# **Power Converter Systems**

#### **Graduate Course EE8407**

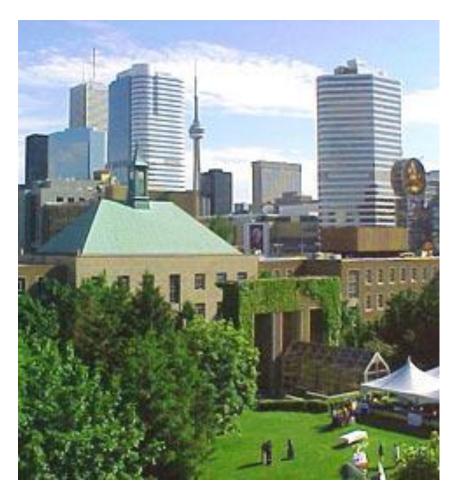
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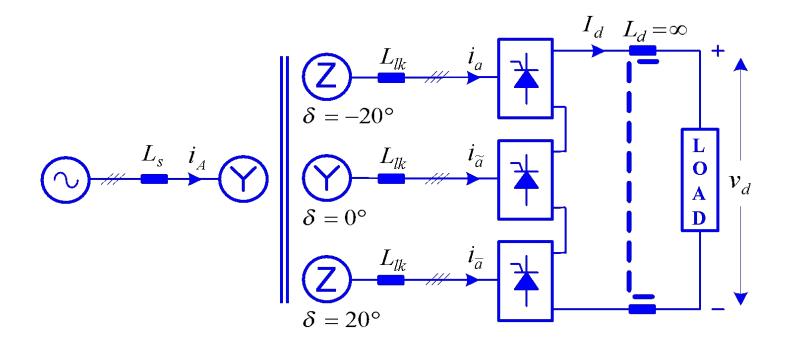
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**Ryerson Campus** 

## Topic 4

#### **Multi-pulse SCR Rectifiers**



## **Multi-pulse SCR Rectifiers**

#### **Lecture Topics**

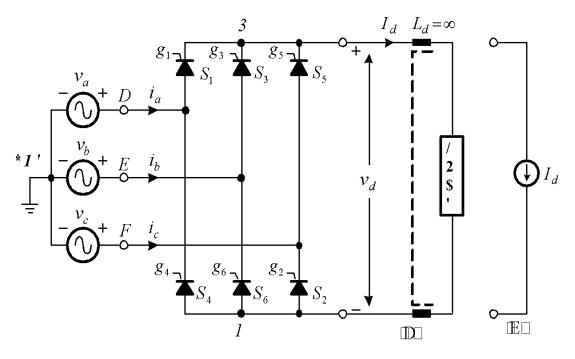
- Six-pulse SCR Rectifier (Building Block)
- 12-, 18- and 24-pulse SCR Rectifiers
- THD and PF

## **Multi-pulse SCR Rectifiers**

## Why Use Multi-pulse SCR Rectifiers?

- To reduce line THD
- To improve input power factor
- To avoid semiconductor devices in series.

Converter Configuration



- Assumption: 1) Ideal SCRs no power loss
  - 2) de current ripple free
- Use constant current source I<sub>d</sub> as a load

#### Waveforms

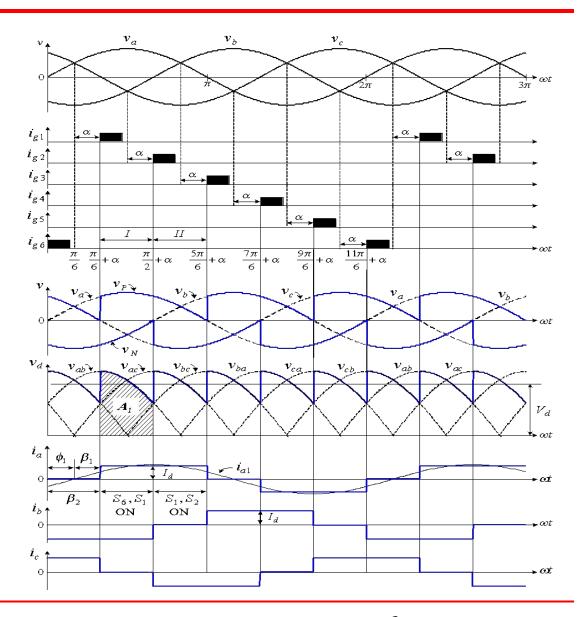
#### Average dc voltage

$$V_{d} = \frac{\text{area } \mathbf{A}_{1}}{\pi/3}$$

$$= \frac{1}{\pi/3} \int_{\pi/6+\alpha}^{\pi/2+\alpha} v_{ab} d(\omega t)$$

$$= \frac{3\sqrt{2}}{\pi} V_{LL} \cos \alpha$$

$$\approx 1.35 V_{LL} \cos \alpha$$



#### Fourier Analysis

$$i_{a} = \frac{2\sqrt{3}}{\pi} I_{d} \left( \sin(\omega t - \phi_{1}) - \frac{1}{5} \sin 5(\omega t - \phi_{1}) - \frac{1}{7} \sin 7(\omega t - \phi_{1}) + \frac{1}{11} \sin 11(\omega t - \phi_{1}) + \frac{1}{13} \sin 13(\omega t - \phi_{1}) - \frac{1}{17} \sin 17(\omega t - \phi_{1}) - \frac{1}{19} \sin 19(\omega t - \phi_{1}) + \dots \right)$$

#### RMS current

$$I_{a} = \left(\frac{1}{2\pi} \int_{0}^{2\pi} (i_{a})^{2} d(\omega t)\right)^{1/2} = \left(\frac{1}{2\pi} \left(\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} (I_{d})^{2} d(\omega t) + \int_{\frac{7\pi}{6} + \alpha}^{\frac{11\pi}{6} + \alpha} (-I_{d})^{2} d(\omega t)\right)\right)^{1/2}$$

$$= \sqrt{\frac{2}{3}} I_{d} = 0.816 I_{d}$$

#### THD

$$THD = \frac{\sqrt{I_a^2 - I_{a1}^2}}{I_{a1}} = \frac{\sqrt{(0.816I_d)^2 - (0.78I_d)^2}}{0.78I_d} = 0.311$$

Power Factor (PF)

Displacement Power Factor

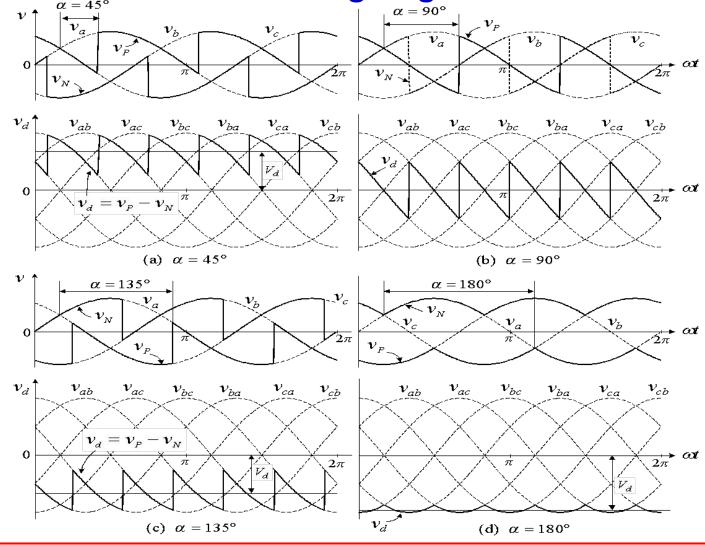
$$\phi_1 = \beta_2 - \beta_1 = \alpha$$

$$DPF = \cos \phi_1 = \cos \alpha$$

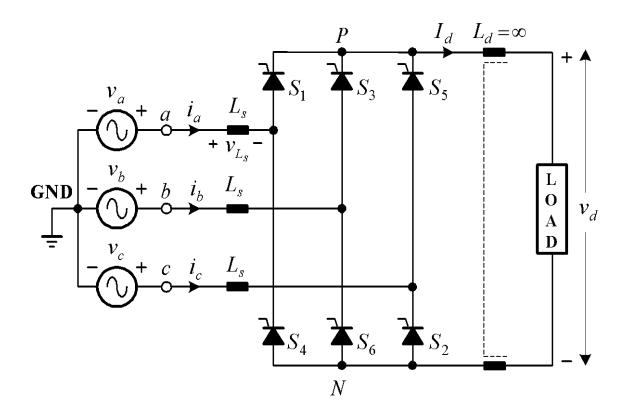
Overall Power Factor

$$PF = \frac{\cos\phi_1}{\sqrt{1 + THD^2}} = 0.955\cos\alpha$$

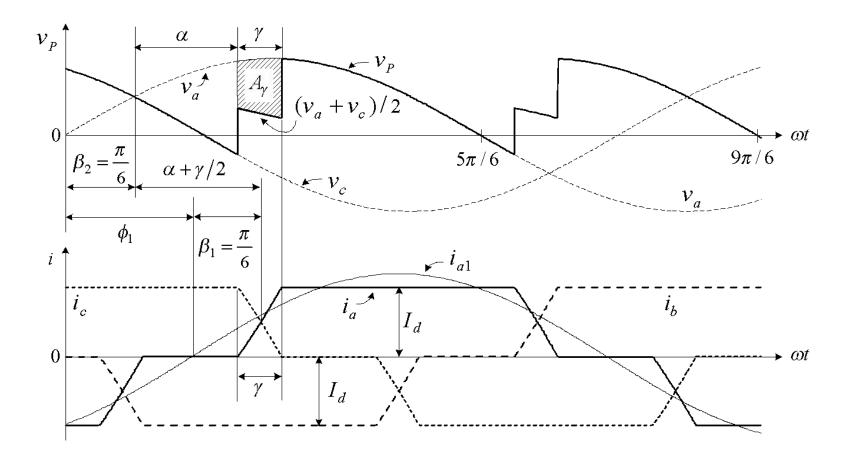
# • Waveforms at Various Firing Angles $\alpha = 45^{\circ}$



Effect of Line Inductance L<sub>s</sub>



#### Waveform During SCR Commutation



- DC Voltage Reduction (caused by commutation)
  - $S_1$  and  $S_5$  on (during commutation)

$$v_{P} = v_{a} - L_{s} \frac{di_{a}}{dt} = v_{c} - L_{s} \frac{di_{c}}{dt}$$

from which

$$v_{P} = \frac{v_{a} + v_{c}}{2} - \frac{L_{s}}{2} \left( \frac{di_{a}}{dt} + \frac{di_{c}}{dt} \right)$$

• Since  $i_a + i_c = I_d$ , and  $i_d$  is constant

$$\frac{di_a}{dt} + \frac{di_c}{dt} = 0$$

from which

$$v_{P} = \frac{v_{a} + v_{c}}{2}$$

#### DC Voltage Reduction

- Shaded Area  $A_{\gamma}$   $A_{\gamma} = \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha + \gamma} (v_a v_p) d(\omega t)$
- Since  $v_a v_p = L_s (di_a/dt)$   $A_{\gamma} = \int_0^{I_d} \omega L_s di_a = \omega L_s I_d$
- Average dc voltage loss

$$\Delta V = \frac{A_{\gamma}}{\pi/3} = \frac{3\omega L_{s}}{\pi} I_{d}$$

Average dc output voltage

$$V_d = 1.35 V_{LL} \cos \alpha - \frac{3\omega L_s}{\pi} I_d$$

#### Commutation Interval

 $ullet A_{\gamma}$  can be rearranged as

$$A_{\gamma} = \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha + \gamma} \frac{v_{ac}}{2} d(\omega t) = \frac{V_{LL}}{\sqrt{2}} (\cos \alpha - \cos(\alpha + \gamma))$$

where 
$$v_{ac} = \sqrt{2}V_{LL} \sin(\omega t - \frac{\pi}{6})d(\omega t)$$

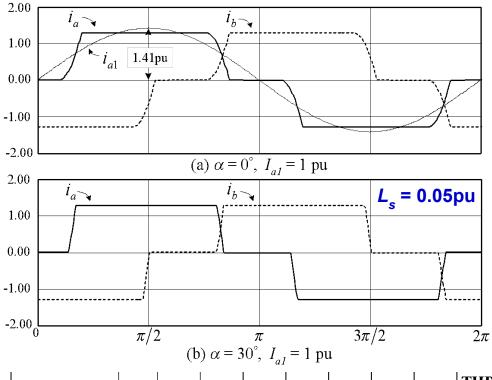
• Substitute  $\omega L_s I_d$  into the above equation

$$\omega L_{s} I_{d} = \frac{V_{LL}}{\sqrt{2}} (\cos \alpha - \cos(\alpha + \gamma))$$

from which

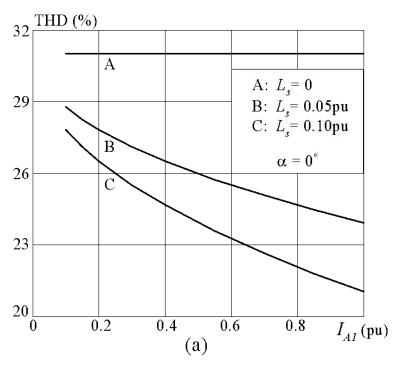
$$\gamma = \cos^{-1} \left( \cos \alpha - \frac{\sqrt{2} \omega L_s}{V_{IL}} I_d \right) - \alpha$$

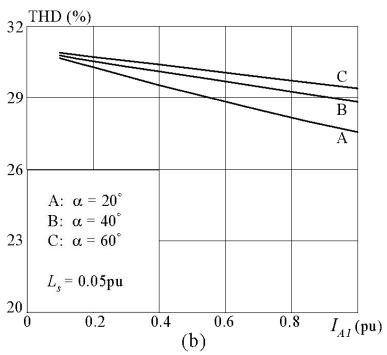
#### Line Current Waveforms



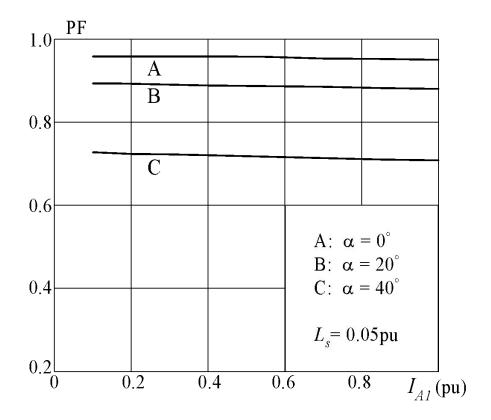
Harmonics n	5	7	11	13	17	19	23	25	THD %
$I_{an}/I_{a1} (\%)$ $\alpha = 0^{\circ}$	18.6	12.4	6.32	4.58	2.40	1.73	1.02	0.87	23.9
$I_{an}/I_{a1} (\%)$ $\alpha = 30^{\circ}$	19.7	14.1	8.58	7.27	5.16	4.62	3.43	3.16	28.3

#### Line Current THD

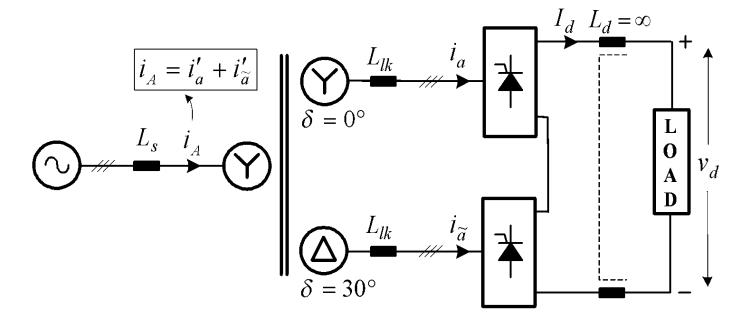




#### Power Factor

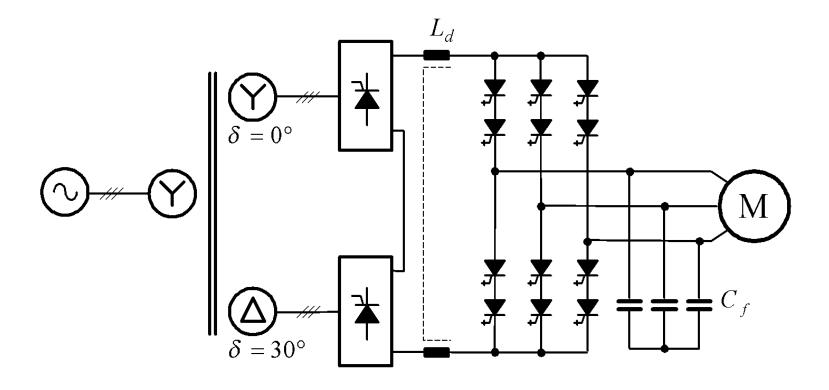


#### Topology



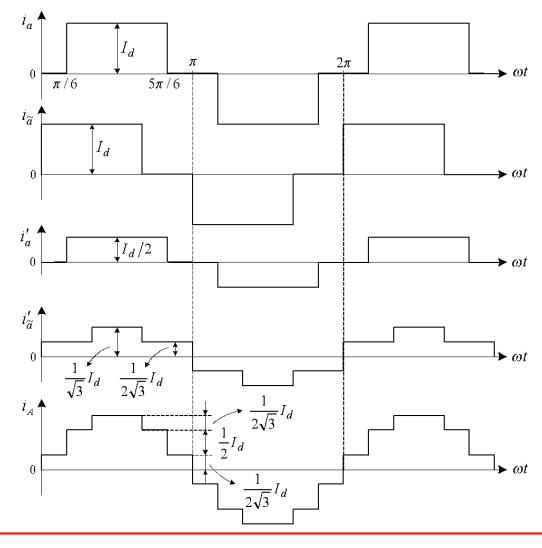
- Main Benefits
  - Low line current THD
  - No SCRs in series

#### Application Example



Used for current source inverter (CSI) fed drive

• Waveforms of Idealized Rectifier  $(L_s = L_{lk} = 0)$ 



#### Fourier Analysis

#### Secondary currents

$$i_{a} = \frac{2\sqrt{3}}{\pi} I_{d} \left( \sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t - \frac{1}{17} \sin 17\omega t - \frac{1}{19} \sin 19\omega t + \dots \right)$$

$$i_{\tilde{a}} = \frac{2\sqrt{3}}{\pi} I_d \left( \sin(\omega t + 30^\circ) - \frac{1}{5} \sin 5(\omega t + 30^\circ) - \frac{1}{7} \sin 7(\omega t + 30^\circ) + \frac{1}{11} \sin 11(\omega t + 30^\circ) + \frac{1}{13} \sin 13(\omega t + 30^\circ) - \frac{1}{17} \sin 17(\omega t + 30^\circ) - \frac{1}{19} \sin 19(\omega t + 30^\circ) + \dots \right)$$

#### Fourier Analysis (continued)

#### Secondary currents referred to the primary side

$$i'_{a} = \frac{\sqrt{3}}{\pi} I_{d} \{ \sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t - \frac{1}{17} \sin 17\omega t - \frac{1}{19} \sin 19\omega t + \dots \}$$

$$i_{\alpha}' = \frac{\sqrt{3}}{\pi} I_{d} \{ \sin \omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t + \frac{1}{17} \sin 17\omega t + \frac{1}{19} \sin 19\omega t + \dots \}$$

#### Primary line current

$$i_{A} = i'_{a} + i'_{a} = \frac{2\sqrt{3}}{\pi} I_{d} \{ \sin \omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t + \frac{1}{23} \sin 23\omega t + \frac{1}{25} \sin 25\omega t + .... \}$$

#### Line Current THD

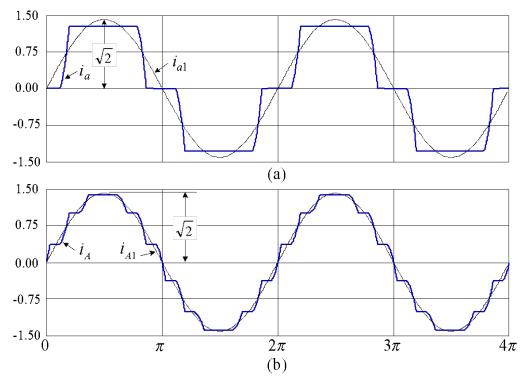
#### Secondary side

$$THD_{i_a} = \frac{\sqrt{I_a^2 - I_{a1}^2}}{I_{a1}} = \frac{\left(I_{a5}^2 + I_{a7}^2 + I_{a11}^2 + I_{a13}^2 + ...\right)^{1/2}}{I_{a1}} = 31.1\%$$

#### Primary side

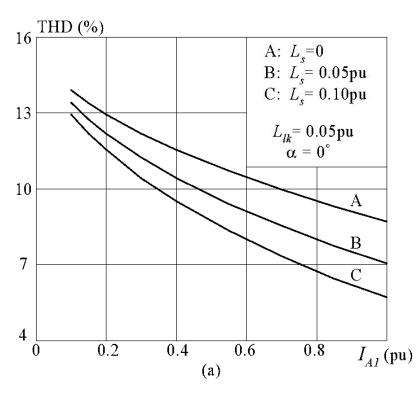
$$THD_{i_A} = \frac{\sqrt{I_A^2 - I_{A1}^2}}{I_{A1}} = \frac{\left(I_{A11}^2 + I_{A13}^2 + I_{A23}^2 + I_{A25}^2 + ...\right)^{1/2}}{I_{A1}} = 15.3\%$$

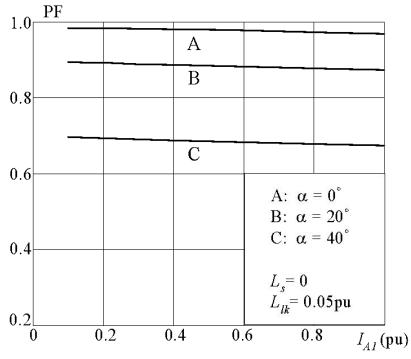
## Line Current Waveform (L<sub>s</sub> ≠ 0 )



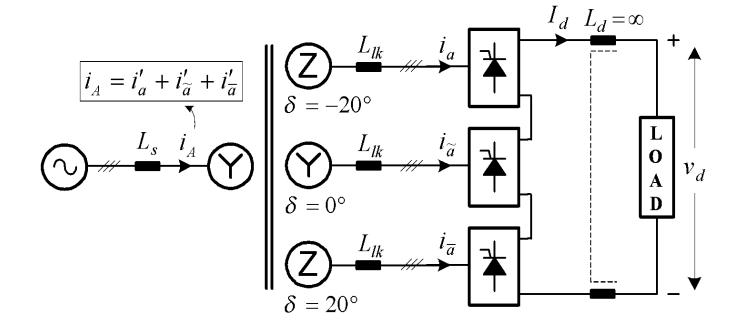
Harmonics n	5	7	11	13	17	19	23	25	<b>THD</b> (%)
$I_{an}/I_{a1}$ (%)	18.8	12.7	6.78	5.05	2.77	2.01	1.01	0.75	24.6
$I_{An}/I_{A1}$ (%)	0	0	6.78	5.05	0	0	1.01	0.75	8.61

#### THD and PF

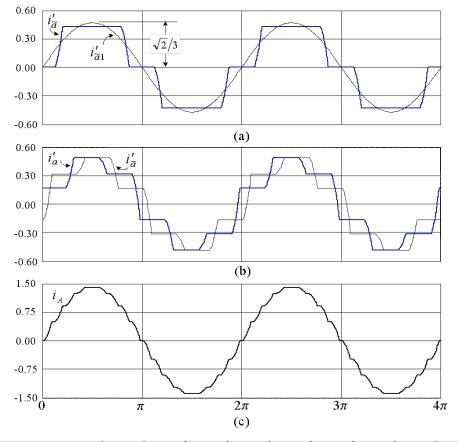




#### Rectifier Configuration

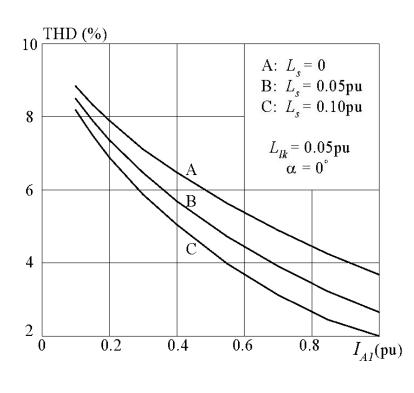


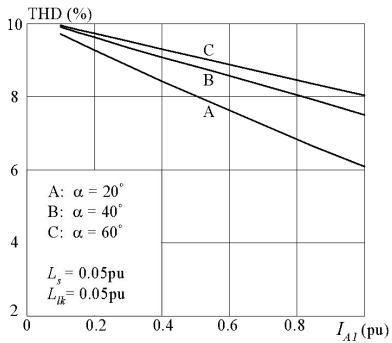
#### Waveforms



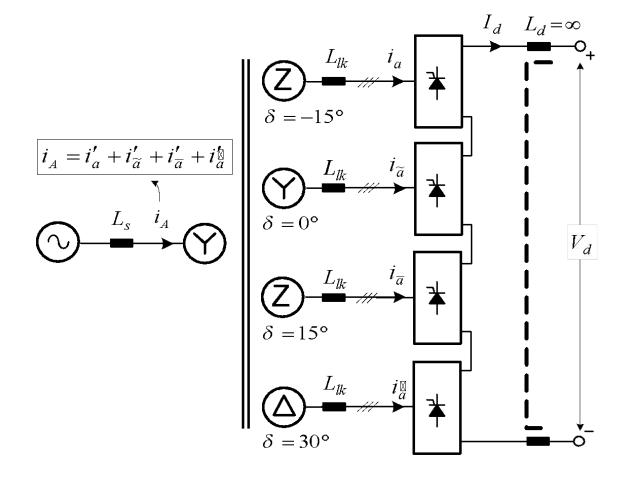
Harmonics <i>n</i>	5	7	11	13	17	19	23	25	THD (%)
$I'_{an} / I'_{a1}$ (%)	18.8	12.7	6.78	5.05	2.77	2.01	1.01	0.75	24.6
$I_{An} / I_{A1}$ (%)	0	0	0	0	2.77	2.01	0	0	3.54

#### Line Current THD

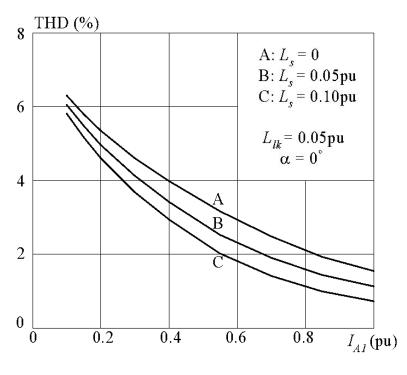


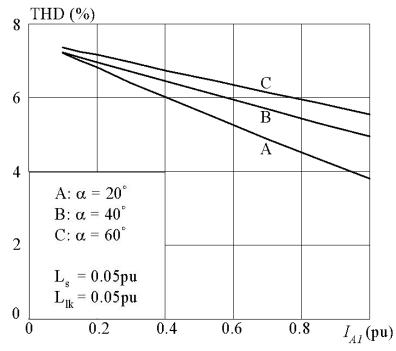


#### Rectifier Configuration



#### Line Current THD





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# Thanks