Проблема необратимости и функциональная механика

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Проблема необратимости заключается в том, как совместить обратимость по времени микроскопической динамики с необратимостью макроскопических уравнений. Эта фундаментальная проблема рассматривалась в известных работах Больцмана, Пуанкаре, Боголюбова, Фейнмана, Ландау и других авторов, и оставалась открытой.

Недавно был предложен следующий подход к решению проблемы необратимости: предложена новая формулировка классической и квантовой механики, которая необратима по времени. Таким образом снимается противоречие между обратимость микроскопической и необратимость макроскопической динамики, поскольку обе динамики в предлагаемом подходе необратимы.

Широко используемое понятие микроскопического состояния системы как точки в фазовом пространстве, а также понятия траектории и микроскопических уравнений движения Ньютона не имеют непосредственного физического смысла, поскольку произвольные вещественные числа не наблюдаемы.

Фундаментальным уравнением микроскопической динамики в предлагаемом неньютоновском "функциональном" подходе является не уравнение Ньютона, а уравнение типа Фоккера—Планка. Показано, что уравнение Ньютона в таком подходе возникает как приближенное уравнение, описывающее динамику средних значений координат для не слишком больших промежутков времени. Вычислены поправки к уравнениям Ньютона.

Такой подход потребовал также пересмотра обычной Копенгагенской интерпретации квантовой механики.

I.V. Volovich, “Randomness in classical mechanics and quantum mechanics”, Found. Phys., 41:3 (2011), 516–528;

http://arxiv.org/pdf/0907.2445.pdf

Time Irreversibility Problem

Non-Newtonian Classical Mechanics

Functional Probabilistic General Relativity

Black Hole Information Paradox

Time Irreversibility Problem

The time irreversibility problem is the problem of how to explain the irreversible behaviour of macroscopic systems from the time-symmetric microscopic laws: $t \to -t$

Newton, Schrodinger Eqs –- reversible

Navier-Stokes, Boltzmann, diffusion, Entropy increasing --- irreversible

$$\frac{\partial u}{\partial t} = \Delta u.$$

Time Irreversibility Problem

Boltzmann, Maxwell, Poincar´e, Bogolyubov, Kolmogorov, von Neumann, Landau, Prigogine, Feynman, Kozlov,…

Poincar´e, Landau, Prigogine, Ginzburg, Feynman: Problem is open.
We will never solve it (Poincare)
Quantum measurement? (Landau)

Lebowitz, Goldstein, Bricmont:
Problem was solved by Boltzmann

Boltzmann`s answers to:

Loschmidt: statistical viewpoint

Poincare—Zermelo: extremely long Poincare recurrence time

Coarse graining

- Not convincing…

Ergodicity

- **Boltzmann, Poincare, Hopf, Kolmogorov, Anosov, Arnold, Sinai,….:**
- **Ergodicity, mixing,… for various important deterministic mechanical and geometrical dynamical systems**

Bogolyubov method

1. Newton to Liouville Eq.
   Bogolyubov (BBGKI) hierarchy
2. Thermodynamic limit (infinite number of particles)

3. The condition of weakening of initial correlations between particles in the distant past

4. Functional conjecture
5. Expansion in powers of density

Divergences.

Why Newton`s mechanics can not be true?

- Newton`s equations of motions use real numbers while one can observe only rationals. (s.i.)

- Classical uncertainty relations

- Time irreversibility problem

- Singularities in general relativity

Classical Uncertainty Relations

$$\Delta q > 0, \quad \Delta p > 0$$

$$\Delta t > 0$$

Newton Equation

$$m\frac{d^2}{dt^2}x = F(x),$$

$$x = x(t),$$

$$(M = R^{2n}, \varphi_t)$$

Phase space (q,p), Hamilton dynamical flow

Newton`s Classical Mechanics

Motion of a point body is described by the trajectory in the phase space.

Solutions of the equations of Newton or Hamilton.

Idealization: Arbitrary real numbers—non observable.

- **Newton`s mechanics deals with non-observable (non-physical) quantities.**

Real Numbers

- **A real number is an infinite series, which is unphysical:**

$$t = \sum_n a_n \frac{1}{10^n}, \ a_n = 0,1,\ldots,9.$$

$$m\frac{d^2}{dt^2}x(t) = F$$

- **Try to solve these problems by developing a new, <u>non-Newtonian</u> mechanics.**

- **And new, non-Einsteinian general relativity**

We attempt the following solution of the irreversibility problem:

a formulation of microscopic dynamics which is irreversible in time: Non-Newtonian Functional Approach.

Functional formulation of classical mechanics

- Here the physical meaning is attributed not to an individual trajectory but only to a bunch of trajectories or to the distribution function on the phase space. The fundamental equation of the microscopic dynamics in the proposed "functional" approach is not the Newton equation but the Liouville or Fokker-Planck-Kolmogorov (Langevin, Smoluchowski) equation for the distribution function of the single particle.
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States and Observables in Functional Classical Mechanics

$$(q, p) \quad \in \quad \mathbb{R}^2 \qquad \text{(phase space)}$$

$\rho = \rho(q, p, t)$ state of a classical particle

$$\rho \geq 0, \quad \int_{\mathbb{R}^2} \rho(q, p, t) dq dp = 1, \; t \in \mathbb{R} \,.$$

States and Observables in Functional Classical Mechanics

$$\overline{f}(t) = \int f(q,p)\rho(q,p,t)dqdp\,.$$

$f(q,p)$ is a function

Not a generalized function

Fundamental Equation in Functional Classical Mechanics

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V(q)}{\partial q}\frac{\partial \rho}{\partial p}\,.$$

Looks like the Liouville equation which is used in statistical physics
to describe a gas of particles but here we use it to describe a single particle.(moon,…)

Instead of Newton equation. No trajectories!

Cauchy Problem for Free Particle

$$\rho|_{t=0} = \rho_0(q,p)\,.$$

$$\rho_0(q,p) = \frac{1}{\pi ab} e^{-\frac{(q-q_0)^2}{a^2}} e^{-\frac{(p-p_0)^2}{b^2}}\,.$$

Poincare, Langevin, Smolukhowsky ,
Krylov, Bogoliubov, Blokhintsev, Born,…

Free Motion

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q}$$

$$\rho(q, p, t) = \rho_0(q - \frac{p}{m}t, p)\ .$$

Delocalization

$$\rho_c(q,t) = \int \rho(q,p,t)dp = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{b^2t^2}{m^2}}} \exp\{-\frac{(q - q_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{b^2t^2}{m^2})}\}$$

$$\Delta q^2(t) = \frac{1}{2}(a^2 + \frac{b^2t^2}{m^2})$$

Newton`s Equation for Average

$$\overline{q}(t) = \int q\rho_c(q,t)dq = q_0 + \frac{p_0}{m}t\,, \quad \overline{p}(t) = \int p\rho_m(p,t)dp = p_0\,.$$

$$\frac{d^2}{dt^2}\overline{q}(t) = 0\,,$$

Comparison with Quantum Mechanics

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$\rho_q(x,t) = |\psi(x,t)|^2 = \frac{1}{\sqrt{\pi}\sqrt{a^2+\frac{\hbar^2 t^2}{a^2 m^2}}}\exp\{-\frac{(x-x_0-\frac{p_0}{m}t)^2}{(a^2+\frac{\hbar^2 t^2}{a^2 m^2})}\}$$

$$a^2 b^2 = \hbar^2$$

$$W(q,p,t) = \rho(q,p,t)$$

Liouville and Newton. Characteristics

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{k} \frac{\partial}{\partial x^i}(\rho v^i) = 0$$

$$\dot{x} = v(x)$$

$$\rho|_{t=0} = \rho_0(x)$$

$$\rho(x, t) = \rho_0(\varphi_{-t}(x))$$

Corrections to Newton`s Equations
Non-Newtonian Mechanics

$$\rho_0(q,p) = \delta_\epsilon(q-q_0)\delta_\epsilon(p-p_0)$$

$$\delta_\epsilon(q) = \frac{1}{\sqrt{\pi}\epsilon} e^{-q^2/\epsilon^2} ,$$

Proposition 1. Newton's Equations

$$\lim_{\epsilon \longrightarrow 0} \int f(q,p)\rho(q,p,t)dqdp = f(\varphi_t(q_0,p_0)) .$$

Corrections to Newton`s Equations

$$\frac{\partial \rho}{\partial t} = -p\frac{\partial \rho}{\partial q} + \lambda q^2 \frac{\partial \rho}{\partial p}$$

$$\dot{p}(t) + \lambda q(t)^2 = 0\,,\ \ \dot{q}(t) = p(t)\,.$$

Corrections to Newton`s Equations

Proposition 2.

$$< q(t) >= q_{\mathrm{Newton}}(t) - \frac{\lambda}{4}\epsilon^2 t^2$$

$$q_{\mathrm{Newton}}(t) = q_0 + p_0 t - \frac{\lambda}{2} q_0^2 t^2$$

$-\frac{\lambda}{4}\epsilon^2 t^2$ is the correction to the Newton solution

Corrections

$$m\frac{d^2}{dt^2} < q(t) > = < F(q)(t) >$$

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- **The Newton equation in this approach appears as an approximate equation describing the dynamics of the expected value of the position and momenta for not too large time intervals.**
- **Corrections to the Newton equation are computed.**
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Fokker-Planck-Kolmogorov versus Newton

$$\frac{\partial f}{\partial t} - \{H, f\} = \sigma \frac{\partial^2 f}{\partial p^2} + \gamma \frac{\partial (pf)}{\partial p}$$

Boltzmann and Bogolyubov Equations

- A method for obtaining kinetic equations from the Newton equations of mechanics was proposed by Bogoliubov. This method has the following basic stages:
- Liouville equation for the distribution function of particles in a finite volume, derive a chain of equations for the distribution functions,
- pass to the infinite-volume, infinite number of particles limit,
- postulate that the initial correlations between the particles were weaker in the remote past,
- introduce the hypothesis that all many-particle distribution functions depend on time only via the one-particle distribution function, and use the formal expansion in power series in the density.

- Non-Newtonian Functional Mechanics:
- Finite volume. Two particles.

Liouville equation for two particles

$$\rho = \rho(x_1, x_2, t)$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_1}\frac{\partial \rho}{\partial p_1} - \frac{p_1}{m}\frac{\partial \rho}{\partial q_1} + \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_2}\frac{\partial \rho}{\partial p_2} - \frac{p_2}{m}\frac{\partial \rho}{\partial q_2},$$

Two particles in finite volume

$$f_1(x_1,t) = V\int_{\Omega_V} \rho(x_1,x_2,t)\,dx_2, \qquad f_2(x_1,x_2,t) = V^2\rho(x_1,x_2,t),$$

$$f_2(x_1,x_2,t_0) = f_1(x_1,t_0)f_1(x_2,t_0).$$

$$f_2(x_1,x_2,t) = f_1(\varphi^{(1)}_{t_0-t}(x_1,x_2),t_0)f_1(\varphi^{(2)}_{t_0-t}(x_1,x_2),t_0),$$

If $\rho(x_1, x_2, t)$

satisfies the Liouville equation then

$f_1(x_1, t)$

obeys to the following equation

$$\left(\frac{\partial}{\partial t} + \frac{p_1}{m}\frac{\partial}{\partial q_1}\right) f_1(x_1, t) =$$

$$= \frac{1}{V}\int_{\Omega_V} \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_1} \frac{\partial}{\partial p_1} [f_1(\varphi^{(1)}_{t_0 - t}(x_1, x_2), t_0) f_1(\varphi^{(2)}_{t_0 - t}(x_1, x_2), t_0)] dq_2\, dp_2.$$

Bogolyubov type equation for two particles in finite volume

- **Kinetic theory for two particles**
- **Hydrodynamics for two particles?**

- **No classical determinism**
- **Classical randomness**

- **World is probabilistic (classical and quantum)**

Compare: Bohr, Heisenberg, von Neumann, Einstein,…

Single particle (moon,….)

$$\rho = \rho(q, p, t)$$

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V}{\partial q}\frac{\partial \rho}{\partial p}$$

$$\rho\,|_{t=0} = \frac{1}{\pi ab}\exp\{-\frac{(q-q_0)^2}{a^2} - \frac{(p-p_0)^2}{b^2}\}$$

- **Newton`s approach: Empty space (vacuum) and point particles.**
- **Reductionism: For physics, biology economy, politics (freedom, liberty,…)**

- **This approach: No empty space. Probability distribution. Collective phenomena. Subjective.**

Fixed classical spacetime?

- **A fixed classical background spacetime does not exists (Kaluza—Klein, Strings, Branes). No black hole metric.**

There is a set of classical universes and a probability distribution $\rho(M, g_{\mu\nu})$ **which satisfies the Liouville equation (not Wheeler—De Witt).**

Stochastic inflation?

Functional General Relativity

- Fixed background (M,g).

Geodesics in functional mechanics $\rho(x,p)$

Probability distributions of spacetimes $\rho(M,g)$

- No fixed classical background spacetime.
- No Penrose—Hawking singularity theorems
- Stochastic geometry? <u>Stochastic BH?</u>

Quantum gravity. Superstrings

$$\sum_M \int F(\Phi, g) e^{iS_M(\Phi, g)} [D\Phi Dg]$$

The sum over manifolds is not defined. Algorithmically unsolved problem.

Example

$$\dot{x} = x^2,\ x(t) = \frac{x_0}{1 - x_0 t} \quad \text{singular}$$

$$\rho(x,t) = C\exp\{-(\frac{x}{1-xt} - q_0)^2 / \varepsilon^2\},$$

nonsingular

Fixed classical spacetime?

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Stochastic inflation?

Quantum gravity Bogoliubov Correlation Functions

- Use Wheeler – de Witt formulation for QG.

ρ_{Σ} **Density operator of the universe on** Σ

Correlation functions

$$f^{(s)}_{i_1 j_1 \ldots i_s j_s}(x_1, \ldots, y_s; \Sigma)$$

$$= Tr(\rho_{\Sigma} g_{i_1 j_1}(x_1) \ldots \pi_{i_s j_s}(y_s))$$

Factorization

$$f^{(s)}_{i_1 j_1 \ldots i_s j_s}(x_1,\ldots,y_s;\Sigma)$$

$$\to \prod_r f_{i_r j_r m_r n_r}(x_r, y_r;\Sigma)$$

Th.M. NieuwenhuizenTh.M. Nieuwenhuizen, I.V. (2005)

QG Bogoliubov-Boltzmann Eqs

$$\frac{\delta f}{\delta \sigma} = J(f),$$

$$\sigma = (x, y, g, \pi, \Sigma)$$

Conclusions

BH and BB information loss (irreversibility) problem

Functional formulation (non-Newtonian) of classical mechanics: distribution function instead of individual trajectories. Fundamental equation: Liouville or FPK for a single particle.

Newton equation—approximate for average values. Corrections to Newton`s trajectories.

Stochastic general relativity. BH information problem. QG Bogoliubov-Boltzmann equations.

Спасибо за внимание!

Information Loss in Black Holes

- **Hawking paradox**.

- **Particular case of the Irreversibility problem.**

- **Bogolyubov method of derivation of kinetic equations -- to quantum gravity.**
- Th.M. NieuwenhuizenTh.M. Nieuwenhuizen, I.V. (2005)