Chapter Six

Demand

Properties of Demand Functions

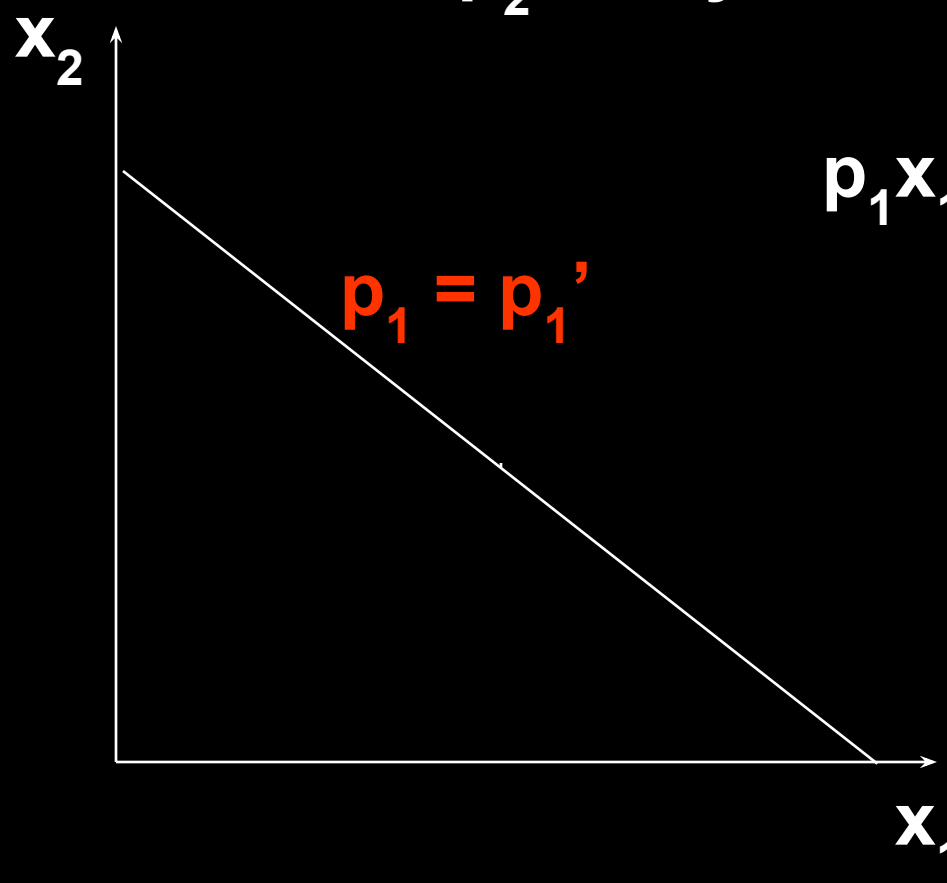
- **Comparative statics analysis** of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices p_1, p_2 and income y change.

Own-Price Changes

- How does $x_1^*(p_1,p_2,y)$ change as p_1 changes, holding p_2 and y constant?
- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

Own-Price Changes

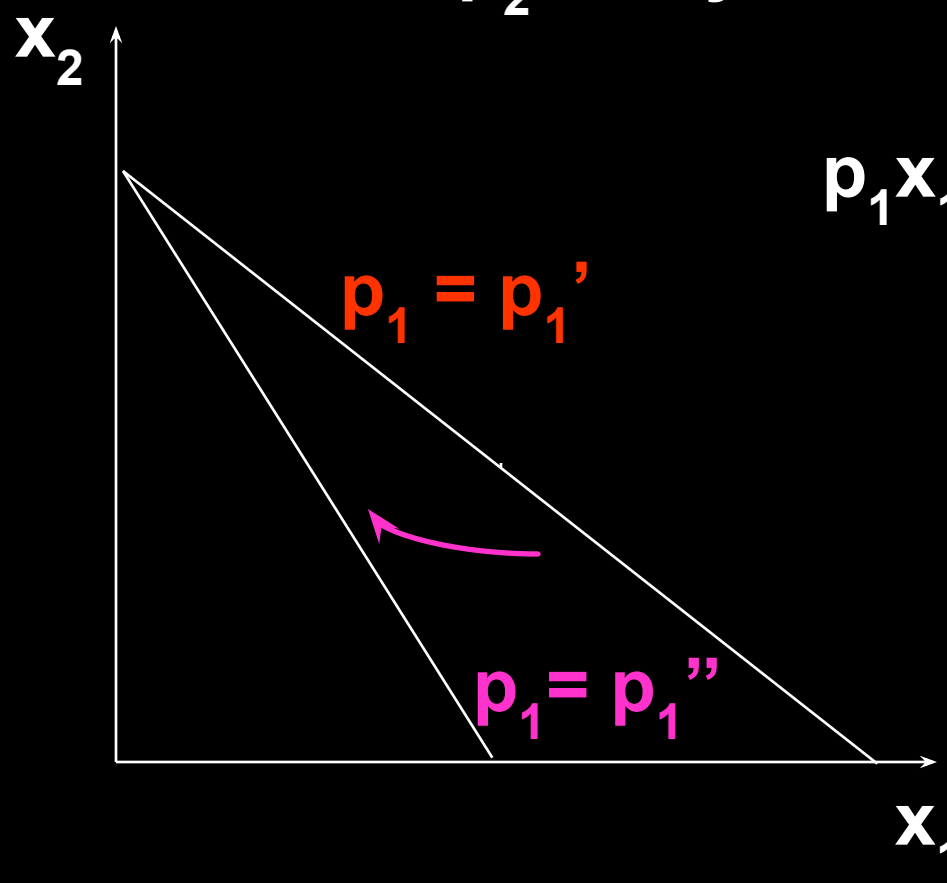
Fixed p_2 and y.



$$p_1x_1 + p_2x_2 = y$$

Own-Price Changes

Fixed p_2 and y.



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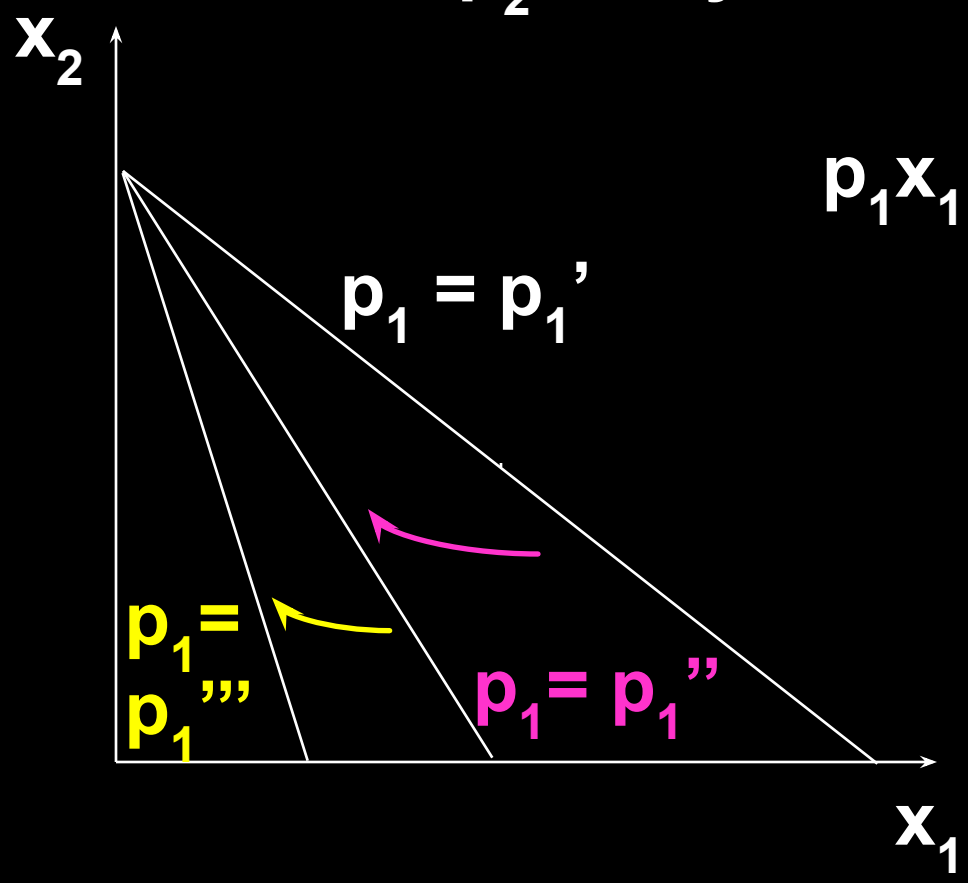
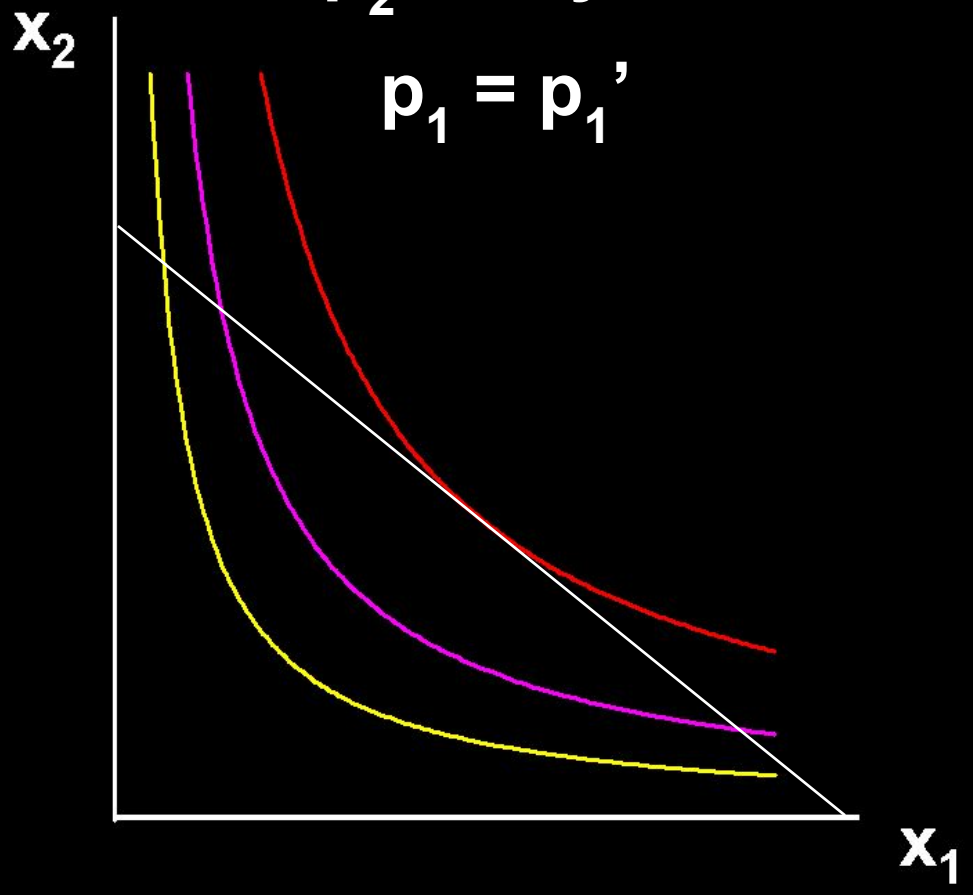


Own-Price Changes

Fixed p_2 and y.



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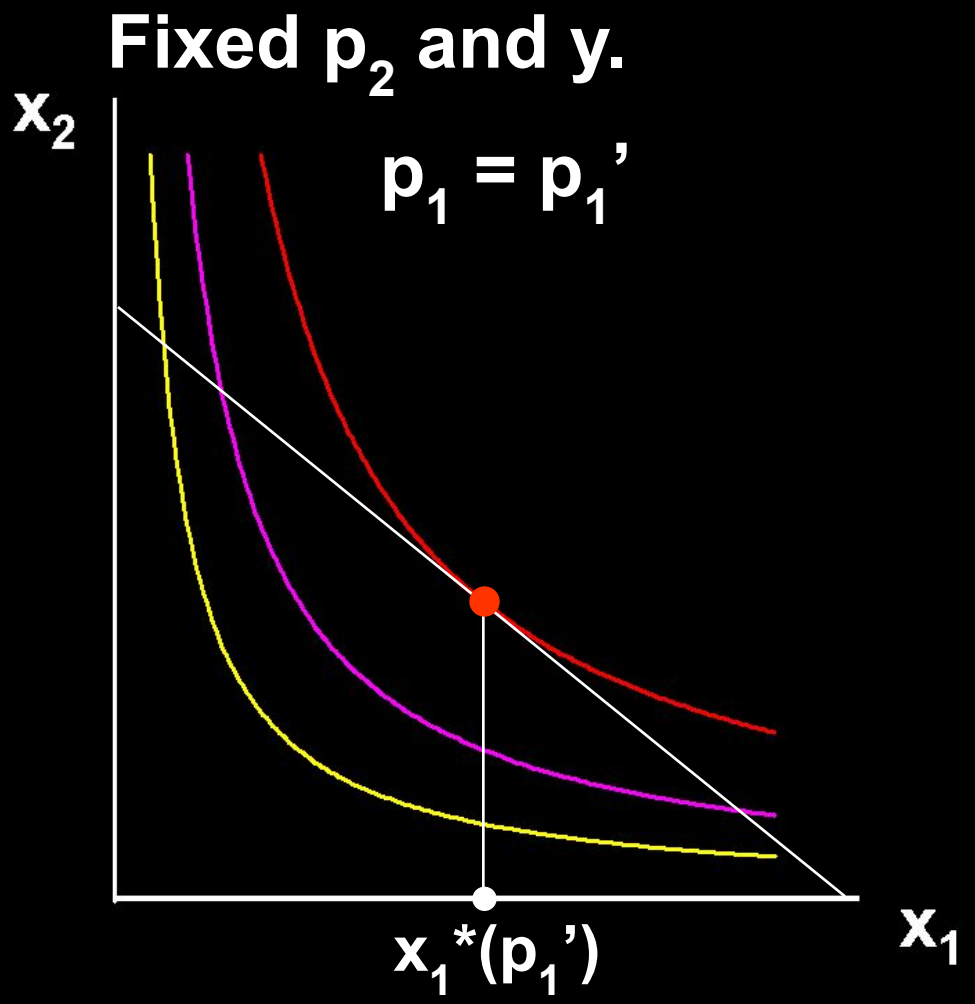

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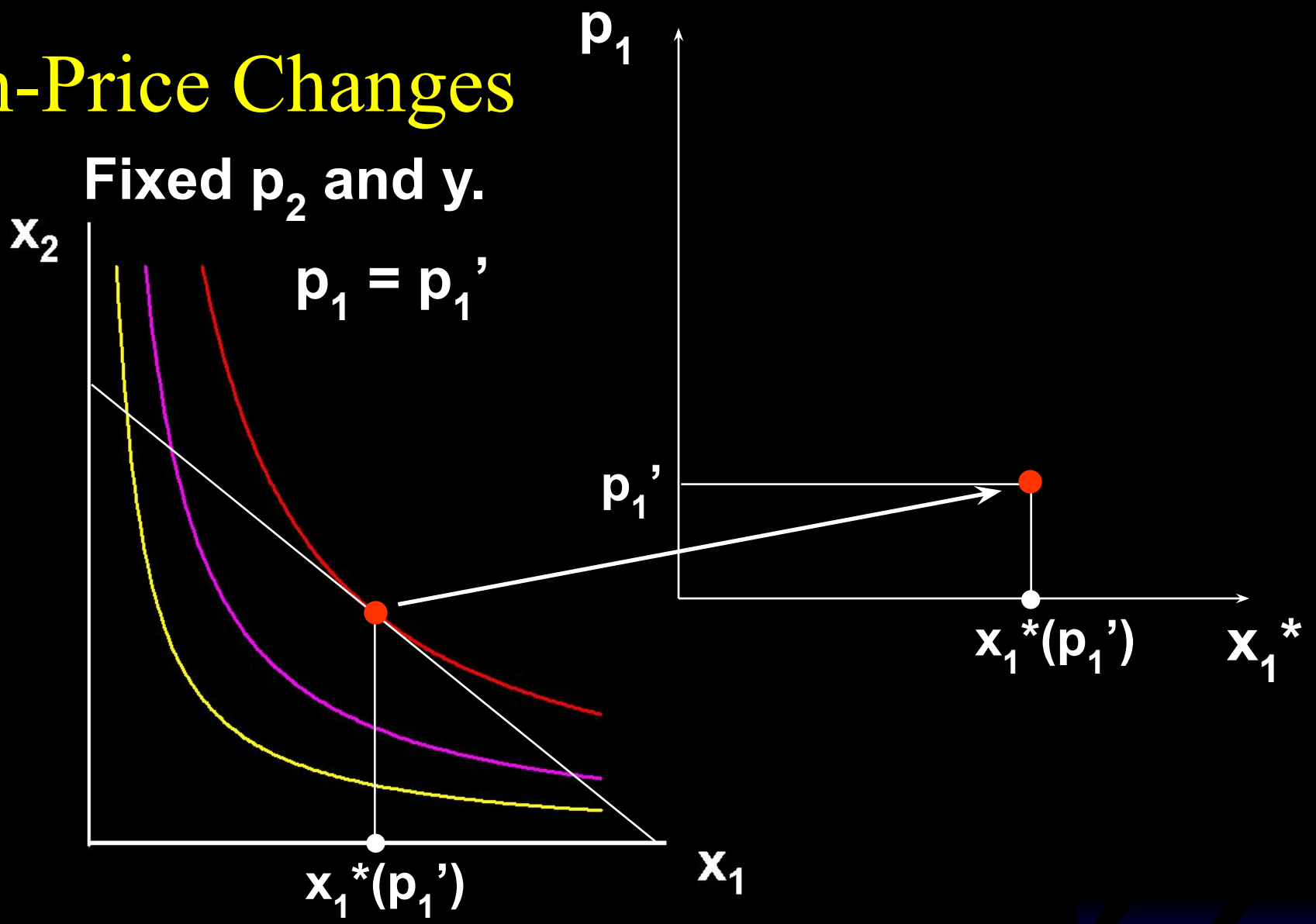

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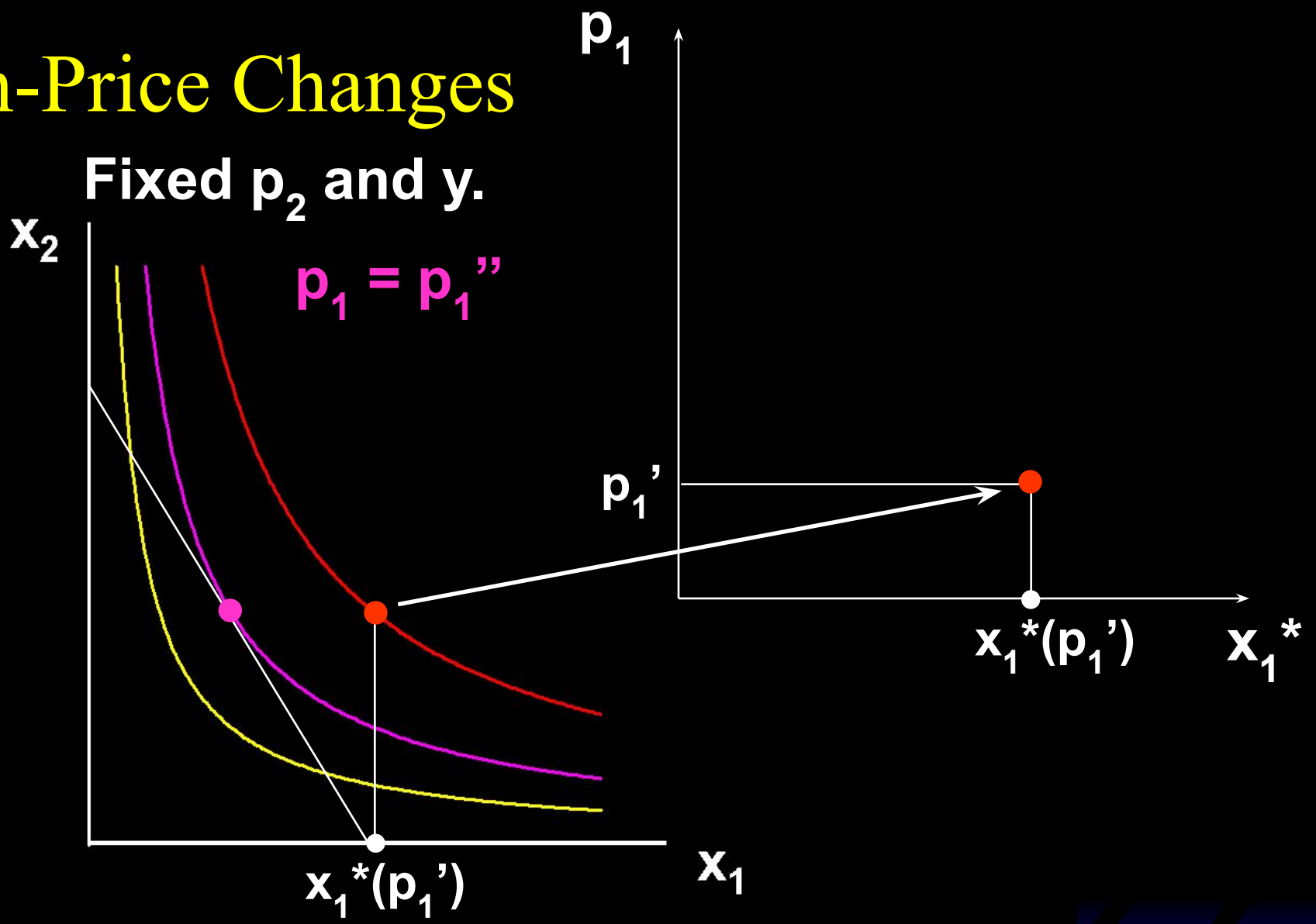
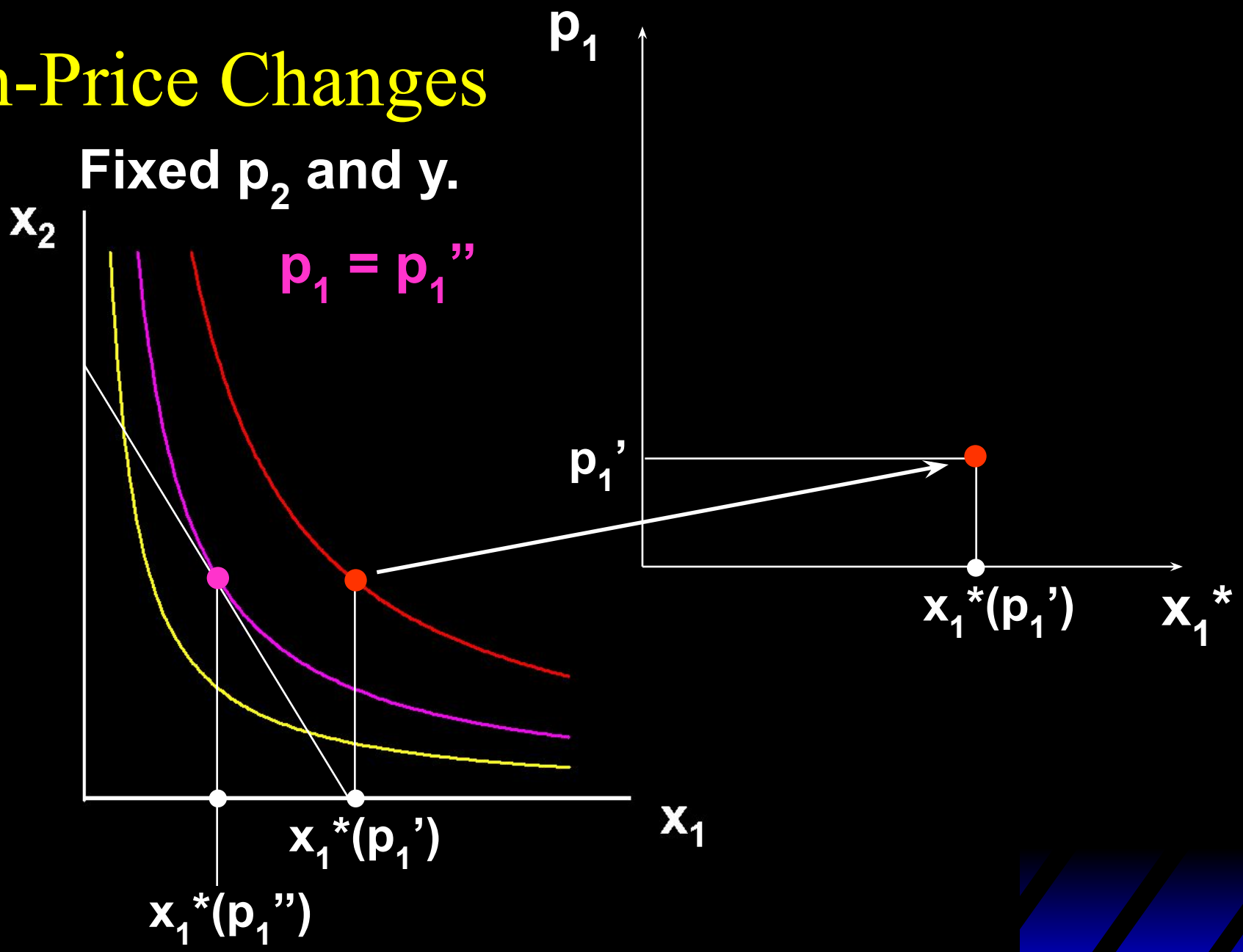


Own-Price Changes



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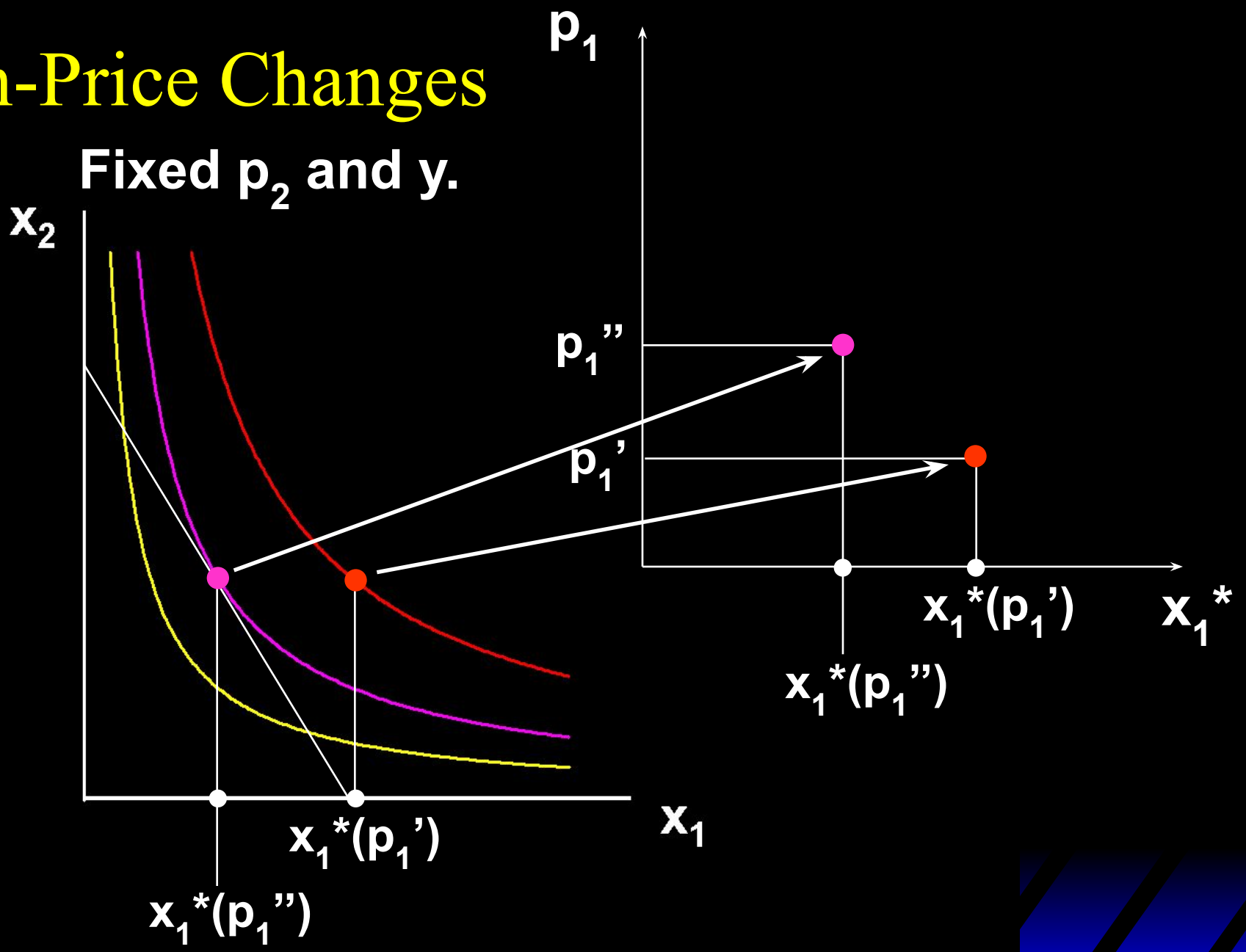

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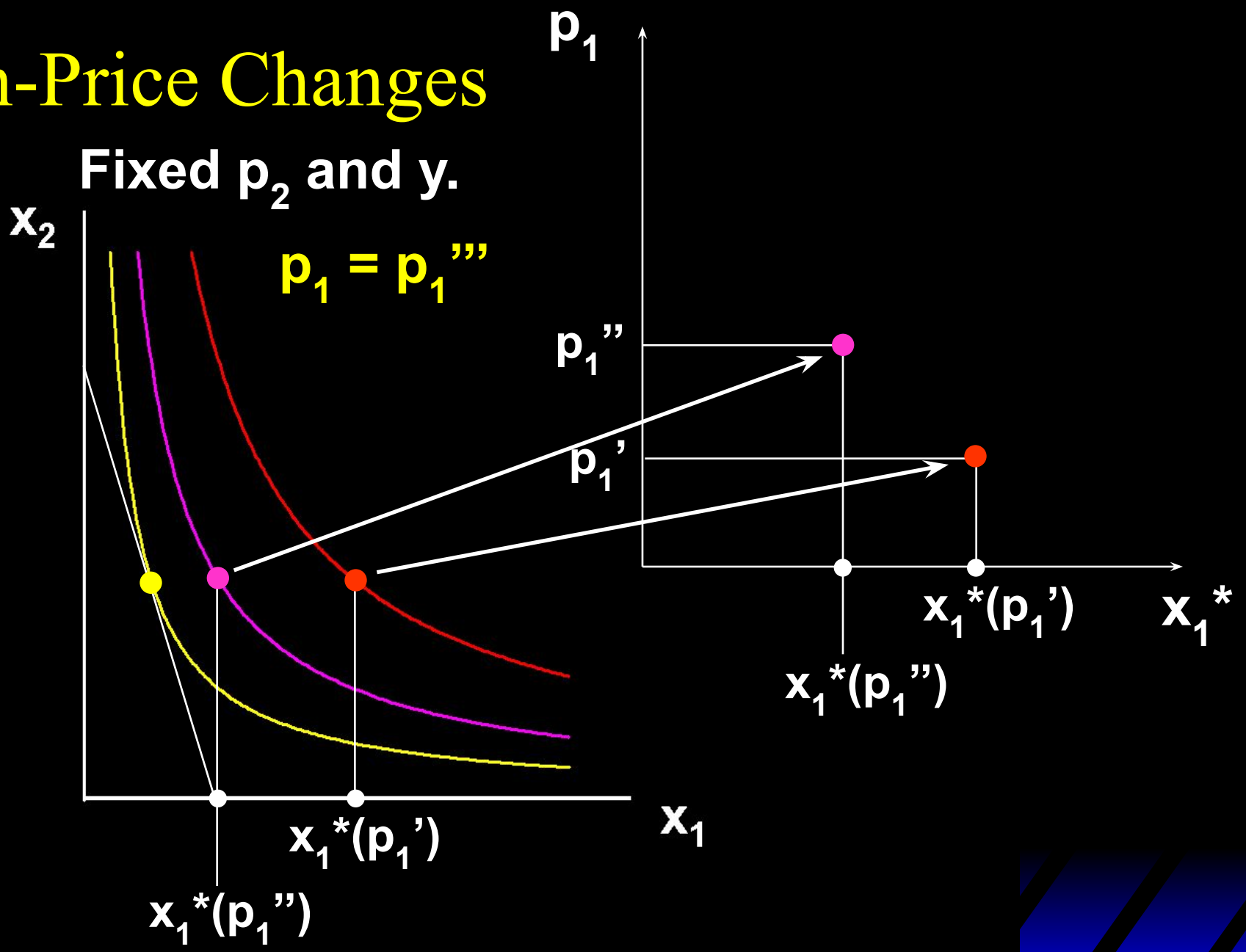

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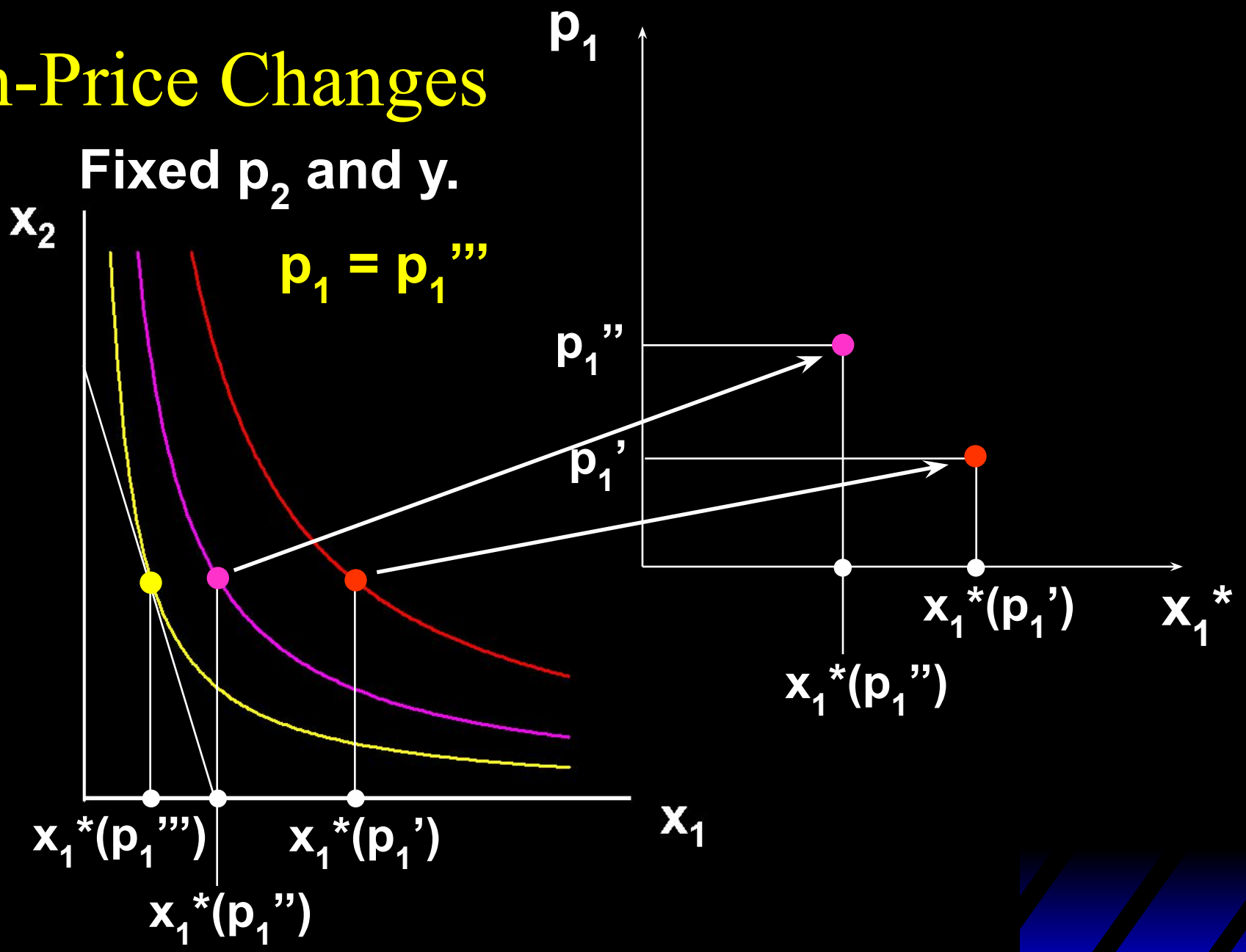
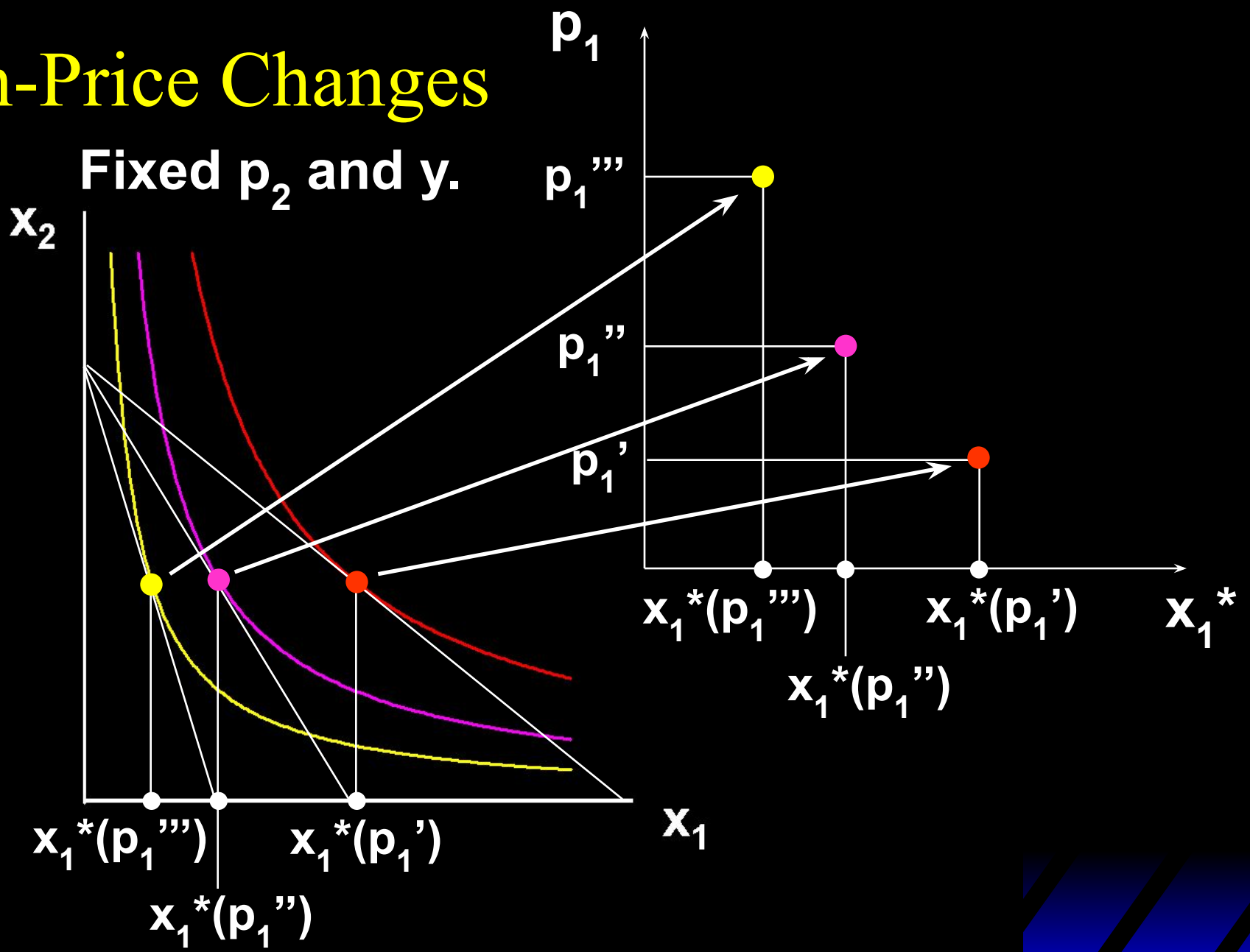


Own-Price Changes



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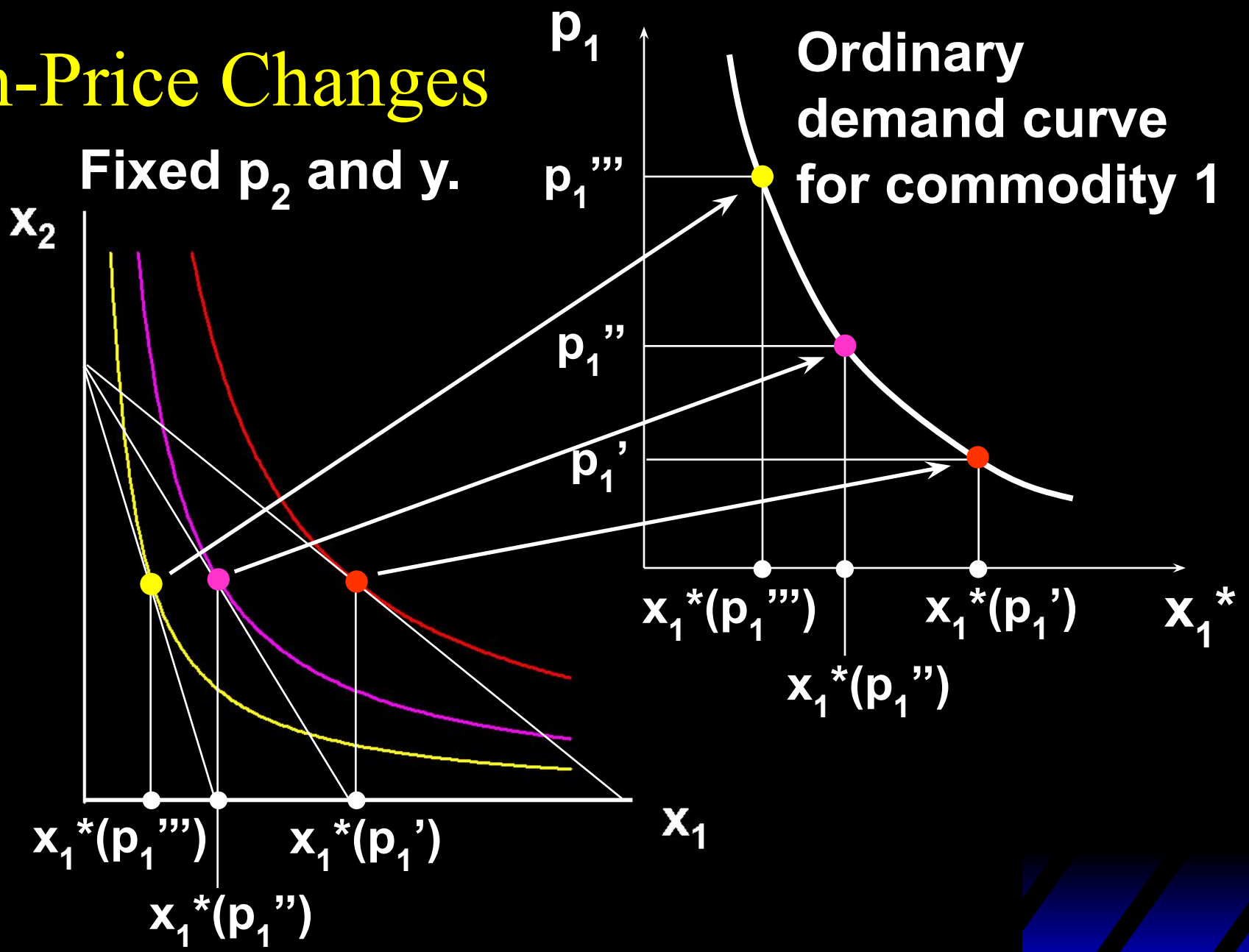

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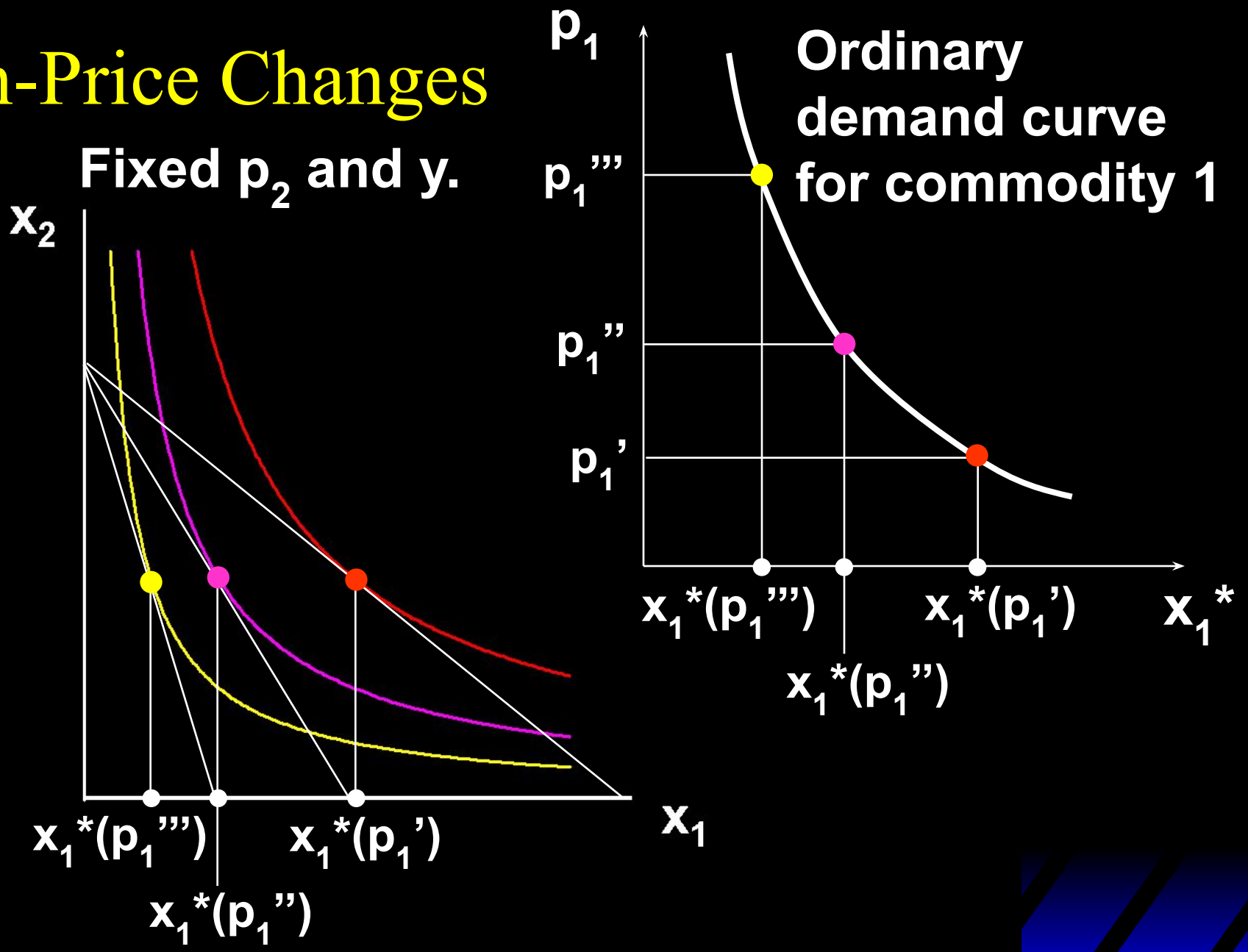
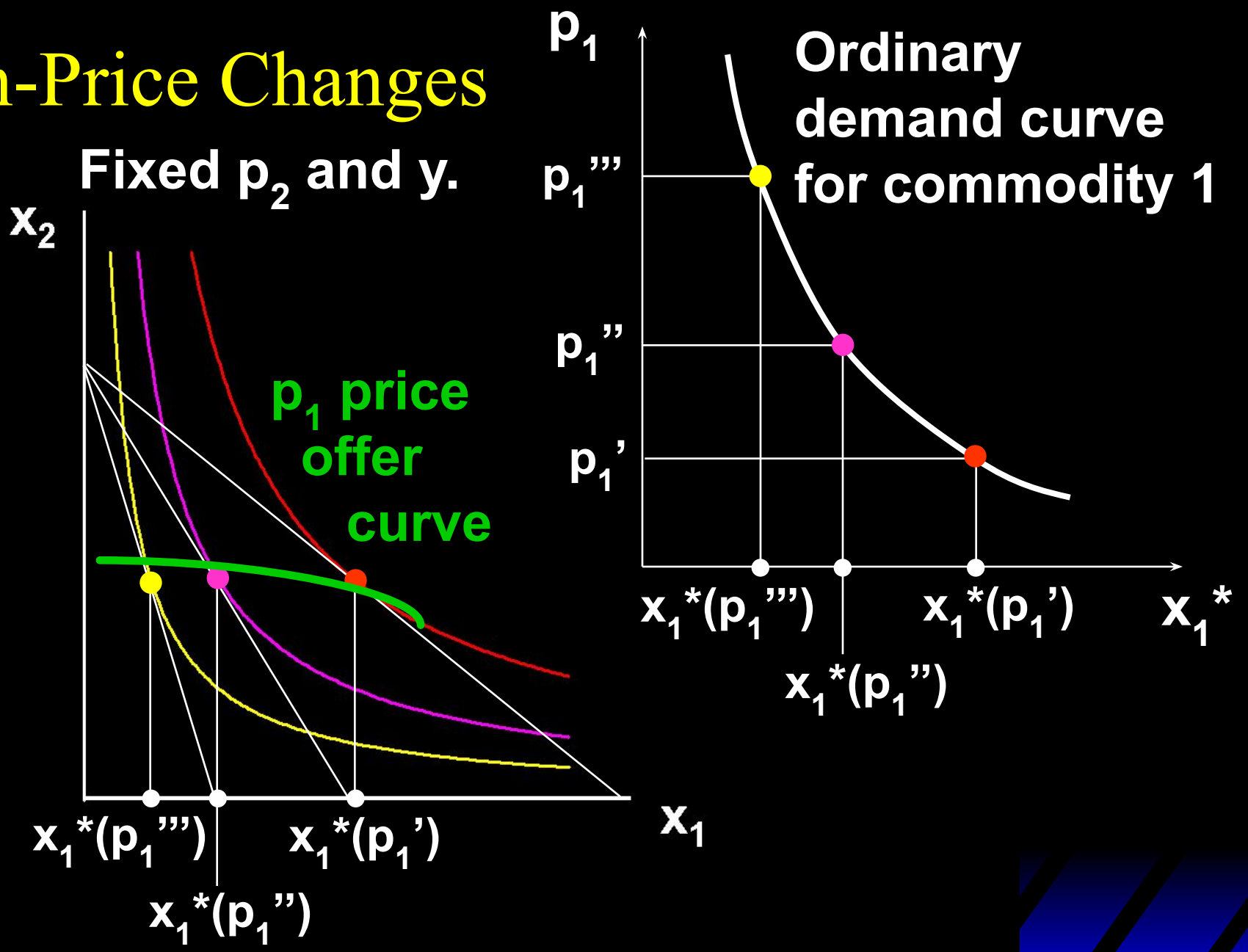


Own-Price Changes



Own-Price Changes

- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the **p_1- price offer curve**.
- The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the **ordinary** demand curve for commodity 1.

Own-Price Changes

- **What does a p_1 price-offer curve look like for Cobb-Douglas preferences?**

Own-Price Changes

- **What does a p_1 price-offer curve look like for Cobb-Douglas preferences?**
- **Take**

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat**

Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a

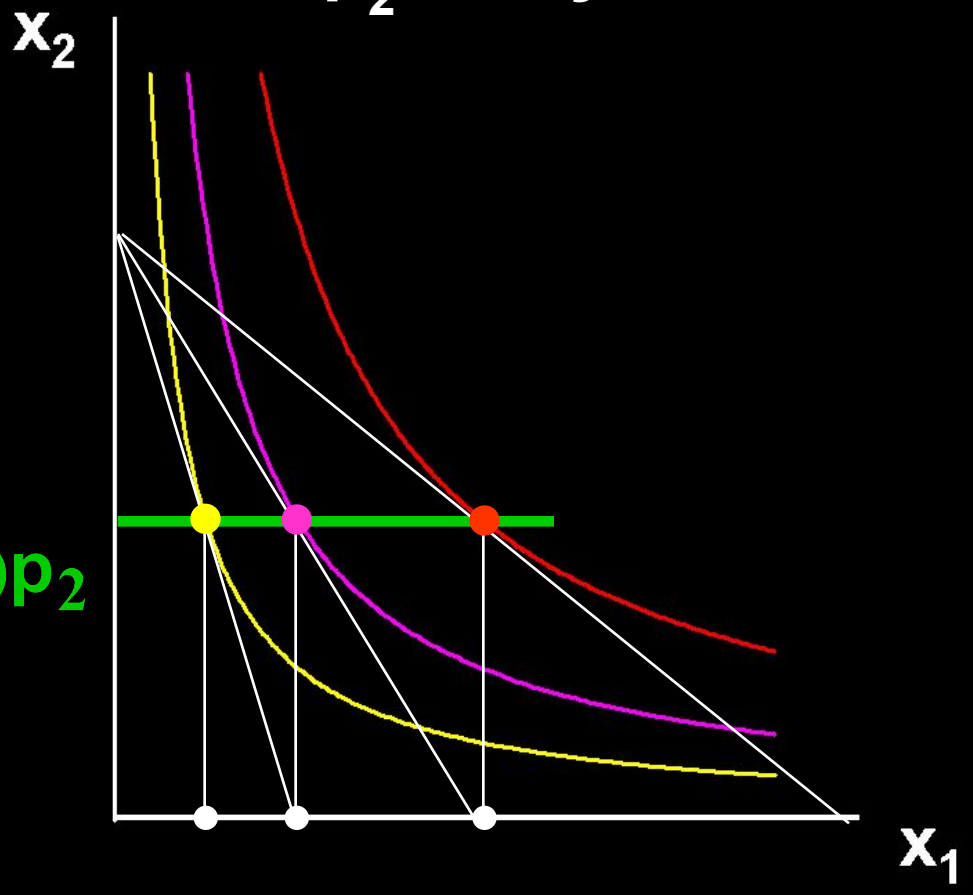
Own-Price Changes

and

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a **rectangular hyperbola**.

Own-Price Changes

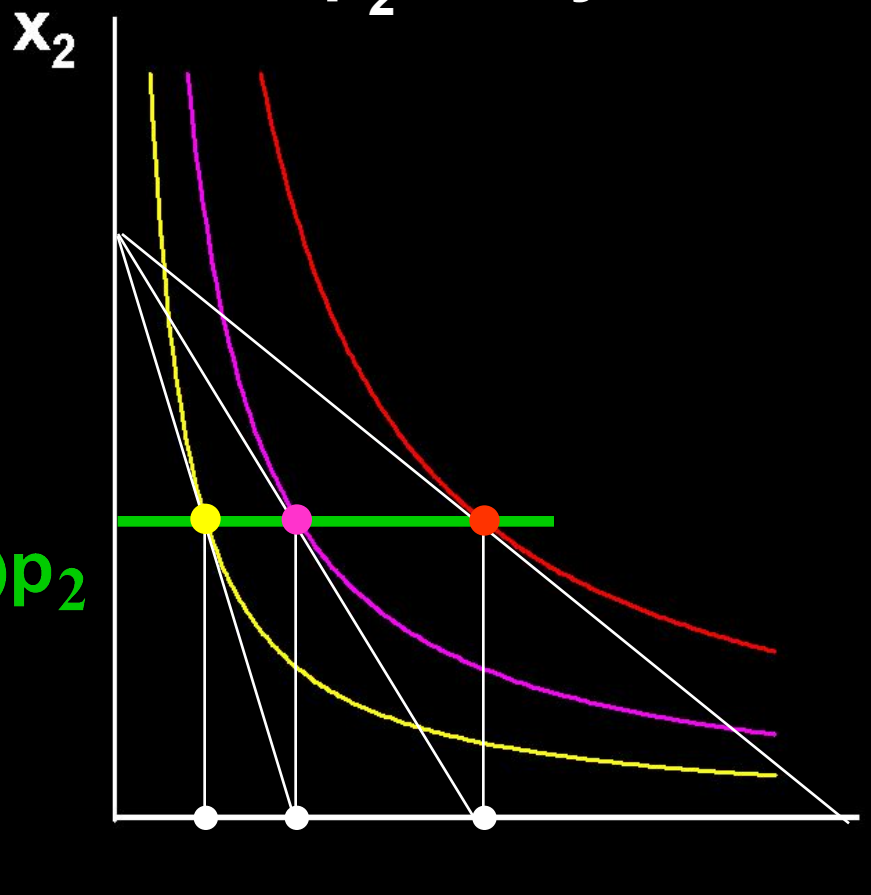
Fixed p_2 and y.



$x_2^* =$ by $(a+b)p_2$

Own-Price Changes

Fixed p_2 and y.



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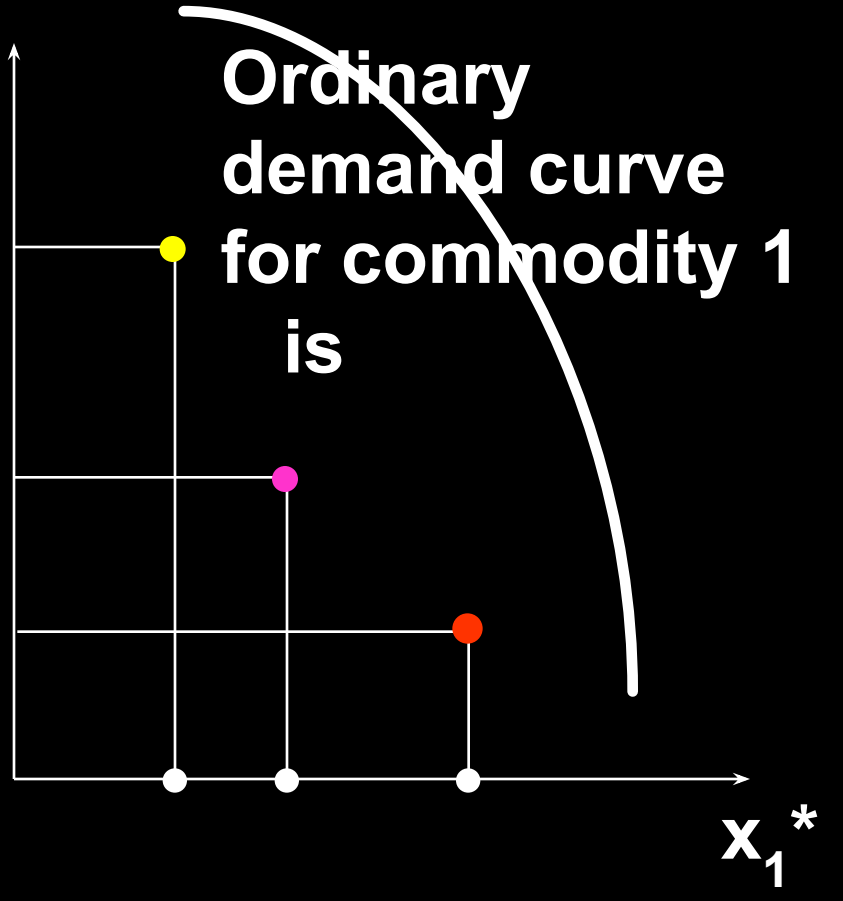

Ordinary demand curve for commodity 1 is

Own-Price Changes

- **What does a p_1 price-offer curve look like for a perfect-complements utility function?**

Own-Price Changes

- **What does a p_1 price-offer curve look like for a perfect-complements utility function?**

  Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

Own-Price Changes

With p_2 and y fixed, higher p_1 causes smaller x_1^* and x_2^*.

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As

Own-Price Changes

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Own-Price Changes

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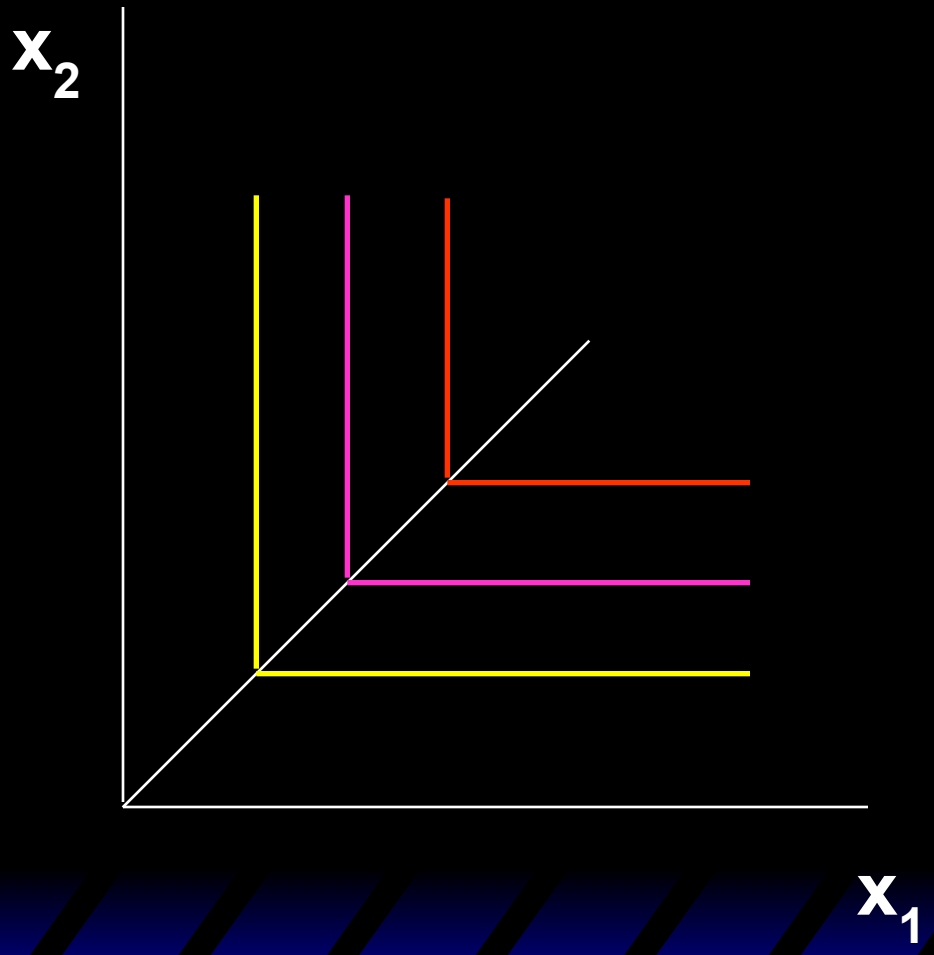

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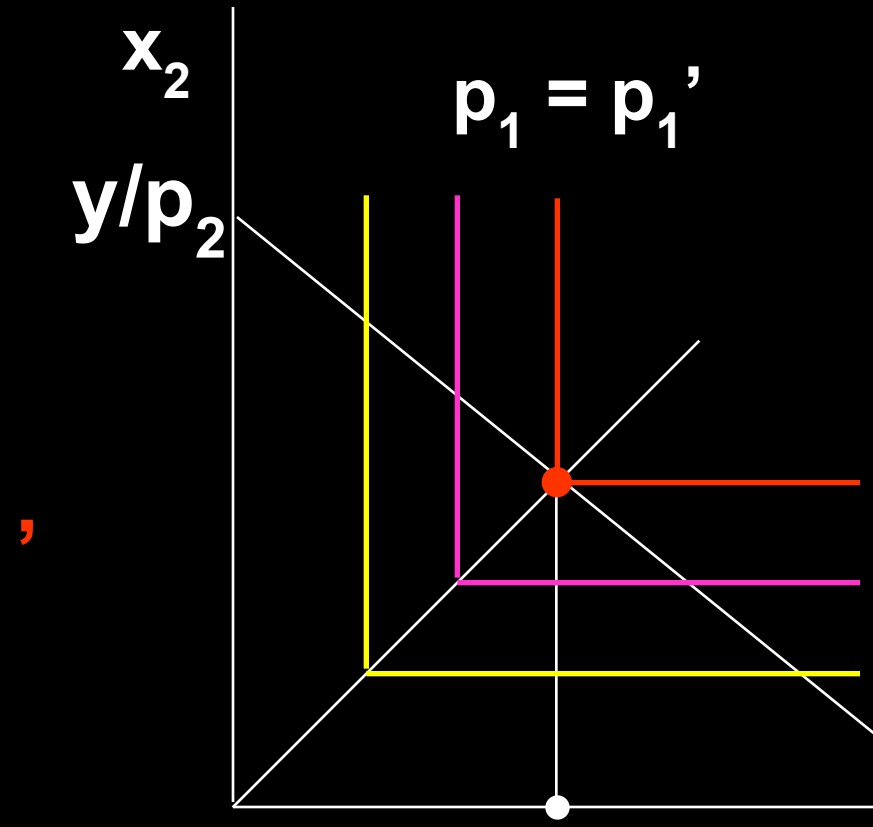
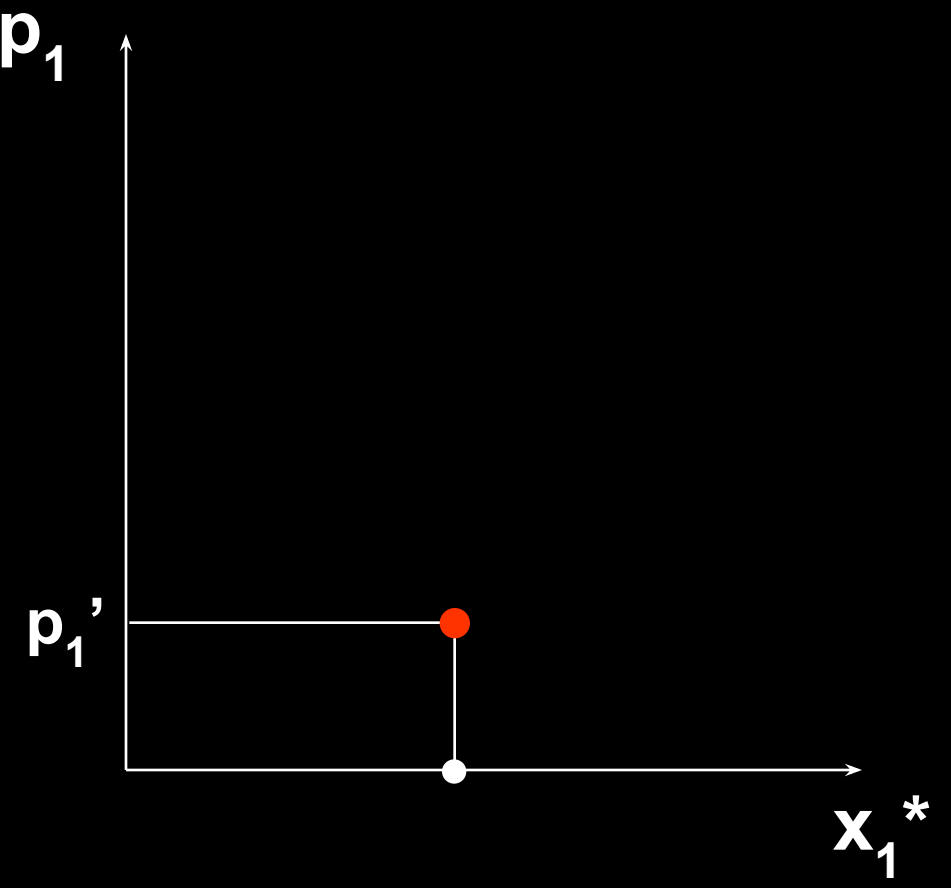
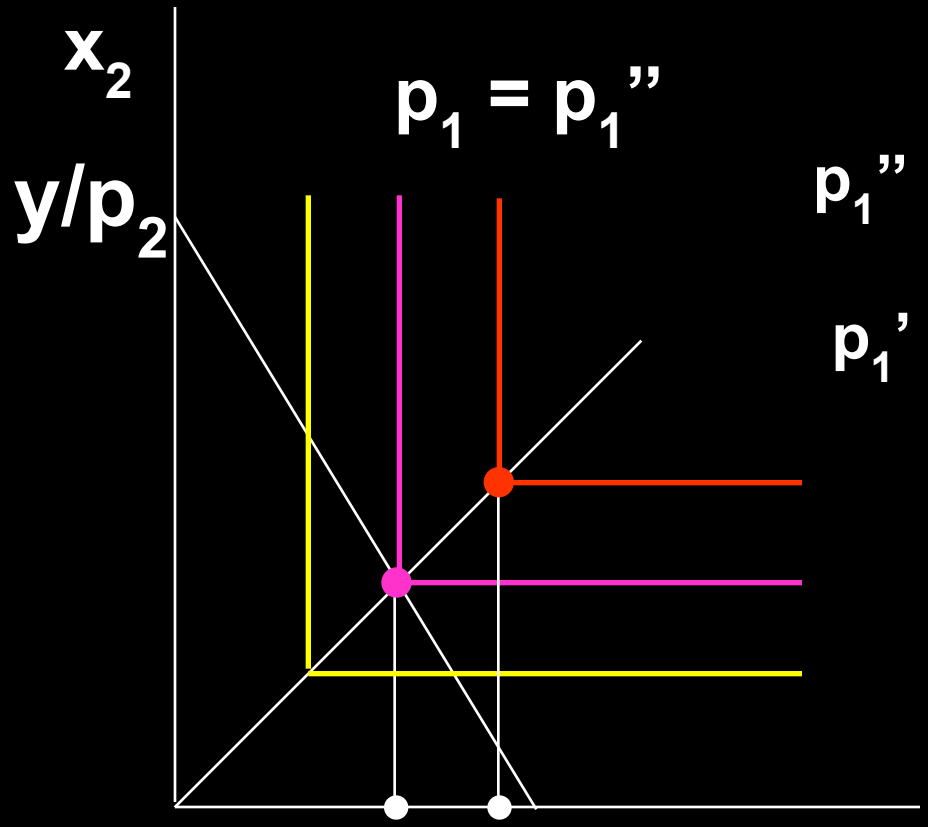


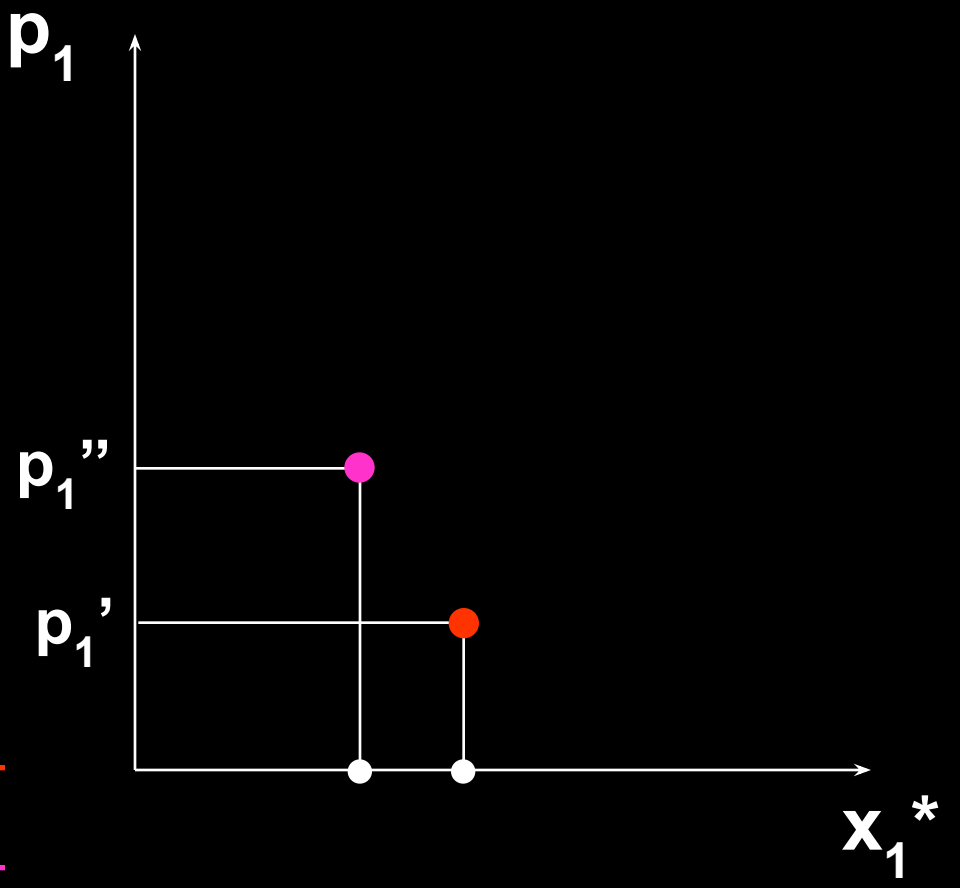


$$x_1 = \frac{y}{p_1' + p_2}$$

Own-Price Changes

Fixed p_2 and y.



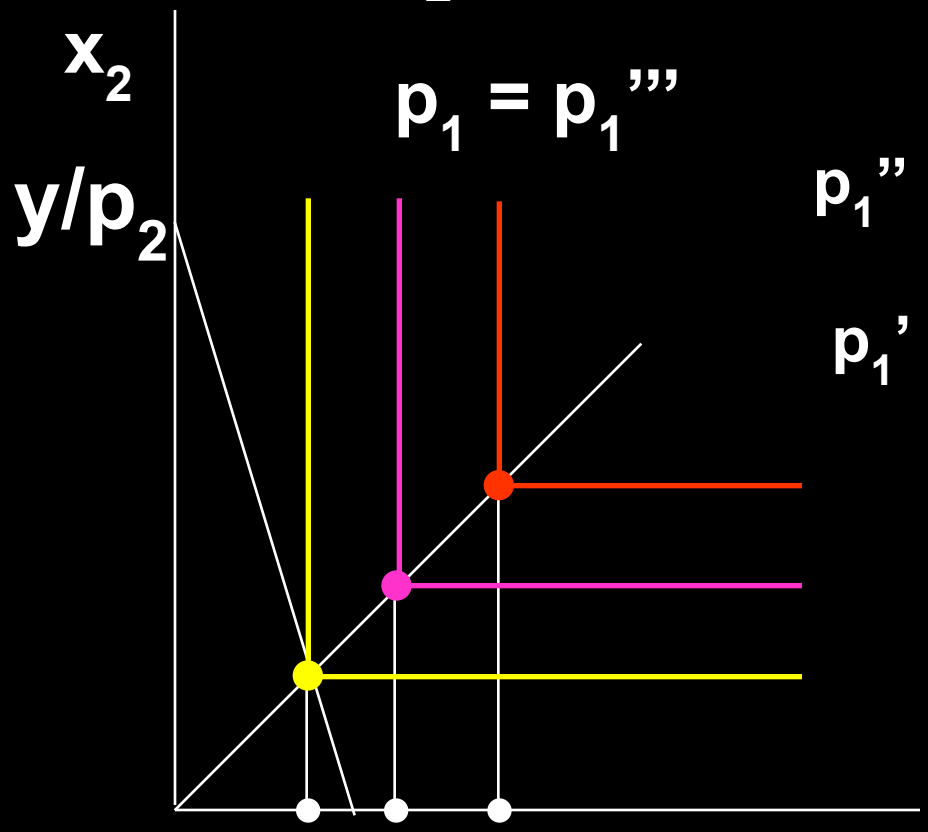


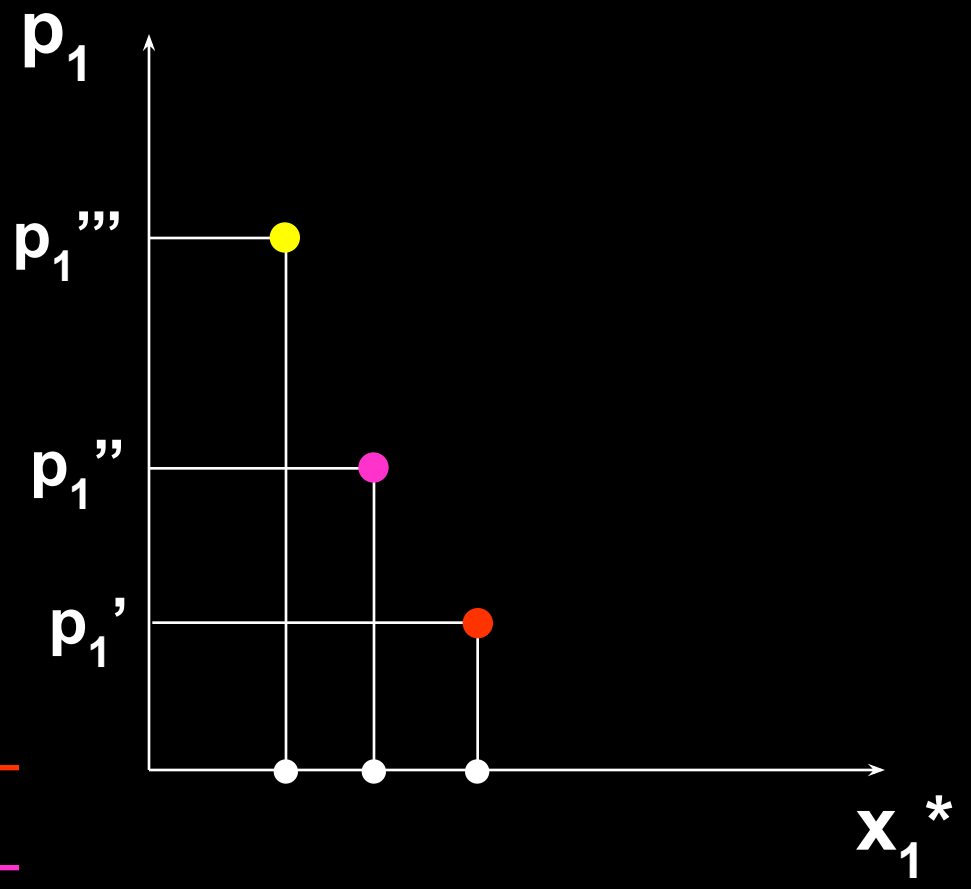
$$x_1'' = \frac{y}{p_1'' + p_2}$$

x_1

Own-Price Changes

Fixed p_2 and y.

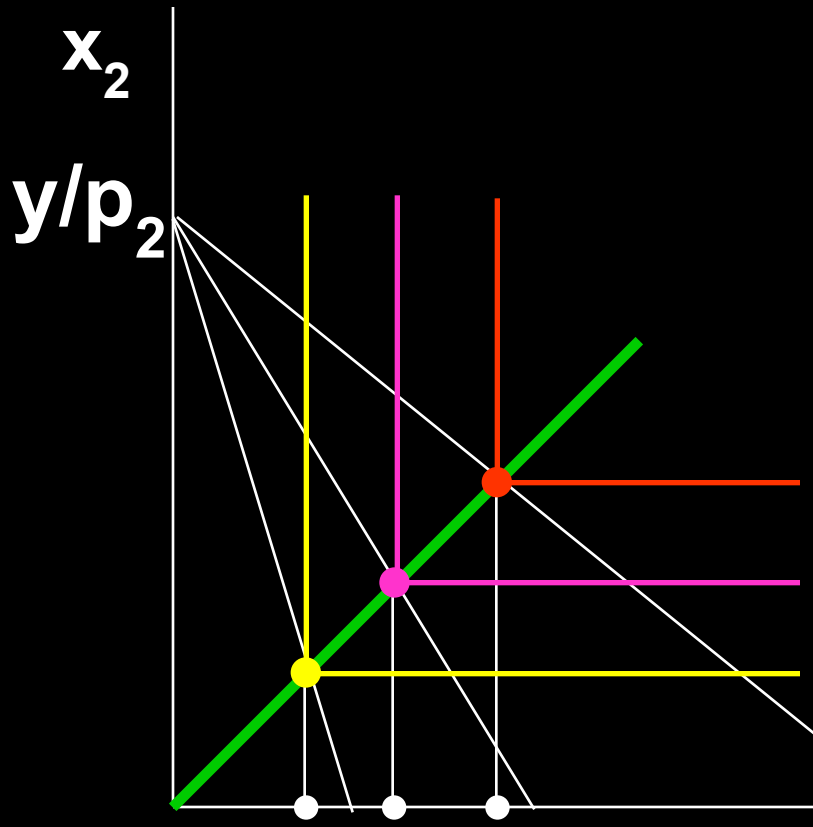


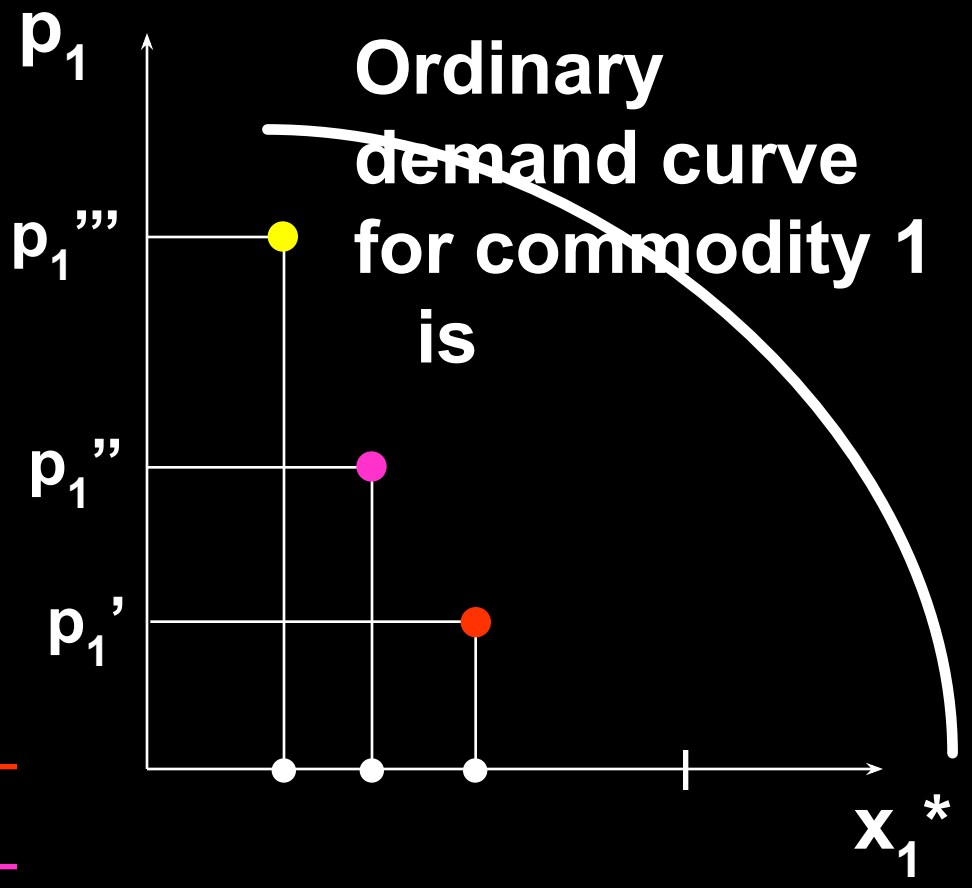


$$x_1 = \frac{y}{p_1''' + p_2}$$

Own-Price Changes

Fixed p_2 and y.





Ordinary demand curve for commodity 1 is

$$x_1 = \frac{y}{p_1 + p_2}$$

Own-Price Changes

- **What does a p_1 price-offer curve look like for a perfect-substitutes utility function?**

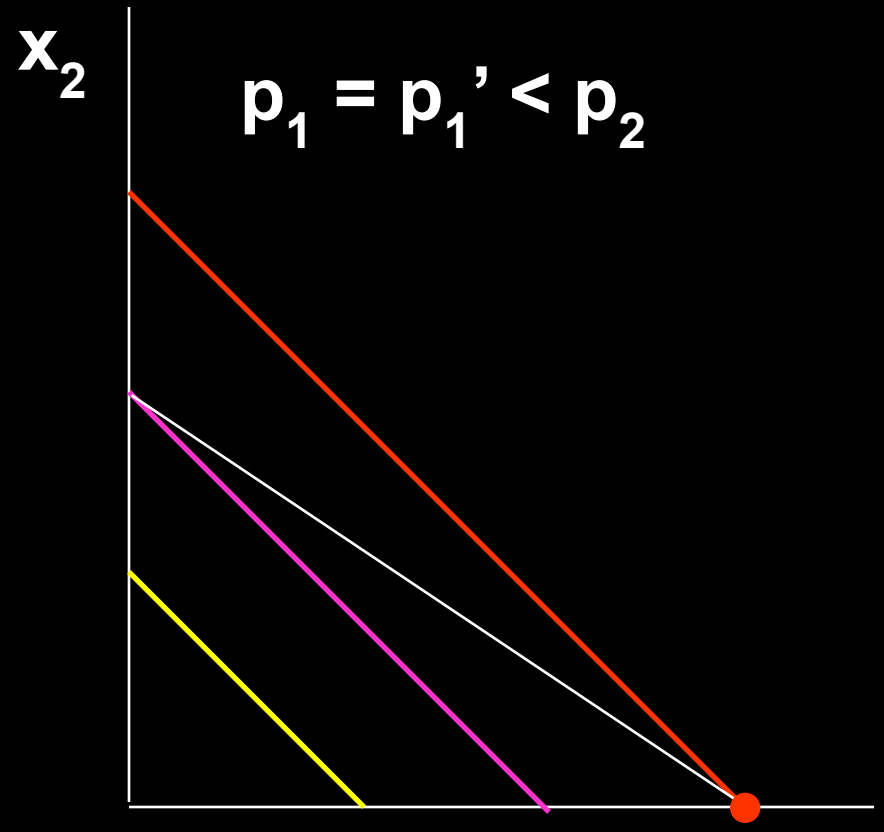
  Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

and

Own-Price Changes

Fixed p_2 and y.



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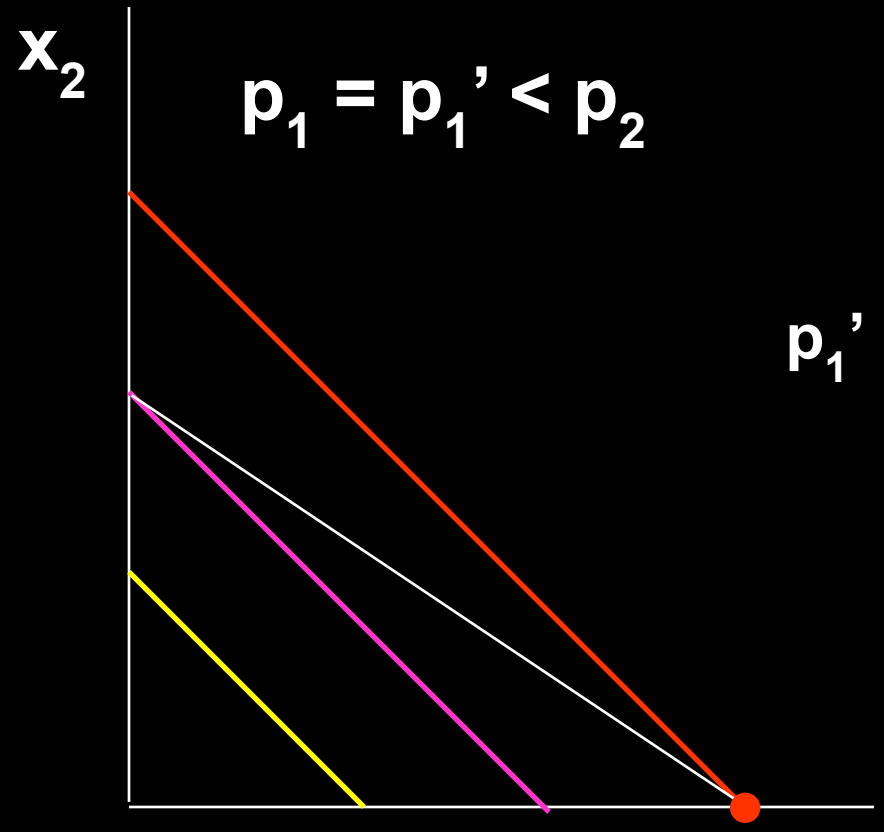

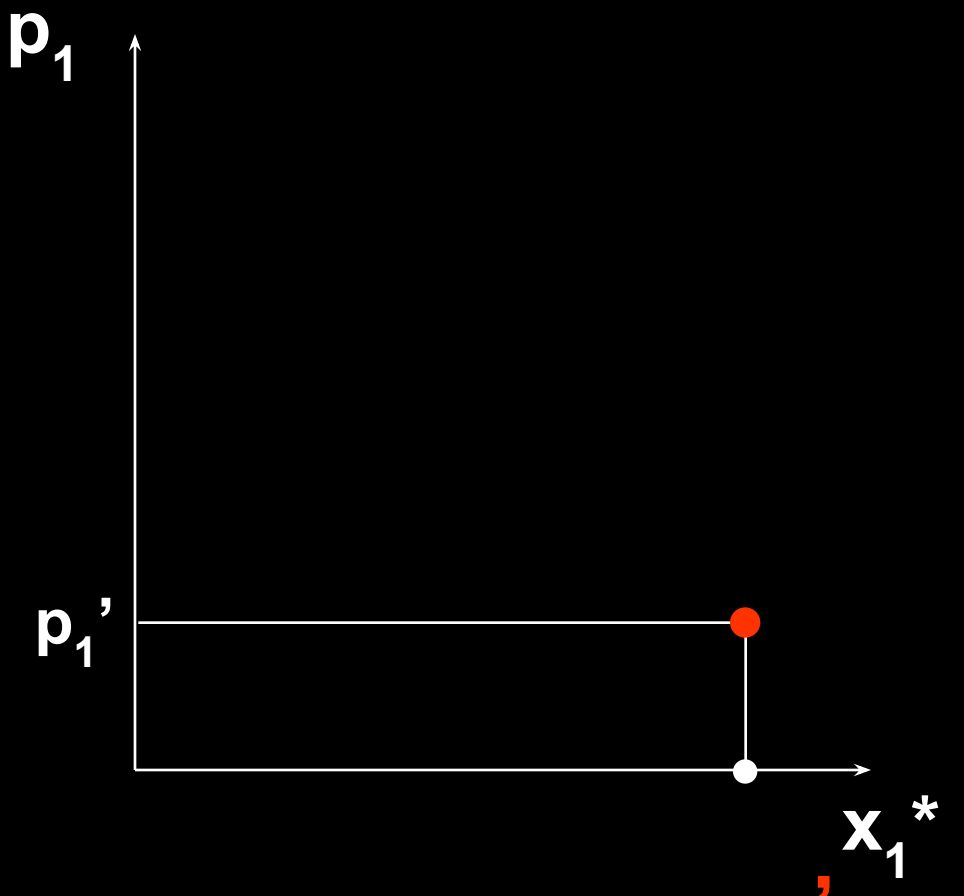

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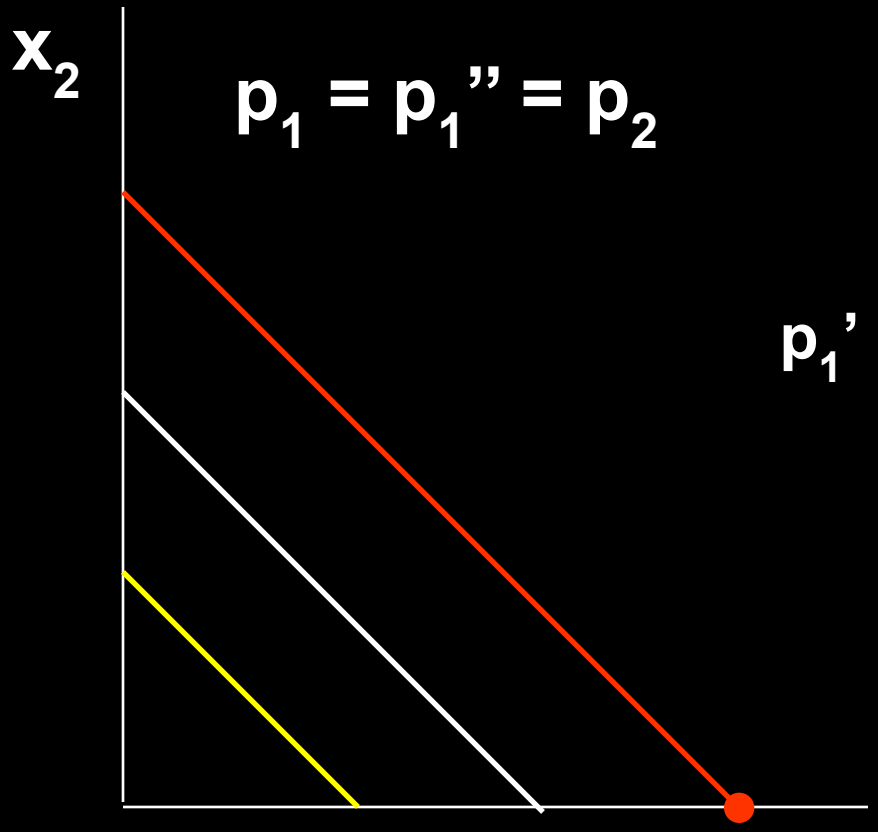

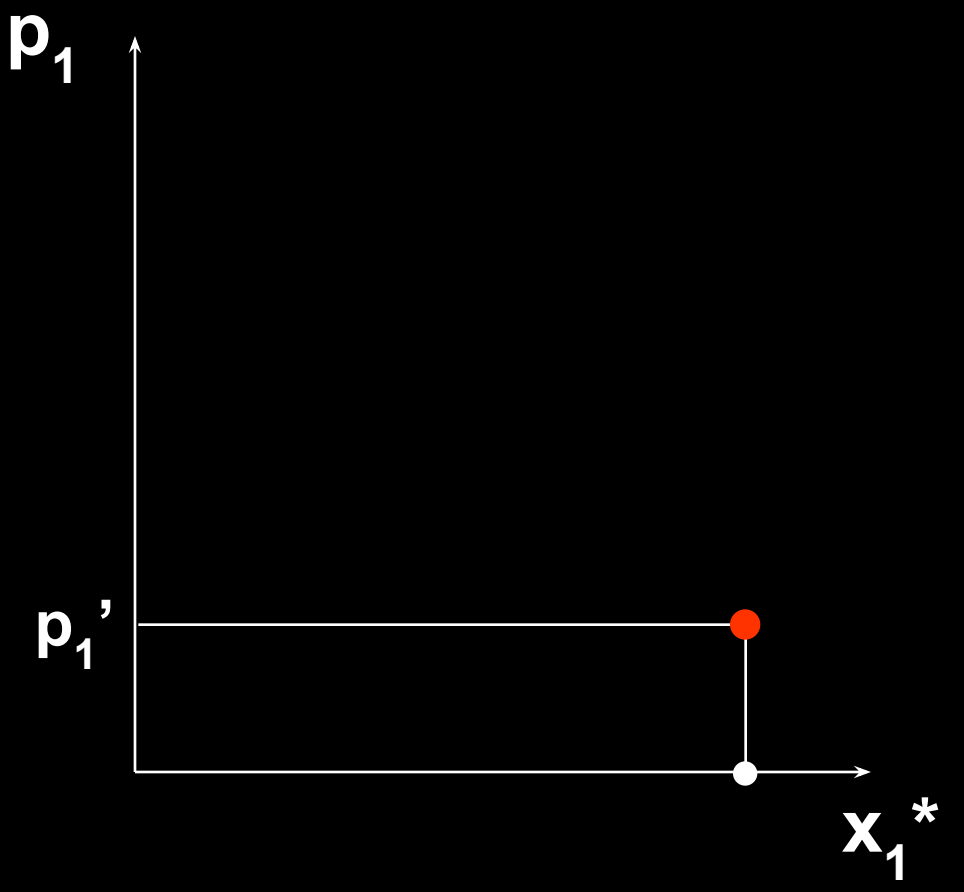

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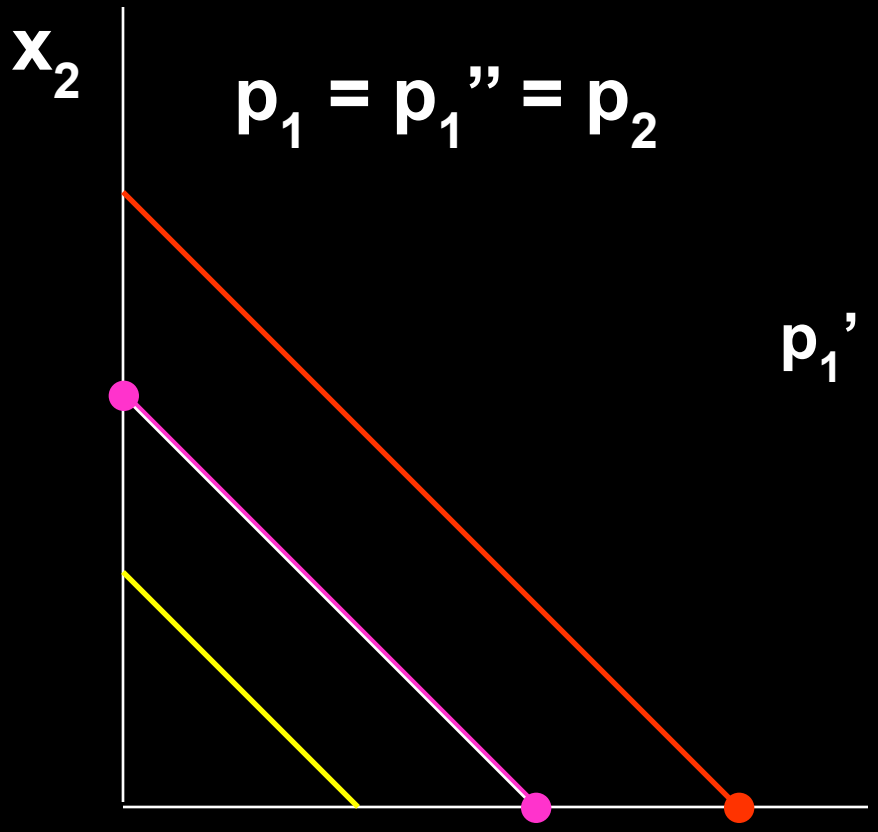

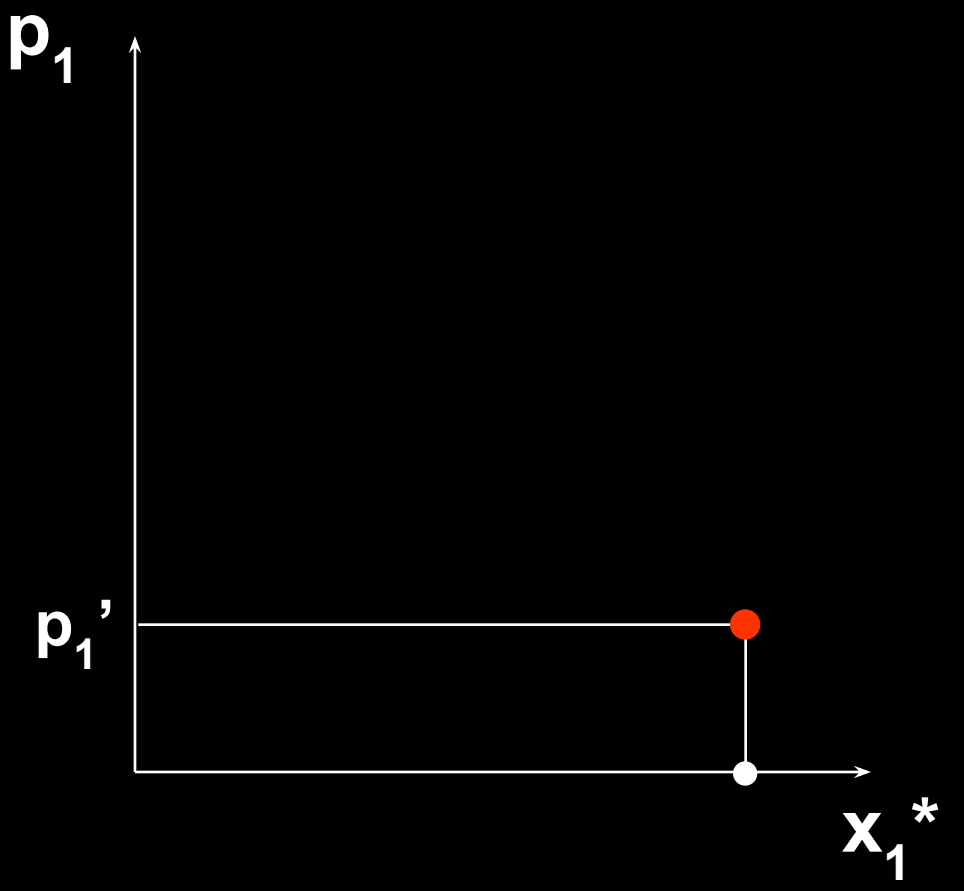

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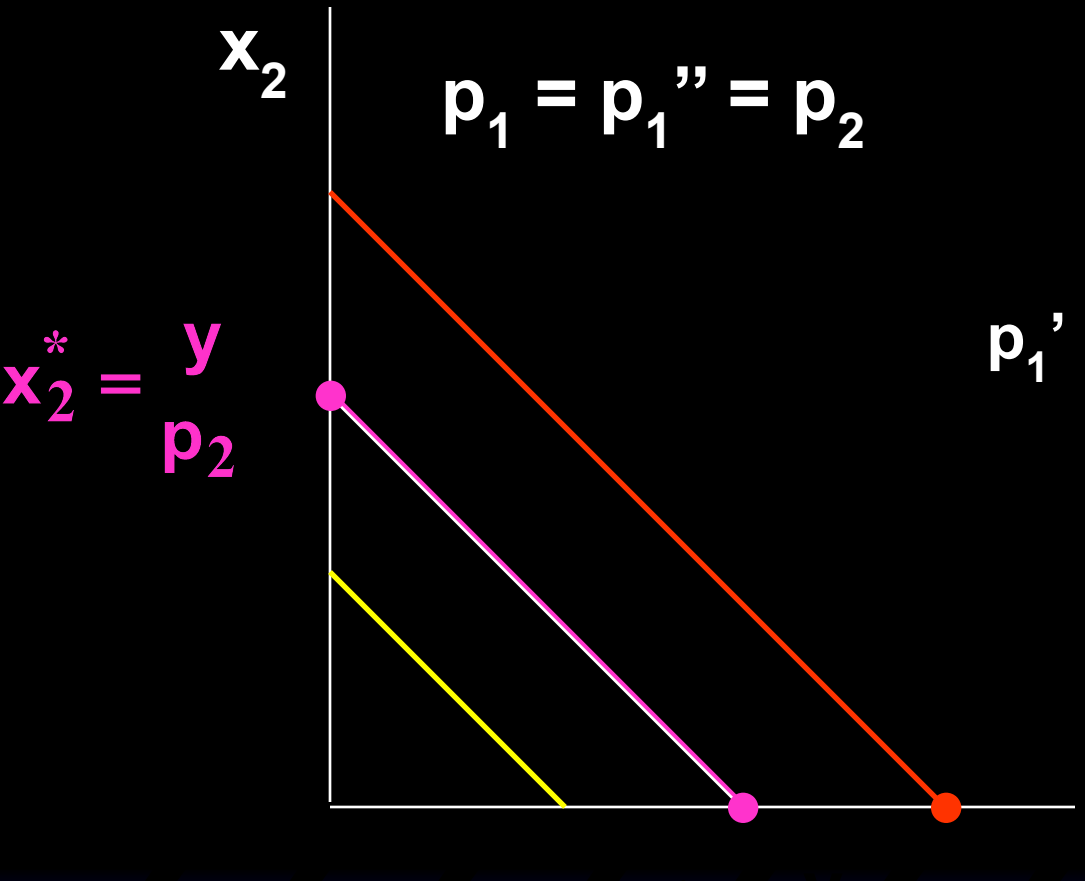

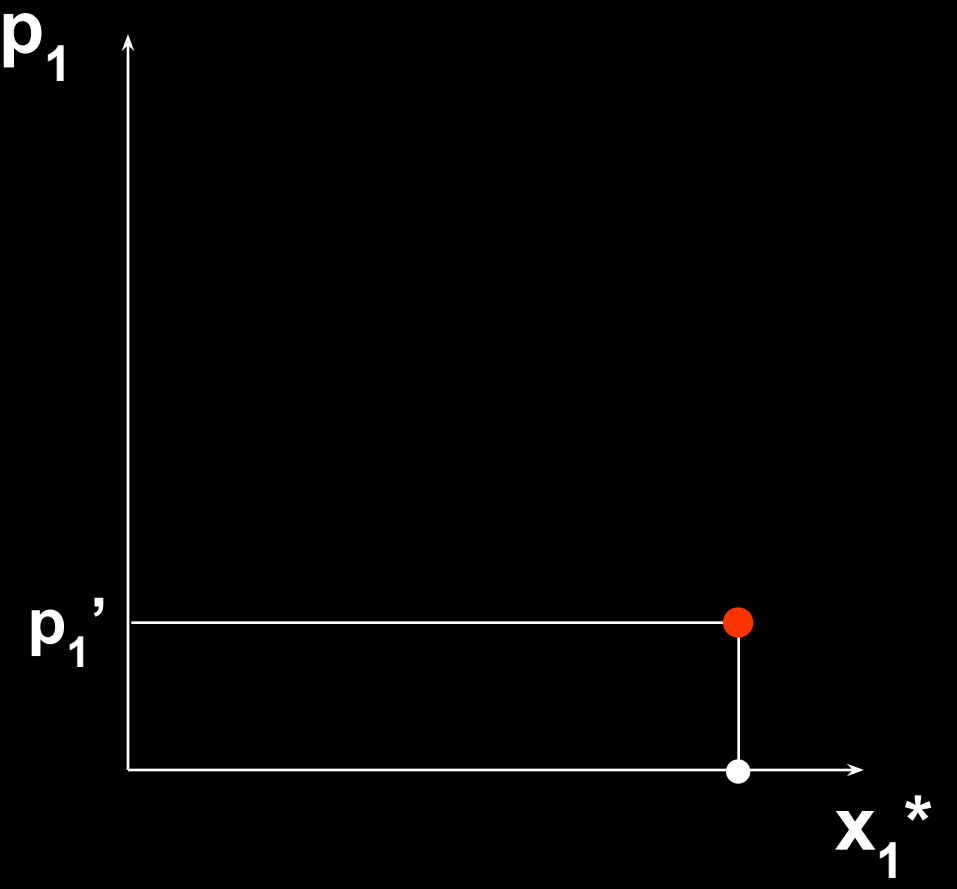

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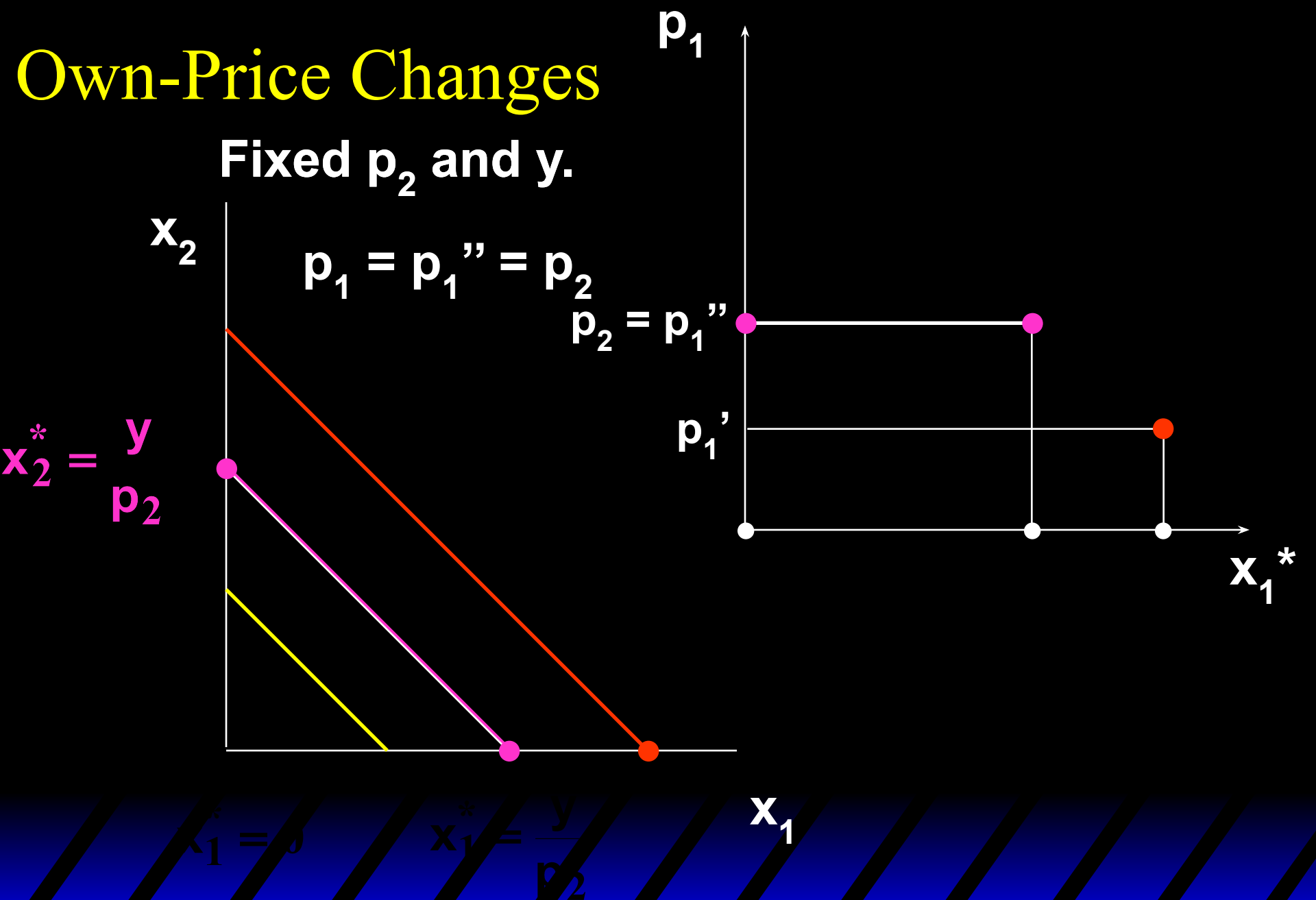

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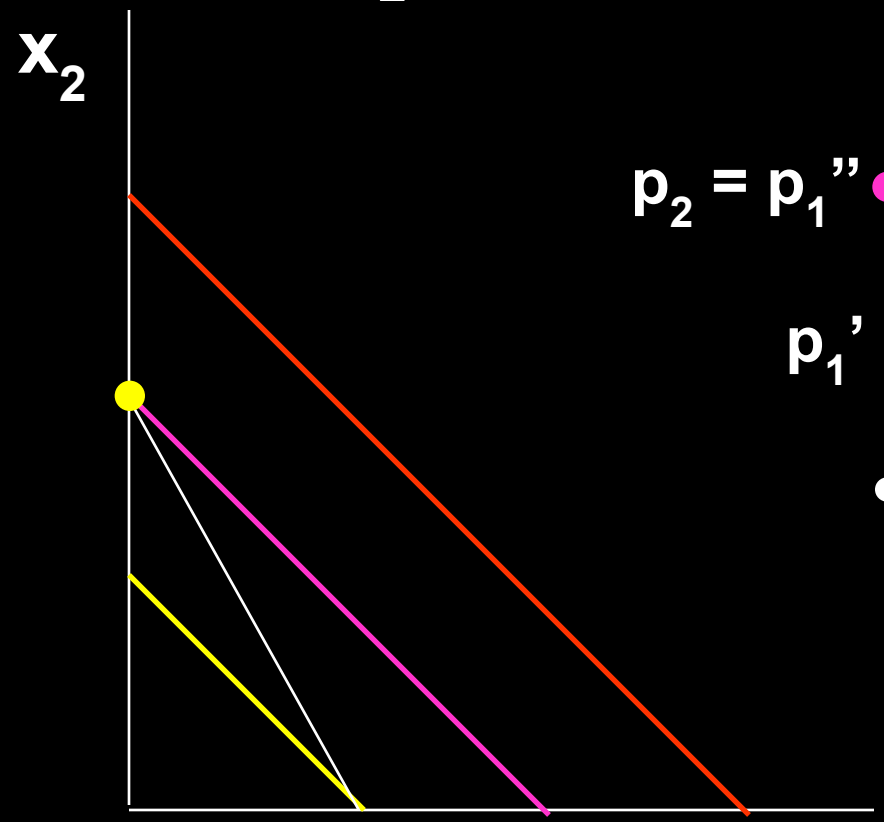

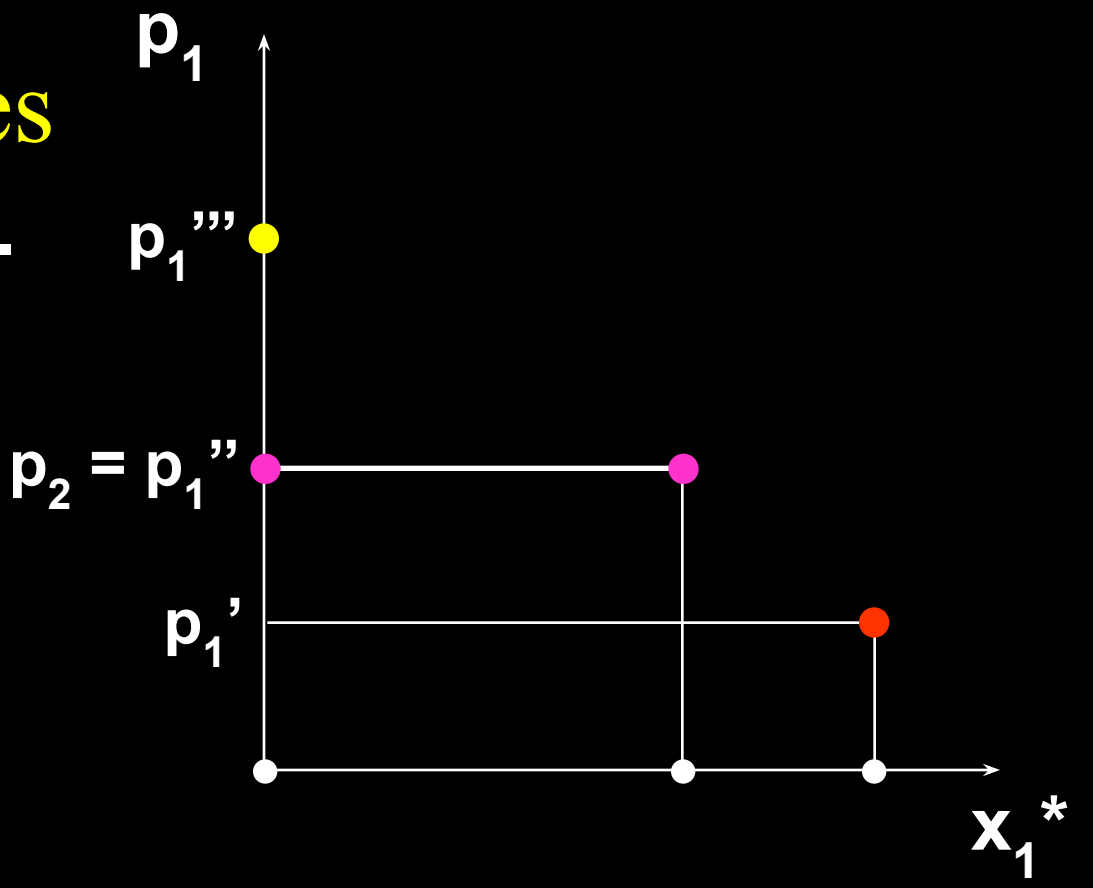

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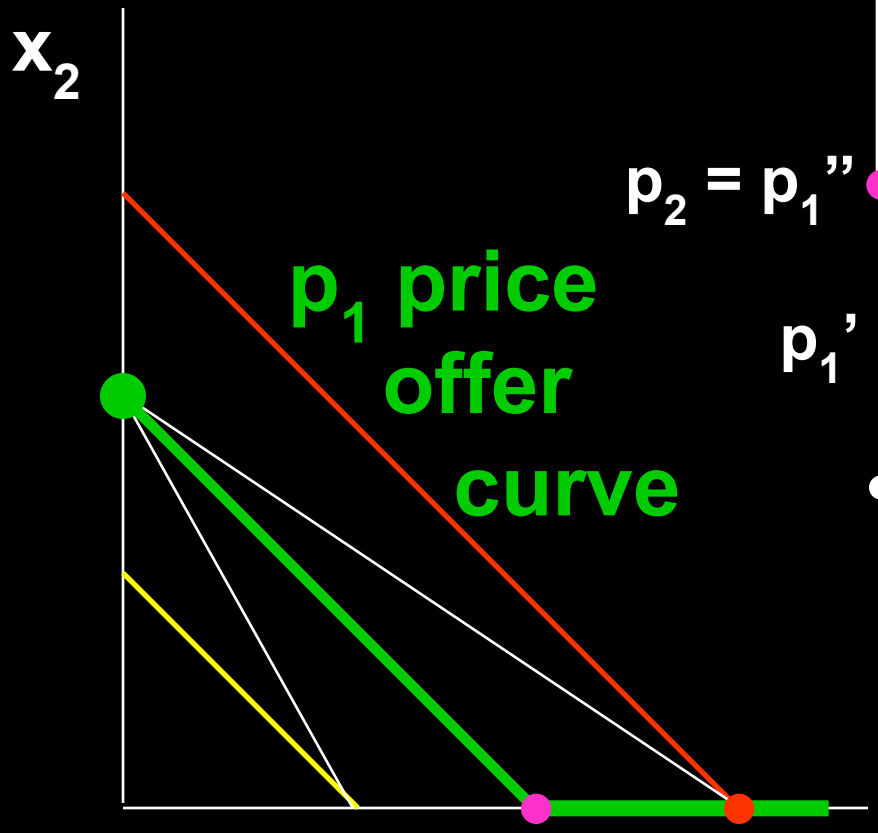
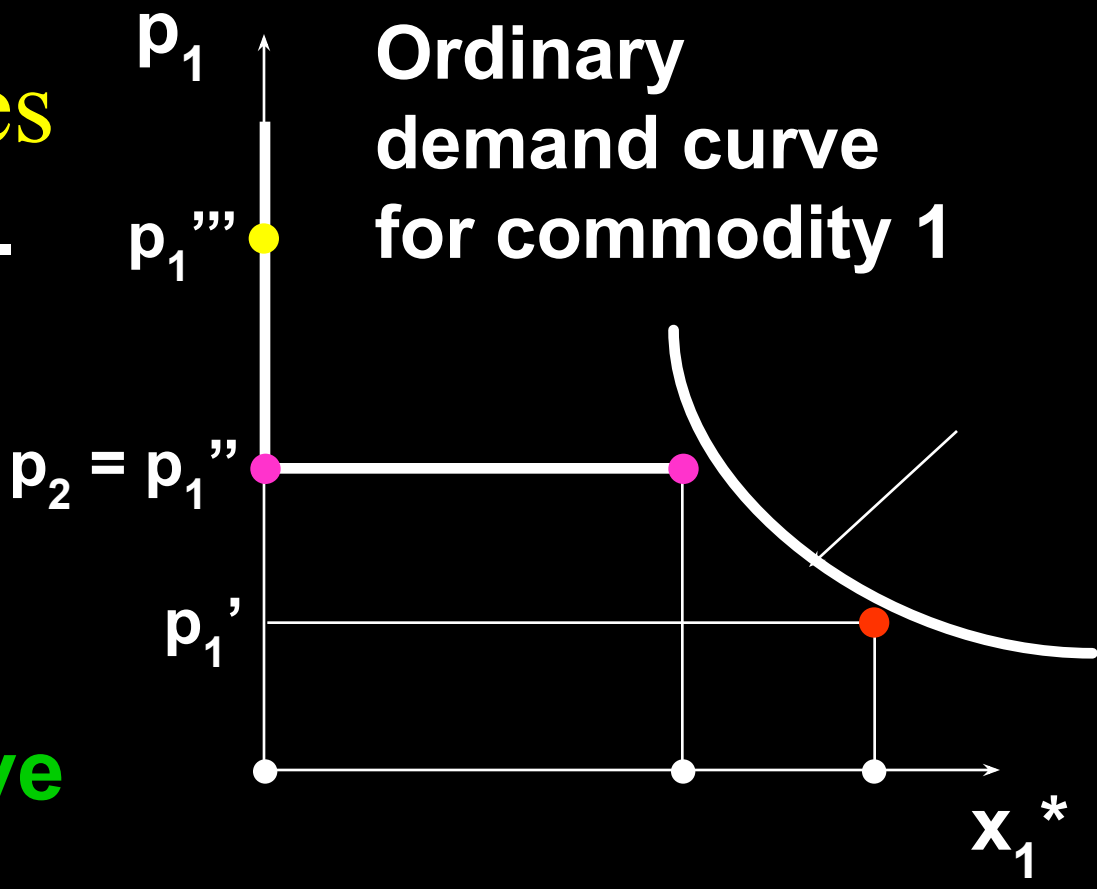


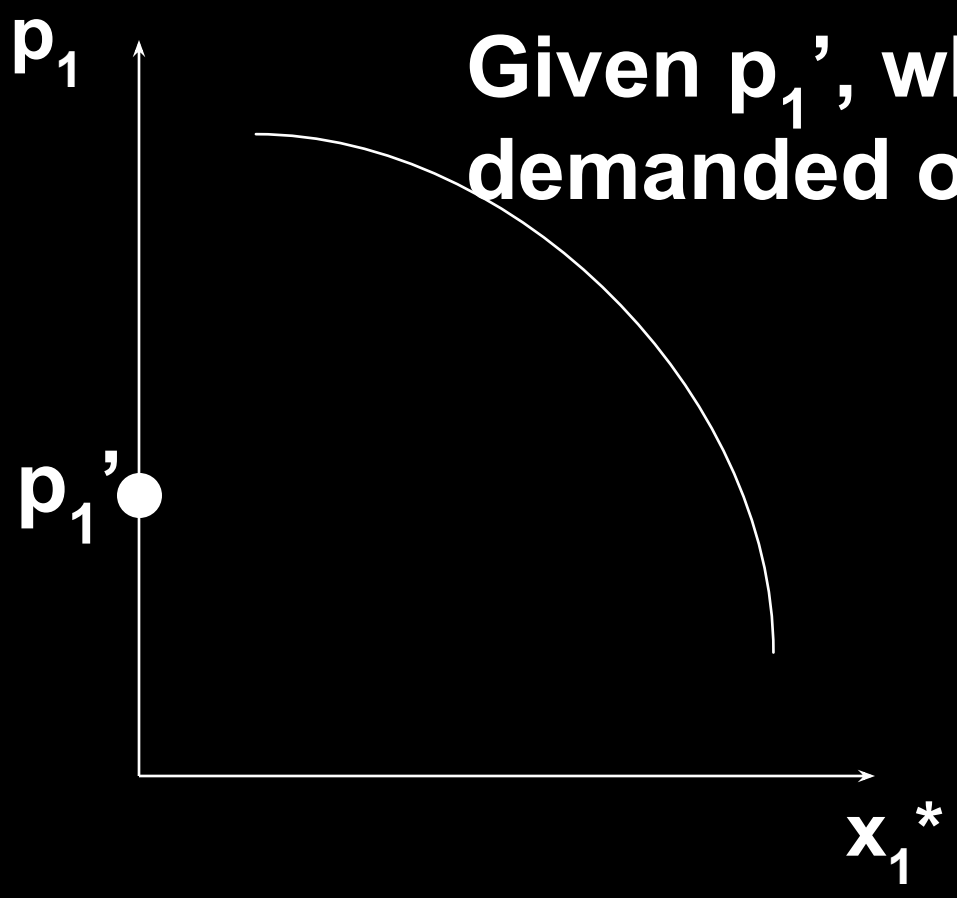


Ordinary demand curve for commodity 1

Own-Price Changes

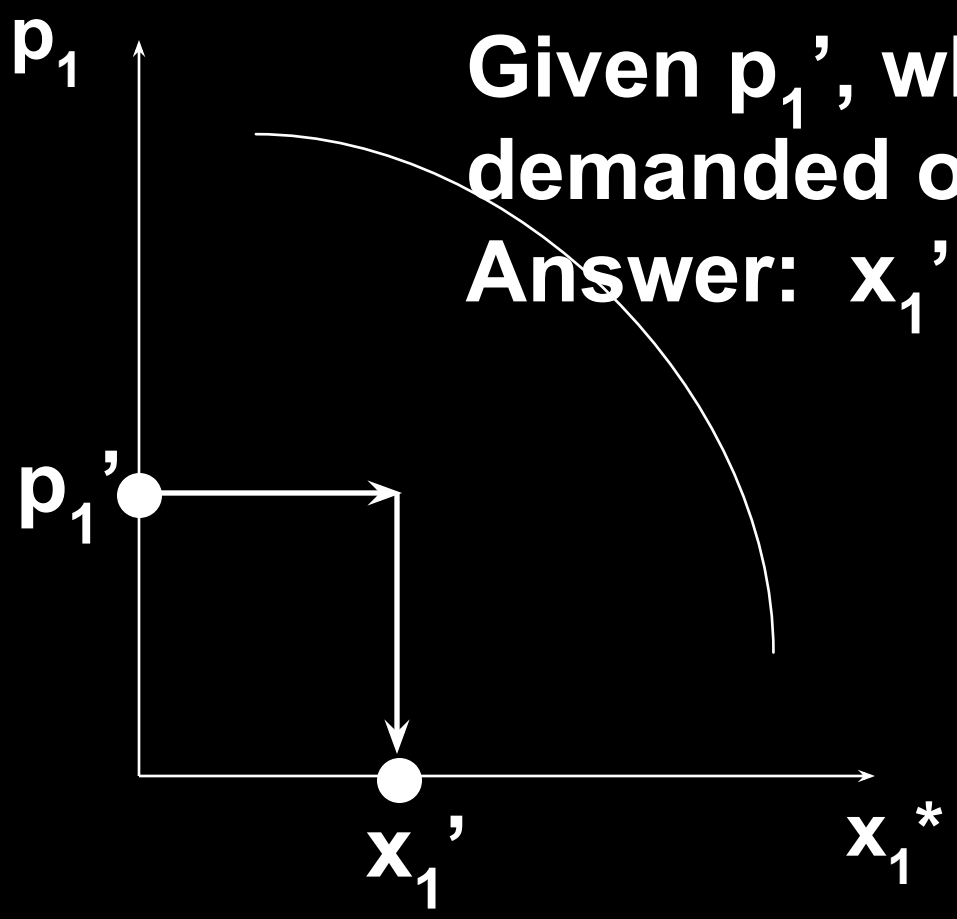
- Usually we ask “Given the price for commodity 1 what is the quantity demanded of commodity 1?”
- But we could also ask the **inverse** question “At what price for commodity 1 would a given quantity of commodity 1 be demanded?”

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?

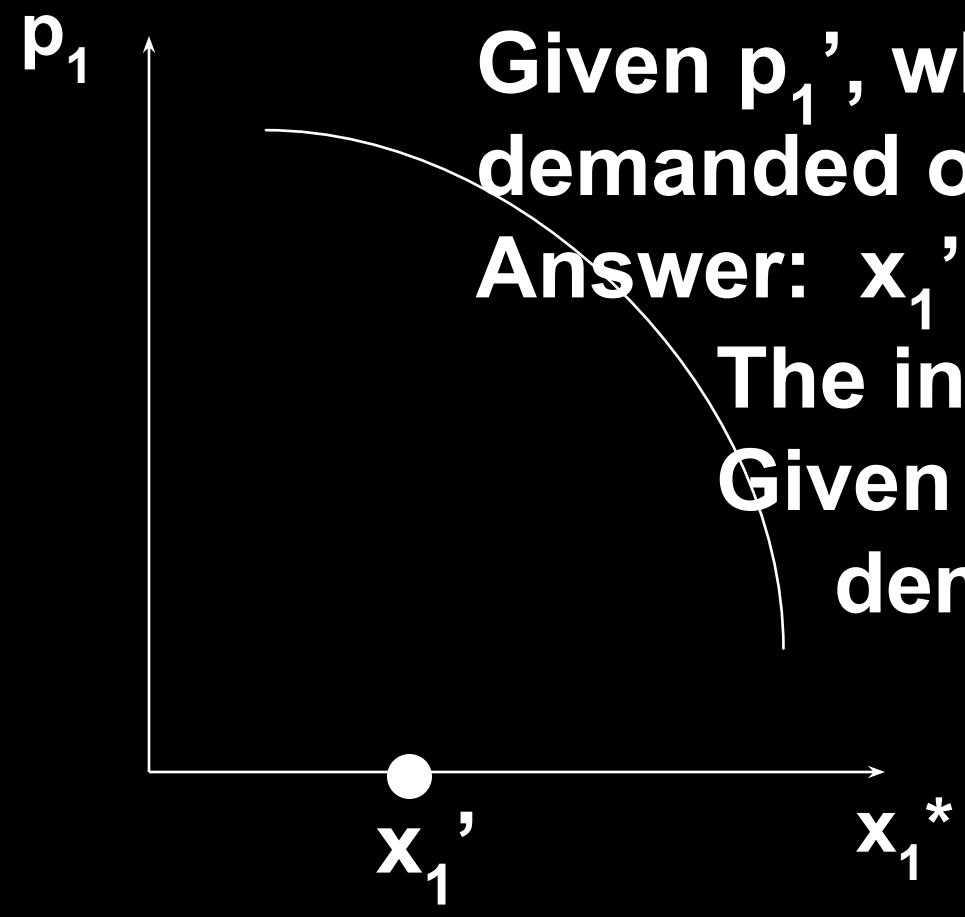
Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?

Answer: x_1' units.

Own-Price Changes

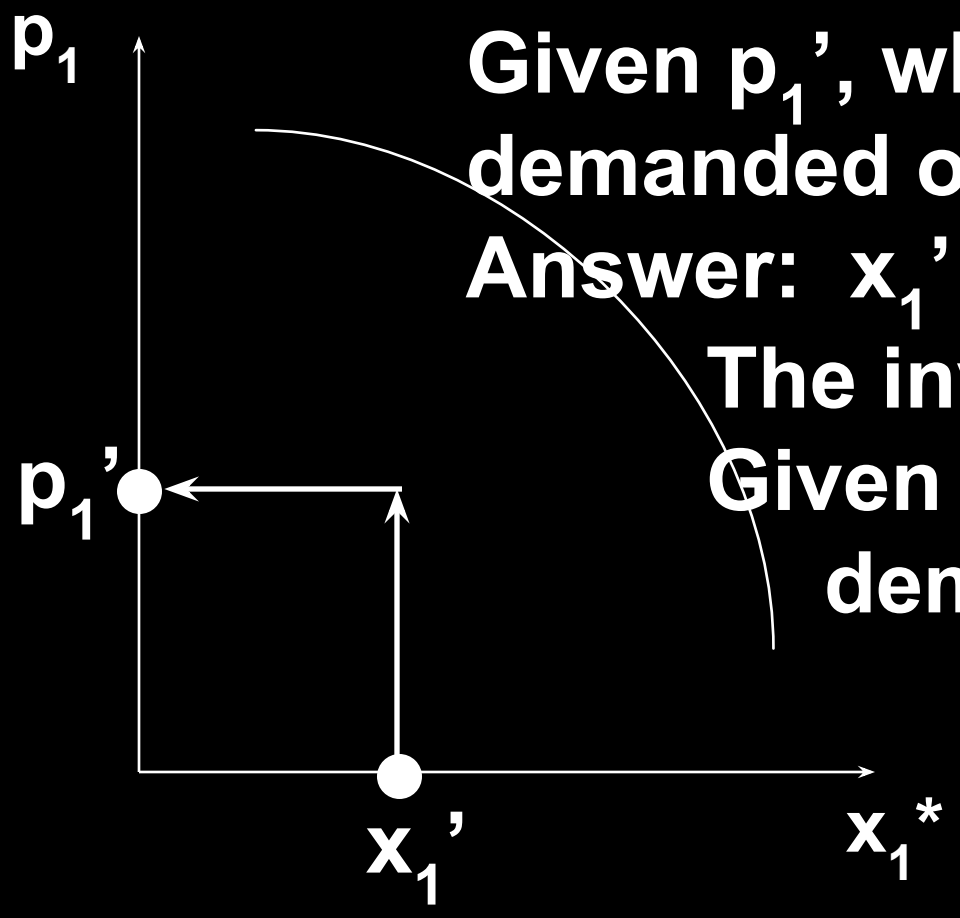


Given p_1', what quantity is demanded of commodity 1?

Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?

Answer: x_1' units.

The inverse question is:

Given x_1' units are demanded, what is the price of commodity 1?

Answer: p_1'

Own-Price Changes

- **Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.**

Own-Price Changes

- **Inverse demand function**

At optimal choice

$|MRS| = p_1/p_2$

Therefore:

$p_1 = p_2 |MRS|$

If $p_2 = 1$, then inverse demand function simply measures MRS, i.e. how much of a good 2 consumer would want to have to compensate for a small reduction in amount of good 1.

Own-Price Changes

- **Inverse demand function**

  If good 2 is money, then MRS (and inverse demand function) measure marginal willingness to pay.

Own-Price Changes

A Cobb-Douglas example:

is the ordinary demand function and

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

is the ordinary demand function and

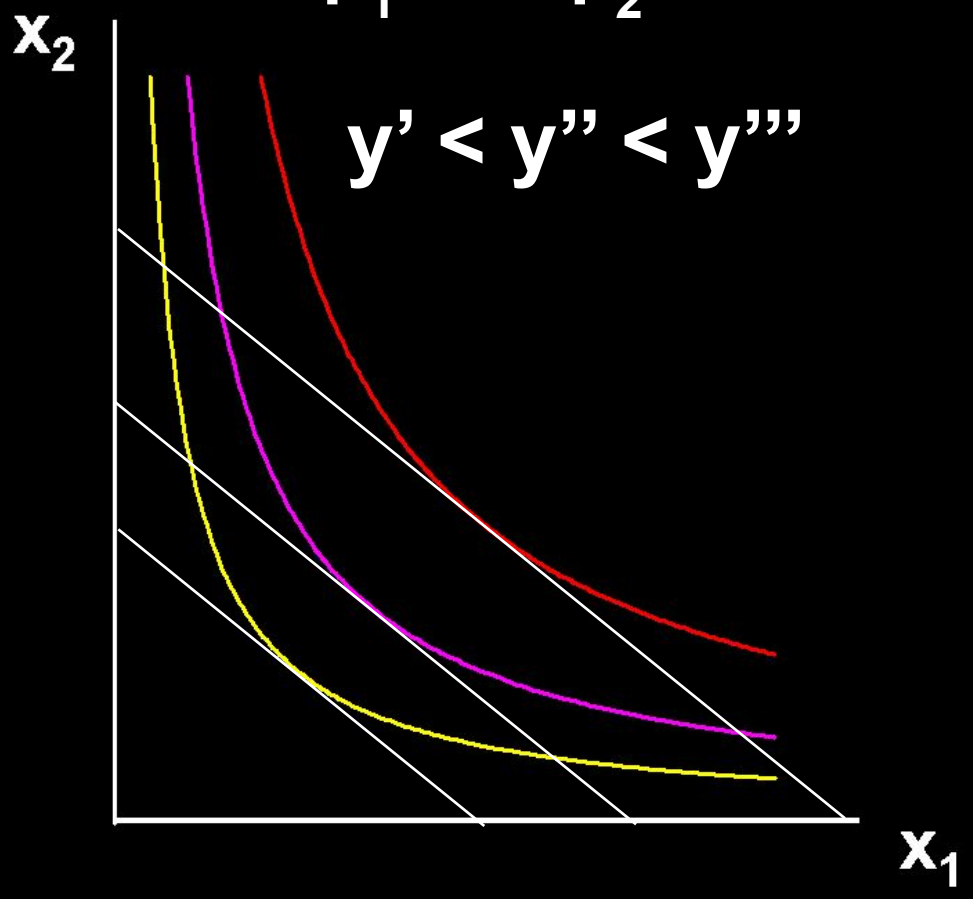
is the inverse demand function.

Income Changes

- How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both p_1 and p_2 constant?

Income Changes

Fixed p_1 and p_2.



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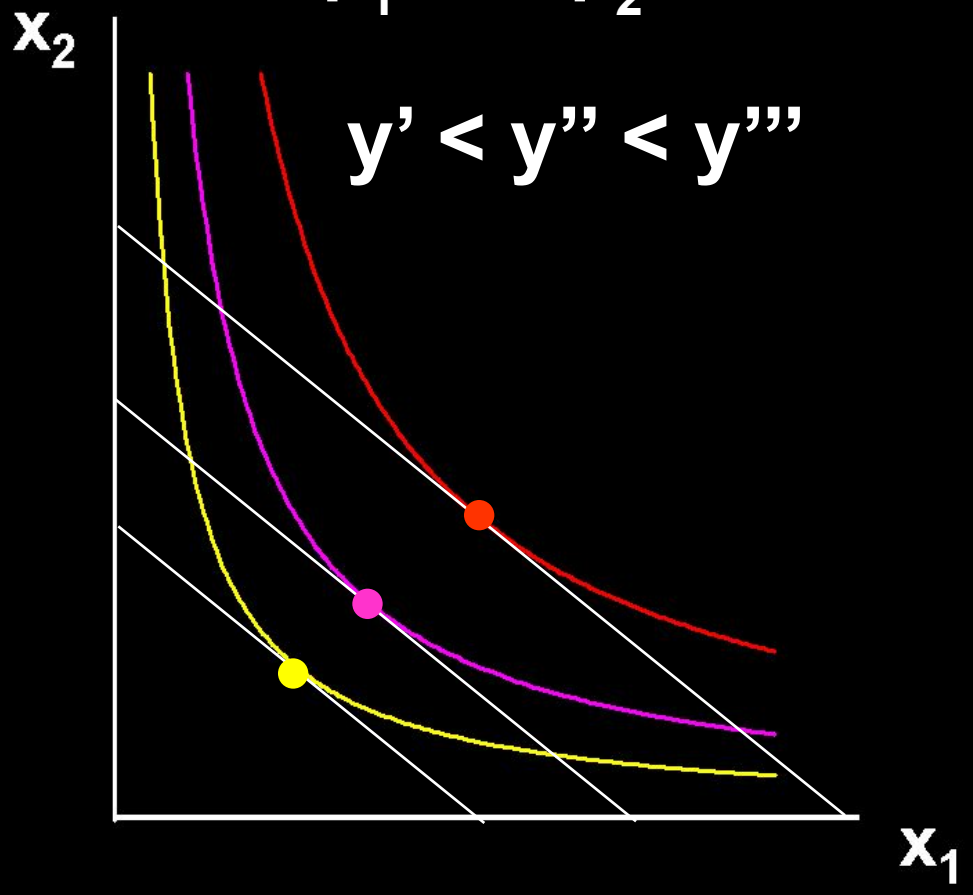

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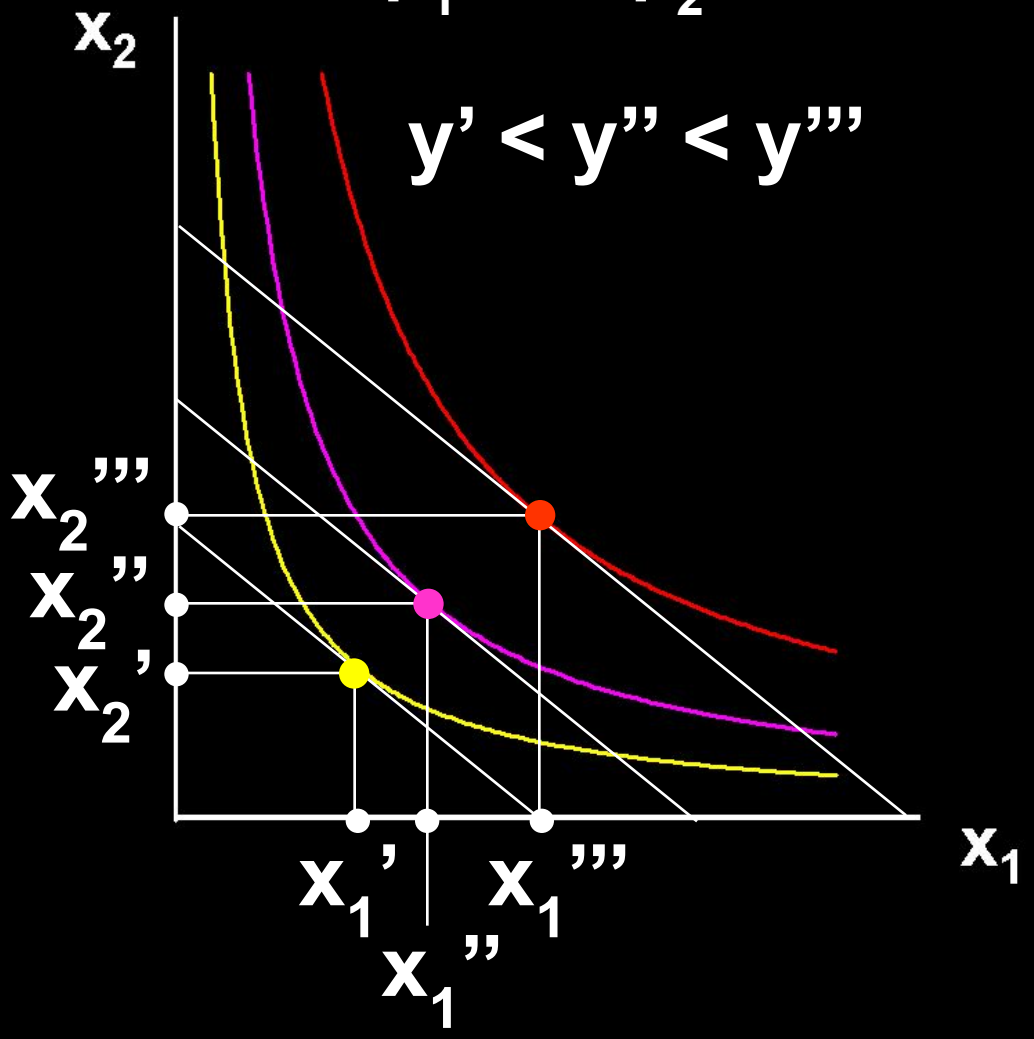
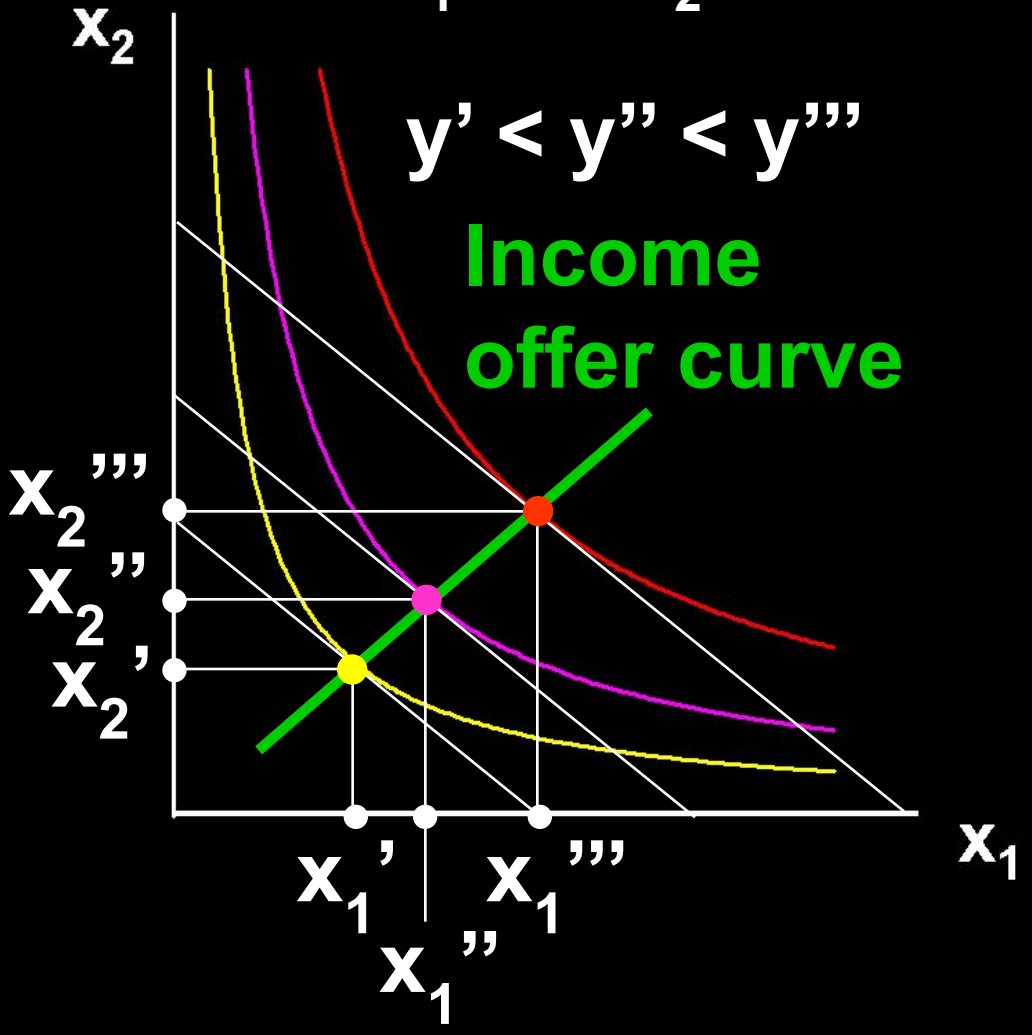


Income Changes

Fixed p_1 and p_2.

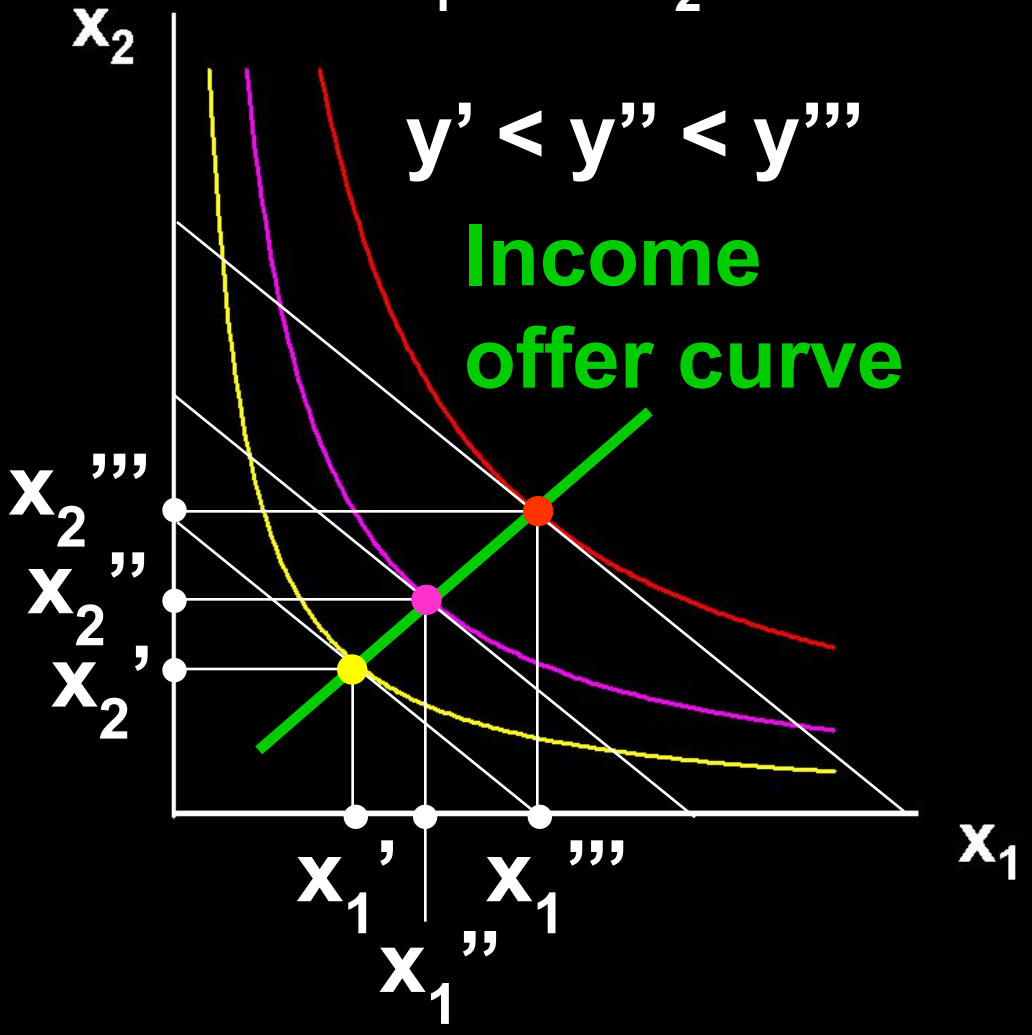


Income Changes

- **A plot of quantity demanded against income is called an Engel curve.**

Income Changes

Fixed p_1 and p_2.



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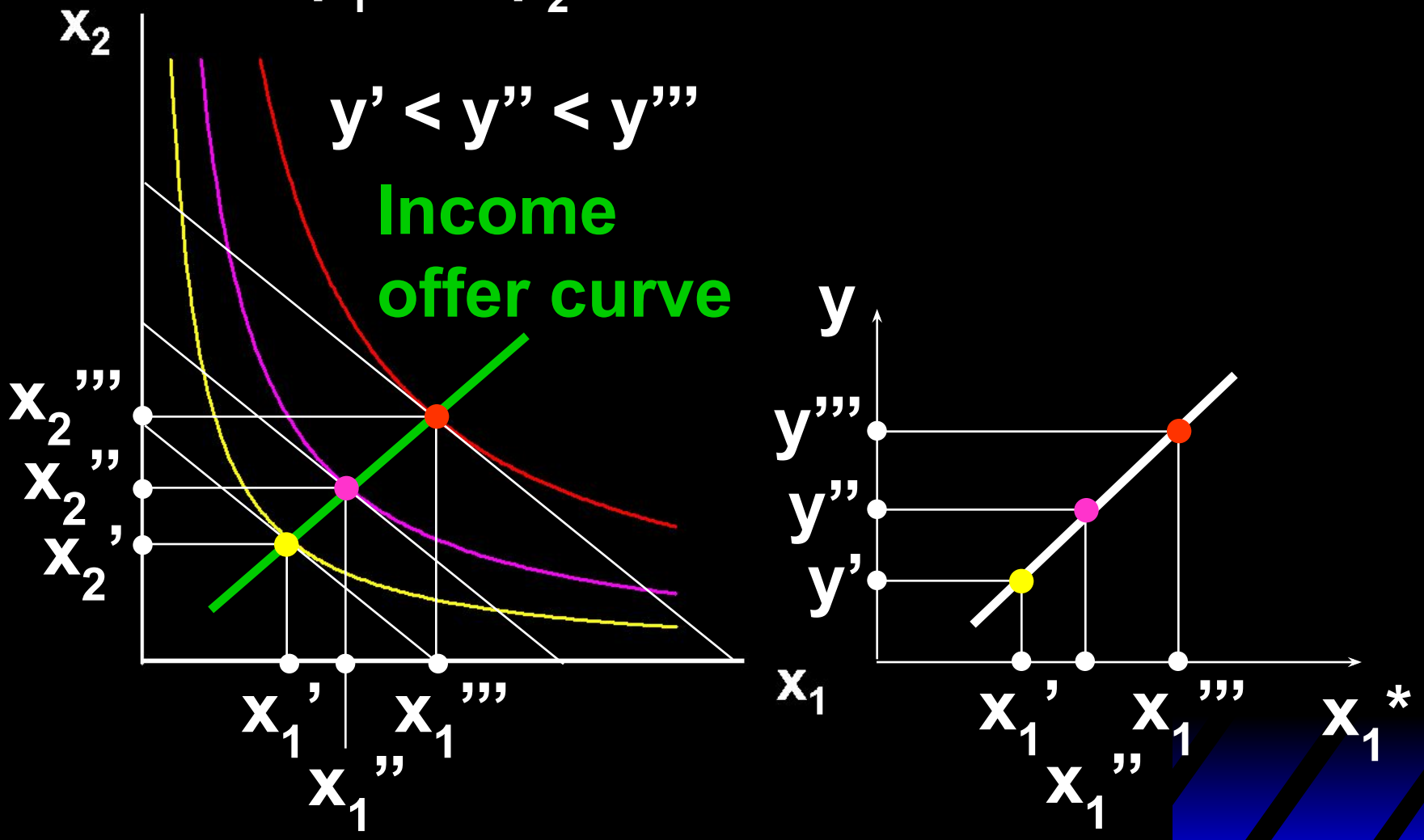
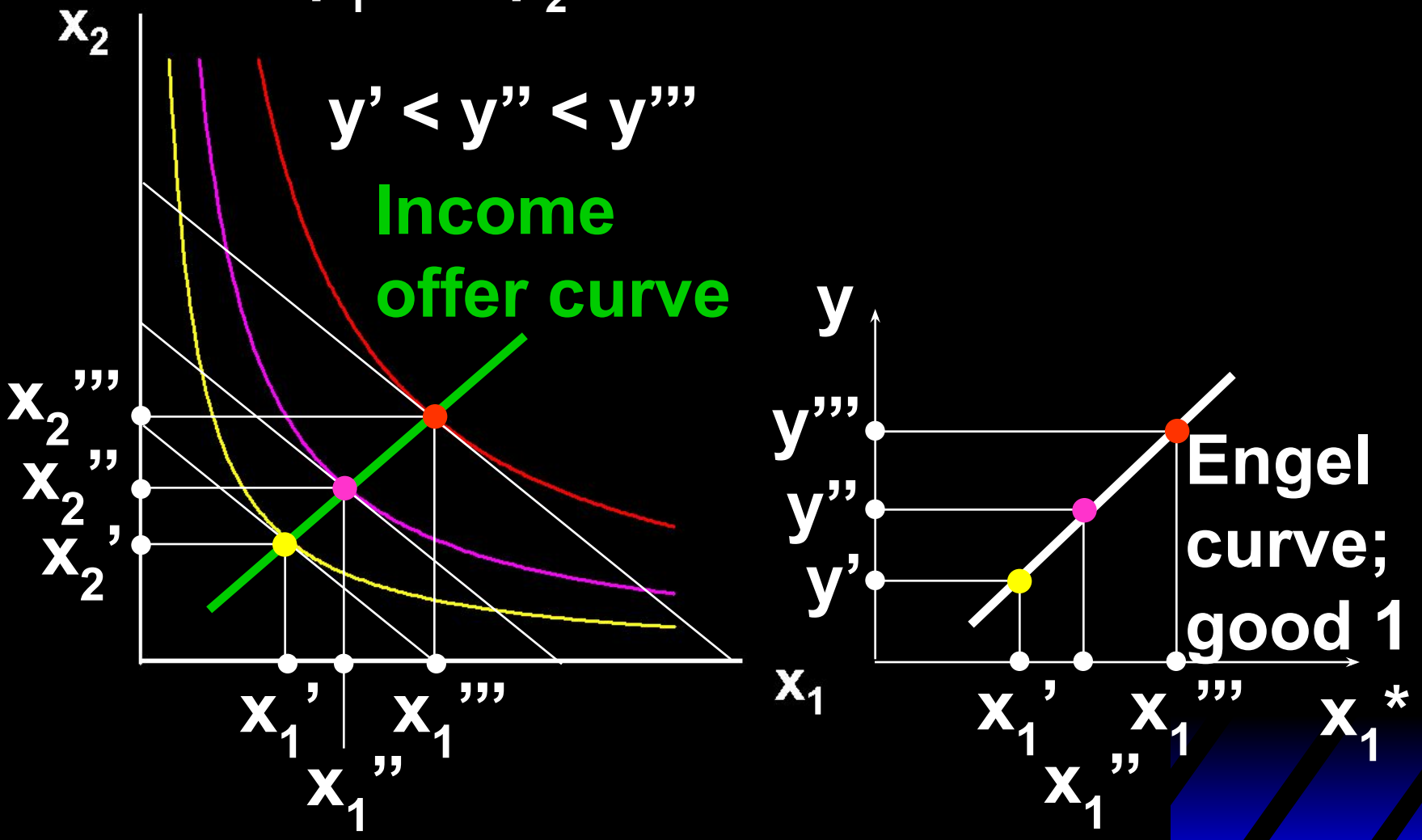


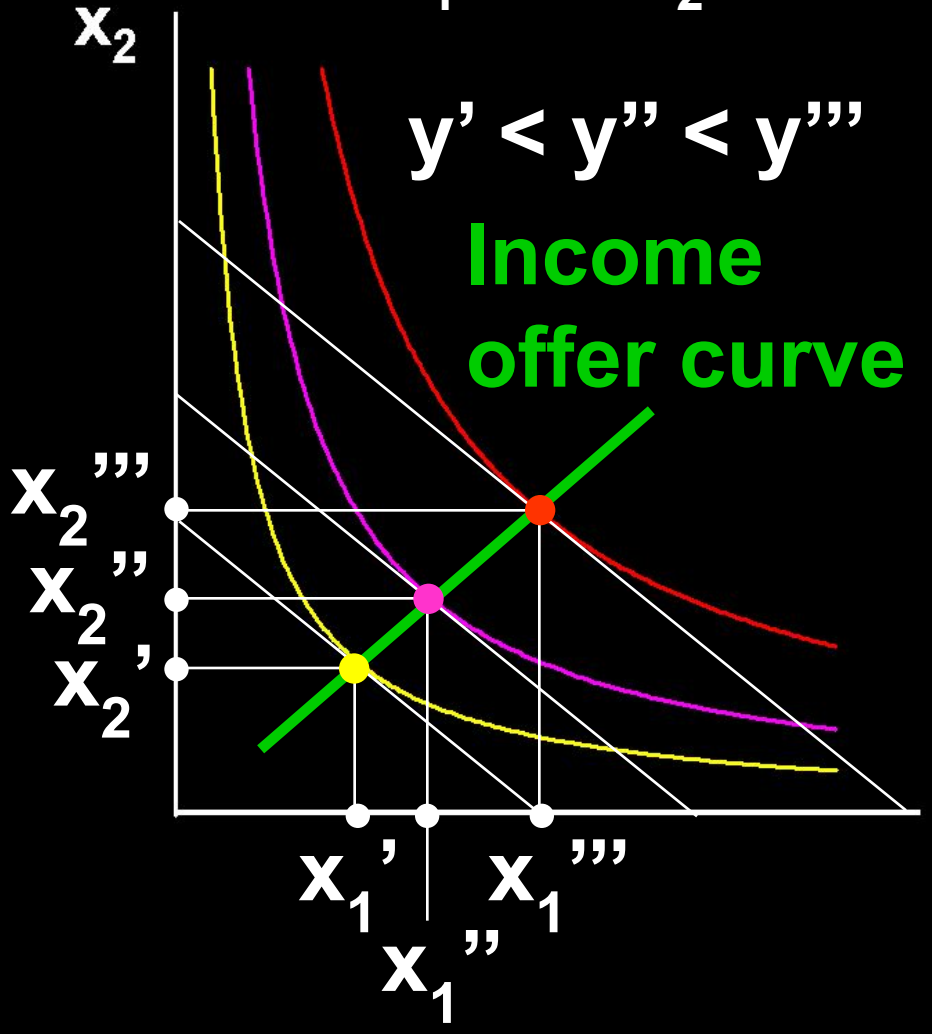
Income Changes

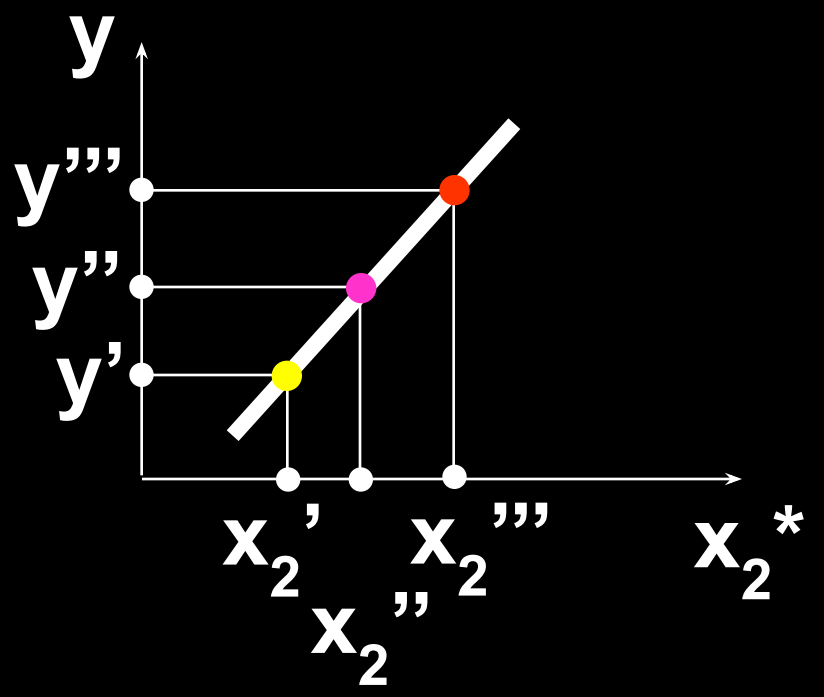
Fixed p_1 and p_2.



Income Changes

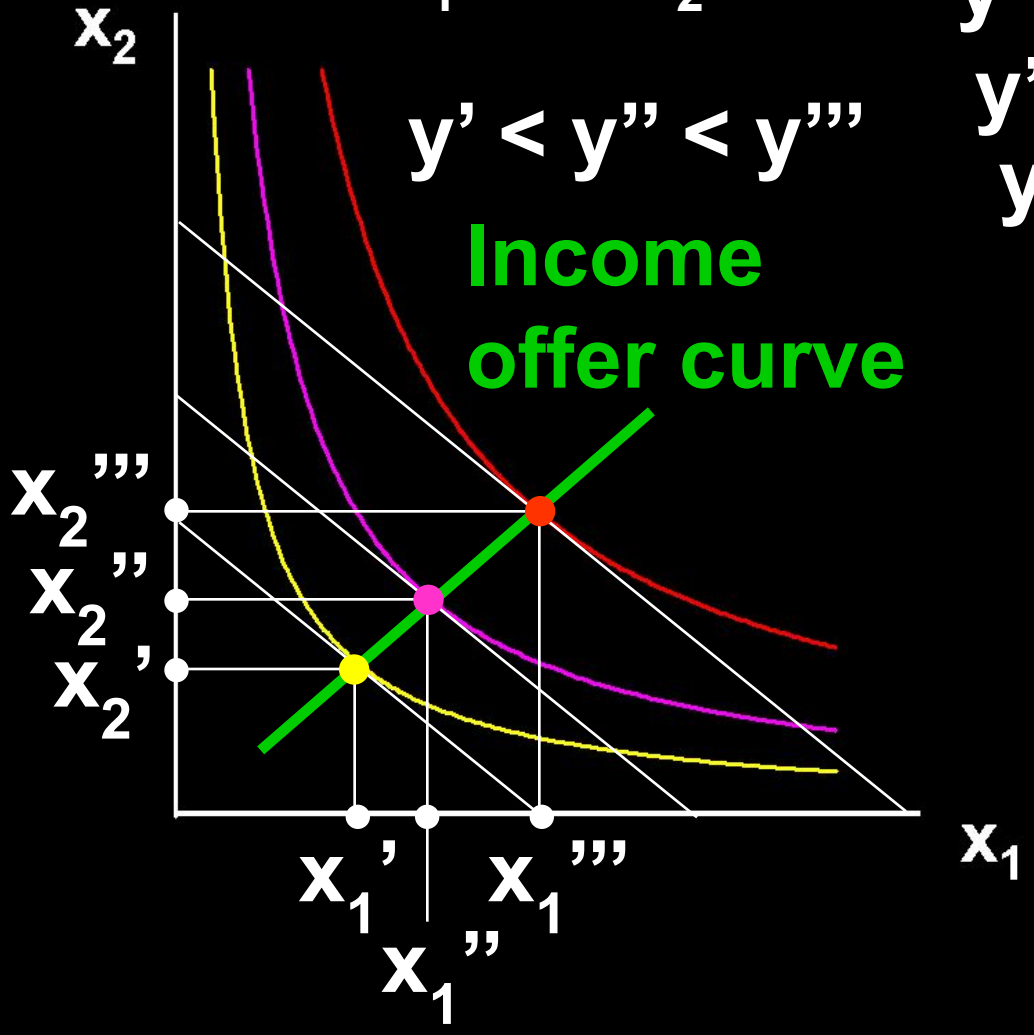
Fixed p_1 and p_2.

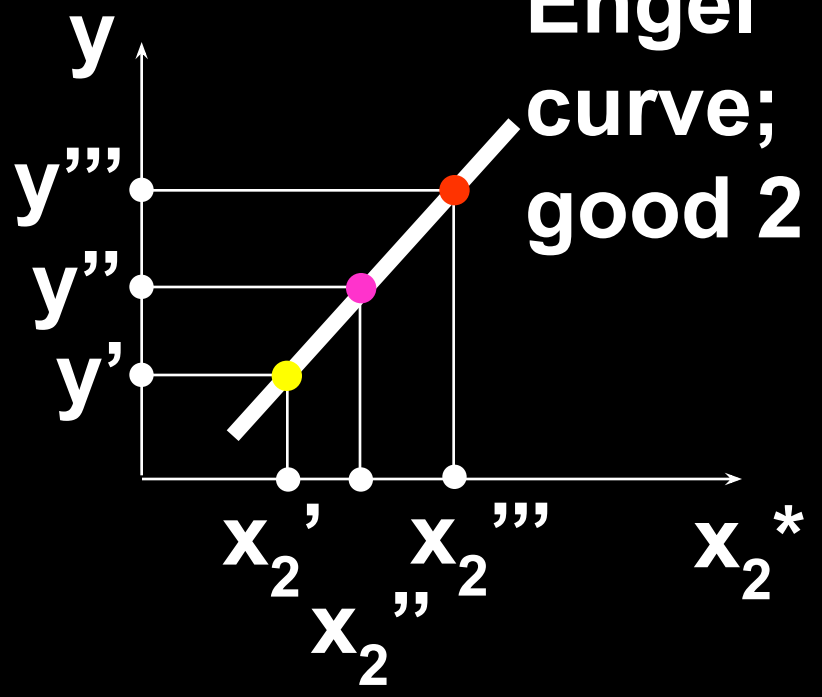




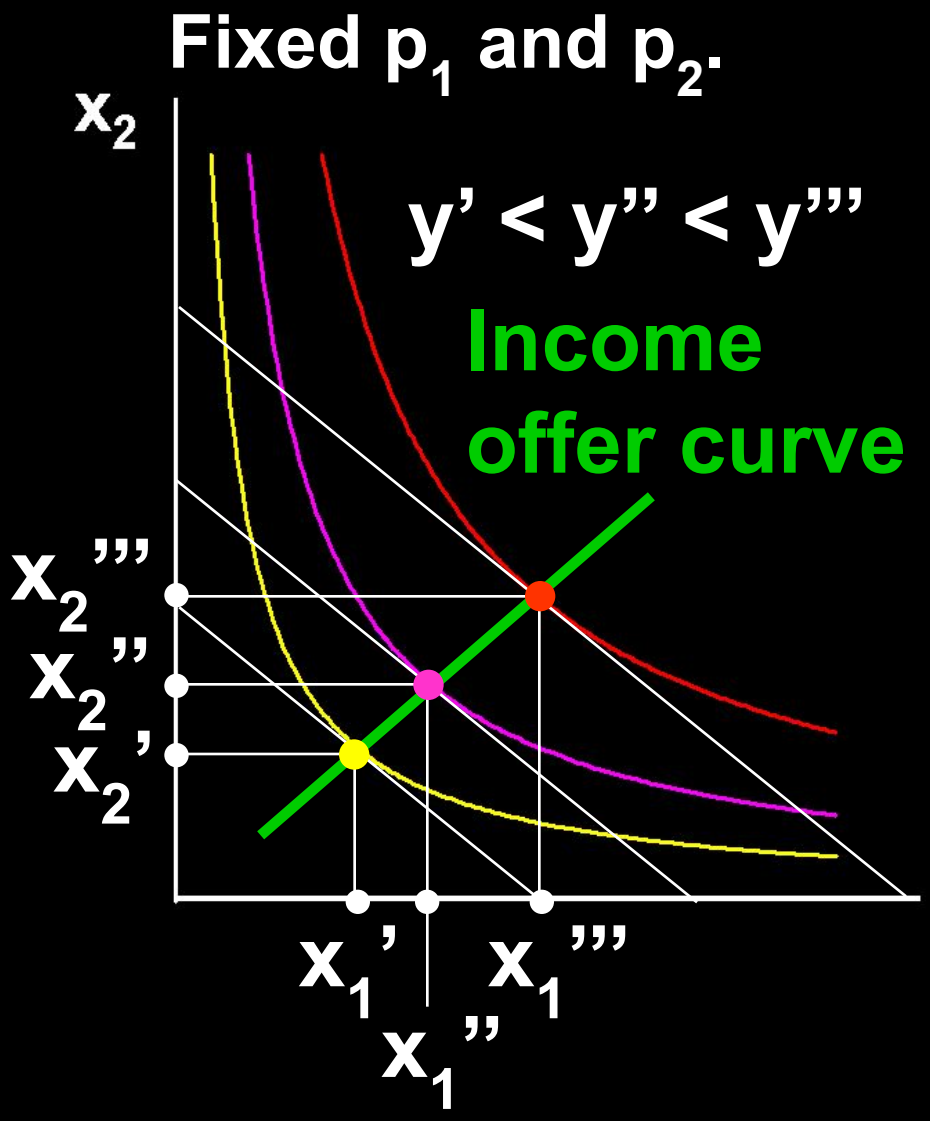
Income Changes

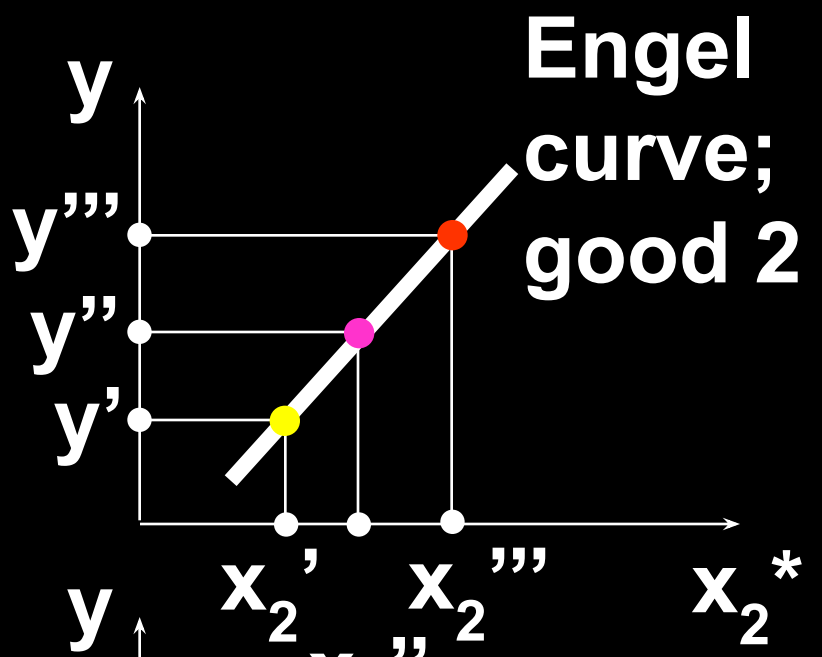
Fixed p_1 and p_2.

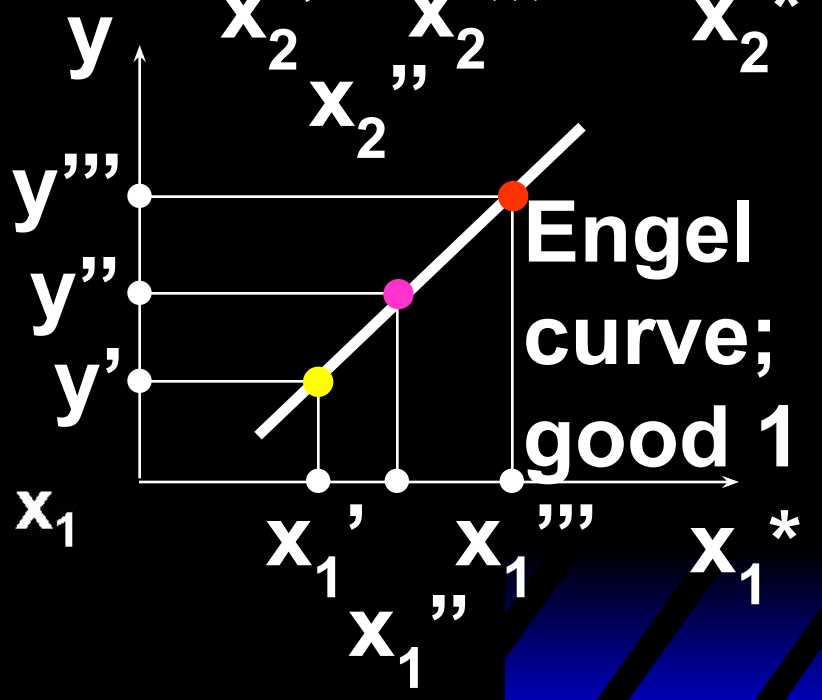




Income Changes







Income Changes and Cobb-Douglas Preferences

- **An example of computing the equations of Engel curves; the Cobb-Douglas case.**

- **The ordinary demand equations are**

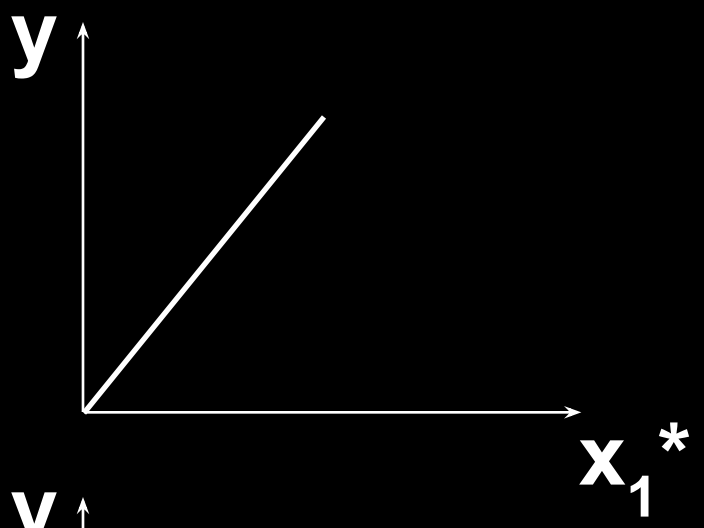
Income Changes and Cobb-Douglas Preferences

Rearranged to isolate y, these are:

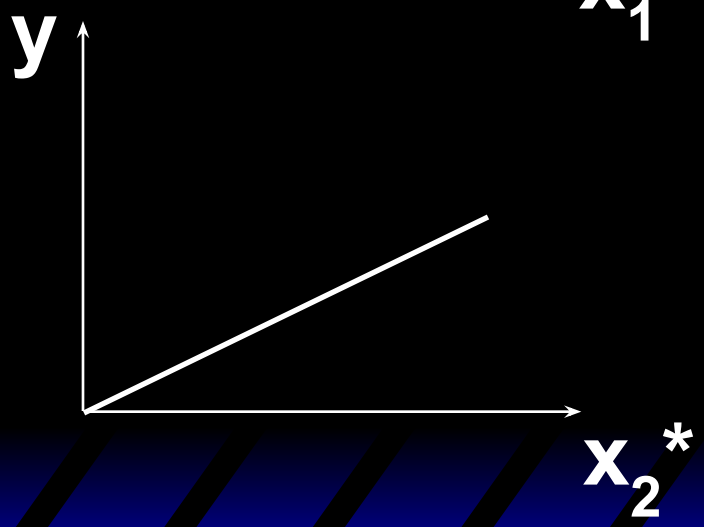
Engel curve for good 1

Engel curve for good 2

Income Changes and Cobb-Douglas Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

- **Another example of computing the equations of Engel curves; the perfectly-complementary case.**

- **The ordinary demand equations are**

Income Changes and Perfectly-Complementary Preferences

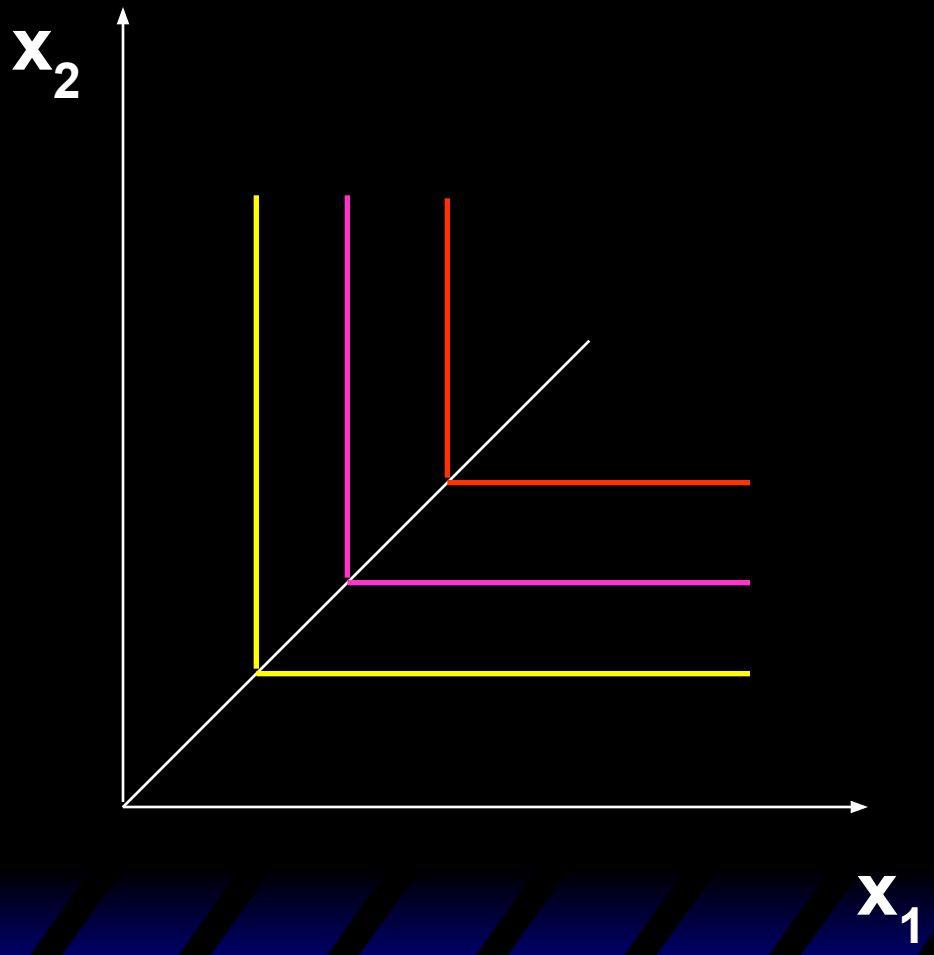
Rearranged to isolate y, these are:

Engel curve for good 1

Engel curve for good 2

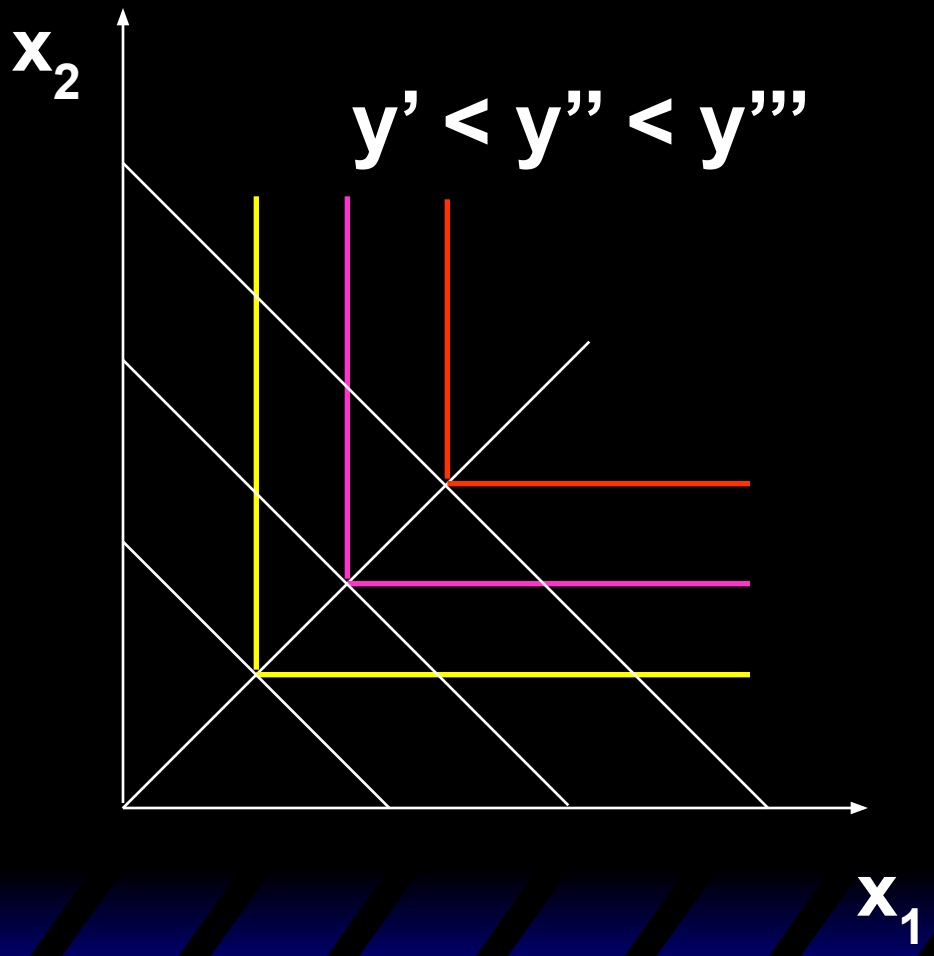
Income Changes

Fixed p_1 and p_2.



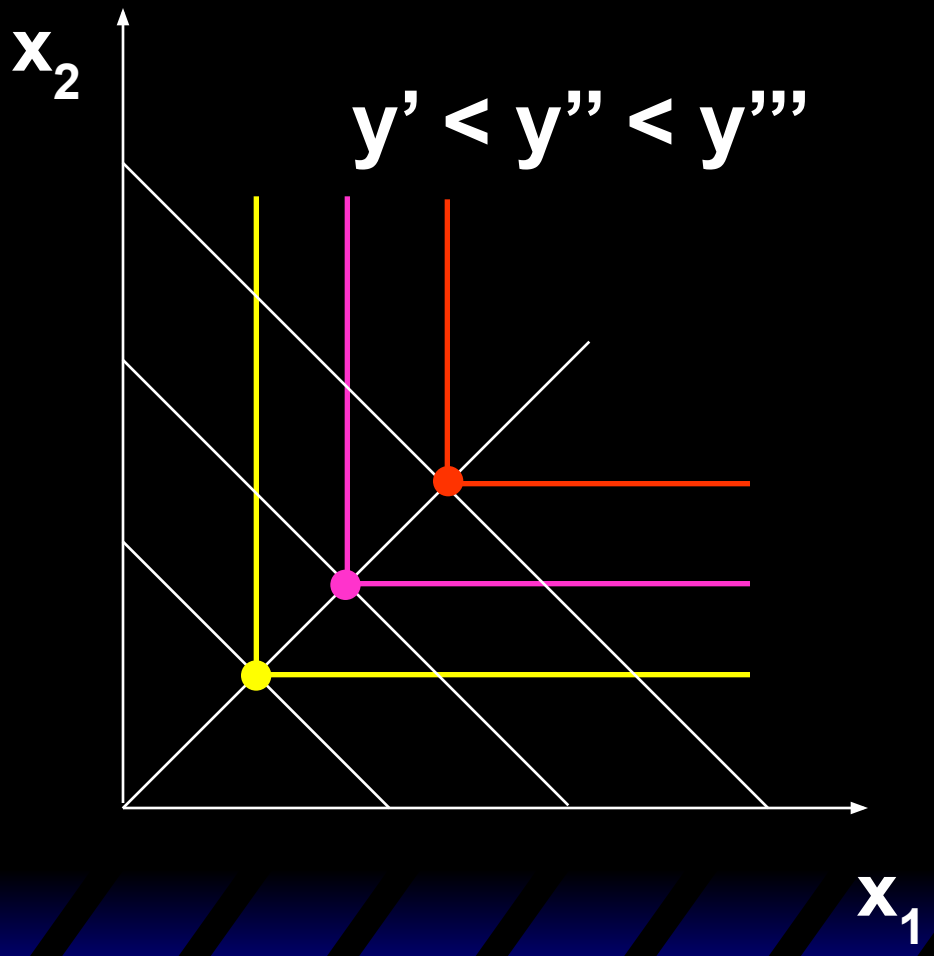
Income Changes

Fixed p_1 and p_2.

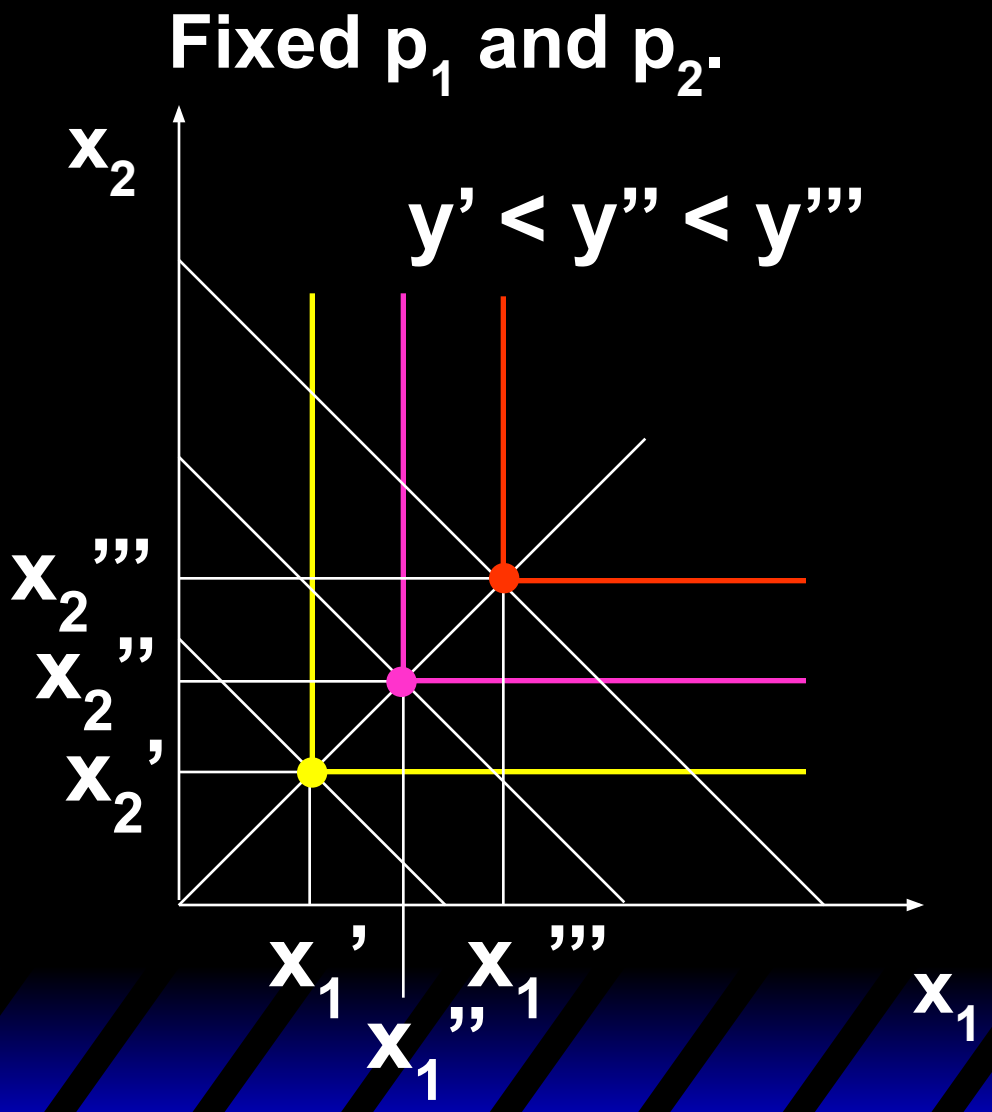


Income Changes

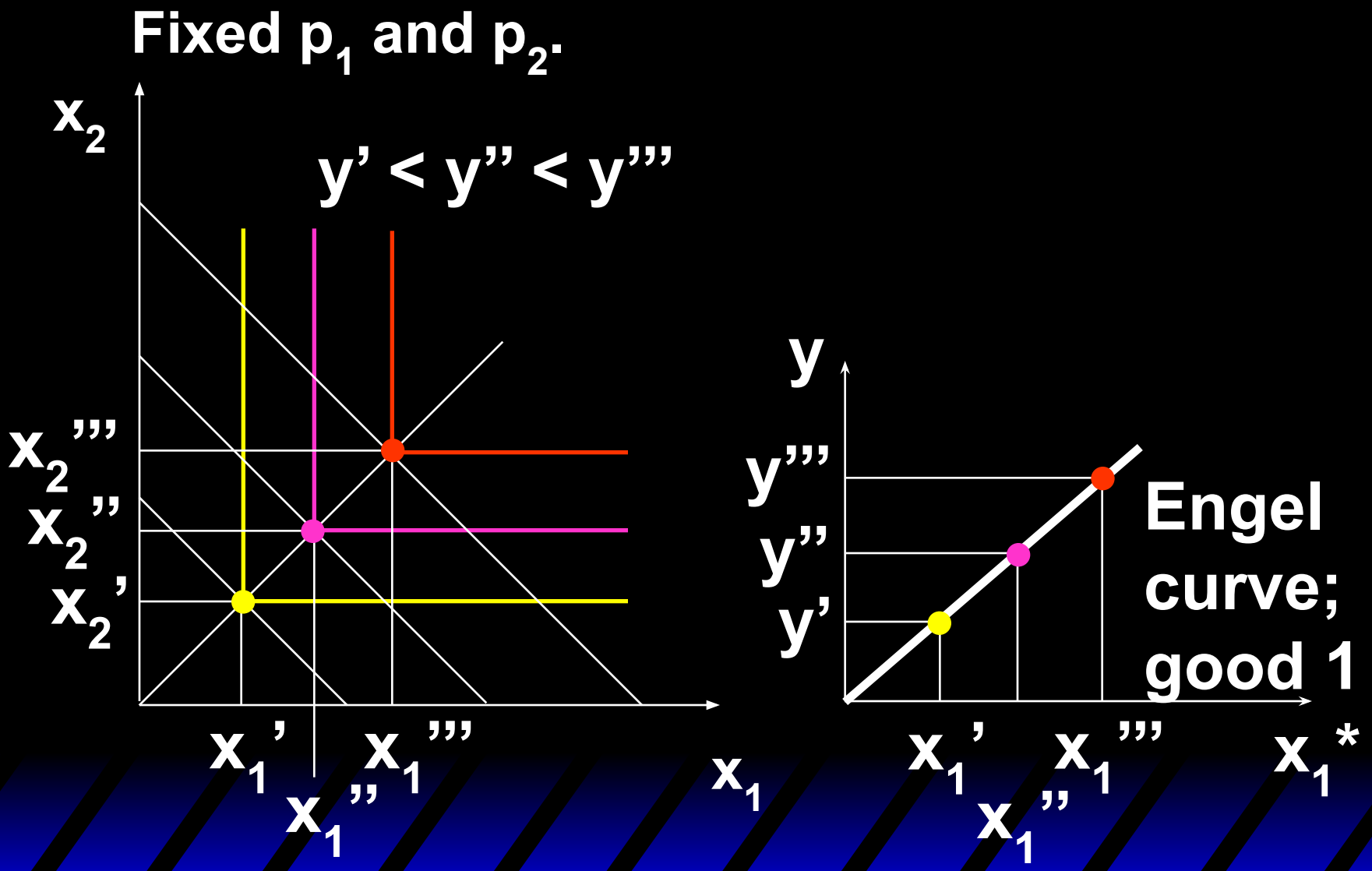
Fixed p_1 and p_2.



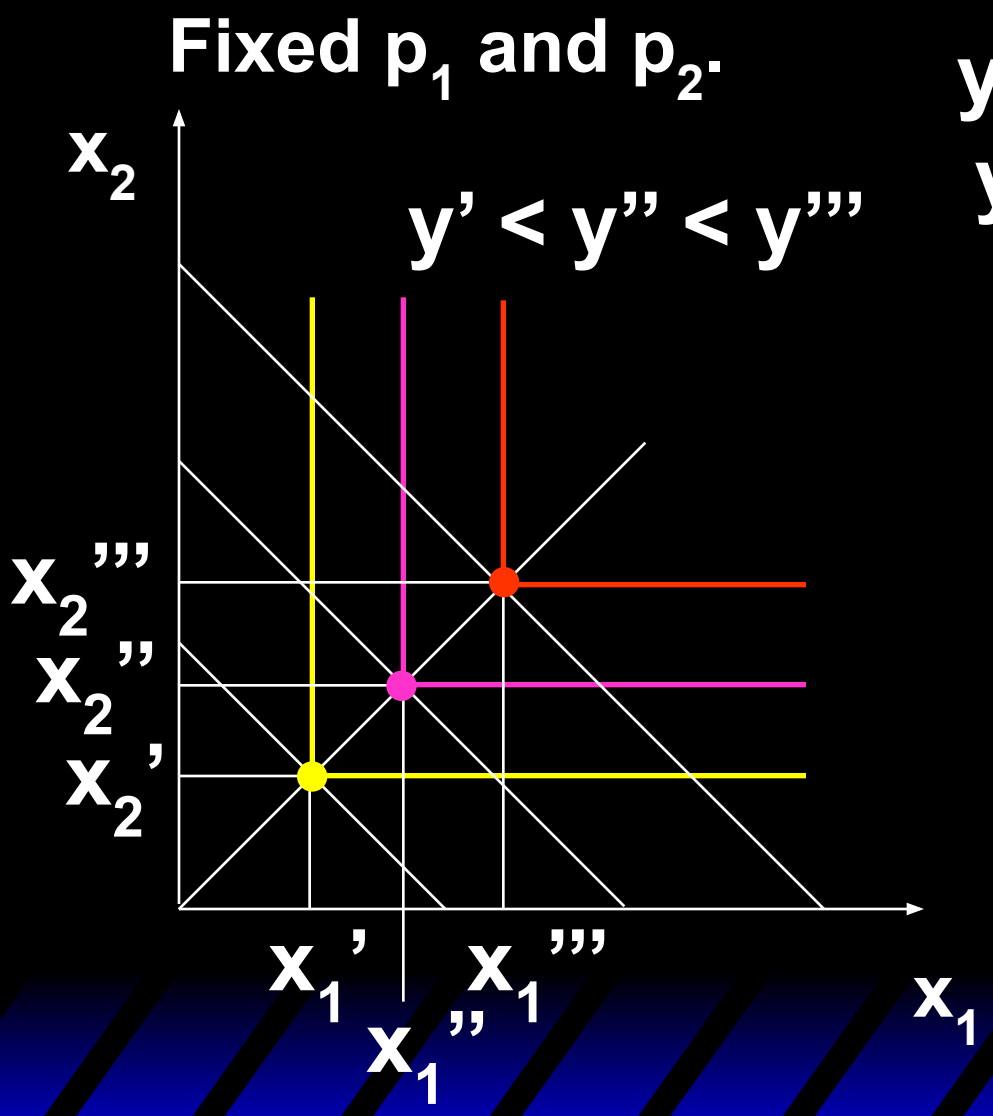
Income Changes

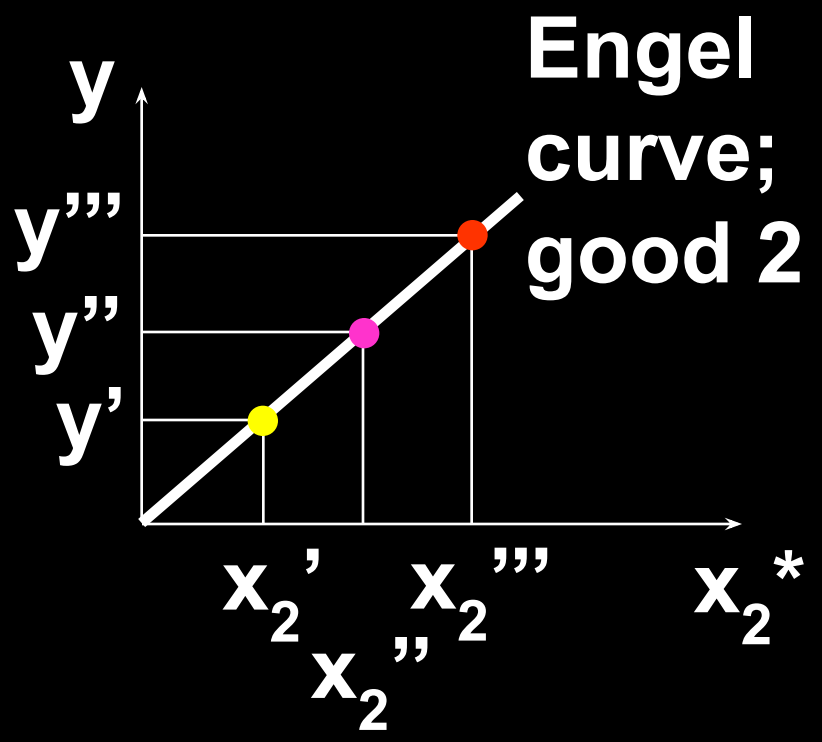


Income Changes

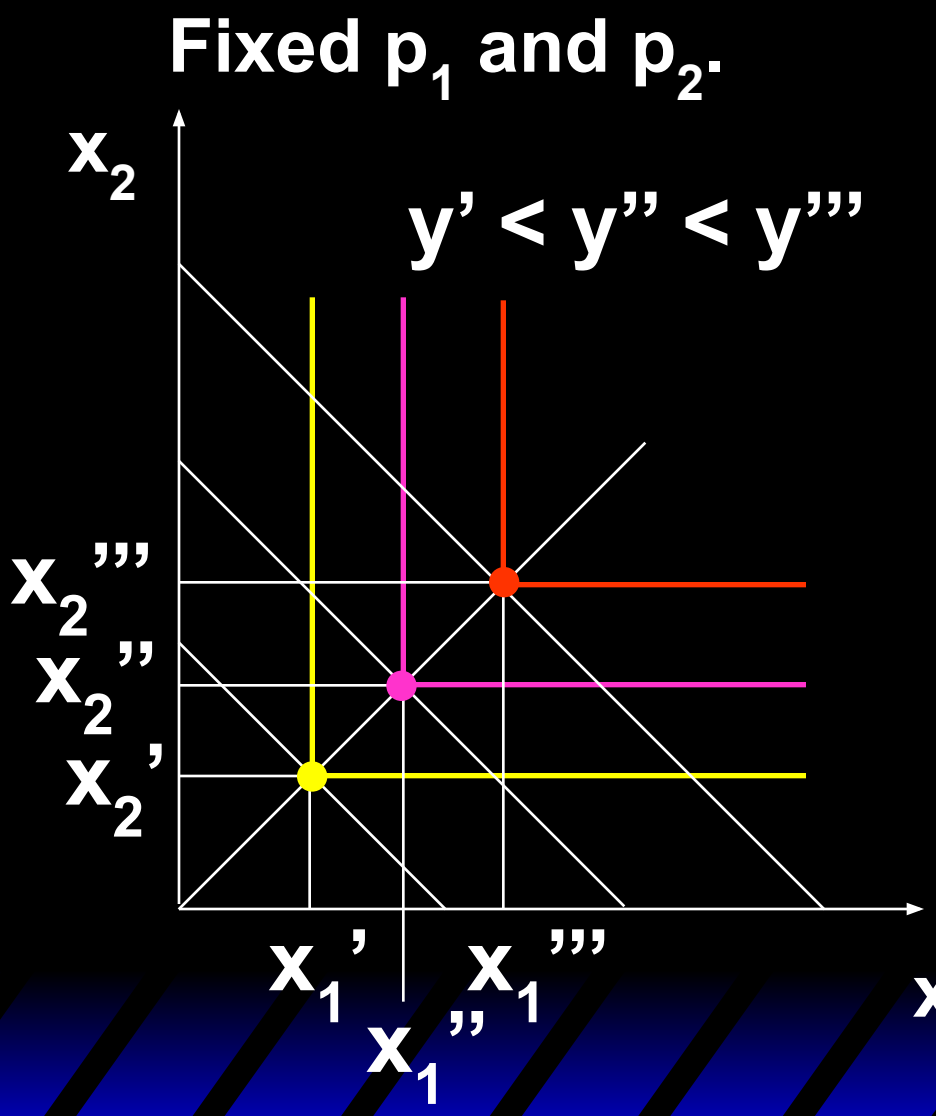


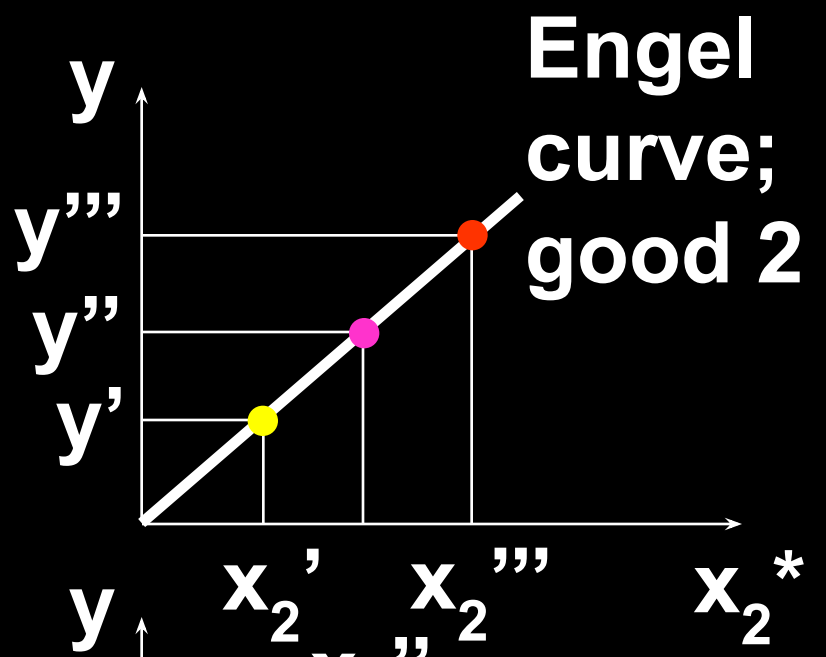
Income Changes

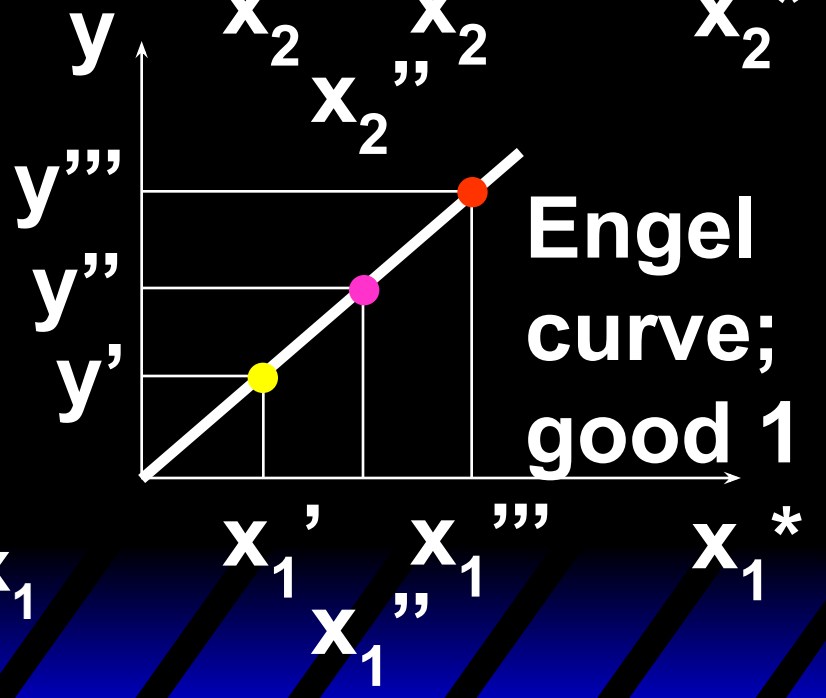




Income Changes



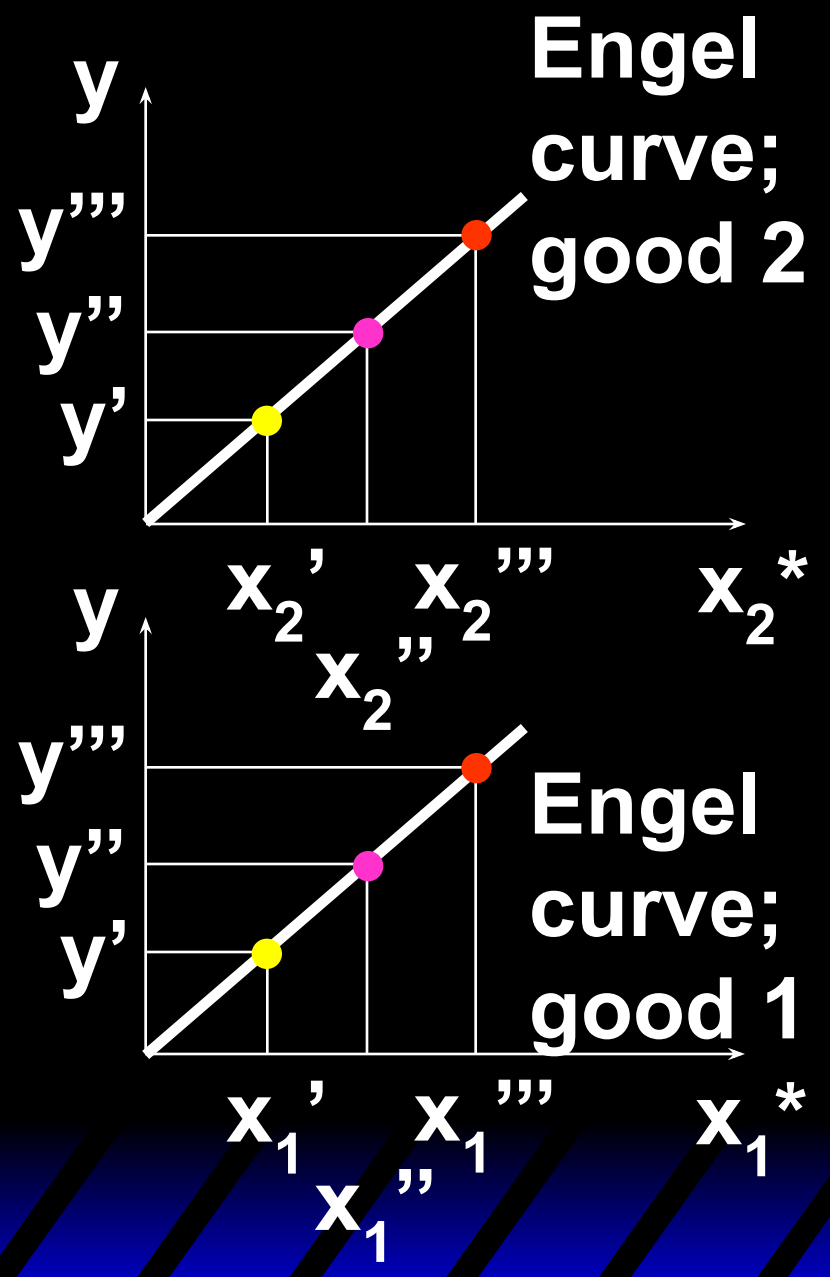




Income Changes

Fixed p_1 and p_2.

$$y = (p_1 + p_2)x_1^*$$



Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**

- **The ordinary demand equations are**

Income Changes and Perfectly-Substitutable Preferences

Income Changes and Perfectly-Substitutable Preferences

Suppose $p_1 < p_2$. Then

Income Changes and Perfectly-Substitutable Preferences

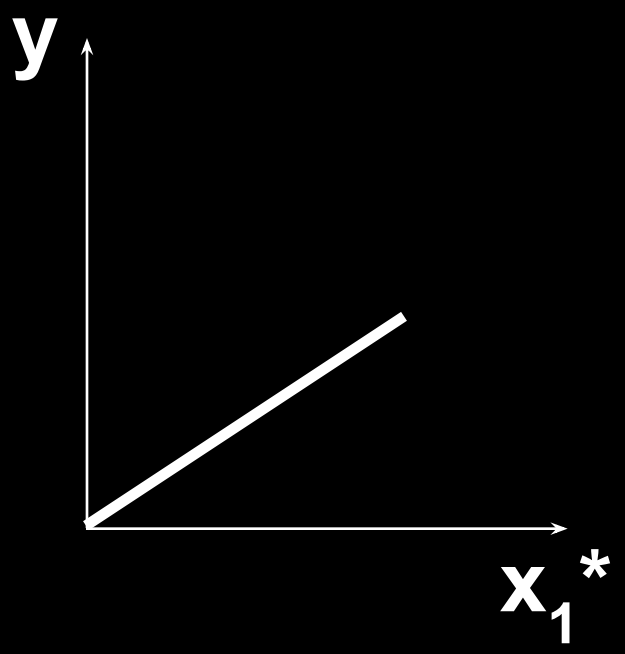
Suppose $p_1 < p_2$. Then and

Income Changes and Perfectly-Substitutable Preferences

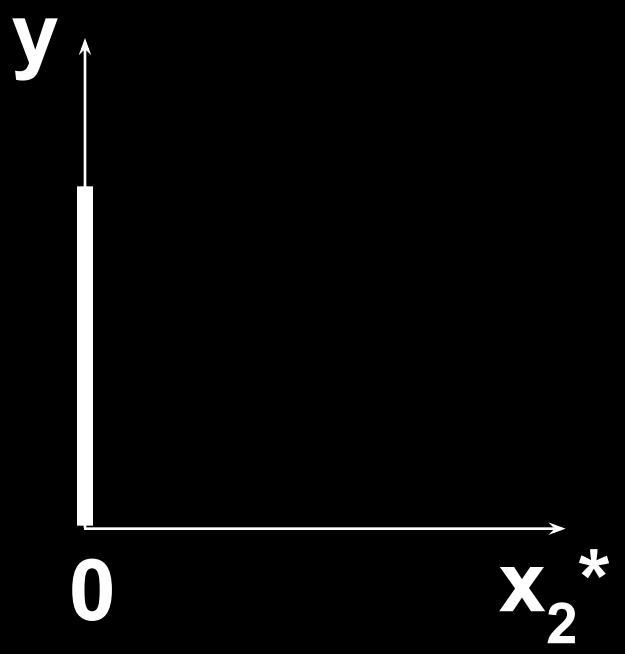
Suppose $p_1 < p_2$. Then and

$\longrightarrow$ $y = p_1 x_1$ and $x_2 = 0$.

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

- In every example so far the Engel curves have all been straight lines? Q: Is this true in general?
- A: No. Engel curves are straight lines if the consumer's preferences are **homothetic**.

Homotheticity

- A consumer's preferences are **homothetic** if and only if

(x_1,x_2) p (y_1,y_2) $\Leftrightarrow$ (kx_1,kx_2) p (ky_1,ky_2) for every $k > 0$.

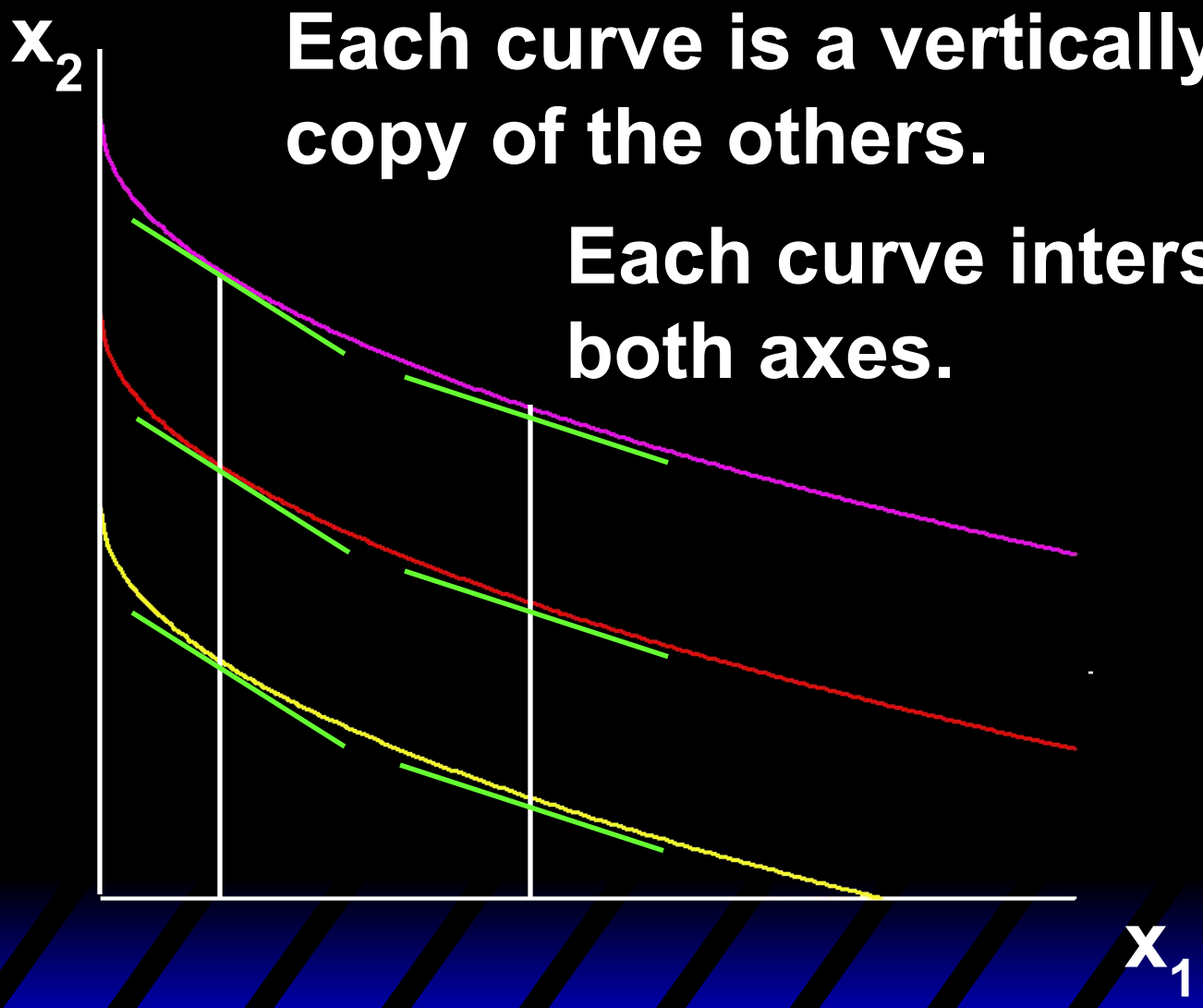
- That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

- **Quasilinear preferences are not homothetic.**

- **For example,**

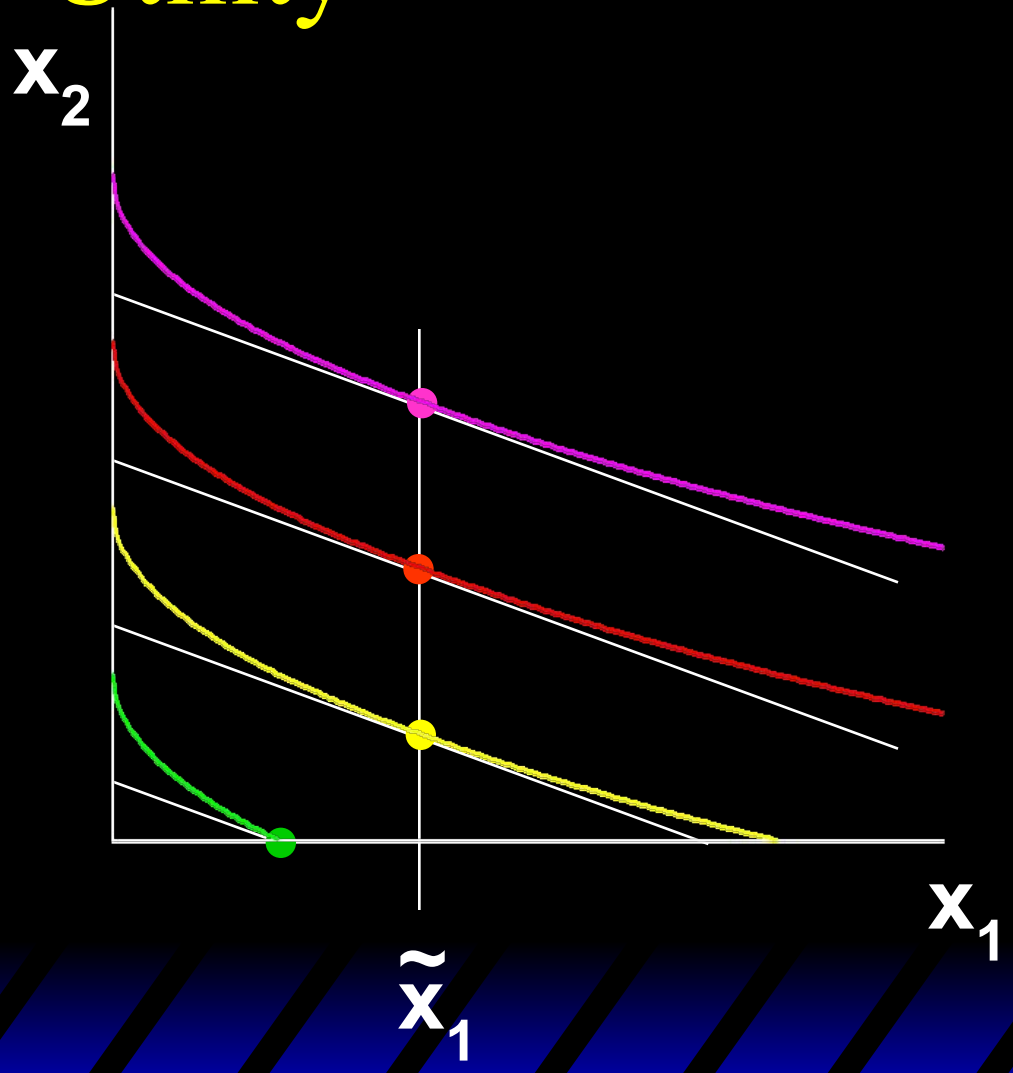
Quasi-linear Indifference Curves



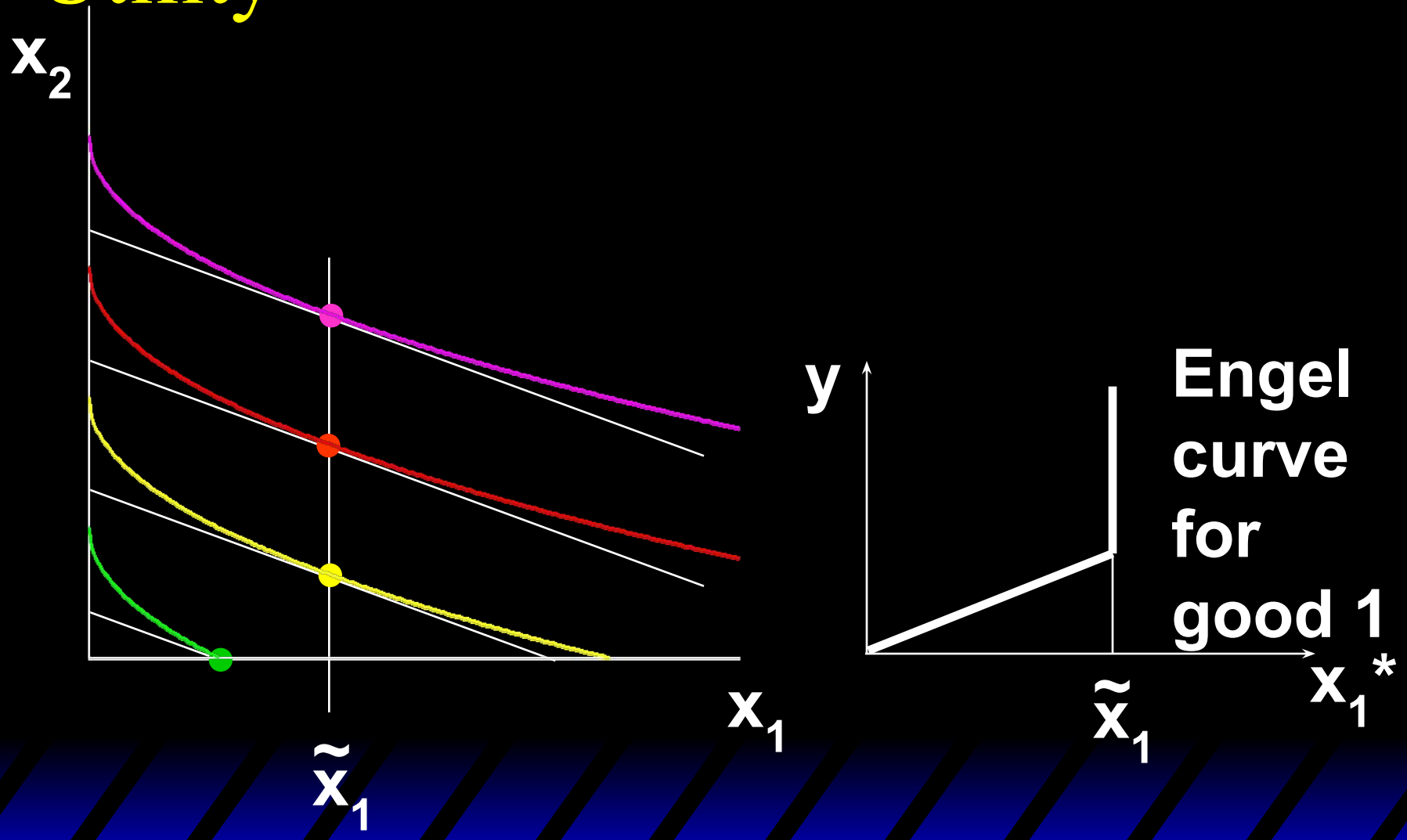
Each curve is a vertically shifted copy of the others.

Each curve intersects both axes.

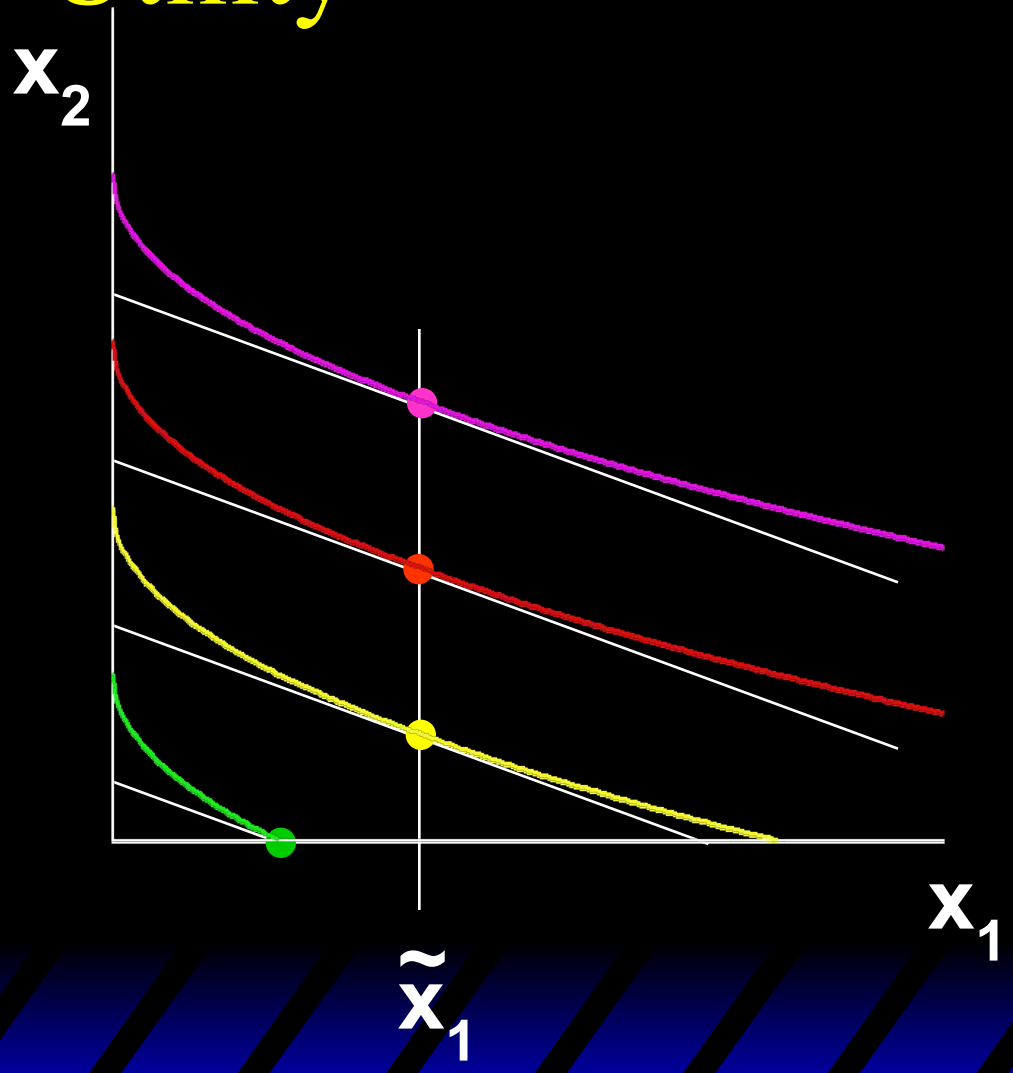
Income Changes; Quasilinear Utility

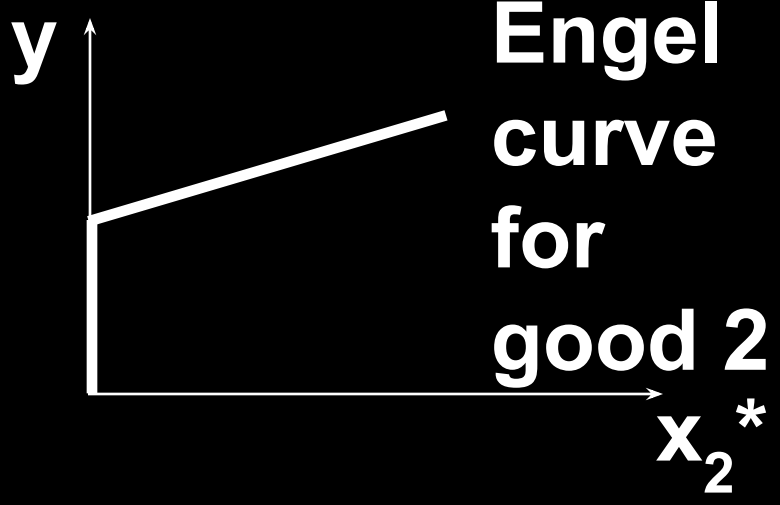


Income Changes; Quasilinear Utility

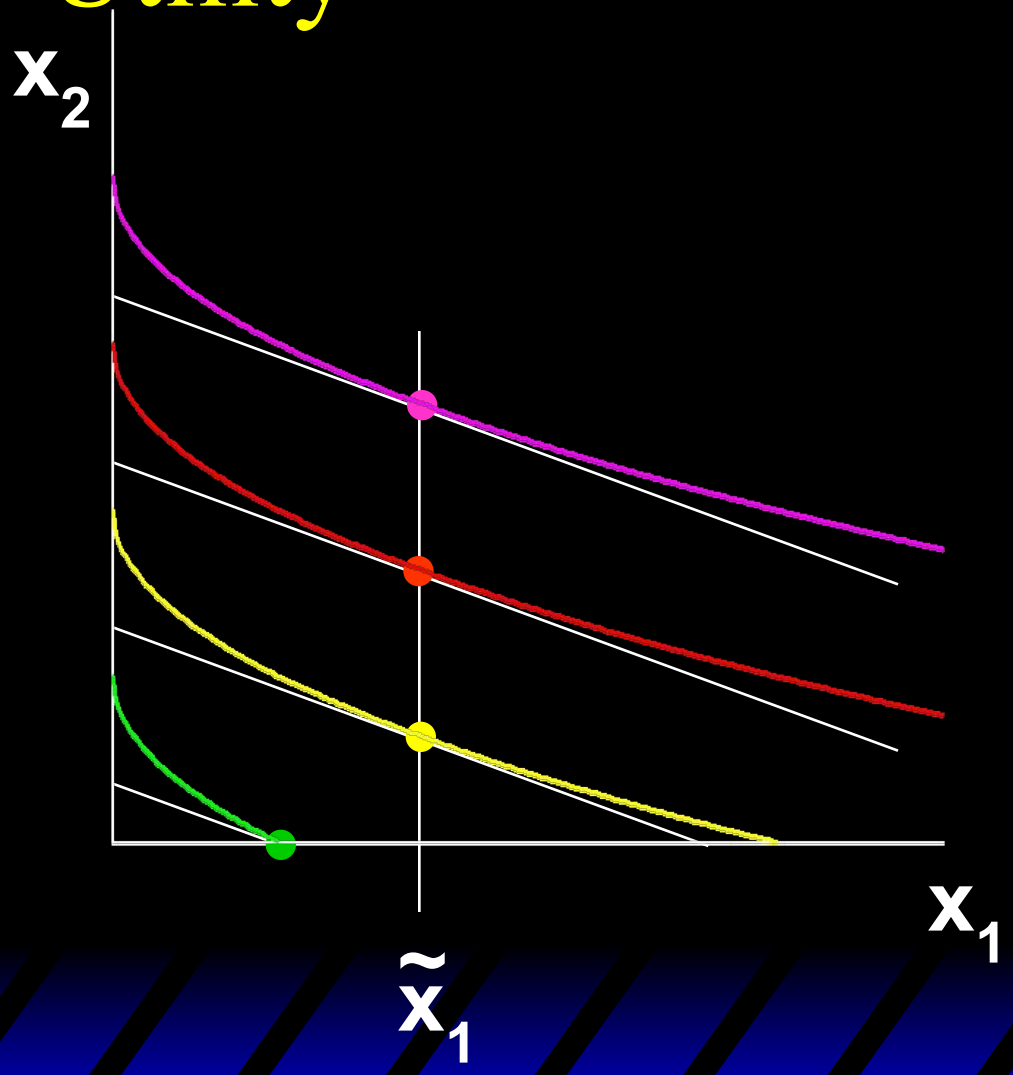


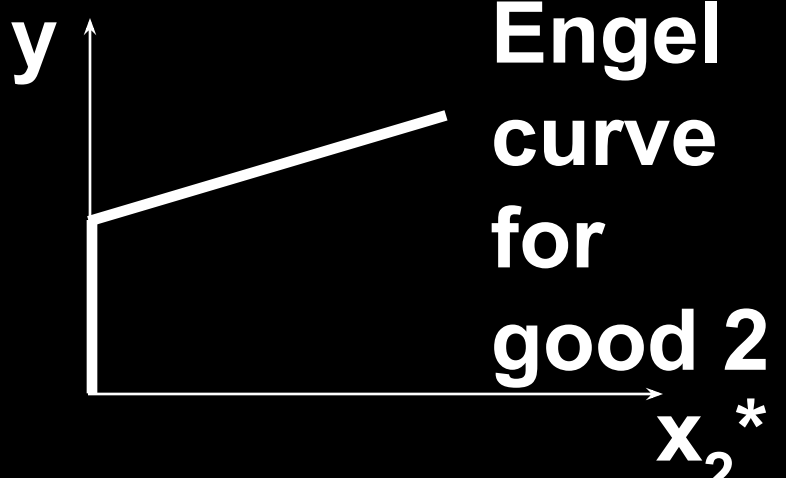
Income Changes; Quasilinear Utility



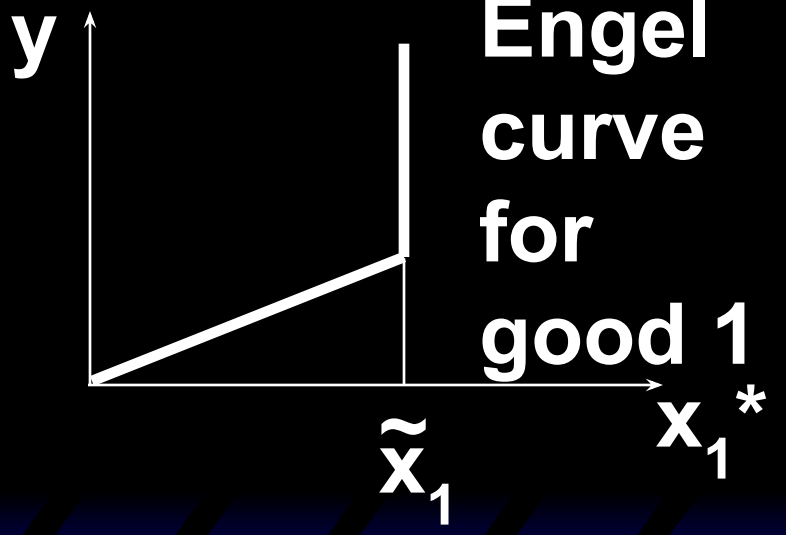


Income Changes; Quasilinear Utility





Engel curve for good 2



Engel curve for good 1

Income Effects

- **A good for which quantity demanded rises with income is called normal.**
- **Therefore a normal good's Engel curve is positively sloped.**

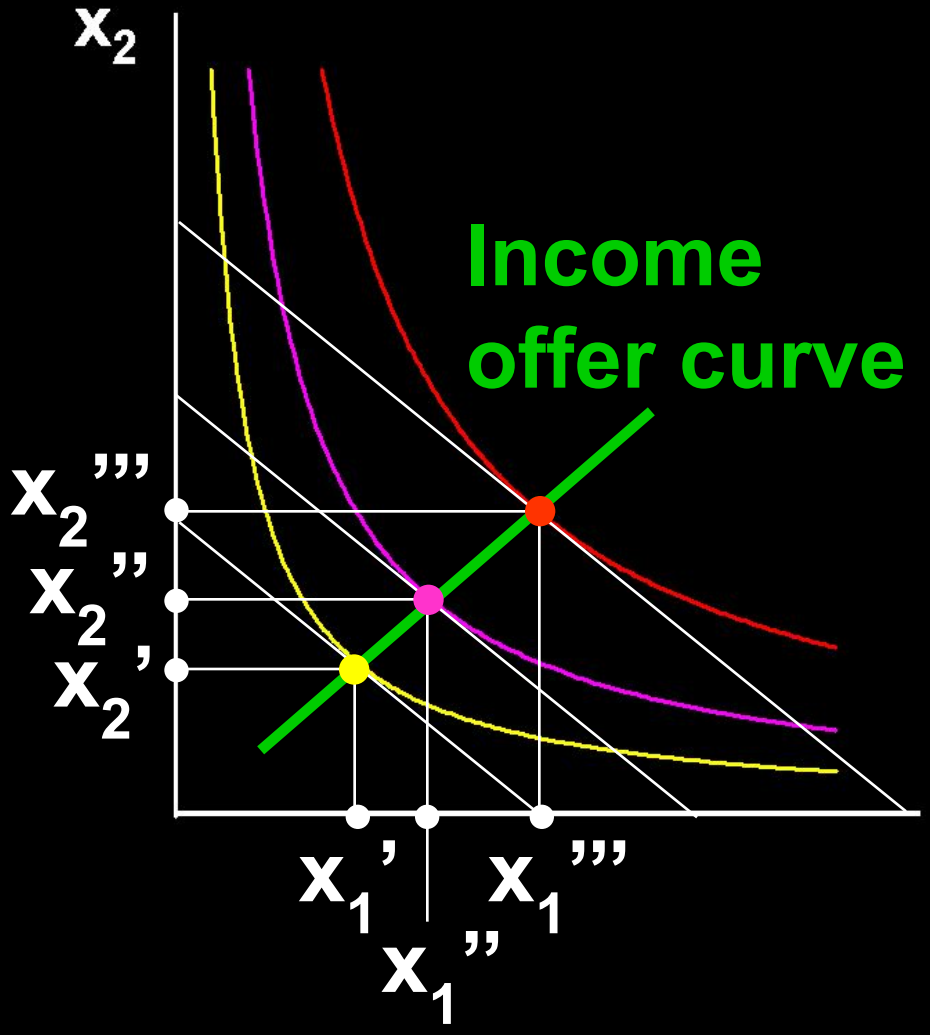
Income Effects

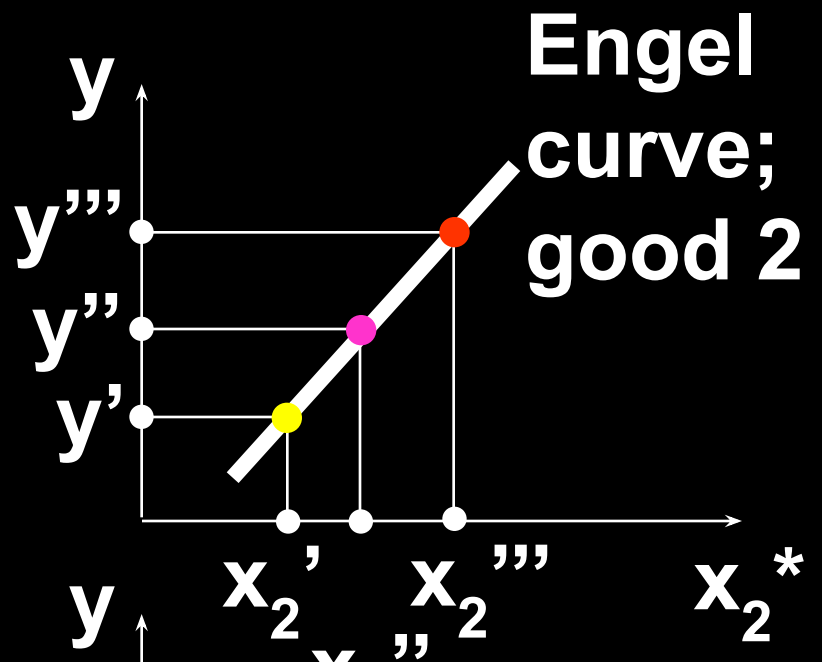
- A good for which quantity demanded falls as income increases is called **income inferior**.
- Therefore an income inferior good's Engel curve is negatively sloped.

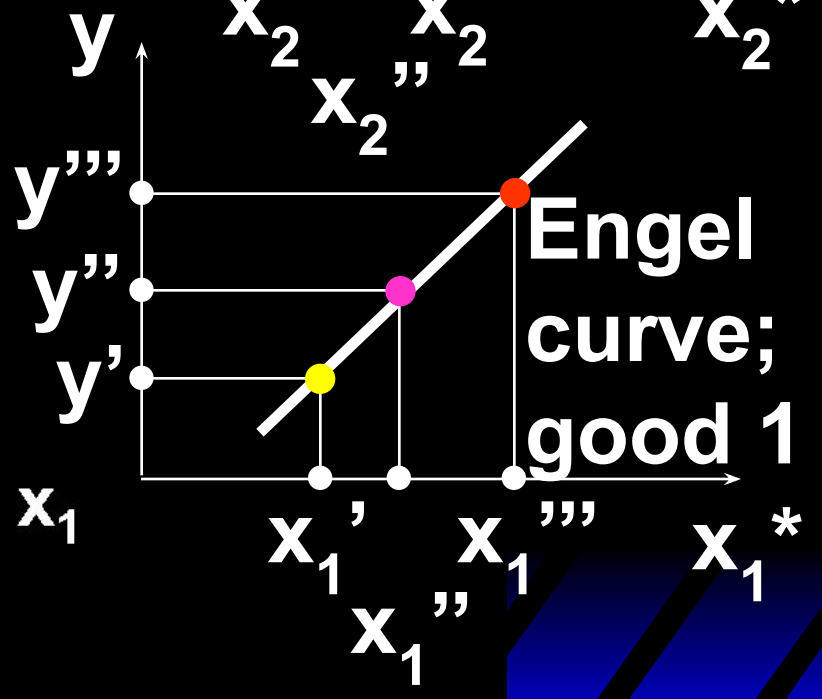
Income Effects

- In the US over last hundred years income increased many times whereas the number of kids per household went down.
- Are children an **inferior good?**

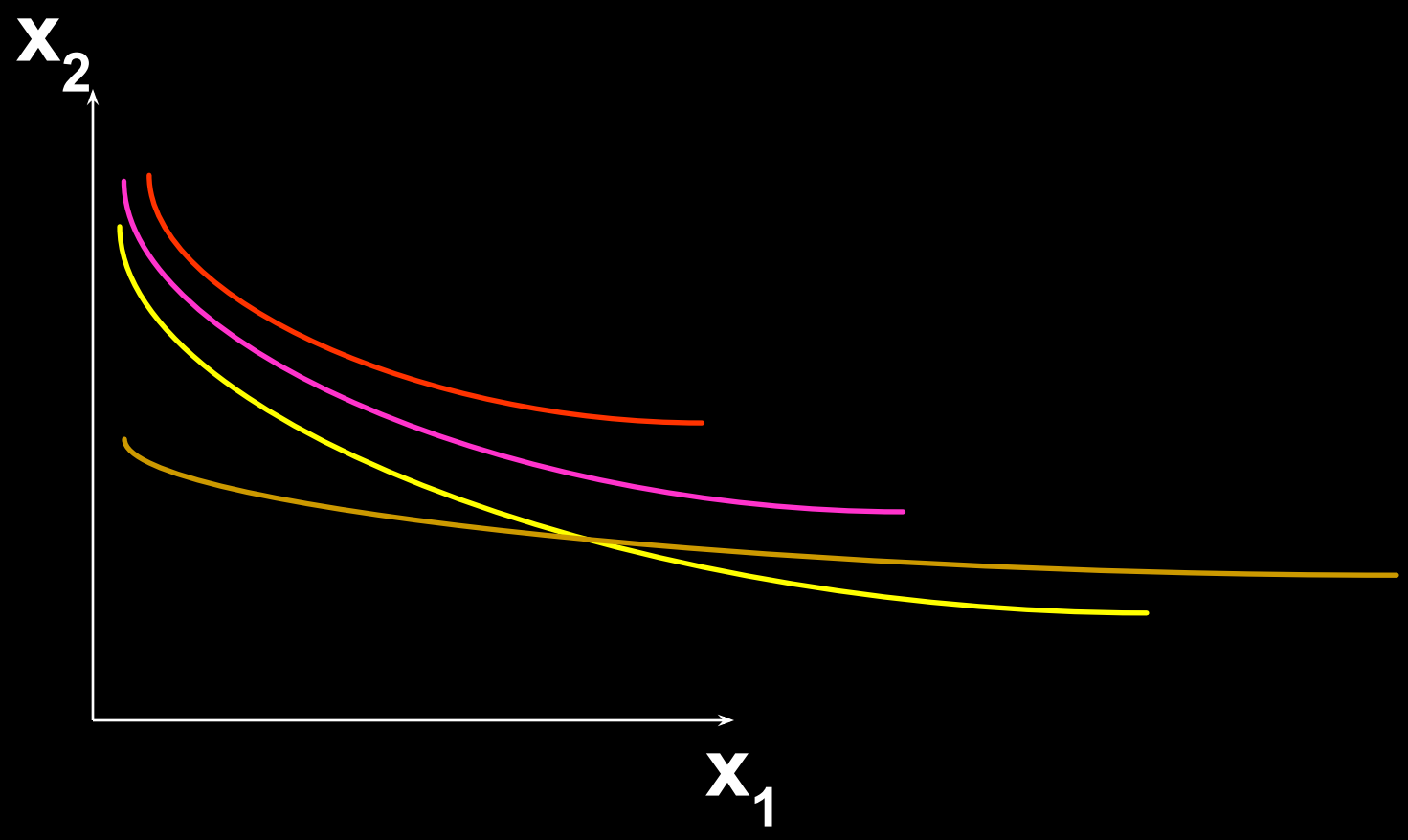
Income Changes; Goods 1 & 2 Normal



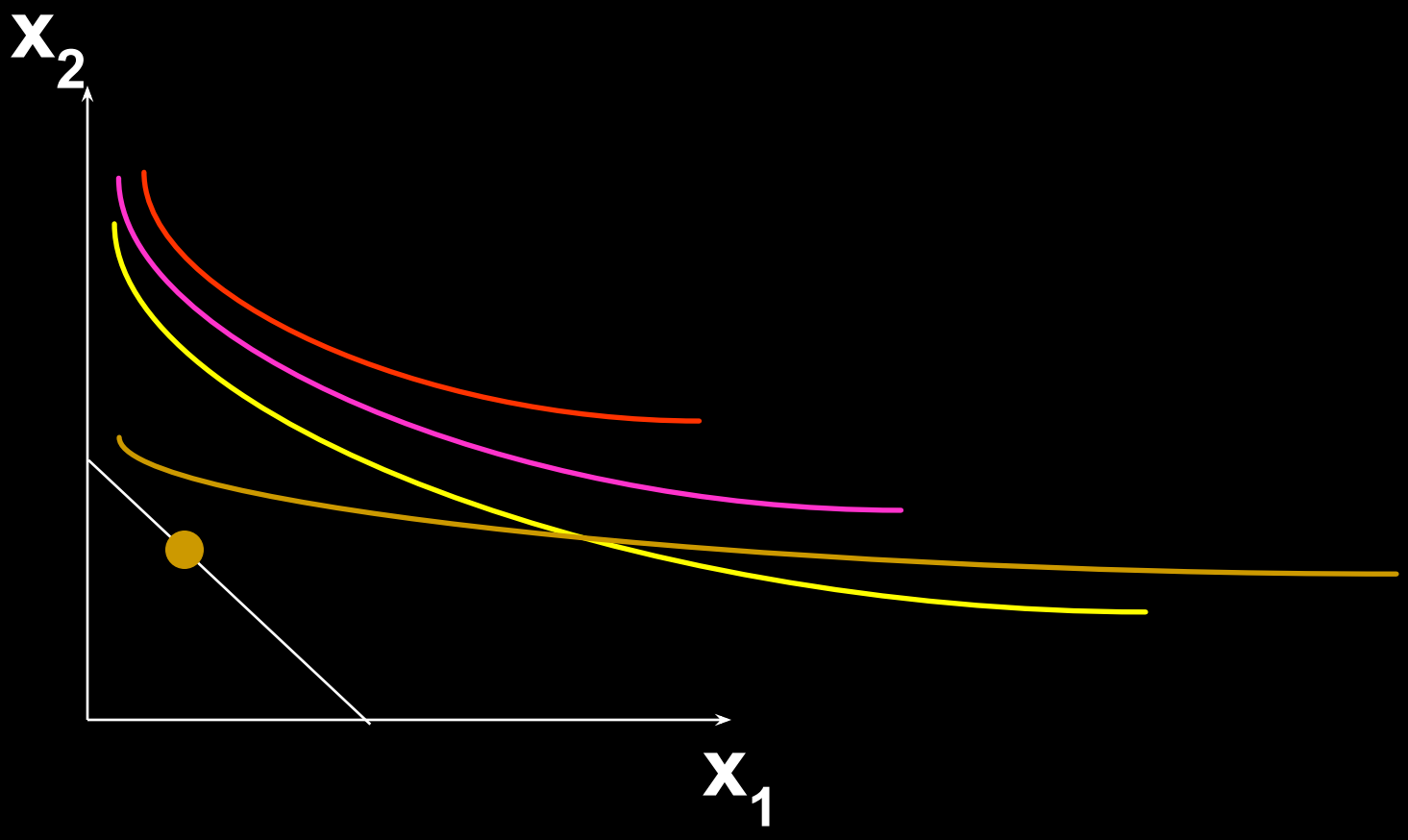




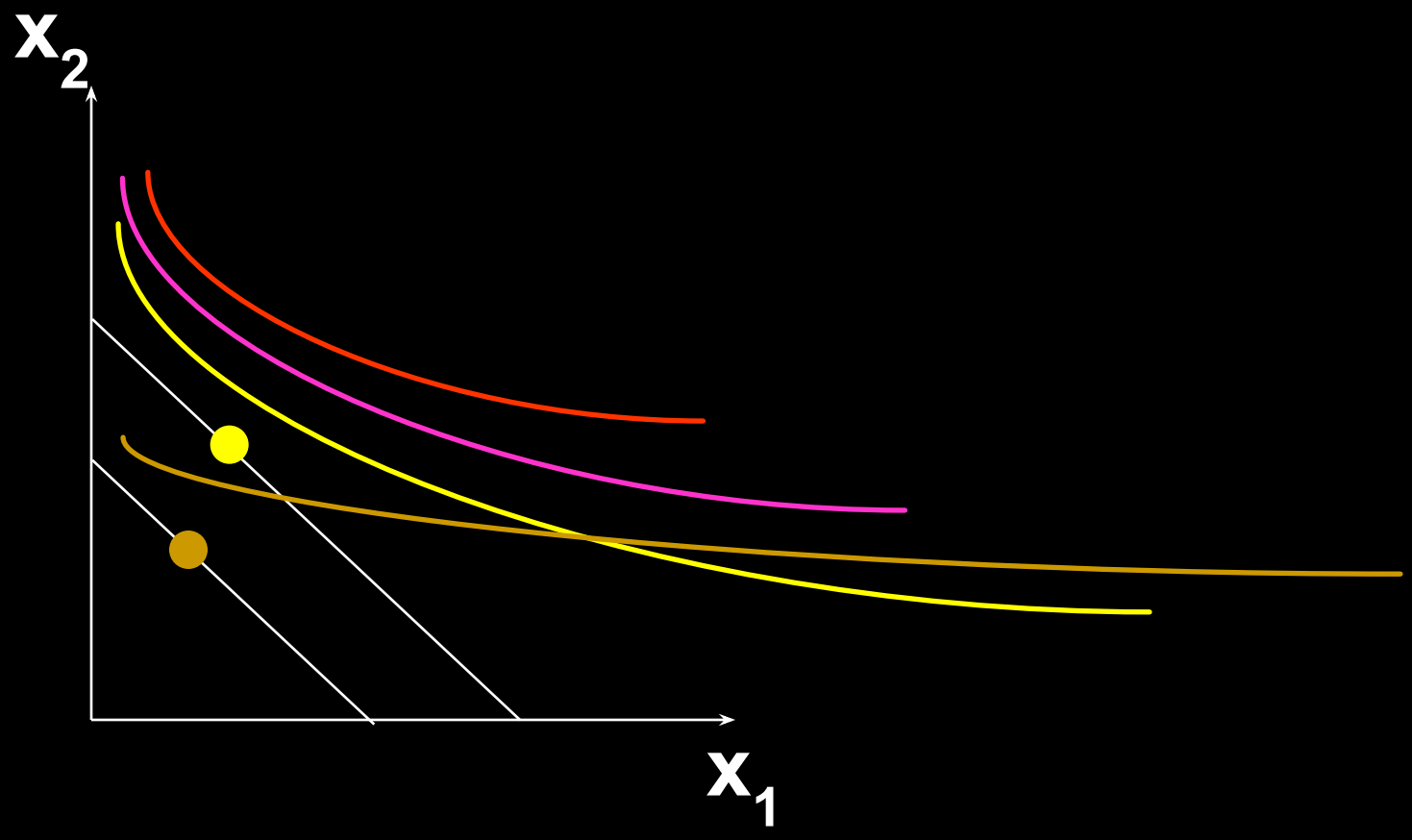
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



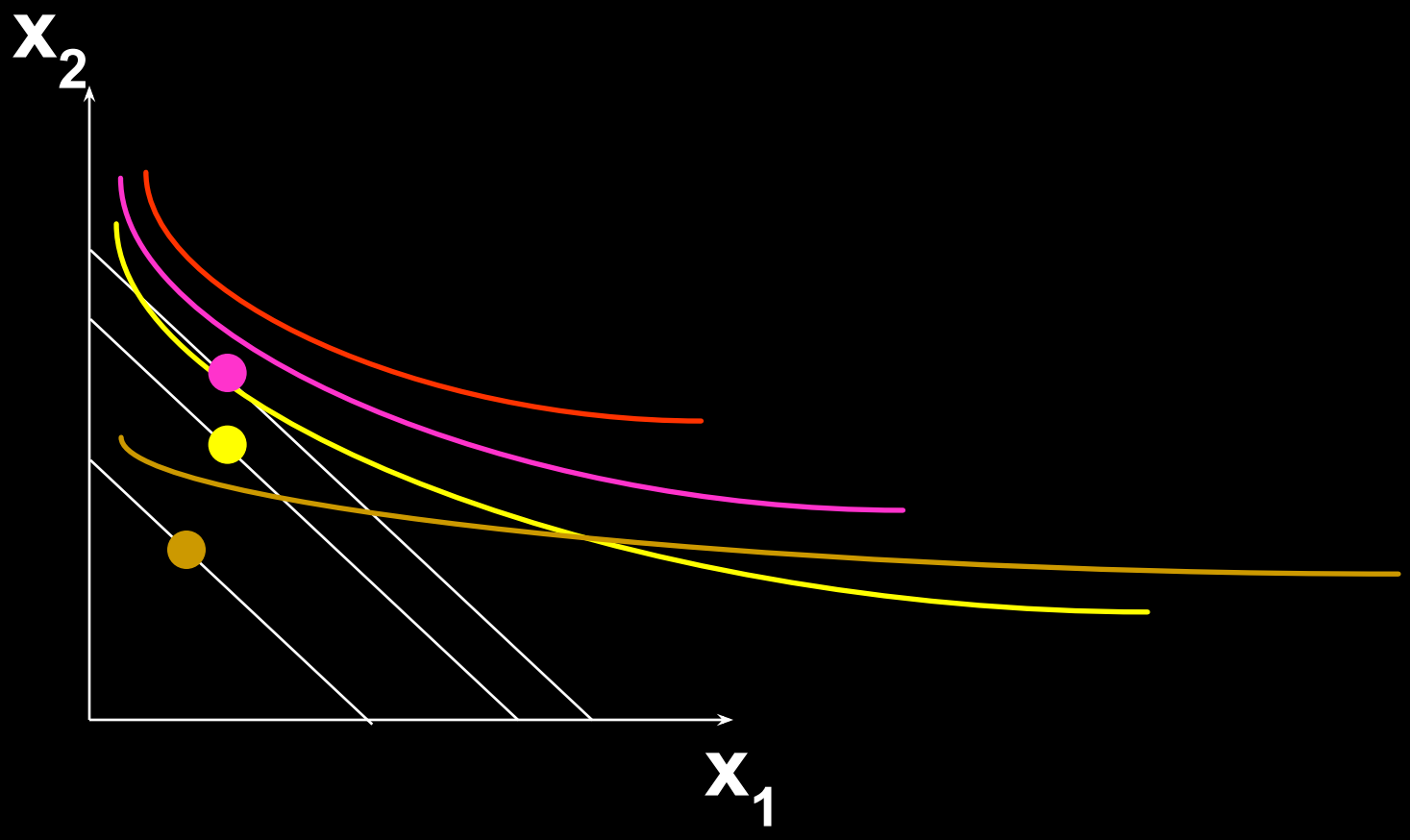
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



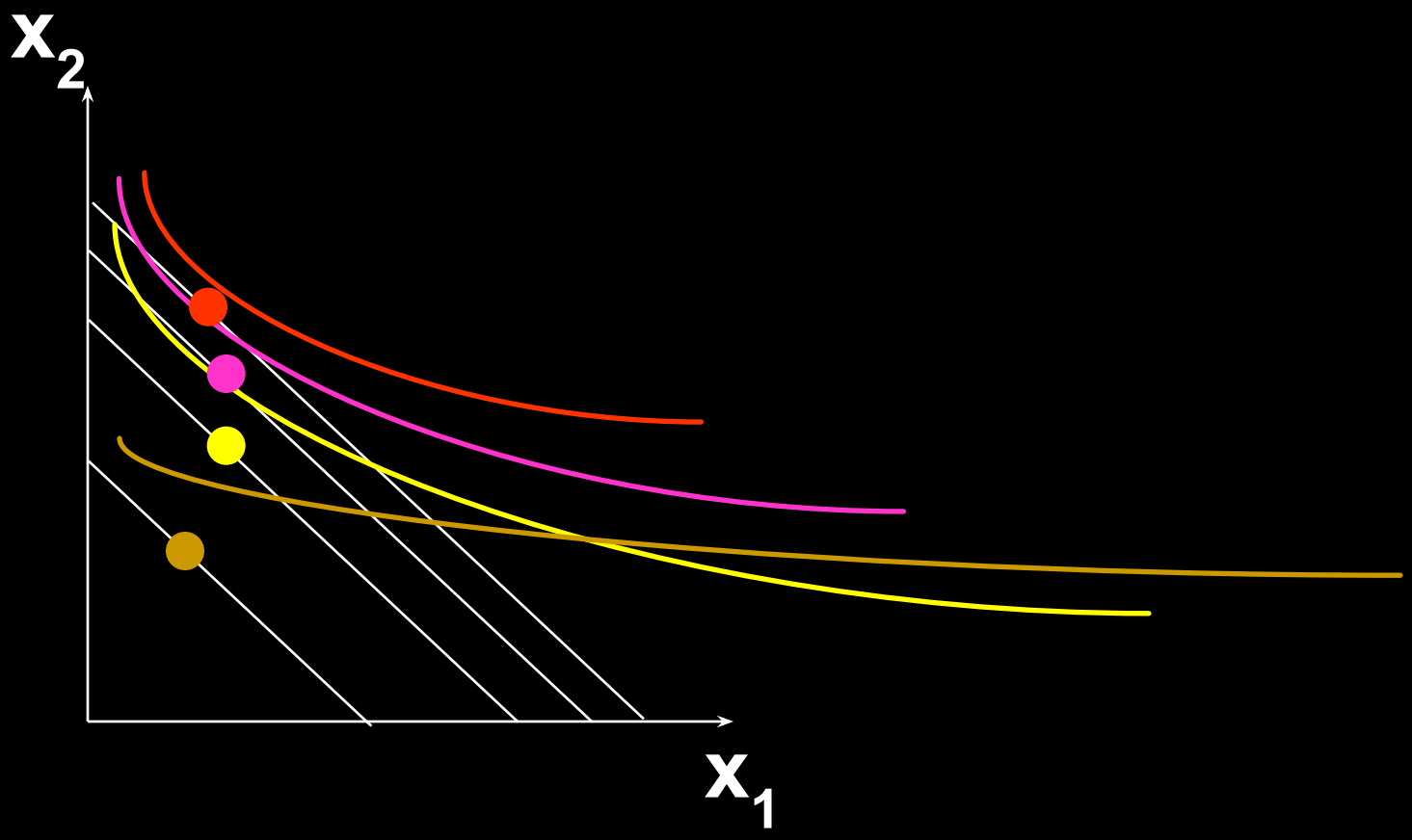
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



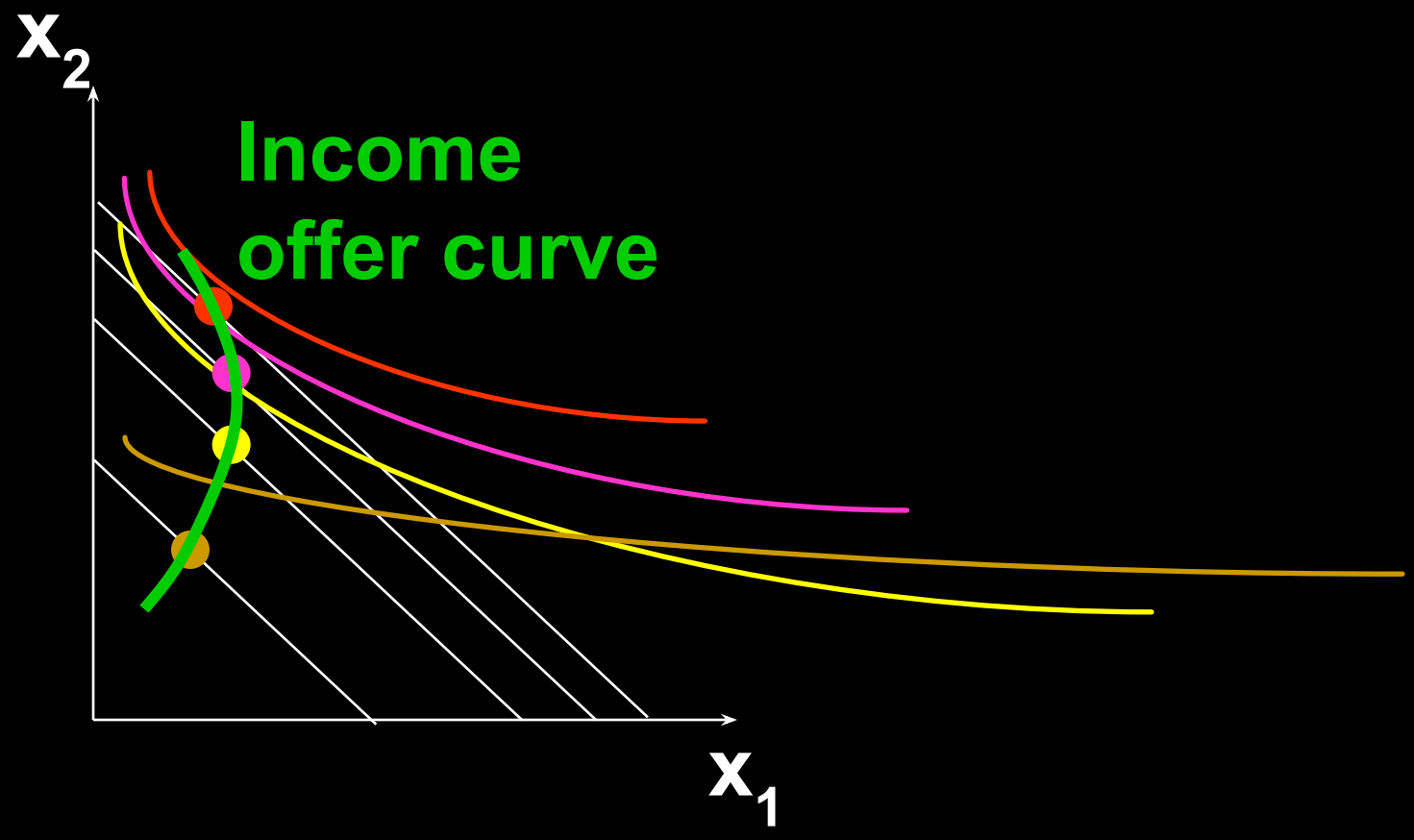
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



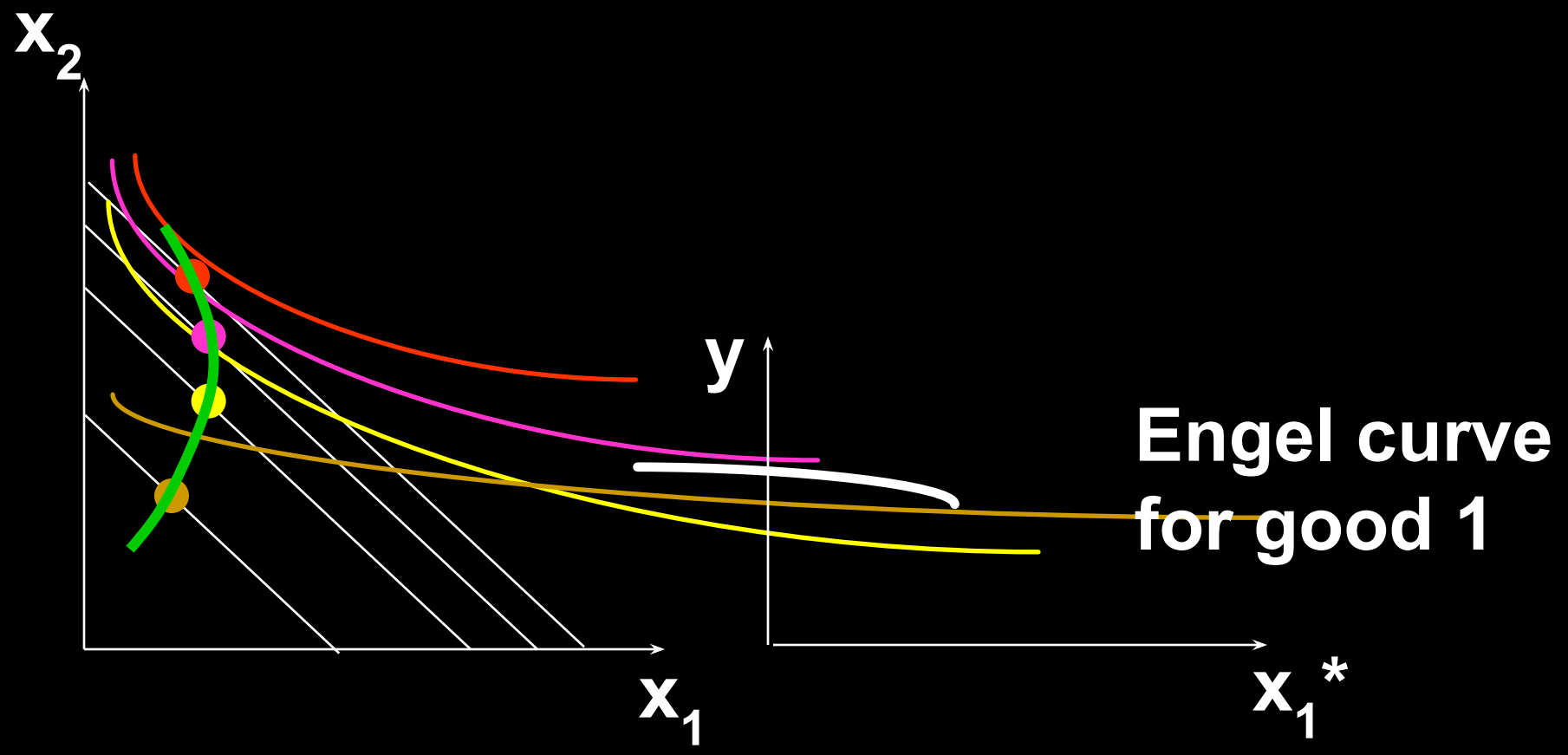
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



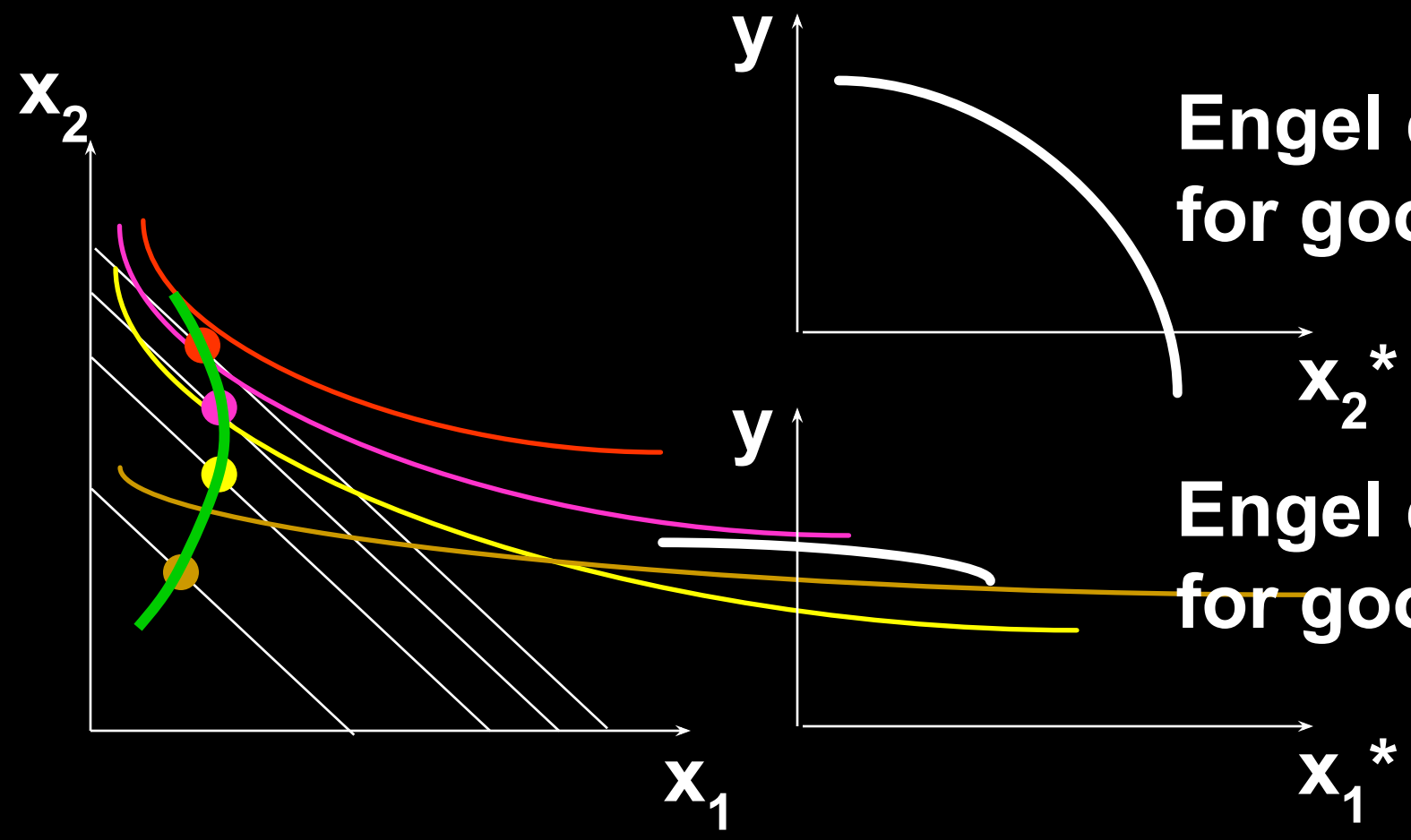
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

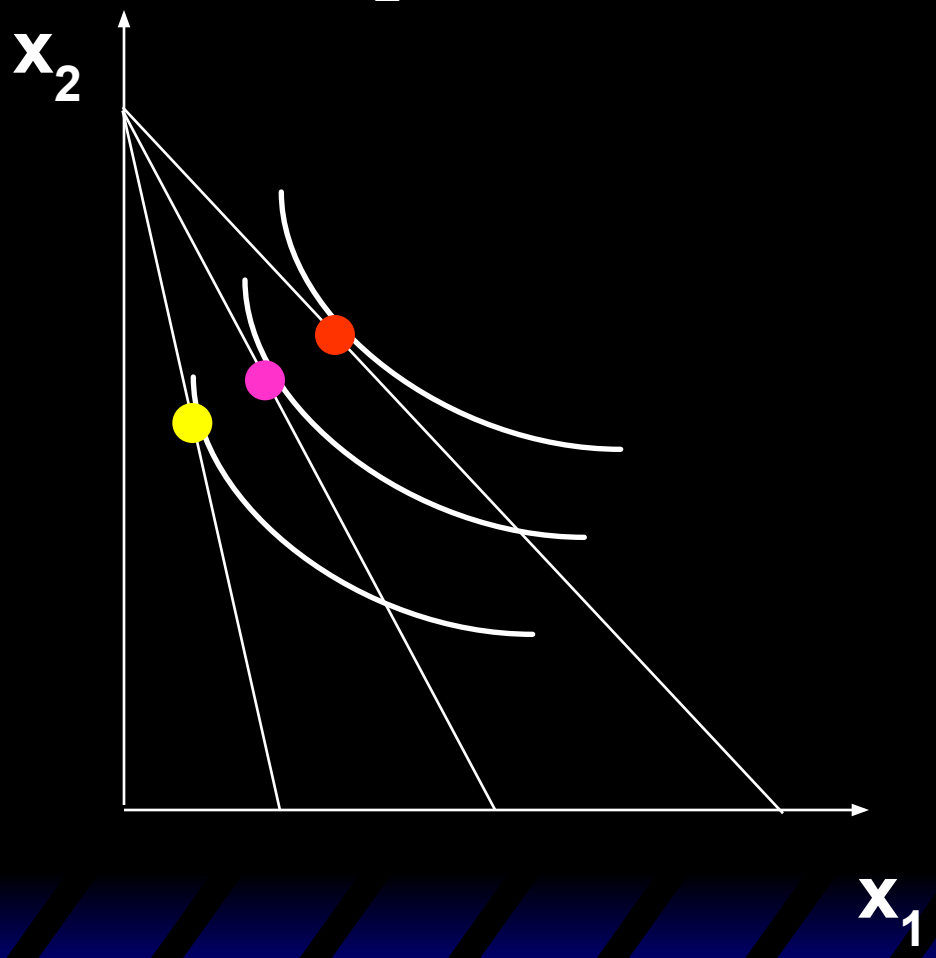


Ordinary Goods

- **A good is called ordinary if the quantity demanded of it always increases as its own price decreases.**

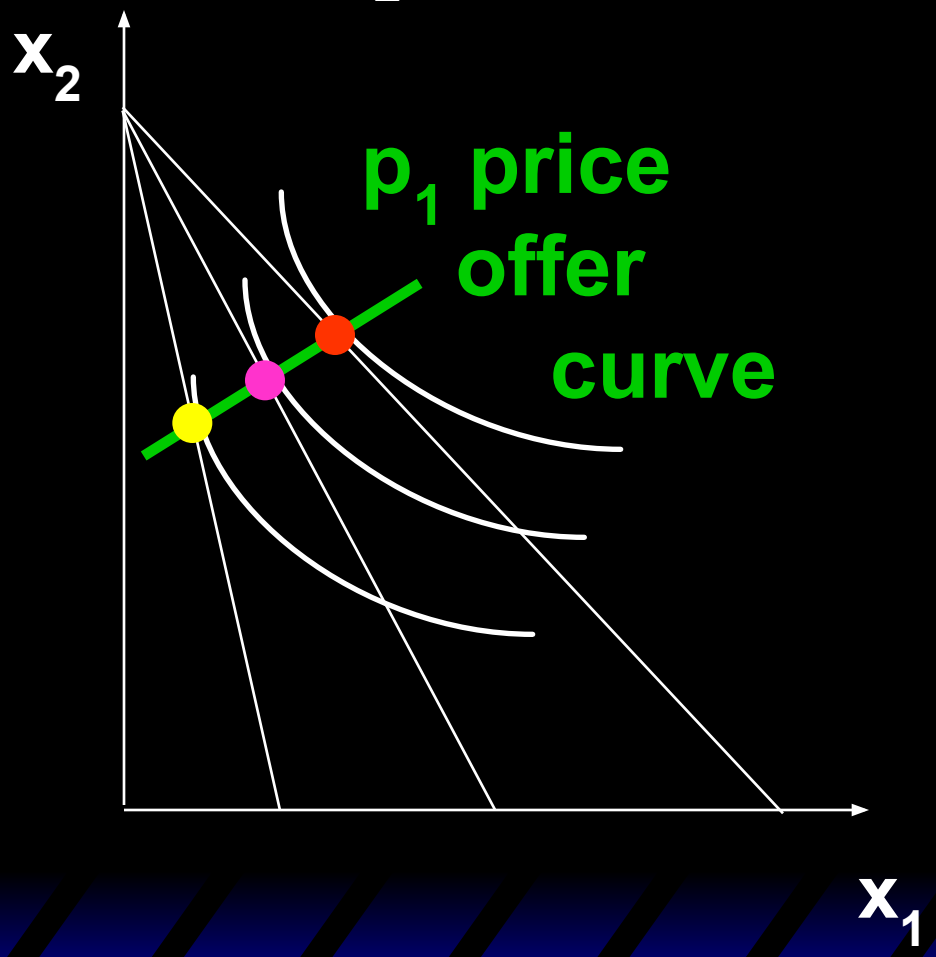
Ordinary Goods

Fixed p_2 and y.



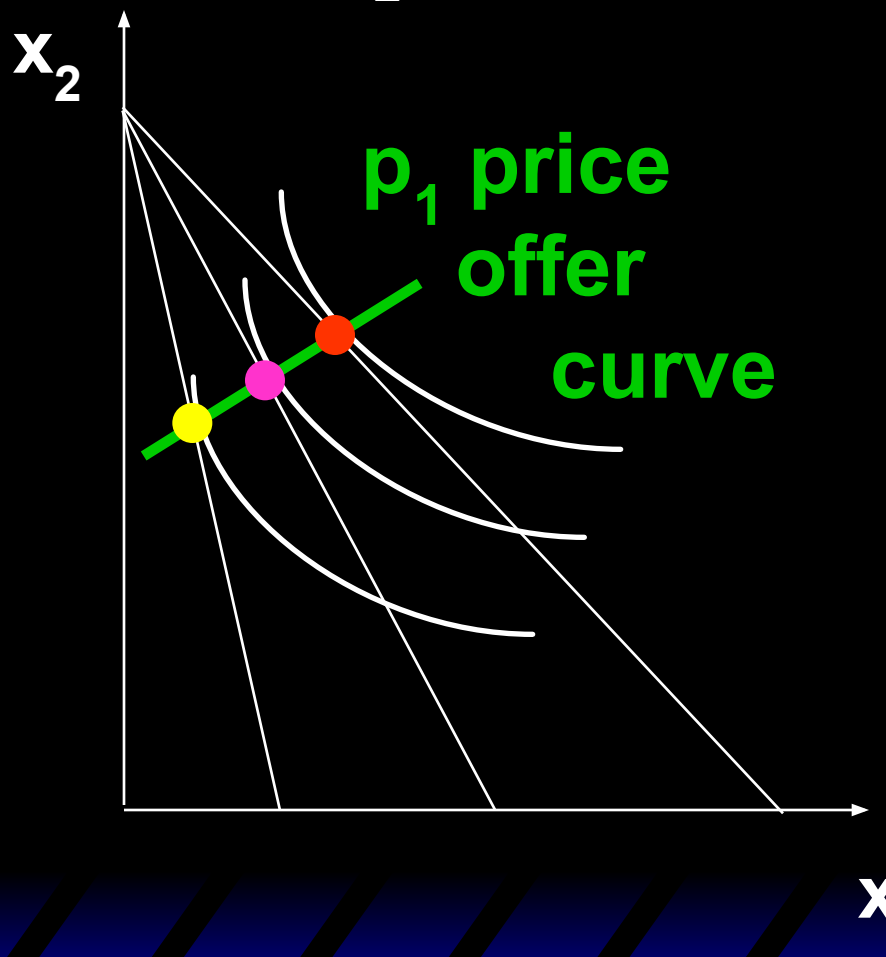
Ordinary Goods

Fixed p_2 and y.

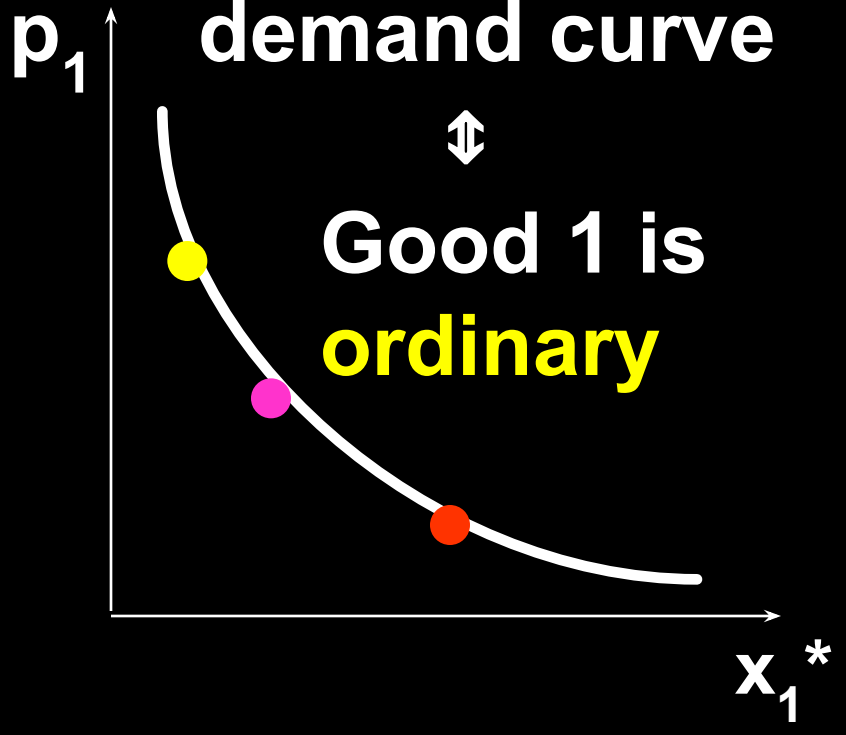


Ordinary Goods

Fixed p_2 and y.



Downward-sloping demand curve



⇕

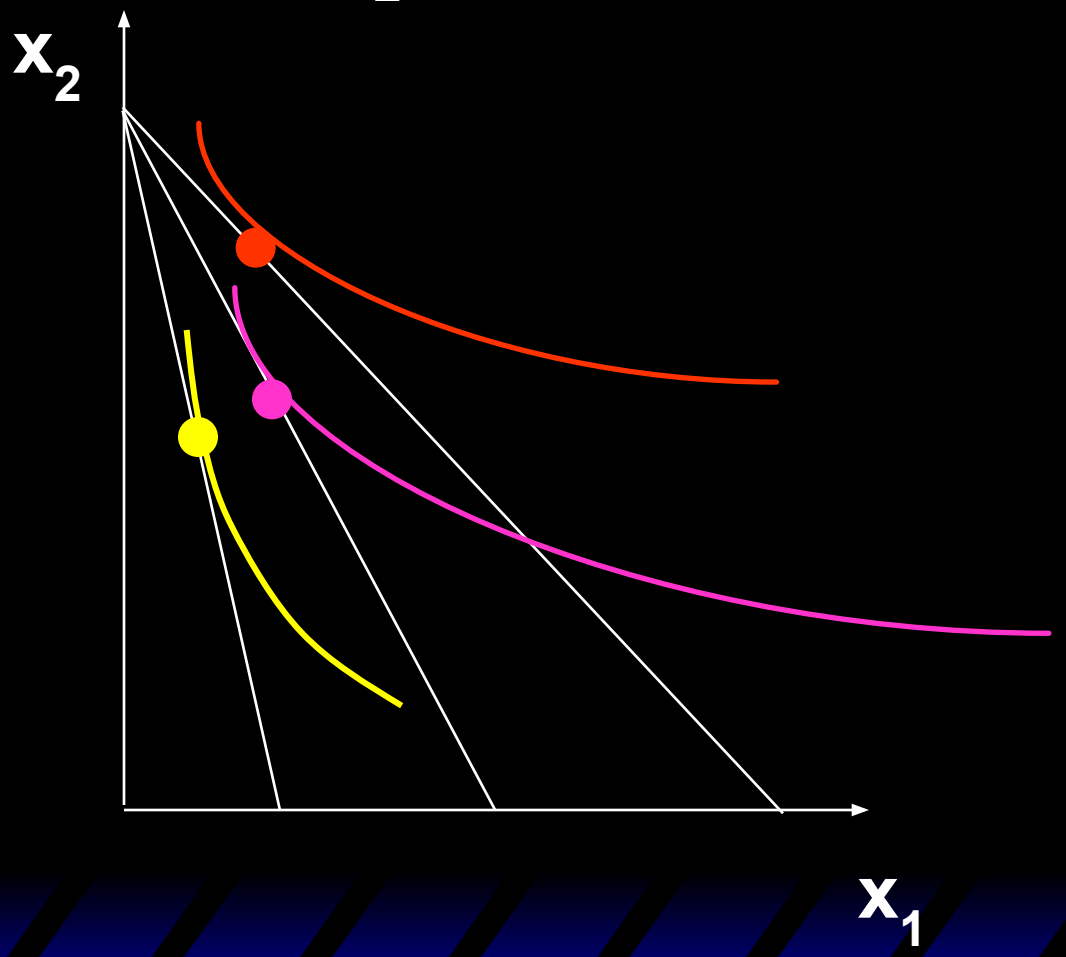
Good 1 is ordinary

Giffen Goods

- **If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.**

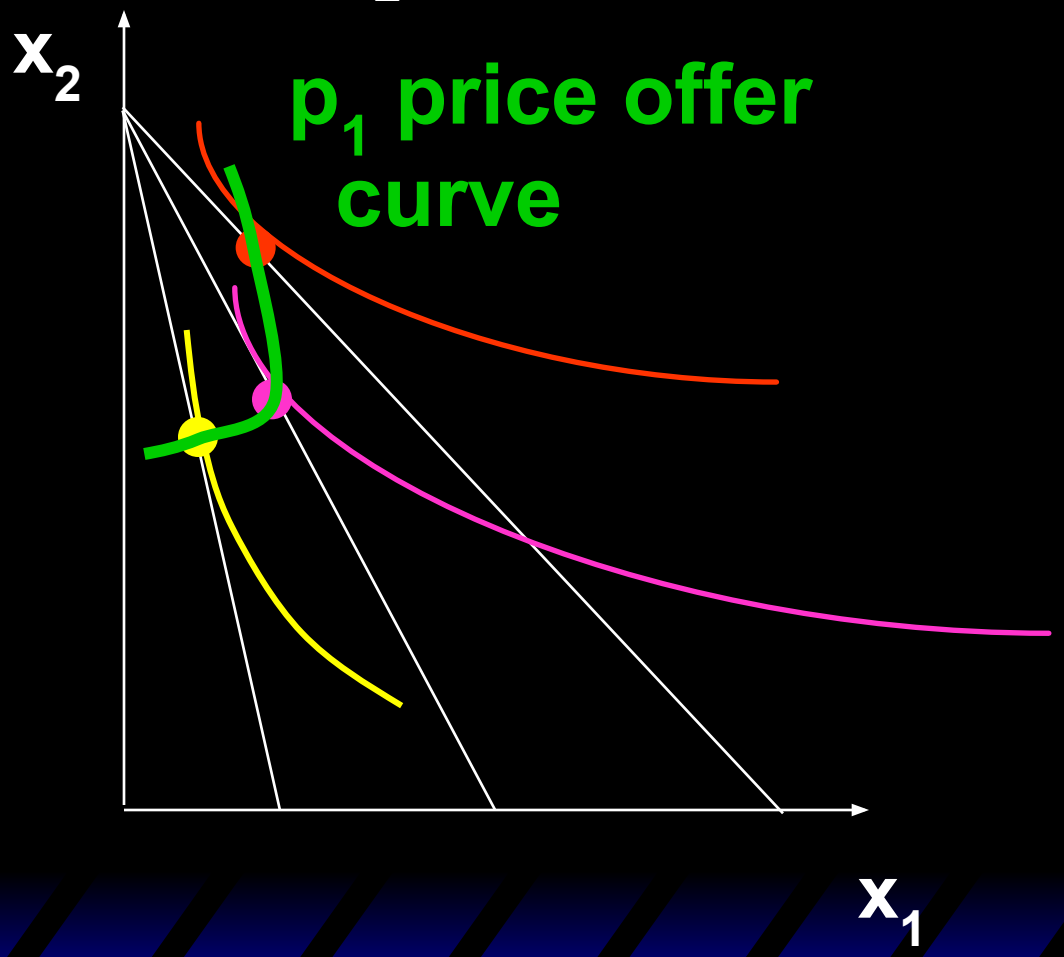
Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

Fixed p_2 and y.

Demand curve has a positively sloped part

⇕

Good 1 is Giffen

Cross-Price Effects

- If an increase in p_2
  - **increases** demand for commodity 1 then commodity 1 is a **gross substitute** for commodity 2.
  - **reduces** demand for commodity 1 then commodity 1 is a **gross complement** for commodity 2.

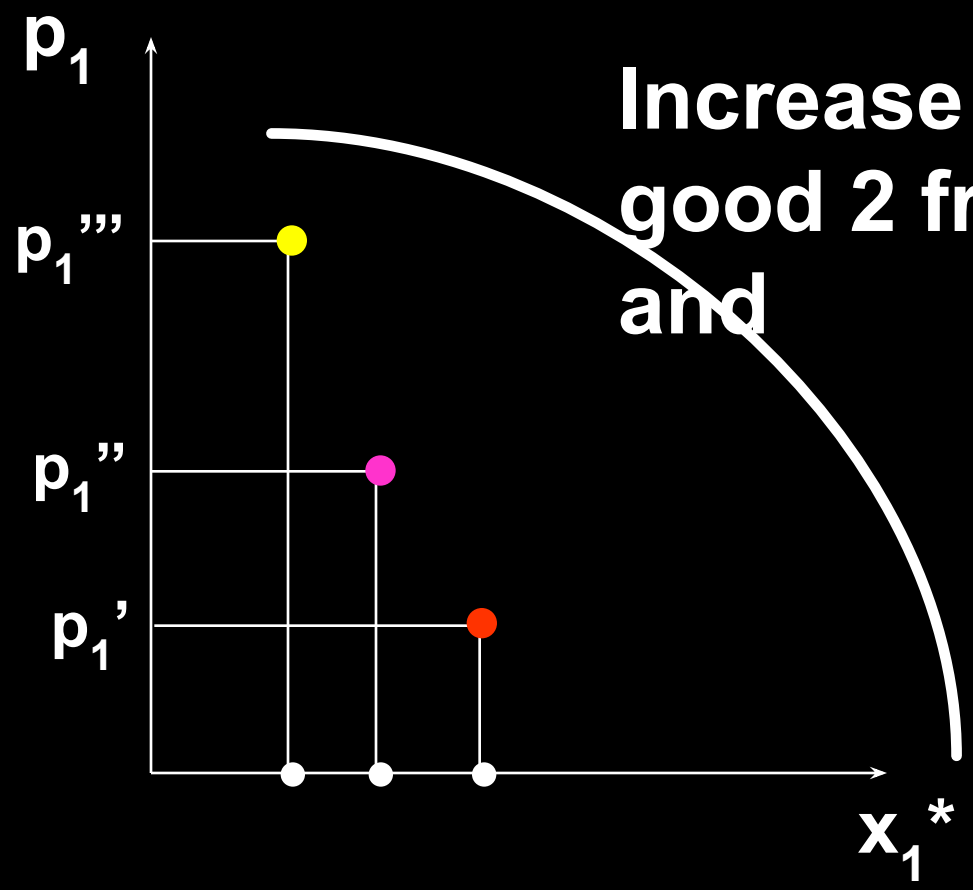
Cross-Price Effects

A perfect-complements example:

so

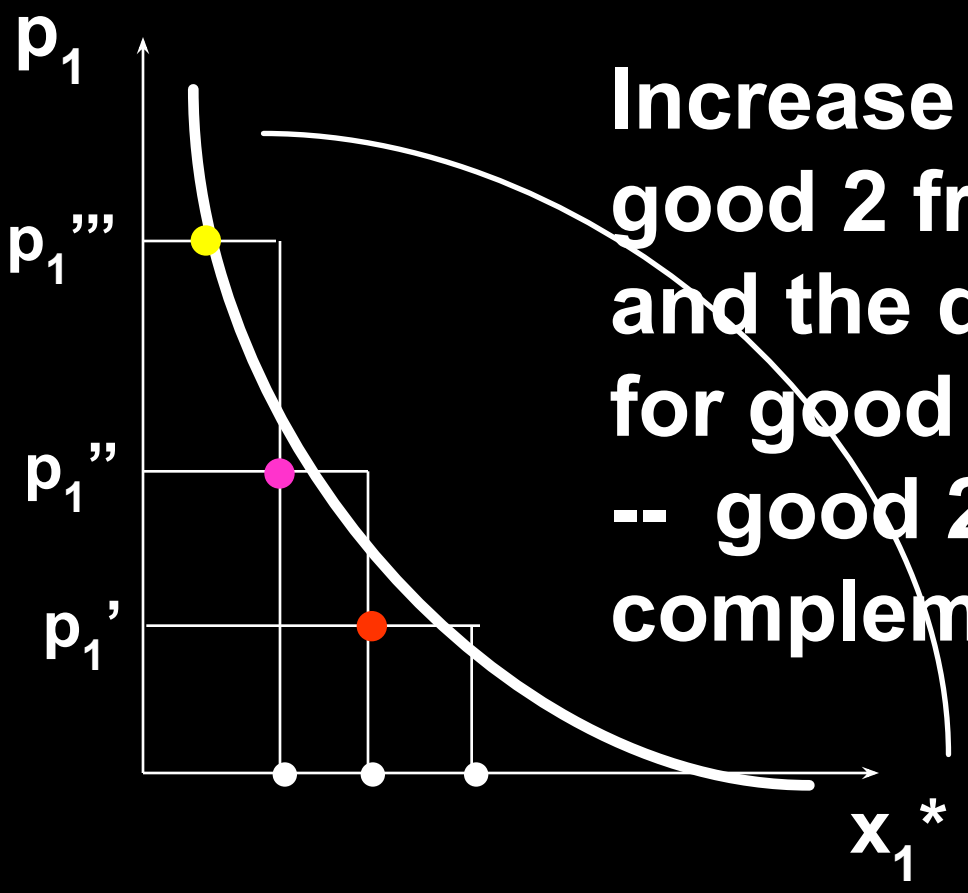
Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

Cross-Price Effects

A Cobb- Douglas example:

so

Cross-Price Effects

A Cobb- Douglas example:

so

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.