ЧИСЛЕННОЕ КОНФОРМНОЕ ОТОБРАЖЕНИЕ В ДВУМЕРНОЙ ГИДРОДИНАМИКЕ

И СМЕЖНЫЕ ПРОБЛЕМЫ ЭЛЕКТРОДИНАМИКИ И ТЕОРИИ УПРУГОСТИ

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Электронная версия доклада В.И. Прохоренко

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И СМЕЖНЫЕ ПРОБЛЕМЫ ЭЛЕКТРОДИНАМИКИ И ТЕОРИИ УПРУГОСТИ

Доклад является представлением одноименной монографии Б.И. Рабиновича и Ю.В. Тюрина, выходящей в ИКИ на английском языке в этом году. Описывается оригинальный численный алгоритм конформного отображения (RT-алгоритм), основанный на двух процедурах (R-процедуре и T-процедуре). Решение проблемы, достигаемое этим методом, охватывает произвольные односвязные и двусвязные области с кусочно-гладкими контурами. RT-алгоритм используется для построения ортогональных сеток и для решения широкого спектра внешних и внутренних двумерных задач гидродинамики, а также некоторых смежных задач электродинамики и теории упругости. В качестве основных инструментов для решения краевых задач на преобразованной области (единичный круг, круговое кольцо, область, ограниченная прямоугольником) применяются методы рядов Тейлора и Лорана, Ритца, конечных элементов, сопряженных вихрей. Все эти методы реализуются в виде программного обеспечения на PC, а результаты представляются средствами машинной графики. Для проверки получаемых численных результатов используются, когда это возможно, аналитические решения, решения, полученные другими численными методами, а также экспериментальные результаты.

NUMERICAL METHODS IN FLUID MECHANICS

Boris I. Rabinovich and Yuri V. Tyurin

Numerical Conformal Mapping in Two-Dimensional Hydrodynamics

SPACE RESEARCH INSTITUTE RUSSIAN ACADEMY OF SCIENCES

Numerical Conformal Mapping in Two-Dimensional Hydrodynamics
&
Related Problems of Electrodynamics and Elasticity Theory

by

Boris I. Rabinovich and Yuri V. Tyurin

English text edited by **Djosef Cherniawsky, PhD,**

Program implementation of the RT-algorithm, the numerical solutions and their graphical representation has been developed by **Yuri Tyurin, PhD, Rudolf Ashkinazy, Alexander Leviant, PhD, Arcadi Livshits, PhD, Eugeni Sokolin PhD**

Acknowledgements

The authors take this opportunity to express deep gratitude to their colleagues mentioned above for creatively purposefully carrying out this complex task. Their role can scarcely be exaggerated.

The authors thanks **Alexander Rabinovich** for the great assistance in composing references on the numerical methods for conformal mapping, as well as **Boris Kruglov** and **Yuri Shilov** for their help in translation the manuscript into English and **Josef Cherniawsky** - for his productive comments on the manuscript.

The authors also would like to thank **Natalia Komarova** for her careful editing the camera-ready version of the book.

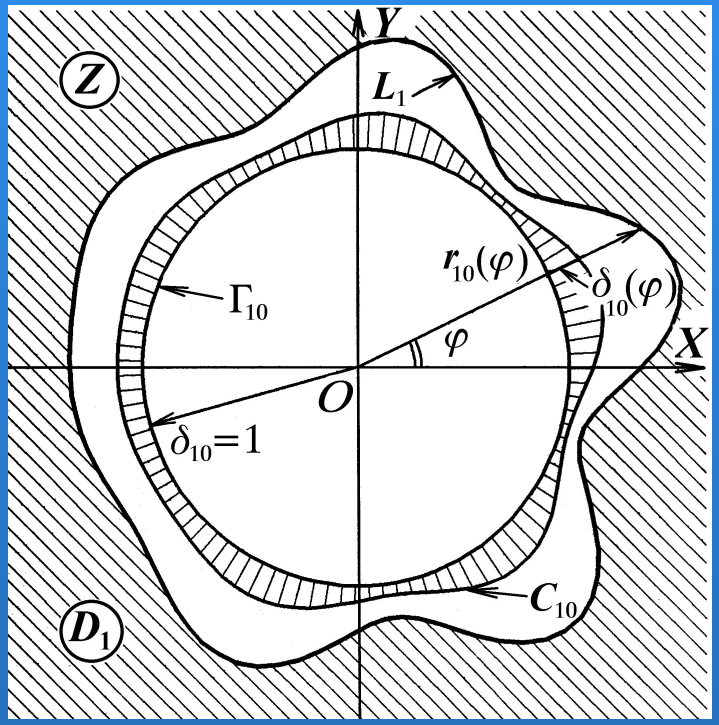
Chapter 1
Lavrentiev Variational Principle and Auxiliary Transformations

Lavrentiev variational principle and representation of mapping functions by Taylor and Laurent series. Construction of the image of variation of an arbitrary contour on a unit circle. Preliminary analytical transforms. General formulation of linear boundary-value problems for simply and doubly connected domains.

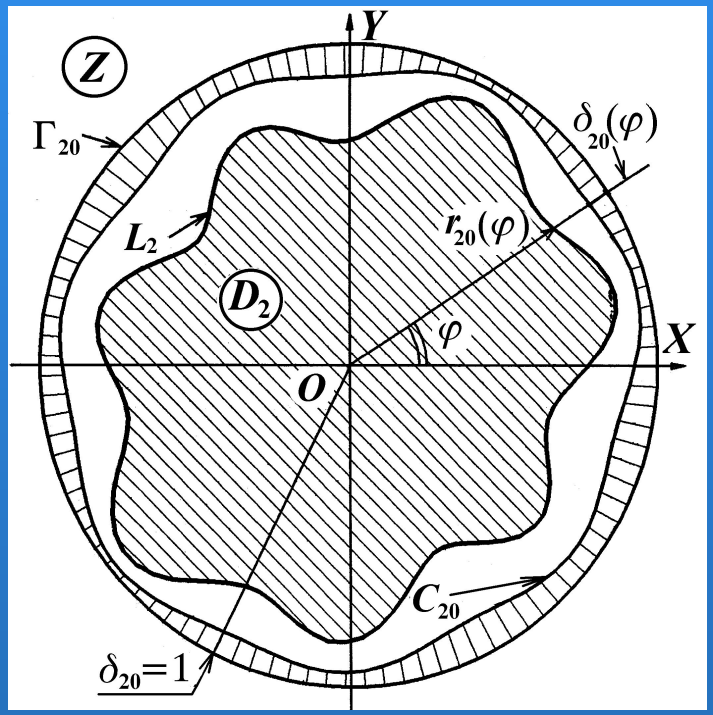
Chapter 2
RT-Algorithm for Conformal Mapping of an Arbitrary Domain onto the Unit Circle and of a Doubly Connected Domain onto the Annulus

General idea and informal description of the RT-algorithm. Mapping of a quasi-circular domain onto a circle (R-procedure). Mapping of an arbitrary simply connected domain onto the unit circle and of a doubly connected domain onto the annulus (T-procedure). Convergence of the RT-algorithm. Methodological examples.

Construction on the unit circle of function variations characterising contour deviation from a circle



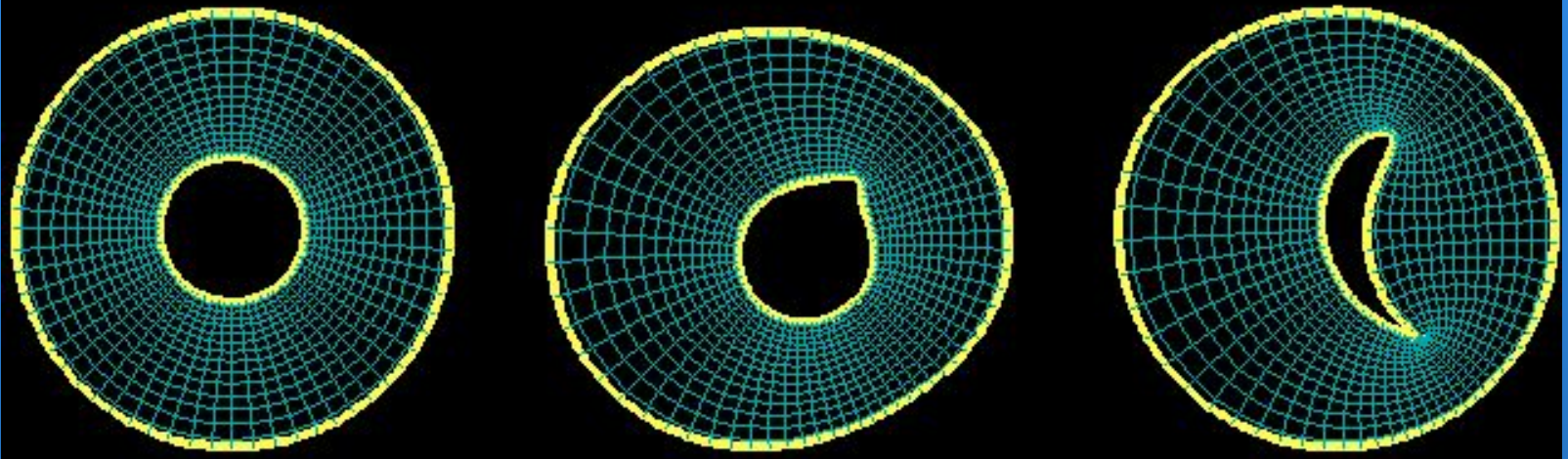
Exterior problem



Interior problem

RT-algorithm. Conformal map example

Double connected domain. Inverse mapping



RT- algorithm

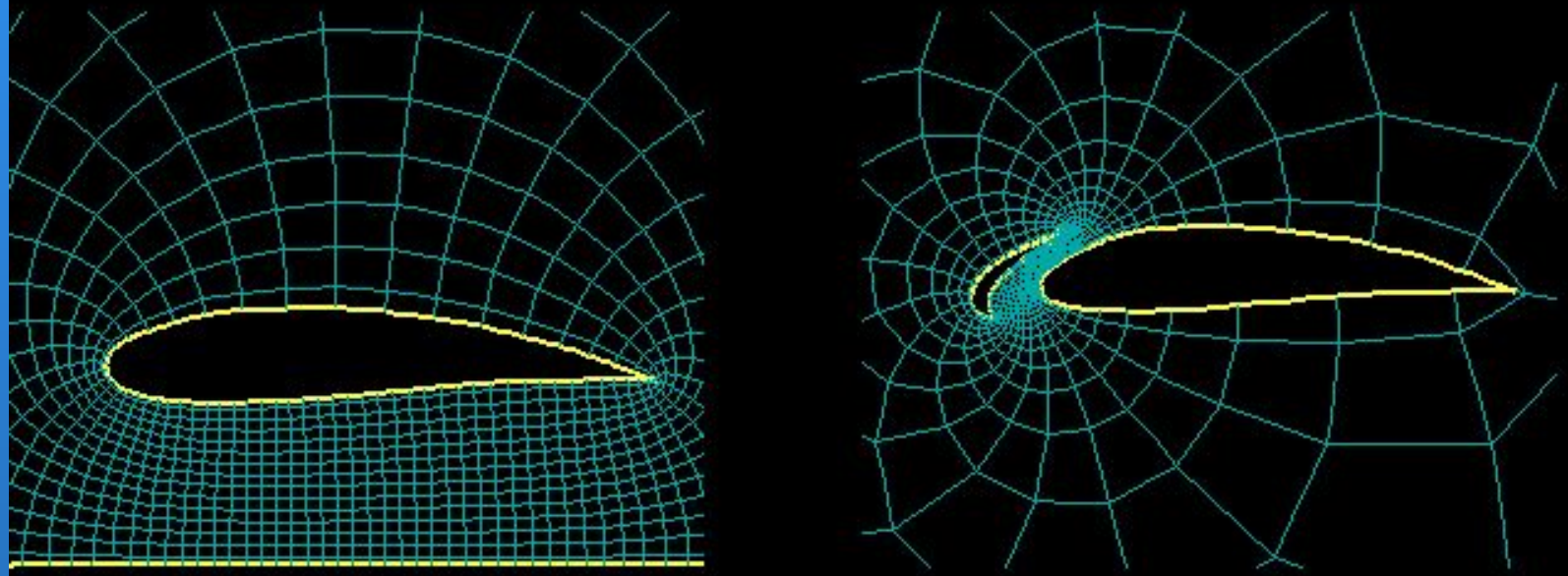
Joukowski transform

SPACE – continue ; ESC – abort

Chapter 3
Exterior problems of Hydrodynamics

Two-dimensional flow problems: Complex potential and formulas of Blasius-Chaplygin and Kutta-Joukowski. Electrostatic and magnetostatic analogies. Complex potentials. Examples of electric and magnetic fields. Unsteady two-dimensional motion of a contour without circulation. Solution of a boundary-value problem for complex potentials. Circulatory flow past two contours. Flow about a contour near a rectilinear boundary. A wing in motion near the ground. General problem on the flow around two contours. Vortices in the Jovian atmosphere. Slatted wing. Generalization to multiply connected domains. Flow about a lattice made of a single row of contours. Airfoil cascade. Flow around a contour with two sharp edges in the presence of two free vortices. Conjugate-vortex method.

Aerodynamic problems :



Conformal map grids

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Hydrodynamic problem

Potential Flow with two Vortices around the Arc

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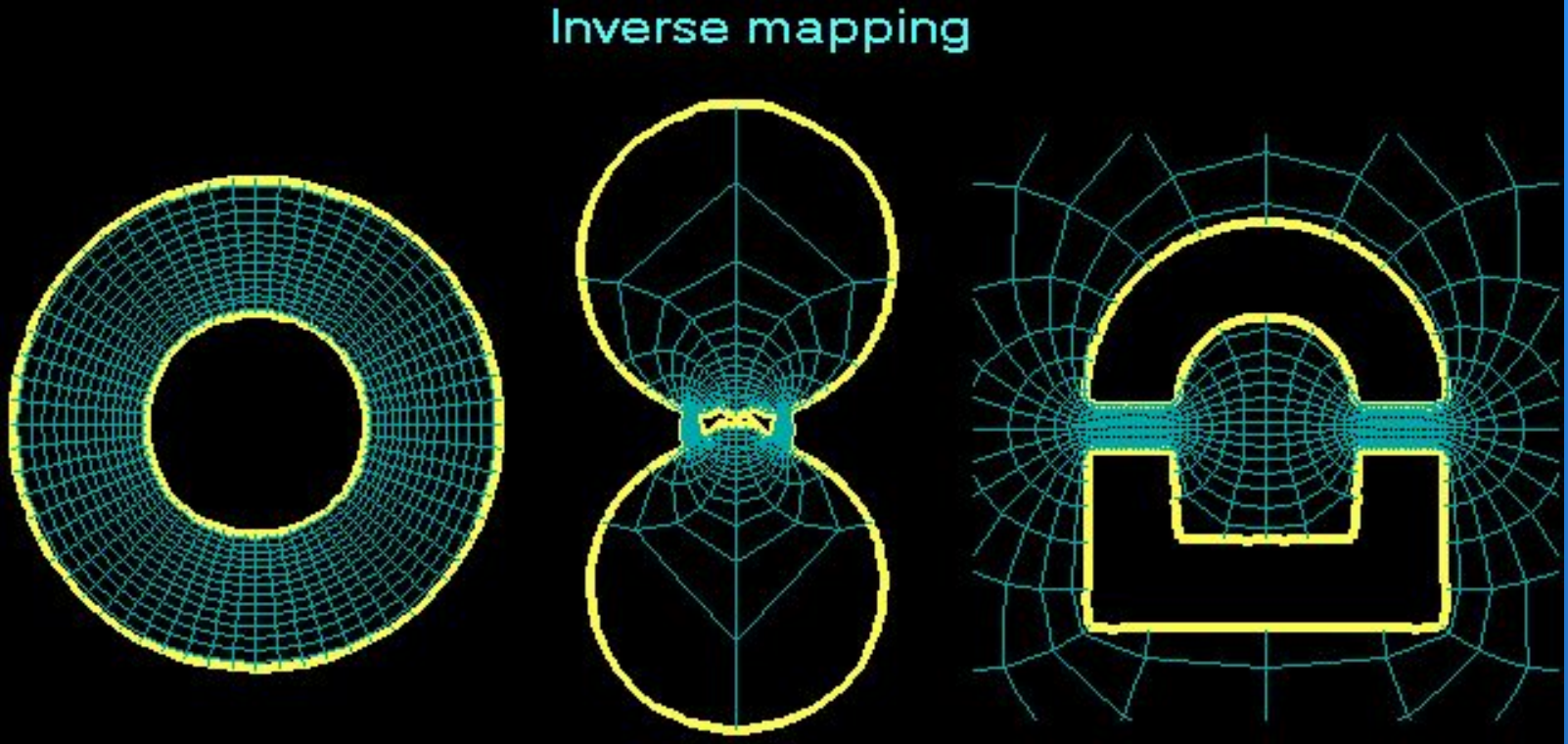
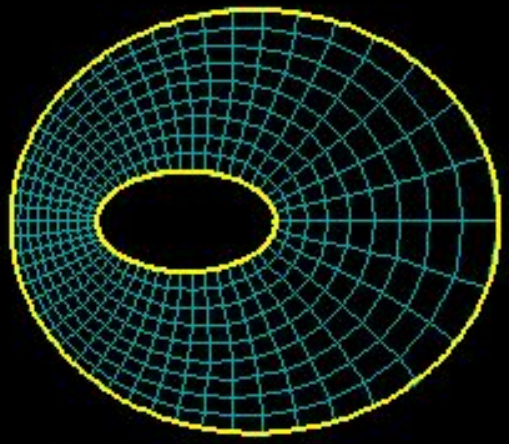
Magnetostatic problem - conformal mapping



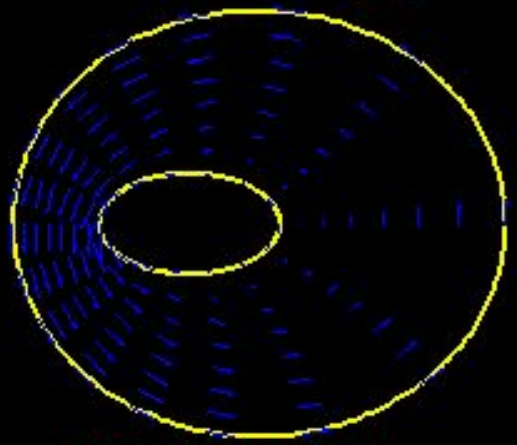
Chapter 4
Interior problems of Hydrodynamics

Stokes-Joukowski problem. Interior boundary-value problems of fluid dynamics for moving cavities. Two-dimensional boundary-value problems. Complex velocity potential and associated moments of inertia of liquid. The Saint-Venant problem as an analogue of the two-dimensional Stokes-Joukowski problem. Examples of solving to the Stokes-Joukowski and Saint-Venant problems. The axisymmetric cavity. Prediction of circulatory flows of liquid in closed domains. An example of calculating of the associated moment of inertia in the presence of local vortex regions.

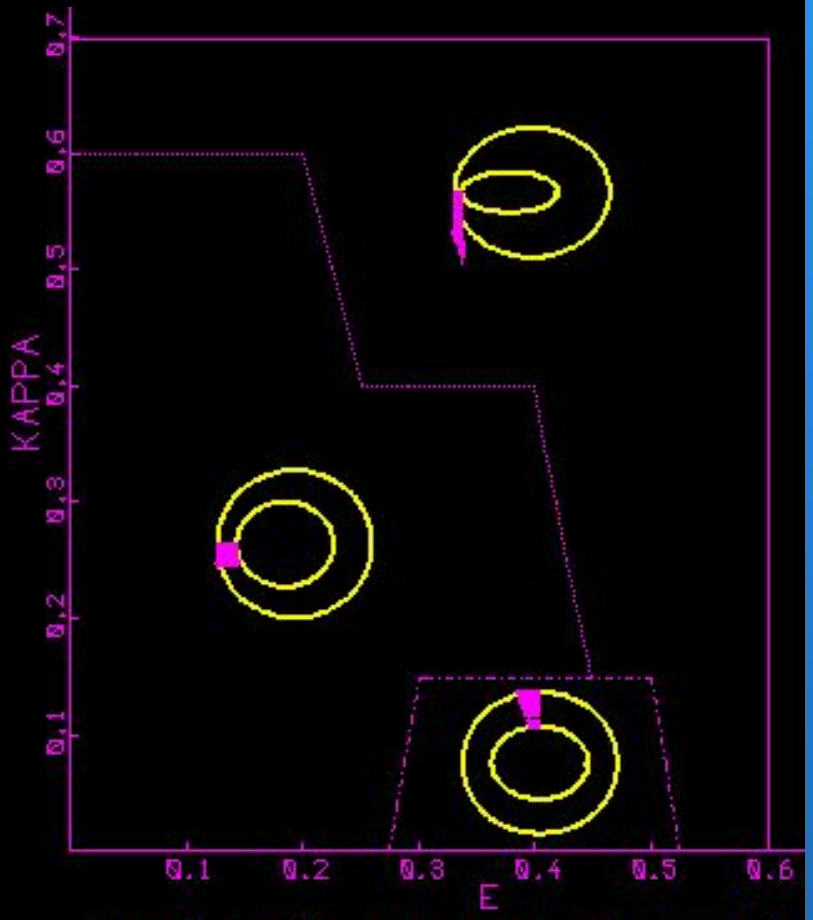
Fluid stream in the rotating tank or the torsion of an elliptical bar



Conformal map grid



Fluid velocity or
Shearing stresses



Location of maximum vector point

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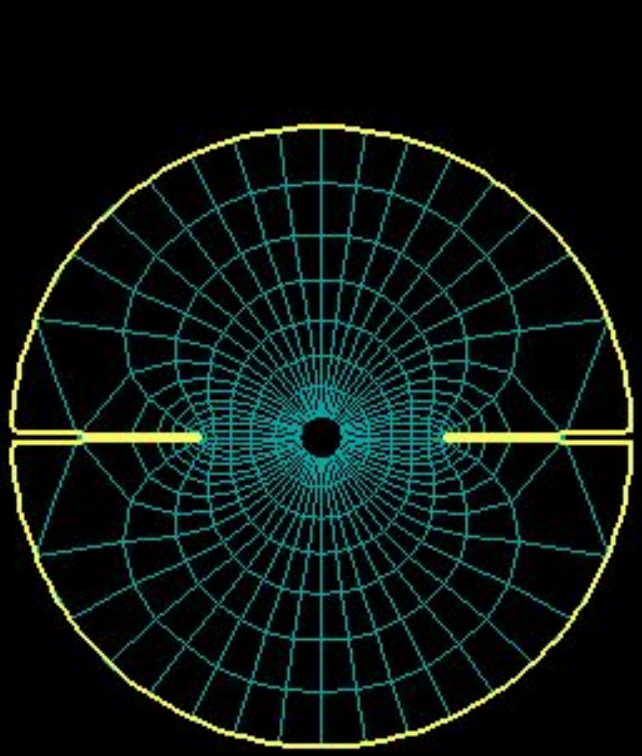
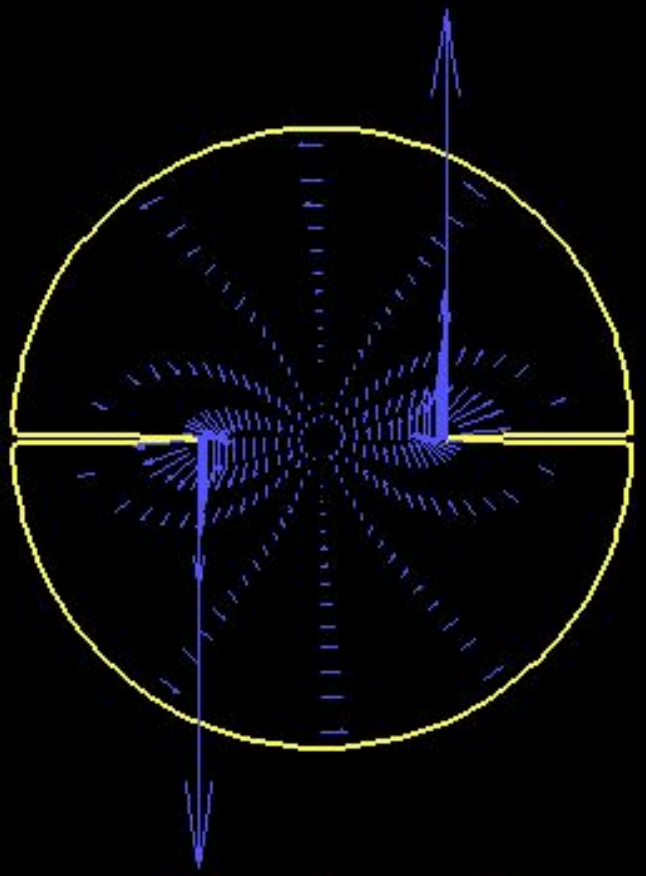
Rotation of the circular tank with two inner ribs



Conformal map grid



Velocity field

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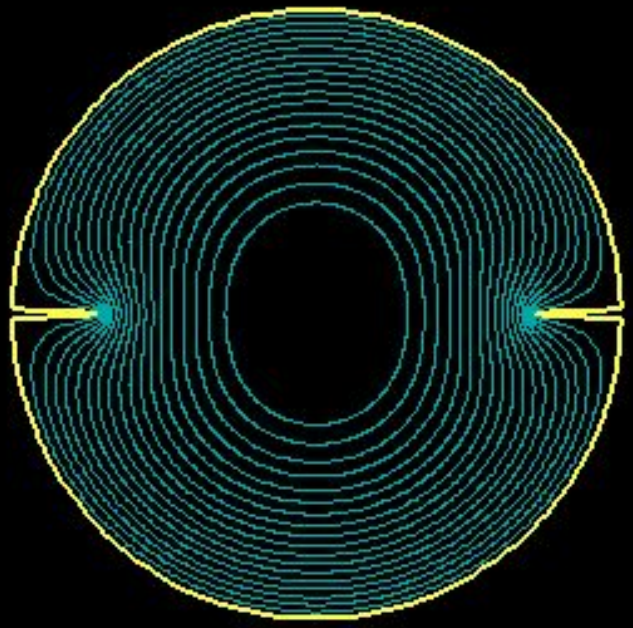
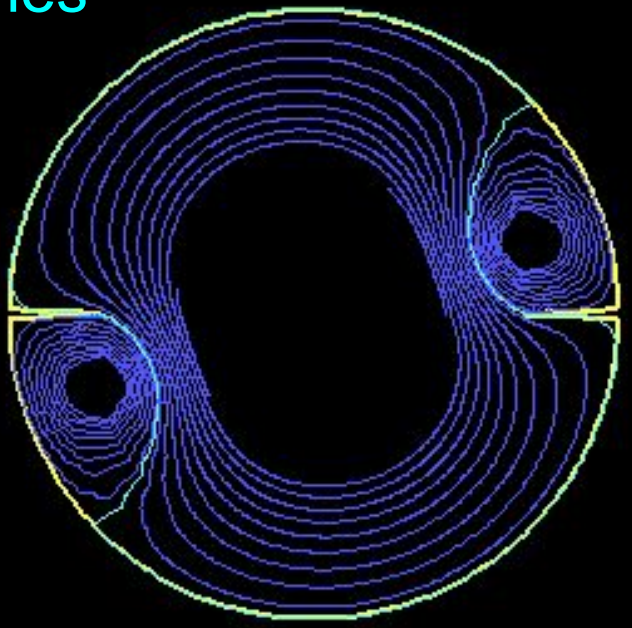
Rotation of the circular tank with two inner ribs

Stream lines



Without Vortices



With Vortices

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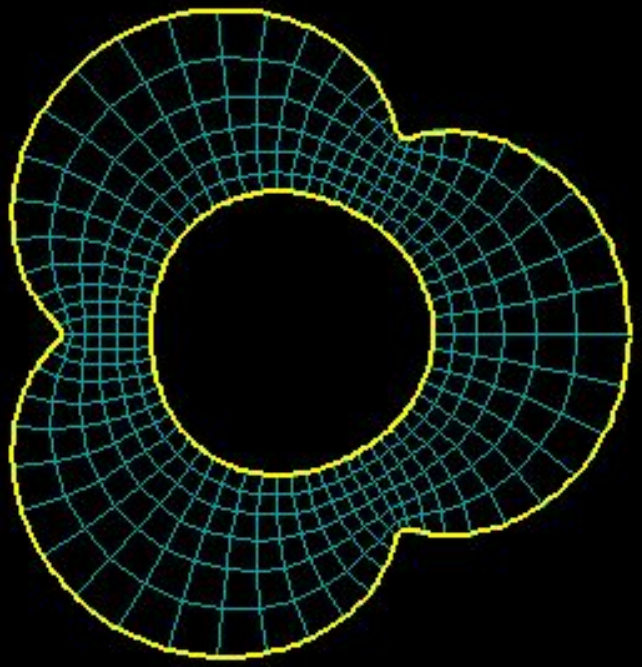
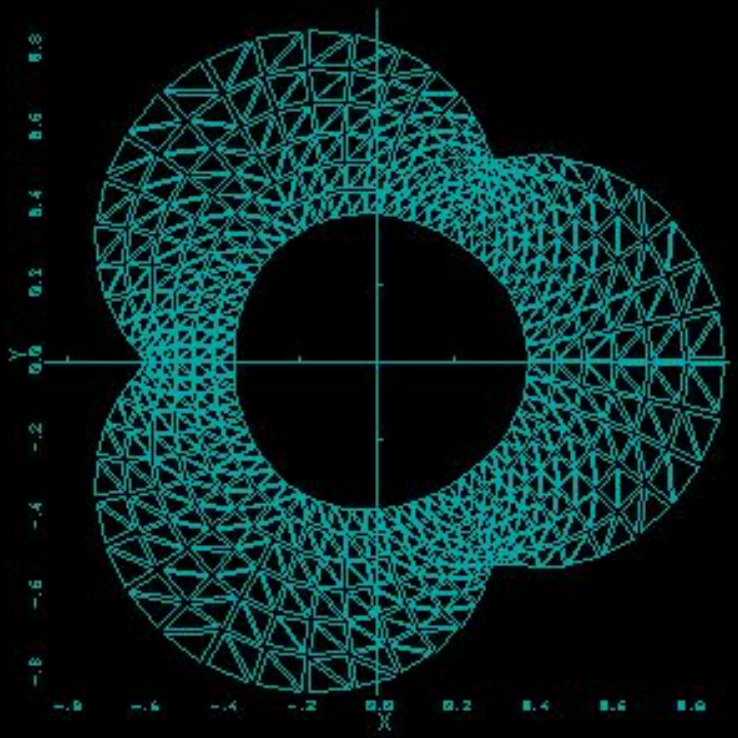
Automatic Finite Element Mesh Generation





Conformal map grid

FE mesh

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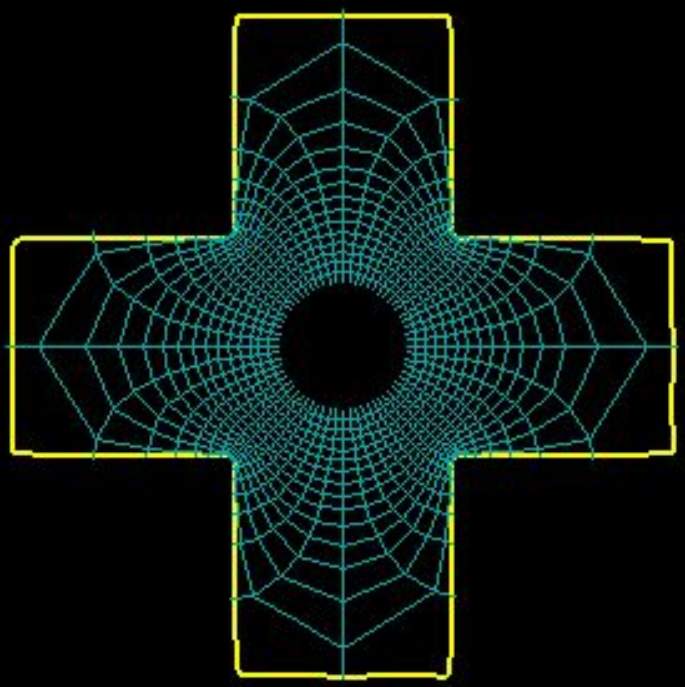
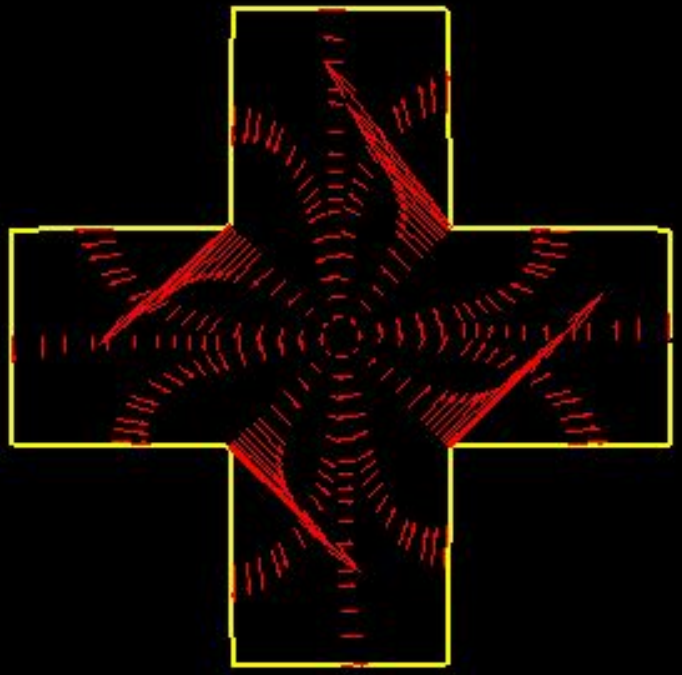
Torsion of the beam of cross-shape section



Conformal map grid



Shearing stresses

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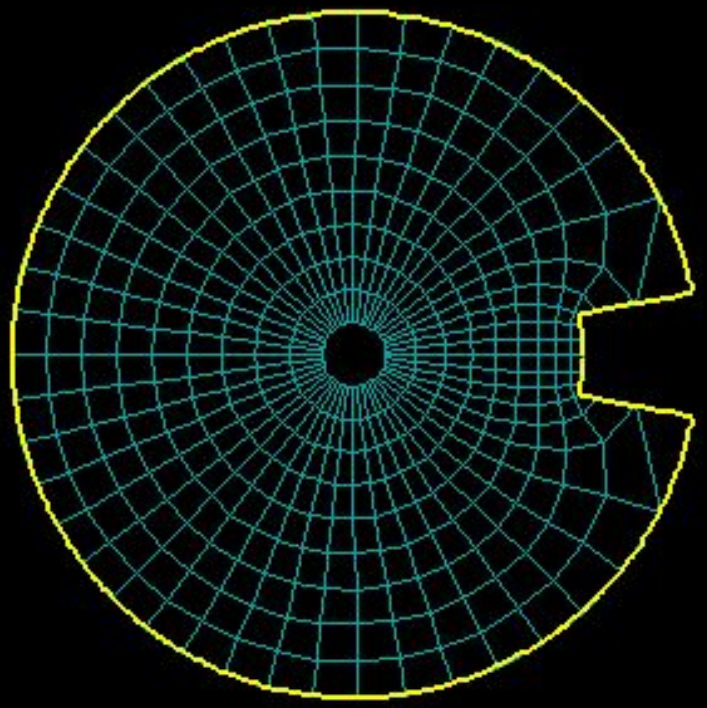
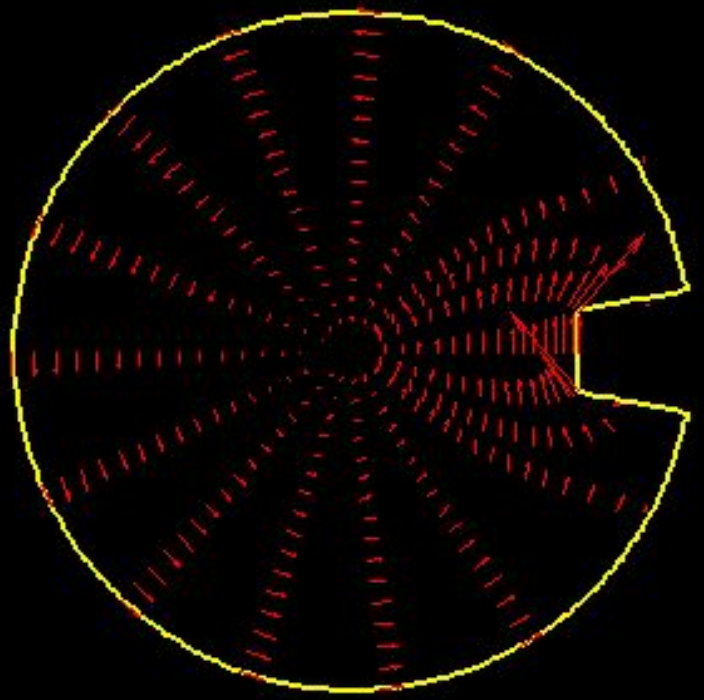
Torsion of the shaft with sectarian cut



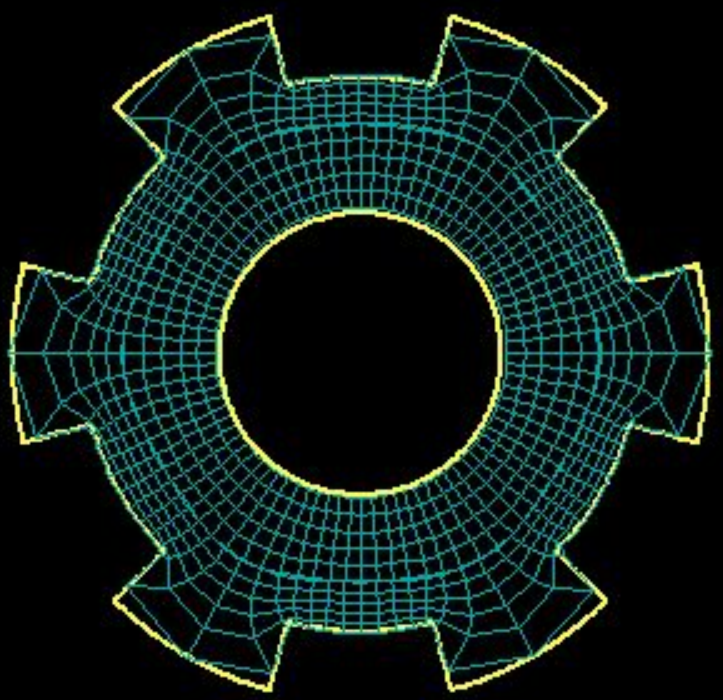
Conformal map grid



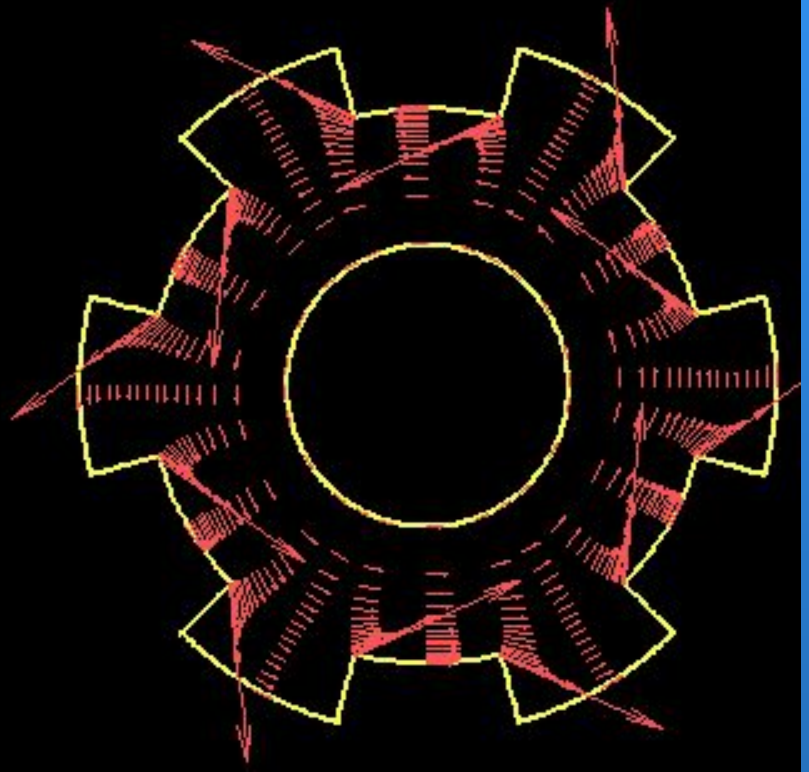
Shearing stresses

– Press "Esc" to abort –

Torsion of the castellated hollow shaft

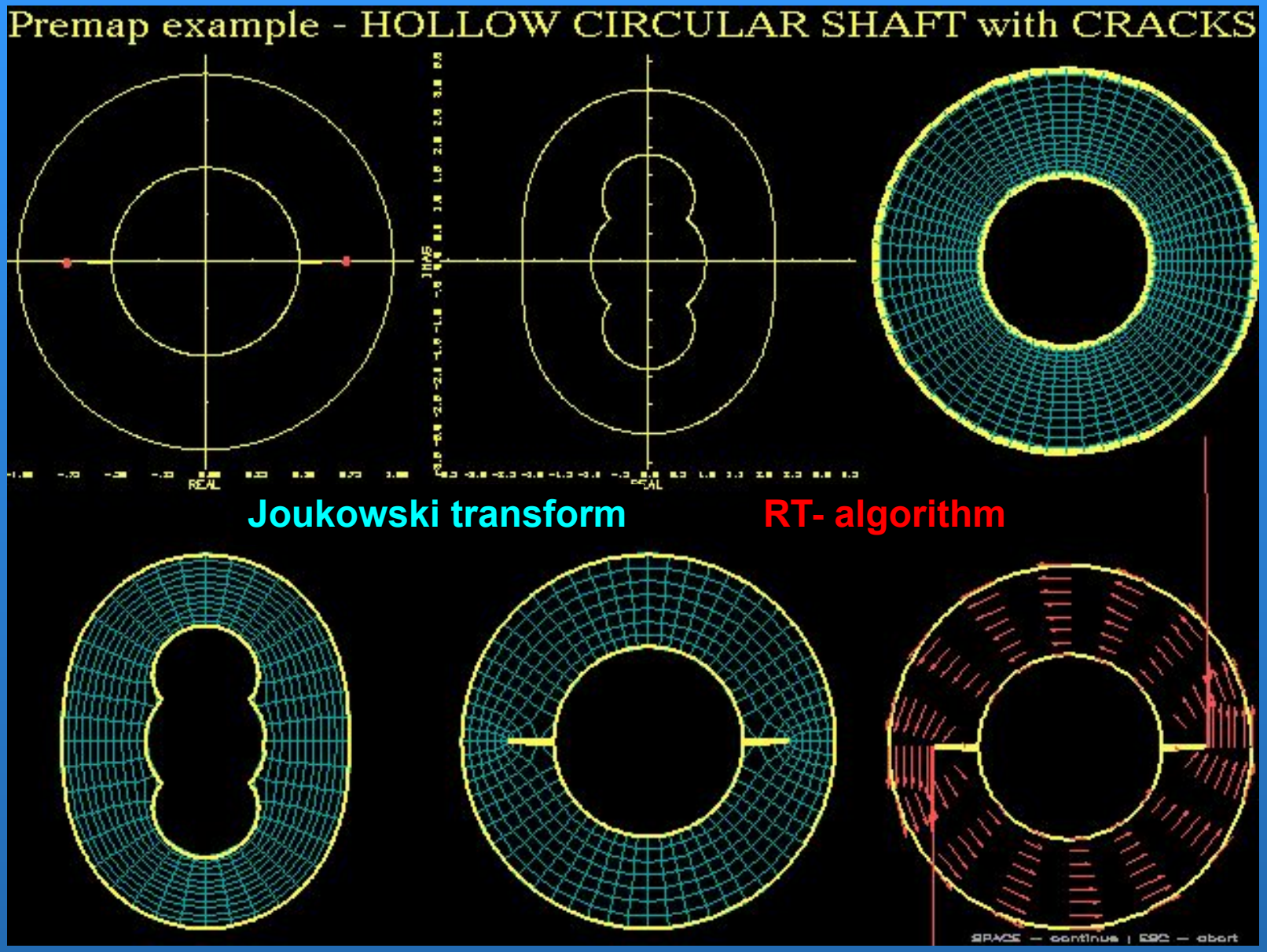


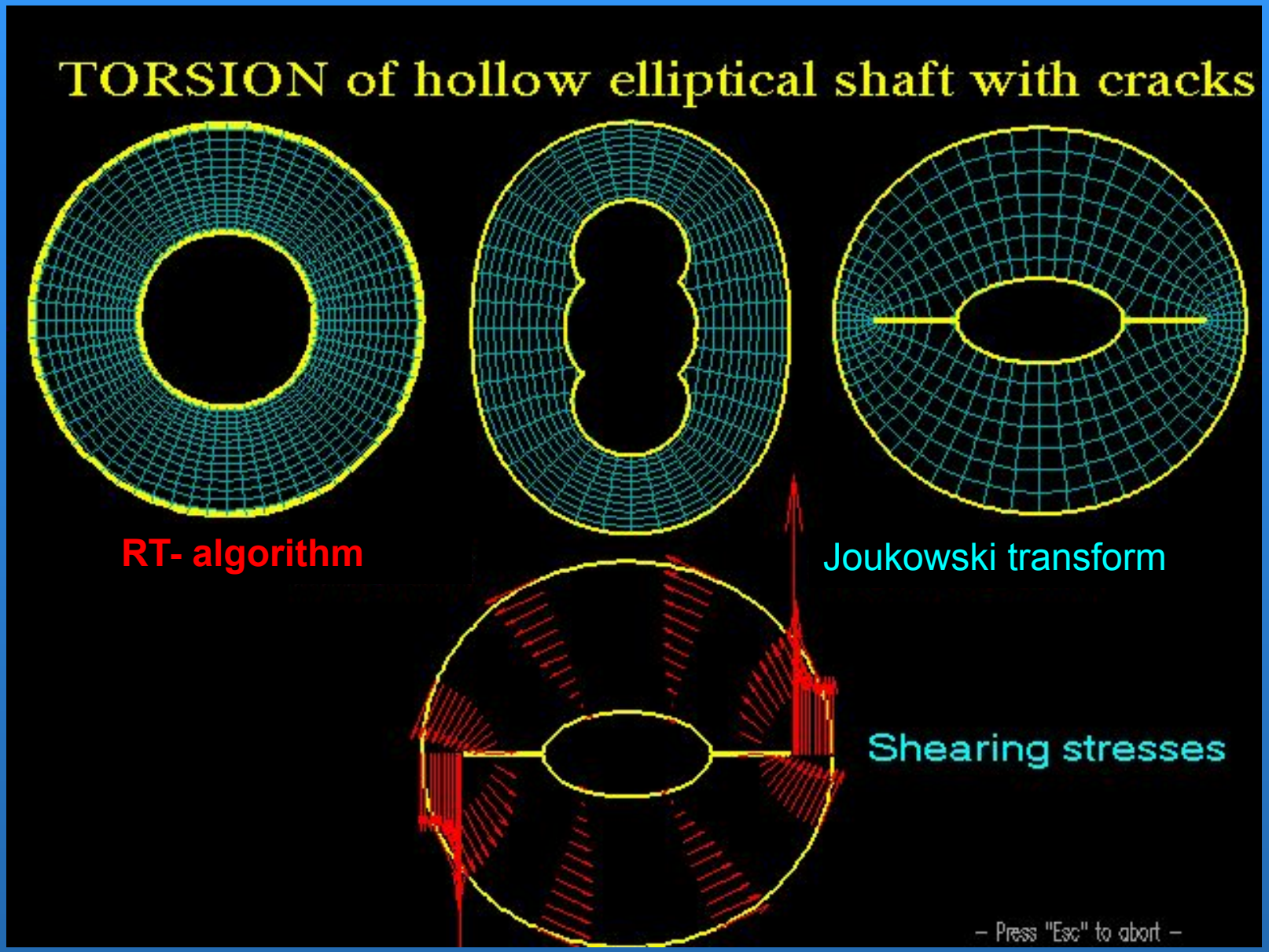
Conformal map grid

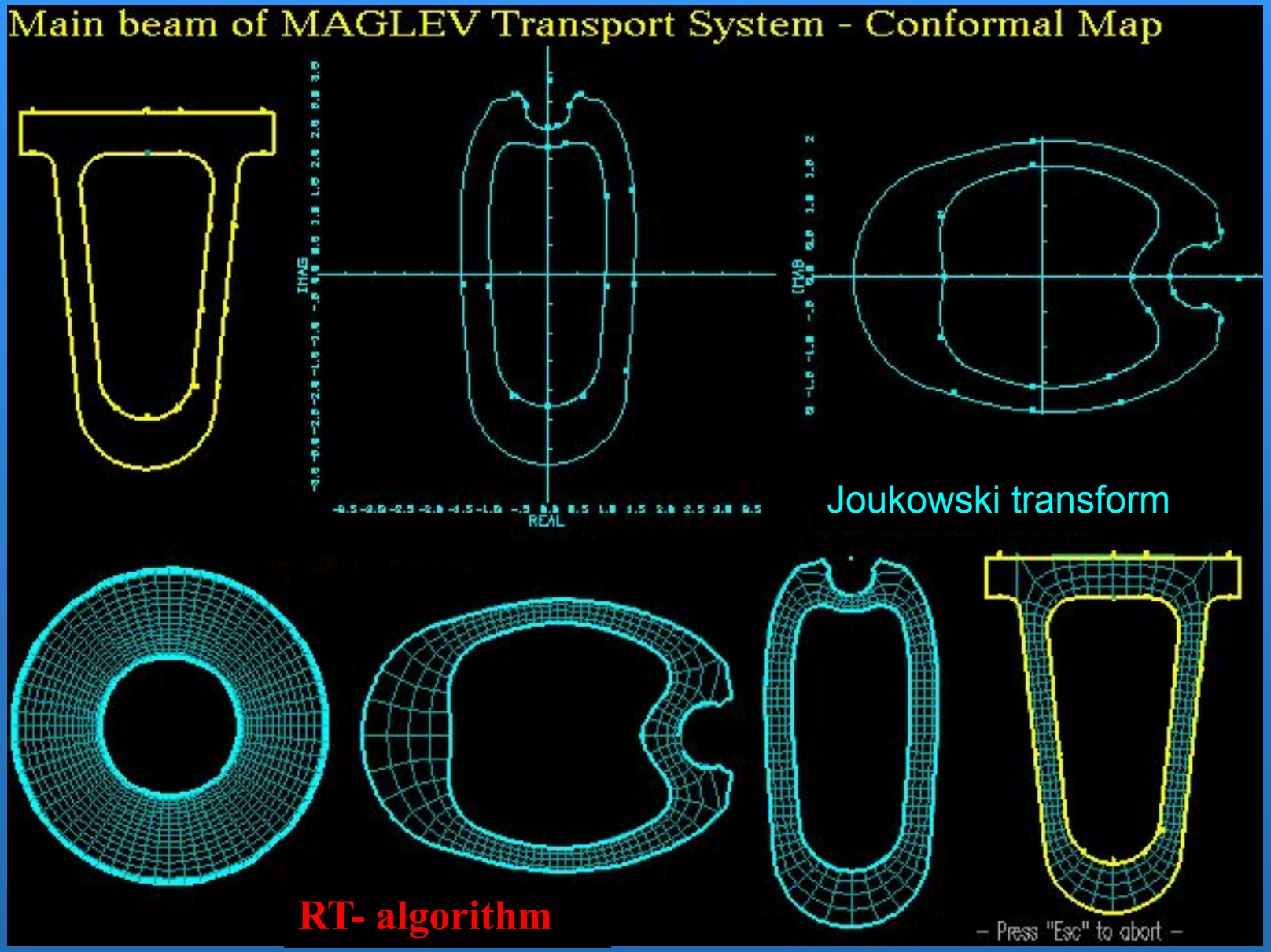


Shearing stresses

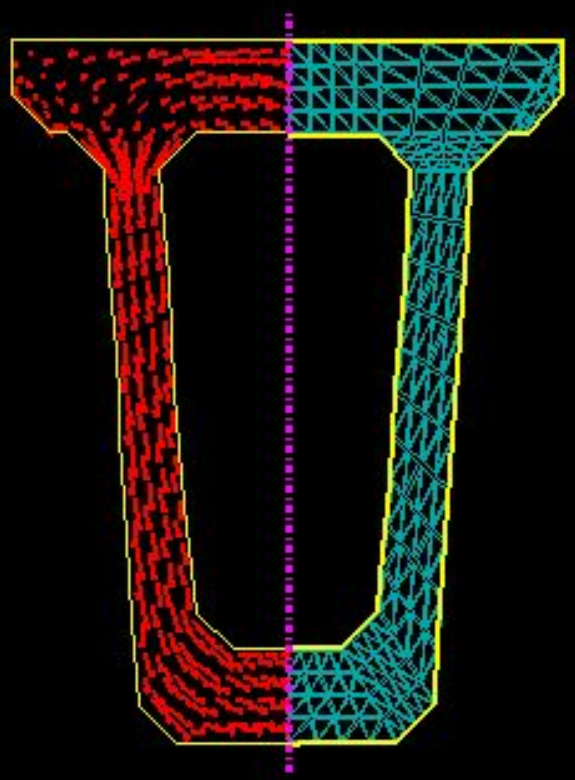
SPACE – continue ; ESC – abort


Premap example - HOLLOW CIRCULAR SHAFT with CRACKS
REAL
IMAG
REAL
Joukowski transform
RT- algorithm
SPACE — continue | ESC — abort

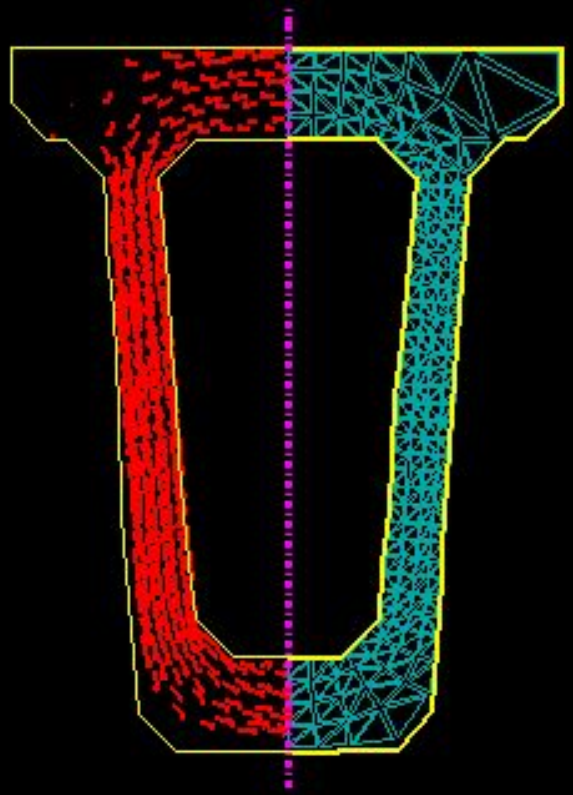

TORSION of hollow elliptical shaft with cracks
RT- algorithm
Joukowski transform
Shearing stresses
– Press "Esc" to abort –


Main beam of MAGLEV Transport System - Conformal Map
IMAG
REAL
Joukowski transform
RT- algorithm
– Press "Esc" to abort –

Torsion of MAGLEV beam - FEM Solution



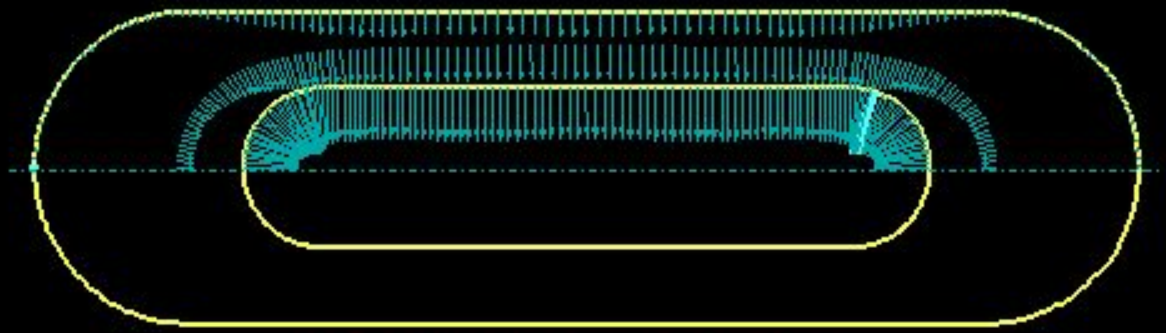
Zone FE mesh



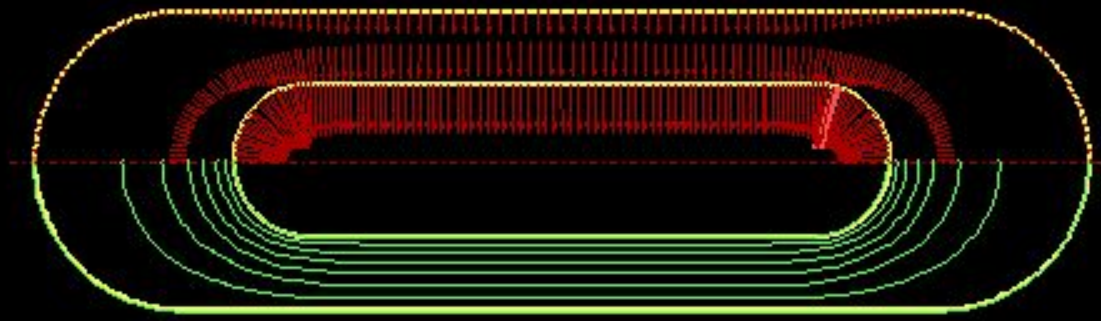
RT-FE mesh

– Press "Esc" to abort –

Axi-Symmetrical Capacitor



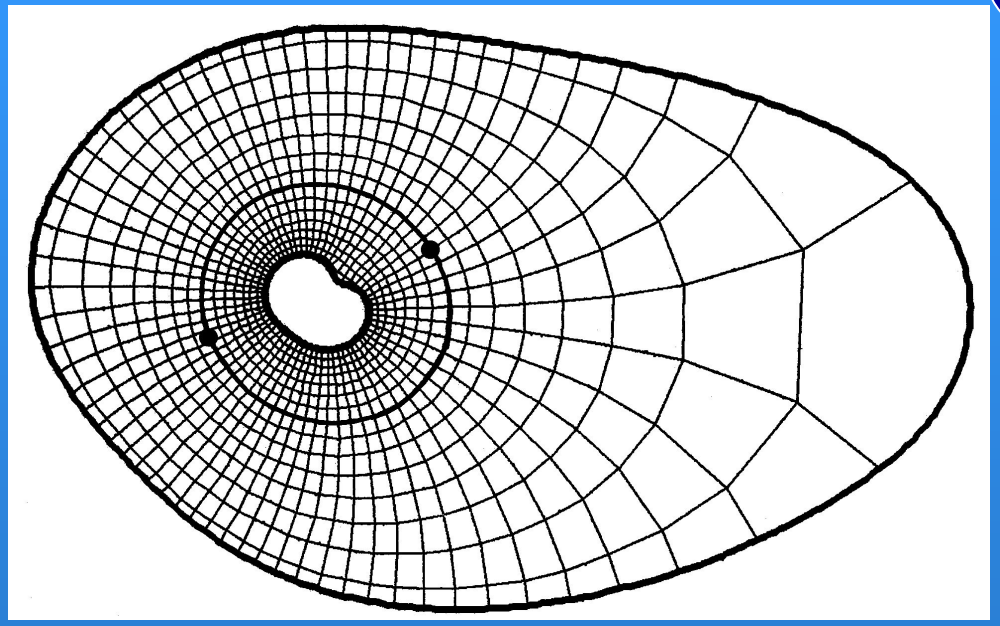
Conformal map grid

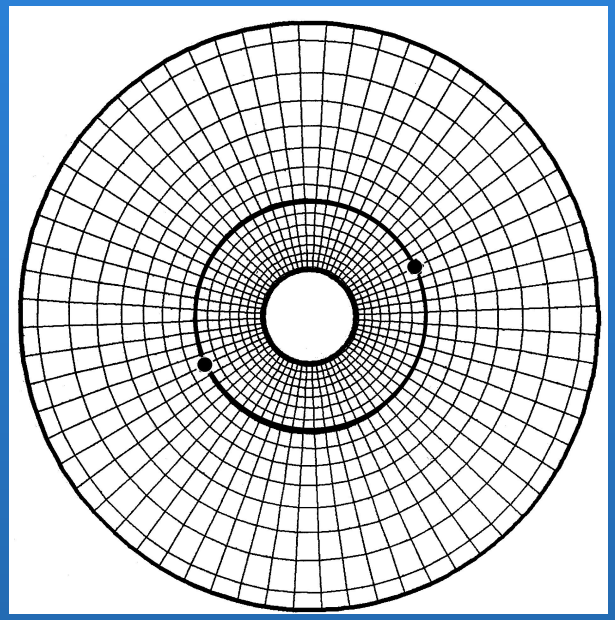


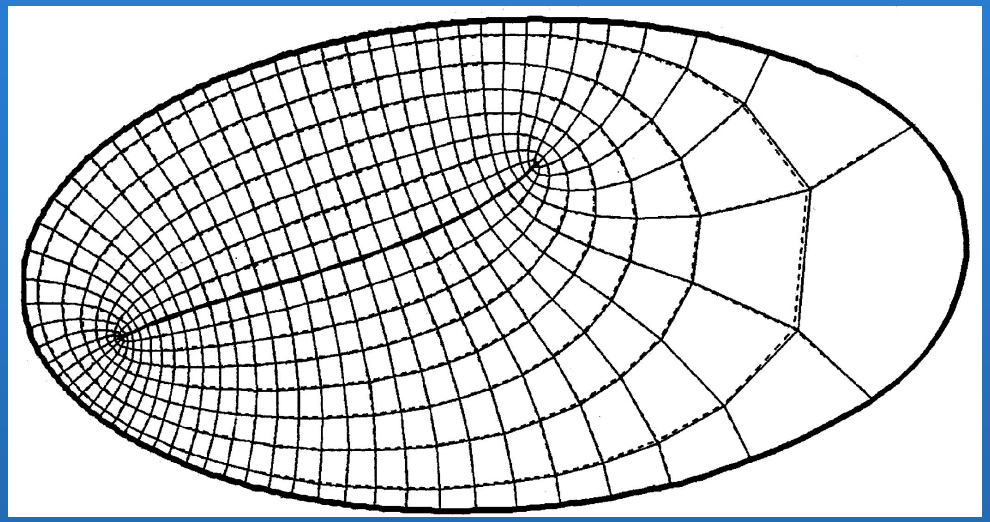
Electrostatic Field Gradient and Equipotential Lines

SPACE – continue ; ESC – abort

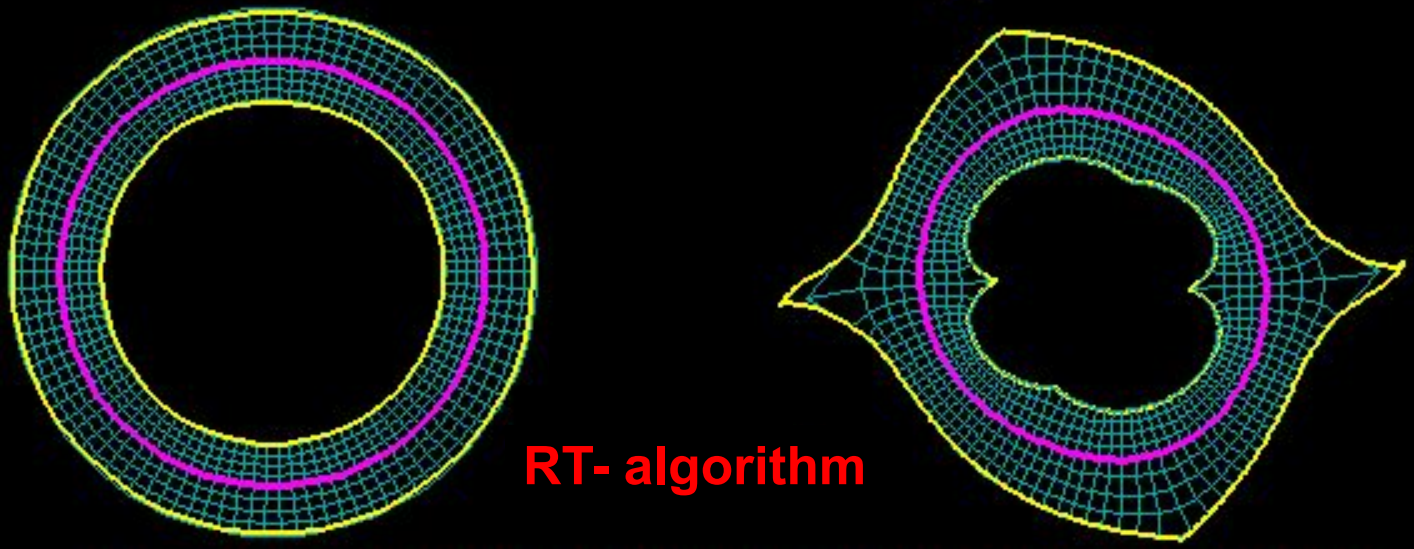
Sequential phase of the construction of conformal mapping and the grid conformally equivalent to a polar grid with a cut





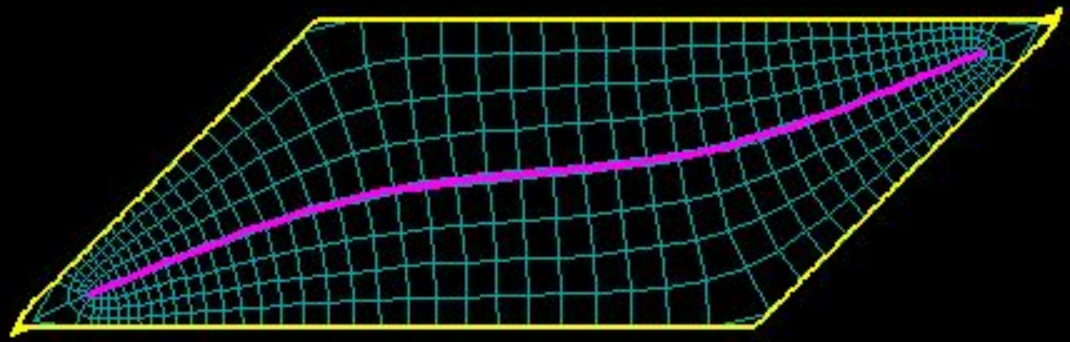


Torsion of the parallelogram bar



RT- algorithm

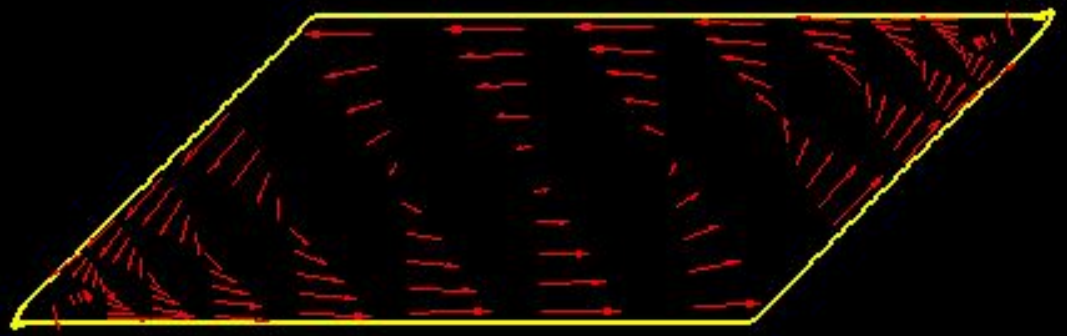
DOUBLY-connected domain is mapped onto SIMPLY-connected domain



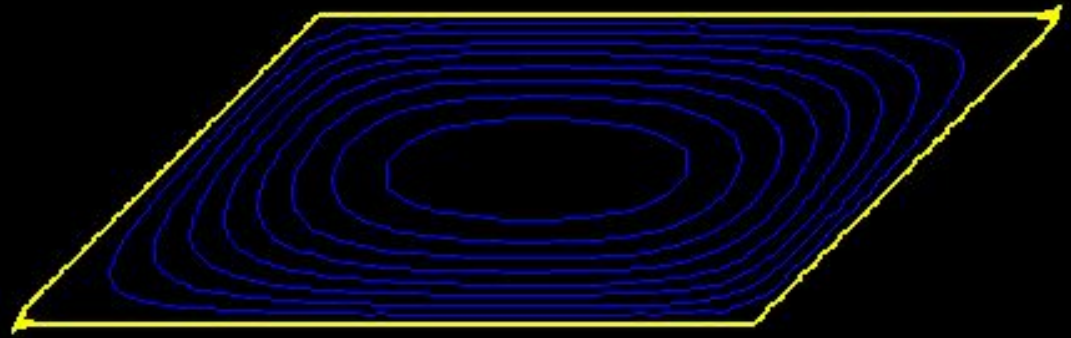
Conformal map grid with inner cut

– Press "Esc" to abort –

Torsion of the parallelogram bar



Shearing stresses field



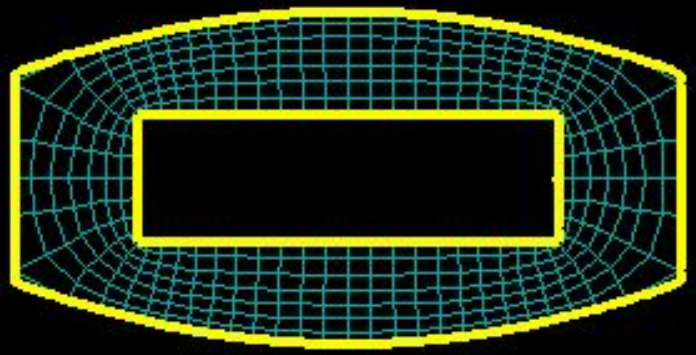
Stress function level lines

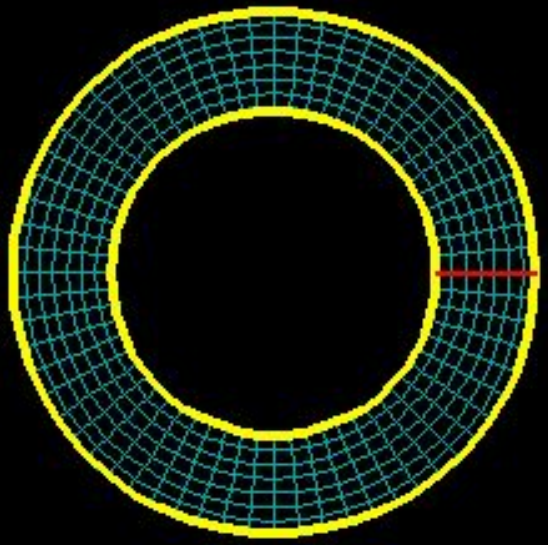
– Press "Esc" to abort –

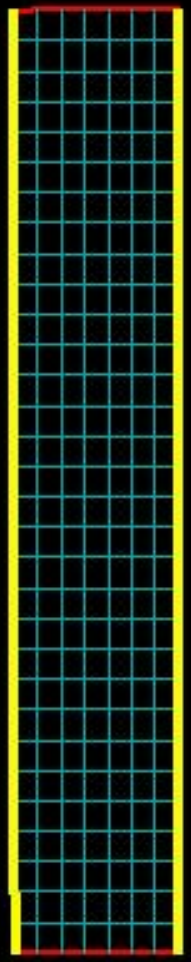
Chapter 5
Liquid Sloshing in Cavities

Boundary-value problems of fluid dynamics in cavities and equations of disturbed motion of the body-liquid system. Natural oscillations of a liquid within vertical cylindrical cavities. Model problems. Oscillations of liquid in cavities in the form of bodies of revolution. Natural oscillations of liquid in horizontal cylindrical cavities. Shallow water approximation. Numerical experiments. Certain generalisations

Conformal Mapping for Ritz Method







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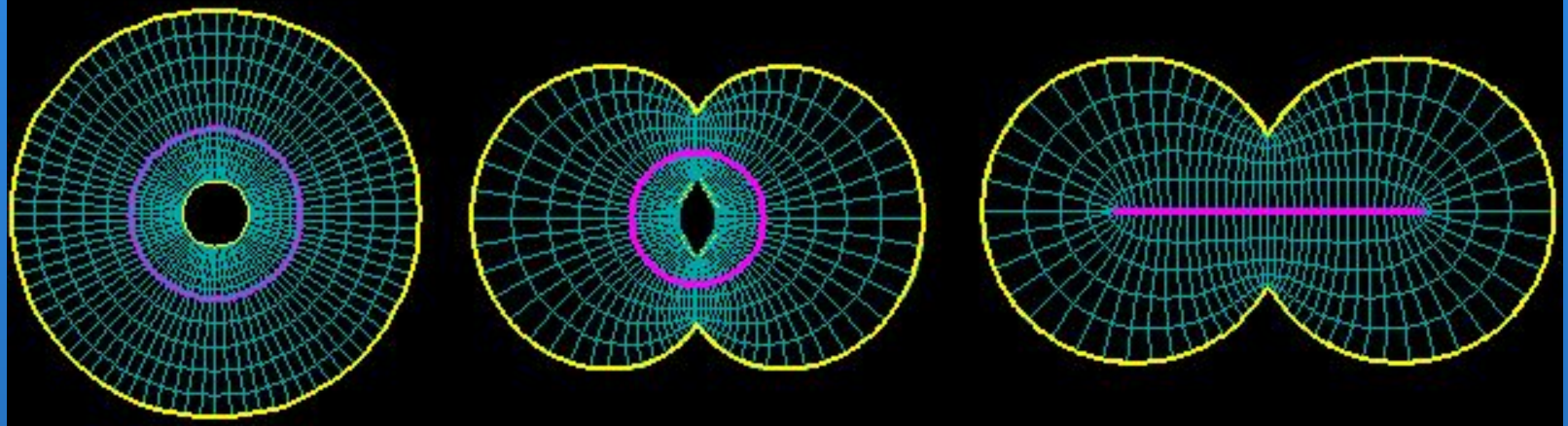
Liquid Sloshing: Sample Basin - Conformal Map

Simply connected domain with inner cut
Inverse mapping



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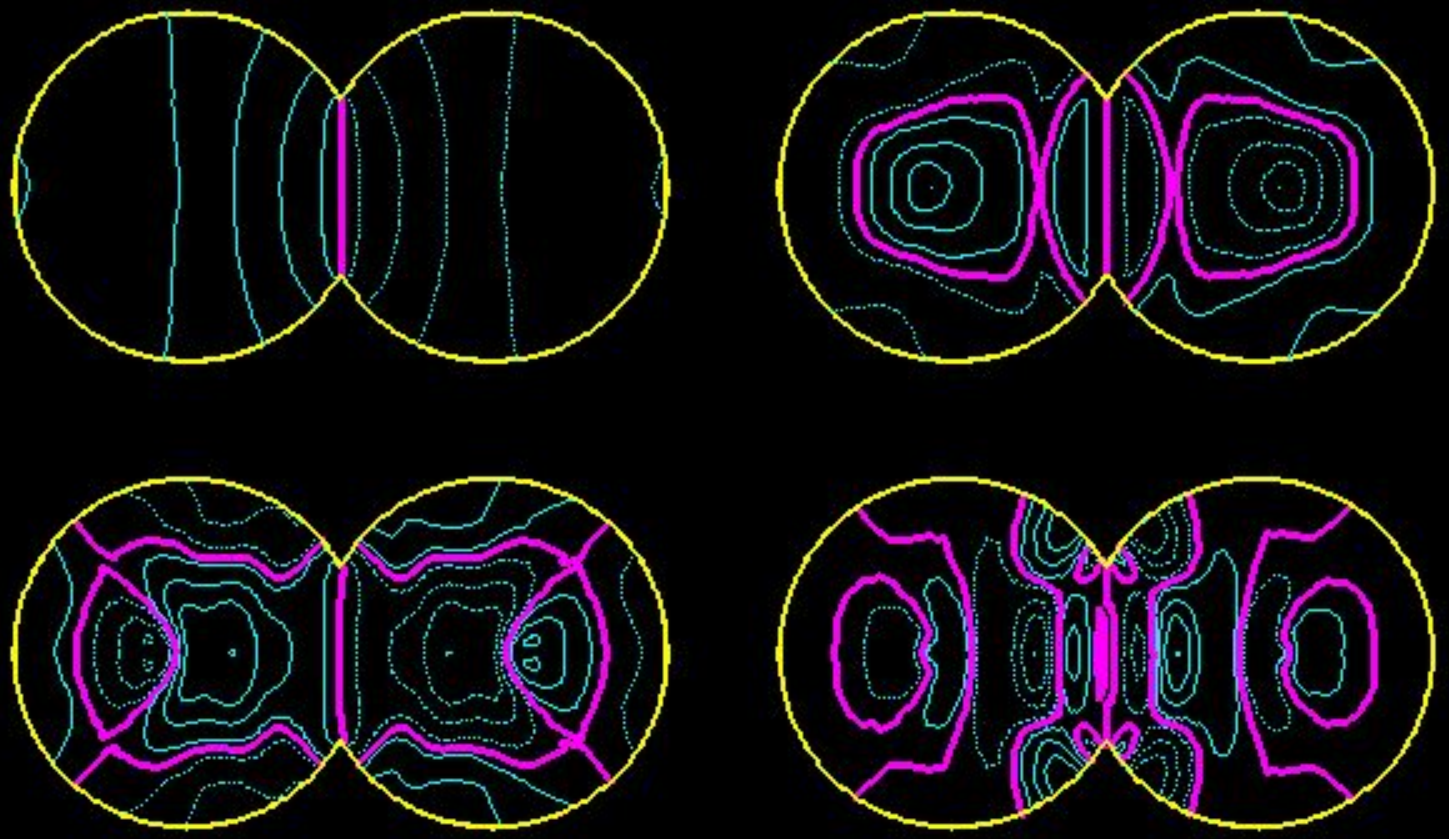
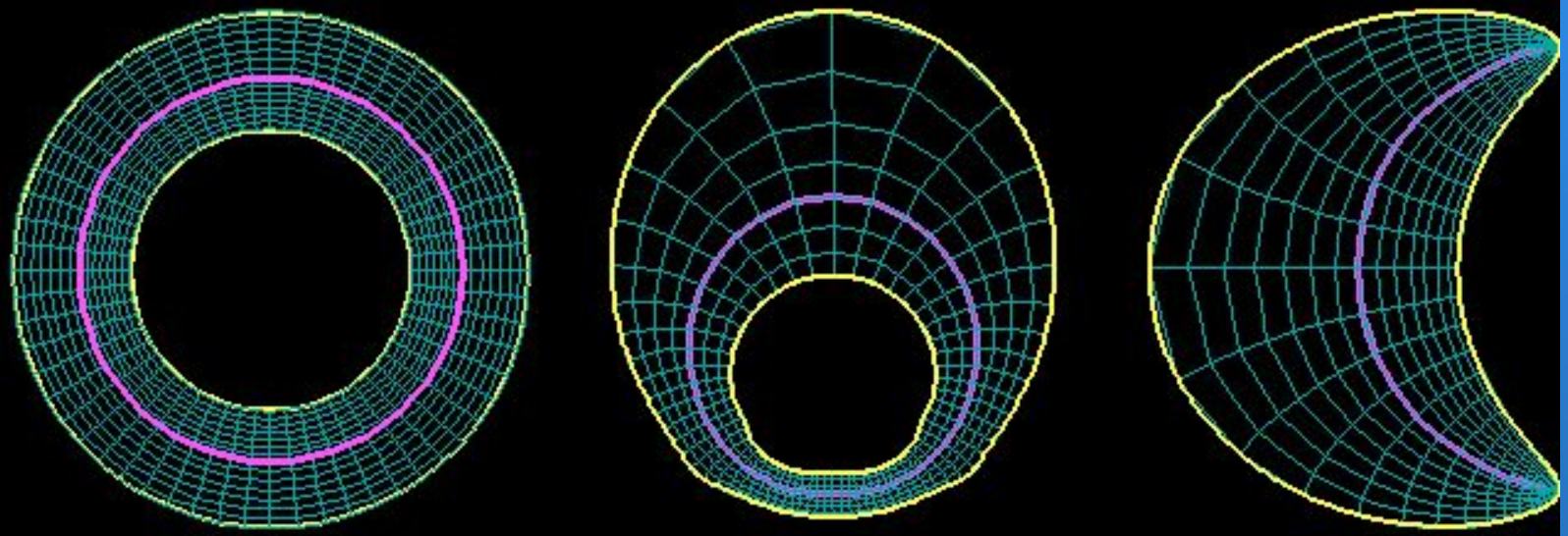
Liquid Sloshing: Sample Basin - Normal Modes



Liquid Sloshing: Sample Basin - Conformal Map

Simply connected domain with inner cut
Inverse mapping



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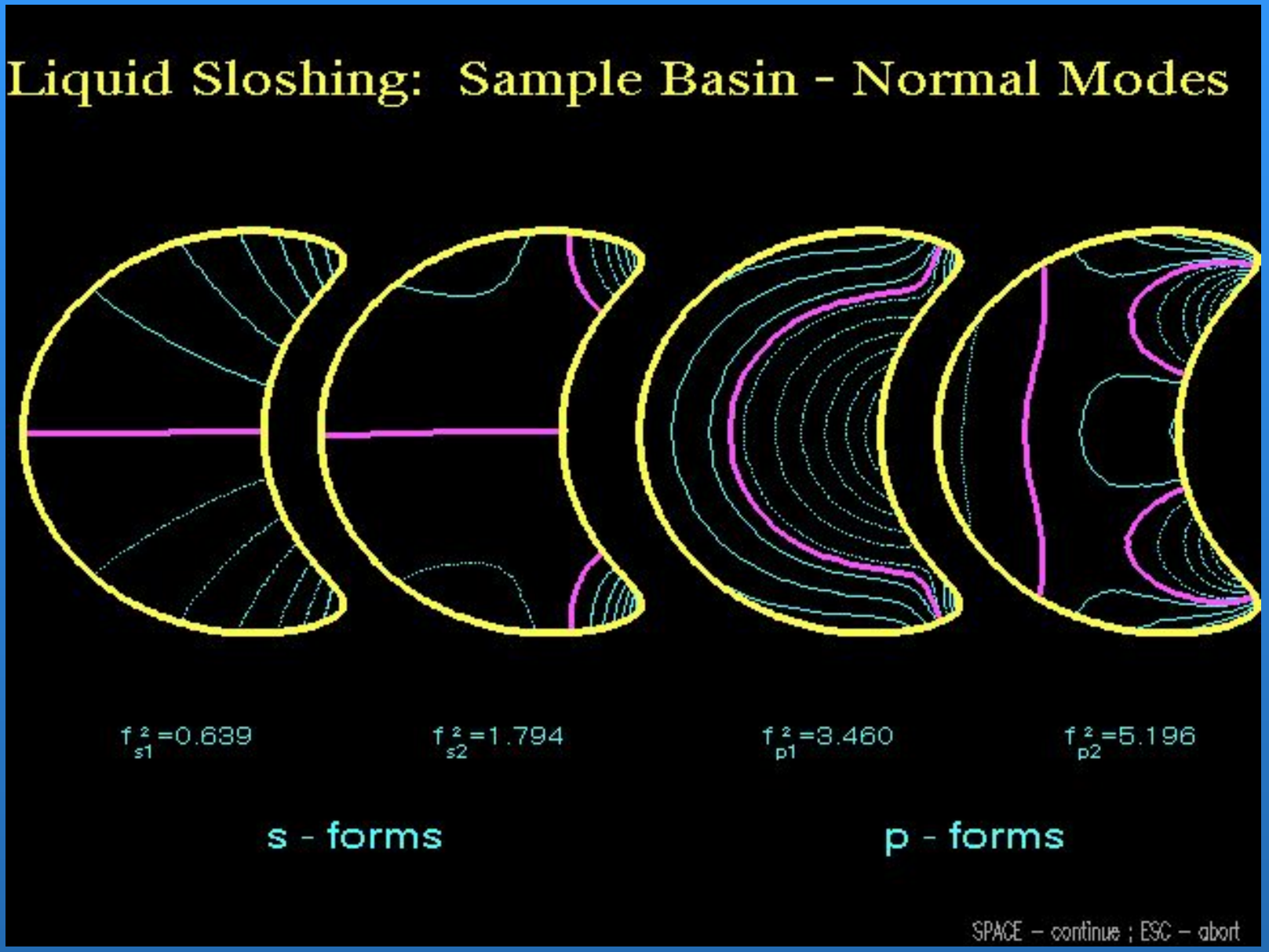
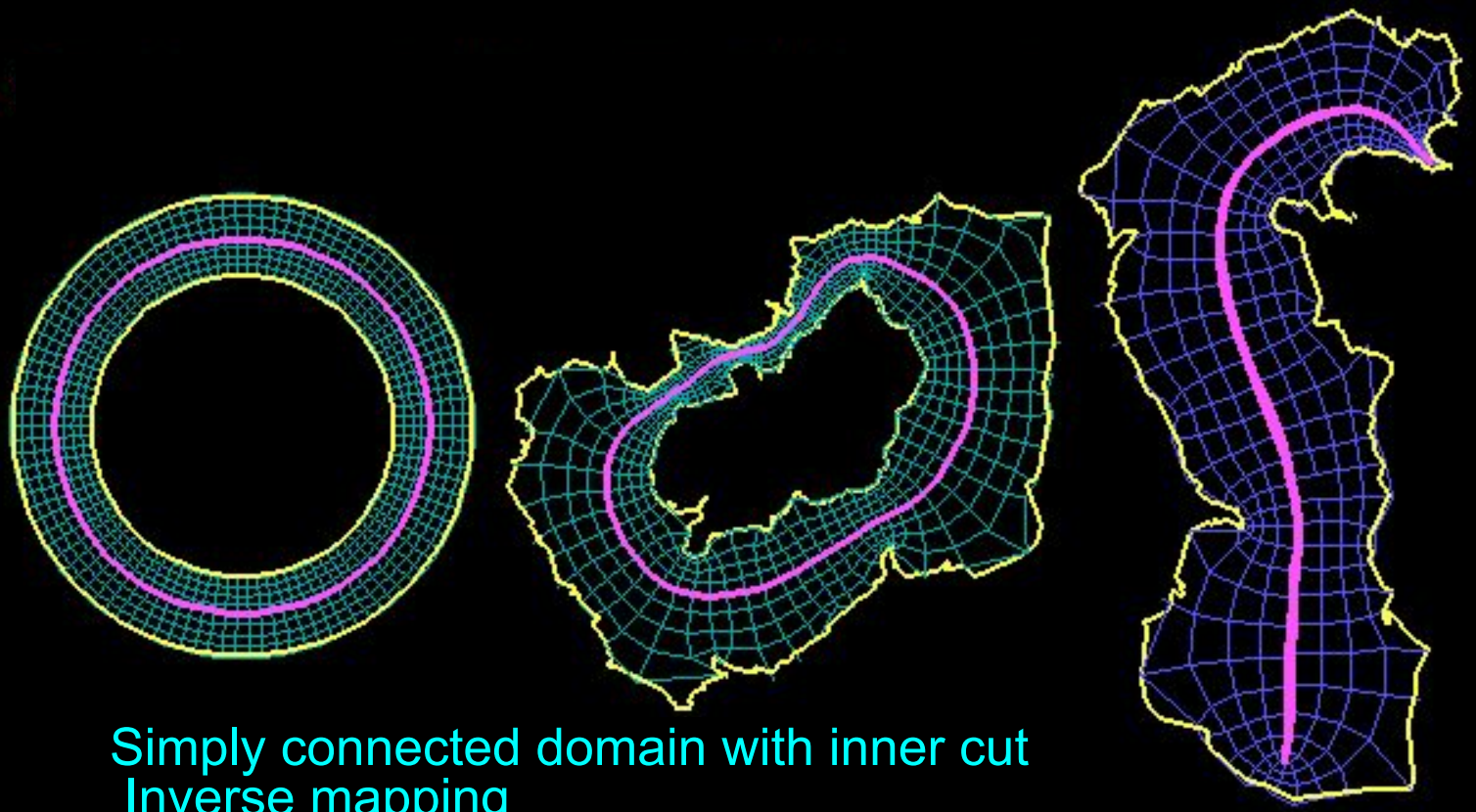
Liquid Sloshing: Sample Basin - Normal Modes



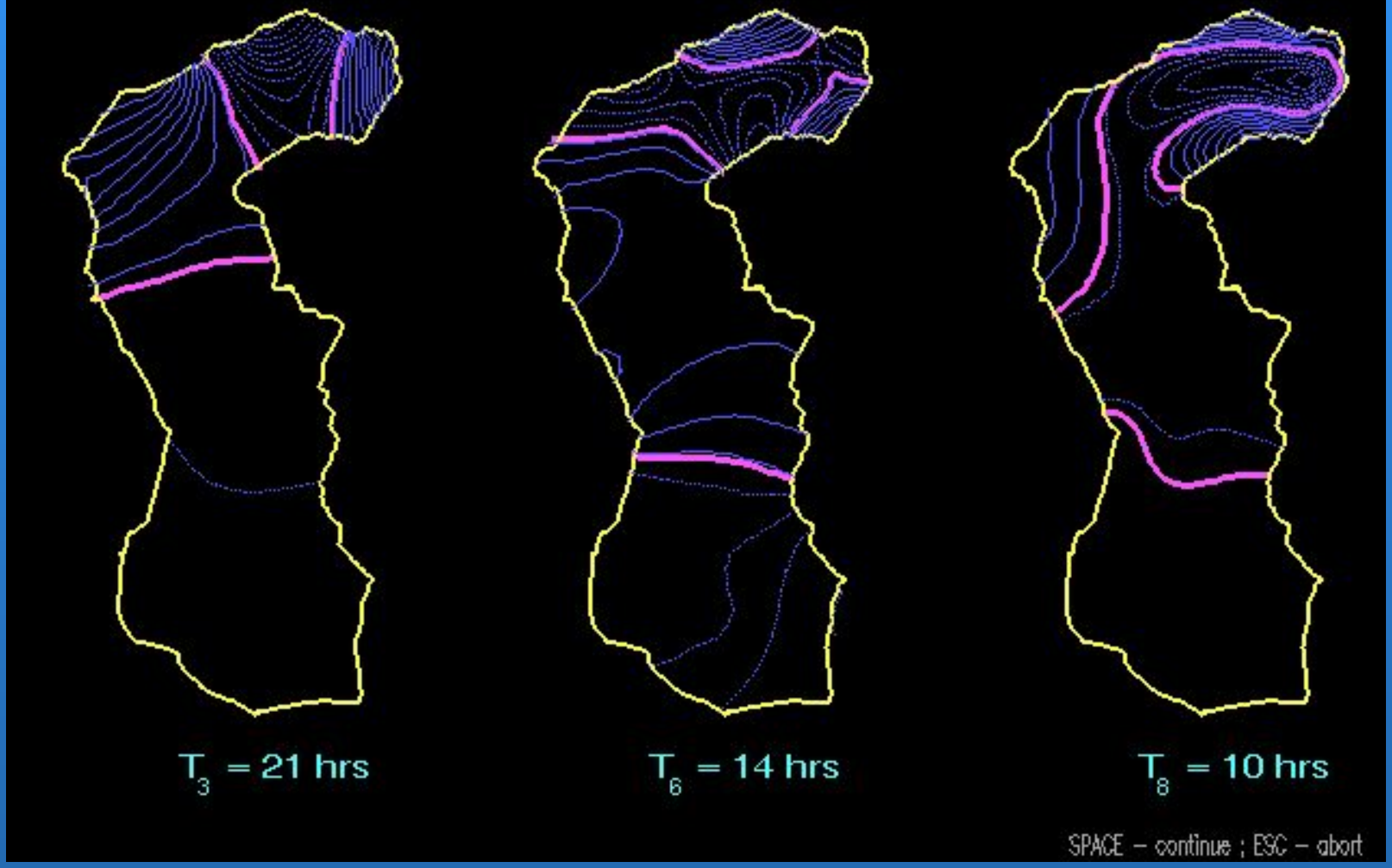
Seiches: The Caspian sea - conformal mapping



Simply connected domain with inner cut
Inverse mapping

SPACE – continue ; ESC – abort

Seiches: The Caspian sea - normal modes



Conclusions

Table 1

The methods used for solution of the boundary-value problems

Chapter №	Method combined with RT-algorithm	Chapter №	Method without RT-algorithm
3	O ◎ ⊕	3	A
4	O ◎ ⊕ ∇	4	A T F
5	□ ⊕	5	A Ω R

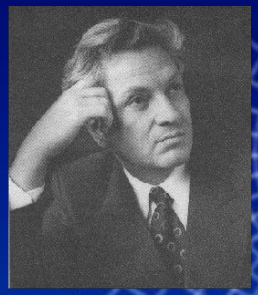
The Notation

O	Taylor series	A	Analytical
◎	Laurent series	T	Trigonometric Series
⊕	Conjugate-vortex	Ω	Frequency
∇	Finite Element	F	Finite Element
□	Ritz	R	Ritz

Table 2
Typical accuracy characteristics of numerical computations using the RT-algorithm of conformal mapping

Parameter	Smooth contour	Contour with corners
Given tolerance for the R-procedure, ε_R	$10^{-5} - 10^{-6}$	
Number of steps of the R-procedure, N_R	3 – 5	
Given tolerance for the T-procedure, ε_T	$10^{-4} - 10^{-5}$	$10^{-3} - 10^{-4}$
Number of steps of the R-procedure, N_T	10 – 30	30 – 50
Relative computational error for integral characteristics (moment of inertia, torsional rigidity), ε_I	$10^{-3} - 10^{-6}$	$10^{-2} - 10^{-3}$
Relative computational error for velocity and stress field characteristics, ε_V	10^{-2}	

NUMERICAL METHODS IN FLUID MECHANICS



This book presents a new numerical method of conformal mapping (the RT-algorithm) based on the R and T procedures. Solutions to problems for this method are given for arbitrary simply connected and doubly connected domains with piecewise-smooth contours. The RT-algorithm is used to solve two-dimensional problems of different physical disciplines, including hydrodynamics, electrodynamics, magnetodynamics, and elasticity theory.

The book encompasses a wide range of numerical methods and computer techniques, including computer graphics. Problems analyzed are at the forefront of applied problems in modern technology.

Solutions to boundary-value problems on the unit circle and circular annulus are obtained by systematic use of Tailor and Laurent series, the Ritz method, Finite Elements, and the Conjugate-Vortex method. Analytical solutions and experimental data are used to verify results of the numerical computations obtained by the RT-algorithm.

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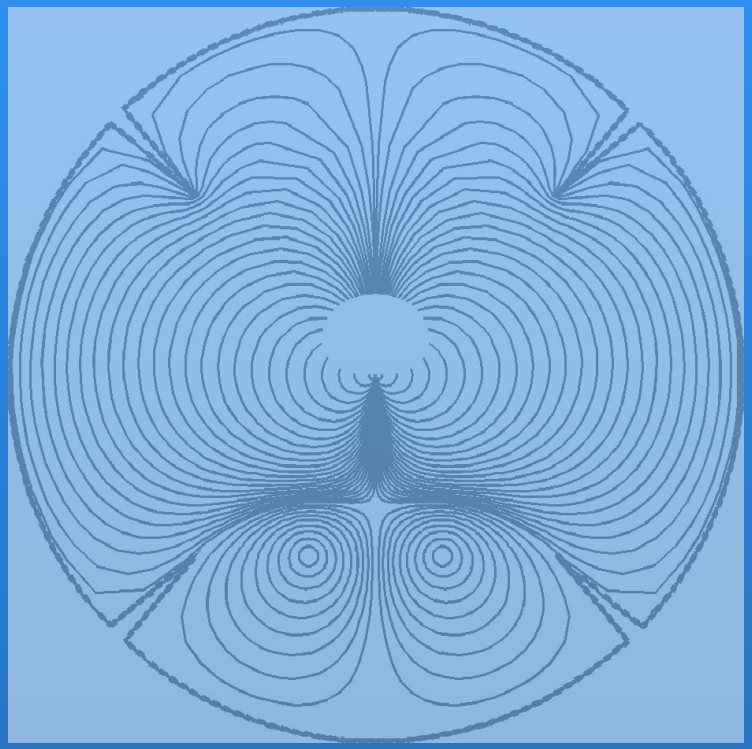

Web site: http://iki.cosmos.ru/people/br_cw.htm