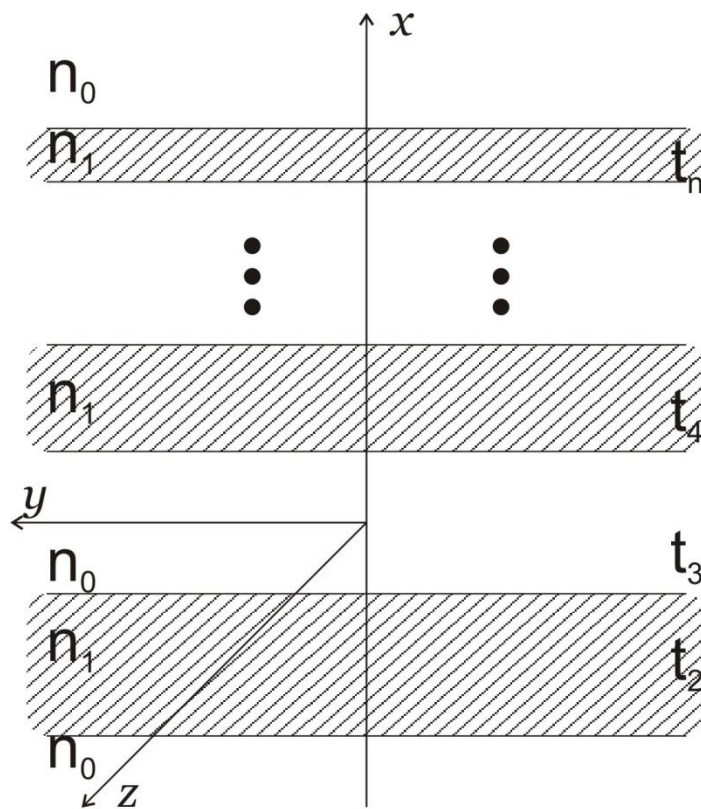


Multilayer model in optics. New analitic results.

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Planar multilayer waveguide



Фигура 1

For TE-waves propagating along Oz axis this is a boundary-value problem for the equation

$$\frac{c^2}{\omega^2} \frac{d^2 E_y}{dx^2} + n_j^2 E_y = \beta^2 E_y,$$

$$\tilde{n} \quad \acute{o}\tilde{n}\grave{e}\hat{i}\hat{a}\grave{e}\grave{y}\grave{i}\grave{e}$$

$$E_y(x_j - 0) = E_y(x_j + 0),$$

$$E'_y(x_j - 0) = E'_y(x_j + 0),$$

$$E_y(\pm\infty) = 0.$$

Reduced variables

$$\frac{d^2 E_y}{d\xi^2} + \eta_j^2 E_y = \sigma^2 E_y$$

$$m + 1 \leq j \leq n, \quad n_1 \geq n_{m+1}, \quad \max n_j = n_k > n_1, \quad \xi = \frac{2\pi n_1}{\lambda_0} x$$

$$\eta_j = \frac{n_j}{n_1}, \quad \sigma = \frac{\beta}{n_1} \quad - \quad \text{íðèââä, íúú é ýóôâêòèáíú é}$$

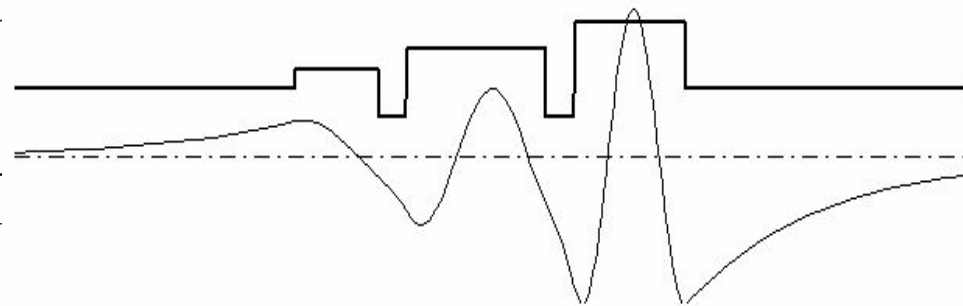
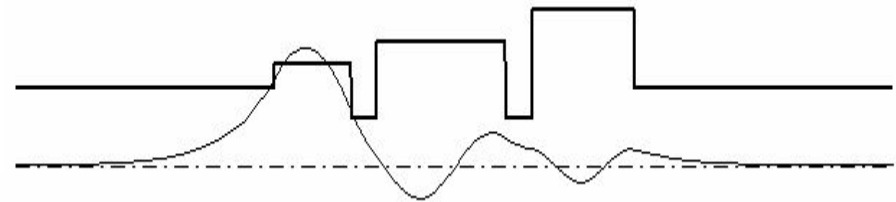
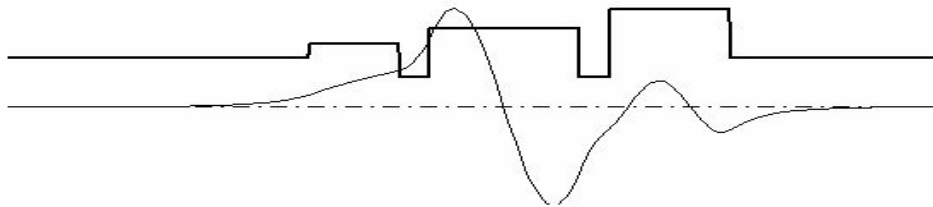
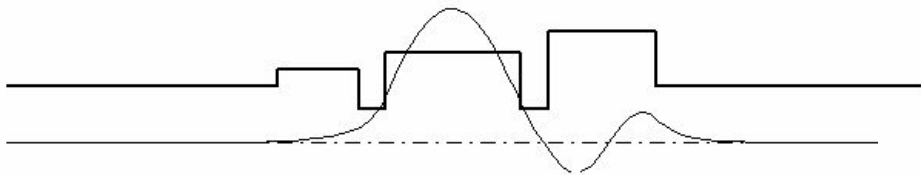
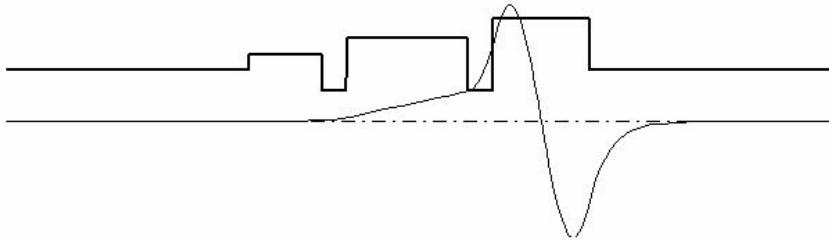
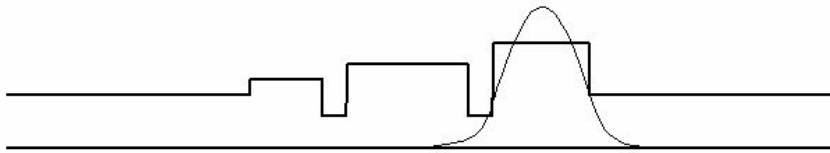
$$\text{ïêàçàðâëü} \quad \text{íðâëîèäíè} \quad \ddot{y},$$

$$1 < \sigma < \eta_k = \alpha, \quad t_j - \text{íðèââä, íá} \quad \ddot{y} \quad \text{òîëüèíà}$$

First example. 7 layers. The number of TE-modes: $K=6$.

$$t_2 = 3, t_3 = t_5 = 1, t_4 = 5, t_6 = 4$$

$$\alpha_1 = \alpha_7 = 1, \alpha_3 = \alpha_5 = 0,6, \alpha_2 = 1,3, \alpha_4 = 1,6, \alpha_6 = 2;$$



$$\sigma = 1,9039 \square ; 1,6003\dots; 1,5196\dots; 1,2804\dots; 1,1474\dots; 1,0216\dots$$

Traditional dispersion equations – equations for the eigenvalues of the propagation constant σ

- Type 1 – equation, is obtained by equating to zero of the determinant of homogenous linear system due to boundary conditions.
- Type 2 — equation, obtained by the known method of characteristic matrices
- These equations have too many terms if the number of layers is more than 4.
- Investigation of waveguides with many layers is now actual.

The properties of the dispersion equations

- Th.1. Type 1 equation has roots, coinciding with the refraction indexes of the inner layers of the waveguide. This roots may not be the eigenvalues of propagation constant.
- Th.2. The set of roots of type 2 equation is exactly the set of the eigenvalues of propagation constant.
- We propose a new one form of the dispersion equation. This equation in some known cases have no parasitic roots, and moreover it may be treated geometrically.

Homogeneous variables, vectors \mathbb{a}_j

$$\mathcal{Q}_j^* = \frac{\mathcal{Q}_j}{\sqrt{\alpha^2 - \sigma^2}}; \quad \mathcal{Q}_j^* = \frac{a_j}{b_j},$$

$$a_2 = \sqrt{\sigma^2 - 1}, \quad b_2 = \sqrt{\alpha^2 - \sigma^2}$$

$$\mathbb{a}_{j+1} = \begin{pmatrix} a_{j+1} \\ b_{j+1} \end{pmatrix} = V_j(\sigma) \begin{pmatrix} a_j \\ b_j \end{pmatrix}$$

Vectors $\overset{\boxtimes}{A}_j$

$$P_j^* = \frac{P_j}{\sqrt{\alpha^2 - \sigma^2}}; \quad P_j^* = \frac{A_j}{B_j},$$

$$A_m = \sqrt{\sigma^2 - \eta_{m+1}^2}, \quad B_m = \sqrt{\alpha^2 - \sigma^2}$$

$$\overset{\boxtimes}{A}_{j-1} = \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} = V_j(\sigma) \begin{pmatrix} A_j \\ B_j \end{pmatrix}$$

Theorem 3. Vectors $\vec{a}_j(\sigma)$ è $\vec{A}_j(\sigma)$

- rotate counter-clockwise when σ is decreasing.

- Theorem 4. If $\sigma \rightarrow \alpha - 0$

- then the directions of this vectors are converging to the direction of Ox axis.

The multilayer equation in vector form

$$\frac{q_j(\overset{\boxtimes}{a}_{j+1}, \overset{\boxtimes}{A}_j^o)}{\sqrt{\alpha^2 - \sigma^2} (\overset{\boxtimes}{a}_{j+1}, \overset{\boxtimes}{A}_j^r)} = 0$$

The formulae for the number of TE-modes.

$$\left(\overset{\boxtimes}{a}_{j+1}(\sigma), \overset{\boxtimes}{A}_j^0(\sigma) \right) = 0$$

$$K = \left[\frac{P_{m+1} + P_m}{\pi} \right]_-$$

$$P_m = \lim_{\sigma \rightarrow 1+0} \operatorname{arctg} \sqrt{\frac{\alpha^2 - \sigma^2}{\sigma^2 - \eta_{m+1}^2}}$$

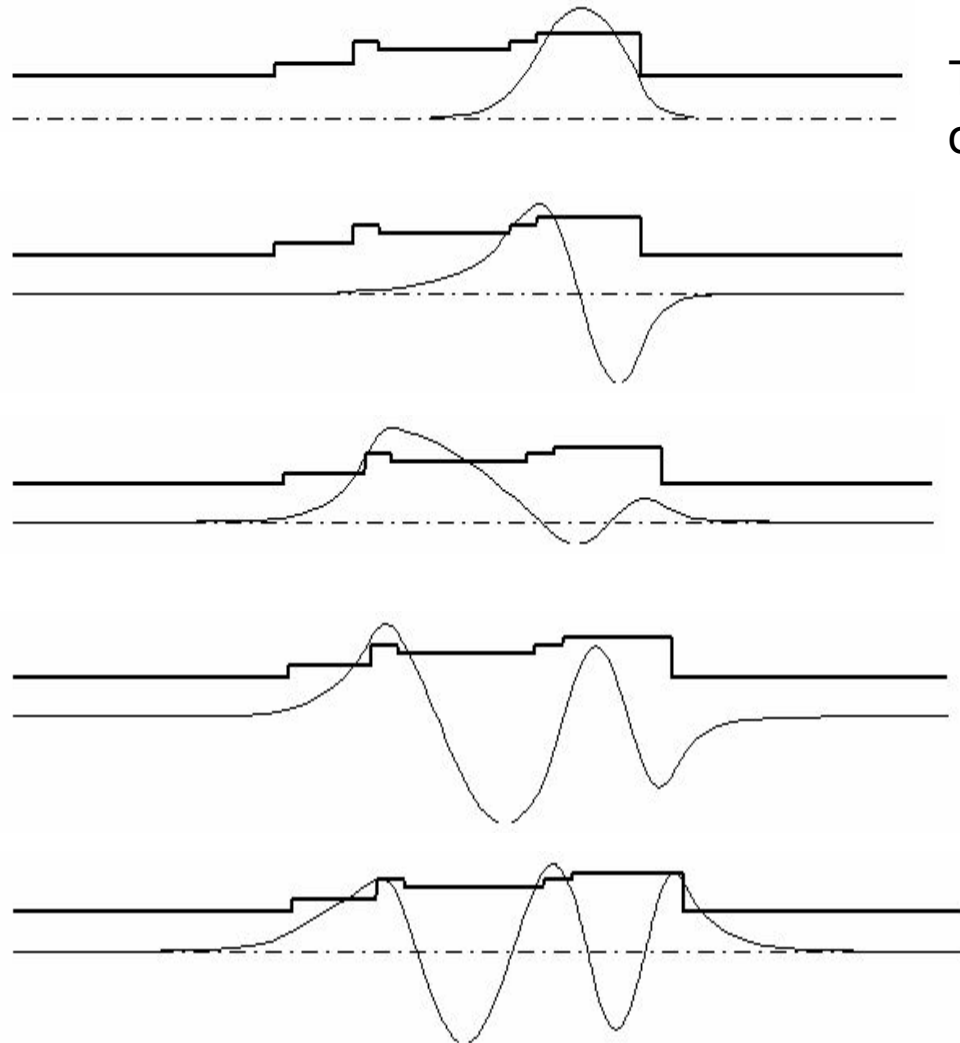
Transform

 $V_j(\sigma)$

$$\begin{pmatrix} ch\gamma_j & r_j sh\gamma_j \\ \frac{sh\gamma_j}{r_j} & ch\gamma_j \end{pmatrix}$$

$$r_j = \frac{\sqrt{\sigma^2 - \eta_j^2}}{\sqrt{\alpha^2 - \sigma^2}}, \quad \gamma_j = t_j \sqrt{\sigma^2 - \eta_j^2}.$$

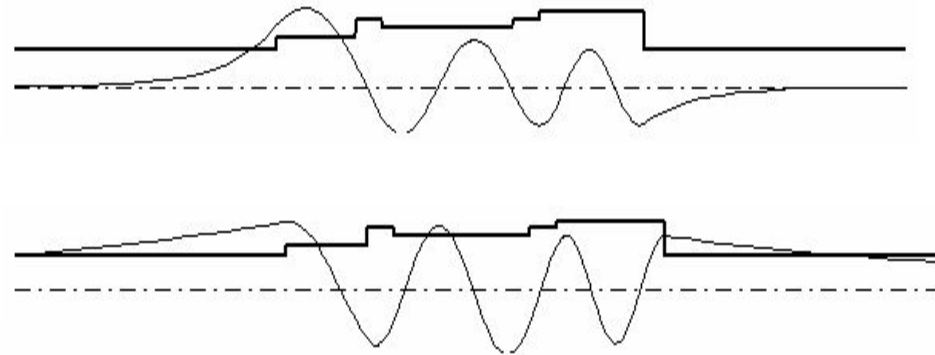
The second example, $K=7$.



The difference from the first example is only

$$\alpha_3 = \alpha_5 = 1,8$$

$\sigma = 1,9232\dots, 1,7046\dots, 1,5802\dots, 1,4681\dots,$
 $1,3068\dots, 1,1263\dots, 1,0022\dots$



Thank you for the attention

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