Power Converter Systems

Graduate Course EE8407

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Textbook: Bin Wu, 'High-Power Converters and AC Drives', Wiley - IEEE Press, 2006

Topic 5

Two-Level Voltage Source Inverter (VSI)

Source:
Alstom

VDM5000 Two-level VSI

Two Level Voltage Source Inverter

Lecture Topics

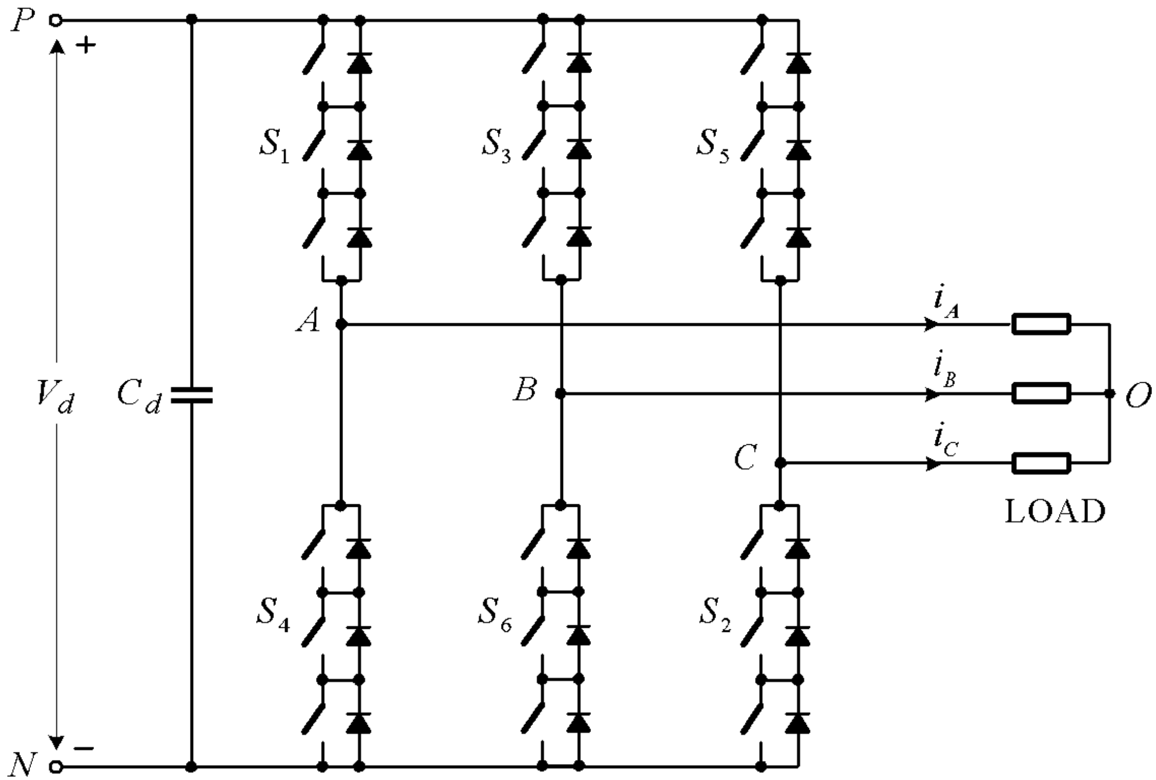
- Sinusoidal PWM
- Space vector modulation

Why Use PWM Techniques?

- To control inverter output frequency (fundamental)
- To control inverter output voltage (fundamental)
- To minimize harmonic distortion

Sinusoidal PWM

- **Inverter Configuration**

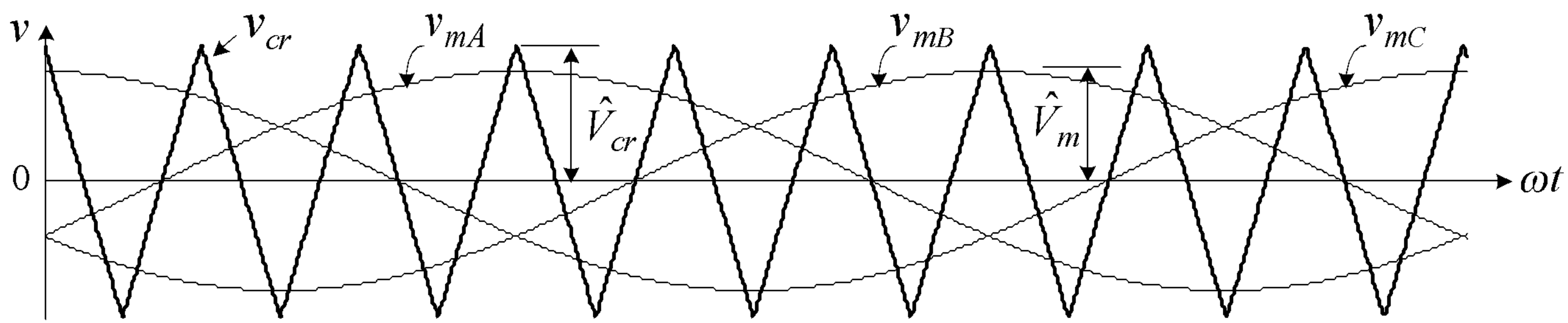


Assumption:

dc capacitor very large → dc voltage ripple free

Sinusoidal PWM

- **Modulating and Carrier Waves**



- **v_{cr} – Carrier wave (triangle)**
- **v_m – Modulating wave (sine)**
- **Amplitude modulation index**

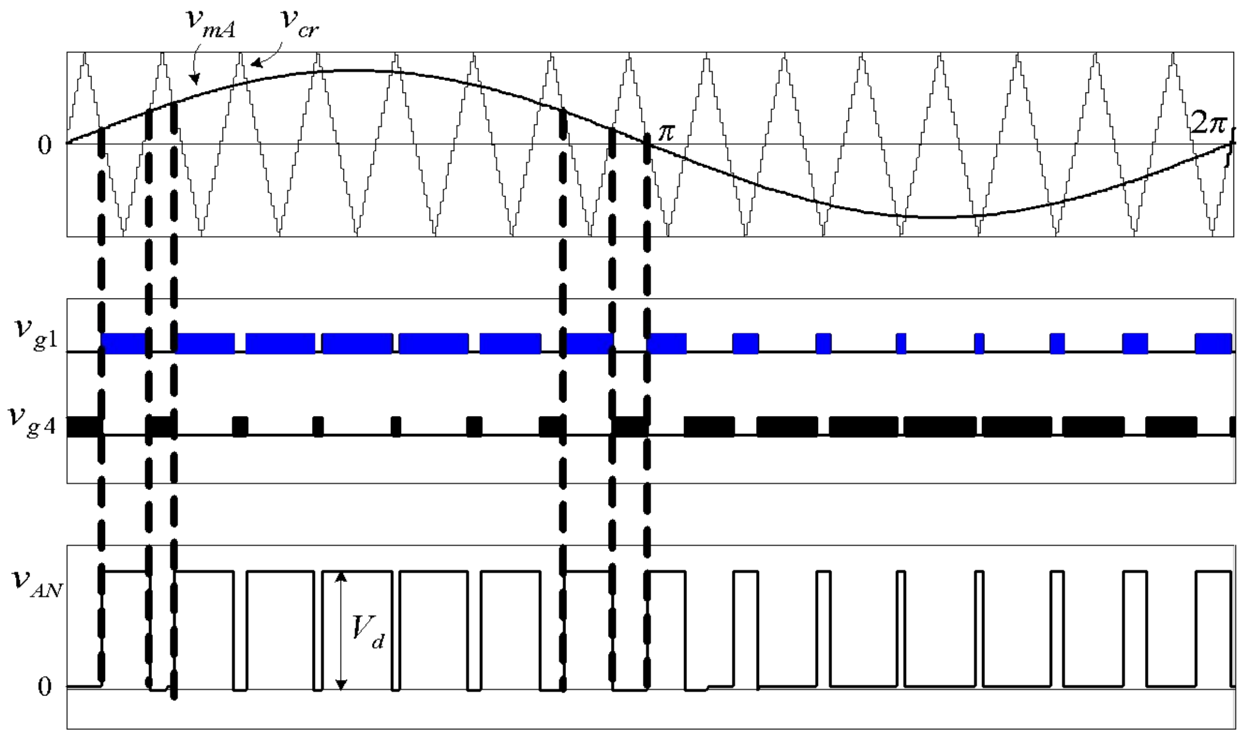
$$m_a = \frac{\hat{V}_m}{\hat{V}_{cr}}$$

- **Frequency modulation index**

$$m_f = \frac{f_{cr}}{f_m}$$

Sinusoidal PWM

- **Gate Signal Generation**

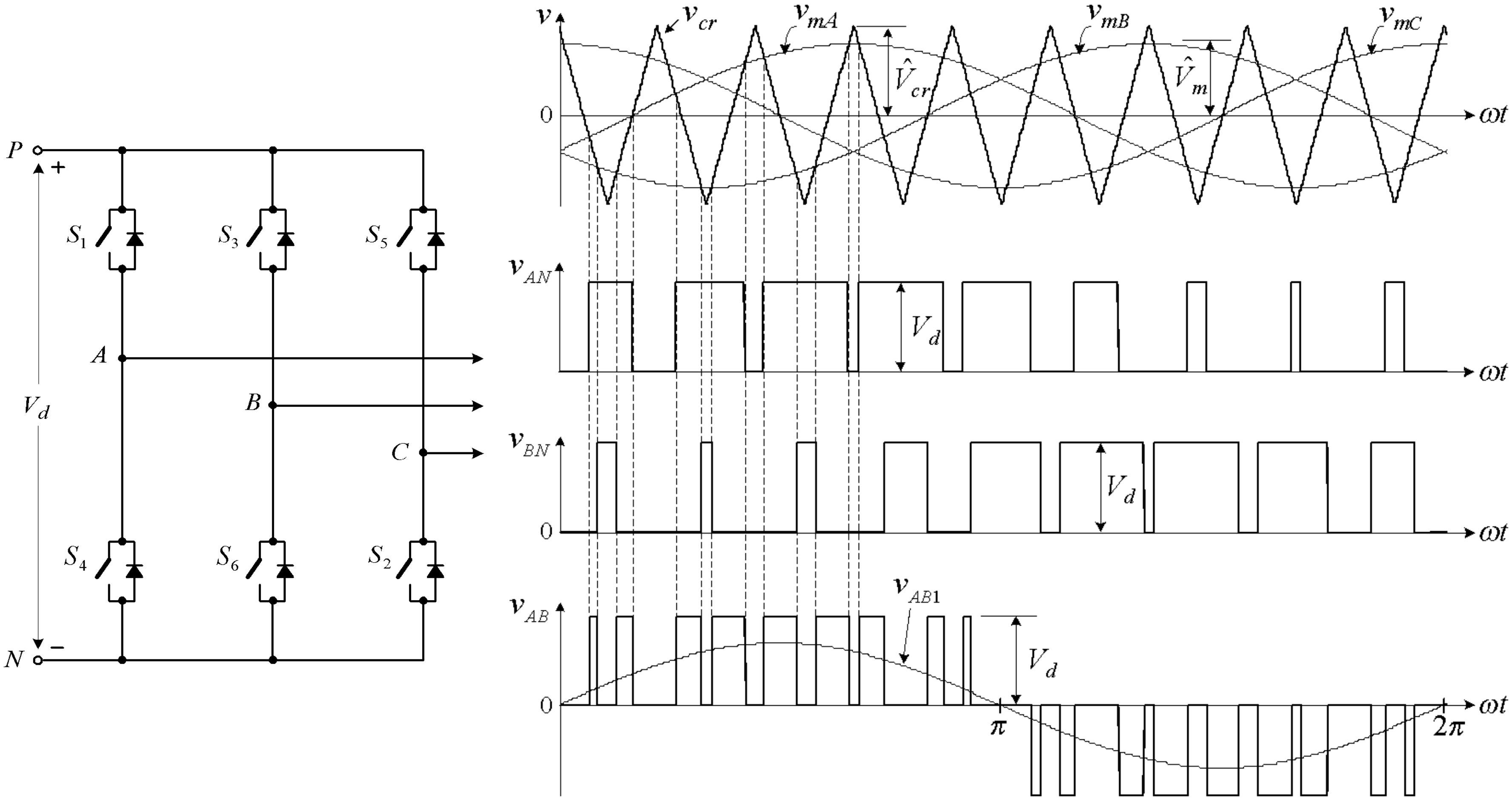


Phase A	$v_{mA} > v_{cr}$	$v_{g1} > 0 \quad (v_{g4} < 0)$	S_1 on $(S_4$ off$)$	$v_{AN} = V_d$
	$v_{mA} < v_{cr}$	$v_{g4} > 0 \quad (v_{g1} < 0)$	S_4 on $(S_1$ off$)$	$v_{AN} = 0$

V_{g1} and V_{g4} are complementary

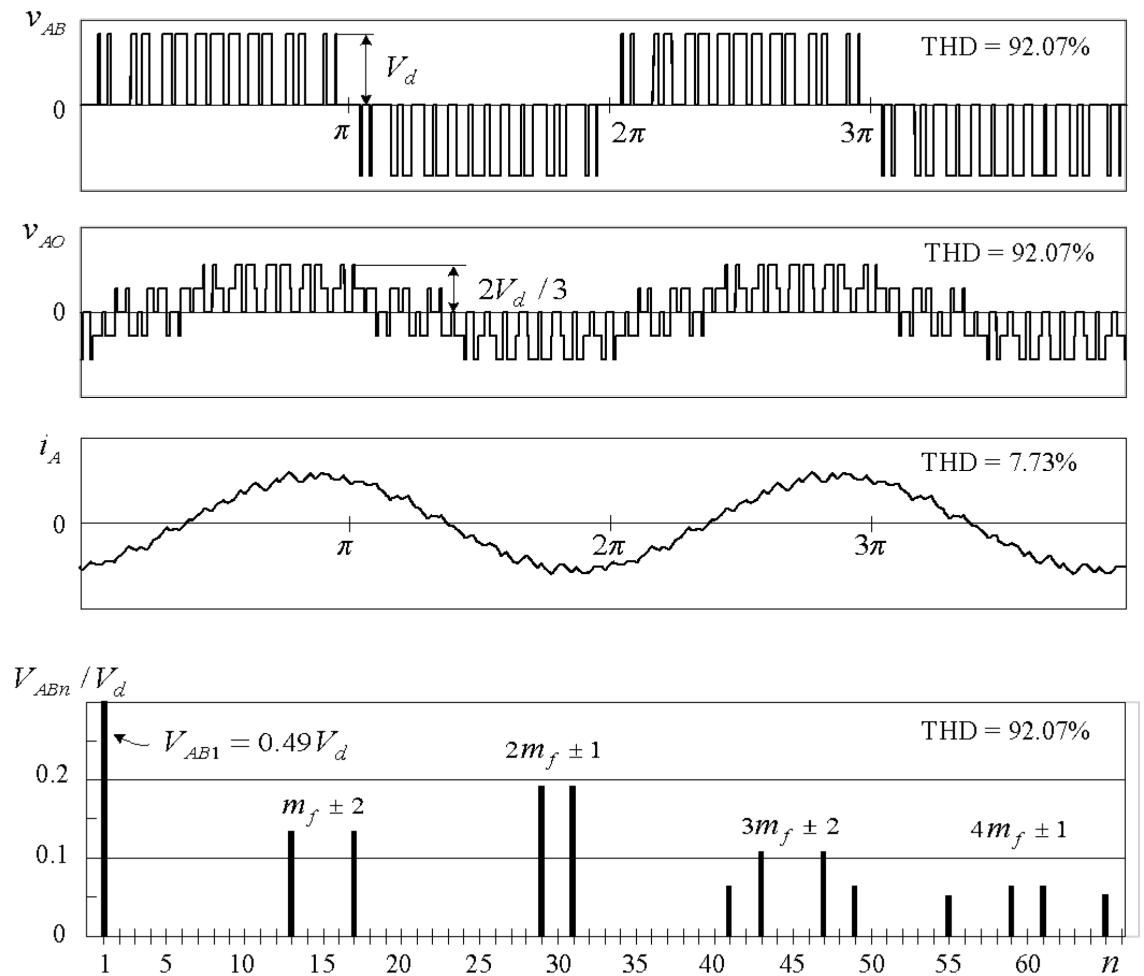
Sinusoidal PWM

- Line-to-Line Voltage V_{AB}



Sinusoidal PWM

- **Waveforms and FFT**



- $m_a = 0.8$, $m_f = 15$, $f_m = 60\text{Hz}$, $f_{cr} = 900\text{Hz}$

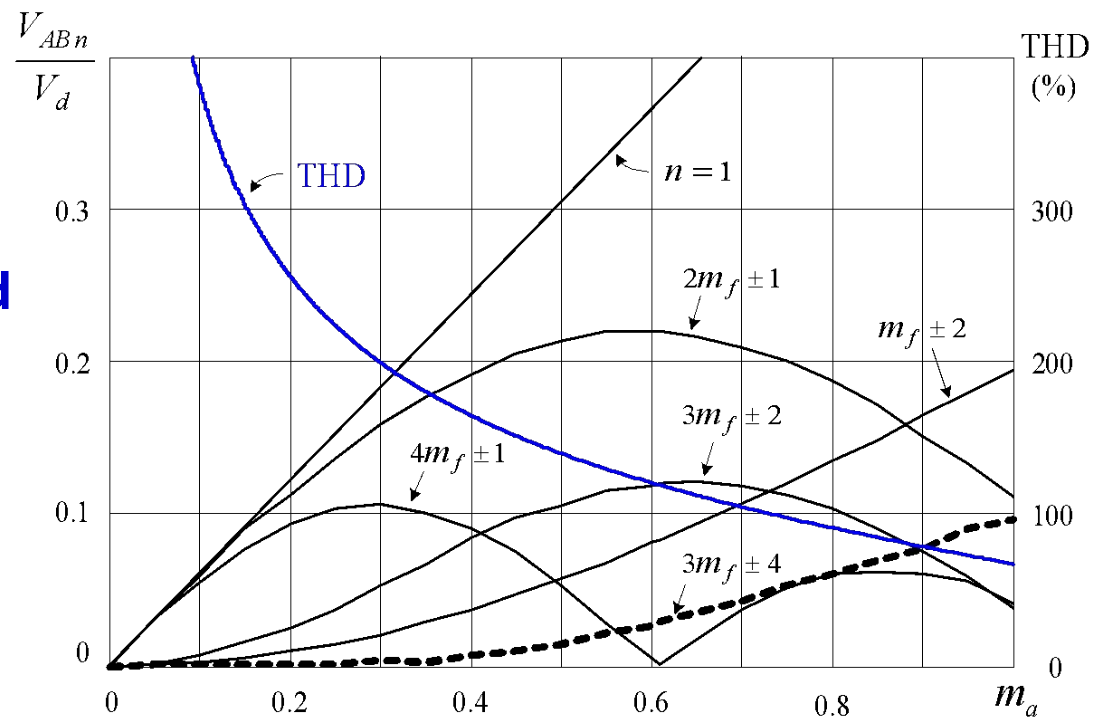
- **Switching frequency** $f_{sw} = f_{cr} = 900\text{Hz}$

Sinusoidal PWM

• Harmonic Content

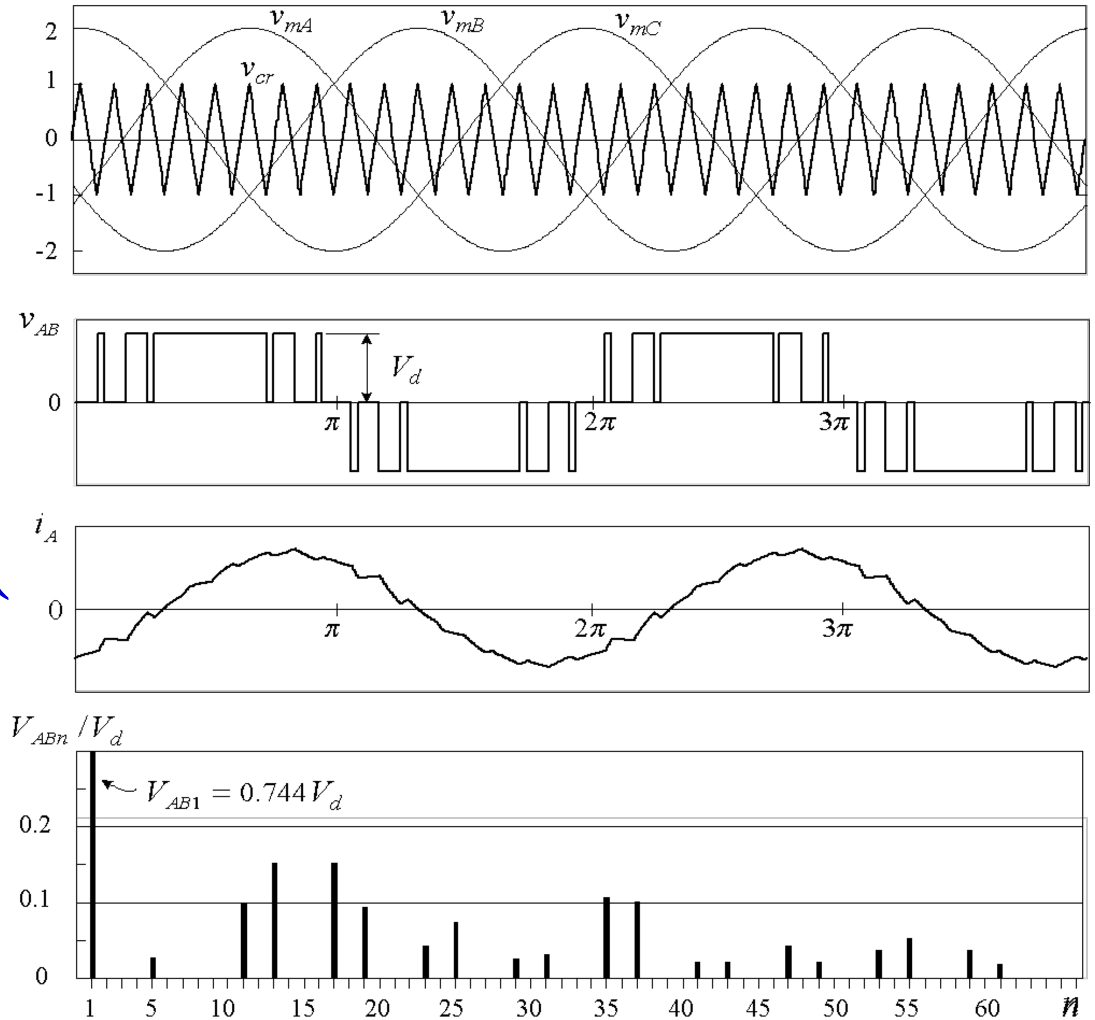
- Low order harmonics $n < (m_f\text{-}2)$ are eliminated

- V_{AB1} versus m_a is linear
- $V_{AB1,max} = 0.612V_d$



Sinusoidal PWM

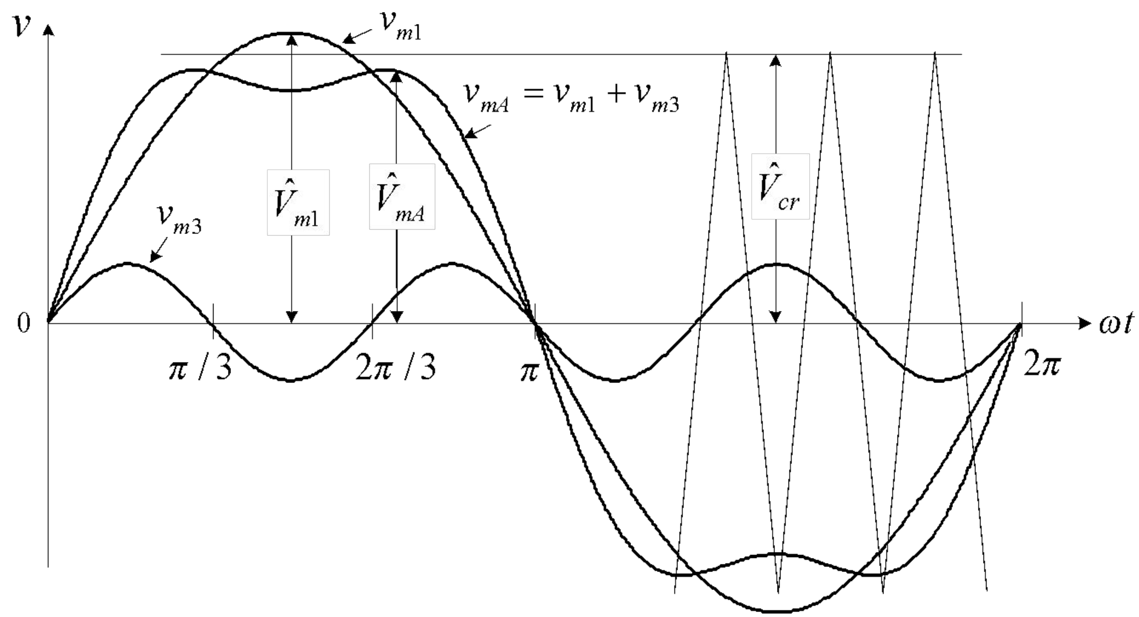
- **Over-Modulation**



- **Fundamental voltage ↑**
- **Low-order harmonics ↑**

Sinusoidal PWM

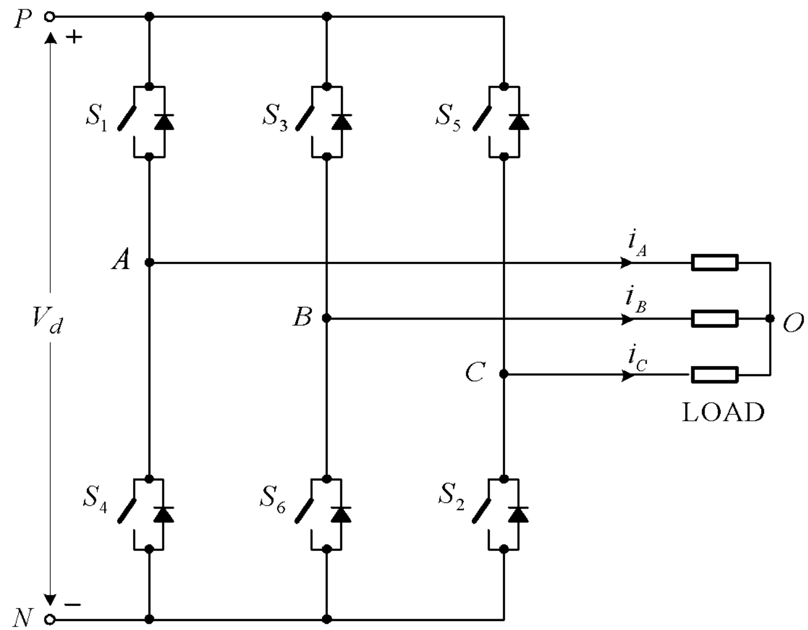
- **Third Harmonic Injection PWM**



- $\hat{V}_{m1} > \hat{V}_{cr}$ **Fundamental voltage increased**
- $\hat{V}_{mA} < \hat{V}_{cr}$ **No low order harmonics produced**
- **3rd harmonic – zero sequence (to appear in v_{AN} and v_{BN})**
- **No triplen harmonics in v_{AB} ($v_{AB} = v_{AN} - v_{BN}$)**

Space Vector Modulation

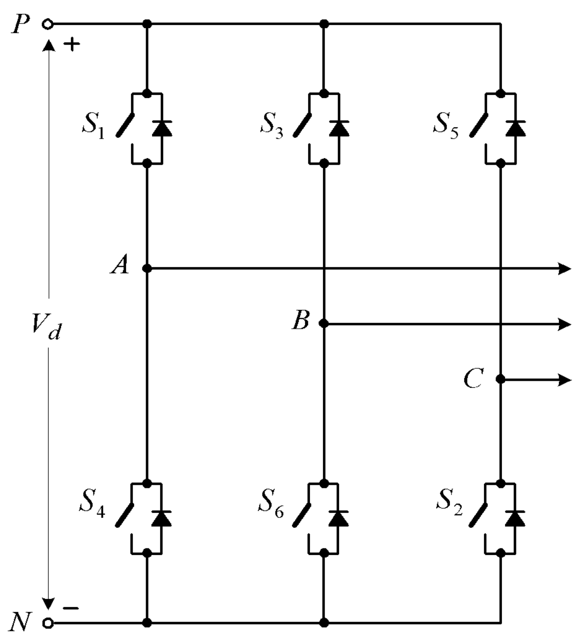
• Switching States



Switching State	Leg A			Leg B			Leg C		
	S_1	S_4	V_{AN}	S_3	S_6	V_{BN}	S_5	S_2	V_{CN}
P	On	Off	V_d	On	Off	V_d	On	Off	V_d
O	Off	On	0	Off	On	0	Off	On	0

Space Vector Modulation

• Switching States (Three-Phase)



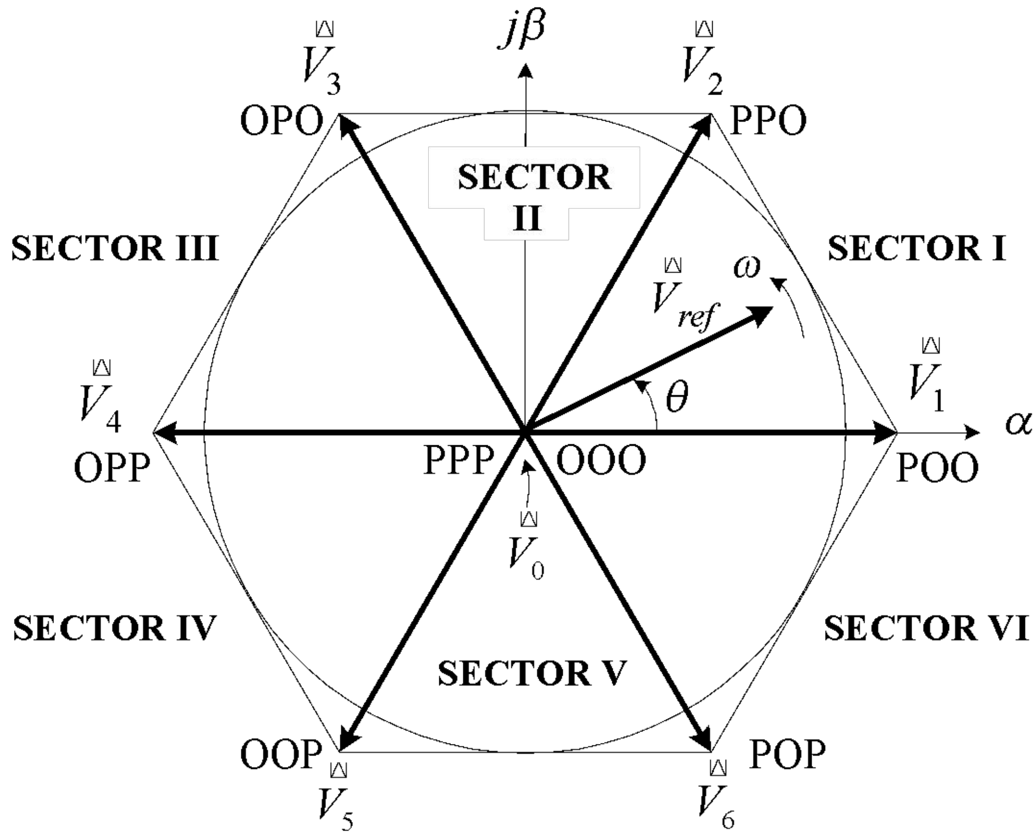
Switching State (Three Phases)	On-state Switch
[PPP]	S_1, S_3, S_5
[OOO]	S_4, S_6, S_2
[POO]	S_1, S_6, S_2
[PPO]	S_1, S_3, S_2
[OPO]	S_4, S_3, S_2
[OPP]	S_4, S_3, S_5
[OOP]	S_4, S_6, S_5
[POP]	S_1, S_6, S_5

• **Eight switching states**

Space Vector Modulation

• Space Vector Diagram

- Active vectors: $\vec{V}_1$ to $\vec{V}_6$ (stationary, not rotating)
- Zero vector: $\vec{V}_0$
- Six sectors: *I* to *VI*



Space Vector Modulation

• Space Vectors

• Three-phase voltages

$$v_{AO}(t) + v_{BO}(t) + v_{CO}(t) = 0 \qquad (1)$$

• Two-phase voltages

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos 0 & \cos\frac{2\pi}{3} & \cos\frac{4\pi}{3} \\ \sin 0 & \sin\frac{2\pi}{3} & \sin\frac{4\pi}{3} \end{bmatrix}\begin{bmatrix} v_{AO}(t) \\ v_{BO}(t) \\ v_{CO}(t) \end{bmatrix} \qquad (2)$$

• Space vector representation

$$\vec{V}(t) = v_\alpha(t) + j\,v_\beta(t) \qquad (3)$$

(2) → (3)

$$\vec{V}(t) = \frac{2}{3}\left[v_{AO}(t)e^{j0} + v_{BO}(t)e^{j2\pi/3} + v_{CO}(t)e^{j4\pi/3}\right] \qquad (4)$$

where $e^{jx} = \cos x + j\sin x$

Space Vector Modulation

• Space Vectors (Example)

Switching state [POO] → S_1, S_6 and S_2 ON

$$v_{AO}(t) = \frac{2}{3}V_d,\quad v_{BO}(t) = -\frac{1}{3}V_d \quad \text{and} \quad v_{CO}(t) = -\frac{1}{3}V_d \qquad (5)$$

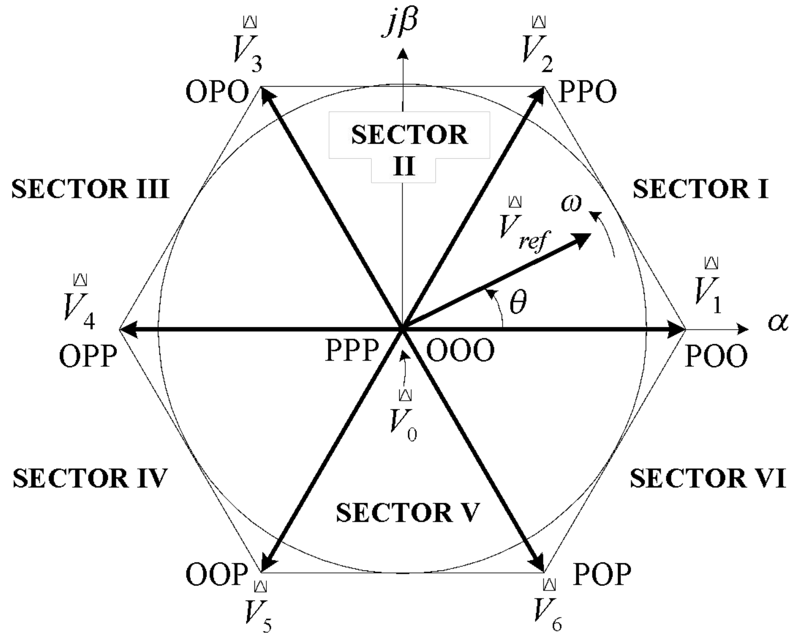
(5) → (4)

$$\vec{V}_1 = \frac{2}{3}V_d\, e^{j0} \qquad (6)$$

Similarly,

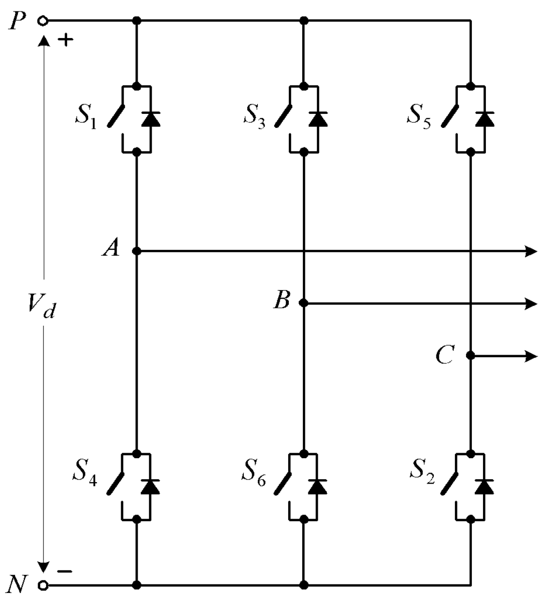
$$\vec{V}_k = \frac{2}{3}V_d\, e^{j(k-1)\frac{\pi}{3}} \qquad (7)$$

$$k = 1, 2, \ldots, 6.$$



Space Vector Modulation

• Active and Zero Vectors



Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_d e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_d e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_d e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_d e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_d e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_d e^{j\frac{5\pi}{3}}$

- Active Vector: 6
- Zero Vector: 1

- Redundant switching states: [PPP] and [OOO]

Space Vector Modulation

• Reference Vector V_{ref}

• **Definition**

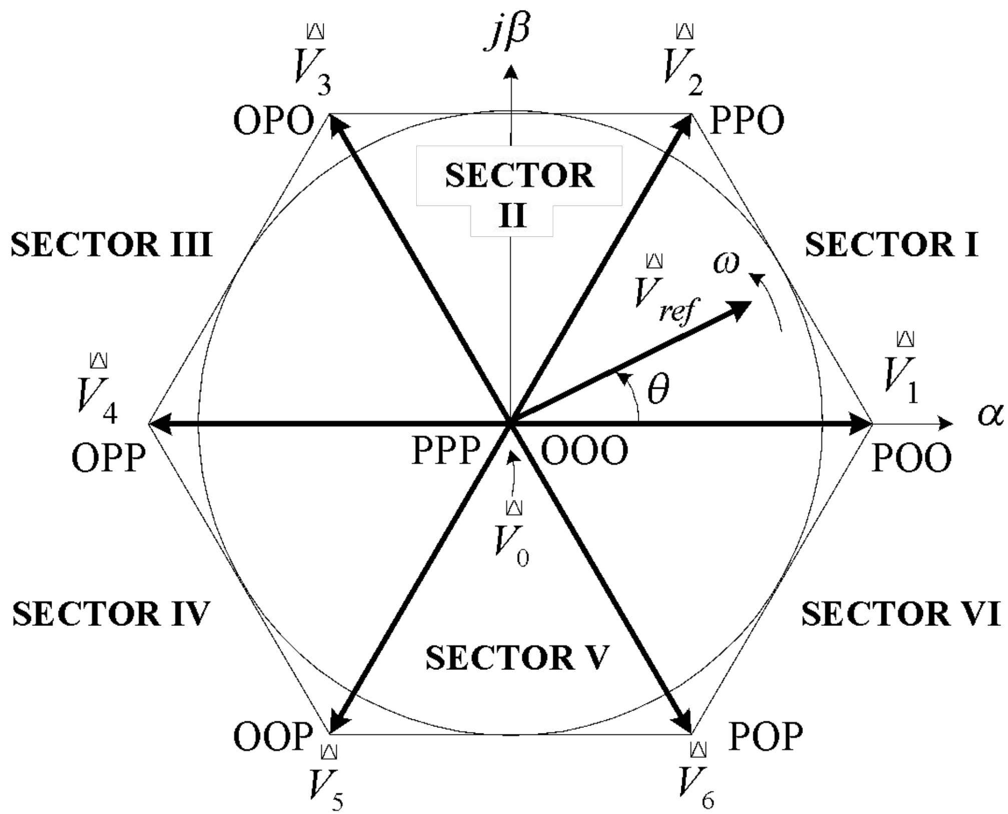
$$\vec{V}_{ref} = V_{ref}\, e^{j\theta}$$

• **Rotating in space at ω**

$$\omega = 2\pi f \qquad \textbf{(8)}$$

• **Angular displacement**

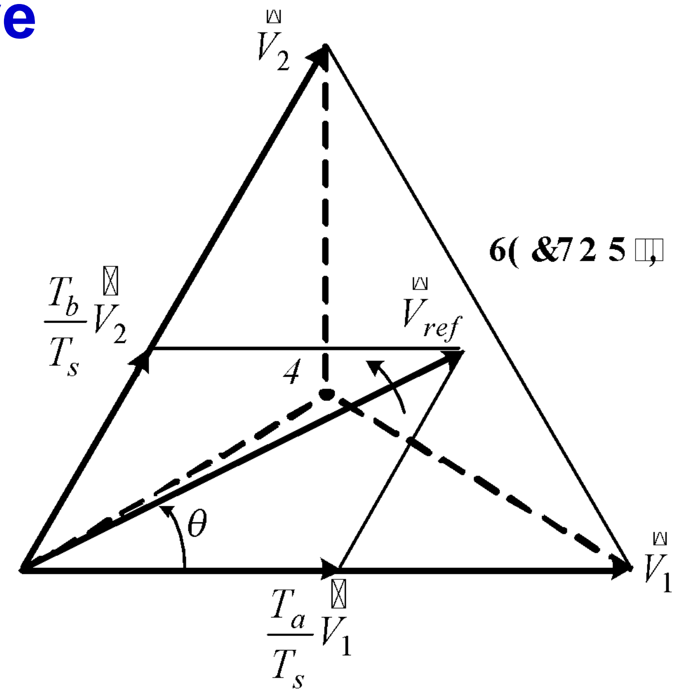
$$\theta(t) = \int_0^t \omega\, dt \qquad \textbf{(9)}$$



Space Vector Modulation

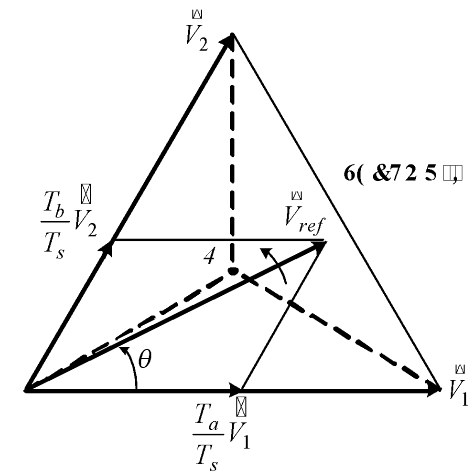
- Relationship Between V_{ref} and V_{AB}

  - V_{ref} is approximated by two active and a zero vectors
  - V_{ref} rotates one revolution, V_{AB} completes one cycle
  - Length of V_{ref} corresponds to magnitude of V_{AB}



Space Vector Modulation

• Dwell Time Calculation



- Volt-Second Balancing

$$\begin{cases} \vec{V}_{ref}\,T_s = \vec{V}_1\,T_a + \vec{V}_2\,T_b + \vec{V}_0\,T_0 \\ T_s \;=\; T_a + T_b + T_0 \end{cases} \quad \textbf{(10)}$$

- T_a, T_b and T_0 – dwell times for $\vec{V}_1, \vec{V}_2$ and $\vec{V}_0$

- T_s – sampling period

- Space vectors

$$\vec{V}_{ref} = V_{ref}\,e^{j\theta},\; \vec{V}_1 = \frac{2}{3}V_d\,,\; \vec{V}_2 = \frac{2}{3}V_d\,e^{j\frac{\pi}{3}} \text{ and } \vec{V}_0 = 0 \qquad \textbf{(11)}$$

(11) → (10)

$$\begin{cases} \mathbf{Re:} \quad V_{ref}(\cos\theta)\,T_s \;=\; \dfrac{2}{3}V_d\,T_a + \dfrac{1}{3}V_d\,T_b \\ \mathbf{Im:} \quad V_{ref}(\sin\theta)\,T_s \;=\; \dfrac{1}{\sqrt{3}}V_d\,T_b \end{cases} \qquad \textbf{(12)}$$

Space Vector Modulation

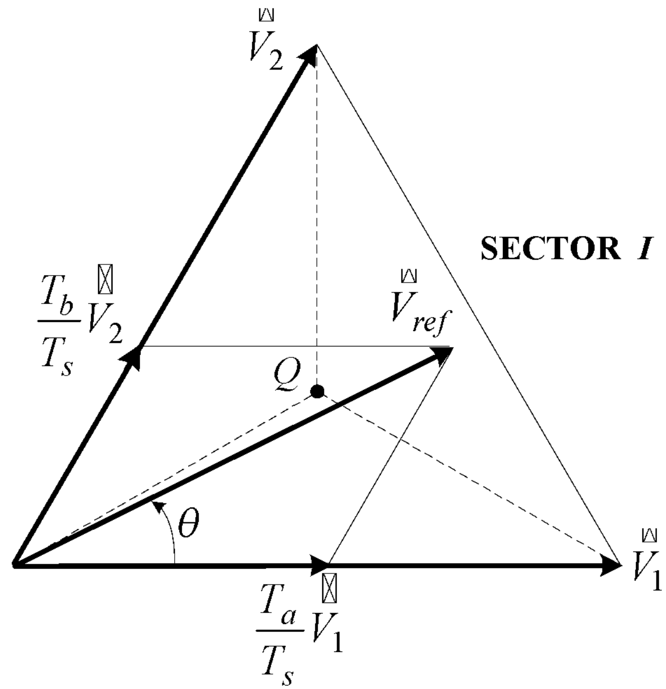
• Dwell Times

Solve (12)

$$\begin{cases} T_a = \dfrac{\sqrt{3}\, T_s V_{ref}}{V_d} \sin\left(\dfrac{\pi}{3} - \theta\right) \\ T_b = \dfrac{\sqrt{3}\, T_s V_{ref}}{V_d} \sin\theta \\ T_0 = T_s - T_a - T_b \end{cases} \qquad 0 \le \theta < \pi/3 \qquad \textbf{(13)}$$

Space Vector Modulation

- V_{ref} Location versus Dwell Times



$\vec{V}_{ref}$ **Location**	$\theta = 0$	$0 < \theta < \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	$\frac{\pi}{6} < \theta < \frac{\pi}{3}$	$\theta = \frac{\pi}{3}$
Dwell Times	$T_a > 0$ $T_b = 0$	$T_a > T_b$	$T_a = T_b$	$T_a < T_b$	$T_a = 0$ $T_b > 0$

Space Vector Modulation

• Modulation Index

$$\begin{cases} T_a = T_s m_a \sin\left(\dfrac{\pi}{3} - \theta\right) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_b - T_c \end{cases} \qquad (15)$$

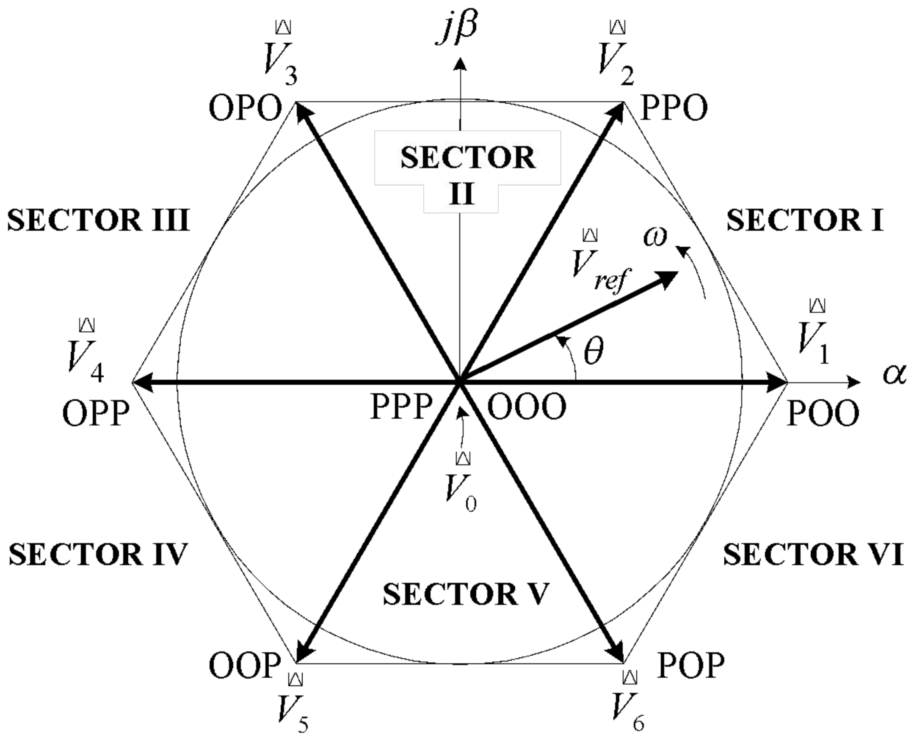
$$m_a = \frac{\sqrt{3}\, V_{ref}}{V_d} \qquad (16)$$

Space Vector Modulation

- **Modulation Range**

  - $V_{ref,max}$

$$V_{ref,\max} = \frac{2}{3} V_d \times \frac{\sqrt{3}}{2} = \frac{V_d}{\sqrt{3}} \qquad (17)$$



(17) → (16)

  - $m_{a,max} = 1$ →

  - Modulation range: $0 \le m_a \le 1$ (18)

Space Vector Modulation

- **Switching Sequence Design**
  - **Basic Requirement:**

    Minimize the number of switchings per sampling period T_s

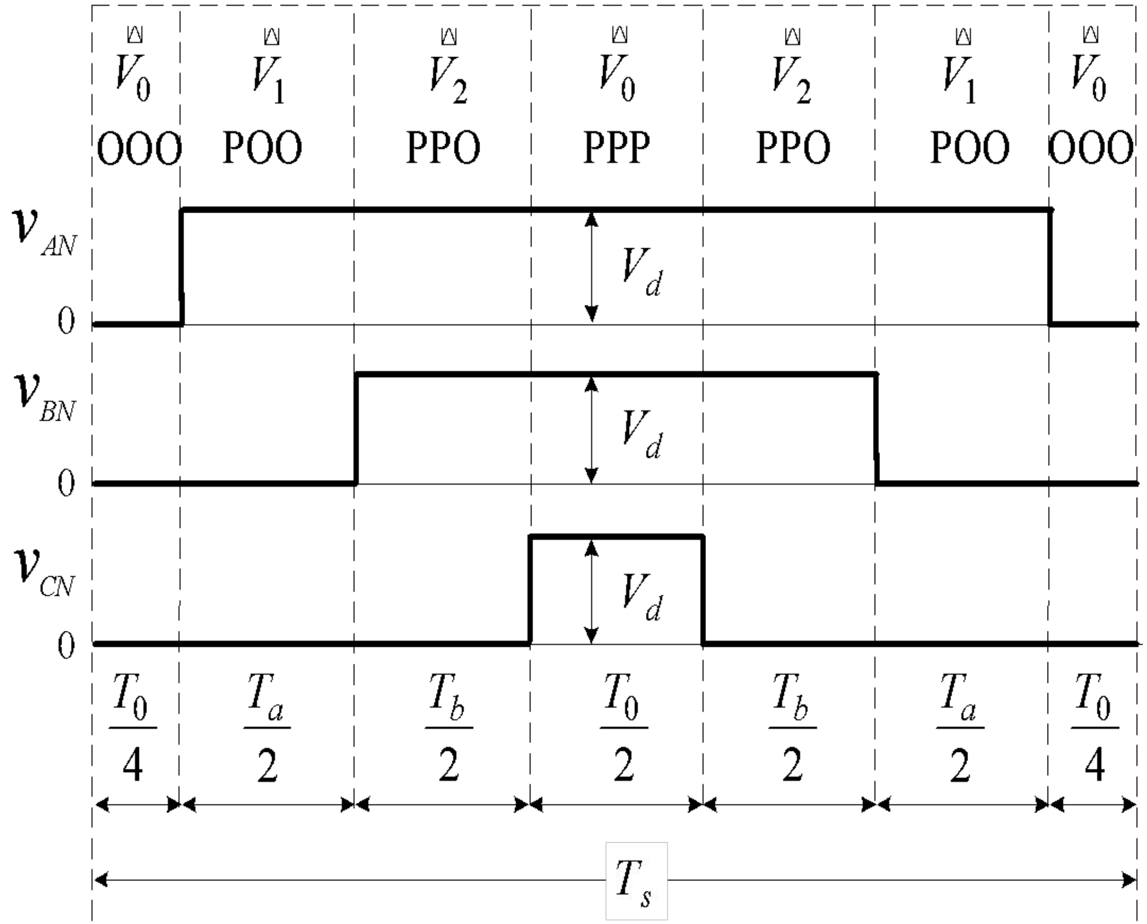
  - **Implementation:**

    Transition from one switching state to the next involves only two switches in the same inverter leg.

Space Vector Modulation

• Seven-segment Switching Sequence

- **Selected vectors:** V_0, V_1 and V_2
- **Dwell times:** $T_s = T_0 + T_a + T_b$

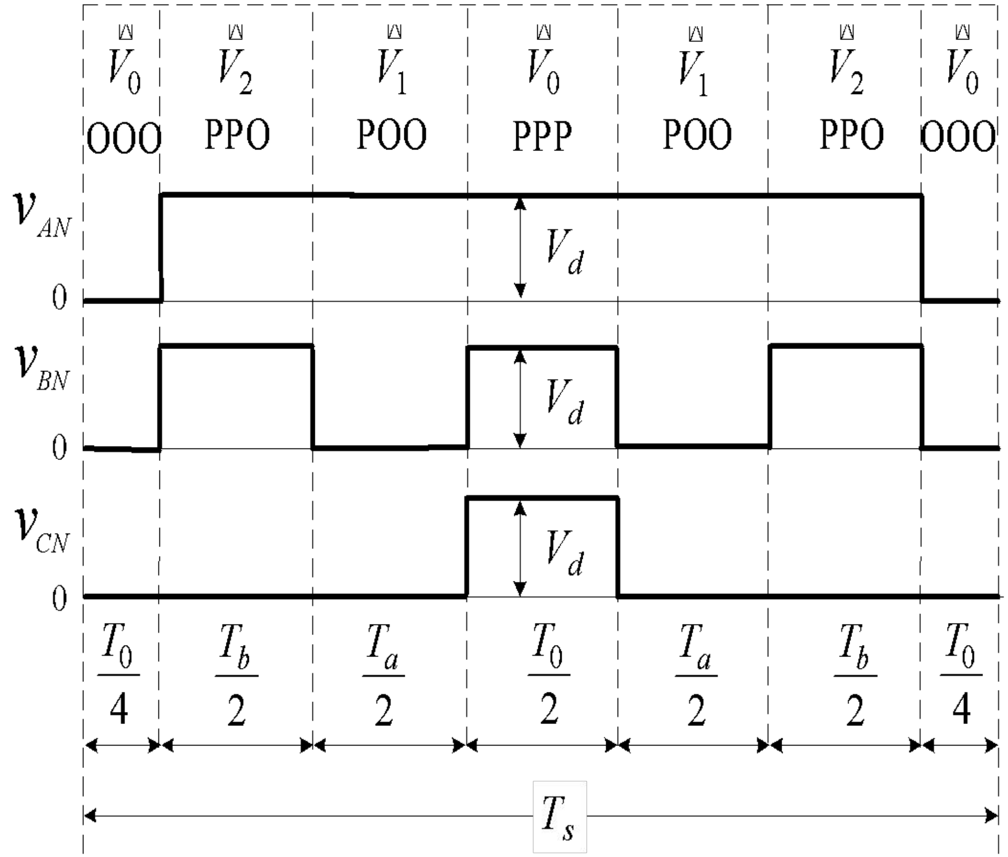


- **Total number of switchings: 6**

Space Vector Modulation

• Undesirable Switching Sequence

- Vectors V_1 and V_2 swapped



- Total number of switchings: 10

Space Vector Modulation

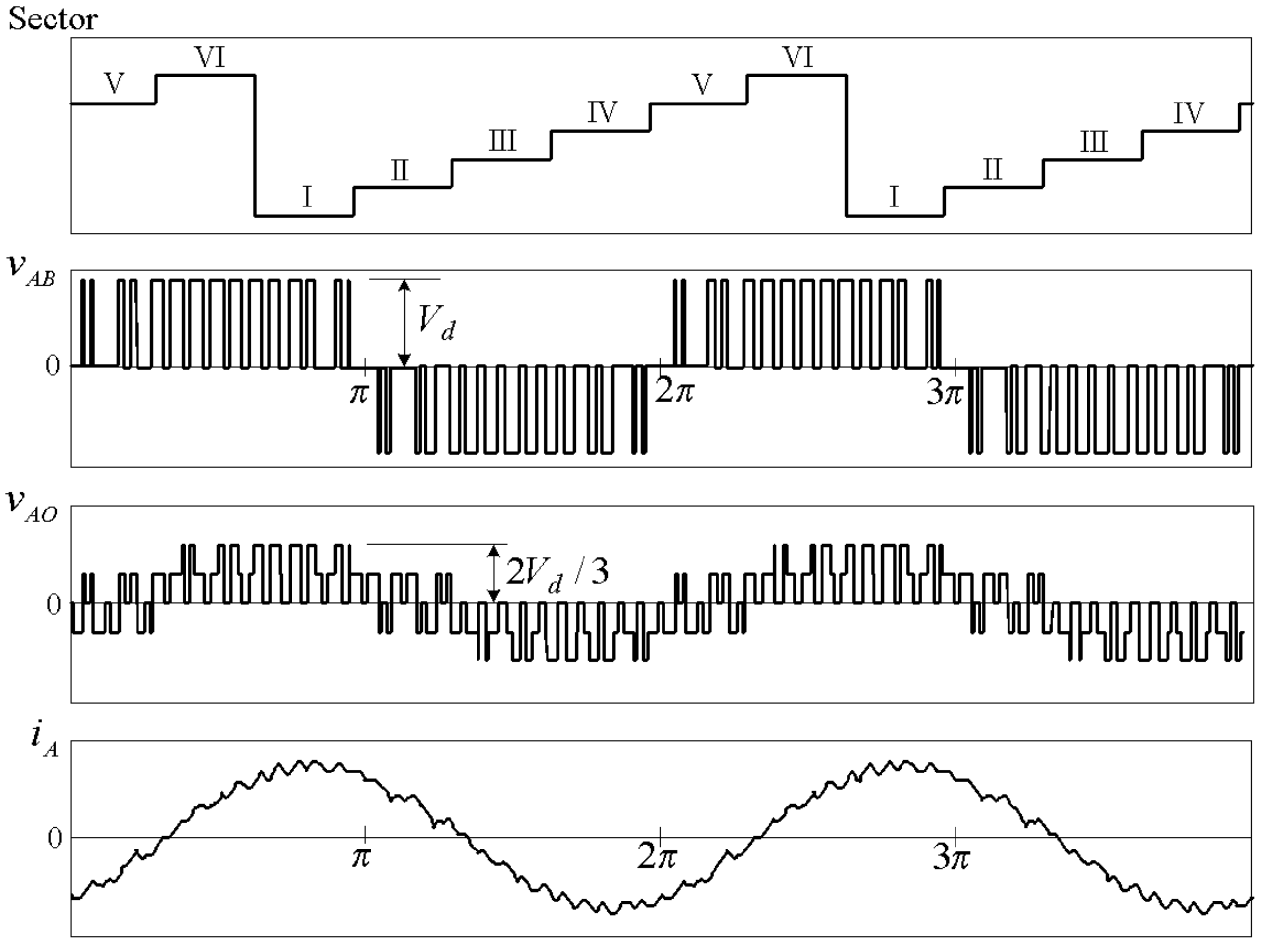
• Switching Sequence Summary (7–segments)

Sector	Switching Sequence						
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	PPO	PPP	PPO	POO	OOO
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	PPO	PPP	PPO	OPO	OOO
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	OOO	OPO	OPP	PPP	OPP	OPO	OOO
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	OPP	PPP	OPP	OOP	OOO
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	OOO	OOP	POP	PPP	POP	OOP	OOO
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	OOO	POO	POP	PPP	POP	POO	OOO

Note: The switching sequences for the odd and ever sectors are different.

Space Vector Modulation

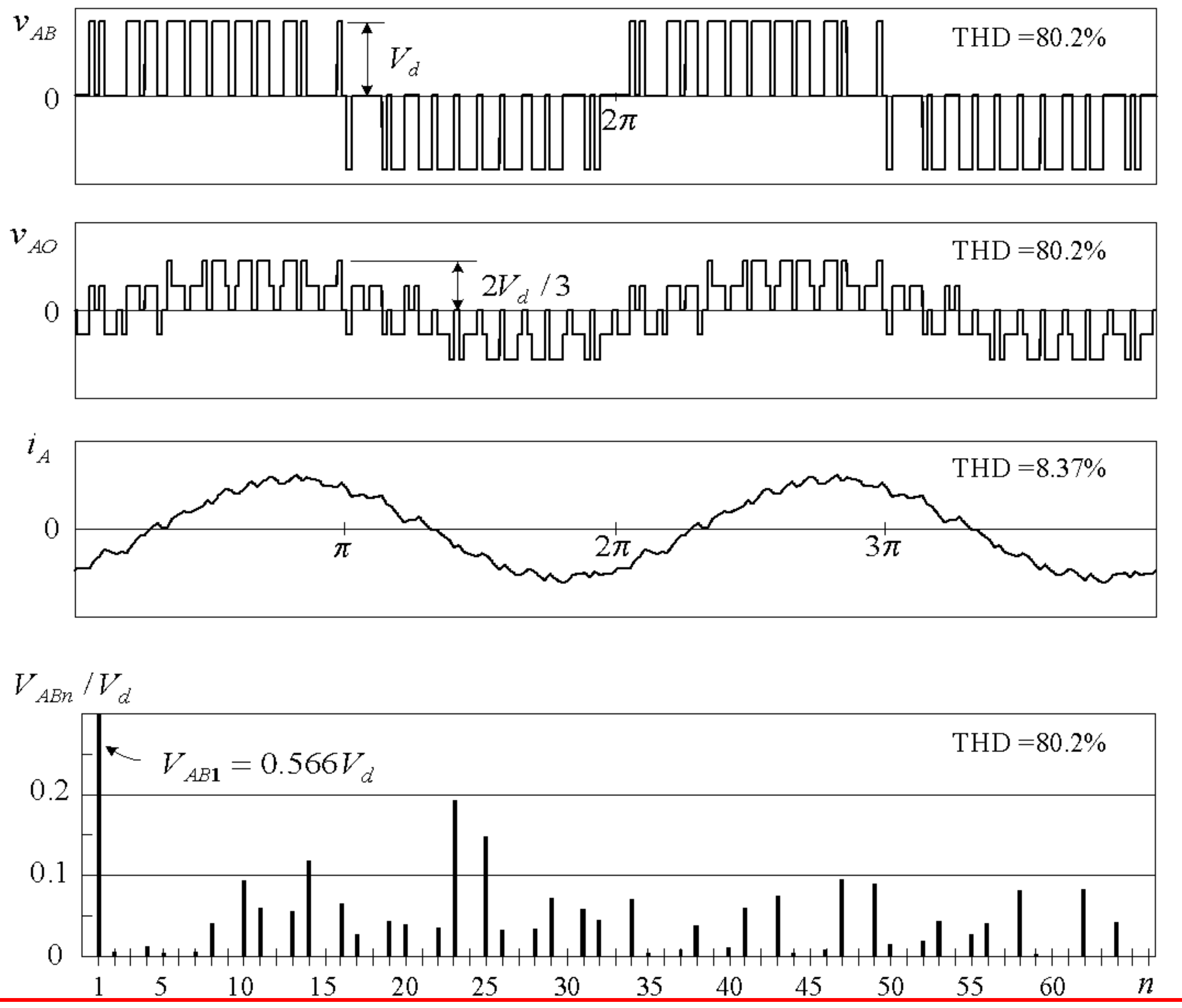
- **Simulated Waveforms**



f_1 = 60Hz, f_{sw} = 900Hz, m_a = 0.696, T_s = 1.1ms

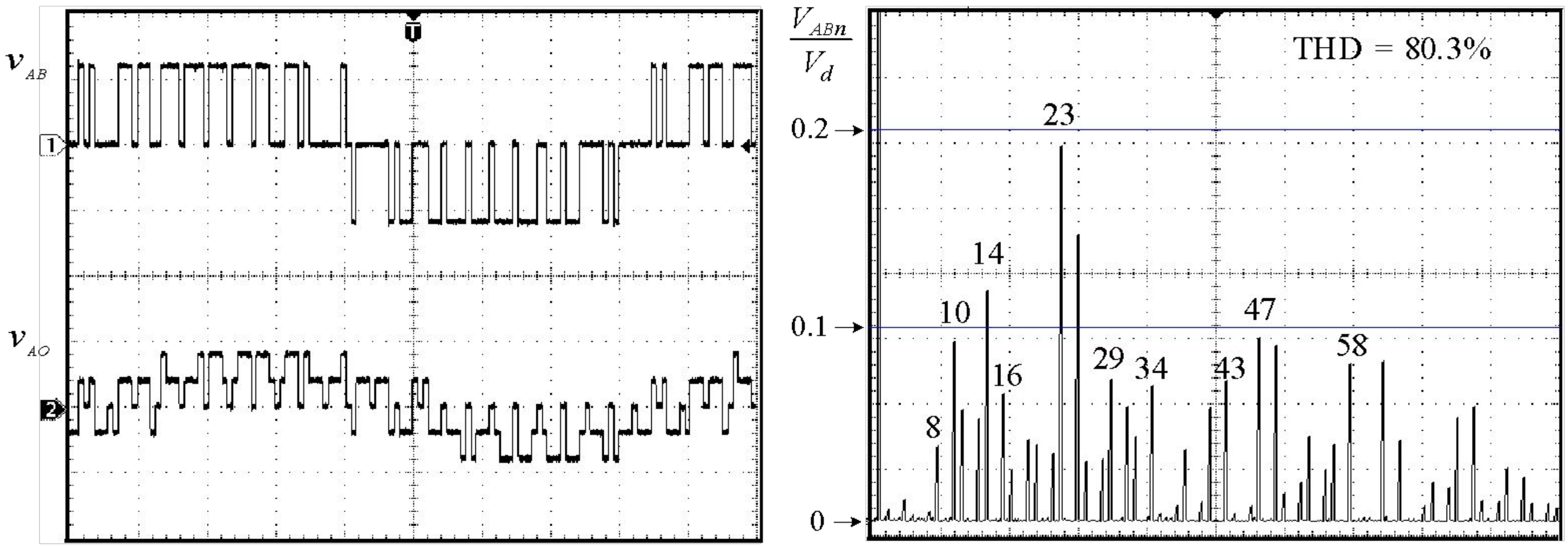
Space Vector Modulation

- **Waveforms and FFT**



Space Vector Modulation

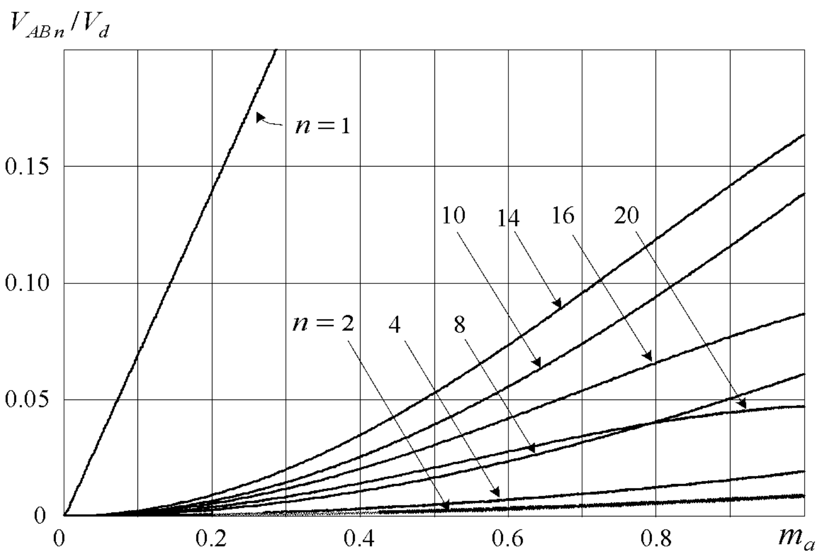
- **Waveforms and FFT** (Measured)



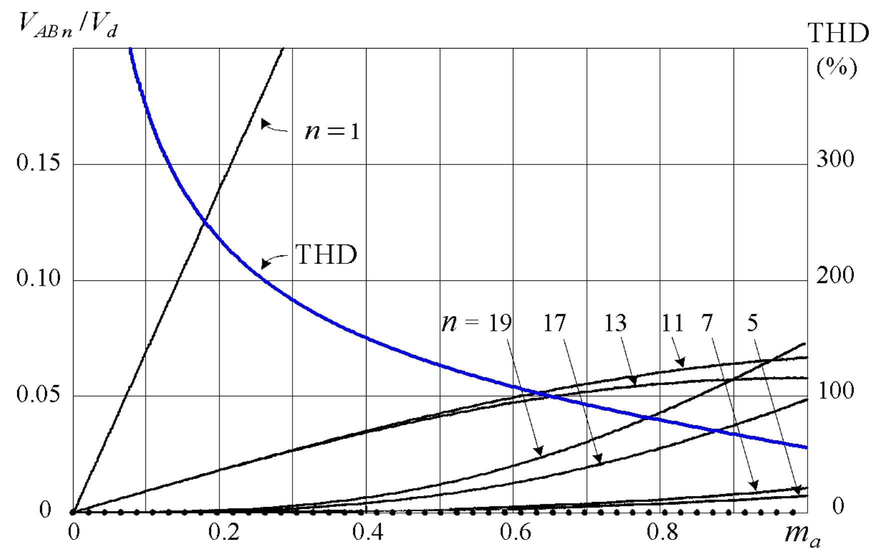
(a) Waveforms 2ms/div (b) Spectrum (500Hz/div)

Space Vector Modulation

• Waveforms and FFT (Measured)



(a) Even order harmonics

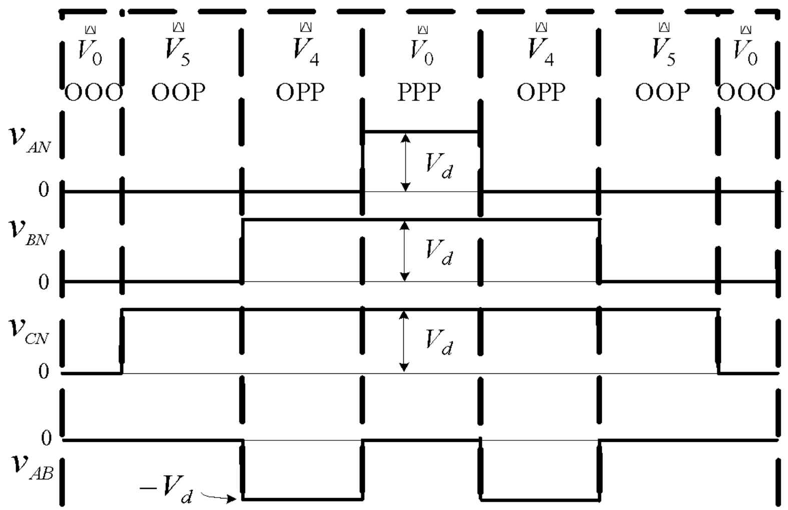


(b) Odd order harmonics

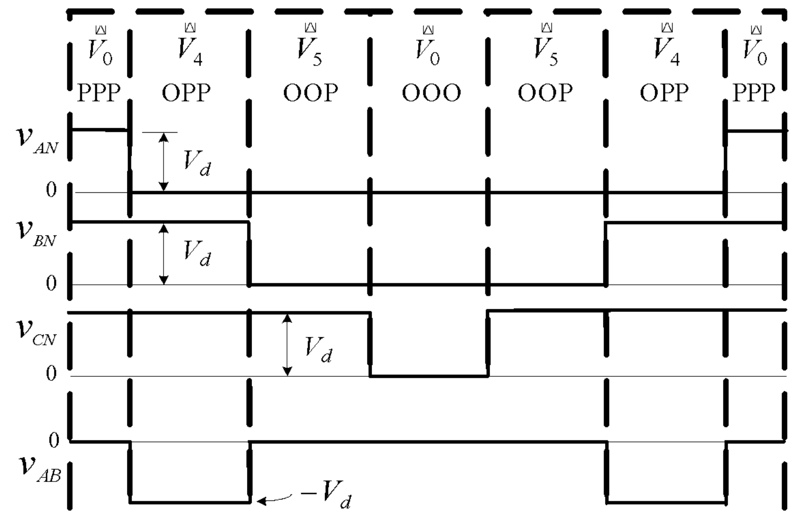
($f_1 = 60\text{Hz}$ and $T_s = 1/720$ sec)

Space Vector Modulation

- **Even-Order Harmonic Elimination**



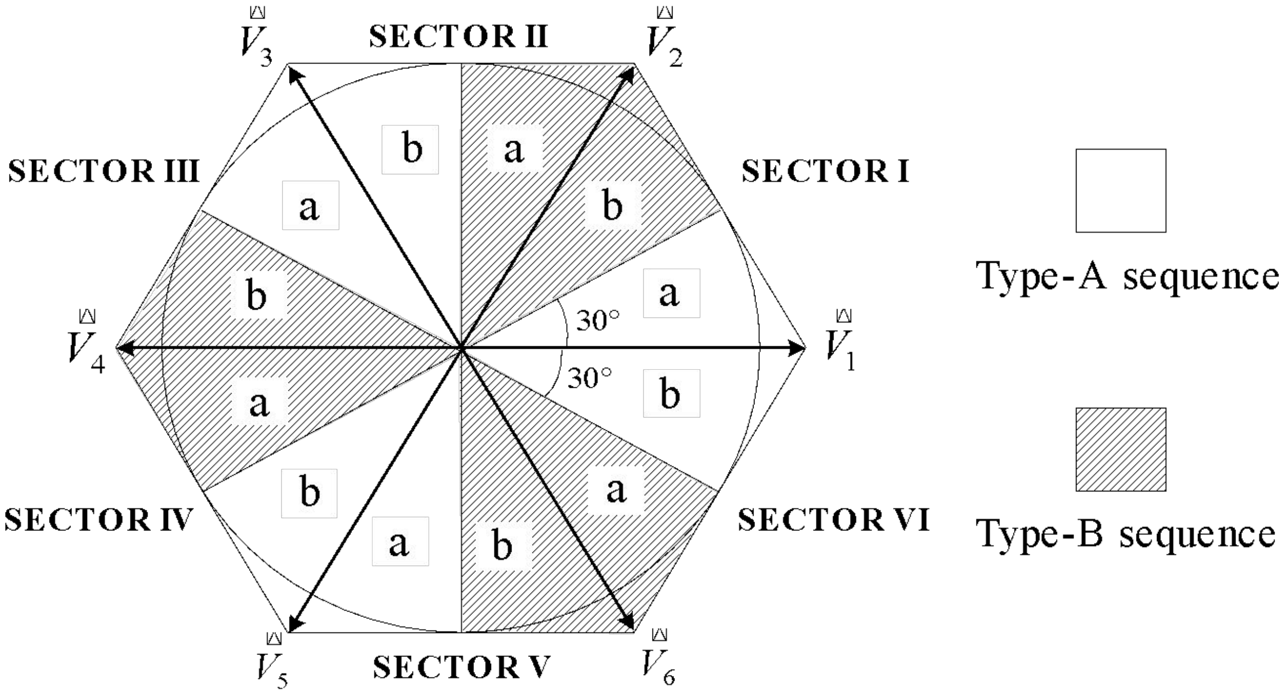
Type-A sequence
(starts and ends with [OOO])



Type-B sequence
(starts and ends with [PPP])

Space Vector Modulation

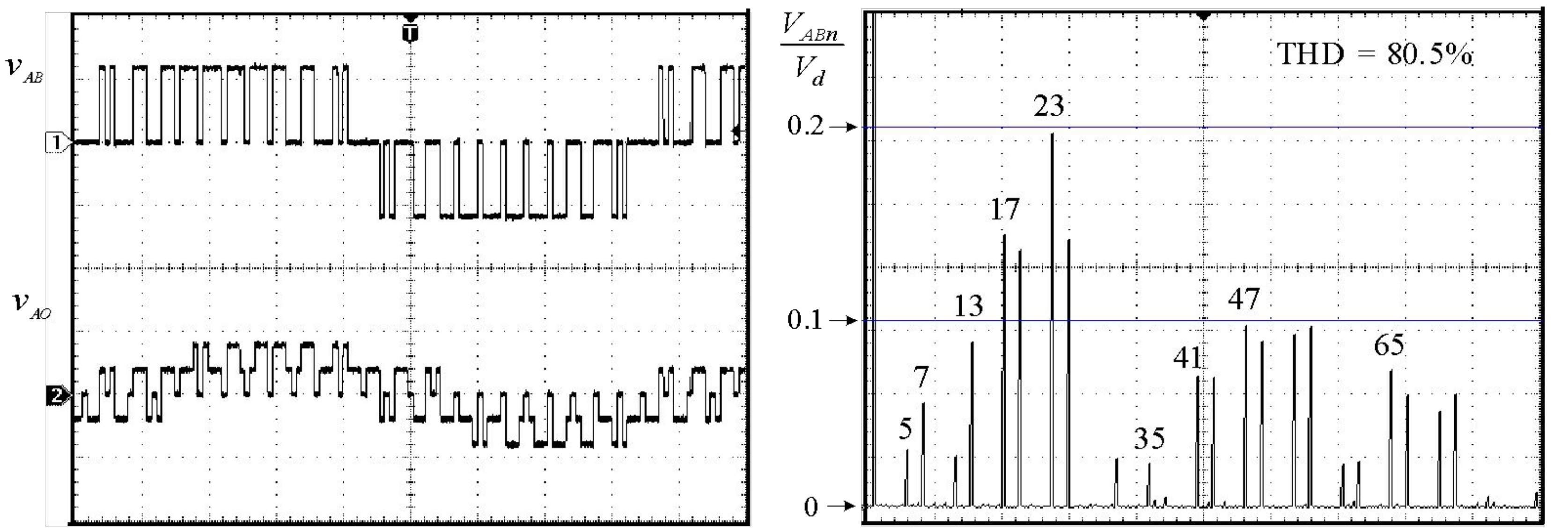
- **Even-Order Harmonic Elimination**



Space vector Diagram

Space Vector Modulation

- **Even-Order Harmonic Elimination**

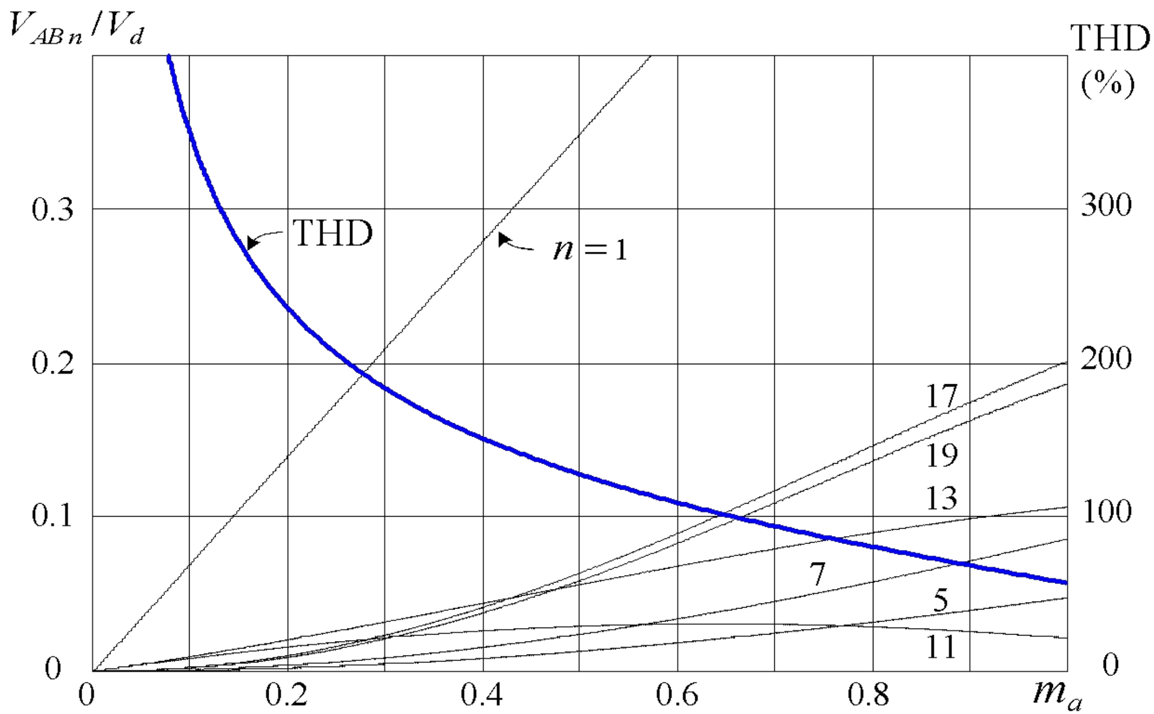


(a) Waveforms 2ms/div (b) Spectrum (500Hz/div)

- **Measured waveforms and FFT**

Space Vector Modulation

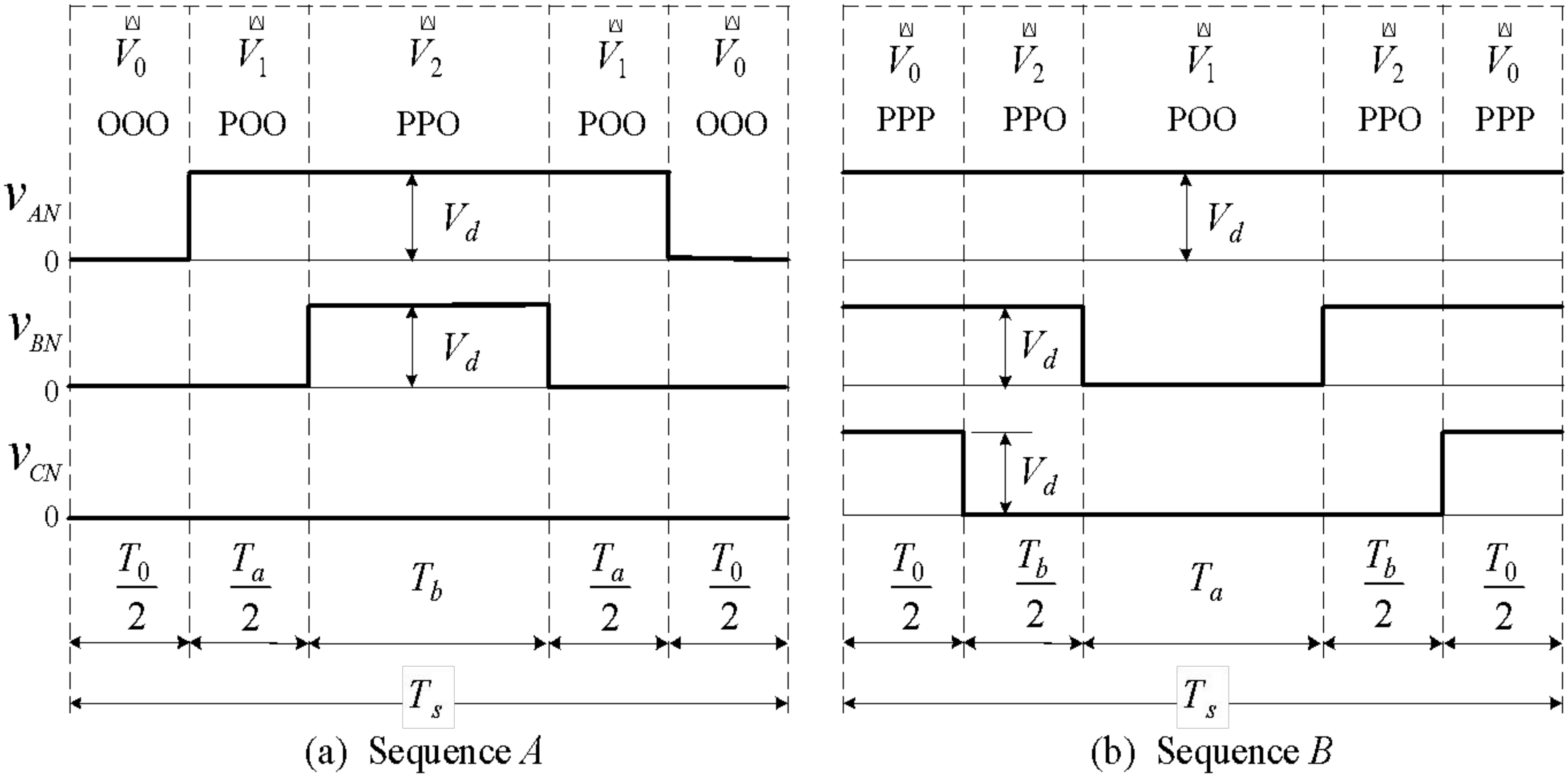
- **Even-Order Harmonic Elimination**



($f_1 = 60\text{Hz}$ **and** $T_s = 1/720$ sec)

Space Vector Modulation

- **Five-segment SVM**



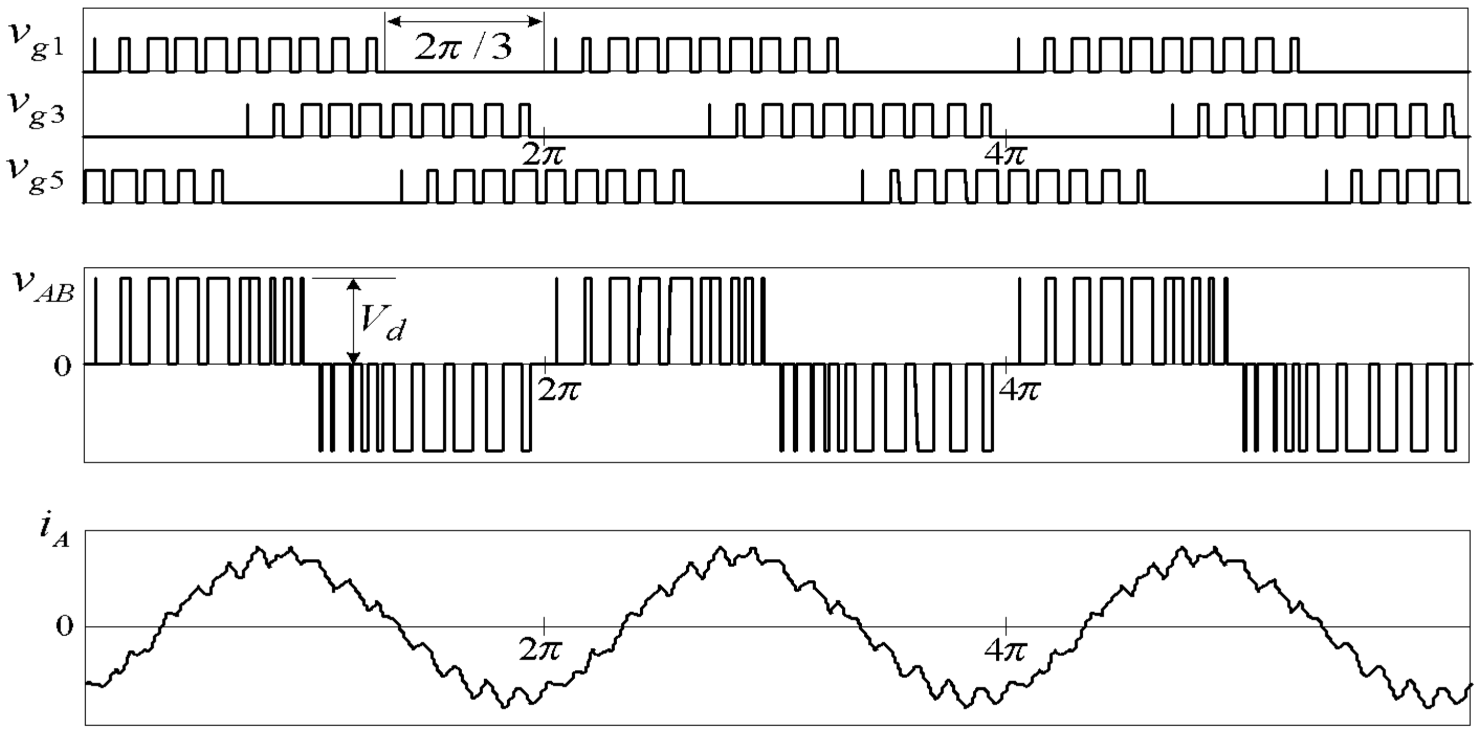
Space Vector Modulation

• Switching Sequence (5-segment)

Sector	Switching Sequence (A)					
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$	$v_{CN} = 0$
	OOO	POO	PPO	POO	OOO	
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$	$v_{CN} = 0$
	OOO	OPO	PPO	OPO	OOO	
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$	$v_{AN} = 0$
	OOO	OPO	OPP	OPO	OOO	
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$	$v_{AN} = 0$
	OOO	OOP	OPP	OOP	OOO	
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$	$v_{BN} = 0$
	OOO	OOP	POP	OOP	OOO	
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$	$v_{BN} = 0$
	OOO	POO	POP	POO	OOO	

Space Vector Modulation

• Simulated Waveforms (5-segment)



• f_1 = 60Hz, f_{sw} = 600Hz, m_a = 0.696, T_s = 1.1ms

• No switching for a 120° period per cycle.
• Low switching frequency but high harmonic distortion

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Thanks