Консультация 3

25.03.2012

Вычисляем пределы

$$\lim_{x \to 0} \frac{\ln(1+\sin x)}{\sin 4x}$$

ЗАДАЧА 1

Решение.

$$\lim_{x \to 0} \frac{\ln(1+\sin x)}{\sin 4x} = \left[\frac{0}{0}\right] = \left| \begin{array}{l} \ln(1+\sin x) \sim \sin x \sim x \\ \quad\quad\quad\quad\quad\quad x \to 0 \\ \sin 4x \sim 4x \end{array} \right| =$$

$$= \lim_{x \to 0} \frac{x}{4x} = \frac{1}{4}$$

ЗАДАЧА 2

$$\lim_{x \to 0} \frac{1 - \cos 10x}{e^{x^2} - 1}$$

Решение.

$$\lim_{x \to 0} \frac{1 - \cos 10x}{e^{x^2} - 1} = \left[\frac{0}{0}\right] = \left| \begin{array}{l} 1-\cos 10x \sim \frac{100x^2}{2} \\ e^{x^2}-1 \sim x^2 \\ \quad x \to 0 \end{array} \right| =$$

$$= \lim_{x \to 0} \frac{50x^2}{x^2} = 50$$

ЗАДАЧА 3

$$\lim_{x \to 0} \frac{\sin 7x}{x^2 + \pi x}$$

Решение.

$$\lim_{x \to 0} \frac{\sin 7x}{x^2 + \pi x} = \left[\frac{0}{0}\right] = \left| \begin{matrix} \sin 7x \sim 7x \\ x^2 + \pi x \sim \pi x \\ x \to 0 \end{matrix} \right| =$$

$$= \lim_{x \to 0} \frac{7x}{\pi x} = \frac{7}{\pi}$$

$$\lim_{x \to 0} \frac{x^2 + \pi x}{\pi x} = \lim_{x \to 0} \left(1 + \frac{x}{\pi}\right) = 1$$

$$\Rightarrow x^2 + \pi x \sim \pi x$$

$$\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{x\sin x}$$

ЗАДАЧА 4

Решение.

$$\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{x\sin x}=\left[\frac{0}{0}\right]=\left|\begin{array}{l} 1-\sqrt{\cos x}=-(\cos x-1+1)^{1/2}+1= \\ =-\left[\left(1+(\cos x-1)\right)^{1/2}-1\right]\sim \\ \sim-\frac{1}{2}(\cos x-1)=\frac{1}{2}(1-\cos x)\sim \\ \sim\frac{1}{2}\,\frac{x^2}{2}=\frac{x^2}{4}\quad x\to 0 \\ x\sin x\sim x^2\quad x\to 0 \end{array}\right|=$$

$$=\lim_{x\to 0}\frac{x^2/4}{x^2}=\frac{1}{4}$$

ЗАДАЧА 5

$$\lim_{x \to 0} \frac{\arcsin 3x}{\sqrt{2+x} - \sqrt{2}}.$$

Решение.

$$\lim_{x \to 0} \frac{\arcsin 3x}{\sqrt{2+x} - \sqrt{2}} = \left[\frac{0}{0}\right] = \left|\begin{array}{l} \arcsin 3x \sim 3x \\ \sqrt{2+x} - \sqrt{2} = \sqrt{2}\left(\sqrt{\frac{2+x}{2}} - 1\right) = \\ = \sqrt{2}\left(\sqrt{1 + \frac{x}{2}} - 1\right) \sim \sqrt{2}\,\frac{1}{2}\,\frac{x}{2} \\ \qquad x \to 0 \end{array}\right.$$

$$= \lim_{x \to 0} \frac{3x}{x/2\sqrt{2}} = 6\sqrt{2}$$

$$\lim_{x \to 0} \frac{1-\cos 2x}{\cos 7x - \cos 3x}$$

ЗАДАЧА 6

Решение.

$$\lim_{x \to 0} \frac{1-\cos 2x}{\cos 7x - \cos 3x} = \left[\frac{0}{0}\right] = \left| \begin{array}{l} 1-\cos 2x \sim \frac{4x^2}{2} = 2x^2 \\ \cos 7x - \cos 3x = -\sin\frac{7x+3x}{2} \cdot \\ \cdot \sin\frac{7x-3x}{2} = -\sin 5x \cdot \sin 2x \\ \sim 5x \cdot 2x = 10x^2 \\ \quad x \to 0 \end{array} \right|$$

$$= \lim_{x \to 0} \frac{2x^2}{10x^2} = \frac{1}{5}$$

ЗАДАЧА 7

$$\lim_{x \to 1} \frac{x^2 - 1}{\ln x}$$

Решение.

$$\lim_{x \to 1} \frac{x^2 - 1}{\ln x} = \left[\frac{0}{0}\right] = \left| \begin{matrix} \ln x = \ln(1 + x - 1) \sim \\ \sim (x-1) \quad x \to 1 \end{matrix} \right| =$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x = 1} (x+1) = 2$$

$$\lim_{x \to 1} \frac{x^2 - 1}{\ln x} = \left[\frac{0}{0}\right] = \left| \begin{matrix} t = x - 1 \quad x = t + 1 \\ x \to 1 \implies t \to 0 \end{matrix} \right| = \lim_{t \to 0} \frac{(t+1)^2 - 1}{\ln(1+t)}$$

$$= \left| \begin{matrix} (t+1)^2 - 1 = t^2 + 2t \sim 2t \\ \ln(1+t) \sim t \\ t \to 0 \end{matrix} \right| = \lim_{t \to 0} \frac{2t}{t} = 2$$

$$\lim_{x\to\pi}\frac{1+\cos 3x}{\sin^2 7x}$$

ЗАДАЧА 8

Решение.

$$\lim_{x\to\pi}\frac{1+\cos 3x}{\sin^2 7x}=\left[\frac{0}{0}\right]=\left|\begin{array}{l} 1+\cos 3x=1-\cos(3\pi-3x)= \\ =1-\cos 3(\pi-x)\sim \\ \sim\dfrac{9(\pi-x)^2}{2} \\ \sin^2 7x=\left[\sin(7\pi-7x)\right]^2\sim \\ \sim\left[7(\pi-x)\right]^2=49(\pi-x)^2 \end{array}\right|=$$

$$=\lim_{x\to\pi}\frac{9(\pi-x)^2}{2\cdot 49\cdot(\pi-x)^2}=\frac{9}{98}$$

$$\lim_{x \to \pi} \frac{1+\cos 3x}{\sin^2 7x}$$

$$= \left[\frac{0}{0}\right] = \left| \begin{matrix} t = x - \pi \quad x = t + \pi \\ x \to \pi \Rightarrow t \to 0 \end{matrix} \right| = \lim_{t \to 0} \frac{1 + \cos(3t + 3\pi)}{\sin^2(7t + 7\pi)} =$$

$$= \lim_{t \to 0} \frac{1 - \cos 3t}{\sin^2 7t} = \left| \begin{matrix} 1 - \cos 3t \sim \frac{9t^2}{2} \\ \sin^2 7t \sim 49t^2 \\ t \to 0 \end{matrix} \right| =$$

$$= \lim_{t \to 0} \frac{9t^2}{4 \cdot 49 t^2} = \frac{9}{98}$$

$$\lim_{x \to 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

ЗАДАЧА 9

Решение.

$$\lim_{x \to 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

$$= \left[\frac{0}{0}\right] = \left| \begin{array}{l} \ln(5-2x) = \ln(1+(4-2x)) \sim (4-2x) \\ \sqrt{10-3x}-2 = 2\left(\sqrt{\frac{10-3x}{4}}-1\right) = \\ = 2\left(\sqrt{1+\frac{6-3x}{4}}-1\right) \sim 2\cdot\frac{1}{2}\cdot\frac{6-3x}{4} \\ \qquad\qquad x \to 2 \end{array} \right| =$$

$$= \lim_{x \to 2} \frac{2(2-x)\cdot 4}{3(2-x)} = \frac{8}{3}$$

$$\lim_{x \to 3} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

ЗАДАЧА 9

Решение.

$$\lim_{x \to 3} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

$$= \left| \begin{array}{ll} t = x-2 & x = t+2 \\ x \to 2 \Rightarrow t \to 0 & \end{array} \right| = \lim_{t \to 0} \frac{\ln(5-2t-4)}{\sqrt{10-3t-6}-2}$$

$$= \lim_{t \to 0} \frac{\ln(1-2t)}{\sqrt{4-3t}-2} = \lim_{t \to 0} \frac{\ln(1-2t)}{2\left(\sqrt{1-\frac{3}{4}t}-1\right)} =$$

$$= \left| \begin{array}{l} \ln(1-2t) \sim -2t, \ t \to 0 \\ \sqrt{1-\frac{3}{4}t}-1 \sim \frac{1}{2} \cdot \left(-\frac{3}{4}t\right) \end{array} \right| = \lim_{t \to 0} \frac{-2t}{\left(-\frac{3}{8}t\right)2} = \frac{8}{3}$$

$$\lim_{x \to \pi/2} \frac{2^{\cos^2 x} - 1}{\ln \sin x}$$

ЗАДАЧА 10

Решение.

$$\lim_{x \to \pi/2} \frac{2^{\cos^2 x} - 1}{\ln \sin x} = \left[\frac{0}{0}\right] = \left| \begin{array}{l} 2^{\cos^2 x} - 1 \sim \ln 2 \cdot \cos^2 x \\ \ln \sin x = \ln(1 + \sin x - 1) \sim \\ \sim \sin x - 1 \qquad x \to \frac{\pi}{2} \end{array} \right| =$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x \cdot \ln 2}{\sin x - 1} = \left| \begin{array}{l} t = x - \frac{\pi}{2} \quad x = t + \frac{\pi}{2} \\ x \to \frac{\pi}{2} \Rightarrow t \to 0 \end{array} \right| =$$

$$= \lim_{t \to 0} \frac{\cos^2\left(t + \frac{\pi}{2}\right) \ln 2}{\sin\left(t + \frac{\pi}{2}\right) - 1} = \lim_{t \to 0} \frac{\sin^2 t \ln 2}{\cos t - 1} =$$

$$= \left| \begin{array}{l} \sin^2 t \ln 2 \sim t^2 \ln 2 \\ \cos t - 1 \underset{t \to 0}{\sim} -\frac{1}{2} t^2 \end{array} \right| = \lim_{t \to 0} \frac{\ln 2 \cdot t^2}{-\frac{1}{2} t^2} = -2 \ln 2$$

ЗАДАЧА 11

$$\lim_{x\to 3}\frac{\log_3 x-1}{\operatorname{tg}\pi x}$$

$$\lim_{x\to 3}\frac{\log_3 x-1}{\operatorname{tg}\pi x}$$

$$=\left[\frac{0}{0}\right]=\left|\begin{array}{l}\log_3 x-1=\log_3 x-\log_3 3=\\ =\log_3\frac{x}{3}=\frac{\ln\left(\frac{x}{3}\right)}{\ln 3}=\frac{1}{\ln 3}\ln\left(1+\frac{x}{3}-1\right)=\\ =\frac{1}{\ln 3}\ln\left(1+\frac{x-3}{3}\right)\sim\frac{x-3}{3\ln 3}\\ \operatorname{tg}\pi x=\operatorname{tg}(\pi x-3\pi)\sim\pi(x-3)\\ \qquad\qquad x\to 3\end{array}\right|=$$

$$=\lim_{x\to 3}\frac{(x-3)}{3\ln 3\cdot\pi(x-3)}=\frac{1}{3\pi\ln 3}$$