Multilayer model in optics. New analitic results.

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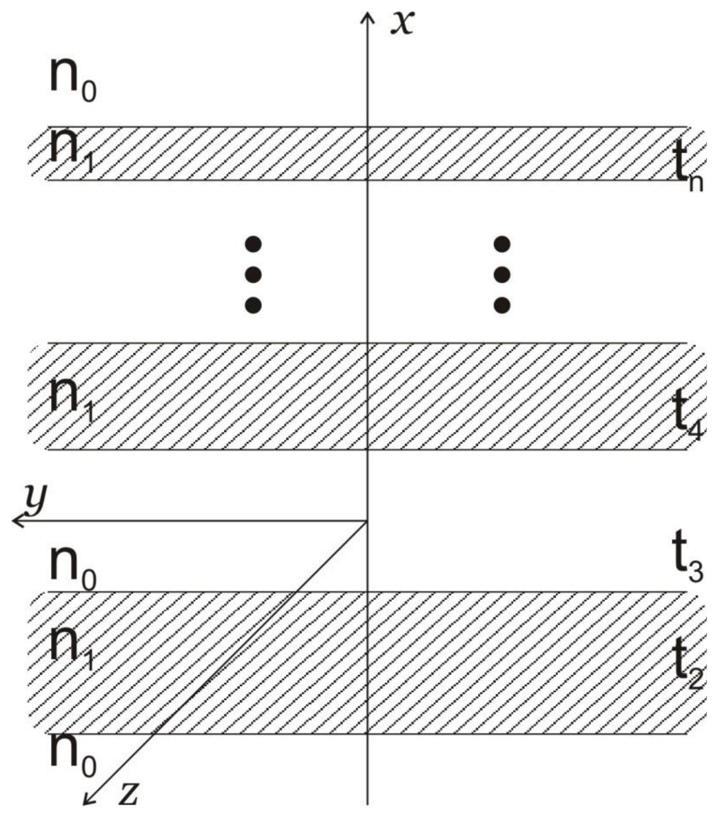
Planar multilayer waveguide



Фигура 1

For TE-waves propagating along Oz axis this is a boundary-value problem for the equation

$$\frac{c^2}{\omega^2}\frac{d^2E_y}{dx^2}+n^2{}_jE_y=\beta^2E_y,$$

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$$E_y(x_j-0)=E_y(x_j+0),$$

$$E'_y(x_j-0)=E'_y(x_j+0),$$

$$E_y(\pm\infty)=0.$$

Reduced variables

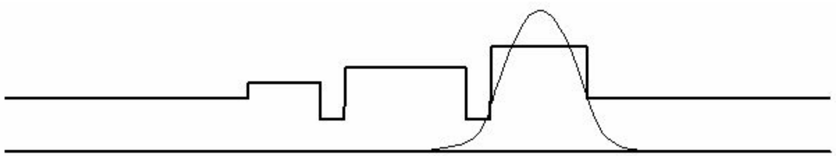
$$\frac{d^2 E_y}{d\xi^2} + \eta_j^2 E_y = \sigma^2 E_y$$

$m+1$ - ÷èñëî ñëî¸â, $n_1 \ge n_{m+1}$, $\max n_j = n_k > n_1$, $\xi = \frac{2\pi n_1}{\lambda_0} x$

$\eta_j = \frac{n_j}{n_1}$, $\sigma = \frac{\beta}{n_1}$ − *ïðèâåä¸ííû é ýôôåêòèâíû é ïîêàçàòåëü ïðåëîìëåíè ÿ,*

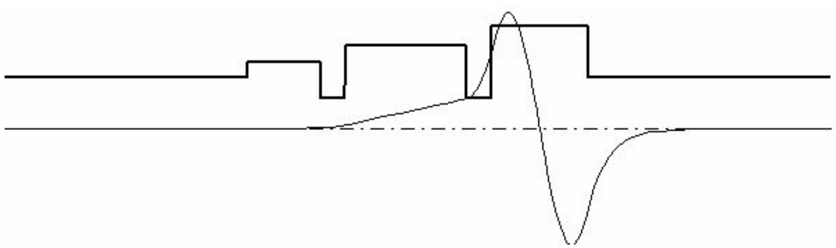
$1 < \sigma < \eta_k = \alpha$, t_j − *ïðèâåä¸ííà ÿ òîëùèíà*

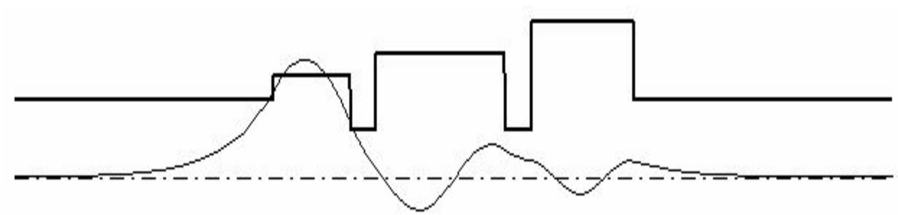
First example. 7 layers. The number of TE-modes: K=6.

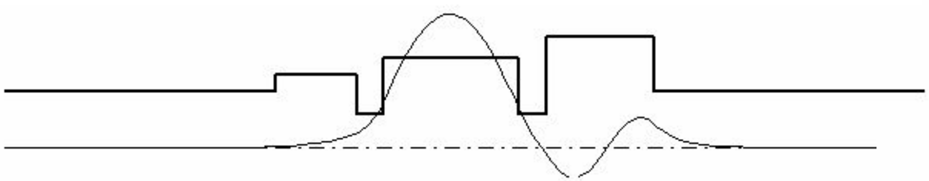


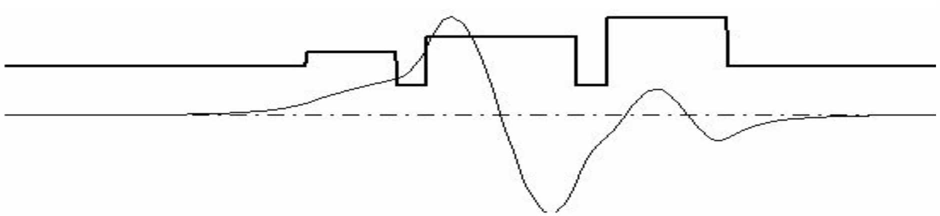
$$t_2 = 3, t_3 = t_5 = 1, t_4 = 5, t_6 = 4$$

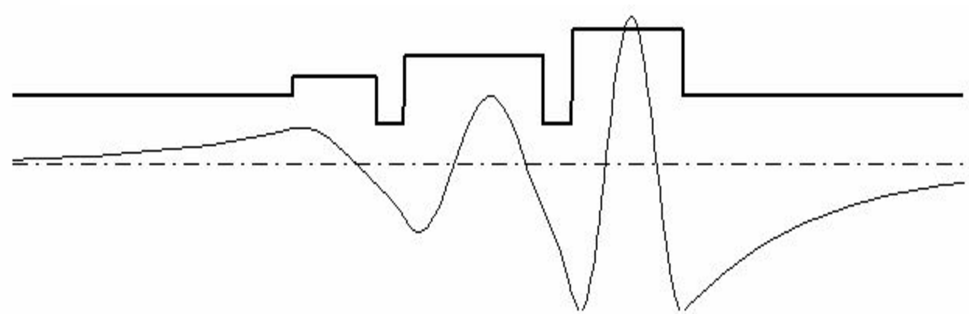
$$\alpha_1 = \alpha_7 = 1, \alpha_3 = \alpha_5 = 0{,}6, \alpha_2 = 1{,}3, \alpha_4 = 1{,}6, \alpha_6 = 2;$$











$\sigma = 1{,}9039\ldots;\quad 1{,}6003\ldots;\quad 1{,}5196\ldots;\quad 1{,}2804\ldots;\quad 1{,}1474\ldots;\quad 1{,}0216\ldots$

Traditional dispersion equations –equations for the eigenvalues of the propagation constant σ

- Type 1 – equation, is obtained by equating to zero of the determinant of homogenius linear system due to boundary conditions.
- Type 2 — equation, obtained by the known method of characteristic matrices
- This equations have too many terms if the number of layers is more then 4.
- Investigation of waveguides with many layers is now actual.

The properties of the dispersion equations

- Th.1. Type 1 equation has roots, coinsiding with the refraction indexes of the inner layers of the waveguide. This roots may not be the eigenvalues of propagation constant.

Th.2. The set of roots of type 2 equation is exactly the set of the eigenvalues of propagation constant.

- We propose a new one form of the dispersion equation. This equation in some known cases have no parasitic roots, and moreover it may be treated geometrically.

Multilayer equation

$$q_j = \sqrt{\sigma^2 - \eta_j^2}, \quad -\textit{õàðàêòåðèñ òèêà} \quad \textit{ñëîÿ}$$

$$Q_2 = q_1 \quad \text{è} \quad Q_j = q_{j-1}\,\mathrm{th}\left(q_{j-1}t_{j-1} + \mathrm{arth}\left(\frac{Q_{j-1}}{q_{j-1}}\right)\right), \quad 3 \le j \le m+1,$$

$$P_{m-1} = q_m \quad \text{è} \quad P_j = q_{j+1}\,\mathrm{th}\left(q_{j+1}t_{j+1} + \mathrm{arth}\left(\frac{P_{j+1}}{q_{j+1}}\right)\right), \quad 1 \le j \le m-2,$$

$$\mathrm{th}\left(q_j t_j + \mathrm{arth}\left(\frac{Q_j}{q_j}\right) + \mathrm{arth}\left(\frac{P_j}{q_j}\right)\right) = 0.$$

Homogenius variables,vectors $\vec{a}_j$

$$Q_j^* = \frac{Q_j}{\sqrt{\alpha^2 - \sigma^2}}; \quad Q_j^* = \frac{a_j}{b_j},$$

$$a_2 = \sqrt{\sigma^2 - 1}, \quad b_2 = \sqrt{\alpha^2 - \sigma^2}$$

$$\vec{a}_{j+1} = \begin{pmatrix} a_{j+1} \\ b_{j+1} \end{pmatrix} = V_j(\sigma) \begin{pmatrix} a_j \\ b_j \end{pmatrix}$$

Vectors $\vec{A}_j$

$$P_j^* = \frac{P_j}{\sqrt{\alpha^2 - \sigma^2}}; \quad P_j^* = \frac{A_j}{B_j},$$

$$A_m = \sqrt{\sigma^2 - \eta_{m+1}^2}, \quad B_m = \sqrt{\alpha^2 - \sigma^2}$$

$$\vec{A}_{j-1} = \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} = V_j(\sigma) \begin{pmatrix} A_j \\ B_j \end{pmatrix}$$

Theorem 3. Vectors $\vec{a}_j(\sigma)$ è $\vec{A}_j(\sigma)$

- rotate counter-clockwise when σ

is decreasing.

- Theorem 4. If $\sigma \to \alpha - 0$
- then the directions of this vectors are converging to the direction of Ox axis.

The multilayer equation in vector form

$$\frac{q_j(\vec{a}_{j+1}, \vec{A}_j^o)}{\sqrt{\alpha^2-\sigma^2}(\vec{a}_{j+1}, \vec{A}_j^r)} = 0$$

The formulae for the number of TE-modes.

$$\left(\vec{a}_{j+1}(\sigma),\ \vec{A}_j^{O}(\sigma)\right) = 0$$

$$K = \left[\frac{p_{m+1} + P_m}{\pi}\right]_-$$

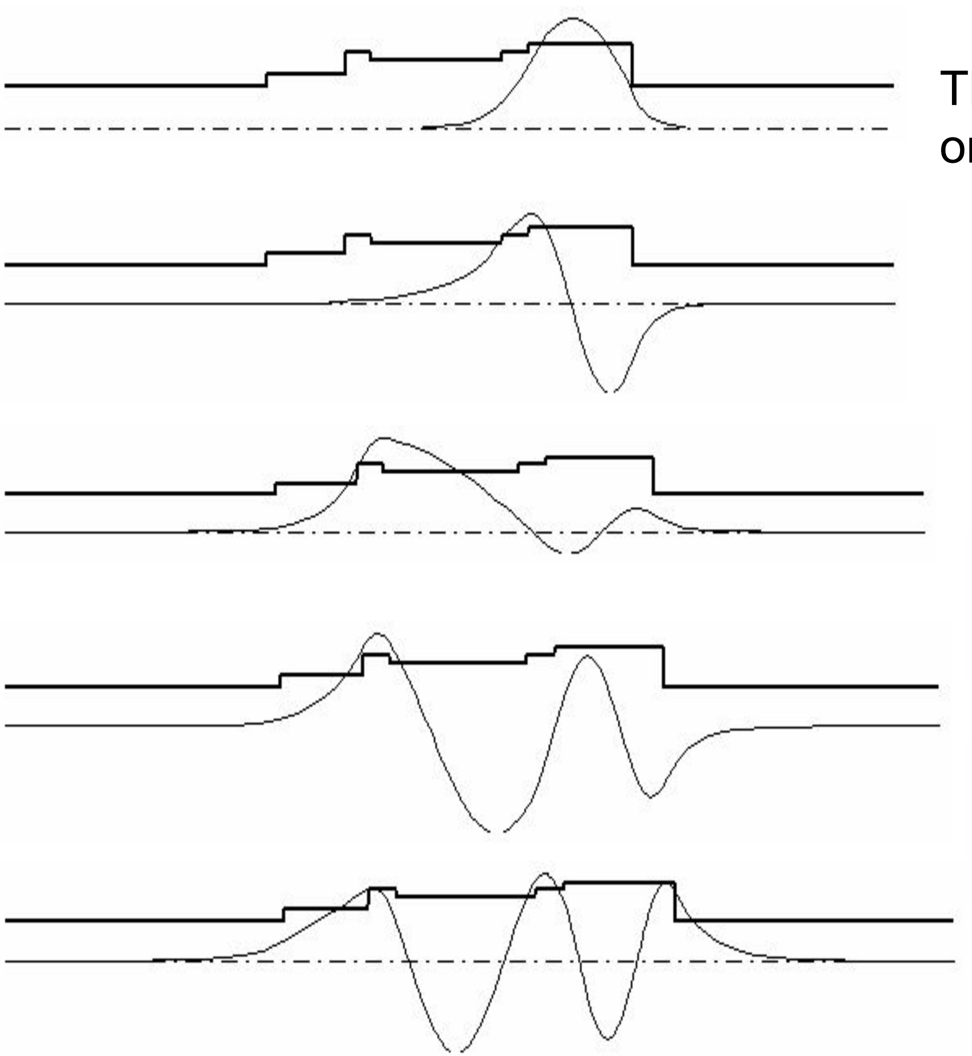
$$P_m = \lim_{\sigma \to 1+0} \operatorname{arctg}\sqrt{\frac{\alpha^2 - \sigma^2}{\sigma^2 - \eta_{m+1}^2}}$$

Transform $V_j(\sigma)$

$$\begin{pmatrix} ch\gamma_j & r_j sh\gamma_j \\ \dfrac{sh\gamma_j}{r_j} & ch\gamma_j \end{pmatrix}$$

$$r_j = \frac{\sqrt{\sigma^2 - \eta_j^2}}{\sqrt{\alpha^2 - \sigma^2}}, \quad \gamma_j = t_j \sqrt{\sigma^2 - \eta_j^2}.$$

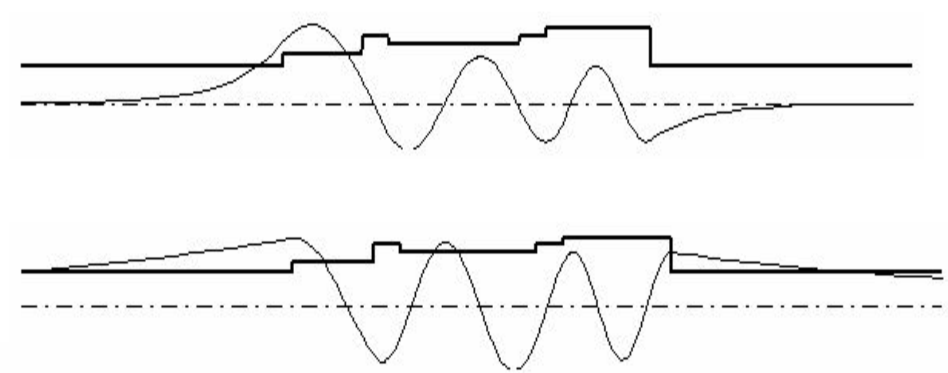
The second example, K=7.



The difference from the first example is only

$$\alpha_3 = \alpha_5 = 1{,}8$$

$\sigma = 1{,}9232\ldots,\quad 1{,}7046\ldots,\quad 1{,}5802\ldots,\quad 1{,}4681\ldots,$
$1{,}3068\ldots,\quad 1{,}1263\ldots,\quad 1{,}0022\ldots$



Thank you for the attention

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