

Консультация 3

25.03.2012

Вычисляем пределы

ЗАДАЧА 1

$$\lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{\sin 4x}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{\sin 4x}$$

$$= \frac{0}{0} = \left(\begin{array}{l} \ln(1 + \sin x) \sim \sin x \sim x \\ \sin 4x \sim 4x \end{array} \right)_{x \rightarrow 0} = \lim_{x \rightarrow 0} \frac{x}{4x} = \frac{1}{4}$$

ЗАДАЧА 2

$$\lim_{x \rightarrow 0} \frac{1 - \cos 10x}{e^{x^2} - 1}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{1 - \cos 10x}{e^{x^2} - 1}$$

$$= \left[\frac{0}{0} \right] = \left| \begin{array}{l} 1 - \cos 10x \sim \frac{100x^2}{2} \\ e^{x^2} - 1 \sim x^2 \\ x \rightarrow 0 \end{array} \right| =$$

$$= \lim_{x \rightarrow 0} \frac{50x^2}{x^2} = 50$$

ЗАДАЧА 3

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{x^2 + \pi x}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{x^2 + \pi x}$$

$$= \left[\frac{0}{0} \right] = \left| \begin{array}{l} \sin 7x \sim 7x \\ x^2 + \pi x \sim \pi x \\ x \rightarrow 0 \end{array} \right| =$$

$$= \lim_{x \rightarrow 0} \frac{7x}{\pi x} = \frac{7}{\pi}$$

$$\lim_{x \rightarrow 0} \frac{x^2 + \pi x}{\pi x} = \lim_{x \rightarrow 0} \left(1 + \frac{x}{\pi} \right) = 1$$
$$\Rightarrow x^2 + \pi x \sim \pi x$$

ЗАДАЧА 4

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x \sin x}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x \sin x} = \left[\frac{0}{0} \right] = \left\{ \begin{array}{l} 1 - \sqrt{\cos x} = -(\cos x - 1 + 1)^{1/2} + 1 = \\ = -\left[(1 + (\cos x - 1))^{1/2} - 1 \right] \sim \\ \sim -\frac{1}{2}(\cos x - 1) = \frac{1}{2}(1 - \cos x) \\ \sim \frac{1}{2} \frac{x^2}{2} = \frac{x^2}{4} \quad x \rightarrow 0 \\ x \sin x \sim x^2 \quad x \rightarrow 0 \end{array} \right.$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x^2}{4}}{x^2} = \frac{1}{4}$$

ЗАДАЧА 5

$$\lim_{x \rightarrow 0} \frac{\arcsin 3x}{\sqrt{2+x} - \sqrt{2}}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{\arcsin 3x}{\sqrt{2+x} - \sqrt{2}} = \left[\frac{0}{0} \right] = \begin{cases} \arcsin 3x \sim 3x \\ \sqrt{2+x} - \sqrt{2} = \sqrt{2} \left(\sqrt{\frac{2+x}{2}} - 1 \right) \\ = \sqrt{2} \left(\sqrt{1 + \frac{x}{2}} - 1 \right) \sim \sqrt{2} \cdot \frac{1}{2} \frac{x}{2} \\ x \rightarrow 0 \end{cases}$$

$$= \lim_{x \rightarrow 0} \frac{3x}{x/2\sqrt{2}} = 6\sqrt{2}$$

ЗАДАЧА 6

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\cos 7x - \cos 3x}$$

Решение.

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\cos 7x - \cos 3x} = \left[\frac{0}{0} \right] = \left\{ \begin{array}{l} 1 - \cos 2x \sim \frac{4x^2}{2} = 2x^2 \\ \cos 7x - \cos 3x = -\sin \frac{7x+3x}{2} \cdot \sin \frac{7x-3x}{2} = -\sin 5x \cdot \sin 2x \\ \sim 5x \cdot 2x = 10x^2 \end{array} \right._{x \rightarrow 0}$$
$$= \lim_{x \rightarrow 0} \frac{2x^2}{10x^2} = \frac{1}{5}$$

ЗАДАЧА 7

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x}$$

Решение.

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x} = \left[\frac{0}{0} \right] = \left| \begin{array}{l} \ln x = \ln(1+x-1) \sim \\ \sim (x-1) \quad x \rightarrow 1 \end{array} \right| =$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x=1} (x+1) = 2$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x} = \left[\frac{0}{0} \right] = \left| \begin{array}{l} t = x-1 \quad x = t+1 \\ x \rightarrow 1 \Rightarrow t \rightarrow 0 \end{array} \right| = \lim_{t \rightarrow 0} \frac{(t+1)^2 - 1}{\ln(1+t)}$$

$$= \left| \begin{array}{l} (t+1)^2 - 1 = t^2 + 2t \sim 2t \\ \ln(1+t) \sim t \\ t \rightarrow 0 \end{array} \right| = \lim_{t \rightarrow 0} \frac{2t}{t} = 2$$

$$1. \lim_{x \rightarrow \pi} \frac{1 + \cos 3x}{\sin^2 7x}$$

ЗАДАЧА 8

Решение.

$$\begin{aligned} \lim_{x \rightarrow \pi} \frac{1 + \cos 3x}{\sin^2 7x} &= \left[\frac{0}{0} \right] = \left\{ \begin{aligned} 1 + \cos 3x &= 1 - \cos(3\pi - 3x) = \\ &= 1 - \cos 3(\pi - x) \sim \\ &\sim \frac{9(\pi - x)^2}{2} \\ \sin^2 7x &= [\sin(7\pi - 7x)]^2 \sim \\ &\sim [7(\pi - x)]^2 = 49(\pi - x)^2 \end{aligned} \right. = \\ &= \lim_{x \rightarrow \pi} \frac{\frac{9(\pi - x)^2}{2}}{49(\pi - x)^2} = \frac{9}{98} \end{aligned}$$

$$\lim_{x \rightarrow \pi} \frac{1 + \cos 3x}{\sin^2 7x}$$

$$= \left[\frac{0}{0} \right] = \left| \begin{array}{l} t = x - \pi \quad x = t + \pi \\ x \rightarrow \pi \Rightarrow t \rightarrow 0 \end{array} \right| = \lim_{t \rightarrow 0} \frac{1 + \cos(3t + 3\pi)}{\sin^2(7t + 7\pi)} =$$

$$= \lim_{t \rightarrow 0} \frac{1 - \cos 3t}{\sin^2 7t} = \left| \begin{array}{l} 1 - \cos 3t \sim \frac{9t^2}{2} \\ \sin^2 7t \sim 49t^2 \\ t \rightarrow 0 \end{array} \right| =$$

$$= \lim_{t \rightarrow 0} \frac{9t^2}{4 \cdot 49t^2} = \frac{9}{98}$$

ЗАДАЧА 9

$$\lim_{x \rightarrow 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

Решение.

$$\lim_{x \rightarrow 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

$$\begin{aligned} &= \left[\frac{0}{0} \right] = \left| \begin{array}{l} \ln(5-2x) = \ln(1+(4-2x)) \sim (4-2x) \\ \sqrt{10-3x}-2 = 2\left(\sqrt{\frac{10-3x}{4}}-1\right) = \\ = 2\left(\sqrt{1+\frac{6-3x}{4}}-1\right) \sim 2 \cdot \frac{1}{2} \cdot \frac{6-3x}{4} \end{array} \right| \\ &= \lim_{x \rightarrow 2} \frac{2(2-x) \cdot 4}{3(2-x)} = \frac{8}{3} \end{aligned}$$

$$\lim_{x \rightarrow 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

ЗАДАЧА 9

Решение.

$$\lim_{x \rightarrow 2} \frac{\ln(5-2x)}{\sqrt{10-3x}-2}$$

$$= \left| \begin{array}{l} t = x - 2 \quad x = t + 2 \\ x \rightarrow 2 \Rightarrow t \rightarrow 0 \end{array} \right| = \lim_{t \rightarrow 0} \frac{\ln(5-2t-4)}{\sqrt{10-3t-6}-2}$$

$$= \lim_{t \rightarrow 0} \frac{\ln(1-2t)}{\sqrt{4-3t}-2} = \lim_{t \rightarrow 0} \frac{\ln(1-2t)}{2\left(\sqrt{1-\frac{3}{4}t}-1\right)}$$

$$= \left| \begin{array}{l} \ln(1-2t) \sim -2t, t \rightarrow 0 \\ \sqrt{1-\frac{3}{4}t}-1 \sim \frac{1}{2} \cdot \left(-\frac{3}{4}t\right) \end{array} \right| = \lim_{t \rightarrow 0} \frac{-2t}{\left(-\frac{3}{8}t\right)^2} = \frac{8}{3}$$

$$\lim_{x \rightarrow \pi/2} \frac{2\cos^2 x - 1}{\ln \sin x}$$

ЗАДАЧА

10

Решение.

$$\lim_{x \rightarrow \pi/2} \frac{2\cos^2 x - 1}{\ln \sin x}$$

$$= \left[\frac{0}{0} \right] = \left| \begin{array}{l} 2\cos^2 x - 1 \sim \ln 2 \cdot \cos^2 x \\ \ln \sin x = \ln(1 + \sin x - 1) \sim \\ \sim \sin x - 1 \quad x \rightarrow \frac{\pi}{2} \end{array} \right| =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x \cdot \ln 2}{\sin x - 1} = \left| \begin{array}{l} t = x - \frac{\pi}{2} \quad x = t + \frac{\pi}{2} \\ x \rightarrow \frac{\pi}{2} \Rightarrow t \rightarrow 0 \end{array} \right| =$$

$$= \lim_{t \rightarrow 0} \frac{\cos^2(t + \frac{\pi}{2}) \ln 2}{\sin(t + \frac{\pi}{2}) - 1} = \lim_{t \rightarrow 0} \frac{\sin^2 t \ln 2}{\cos t - 1} =$$

$$= \left| \begin{array}{l} \sin^2 t \ln 2 \sim t^2 \ln 2 \\ \cos t - 1 \sim -\frac{1}{2} t^2 \\ t \rightarrow 0 \end{array} \right| = \lim_{t \rightarrow 0} \frac{\ln 2 \cdot t^2}{-\frac{1}{2} t^2} = -2 \ln 2$$

$$\lim_{x \rightarrow 3} \frac{\log_3 x - 1}{\operatorname{tg} \pi x}$$

ЗАДАЧА

11

$$\lim_{x \rightarrow 3} \frac{\log_3 x - 1}{\operatorname{tg} \pi x}$$

$$= \left[\frac{0}{0} \right] = \left| \begin{array}{l} \log_3 x - 1 = \log_3 x - \log_3 3 = \\ = \log_3 \frac{x}{3} = \frac{\ln(\frac{x}{3})}{\ln 3} = \frac{1}{\ln 3} \ln(1 + \frac{x}{3} - 1) = \\ = \frac{1}{\ln 3} \ln(1 + \frac{x-3}{3}) \sim \frac{x-3}{3 \ln 3} \\ \operatorname{tg} \pi x = \operatorname{tg}(\pi x - 3\pi) \sim \pi(x-3) \\ x \rightarrow 3 \end{array} \right| =$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)}{3 \ln 3 \cdot \pi(x-3)} = \frac{1}{3\pi \ln 3}$$