

Power Converter Systems

Graduate Course EE8407

Bin Wu PhD, PEng

Professor
ELCE Department
Ryerson University

Contact Info

Office: ENG328

Tel: (416) 979-5000 ext: 6484

Email: bwu@ee.ryerson.ca

<http://www.ee.ryerson.ca/~bwu/>



Ryerson Campus

Topic 9

PWM Current Source Inverters (CSI)



PWM CSI fed MV drive

Courtesy of
Rockwell Automation

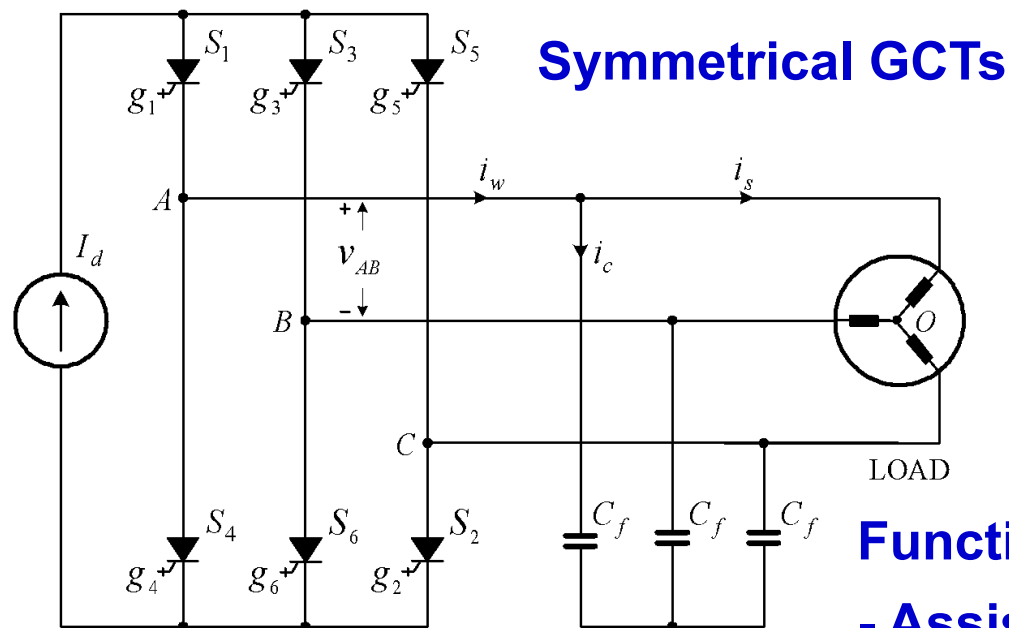
PWM Current Source Inverters

Lecture Topics

- **Single Bridge Inverter**
- **Trapezoidal Modulation (TPWM)**
- **Selective Harmonic Elimination (SHE)**
- **Space Vector Modulation (SVM)**
- **Dual Bridge Inverter**

Single Bridge Inverter

• Inverter Topology



Function of C_f :

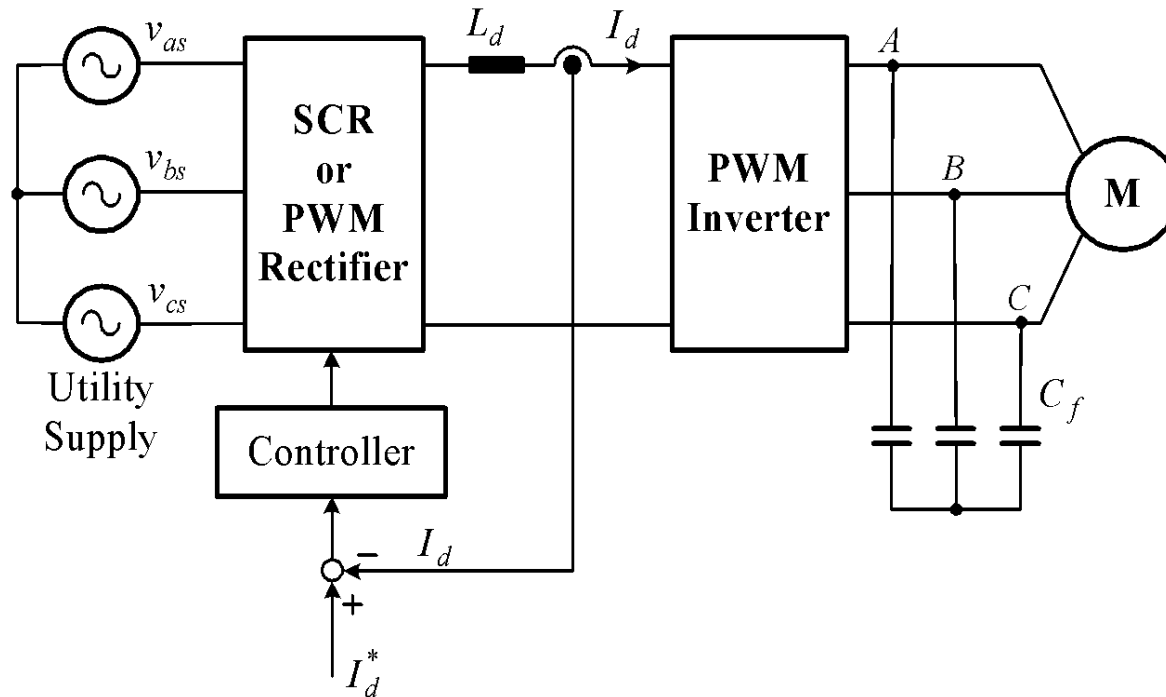
- Assist GCT commutation
- Reduce harmonic distortion

Features:

- Simple topology – no antiparallel diodes
- Reliable short circuit protection – constant dc current
- Very low dv/dt on motor terminals

Single Bridge Inverter

- DC Current Source I_d

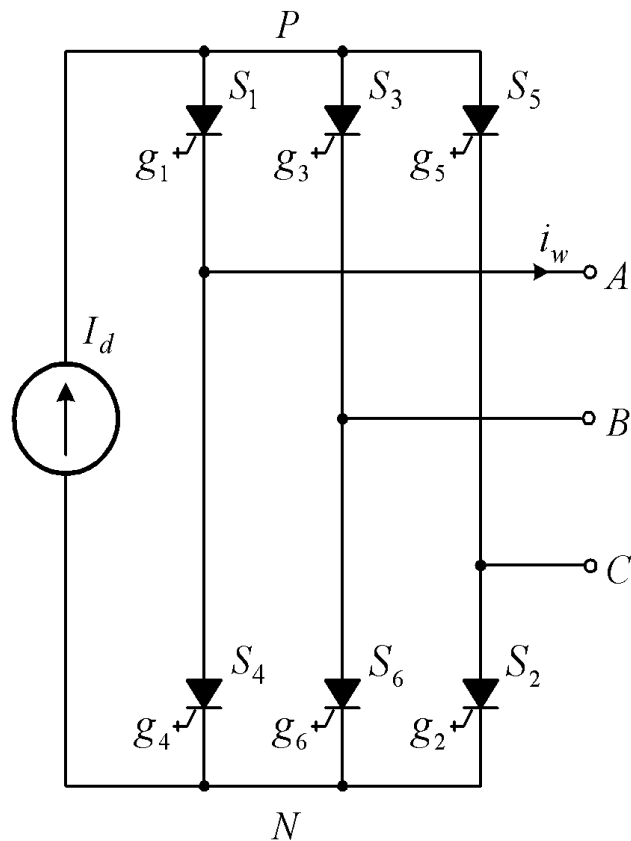


Implementation of dc current source:

- use a large size dc choke – making I_d smooth;
- use dc current feedback control – keeping I_d constant

Single Bridge Inverter

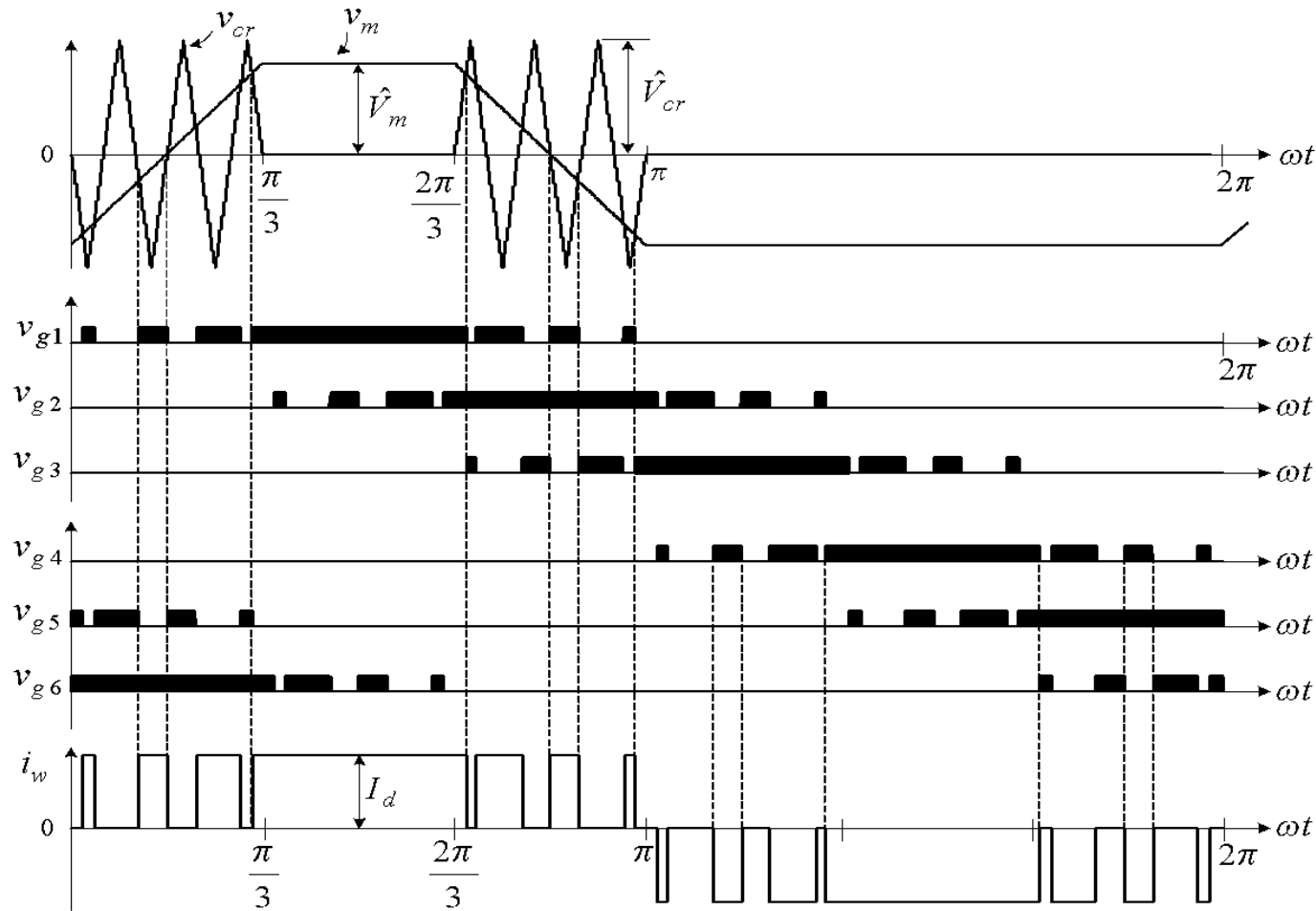
• Modulation Techniques



- Trapezoidal PWM (TPWM)
- Selective Harmonic Elimination (SHE)
- Space Vector Modulation (SVM)
- Constraints on Switching Pattern Design
 - dc current I_d cannot be interrupted
 - inverter output current waveform must be defined.
- At any time instant, only two switches are turned on, one connected to the positive dc bus, and the other to the negative dc bus.

Trapezoidal Modulation

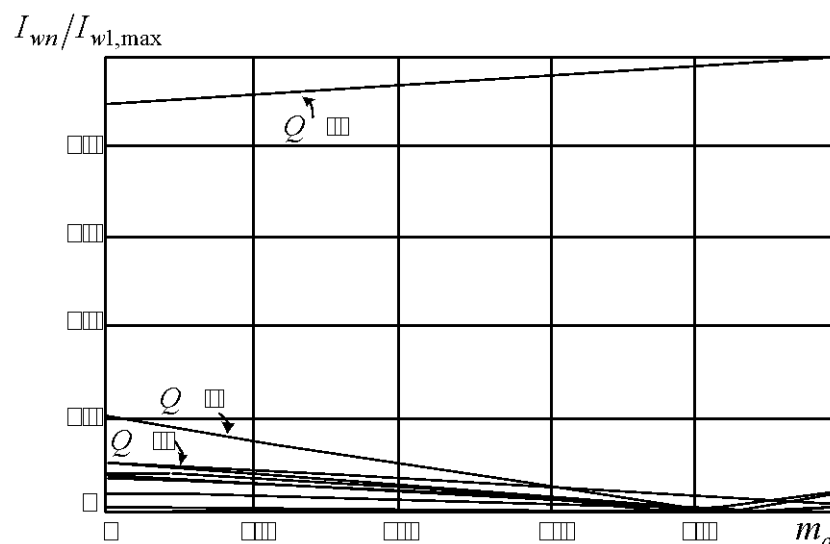
• Trapezoidal PWM (TPWM)



Number of pulses per half cycle: $N_p = 7$

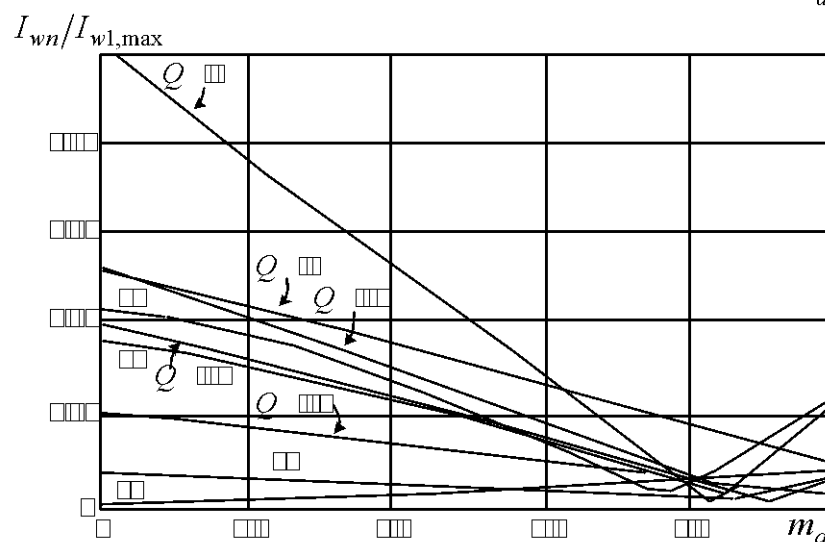
Trapezoidal Modulation

• Harmonic Content (TPWM)



- $N_p = 15$

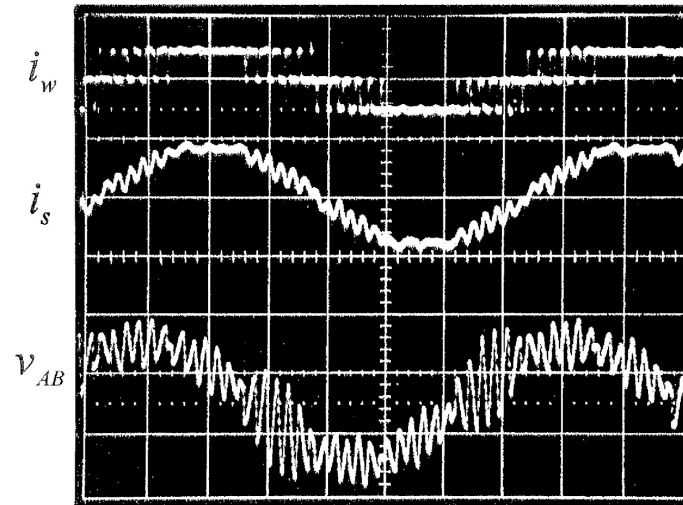
- Fundamental does not change much with m_a



- $m_a = 0.85$

Trapezoidal Modulation

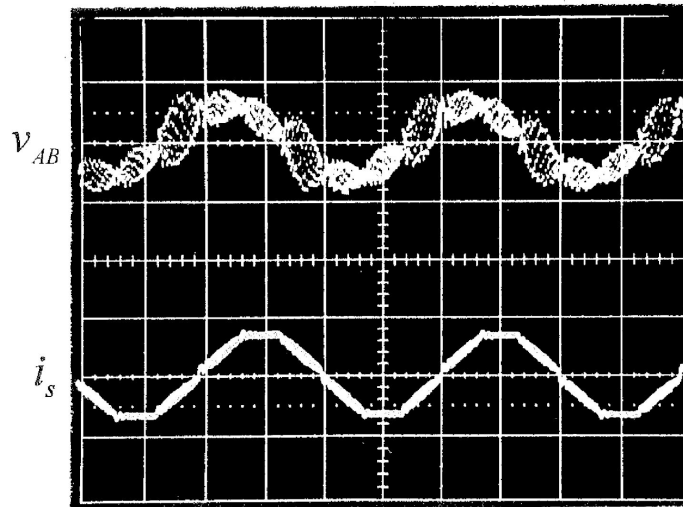
- Waveforms (TPWM)



(a) $f_1 = 13.8\text{Hz}$

$$f_1 = 13.8\text{Hz}, N_p = 13,$$

$$f_{sw} = 180\text{Hz}$$



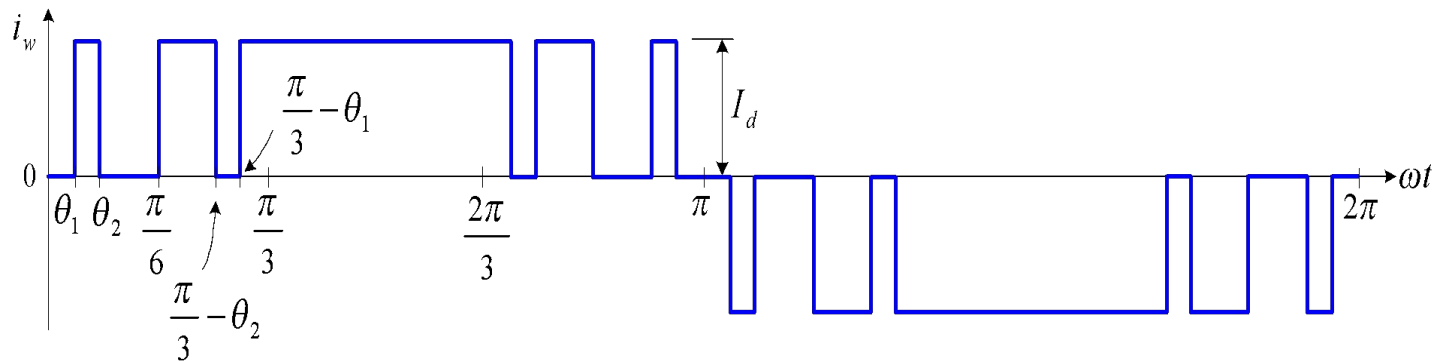
(b) $f_1 = 5\text{Hz}$

$$f_1 = 5\text{Hz}, N_p = 31,$$

$$f_{sw} = 155\text{Hz}$$

Selective Harmonic Elimination (SHE)

• Principle



- Not all the switching angles are independent.
- Only two harmonics can be eliminated with $N_p = 5$.
- Number of harmonics to be eliminated:

$$k = (N_p - 1) / 2$$

- No modulation index control – the magnitude of inverter output current is controlled by I_d

Selective Harmonic Elimination

• Fourier Analysis

$$i_w(\omega t) = \sum_{n=1}^{\infty} a_n \sin(n\omega t)$$

$$a_n = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} i_w(\omega t) \sin(n\omega t) d(\omega t)$$

$$a_n = \frac{4I_{dc}}{\pi} \times \begin{cases} \int_{\theta_1}^{\theta_2} \sin(n\omega t) d(\omega t) + \dots + \int_{\theta_k}^{\frac{\pi}{6}} \sin(n\omega t) d(\omega t) + \\ \int_{\frac{\pi}{3}-\theta_{k-1}}^{\frac{\pi}{3}-\theta_k} \sin(n\omega t) d(\omega t) + \dots + \int_{\frac{\pi}{3}-\theta_1}^{\frac{\pi}{2}} \sin(n\omega t) d(\omega t) & k = \text{odd}; \\ \int_{\theta_1}^{\theta_2} \sin(n\omega t) d(\omega t) + \dots + \int_{\theta_{k-1}}^{\theta_k} \sin(n\omega t) d(\omega t) + \\ \int_{\frac{\pi}{6}}^{\frac{\pi}{3}-\theta_k} \sin(n\omega t) d(\omega t) + \dots + \int_{\frac{\pi}{3}-\theta_1}^{\frac{\pi}{2}} \sin(n\omega t) d(\omega t) & k = \text{even}. \end{cases}$$

Selective Harmonic Elimination

• Fourier Analysis

Expression for a_n

$$a_n = \frac{4I_{dc}}{\pi n} \times \left\{ \begin{array}{l} \cos(n\theta_1) + \cos[n(\frac{\pi}{3} - \theta_1)] - \cos(n\theta_2) - \cos[n(\frac{\pi}{3} - \theta_2)] + \dots \\ + \cos(n\theta_k) + \cos[n(\frac{\pi}{3} - \theta_k)] - \cos(n\frac{\pi}{6}) \quad k = \text{odd}; \\ \cos(n\theta_1) + \cos[n(\frac{\pi}{3} - \theta_1)] - \cos(n\theta_2) - \cos[n(\frac{\pi}{3} - \theta_2)] + \dots \\ - \cos(n\theta_k) - \cos[n(\frac{\pi}{3} - \theta_k)] + \cos(n\frac{\pi}{6}) \quad k = \text{even}. \end{array} \right.$$

Selective Harmonic Elimination

• Switching Angle Calculation

To Eliminate n^{th} harmonic, set $a_n = 0$

$$F_i(\theta_1, \theta_2, \theta_3, \dots, \theta_k) = 0 \quad i = 1, 2, \dots, k$$

For 5th, 7th and 11th harmonic elimination:

$$F_1 = \cos(5\theta_1) + \cos\left[5\left(\frac{\pi}{3} - \theta_1\right)\right] - \cos(5\theta_2) - \cos\left[5\left(\frac{\pi}{3} - \theta_2\right)\right] + \\ + \cos(5\theta_3) + \cos\left[5\left(\frac{\pi}{3} - \theta_3\right)\right] - \cos\left(\frac{5\pi}{6}\right) = 0$$

$$F_2 = \cos(7\theta_1) + \cos\left[7\left(\frac{\pi}{3} - \theta_1\right)\right] - \cos(7\theta_2) - \cos\left[7\left(\frac{\pi}{3} - \theta_2\right)\right] \\ + \cos(7\theta_3) + \cos\left[7\left(\frac{\pi}{3} - \theta_3\right)\right] - \cos\left(\frac{7\pi}{6}\right) = 0$$

$$F_3 = \cos(11\theta_1) + \cos\left[11\left(\frac{\pi}{3} - \theta_1\right)\right] - \cos(11\theta_2) - \cos\left[11\left(\frac{\pi}{3} - \theta_2\right)\right] \\ + \cos(11\theta_3) + \cos\left[11\left(\frac{\pi}{3} - \theta_3\right)\right] - \cos\left(\frac{11\pi}{6}\right) = 0$$

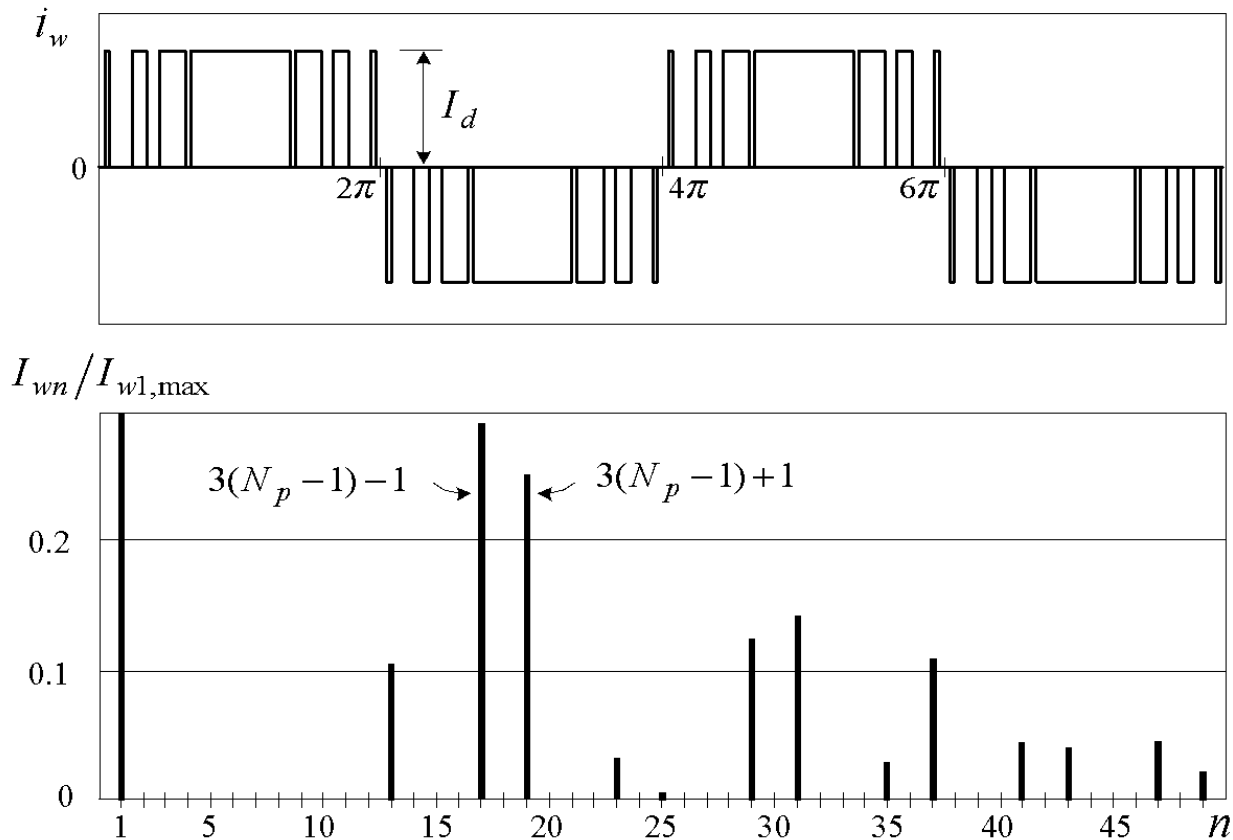
Selective Harmonic Elimination

• Switching Angles (SHE)

Harmonics Eliminated	Switching Angles			Harmonics Eliminated	Switching Angles			
	θ_1	θ_2	θ_3		θ_1	θ_2	θ_3	θ_4
5	18.00	-	-	7,11,17	11.70	14.12	24.17	-
7	21.43	-	-	7,13,17	12.69	14.97	24.16	-
11	24.55	-	-	7,13,19	13.49	15.94	24.53	-
13	25.38	-	-	11,13,17	14.55	15.97	25.06	-
5,7	7.93	13.75	-	11,13,19	15.24	16.71	25.32	-
5,11	12.96	19.14	-	13,17,19	17.08	18.23	25.84	-
5,13	14.48	21.12	-	13,17,23	18.03	19.22	26.16	-
7,11	15.23	19.37	-	5,7,11,13	0.00	1.60	15.14	20.26
7,13	16.58	20.79	-	5,7,11,17	0.07	2.63	16.57	21.80
7,17	18.49	23.08	-	5,7,11,19	1.11	4.01	18.26	23.60
11,13	19.00	21.74	-	5,7,13,17	1.50	4.14	16.40	21.12
11,17	20.51	23.14	-	5,7,13,19	2.56	5.57	17.82	22.33
11,19	21.10	23.75	-	5,7,17,19	4.59	7.96	17.17	20.55
13,17	21.19	23.45	-	5,11,13,17	4.16	6.07	16.79	22.04
13,19	21.71	23.94	-	5,11,13,19	5.13	7.26	17.57	22.72
5,7,11	2.24	5.60	21.26	5,11,17,19	6.93	9.15	17.85	22.77
5,7,13	4.21	8.04	22.45	5,13,17,19	7.80	9.82	18.01	23.25
5,7,17	6.91	11.96	25.57	7,11,13,17	5.42	6.65	18.03	22.17
5,11,13	7.81	11.03	22.13	7,11,13,19	6.35	7.69	18.67	22.74
5,11,17	10.16	14.02	23.34	7,11,17,19	8.07	9.44	19.09	22.93
5,13,17	11.24	14.92	22.98	7,13,17,19	8.88	10.12	19.35	23.22
7,11,13	9.51	11.64	23.27	11,13,17,19	10.39	11.14	20.56	23.60

Selective Harmonic Elimination

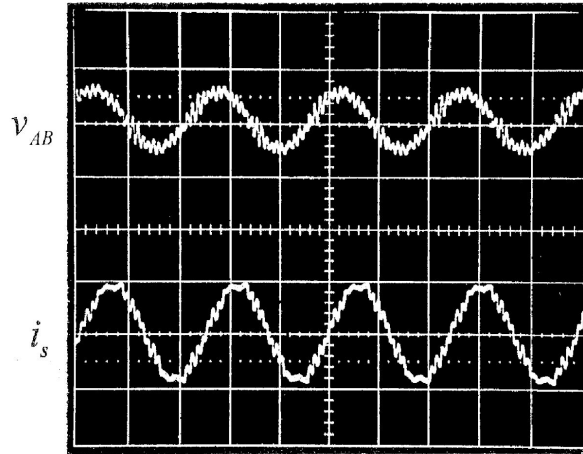
• Inverter Waveforms (SHE)



Harmonic eliminated: 5th, 7th and 11th; $f_{sw} = 420\text{Hz}$

Selective Harmonic Elimination

• Waveforms (SHE)



(a) $f_1 = 20\text{Hz}$

(a)

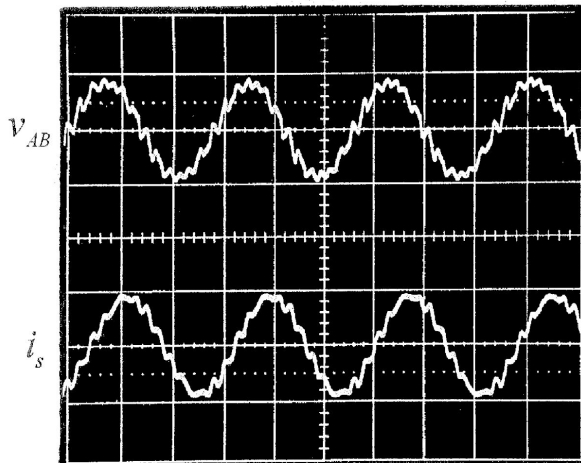
$$f_1 = 20\text{Hz}, N_p = 7$$

$$I_{w5} = I_{w7} = I_{w11} = 0$$

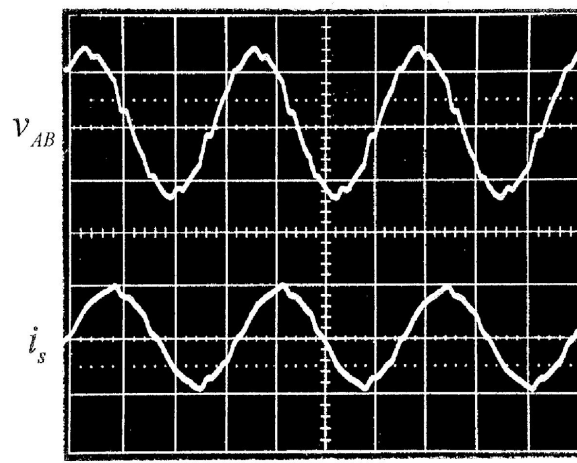
(b)

$$f_1 = 35\text{Hz}, N_p = 5$$

$$I_{w5} = I_{w7} = 0$$



(b) $f_1 = 35\text{Hz}$



(c) $f_1 = 60\text{Hz}$

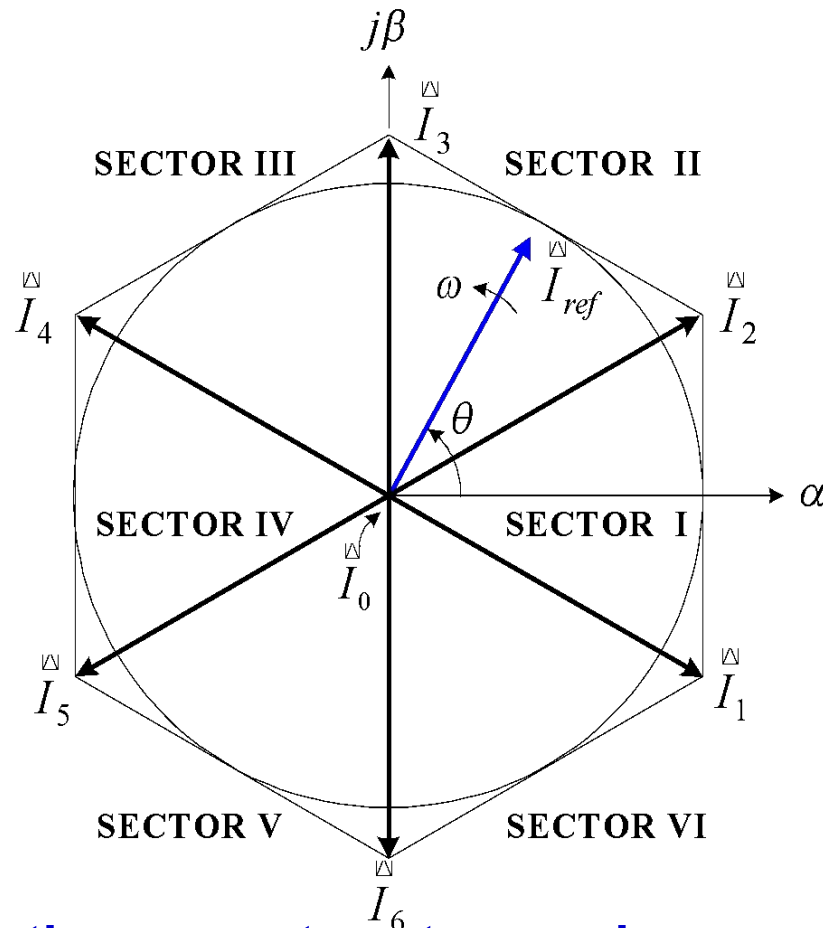
(c)

$$f_1 = 60\text{Hz}, N_p = 3$$

$$I_{w5} = 0$$

Space Vector Modulation

- Space Vector Modulation

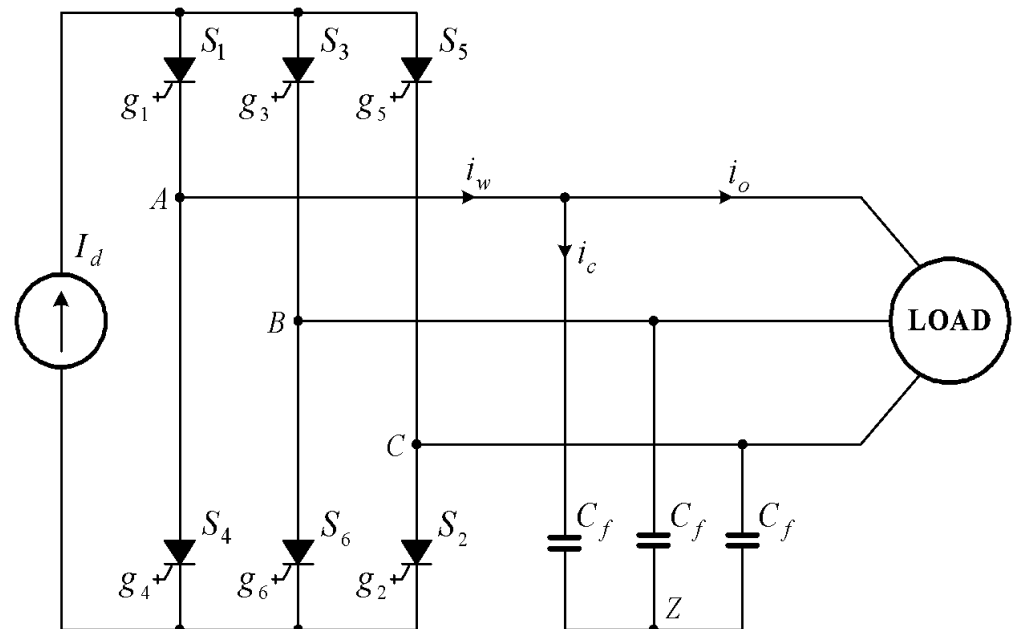


- Six active current vectors and one zero vector
- Six sectors

Space Vector Modulation

- Switching State

Switching State	Leg A		
	S_1	S_4	I_w
[1]	On	Off	I_d
[-1]	Off	On	$-I_d$
[0]	Off	Off	0
[2]	On	On	0



Space Vector Modulation

• Switching States & Space Vectors

Space Vector		Switching State	On-state Switch	Vector Definition
Zero Vector	I_0	[2 0 0]	S_1, S_4	$I_0 = 0$
		[0 2 0]	S_3, S_6	
		[0 0 2]	S_5, S_2	
Active Vector	I_1	[1 -1 0]	S_6, S_1	$I_1 = \frac{2}{\sqrt{3}} I_d e^{-j\pi/6}$
	I_2	[1 0 -1]	S_1, S_2	$I_2 = \frac{2}{\sqrt{3}} I_d e^{j\pi/6}$
	I_3	[0 1 -1]	S_2, S_3	$I_3 = \frac{2}{\sqrt{3}} I_d e^{j3\pi/6}$
	I_4	[-1 1 0]	S_3, S_4	$I_4 = \frac{2}{\sqrt{3}} I_d e^{j5\pi/6}$
	I_5	[-1 0 1]	S_4, S_5	$I_5 = \frac{2}{\sqrt{3}} I_d e^{j7\pi/6}$
	I_6	[0 -1 1]	S_5, S_6	$I_6 = \frac{2}{\sqrt{3}} I_d e^{j9\pi/6}$

[1]: Upper switch on

[-1]: Lower switch on

[0]: None of the switches in a leg turned on

[2]: Both switches in a leg turned on (bypass mode)

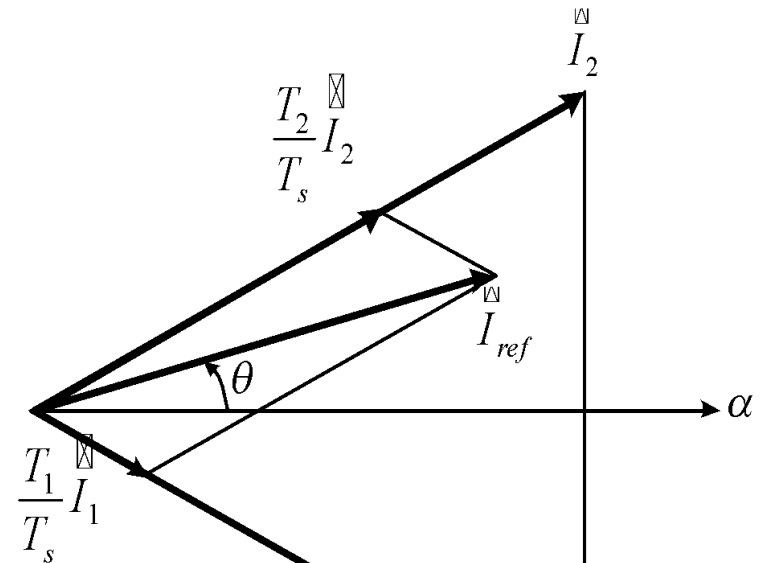
Space Vector Modulation

• Dwell Time Calculation

Following the same principle of SVM for the NPC inverter

$$\begin{cases} \vec{I}_{ref} T_s = \vec{I}_1 T_1 + \vec{I}_2 T_2 + \vec{I}_0 T_0 \\ T_s = T_1 + T_2 + T_0 \end{cases}$$

$$\begin{cases} \text{Re: } I_{ref} (\cos \theta) T_s = I_d (T_1 + T_2) \\ \text{Im: } I_{ref} (\sin \theta) T_s = \frac{1}{\sqrt{3}} I_d (-T_1 + T_2) \end{cases}$$



Space Vector Modulation

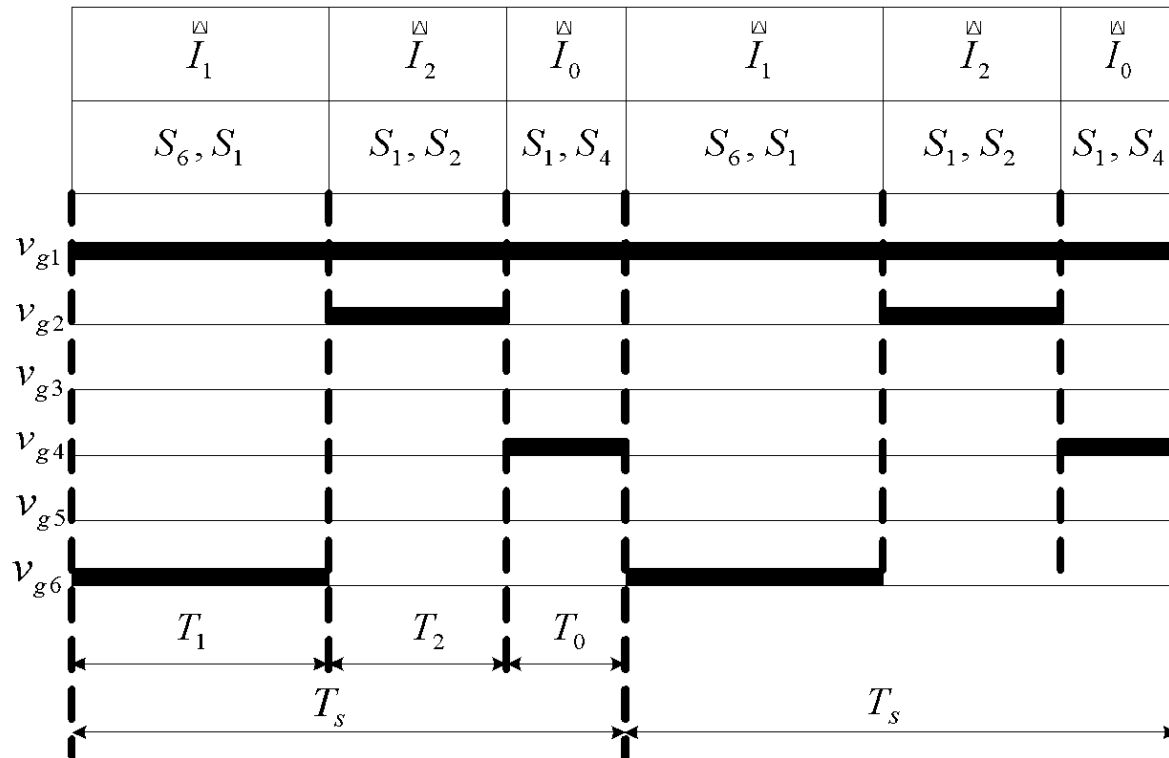
• Dwell Time Calculation

$$\left\{ \begin{array}{l} T_1 = \frac{I_{ref}}{I_d} \sin\left(\frac{\pi}{6} - \theta\right) T_s \\ T_2 = \frac{I_{ref}}{I_d} \sin\left(\frac{\pi}{6} + \theta\right) T_s \\ T_0 = T_s - T_1 - T_2 \end{array} \right. \longrightarrow \left\{ \begin{array}{l} T_1 = m_a \sin\left(\frac{\pi}{6} - \theta\right) T_s \\ T_2 = m_a \sin\left(\frac{\pi}{6} + \theta\right) T_s \\ T_0 = T_s - T_1 - T_2 \end{array} \right.$$

$$m_a = \frac{I_{ref}}{I_d} = \frac{\hat{I}_{w1}}{I_d}$$

Space Vector Modulation

• Switching Sequence

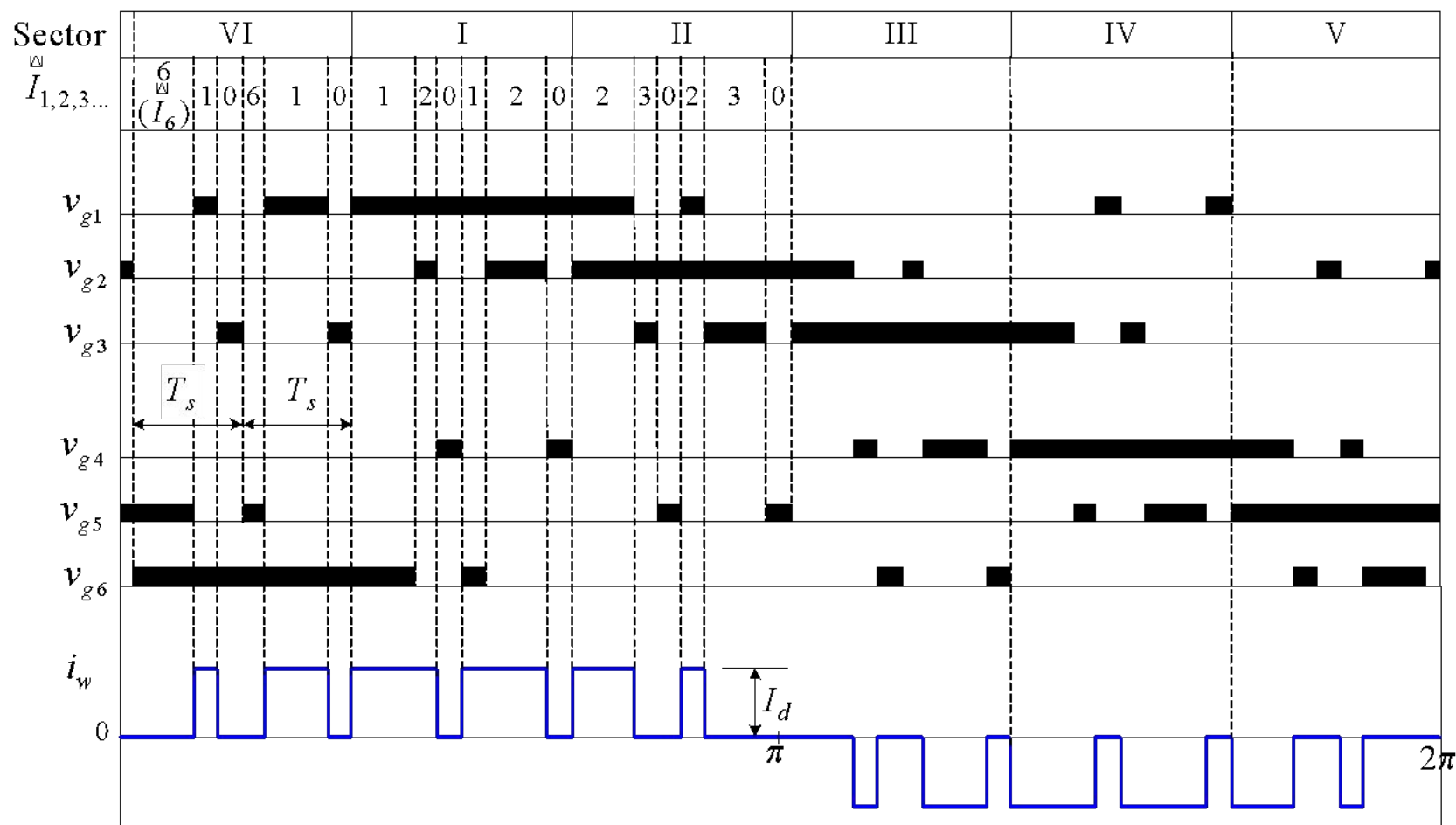


Requirements:

- 1) Transition from one switching state to the next involves only two switches
- 2) At any time only two switches are on.

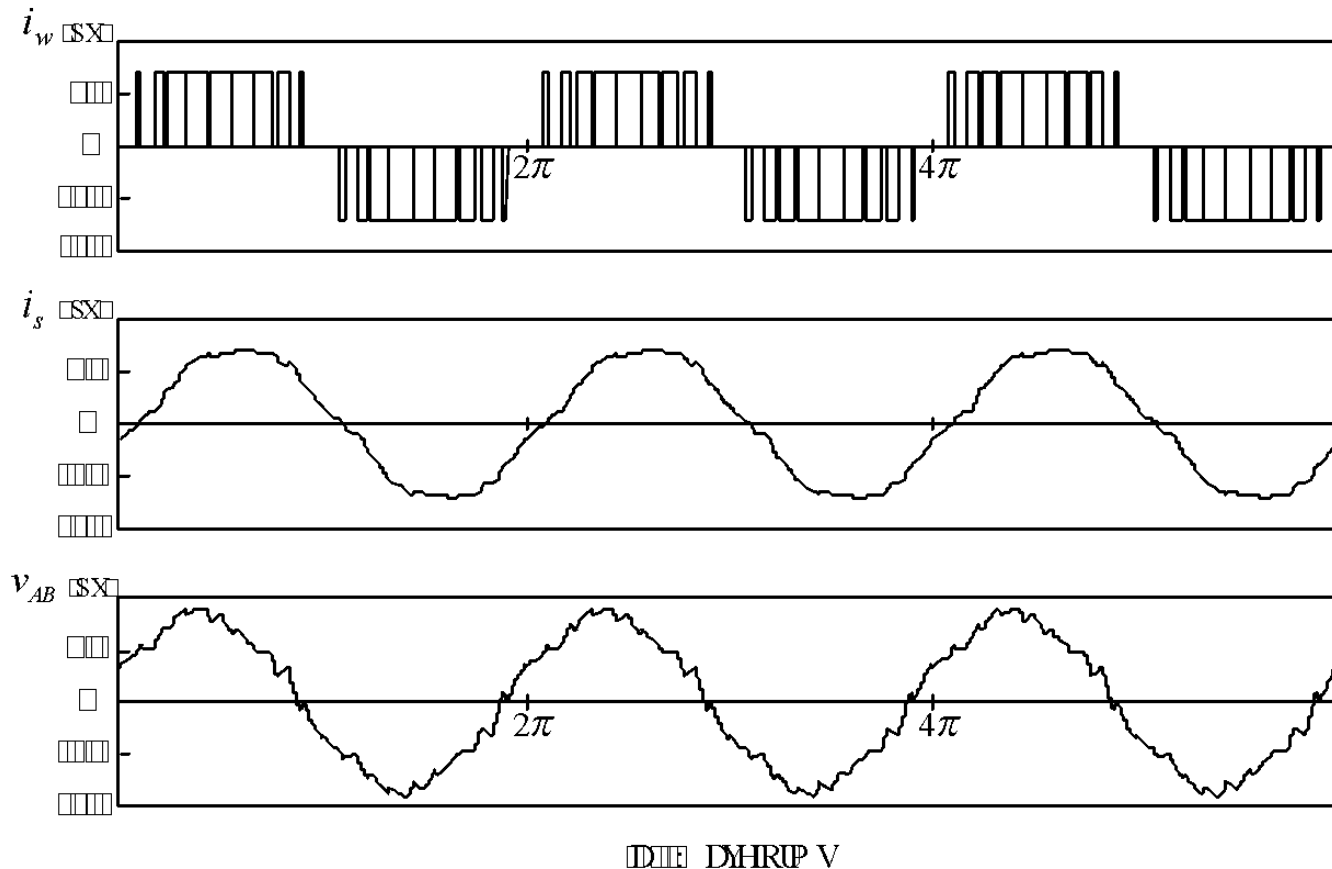
Space Vector Modulation

- Switching Sequence (Over one cycle)



Space Vector Modulation

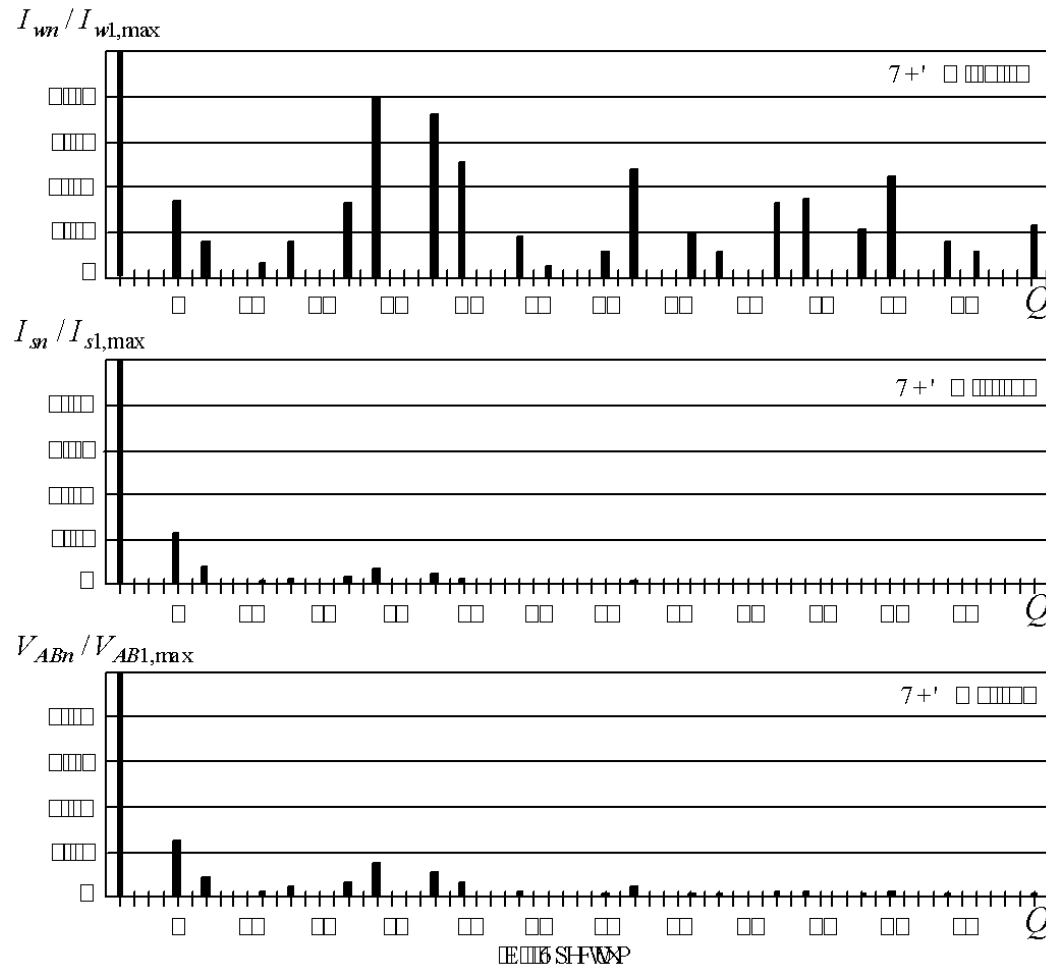
• Inverter Output Waveforms (SVM)



$$f_1 = 60\text{Hz}, f_{sw} = 540\text{Hz}, N_p = 9 \text{ and } m_a = 1$$

Space Vector Modulation

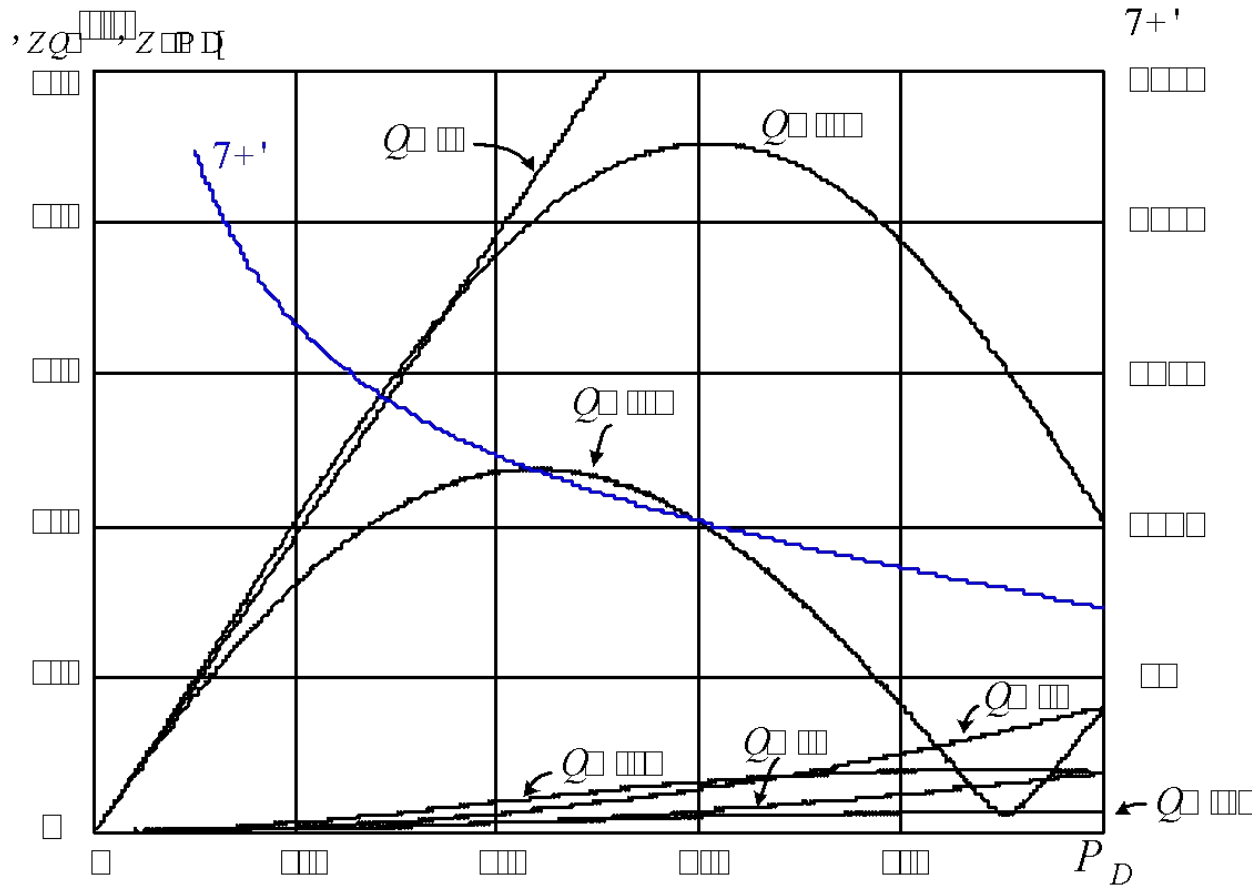
• Harmonic Spectrum (SVM)



$$f_1 = 60\text{Hz}, f_{sw} = 540\text{Hz}, N_p = 9 \text{ and } m_a = 1$$

Space Vector Modulation

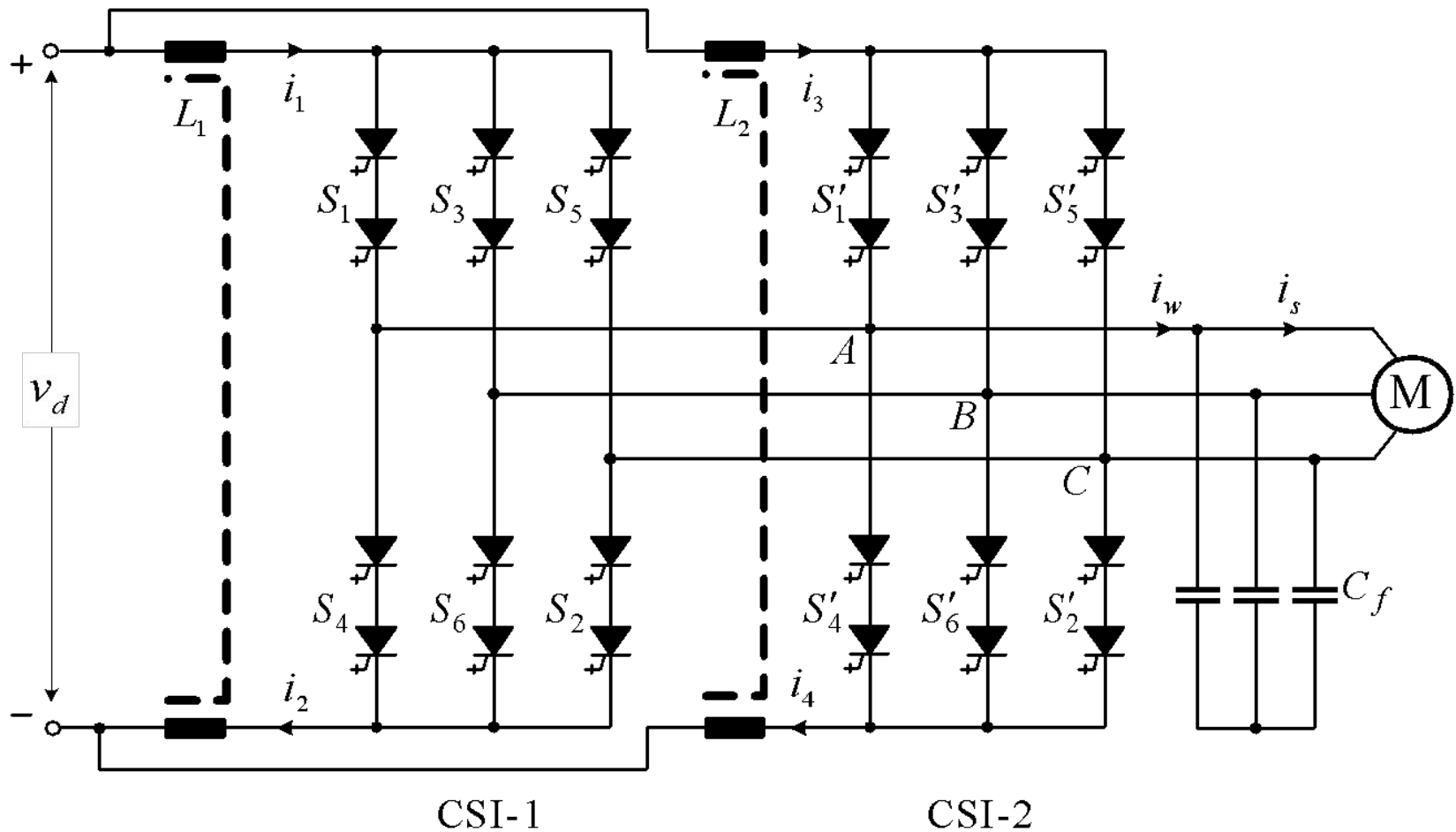
• Harmonic Content (SVM)



(a) $f_{sw} = 540\text{Hz}$, $N_p = 9$

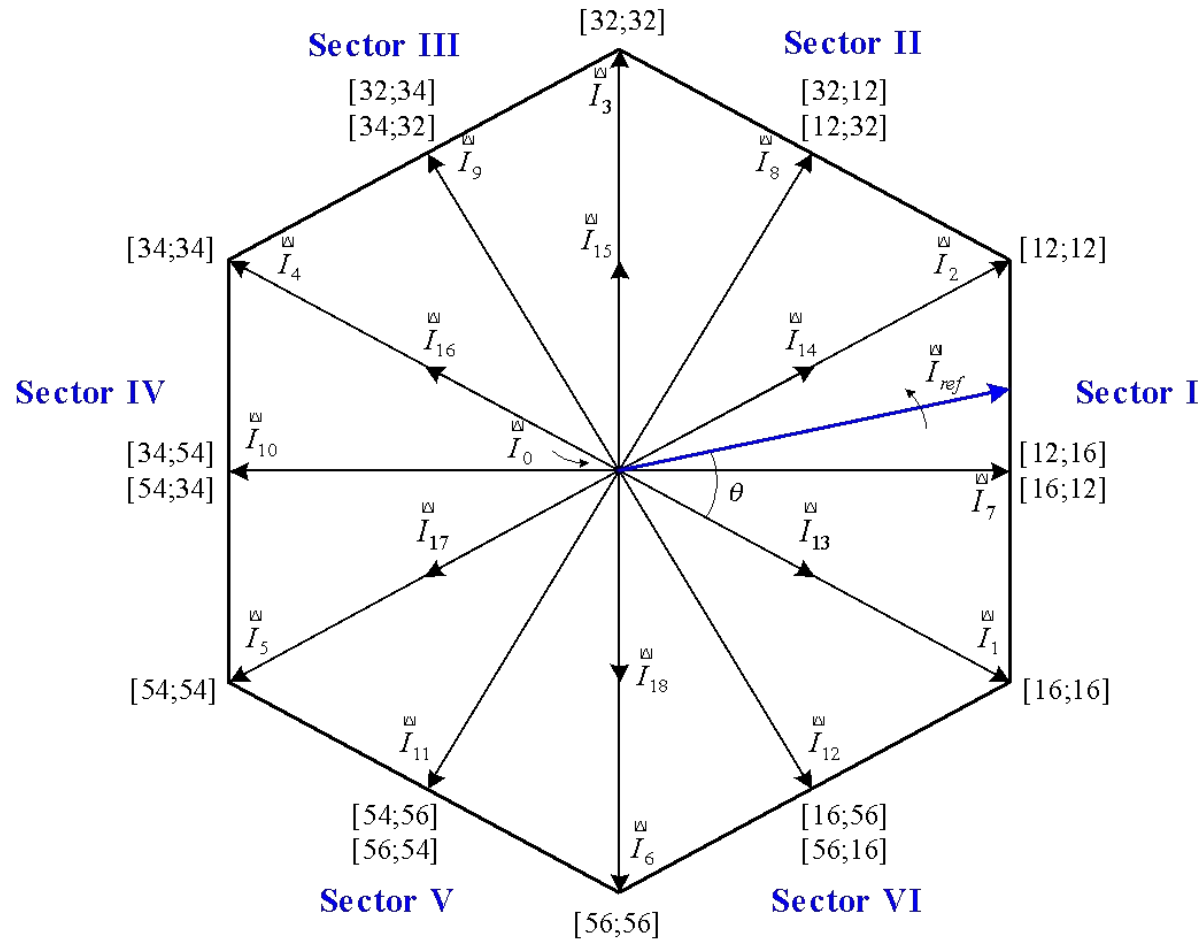
Dual-Bridge Inverter

• Converter Configuration



Dual-Bridge Inverter

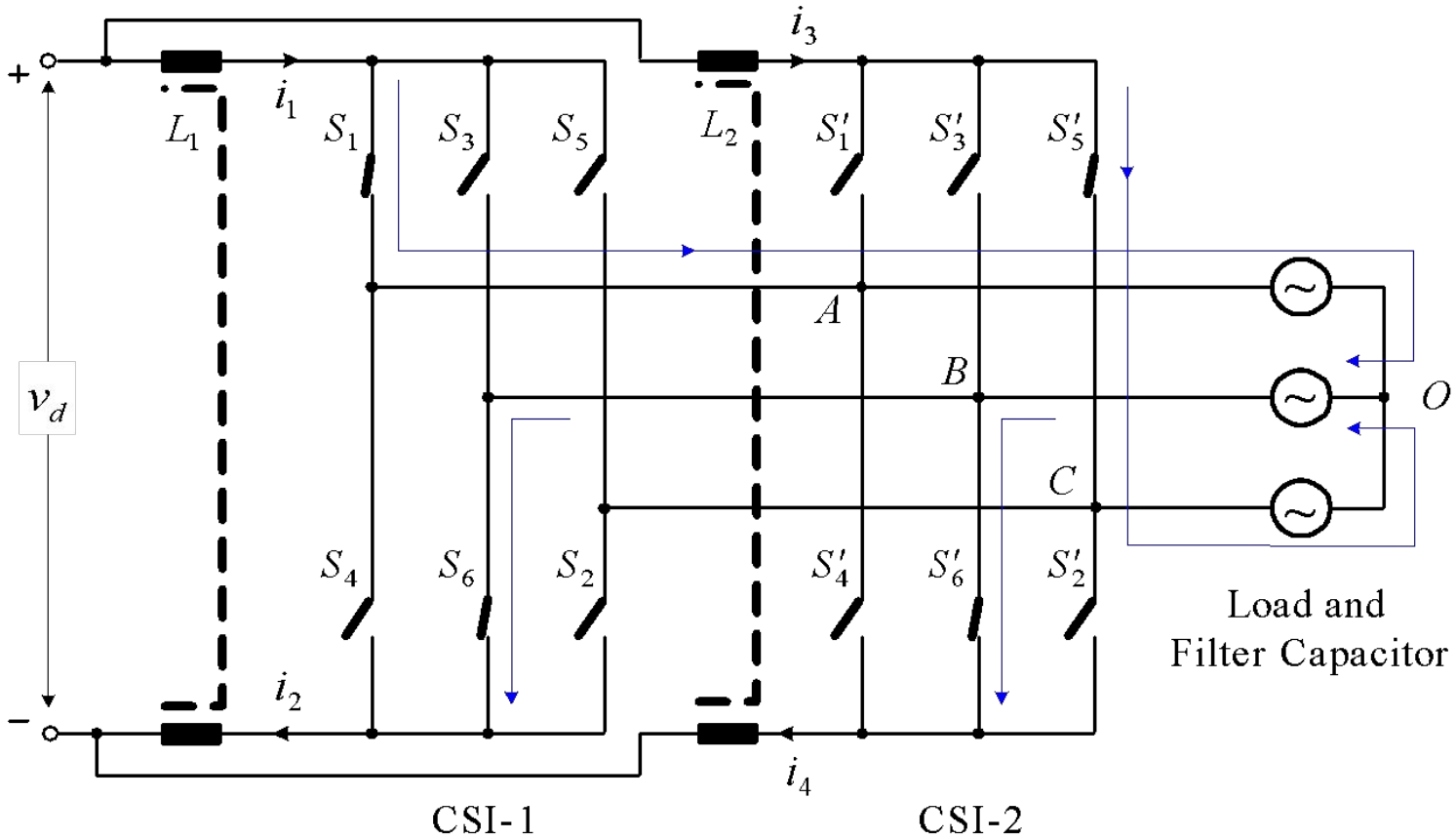
• Space Vector Diagram



19 vectors and 51 switching states

Dual-Bridge Inverter

- DC Current Balance Control



Current paths with switching state [16;56]

Dual-Bridge Inverter

• DC Current Balance Control

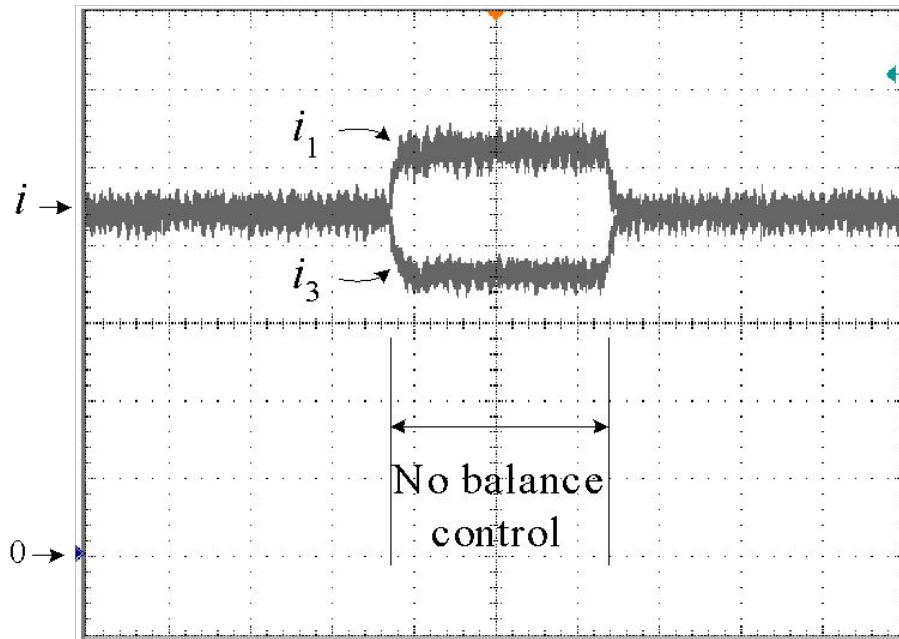
Switching State	Load Voltage	i_1	i_2	i_3	i_4
[16;56] and [16;56]	$v_{AO} = v_{CO}$	×	×	×	×
[16;56]	$v_{AO} > v_{CO}$	↓	×	↑	×
	$v_{AO} < v_{CO}$	↑	×	↓	×
[56;16]	$v_{AO} > v_{CO}$	↑	×	↓	×
	$v_{AO} < v_{CO}$	↓	×	↑	×

Symbol '×': dc currents not affected.

Current paths with switching state [16;56]

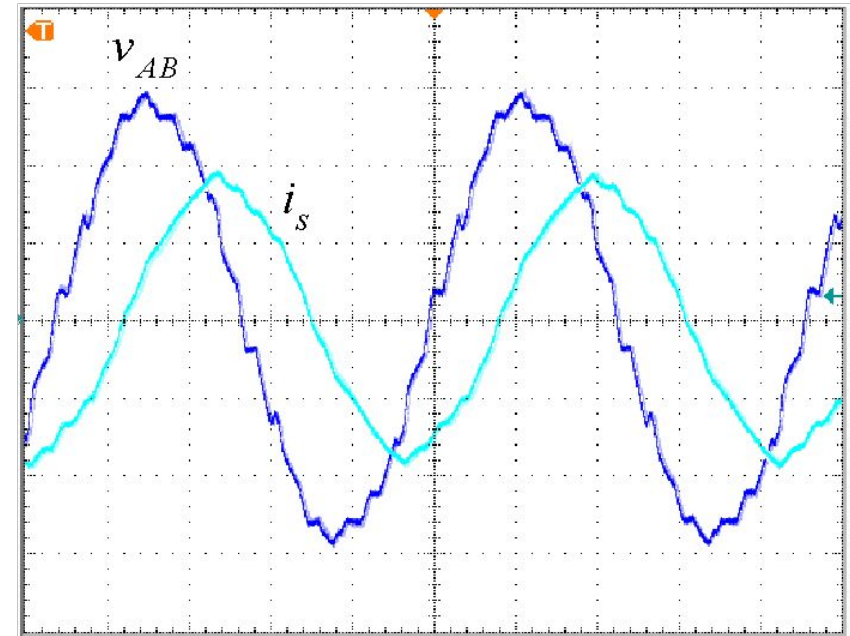
Dual-Bridge Inverter

- Space Vector Diagram



Timebase: 1.0sec/div

(a) dc current transient response



Timebase: 4ms/div

(b) Steady state ac waveforms

Summary

• PWM Scheme Comparison

Item	SVM	TPWM	SHE
DC Current Utilization $I_{wl,max} / I_d$	0.707	0.74	0.73 to 0.78
Dynamic performance	High	Medium	Low
Digital Implementation	Real time	Real time or look-up table	Look-up table
Harmonic Performance	Adequate	Good	Best
dc Current Bypass Operation	Yes	No	Optional



Thanks