

# Power Converter Systems

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## Graduate Course EE8407

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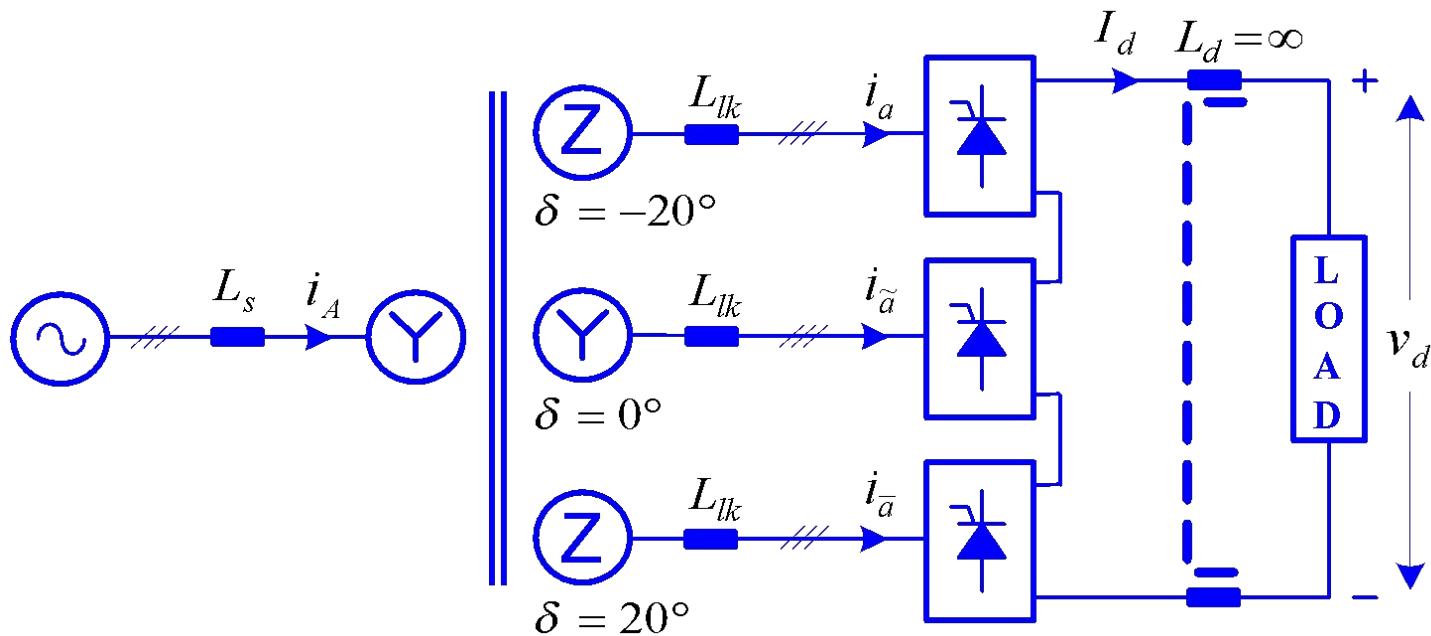
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**Ryerson Campus**

# Topic 4

## Multi-pulse SCR Rectifiers



# Multi-pulse SCR Rectifiers

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## Lecture Topics

- Six-pulse SCR Rectifier (Building Block)
- 12-, 18- and 24-pulse SCR Rectifiers
- THD and PF

# Multi-pulse SCR Rectifiers

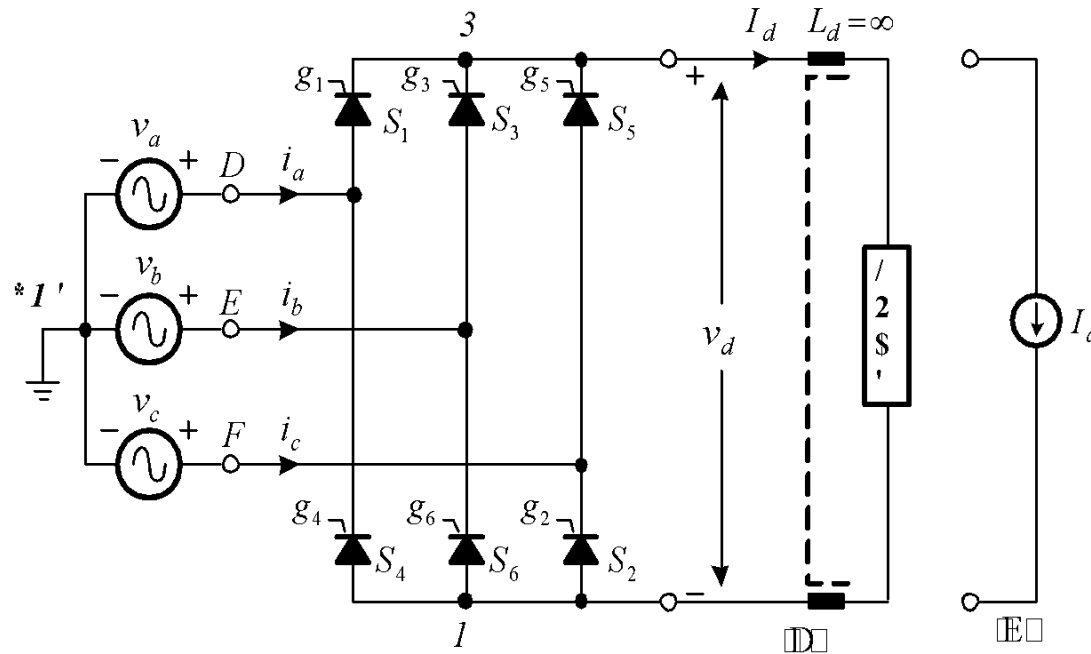
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## Why Use Multi-pulse SCR Rectifiers?

- To reduce line THD
- To improve input power factor
- To avoid semiconductor devices in series.

# Six-pulse SCR Rectifier

## • Converter Configuration



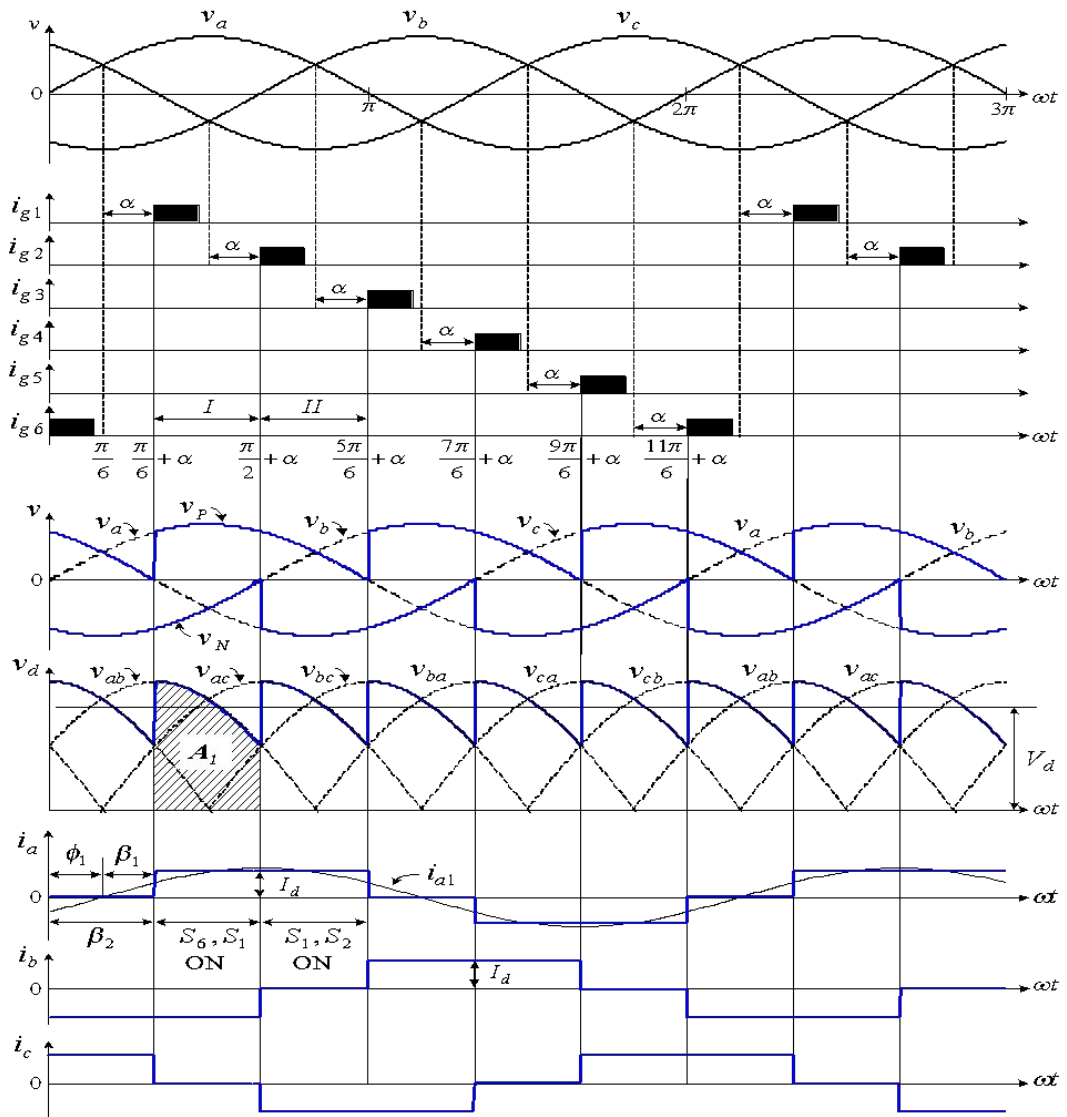
• Assumption: 1) Ideal SCRs - no power loss

2)  $L_d = \infty \rightarrow$  dc current ripple free

• Use constant current source  $I_d$  as a load

# Six-pulse SCR Rectifier

- Waveforms



- Average dc voltage

$$\begin{aligned}
 V_d &= \frac{\text{area } A_1}{\pi / 3} \\
 &= \frac{1}{\pi / 3} \int_{\pi/6 + \alpha}^{\pi/2 + \alpha} v_{ab} d(\omega t) \\
 &= \frac{3\sqrt{2}}{\pi} V_{LL} \cos \alpha \\
 &\approx 1.35 V_{LL} \cos \alpha
 \end{aligned}$$

# Six-pulse SCR Rectifier

## • Fourier Analysis

$$i_a = \frac{2\sqrt{3}}{\pi} I_d \left( \sin(\omega t - \phi_1) - \frac{1}{5} \sin 5(\omega t - \phi_1) - \frac{1}{7} \sin 7(\omega t - \phi_1) + \frac{1}{11} \sin 11(\omega t - \phi_1) \right. \\ \left. + \frac{1}{13} \sin 13(\omega t - \phi_1) - \frac{1}{17} \sin 17(\omega t - \phi_1) - \frac{1}{19} \sin 19(\omega t - \phi_1) + \dots \right)$$

## • RMS current

$$I_a = \left( \frac{1}{2\pi} \int_0^{2\pi} (i_a)^2 d(\omega t) \right)^{1/2} = \left( \frac{1}{2\pi} \left( \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} (I_d)^2 d(\omega t) + \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} (-I_d)^2 d(\omega t) \right) \right)^{1/2} \\ = \sqrt{\frac{2}{3}} I_d = 0.816 I_d$$

## • THD

$$THD = \frac{\sqrt{I_a^2 - I_{a1}^2}}{I_{a1}} = \frac{\sqrt{(0.816 I_d)^2 - (0.78 I_d)^2}}{0.78 I_d} = 0.311$$

# Six-pulse SCR Rectifier

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- Power Factor (PF)

- Displacement Power Factor

$$\phi_1 = \beta_2 - \beta_1 = \alpha$$

$$DPF = \cos \phi_1 = \cos \alpha$$

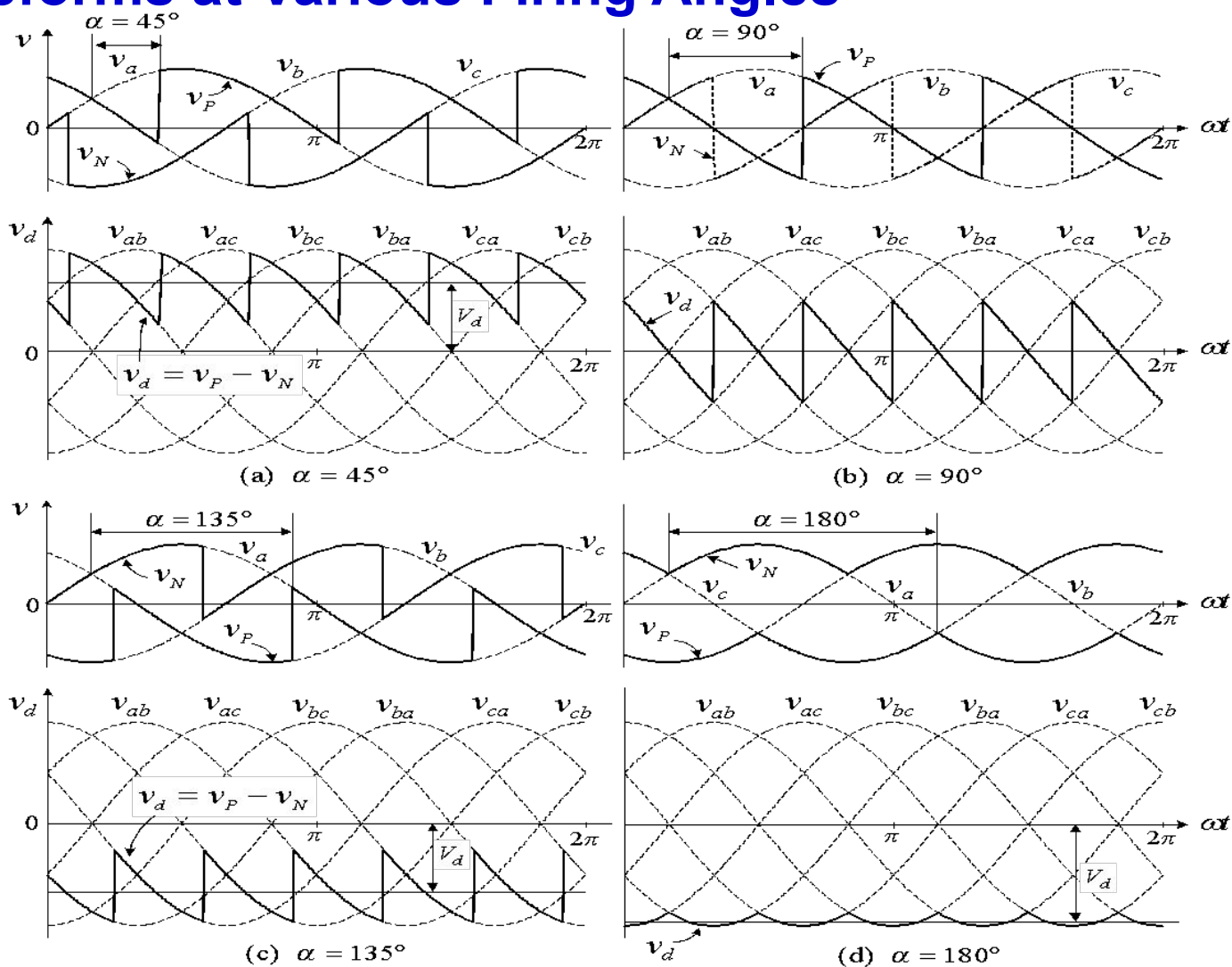
- Overall Power Factor

$$PF = \frac{\cos \phi_1}{\sqrt{1 + THD^2}} = 0.955 \cos \alpha$$



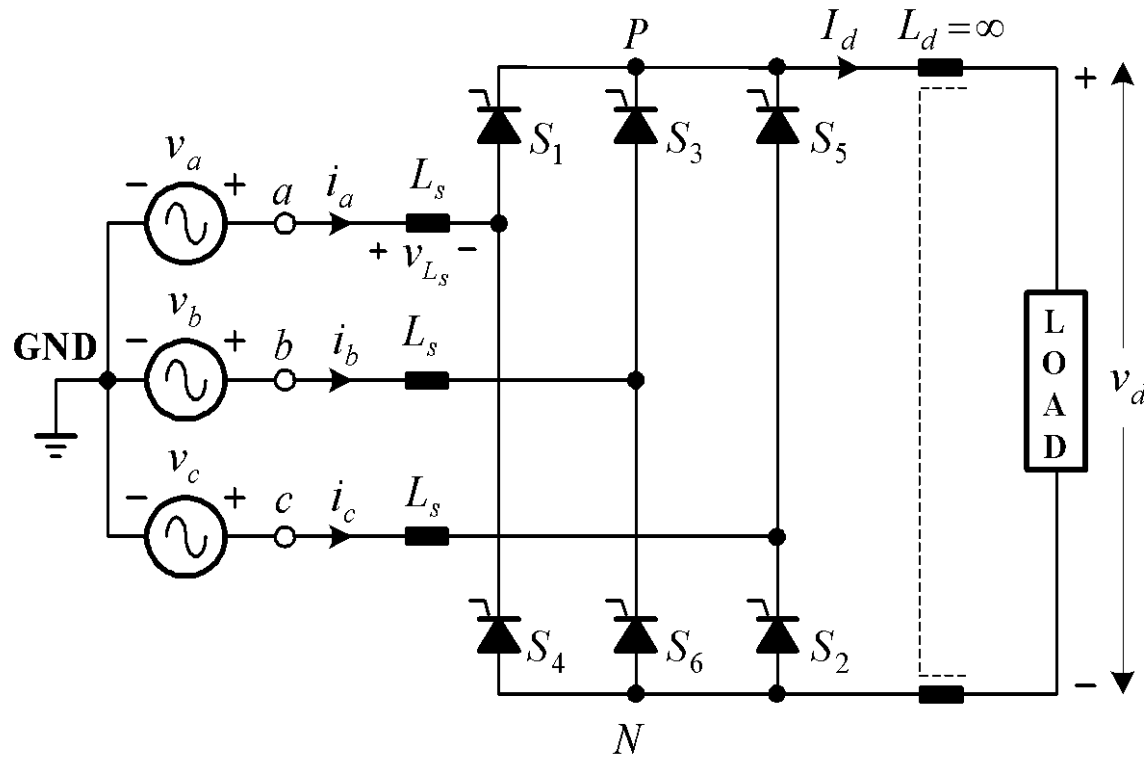
# Six-pulse SCR Rectifier

## Waveforms at Various Firing Angles



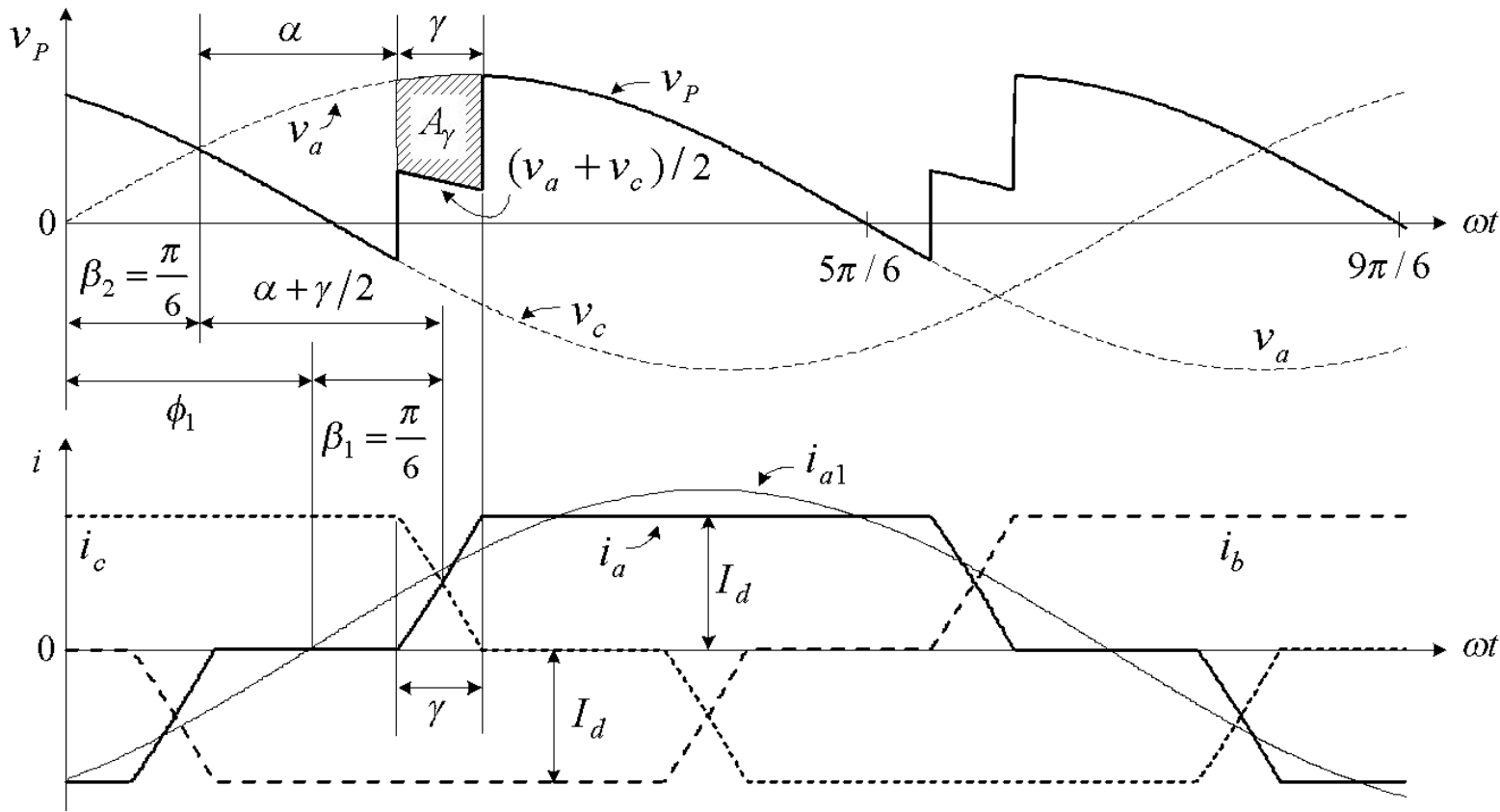
# Six-pulse SCR Rectifier

- Effect of Line Inductance  $L_s$



# Six-pulse SCR Rectifier

## • Waveform During SCR Commutation



# Six-pulse SCR Rectifier

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- **DC Voltage Reduction** (caused by commutation)
  - **$S_1$  and  $S_5$  on (during commutation)**

$$v_P = v_a - L_s \frac{di_a}{dt} = v_c - L_s \frac{di_c}{dt}$$

from which

$$v_P = \frac{v_a + v_c}{2} - \frac{L_s}{2} \left( \frac{di_a}{dt} + \frac{di_c}{dt} \right)$$

- **Since  $i_a + i_c = I_d$ , and  $I_d$  is constant**

$$\frac{di_a}{dt} + \frac{di_c}{dt} = 0$$

from which

$$v_P = \frac{v_a + v_c}{2}$$

# Six-pulse SCR Rectifier

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- DC Voltage Reduction

- Shaded Area  $A_\gamma$

$$A_\gamma = \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha + \gamma} (v_a - v_p) d(\omega t)$$

- Since  $v_a - v_p = L_s (di_a / dt)$

$$A_\gamma = \int_0^{I_d} \omega L_s di_a = \omega L_s I_d$$

- Average dc voltage loss

$$\Delta V = \frac{A_\gamma}{\pi/3} = \frac{3\omega L_s}{\pi} I_d$$

- Average dc output voltage

$$V_d = 1.35V_{LL} \cos\alpha - \frac{3\omega L_s}{\pi} I_d$$

# Six-pulse SCR Rectifier

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- **Commutation Interval**

- $A_\gamma$  can be rearranged as

$$A_\gamma = \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{6} + \alpha + \gamma} \frac{v_{ac}}{2} d(\omega t) = \frac{V_{LL}}{\sqrt{2}} (\cos \alpha - \cos(\alpha + \gamma))$$

where  $v_{ac} = \sqrt{2} V_{LL} \sin(\omega t - \frac{\pi}{6}) d(\omega t)$

- **Substitute  $\omega L_s I_d$  into the above equation**

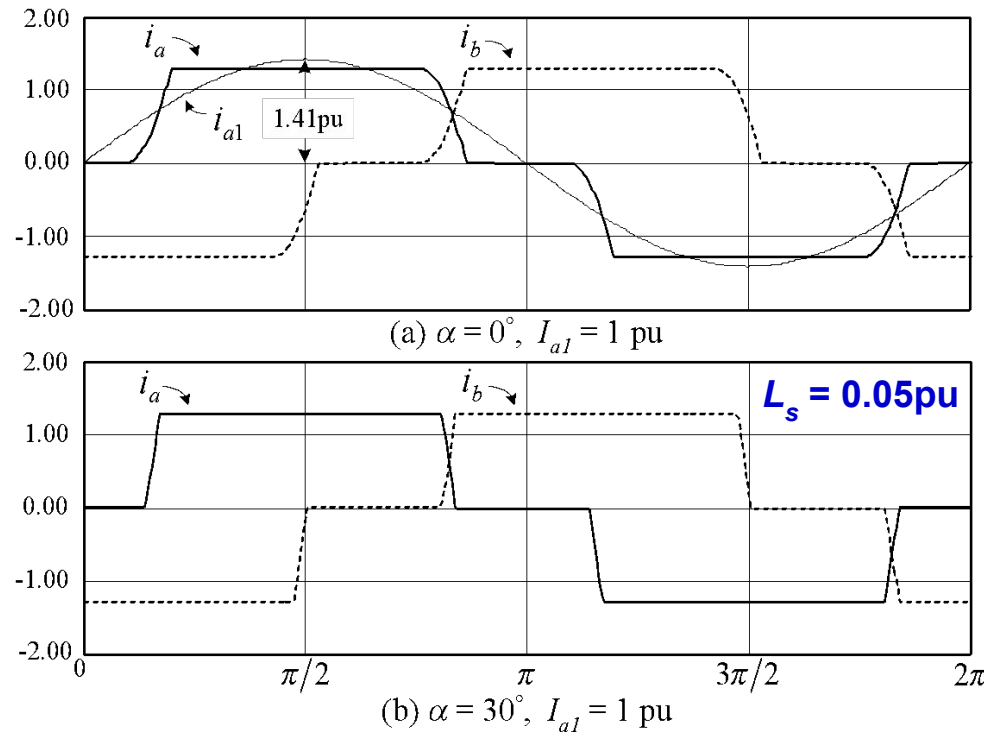
$$\omega L_s I_d = \frac{V_{LL}}{\sqrt{2}} (\cos \alpha - \cos(\alpha + \gamma))$$

from which

$$\gamma = \cos^{-1} \left( \cos \alpha - \frac{\sqrt{2} \omega L_s I_d}{V_{LL}} \right) - \alpha$$

# Six-pulse SCR Rectifier

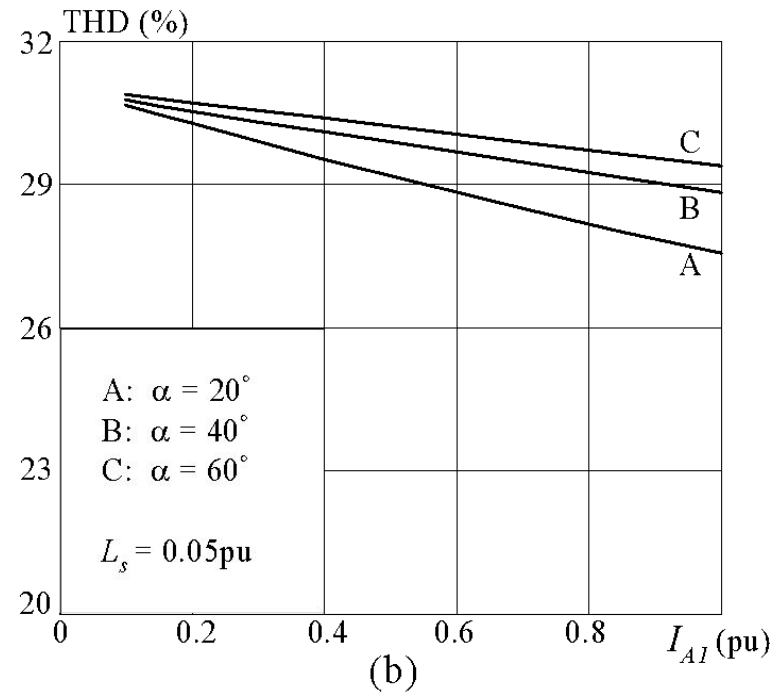
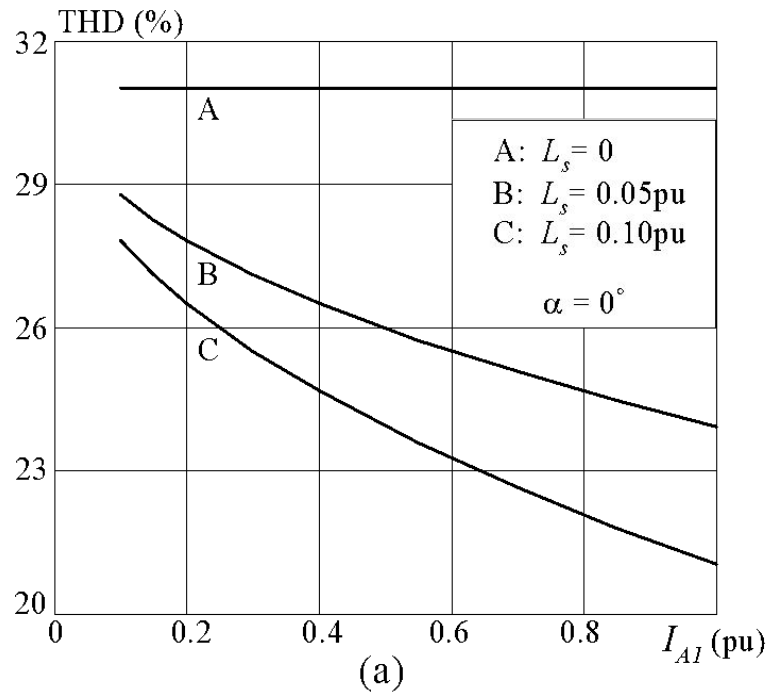
## • Line Current Waveforms



Harmonics $n$	5	7	11	13	17	19	23	25	THD %
$I_{an} / I_{a1}$ (%) $\alpha = 0^\circ$	18.6	12.4	6.32	4.58	2.40	1.73	1.02	0.87	23.9
$I_{an} / I_{a1}$ (%) $\alpha = 30^\circ$	19.7	14.1	8.58	7.27	5.16	4.62	3.43	3.16	28.3

# Six-pulse SCR Rectifier

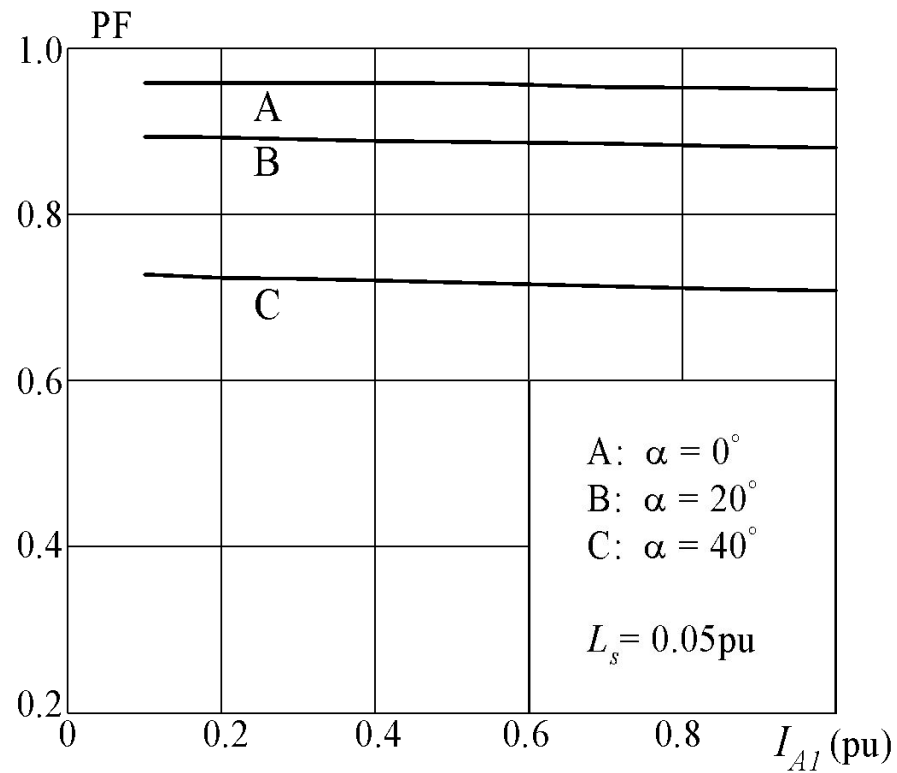
## • Line Current THD





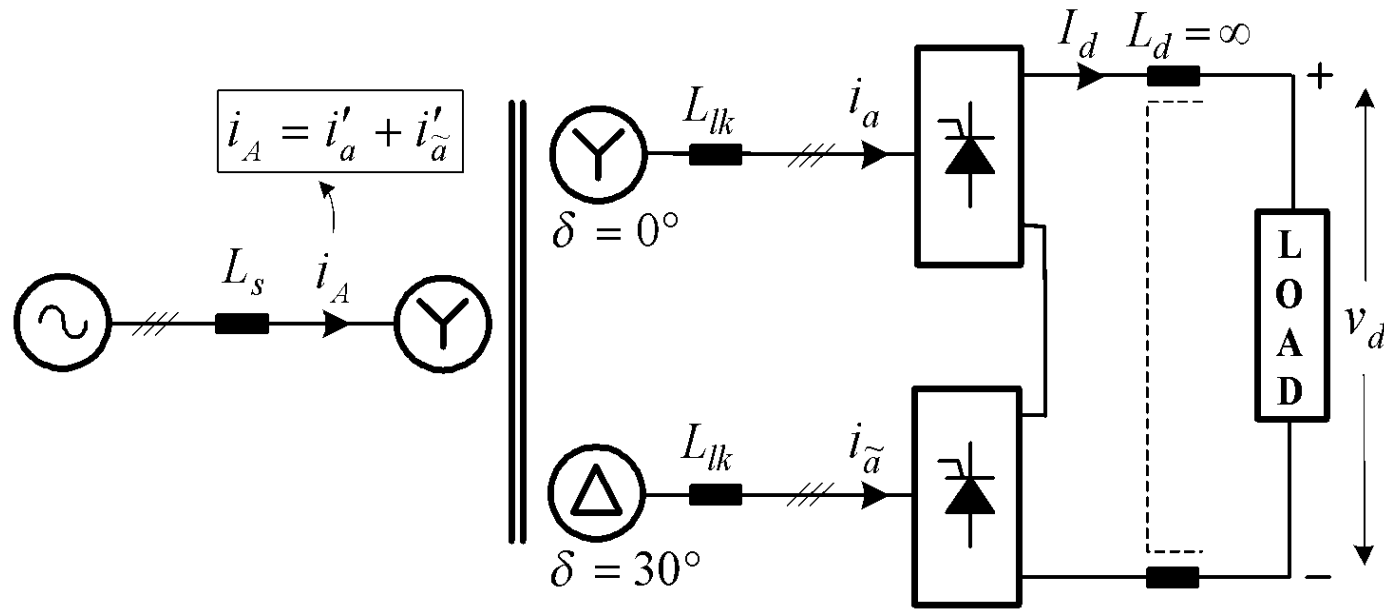
# Six-pulse SCR Rectifier

- Power Factor



# 12-pulse SCR Rectifier

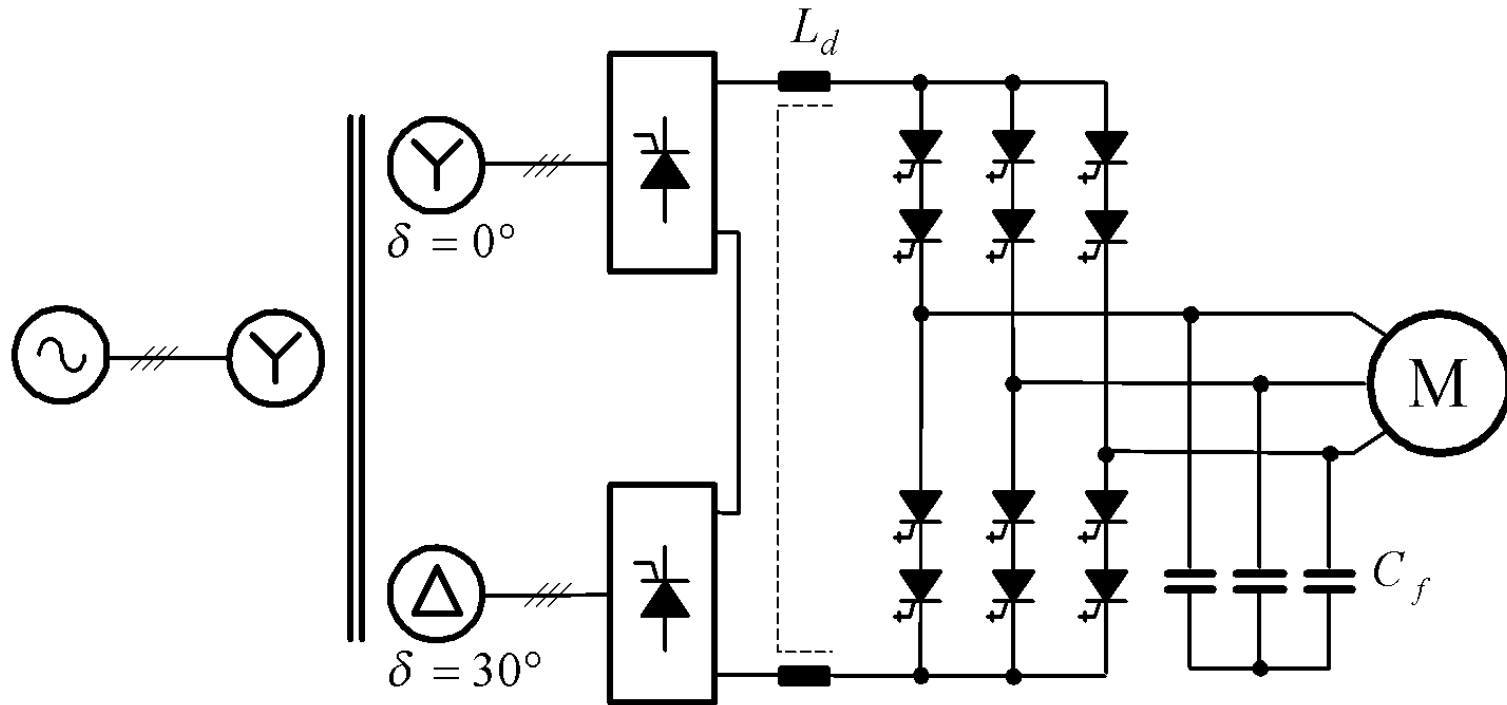
## • Topology



- **Main Benefits**
  - **Low line current THD**
  - **No SCRs in series**

# 12-pulse SCR Rectifier

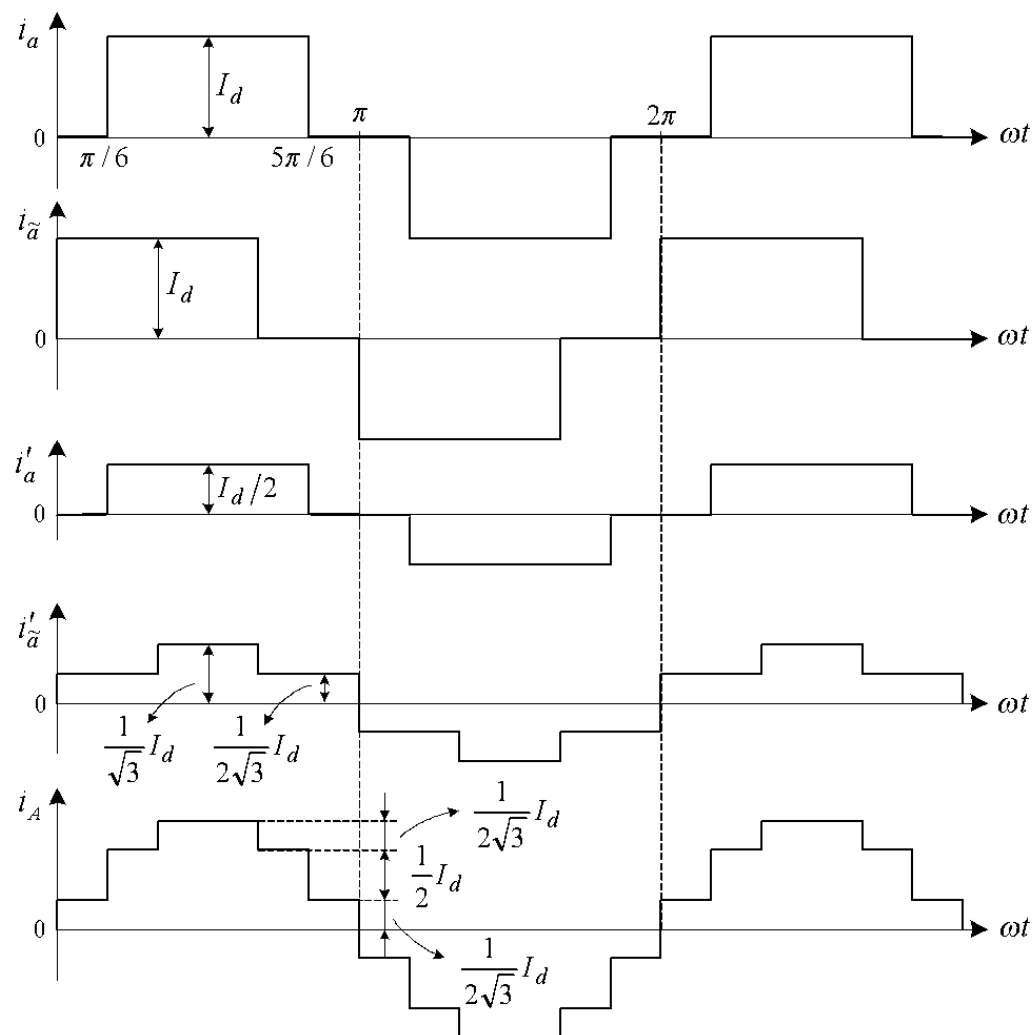
- Application Example



- Used for current source inverter (CSI) fed drive

# 12-pulse SCR Rectifier

- Waveforms of Idealized Rectifier ( $L_s = L_{lk} = 0$ )



# 12-pulse SCR Rectifier

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- Fourier Analysis

- Secondary currents

$$i_a = \frac{2\sqrt{3}}{\pi} I_d \left( \sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t \right. \\ \left. - \frac{1}{17} \sin 17\omega t - \frac{1}{19} \sin 19\omega t + \dots \right)$$

$$i_{\tilde{a}} = \frac{2\sqrt{3}}{\pi} I_d \left( \sin(\omega t + 30^\circ) - \frac{1}{5} \sin 5(\omega t + 30^\circ) - \frac{1}{7} \sin 7(\omega t + 30^\circ) + \frac{1}{11} \sin 11(\omega t + 30^\circ) \right. \\ \left. + \frac{1}{13} \sin 13(\omega t + 30^\circ) - \frac{1}{17} \sin 17(\omega t + 30^\circ) - \frac{1}{19} \sin 19(\omega t + 30^\circ) + \dots \right)$$

# 12-pulse SCR Rectifier

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- **Fourier Analysis (continued)**

- **Secondary currents referred to the primary side**

$$i'_a = \frac{\sqrt{3}}{\pi} I_d \left\{ \sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t \right. \\ \left. - \frac{1}{17} \sin 17\omega t - \frac{1}{19} \sin 19\omega t + \dots \right\}$$

$$i'_a = \frac{\sqrt{3}}{\pi} I_d \left\{ \sin \omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t \right. \\ \left. + \frac{1}{17} \sin 17\omega t + \frac{1}{19} \sin 19\omega t + \dots \right\}$$

- **Primary line current**

$$i_A = i'_a + i''_a = \frac{2\sqrt{3}}{\pi} I_d \left\{ \sin \omega t + \frac{1}{11} \sin 11\omega t + \frac{1}{13} \sin 13\omega t + \frac{1}{23} \sin 23\omega t \right. \\ \left. + \frac{1}{25} \sin 25\omega t + \dots \right\}$$

# 12-pulse SCR Rectifier

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- Line Current THD

- Secondary side

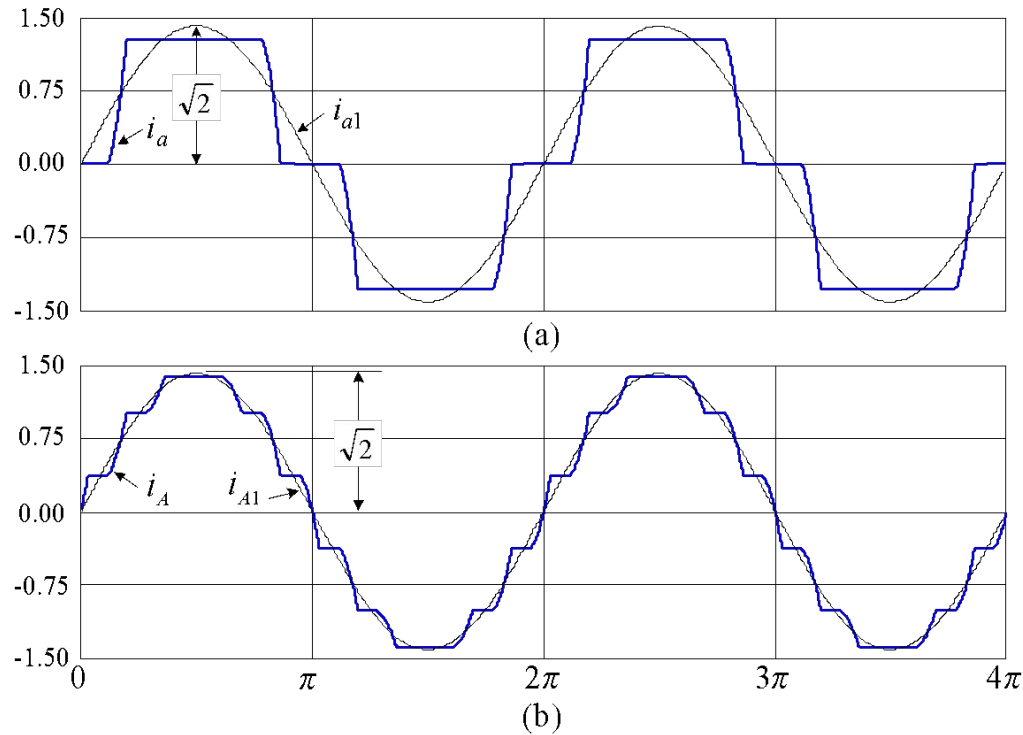
$$THD_{i_a} = \frac{\sqrt{I_a^2 - I_{a1}^2}}{I_{a1}} = \frac{(I_{a5}^2 + I_{a7}^2 + I_{a11}^2 + I_{a13}^2 + \dots)^{1/2}}{I_{a1}} = 31.1\%$$

- Primary side

$$THD_{i_A} = \frac{\sqrt{I_A^2 - I_{A1}^2}}{I_{A1}} = \frac{(I_{A11}^2 + I_{A13}^2 + I_{A23}^2 + I_{A25}^2 + \dots)^{1/2}}{I_{A1}} = 15.3\%$$

# 12-pulse SCR Rectifier

## • Line Current Waveform ( $L_s \neq 0$ )

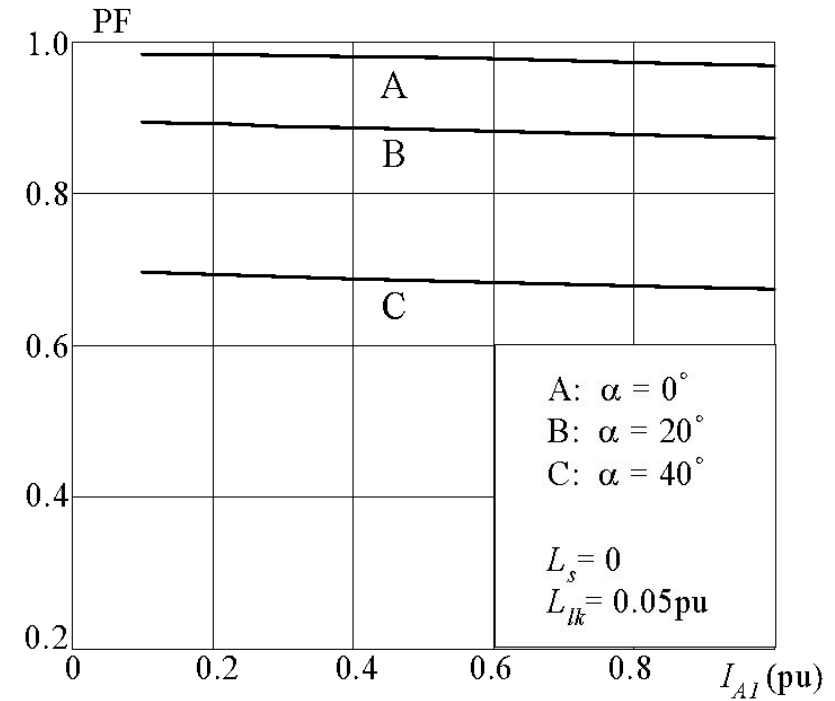
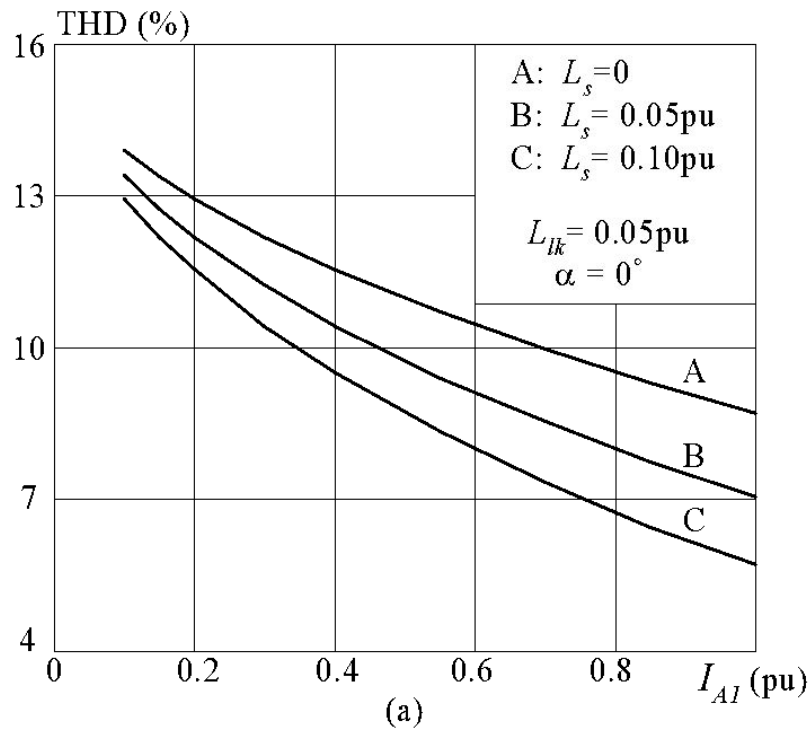


Harmonics $n$	5	7	11	13	17	19	23	25	THD (%)
$I_{an} / I_{a1}$ (%)	18.8	12.7	6.78	5.05	2.77	2.01	1.01	0.75	24.6
$I_{An} / I_{A1}$ (%)	0	0	6.78	5.05	0	0	1.01	0.75	8.61



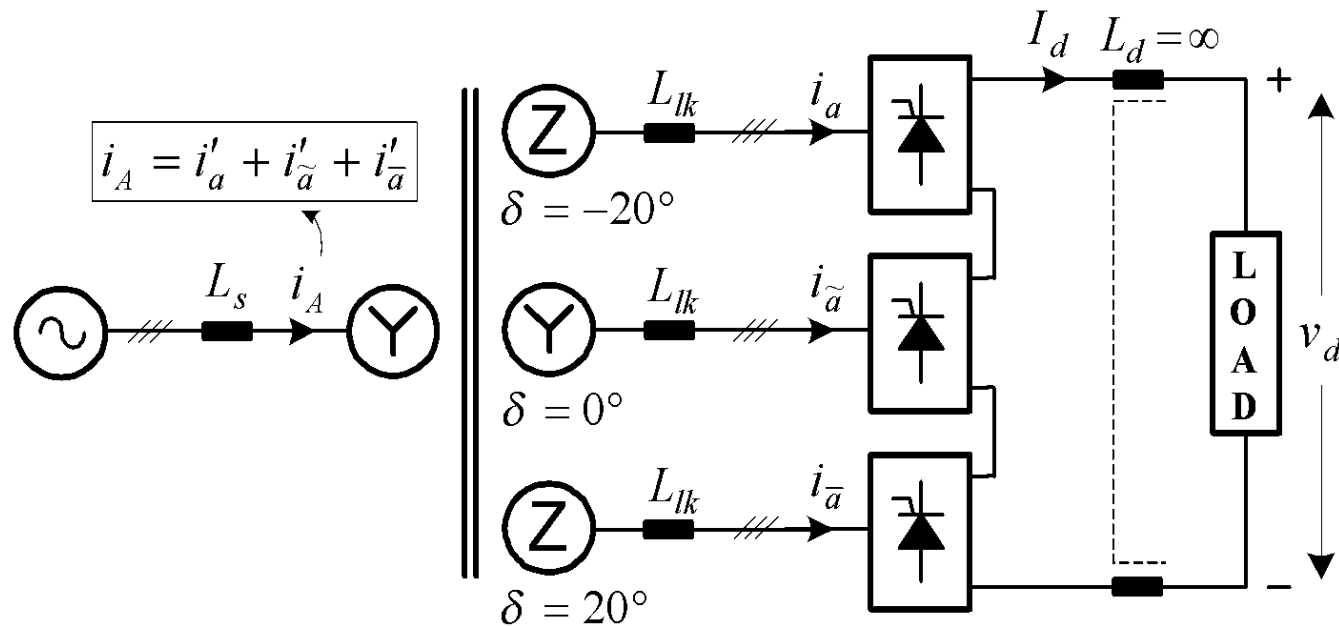
# 12-pulse SCR Rectifier

## • THD and PF



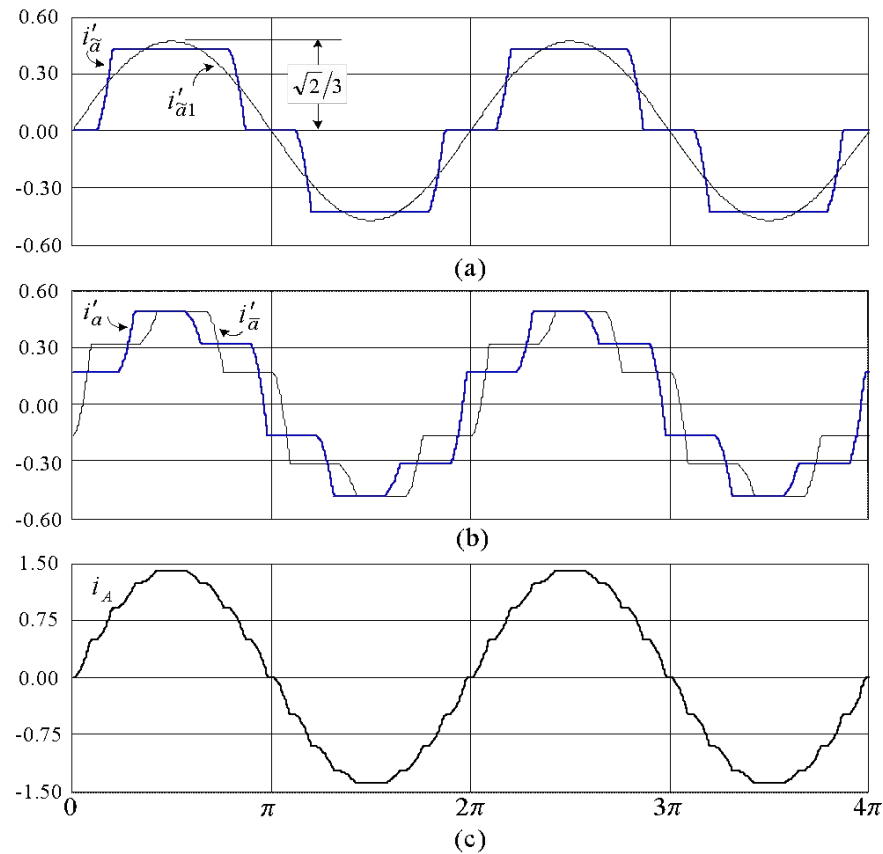
# 18-pulse SCR Rectifier

## • Rectifier Configuration



# 18-pulse SCR Rectifier

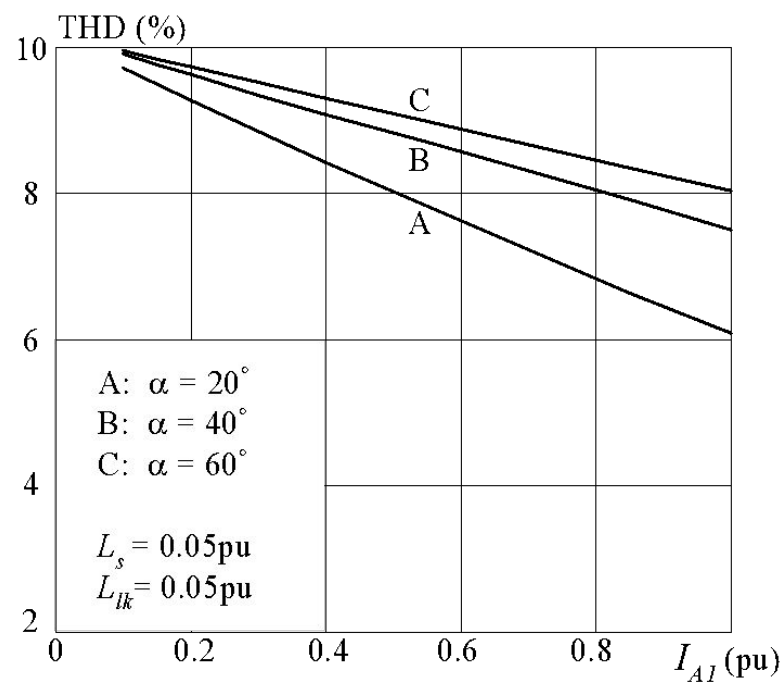
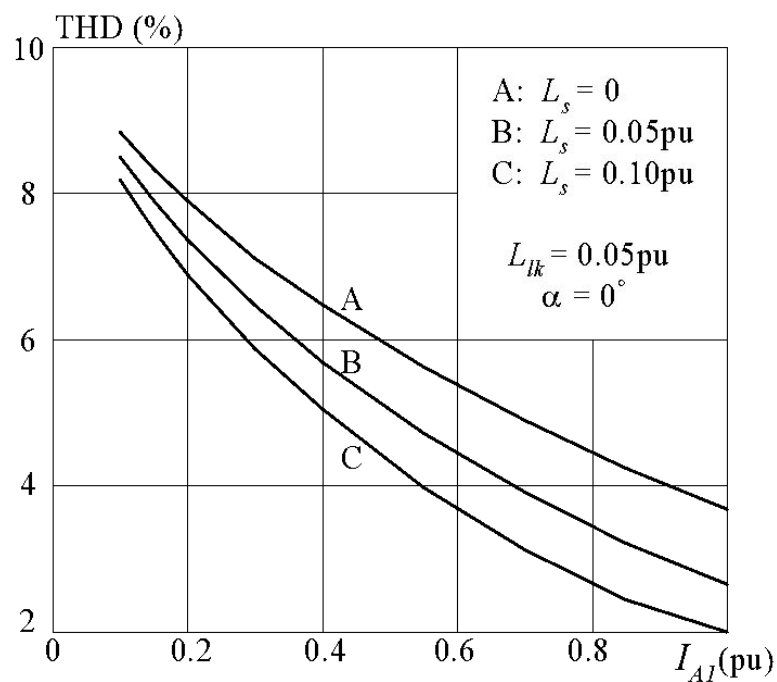
## • Waveforms



Harmonics $n$	5	7	11	13	17	19	23	25	THD (%)
$I'_{an} / I'_{a1}$ (%)	18.8	12.7	6.78	5.05	2.77	2.01	1.01	0.75	24.6
$I_{An} / I_{A1}$ (%)	0	0	0	0	2.77	2.01	0	0	3.54

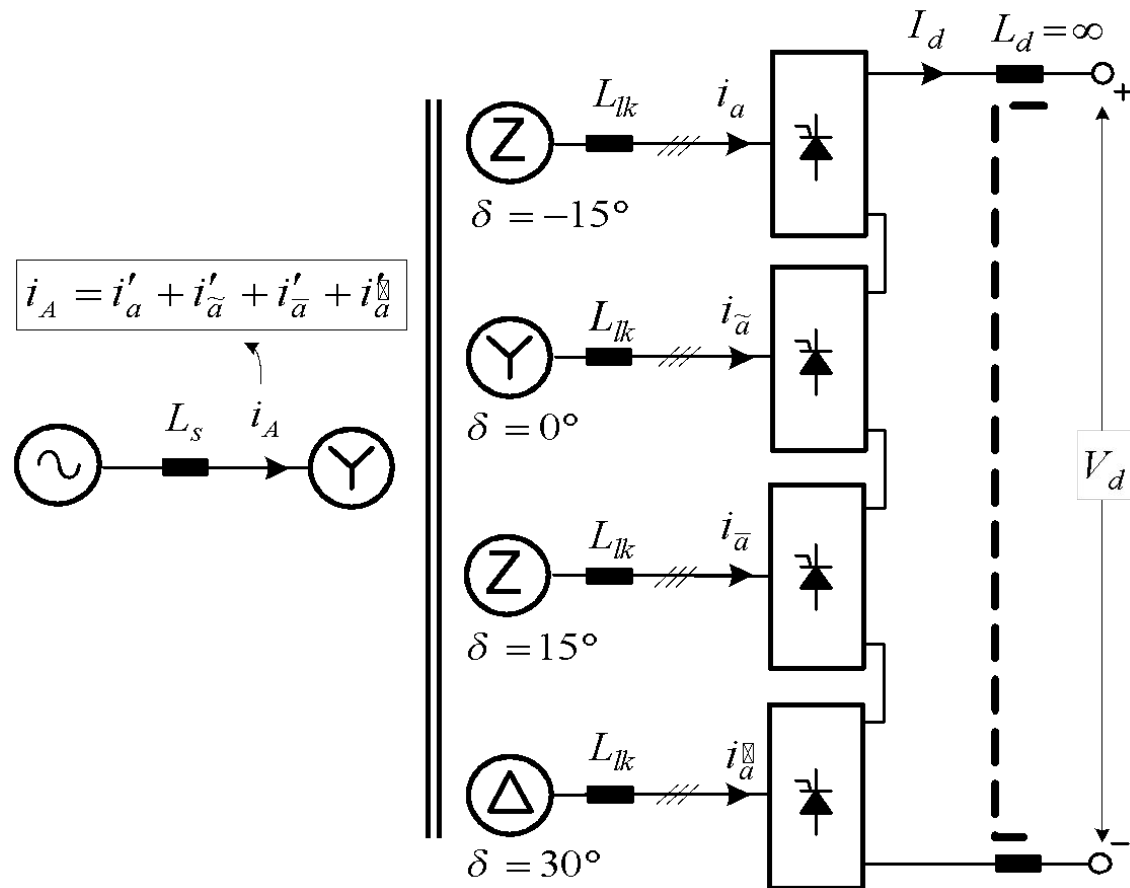
# 18-pulse SCR Rectifier

## • Line Current THD



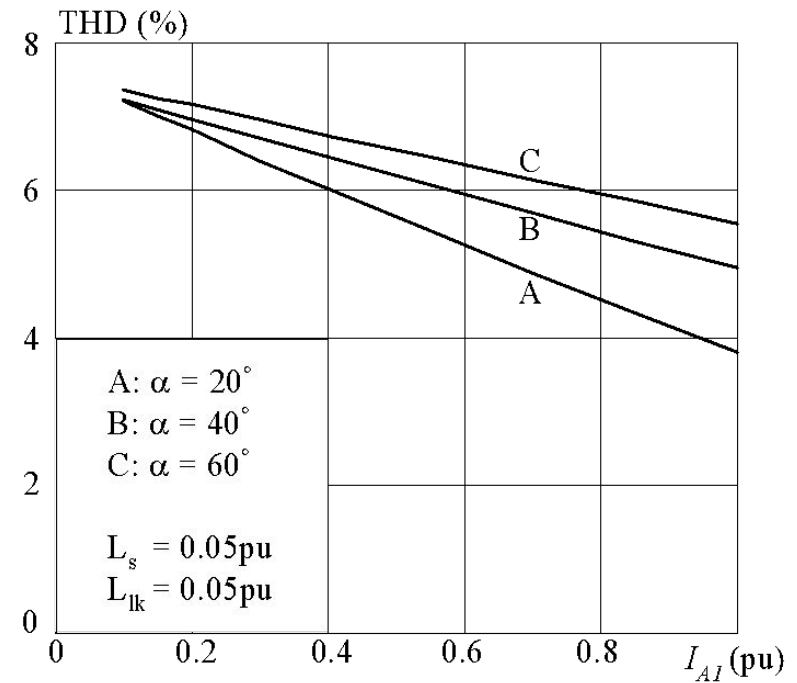
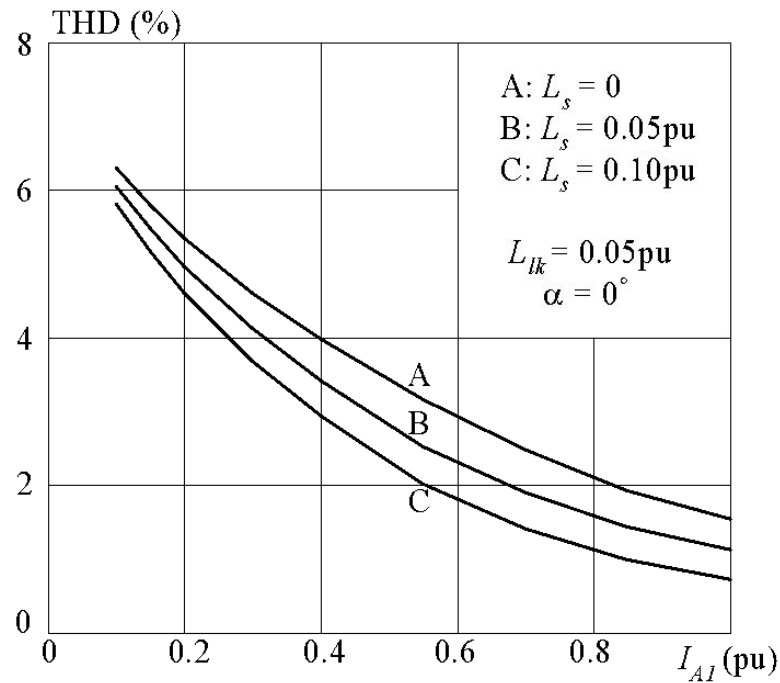
# 24-pulse SCR Rectifier

## • Rectifier Configuration



# 24-pulse SCR Rectifier

## • Line Current THD





Thanks