MECHANICAL WAVE

is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next

MECHANICAL WAVE

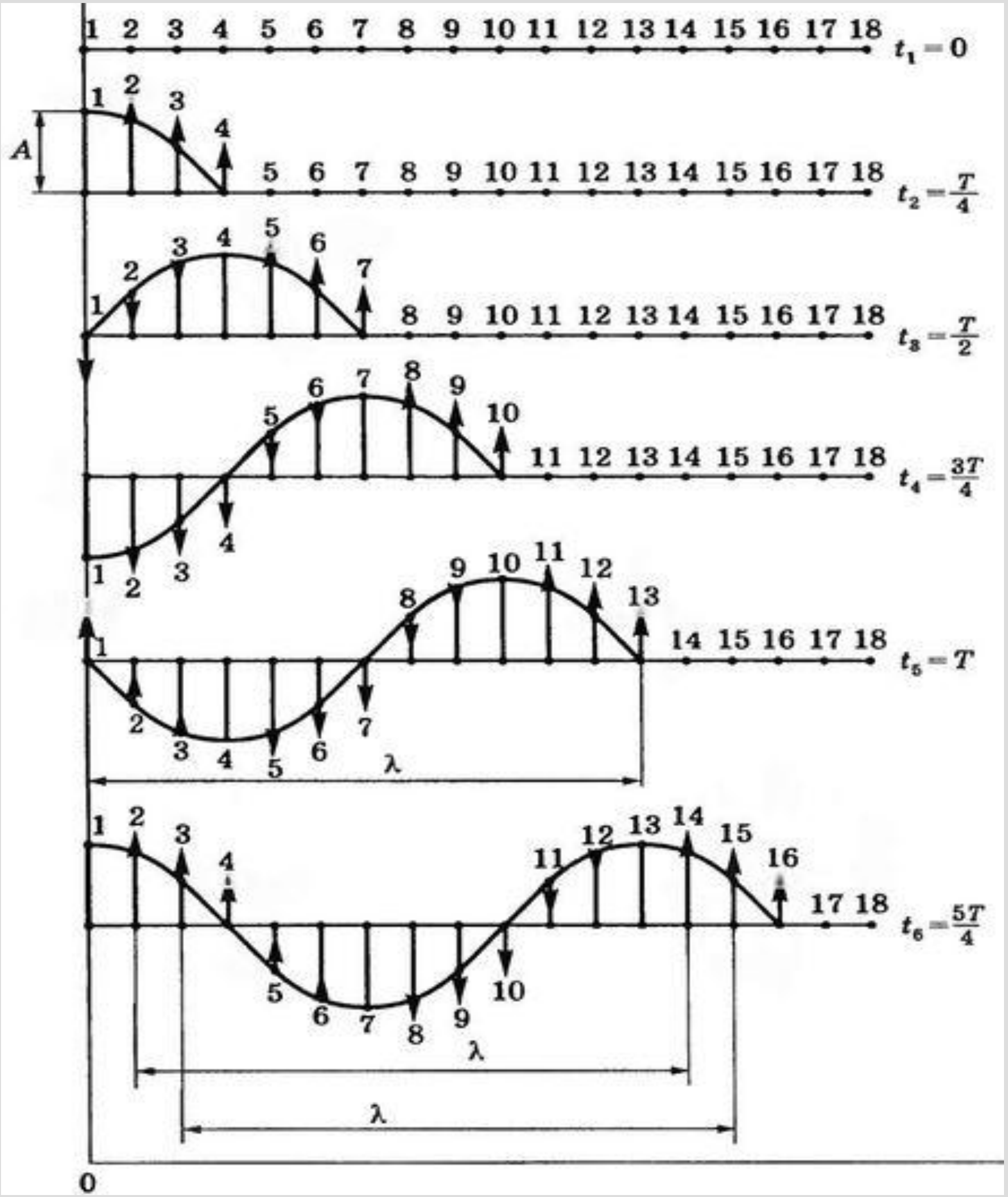
TRANSVERSE WAVE

LONGITUDINAL WAVE

A **transverse wave** is a type of wave motion when the particles of the medium vibrate at right angles to the direction of propagation of the wave

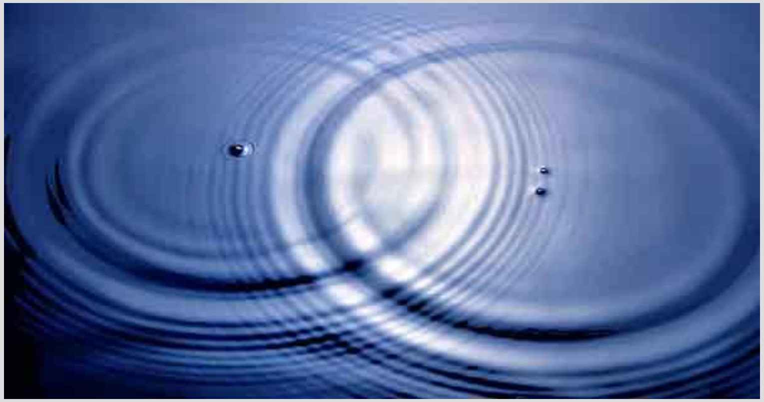
A **longitudinal wave** is one in which vibrations occur in the same direction as the direction the wave travels along

PROGRESSIVE TRANSVERSE WAVE

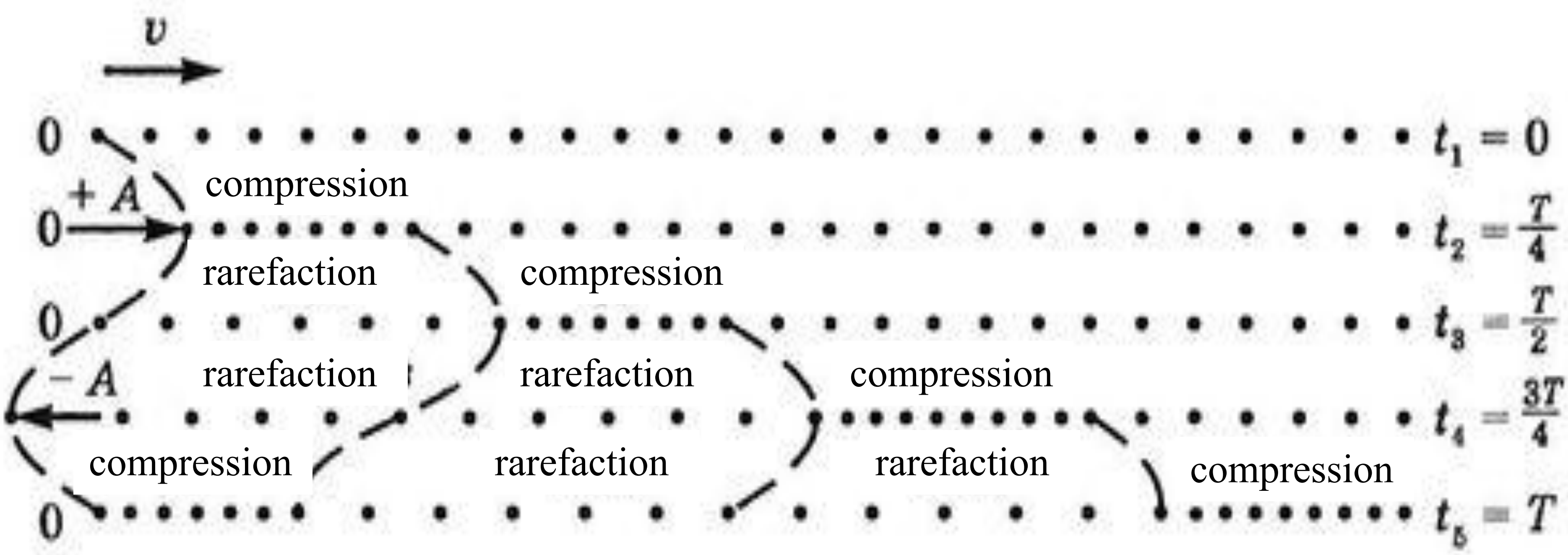


Each particle vibrates perpendicular to the direction of propagation with the same amplitude and frequency

Light and all other electromagnetic waves, and water waves are transverse



PROGRESSIVE LONGITUDINAL WAVE



The displacements of the particles cause regions of high density (compressions) and of low density (rarefactions) to be formed along the wave

Each particle vibrates about its mean position with the same amplitude and frequency

Longitudinal wave is a sound wave. This is propagated by alternate compressions and rarefactions of the air

CHARACTERISTICS of MECHANICAL WAVES

f is frequency of a wave

$\omega = 2\pi f$ is cyclic frequency of a wave

$T = \frac{2\pi}{\omega} = \frac{1}{f}$ is period of a wave

$\vec{v}$ $\quad v = \frac{\omega}{k}$ is phase velocity of a wave

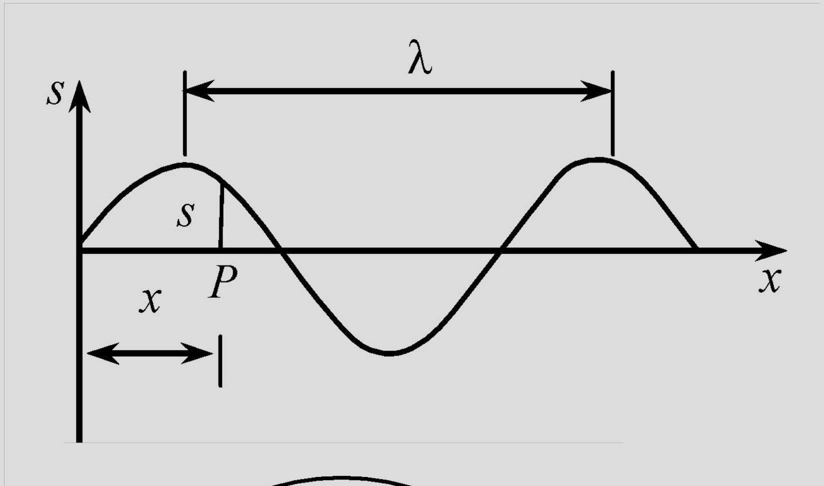
$\vec{u}$ $\quad u = \frac{d\omega}{dk}$ is group velocity of a wave

$\vec{k}$ is wave vector directed along the normal to the wave surface. It demonstrates the direction of wave propagation

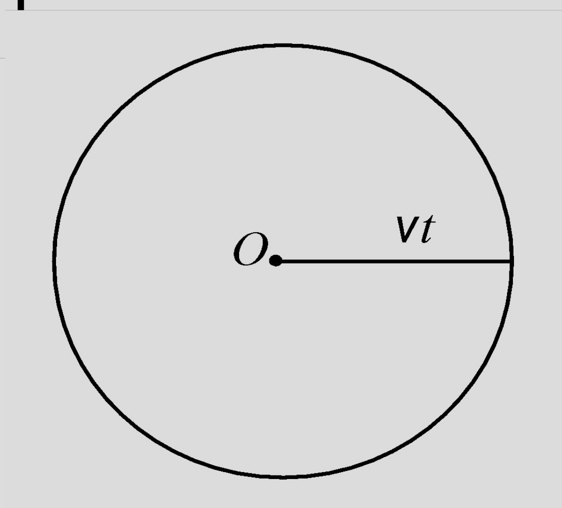
$|k| = 2\pi/\lambda$ is wave number. It is modulus of wave vector

$$\lambda = vT = \frac{v}{f} = \frac{2\pi v}{\omega}$$ **is a wavelength**

$S(A)$ is **amplitude** of a wave



The distance travelled by a wave in the time in which the particle of the medium completes one vibration is the **wavelength**



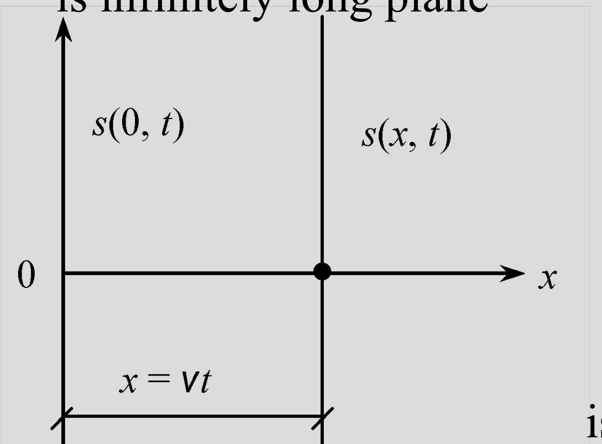
O is point source

wave front (there are spherical, plane, cylindrical and other waves depending on the shape of a wavefront)

After time t the wave has travelled distance υt
υ is the velocity of the wave
The energy of the wave has reached the surface of a sphere of centre O and radius υt
The surface of the sphere is called the **wavefront** of the wave at this instant.
Every point on it is vibrating **in phase** with every other point

WAVE EQUATION

Source of wave
is infinitely long plane



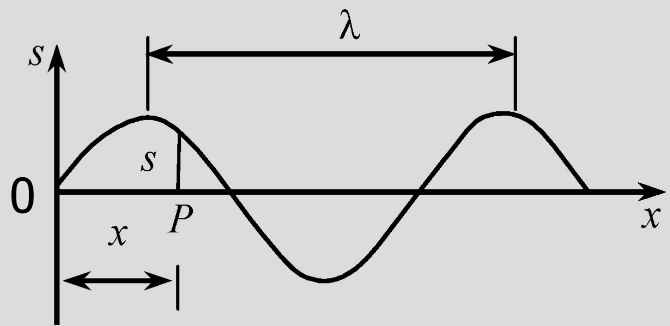
a) a plane-progressive wave

$$s(0,t) = s_0 \cos \omega t \quad \tau = x/v$$

$$(x,t) = s_0 \cos \omega (t-\tau) = s_0 \cos \omega (t - x/v)$$

$$v = \omega / k \qquad s(x,t) = s_0 \cos(\omega t - kx)$$

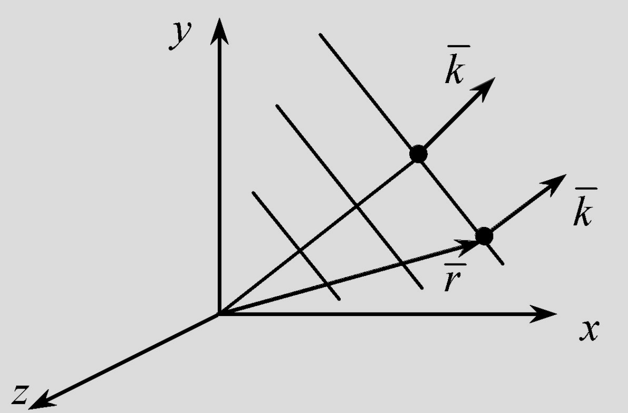
is equation of a ***linear-progressive*** *wave* (a *plane-progressive wave*)



The negative sign indicates that, since the wave moves from left to right, the vibrations at points such as *P* to the right of *O* will lag that at *O*

A wave travelling in the *opposite direction*, arrives at *P* before *O*. Thus, the vibration at *P* leads that at *O*.

$$s(x,t) = s_0 \cos(\omega t + kx)$$

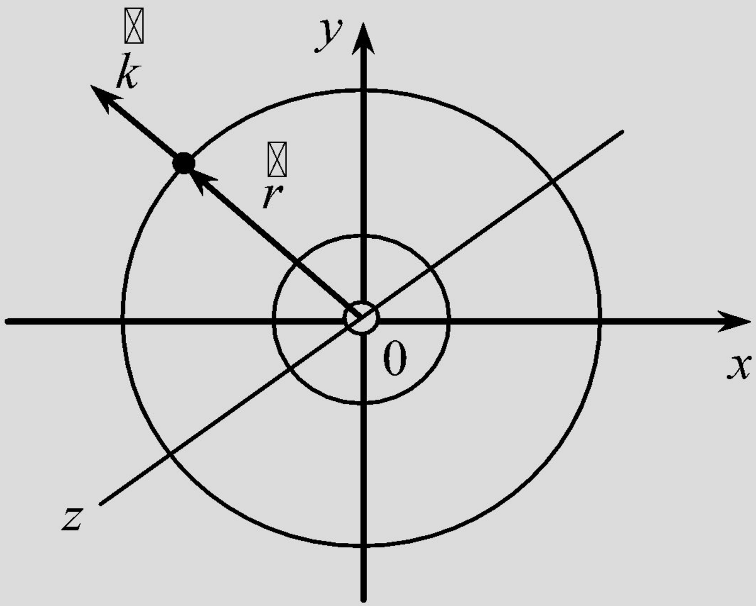


General case: $$s(\vec{r},t) = s_0 \cos\left[\omega t - \left(\vec{k} \cdot \vec{r}\right)\right].$$

$$\left(\vec{k} \cdot \vec{r}\right) = k_x x + k_y y + k_z z$$

$$s(\vec{r},t) = s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right).$$

b) a spherical wave



$$s(\vec{r},t)=\frac{s_0}{r}\cos(\omega t-kr).$$

is equation of a ***spherical wave***

r is the radius of the spherical wavefront at time t

The amplitude of the wave diminishes as distance *r* from the source *O* increases. This is caused by the decrease of energy density at larger distances from the source

$$s(\vec{r},t) = s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right)$$

$$\frac{\partial^2 s}{\partial t^2} = -\omega^2 s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right) = -\omega^2 s$$

$$\frac{\partial^2 s}{\partial x^2} = -k_x^2 s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right)$$

$$\frac{\partial^2 s}{\partial y^2} = -k_y^2 s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right)$$

$$\frac{\partial^2 s}{\partial z^2} = -k_z^2 s_0 \cos\left(\omega t - k_x x - k_y y - k_z z\right)$$

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=-\left(k_x^2+k_y^2+k_z^2\right)s_0\cos\left(\omega t-k_x x-k_y y-k_z z\right)$$

$$k_x^2+k_y^2+k_z^2=k^2$$

$$s_0\cos\left(\omega t-k_x x-k_y y-k_z z\right)=s$$

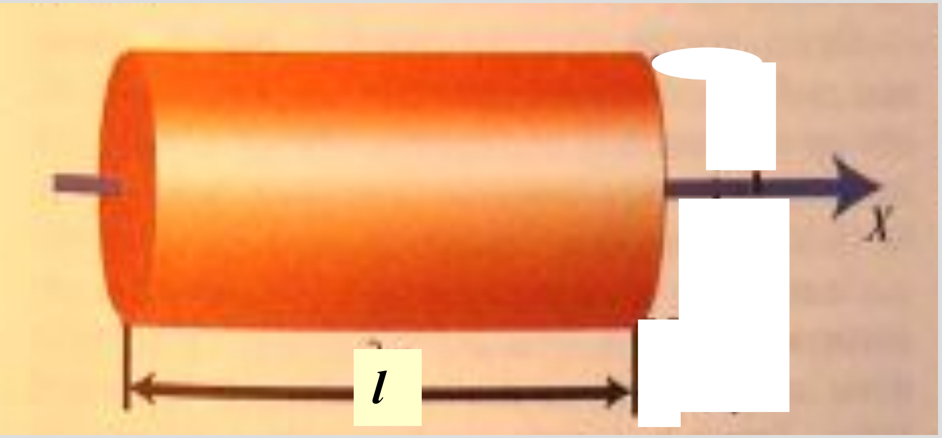
$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=-k^2 s.$$

$$s=-\frac{1}{\omega^2}\cdot\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=\frac{k^2}{\omega^2}\cdot\frac{\partial^2 s}{\partial t^2}=\frac{1}{v^2}\cdot\frac{\partial^2 s}{\partial t^2}$$

$\Delta s=\frac{1}{v^2}\cdot\frac{\partial^2 s}{\partial t^2}$ is the *wave differential equation*

PHASE VELOCITY OF WAVE



If an elastic bar is caused to vibrate by being strucked, an elastic wave begins to propagate along the bar

Phase velocity is $v = l/\Delta t$

Δt is impact duration

l is displacement of a wave front

Velocity of displacement of bar points after impact is $u = \Delta l/\Delta t$

Δl is displacement of points of a bar end per Δt

$$\Delta(mu) = mu = F\Delta t. \qquad F\Delta t = mu = \rho l A u = \rho l A\frac{\Delta l}{\Delta t}$$

$$F = EA\frac{\Delta l}{l}$$ E is *Young's modulus of elasticity*

$$EA\frac{\Delta l}{l}\Delta t = \rho l A\frac{\Delta l}{\Delta t} = \rho A\Delta l\frac{l}{\Delta t}. \qquad \rho\frac{l^2}{\Delta t^2} = \rho v^2 = E.$$

$v = \sqrt{E/\rho}$ is the phase velocity of a longitudinal elastic wave

$v = \sqrt{G/\rho}$ is the phase velocity of a transversal wave

G is *shear modulus*