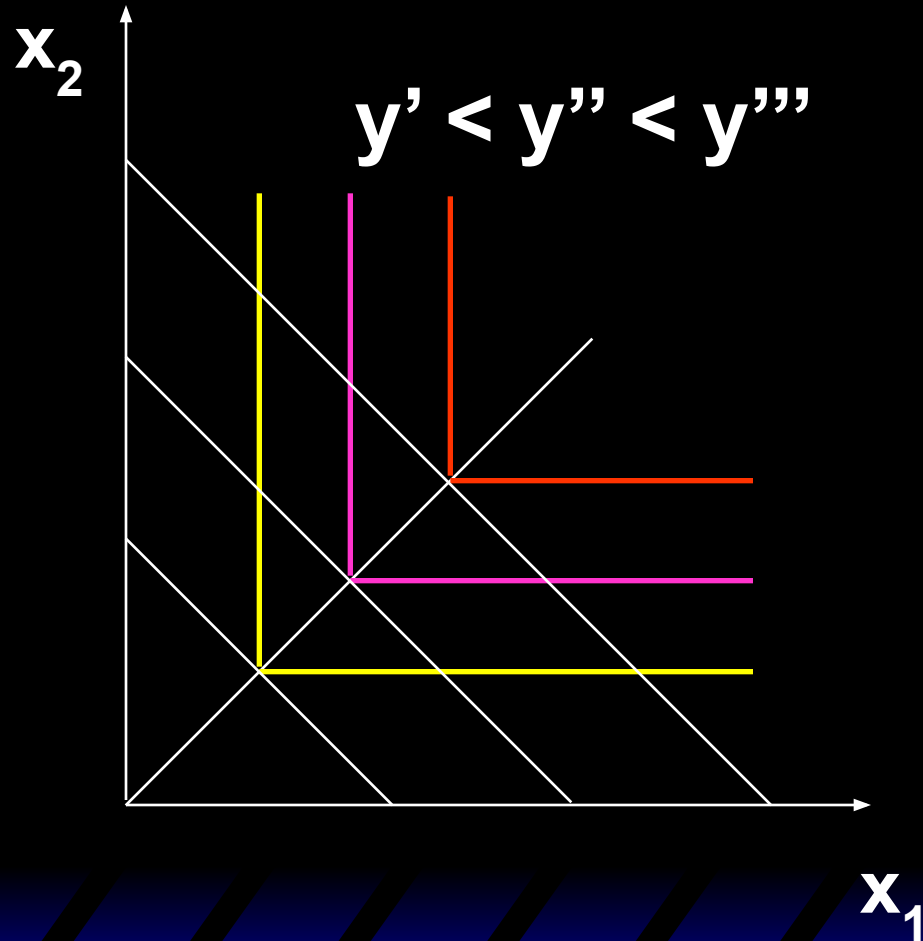
Income Changes

Fixed p_1 and p_2 .



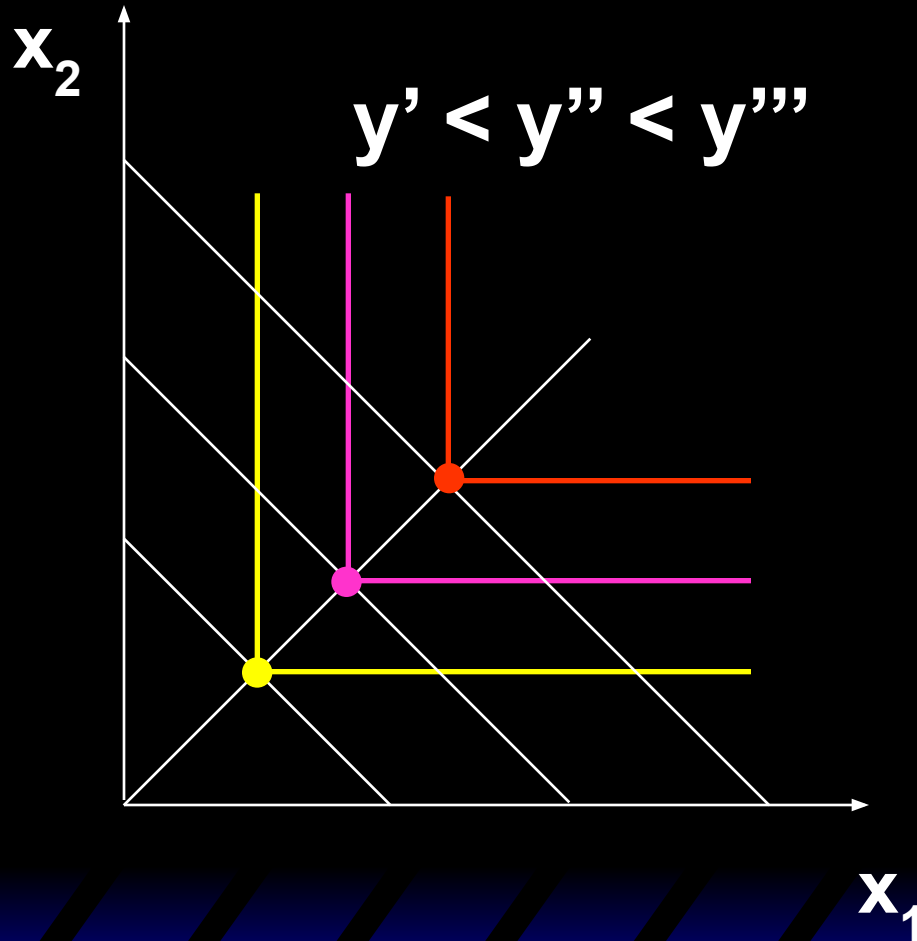
Income Changes

Fixed p_1 and p_2 .



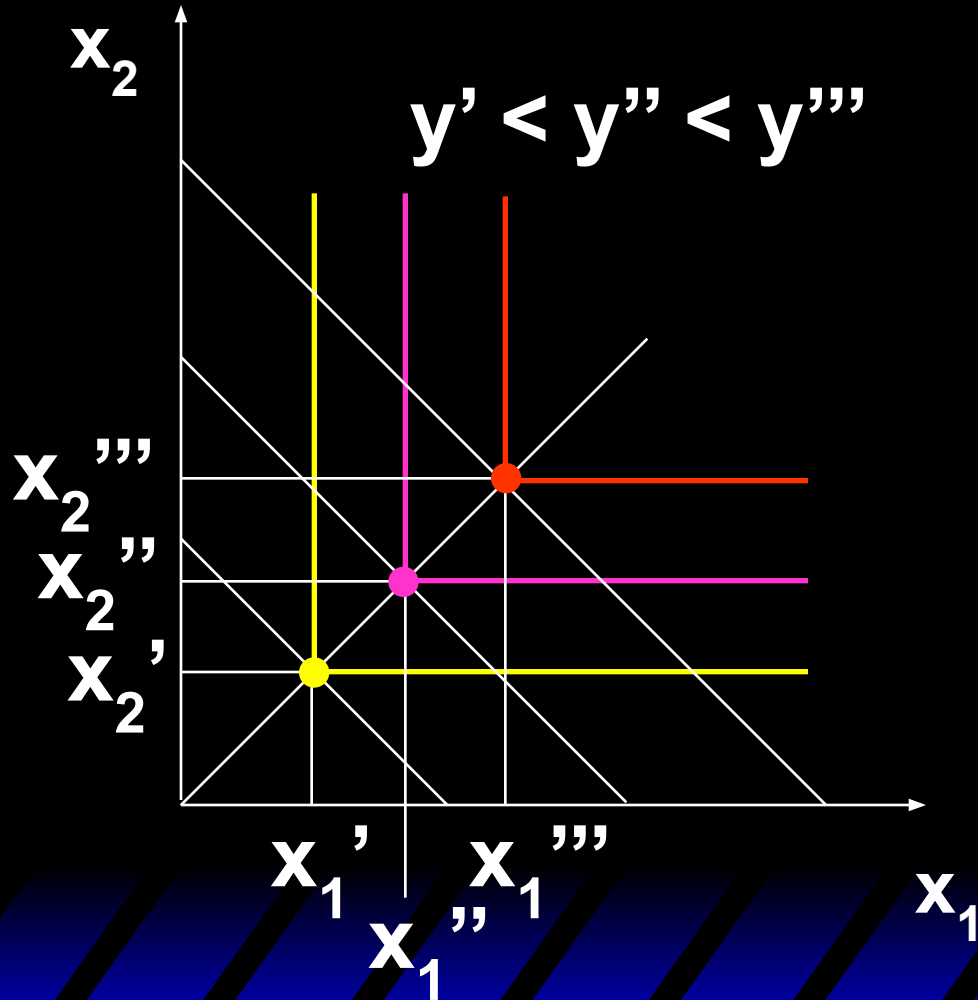
Income Changes

Fixed p_1 and p_2 .



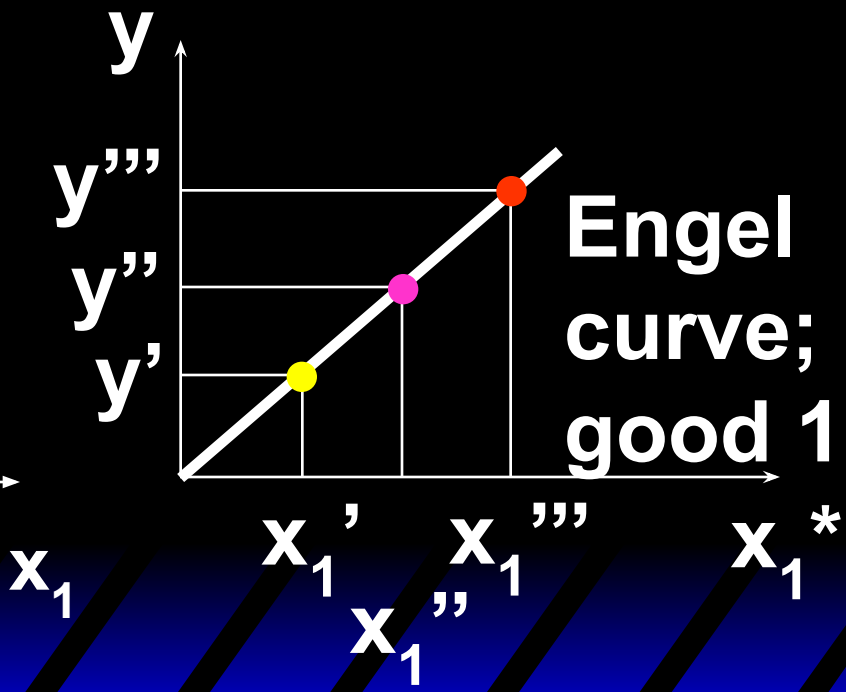
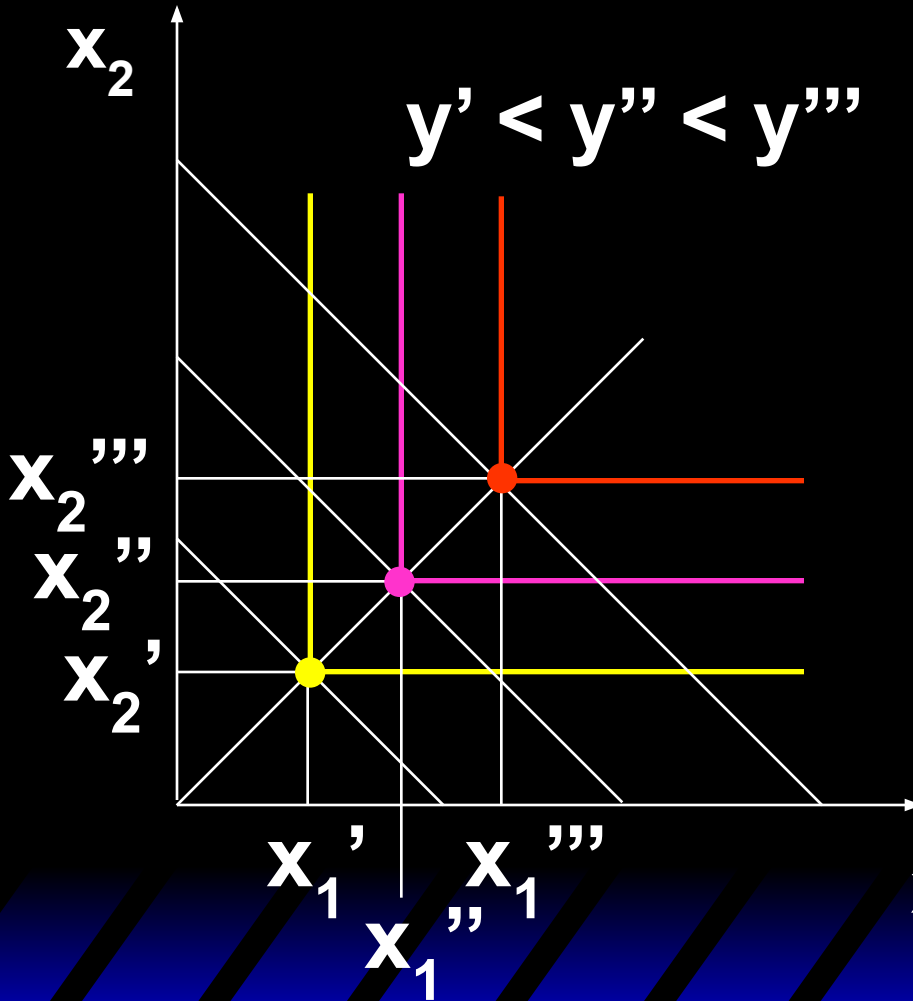
Income Changes

Fixed p_1 and p_2 .



Income Changes

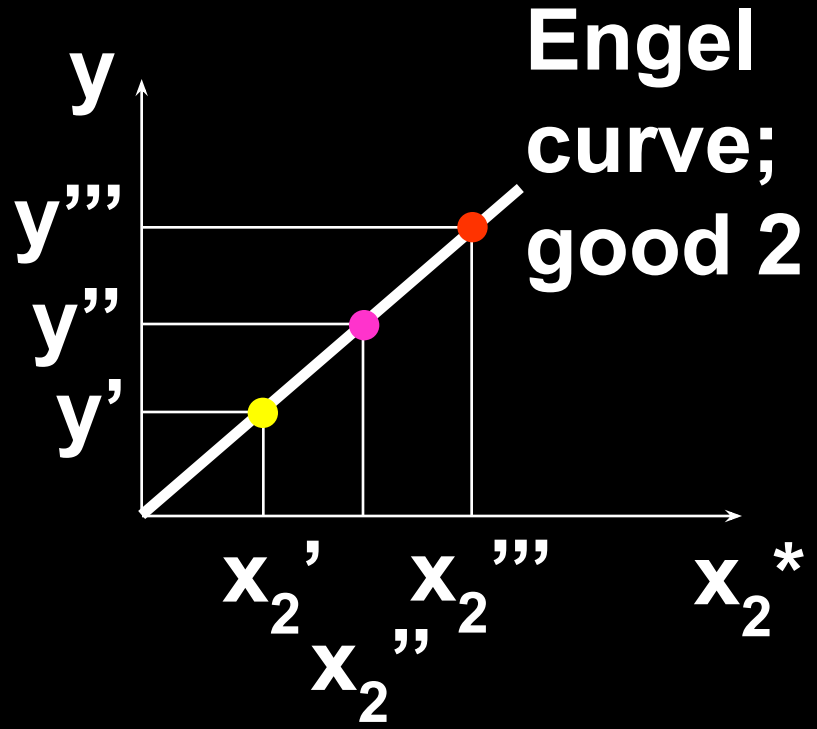
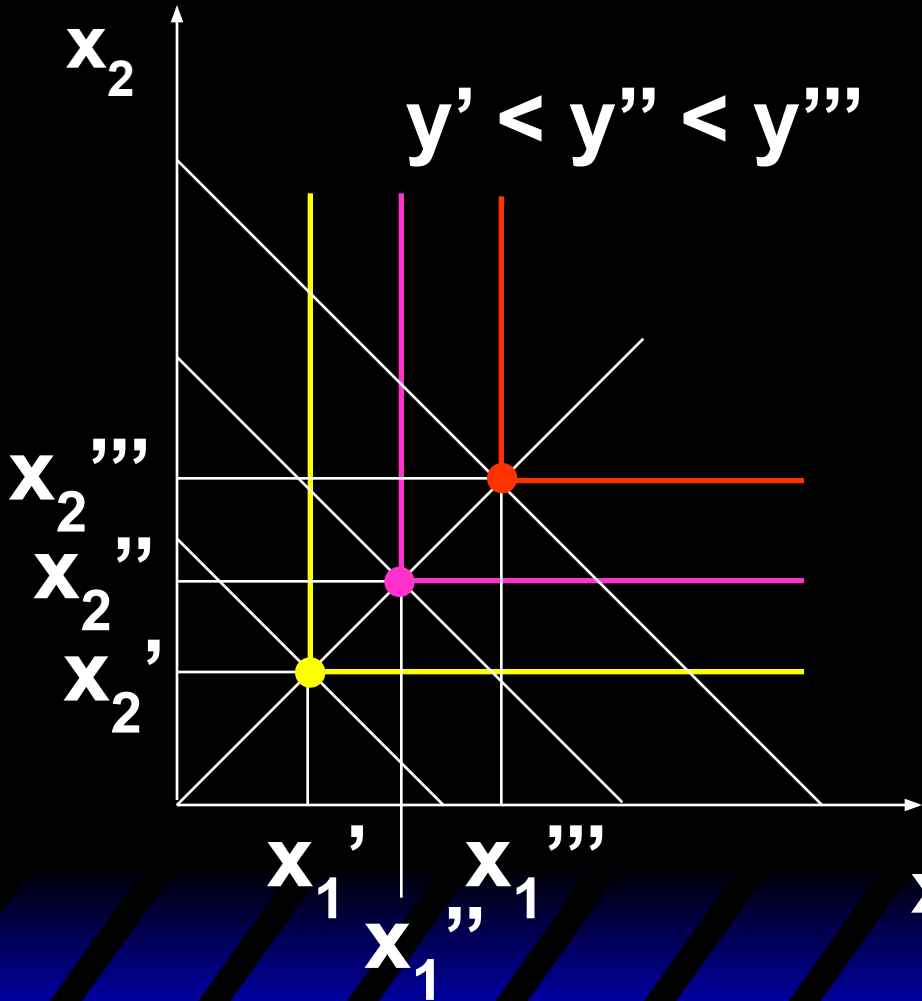
Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .

$$y' < y'' < y'''$$

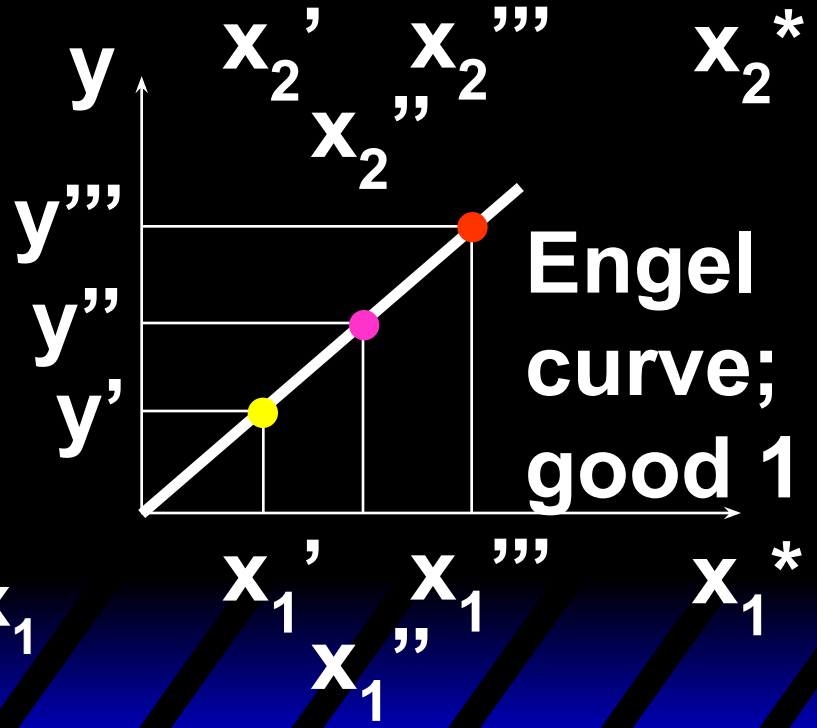
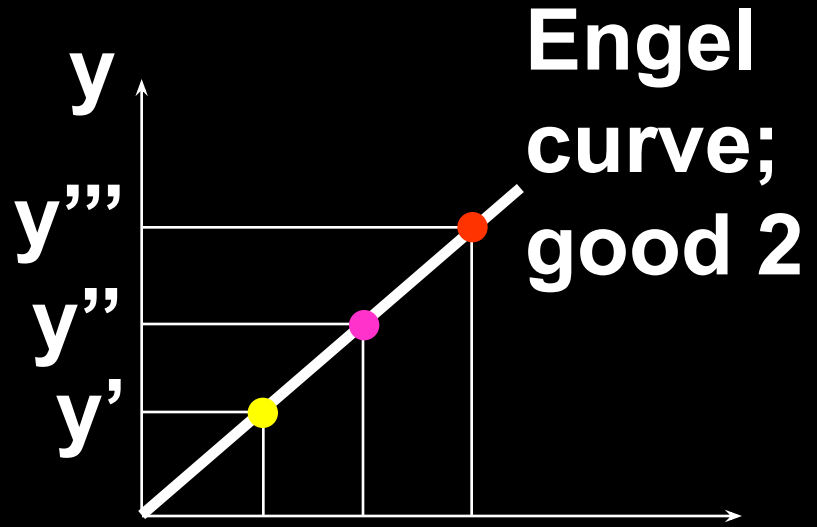
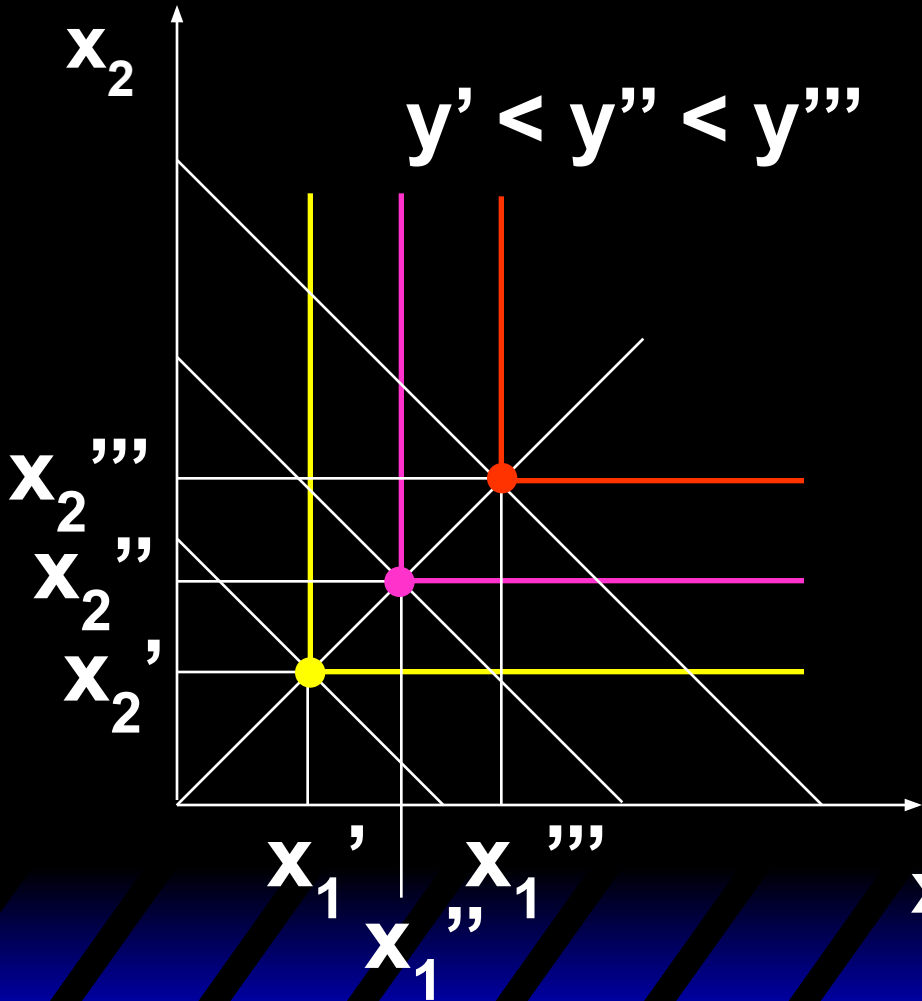


Engel curve; good 2

Income Changes

Fixed p_1 and p_2 .

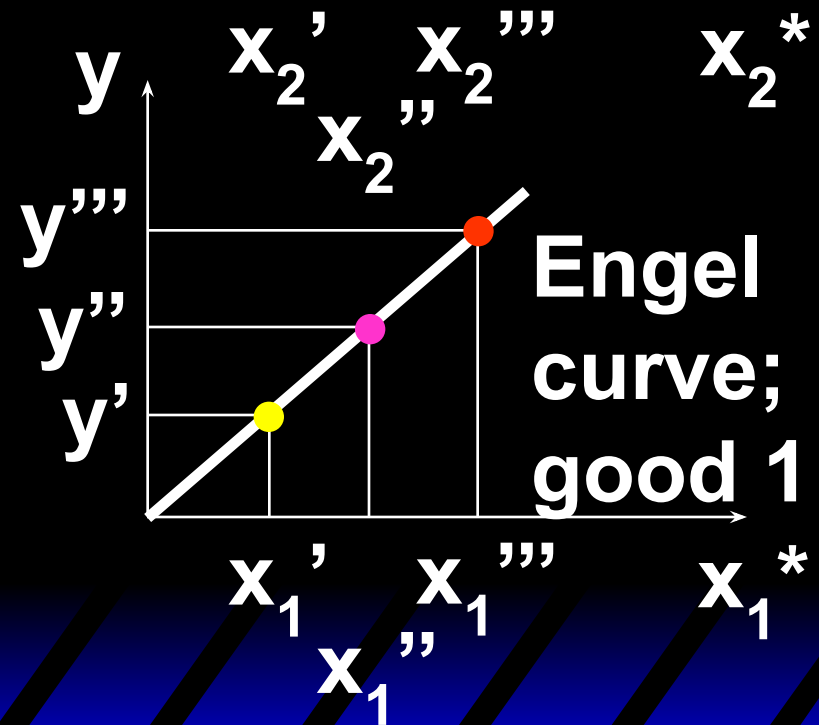
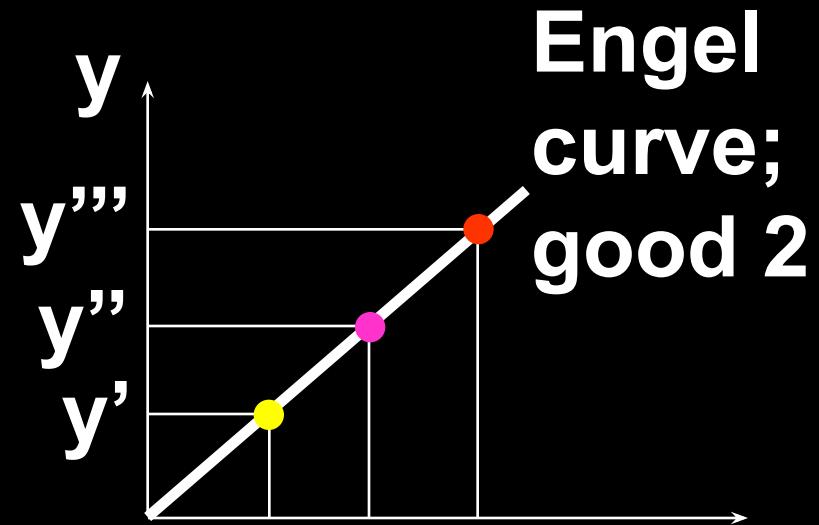
$$y' < y'' < y'''$$



Income Changes

Fixed p_1 and p_2 .

$$y = (p_1 + p_2)x_1^*$$



Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**
- **The ordinary demand equations are**

Income Changes and Perfectly-Substitutable Preferences



Income Changes and Perfectly-Substitutable Preferences

Suppose $p_1 < p_2$. Then

Income Changes and Perfectly-Substitutable Preferences

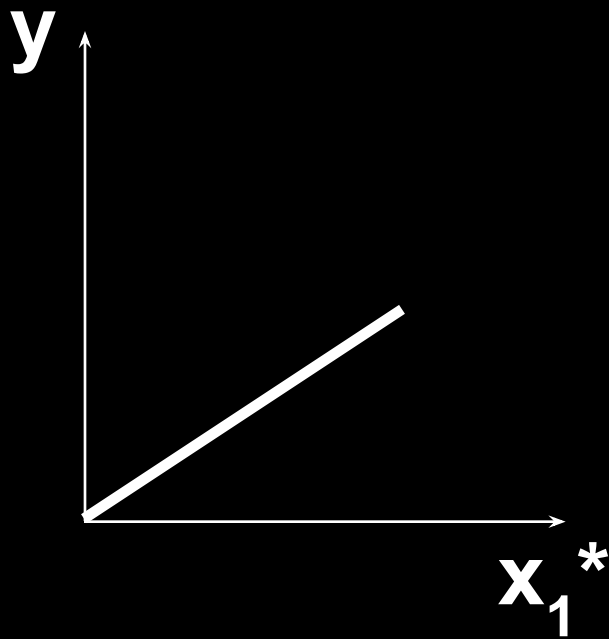
Suppose $p_1 < p_2$. Then and

Income Changes and Perfectly-Substitutable Preferences

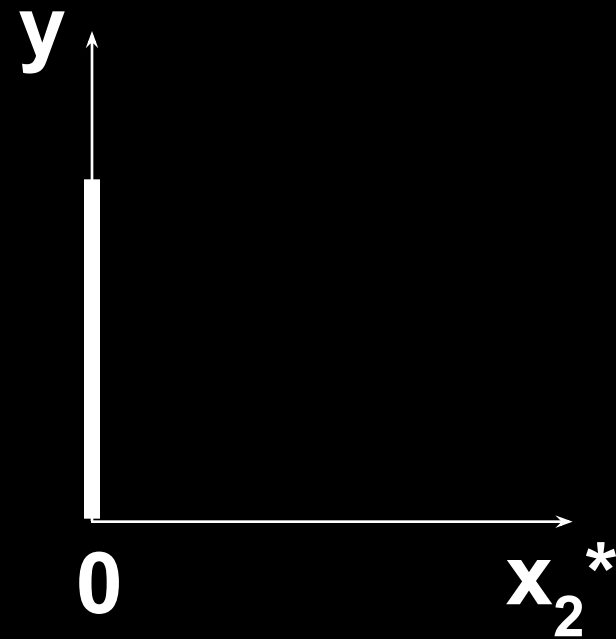
Suppose $p_1 < p_2$. Then and

→ $y = p_1 x_1$ and $x_2 = 0$.

Income Changes and Perfectly-Substitutable Preferences



Engel curve
for good 1



Engel curve
for good 2

Income Changes

- In every example so far the Engel curves have all been straight lines?
Q: Is this true in general?
- A: No. Engel curves are straight lines if the consumer's preferences are **homothetic**.

Homotheticity

- A consumer's preferences are **homothetic** if and only if

$$(x_1, x_2) p (y_1, y_2) \Leftrightarrow (kx_1, kx_2) p (ky_1, ky_2)$$

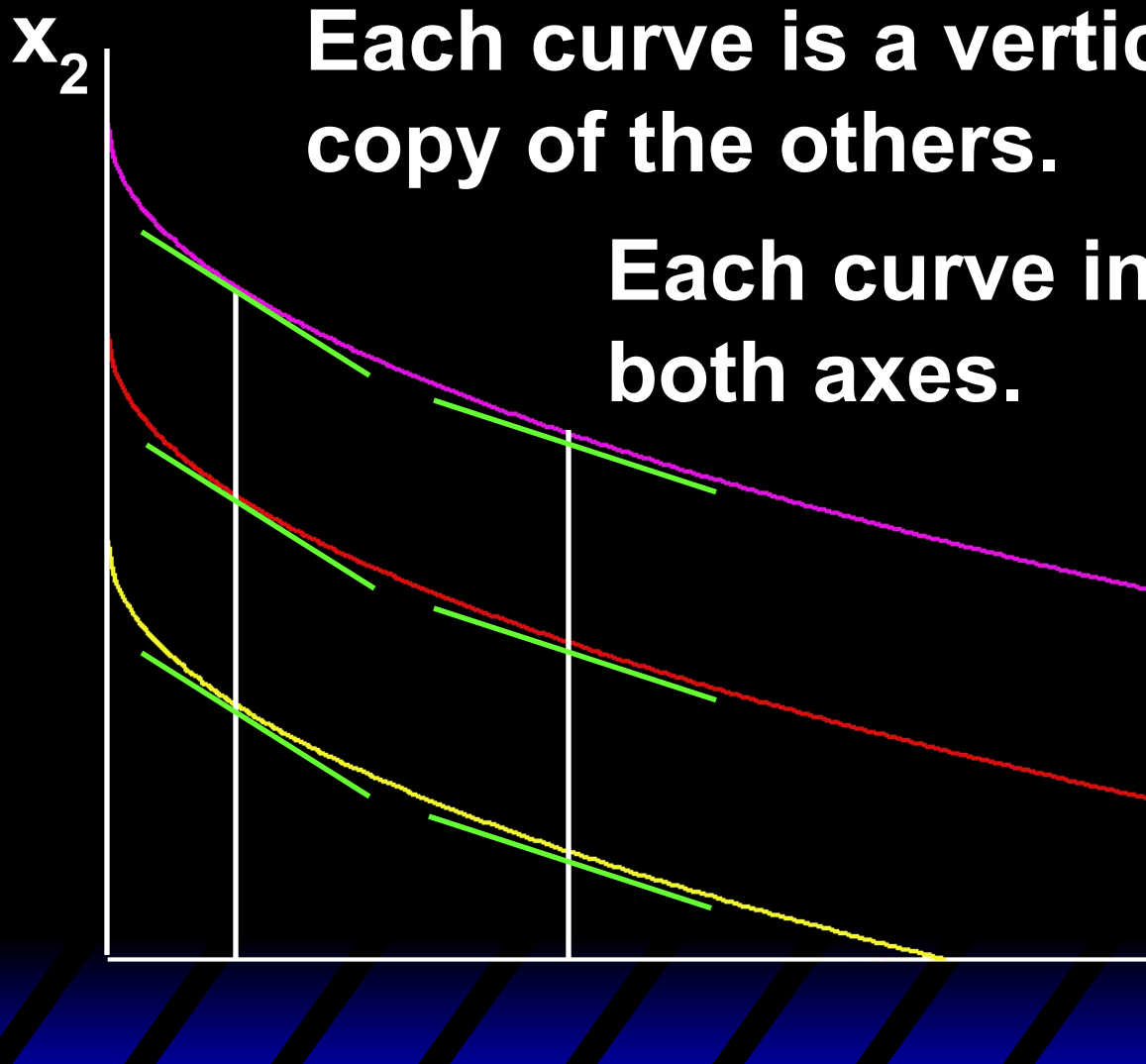
for every $k > 0$.

- That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

- **Quasilinear preferences are not homothetic.**
- **For example,**

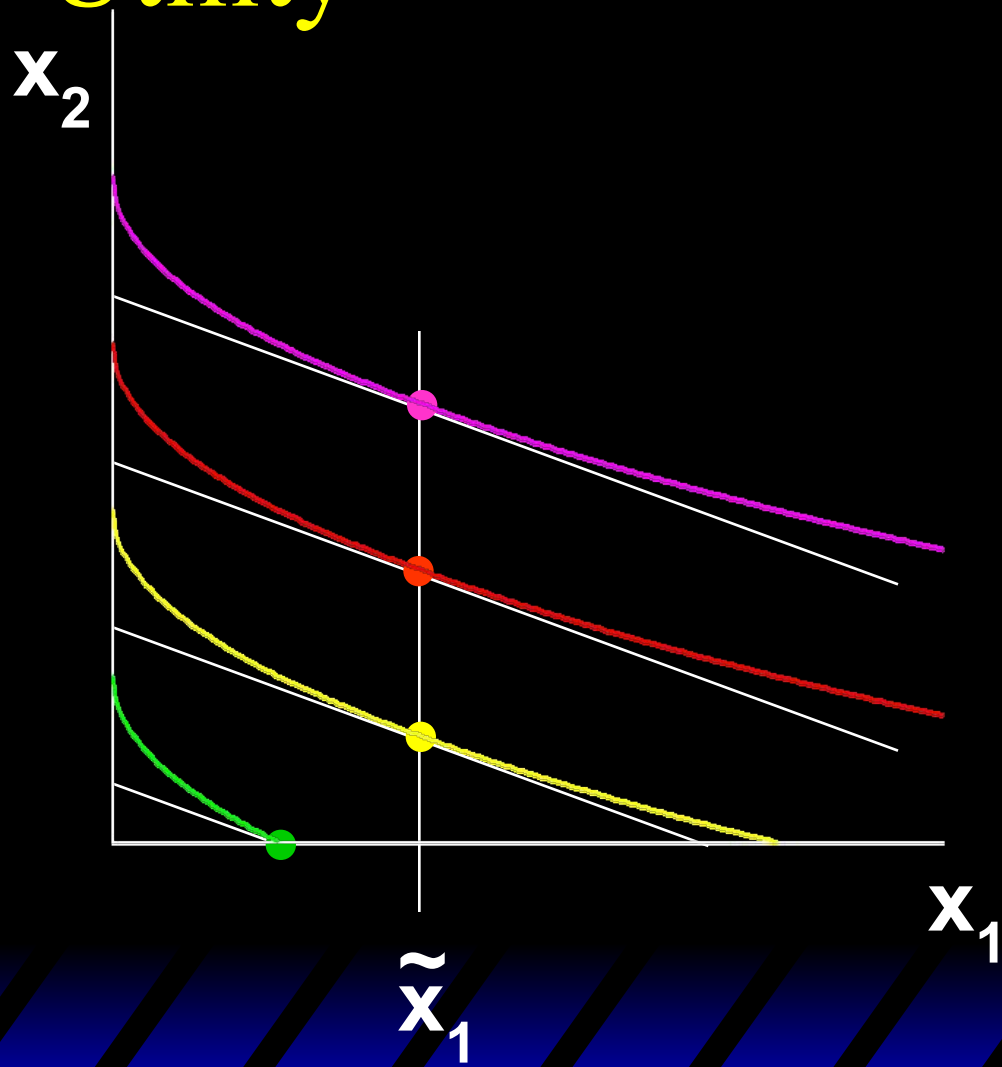
Quasi-linear Indifference Curves



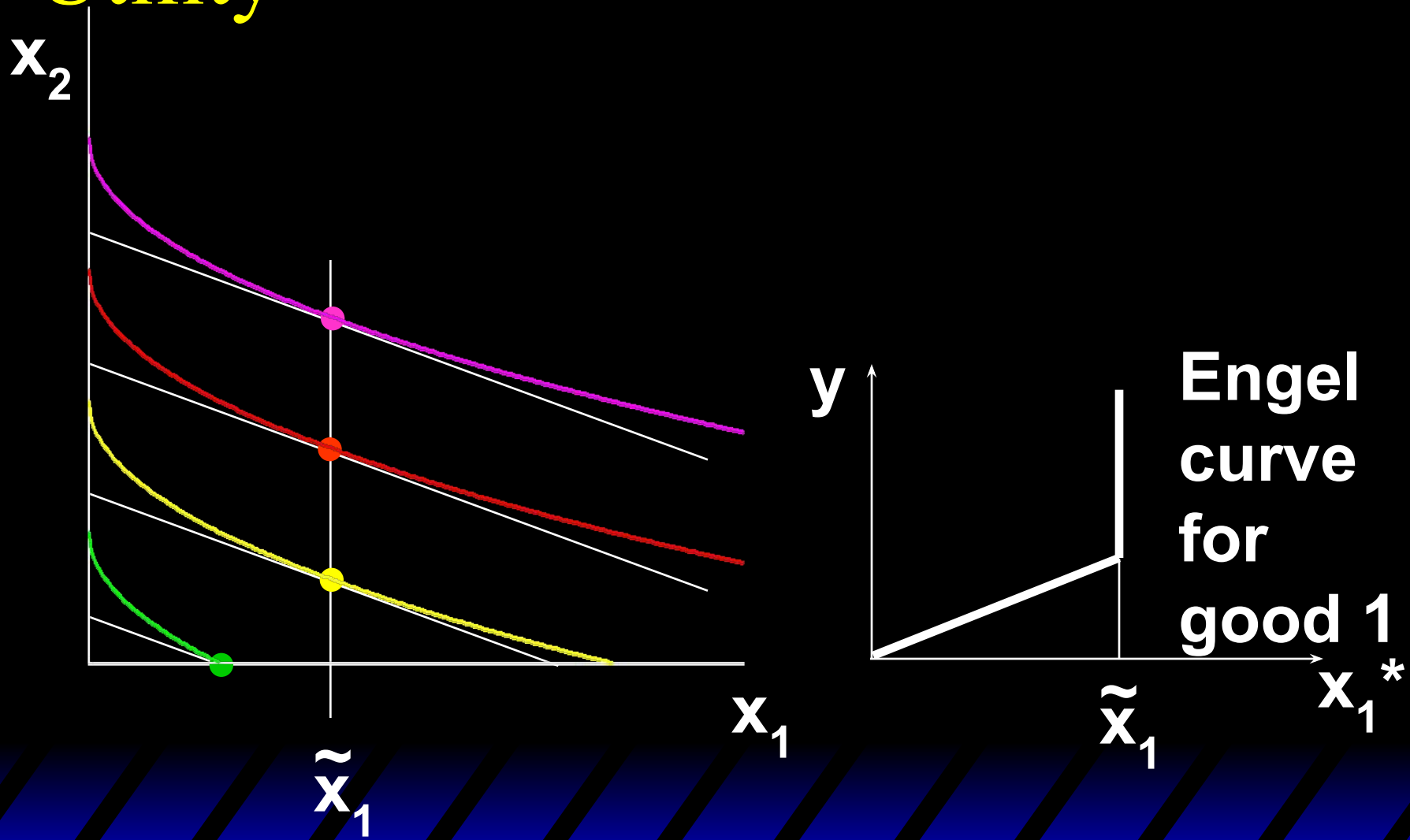
Each curve is a vertically shifted copy of the others.

Each curve intersects both axes.

Income Changes; Quasilinear Utility

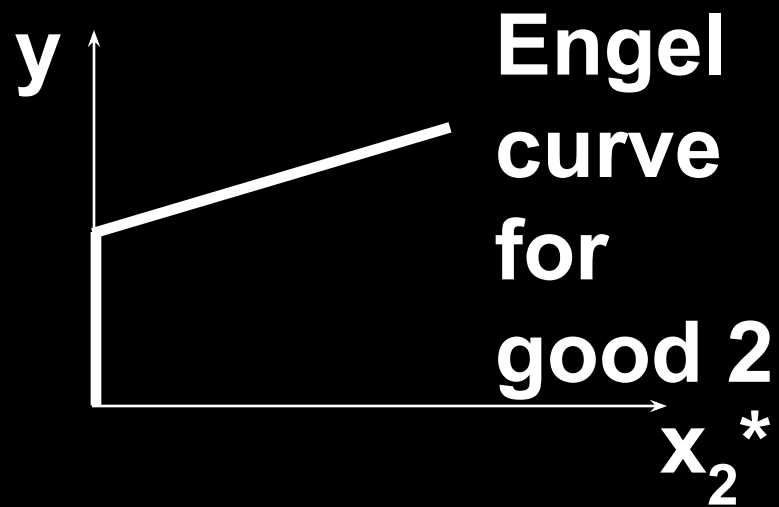
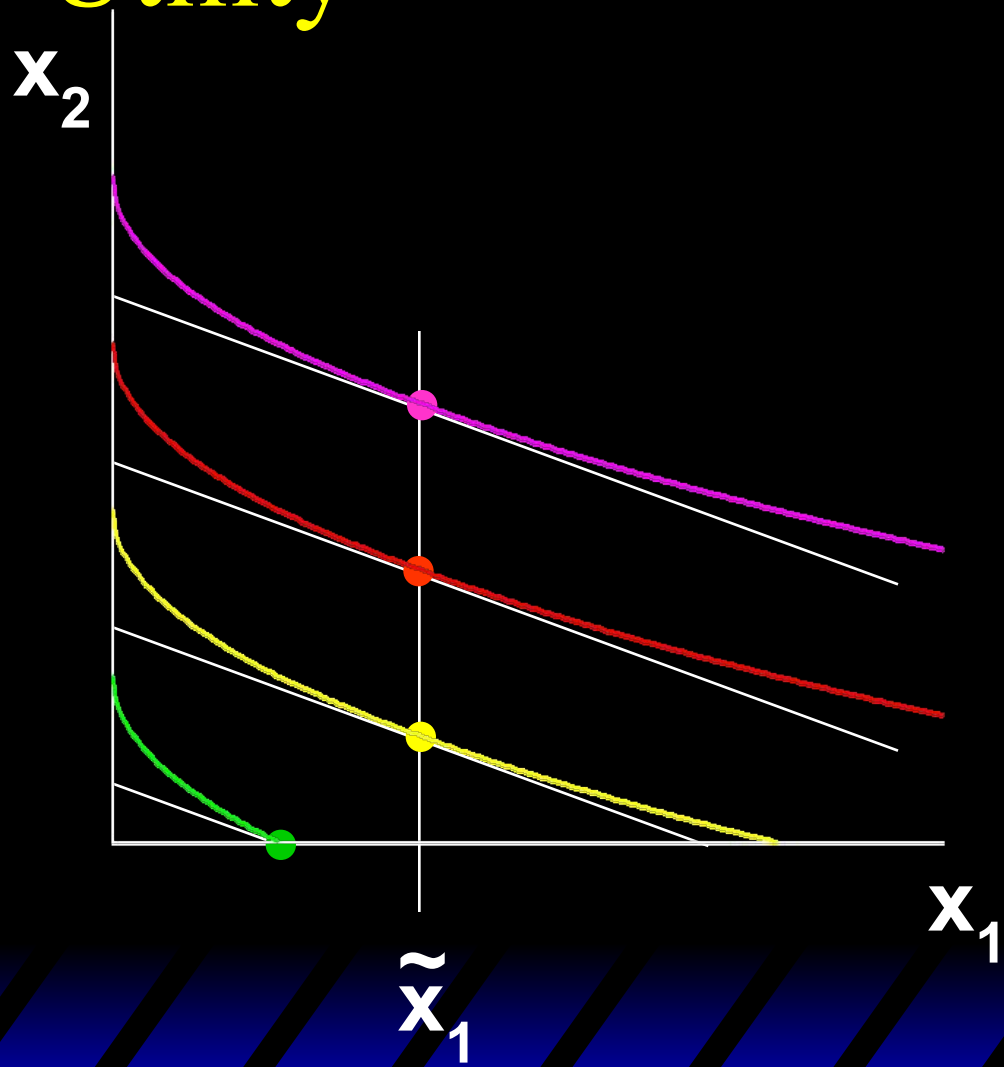


Income Changes; Quasilinear Utility



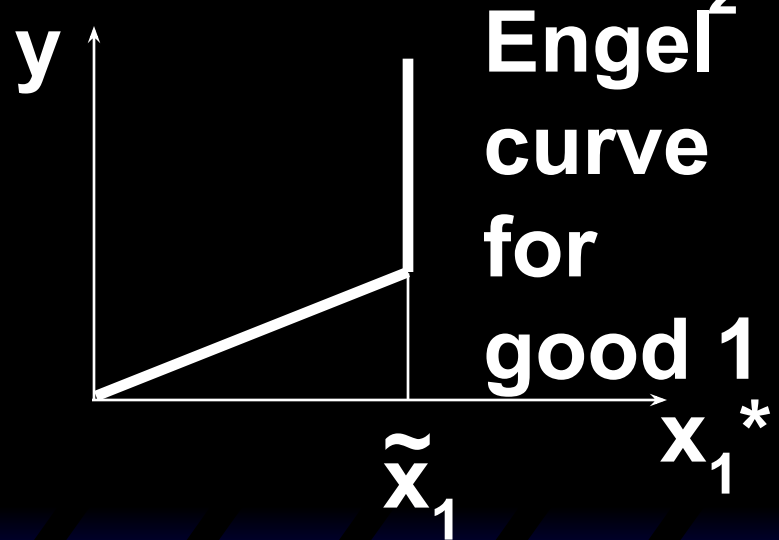
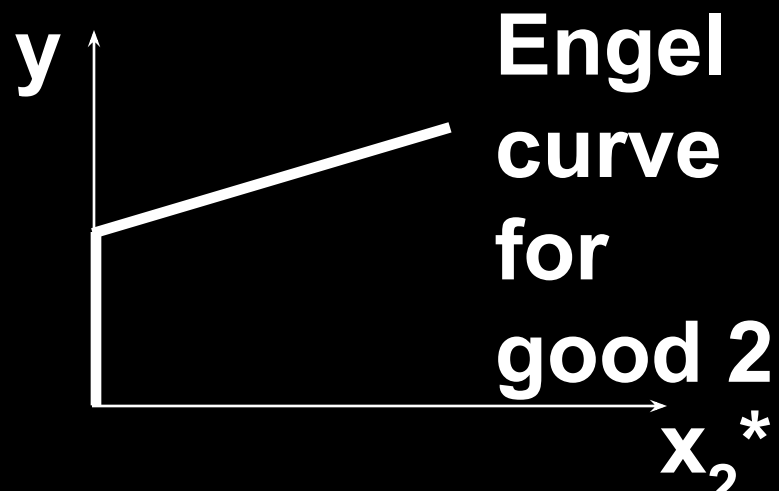
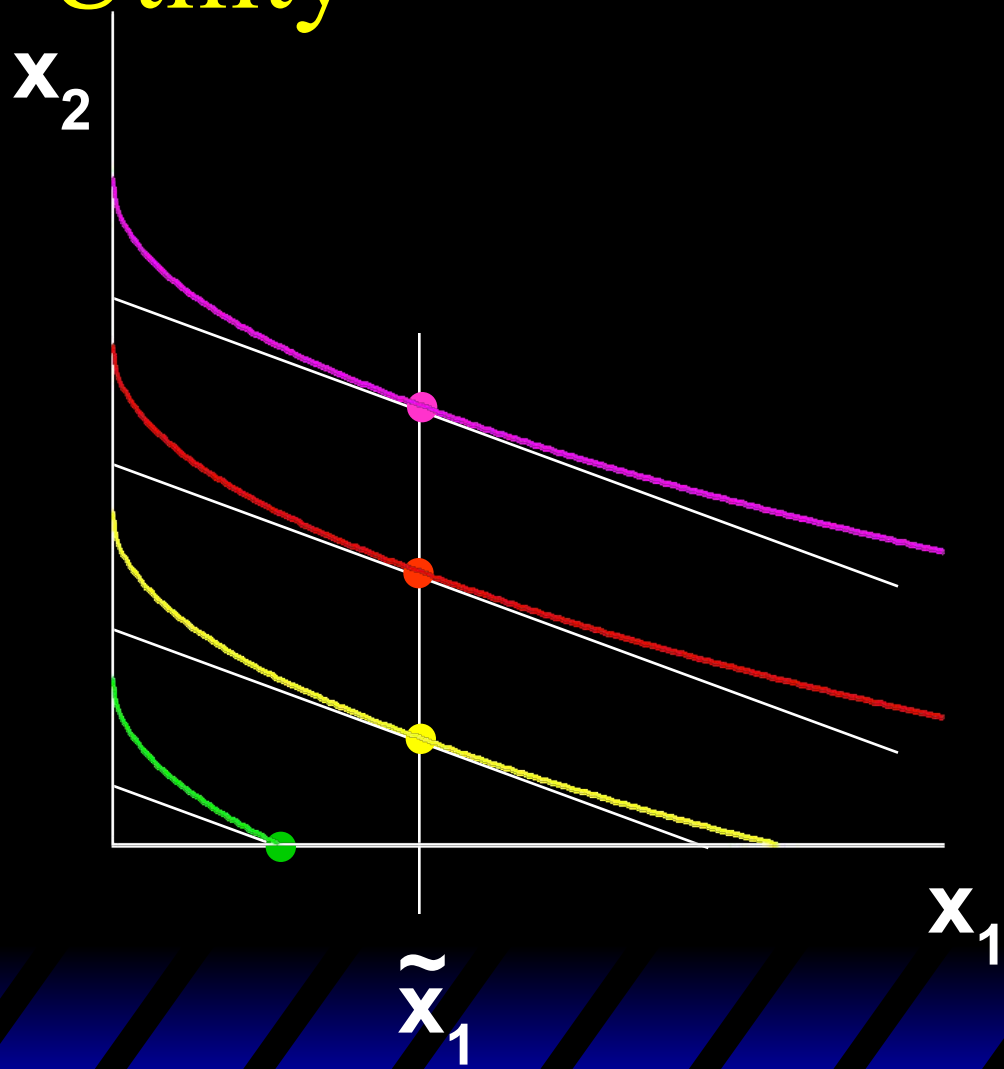
Income Changes; Quasilinear

Utility



Income Changes; Quasilinear

Utility



Income Effects

- A good for which quantity demanded rises with income is called **normal**.
- Therefore a normal good's Engel curve is positively sloped.

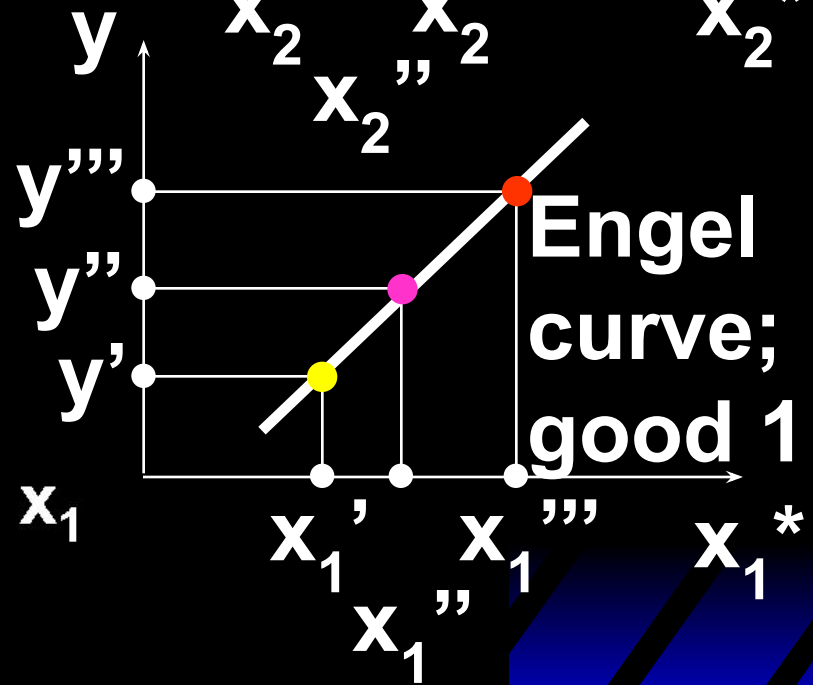
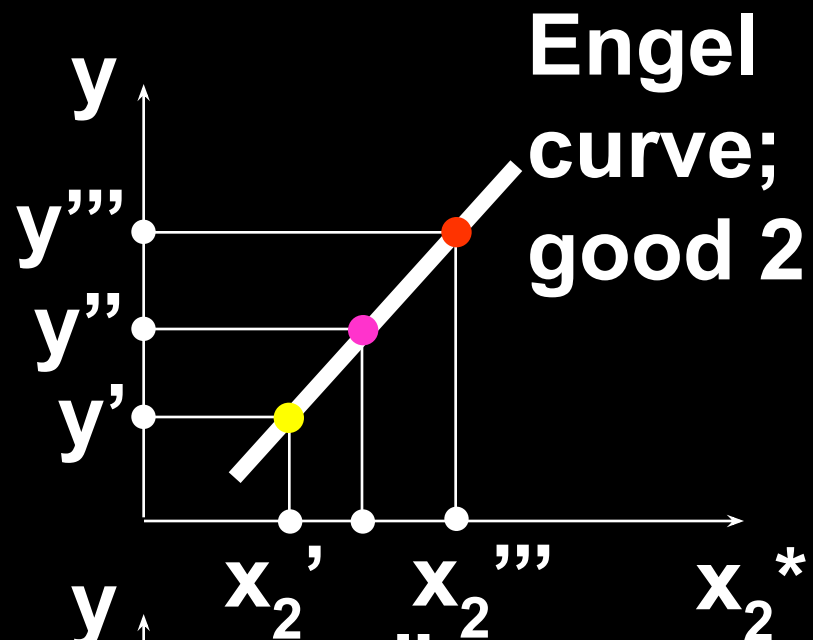
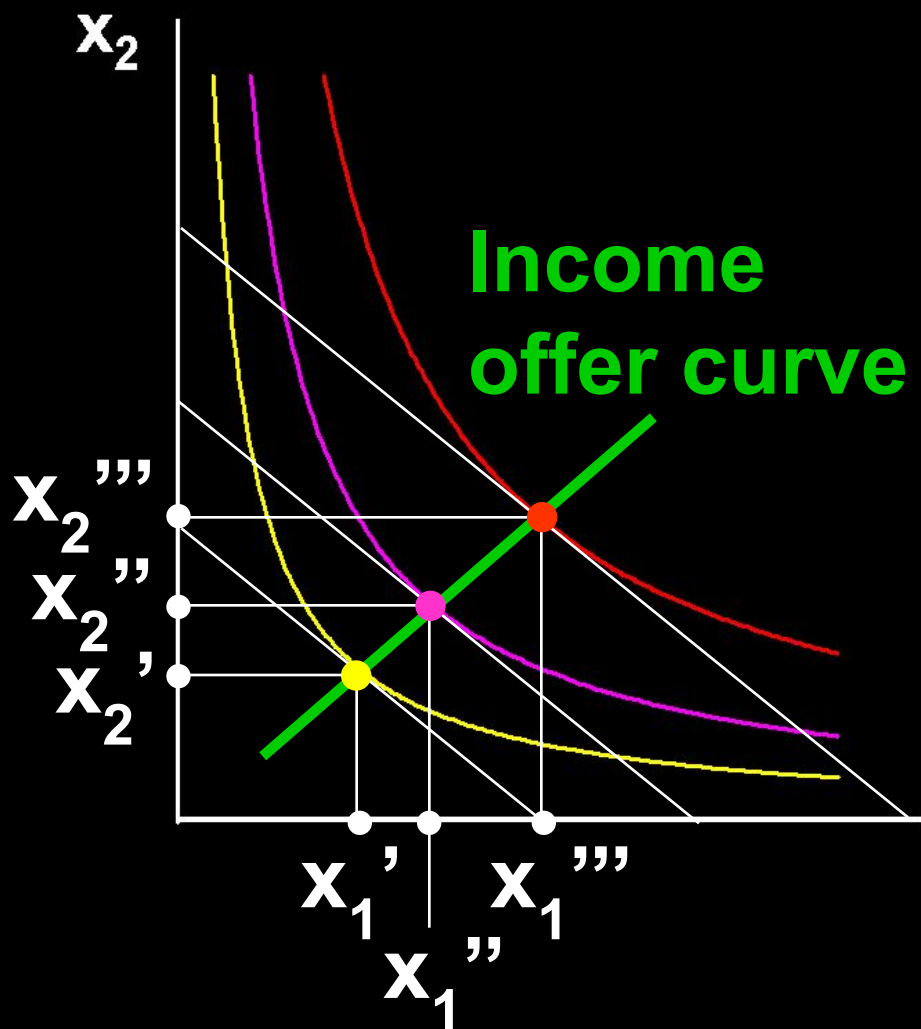
Income Effects

- A good for which quantity demanded falls as income increases is called **income inferior**.
- Therefore an income inferior good's Engel curve is negatively sloped.

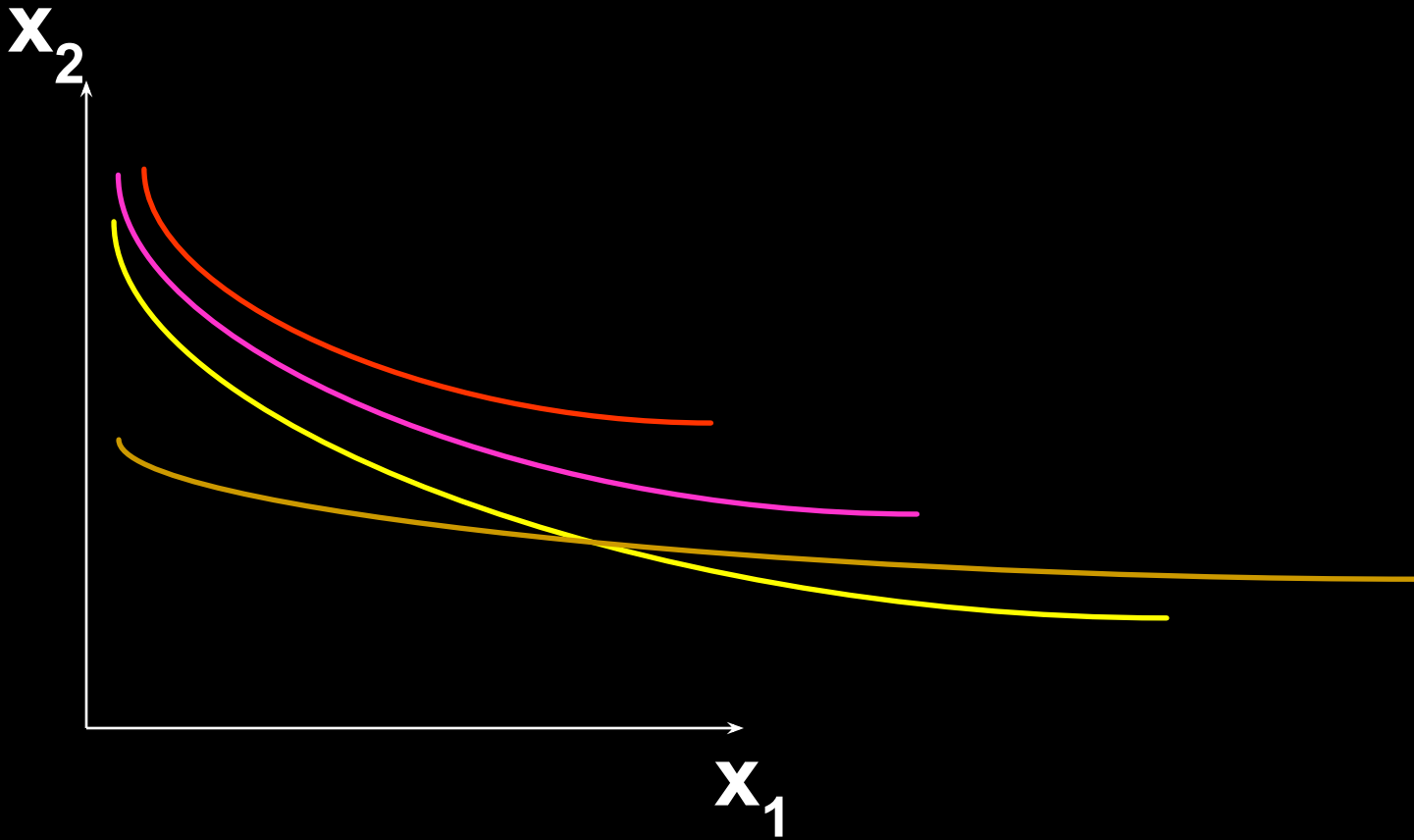
Income Effects

- In the US over last hundred years income increased many times whereas the number of kids per household went down.
- Are children an **inferior good**?

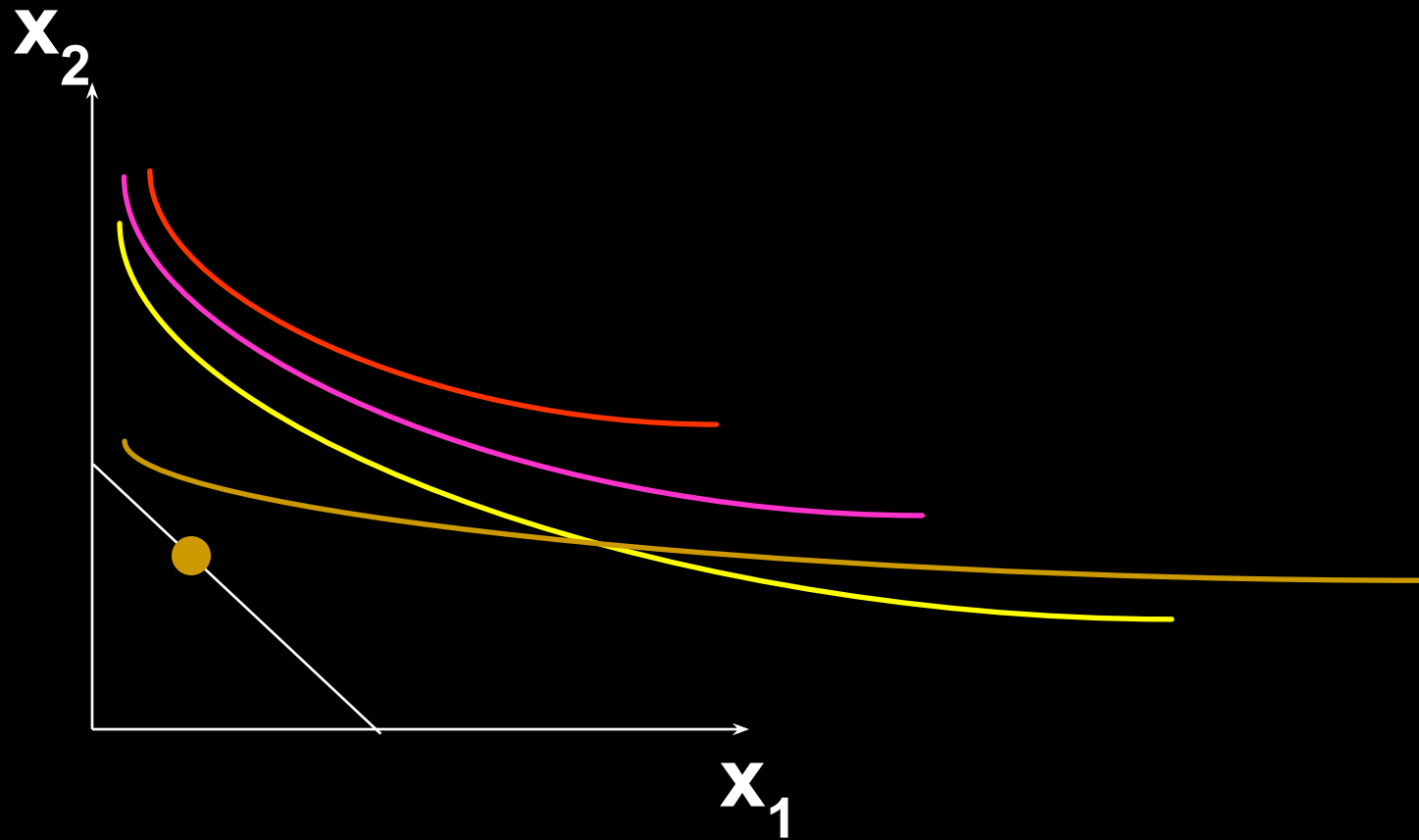
Income Changes; Goods 1 & 2 Normal



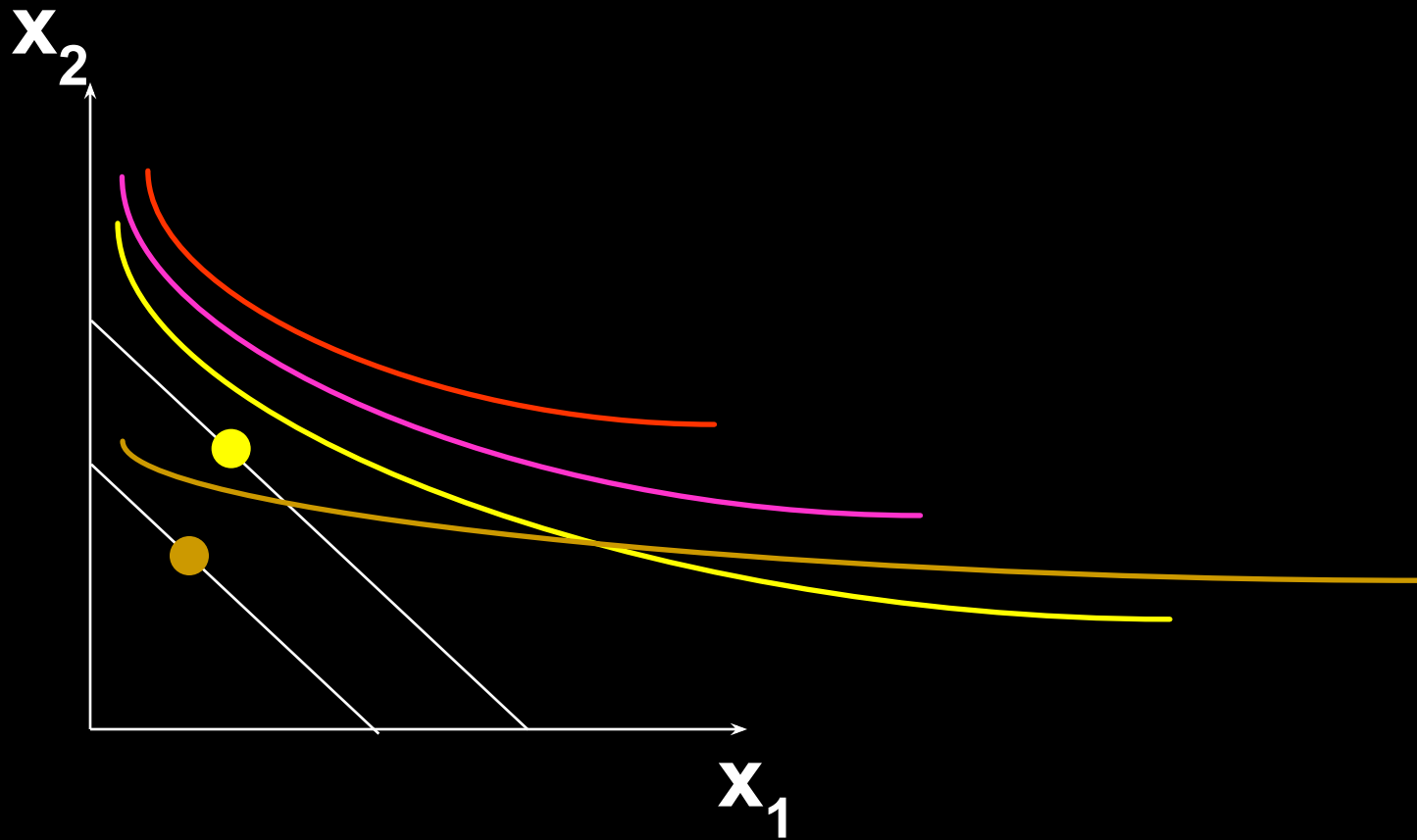
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



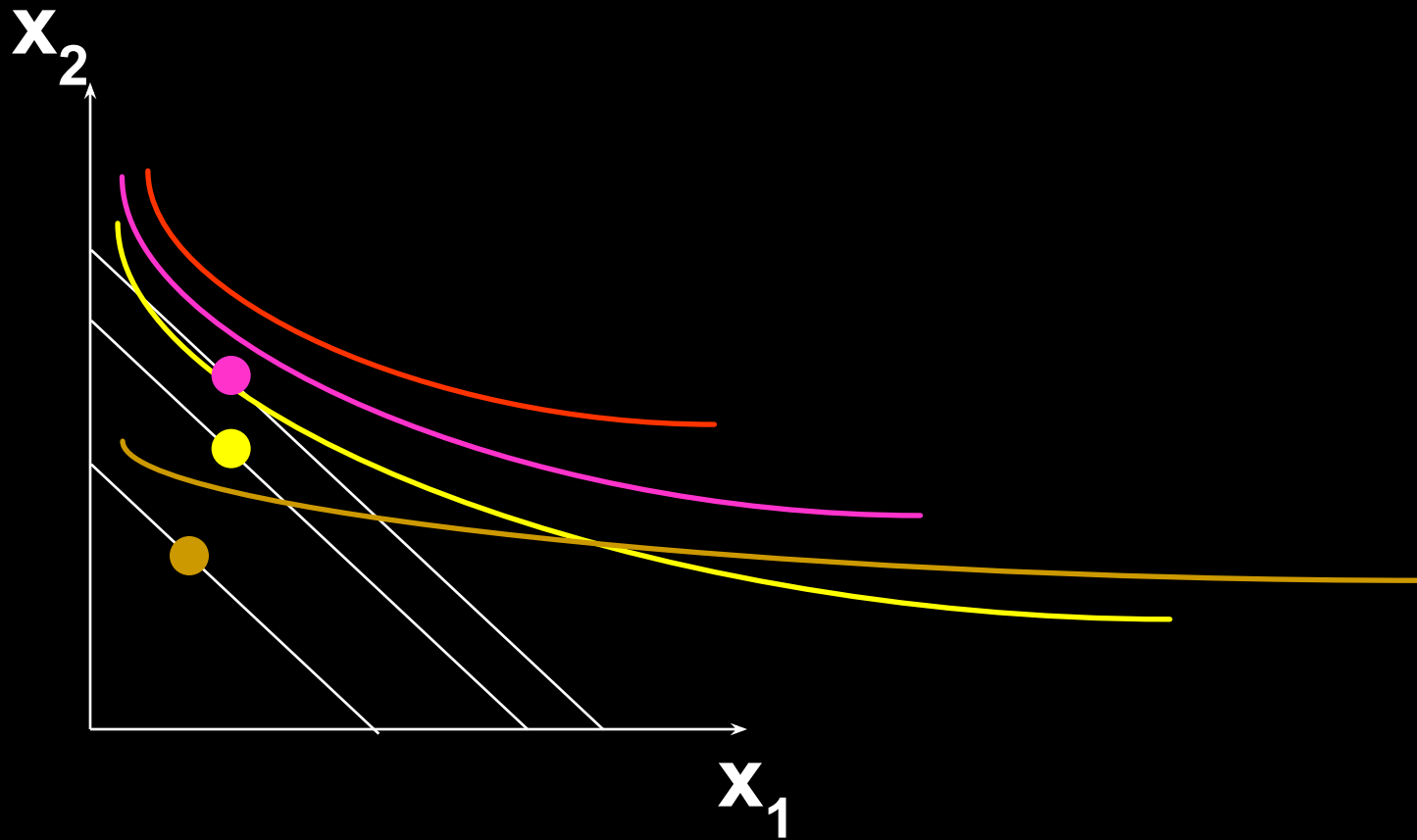
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



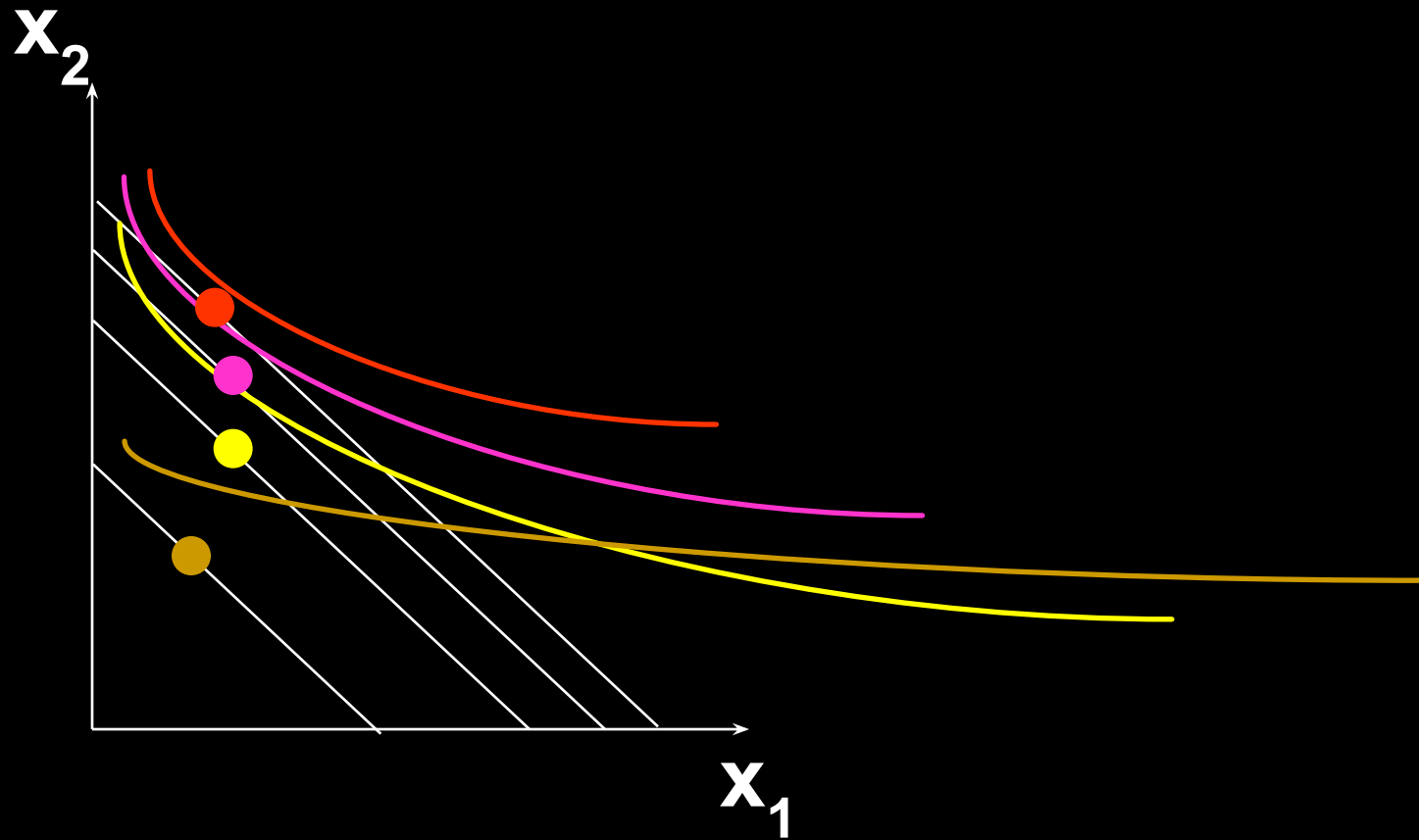
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



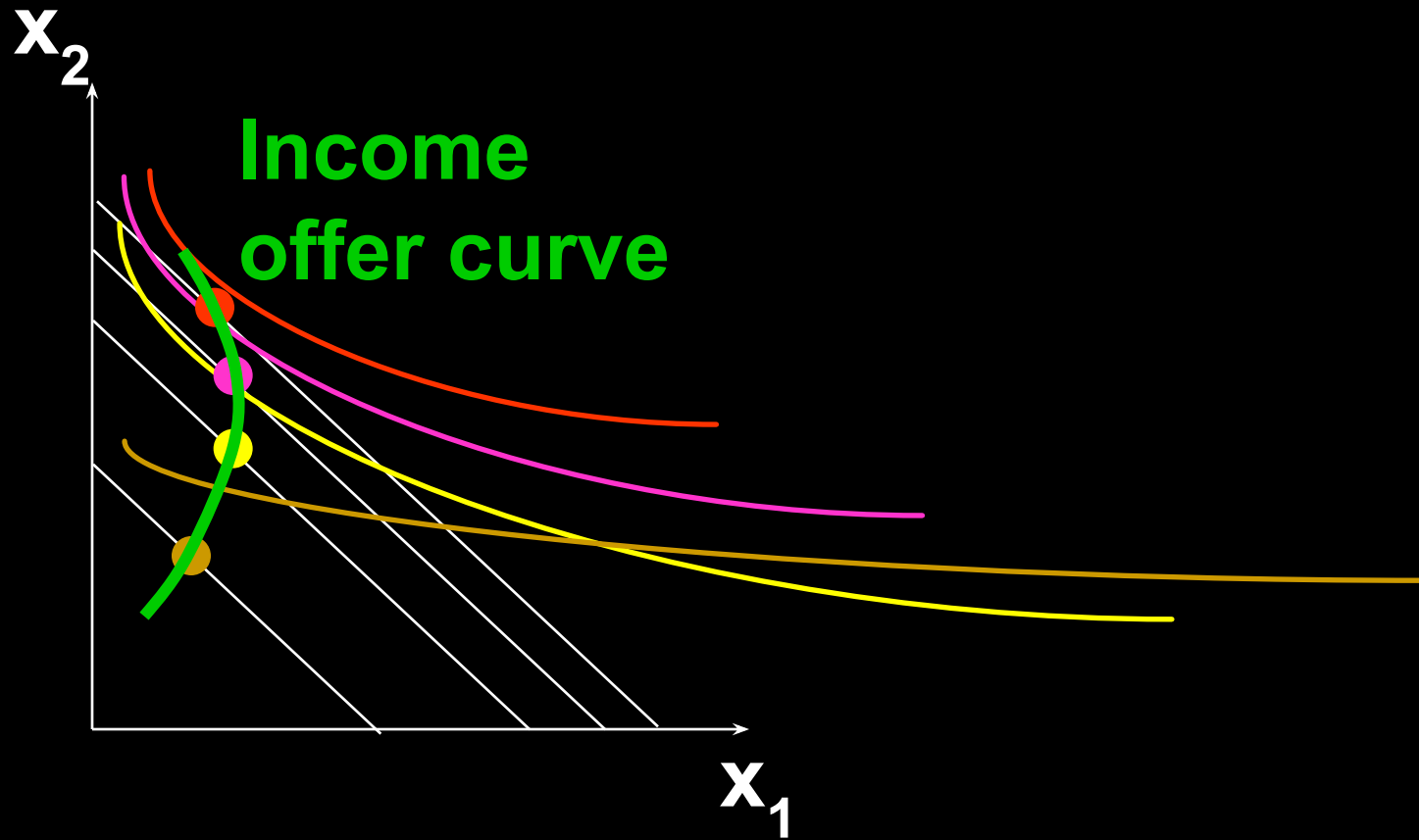
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



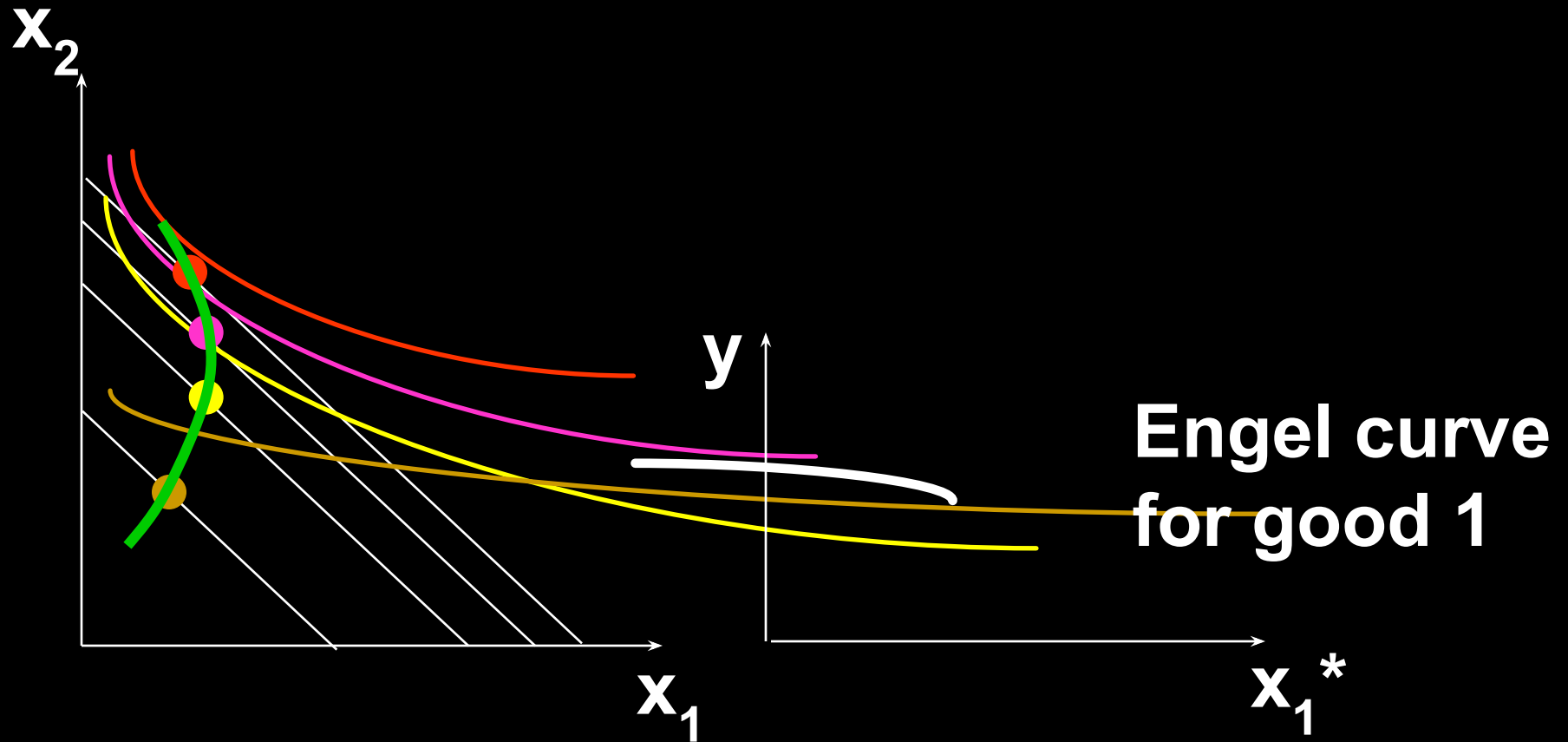
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



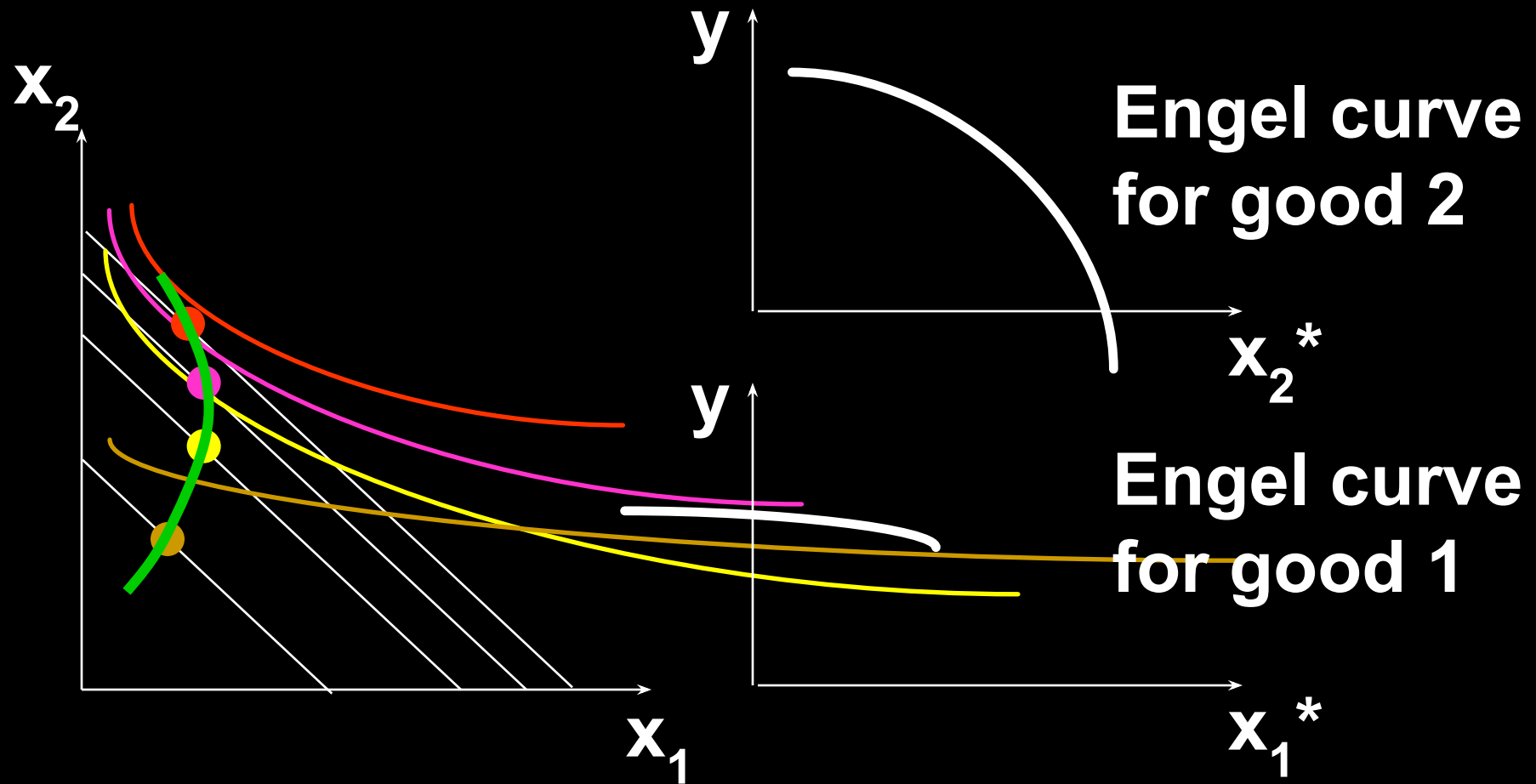
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

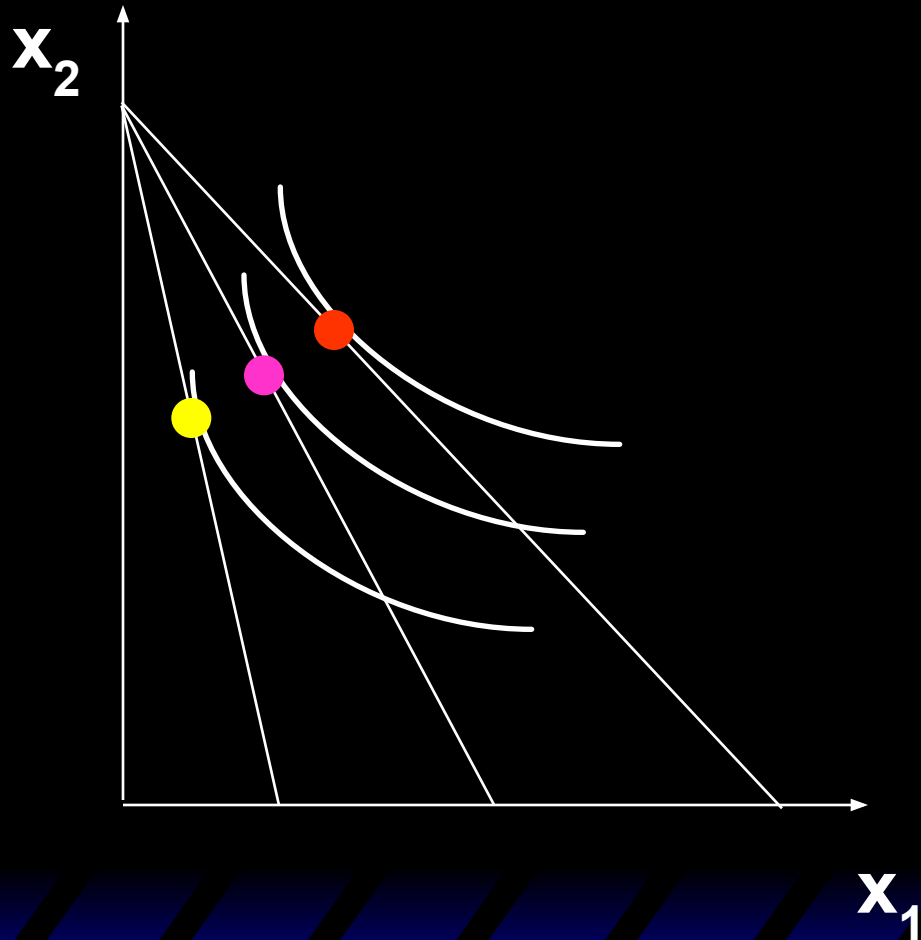


Ordinary Goods

- A good is called **ordinary** if the quantity demanded of it always increases as its own price decreases.

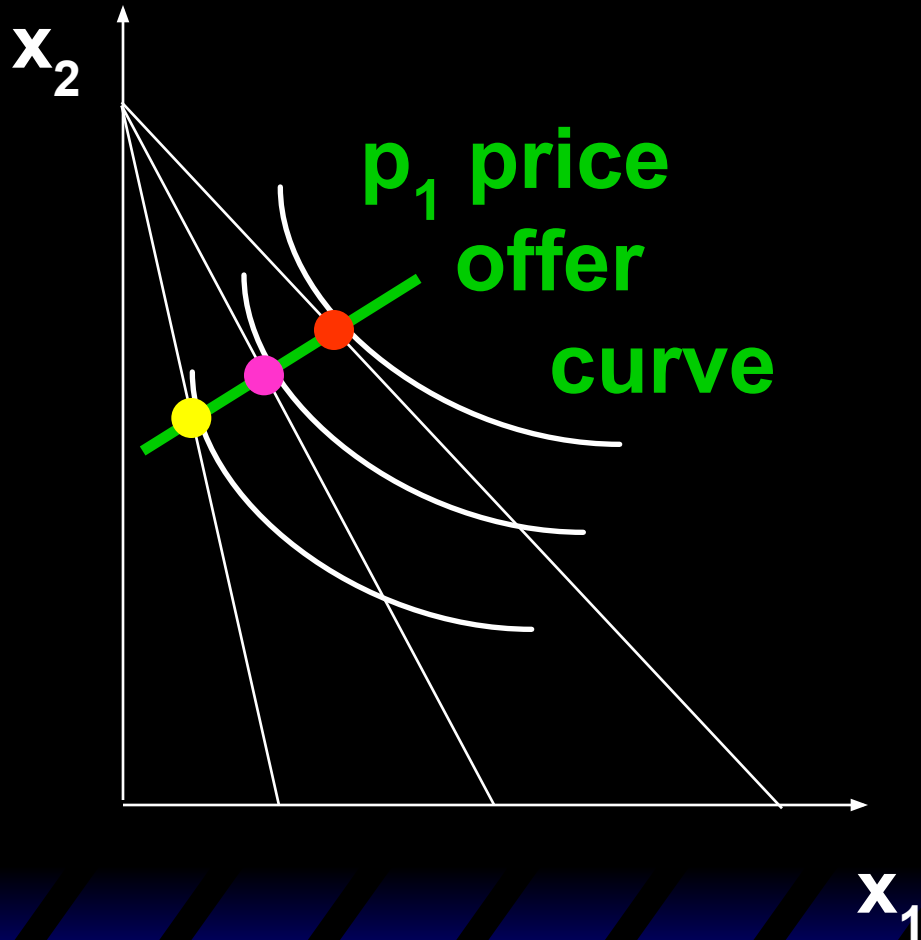
Ordinary Goods

Fixed p_2 and y .

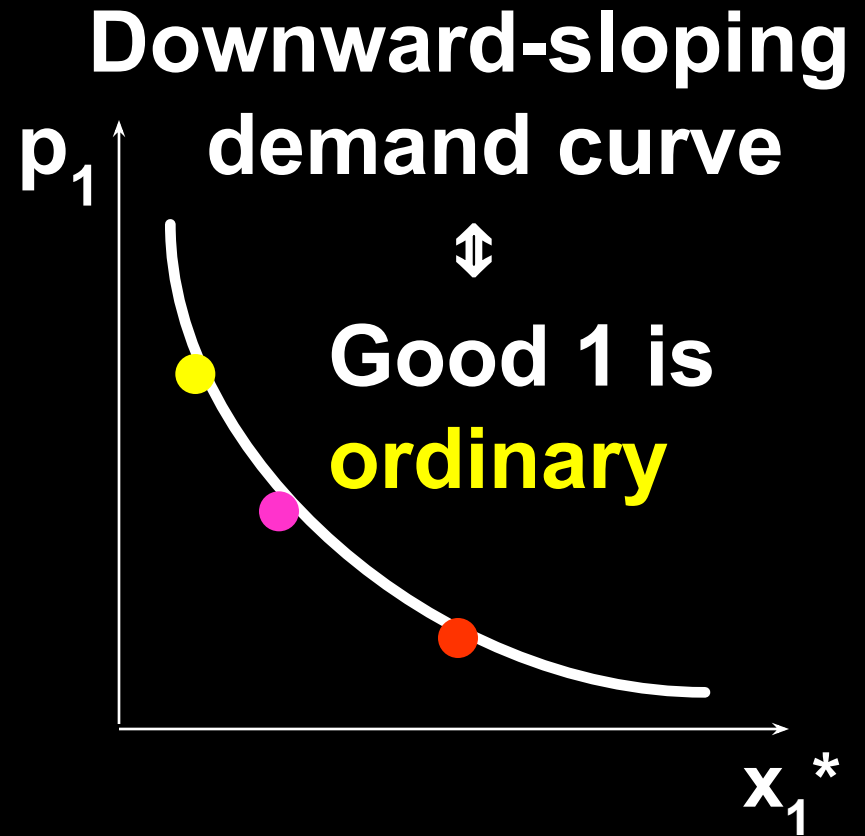
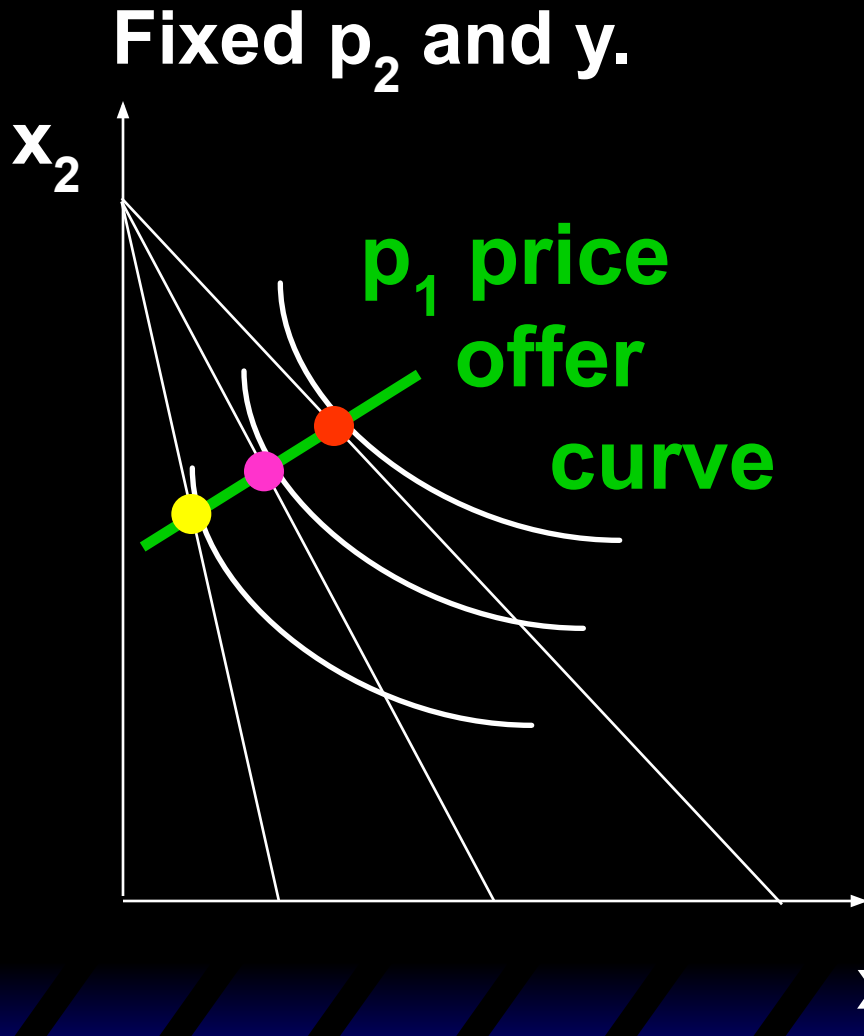


Ordinary Goods

Fixed p_2 and y .



Ordinary Goods

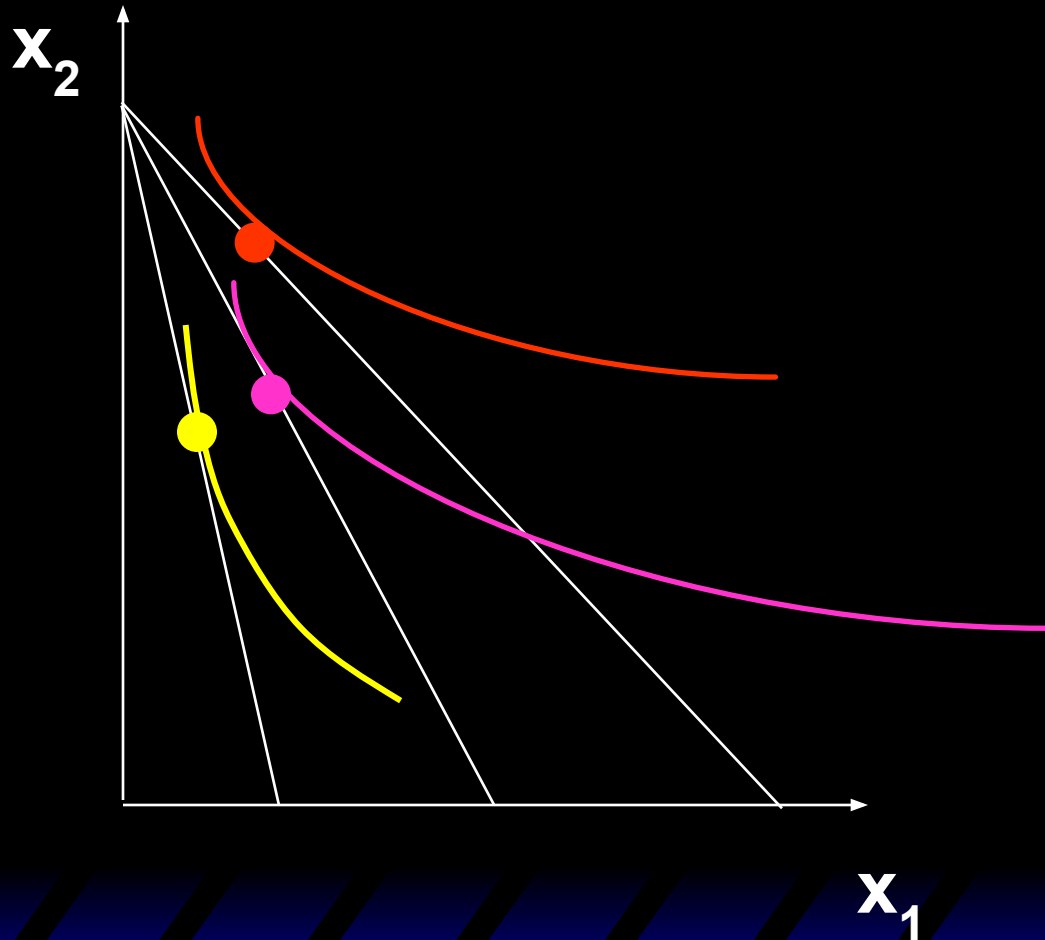


Giffen Goods

- If, for **some** values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called **Giffen**.

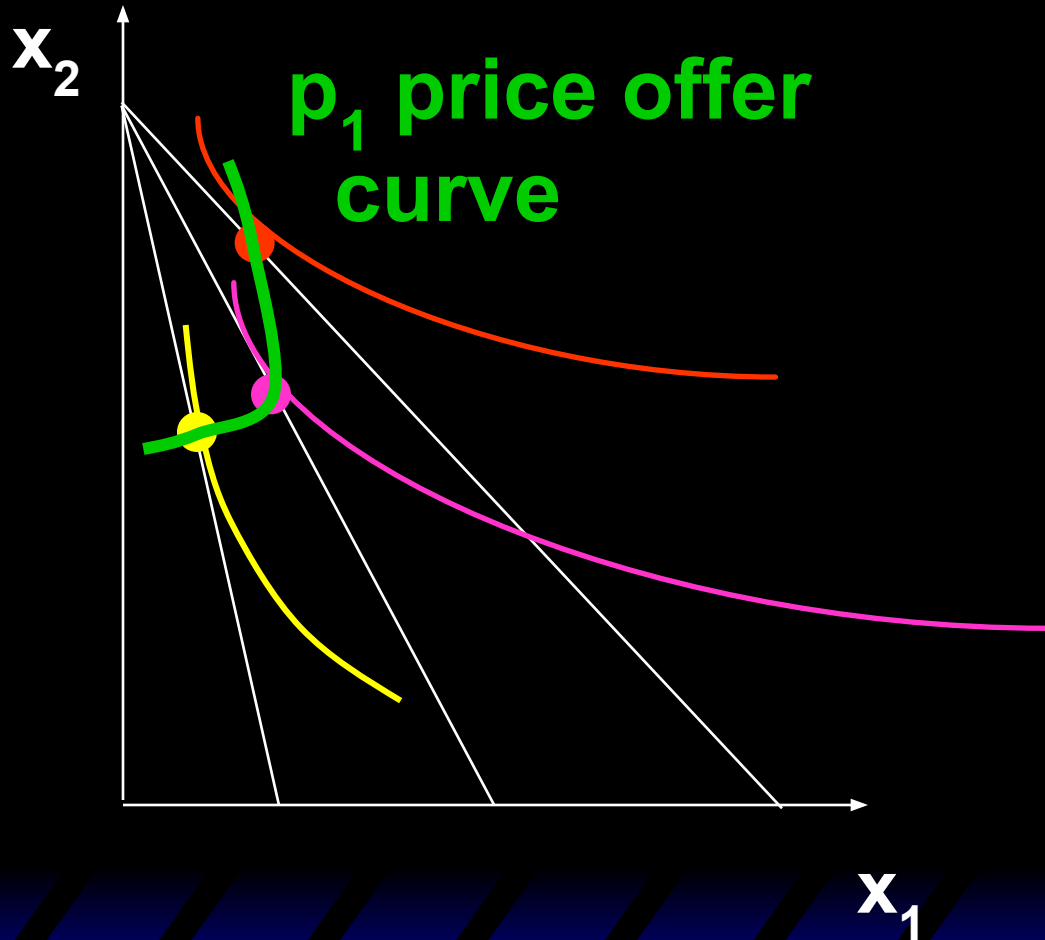
Ordinary Goods

Fixed p_2 and y .



Ordinary Goods

Fixed p_2 and y .



Ordinary Goods

Fixed p_2 and y .



Demand curve has

a positively
sloped part



Good 1 is
Giffen



Cross-Price Effects

- If an increase in p_2
 - **increases** demand for commodity 1 then commodity 1 is a **gross substitute** for commodity 2.
 - **reduces** demand for commodity 1 then commodity 1 is a **gross complement** for commodity 2.

Cross-Price Effects

A perfect-complements example:

so

Therefore commodity 2 is a gross complement for commodity 1.



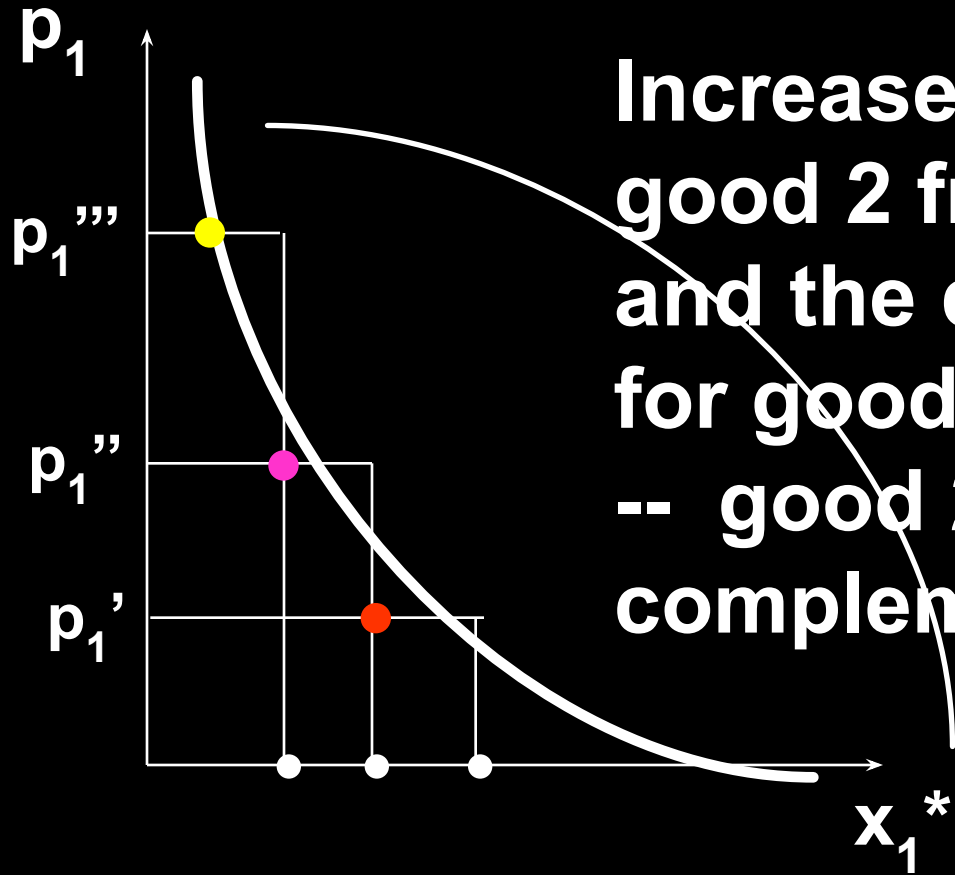
Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and

p_2'

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

p_2''

Cross-Price Effects

A Cobb- Douglas example:

so

Cross-Price Effects

A Cobb- Douglas example:

so

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.