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Гравитационные возмущения и их роль в формировании лунных траекторий нового типа

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АННОТАЦИЯ.

В рамках задачи четырех тел (Земля-Луна-Солнце-точка) представлены результаты исследования лунных траекторий Земля-Луна и Луна-Земля нового, «обходного» типа.

Данные траектории имеют отлет от Земли на большое расстояние (около $1{,}5 \cdot 10^6$ км), где под влиянием Солнечных гравитационных возмущений пассивно меняется перигейное расстояние траектории точки от малого значения у Земли до ~ радиуса Лунной орбиты.

Это позволяет с помощью гравитационных Земных возмущений осуществить в районе залунной точки либрации L_2 пассивное изменение энергии селеноцентрического движения точки от положительной до нулевой, а затем – до отрицательной, что соответствует движению точки у Луны по орбите спутника Луны, т.е. захвату для полета Земля-Луна и освобождению для полета Луна-Земля.

CONTENTS

I. INTRODUCTION. *Trajectories of direct space flight - a*

В своем творчестве Г.Н. Дубошин значительное внимание уделял проблеме вычисления и анализа влияния гравитационных возмущений, в частности, для траекторий полета в системе Земля-Луна. В последнее время были открыты новые классы лунных траекторий, в которых такие возмущения играют особенно большую роль. Кратко опишем их в данном докладе. Исследование космических полетов между Землей и Луной имеют большое значение как для Небесной механики, так и для Космонавтики. Для практически всех полетов, начиная с 1959 г., использовались «прямые» траектории [V.A. Egorov, 1957; V.A. Egorov and L.I. Gusev, 1980; etc.].

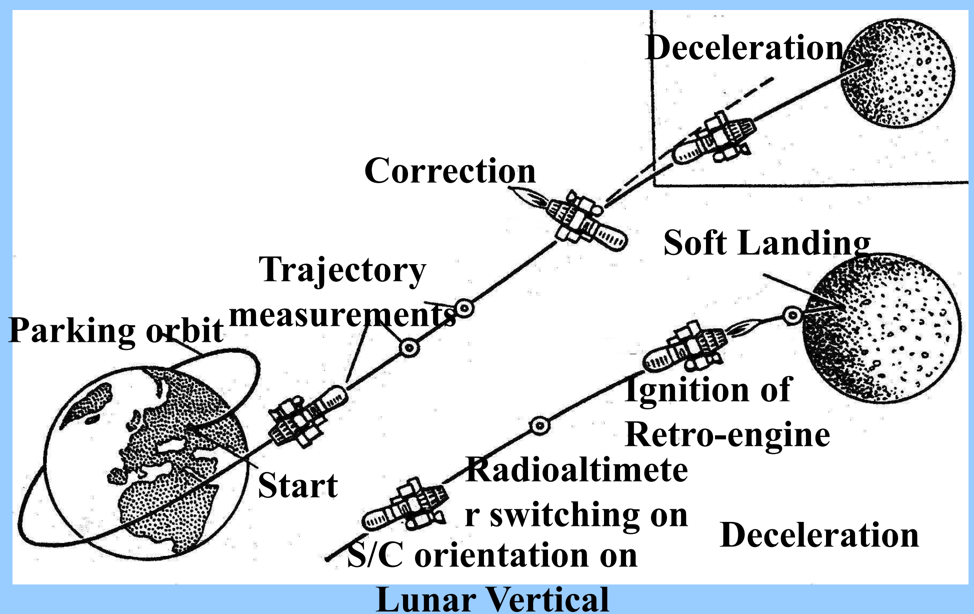


Figure 1. Scheme of the Luna 9 Mission

Схема полета КА Луна-9 для первой мягкой посадки на Луну, а также схема полета КА «Аполлон», первой пилотируемой экспедиции на Луну, приведены здесь для примера.

I. INTRODUCTION. Trajectories of direct space flight - b

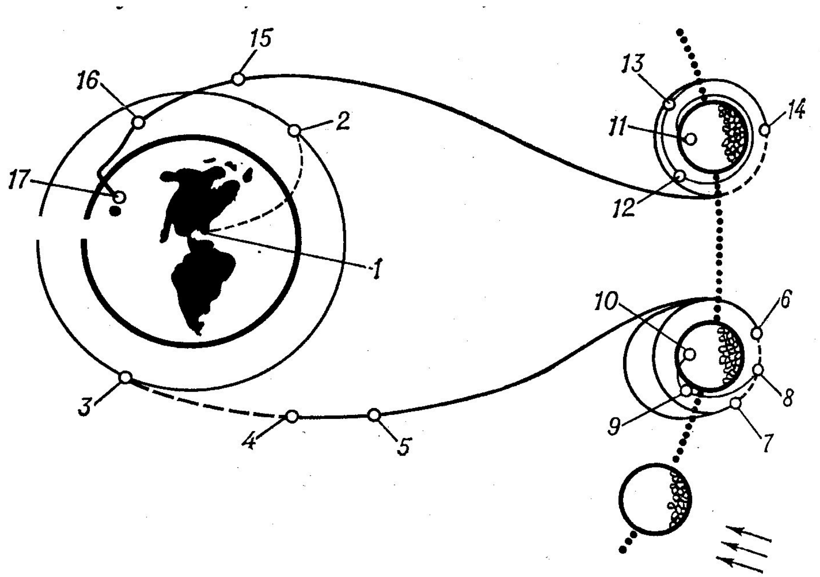


Figure 2. Scheme of the Apollo Mission

For direct flights, trajectories have small enough (several days) time of flight, approach to and departure from the Moon are performed on hyperbolic selenocentric orbits (with velocity at "infinity" $V_\infty \approx 1$ km/s). This results in the large fuel consumption for spacecraft flights under using these trajectories.
It is important to search new low energy lunar flights: a) other schemes; b) Earth-Moon flights with passive capture and Moon-Earth flights with passive escape; c) other types of engines.

***In a central field*, for flight with a high thrust (impulses), there are two main transfers here: two-impulse Hohmann-Tsander Transfer (Figure 3) and Three-Impulse Bi-Elliptical Sternfeld Transfer, Figure 4.**

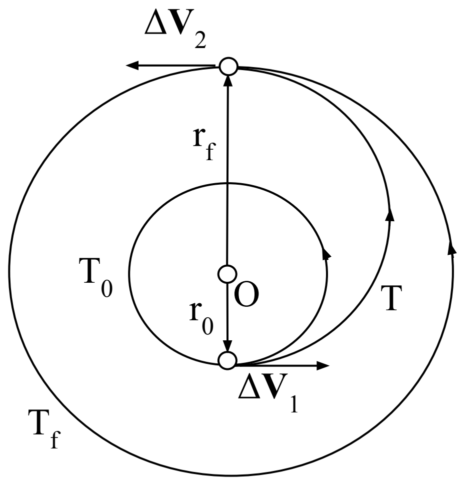


Figure 3. Two-Impulse Hohmann-Tsander Transfer

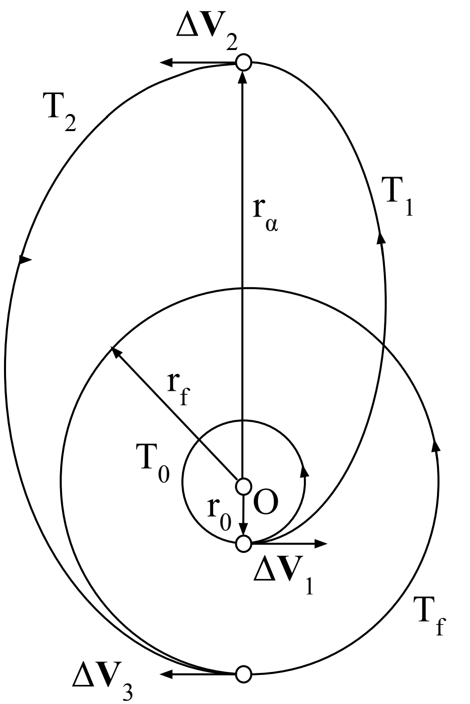


Figure 4. Three-Impulse Bi-Elliptical Sternfeld Transfer

The first scheme leads to the direct lunar flights, the second one produces *Bi-Elliptical* lunar flights.

If maximum distance r_α from the Earth is large enough, this last scheme is better than the direct flight from energy point of view.

But Sun's perturbations have to be considered here.

"Detour" Earth-to-Moon flights

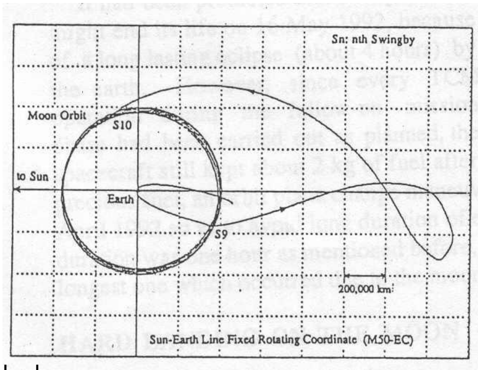


Figure 5. Hiten flight

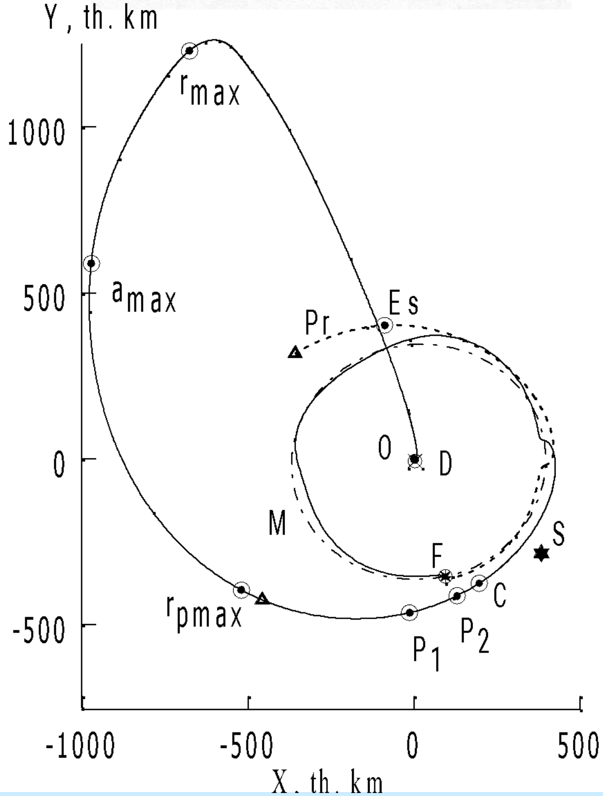


Figure 6. Geocentric Earth-to-Moon trajectory and its passive prolongation (P_1: V_∞ = 0.4 km/s; P_2: V_∞=0.2 km/s; C, Es: V_∞ = 0, E=0)

New indirect "*detour*" Earth-to-Moon flights in frame of the Earth-Moon-Sun-particle system are found recently [Belbruno and Miller 1993; Hiroshi Yamakawa et al 1993; Biesbroek R. and Janin G. (2000); Belló Mora et al 2000; Koon et al 2001; Ivashkin 2002; etc].

They seem to be similar to Bi-Elliptical flights, but from dynamical point of view they differ from the last ones: **ascent of perigee is given by the Sun gravity but not by an impulse and approach the Moon is along the elliptical orbit (with capture) due to the Earth gravity effect**.

This scheme may be also used for the Moon-to-Earth flight to have a gravitational escape from the Moon attraction [Hiroshi Yamakawa, et al.; V.V. Ivashkin]. ***Numerical and theoretical analysis has proved existence of these Moon-Earth "detour" trajectories***.

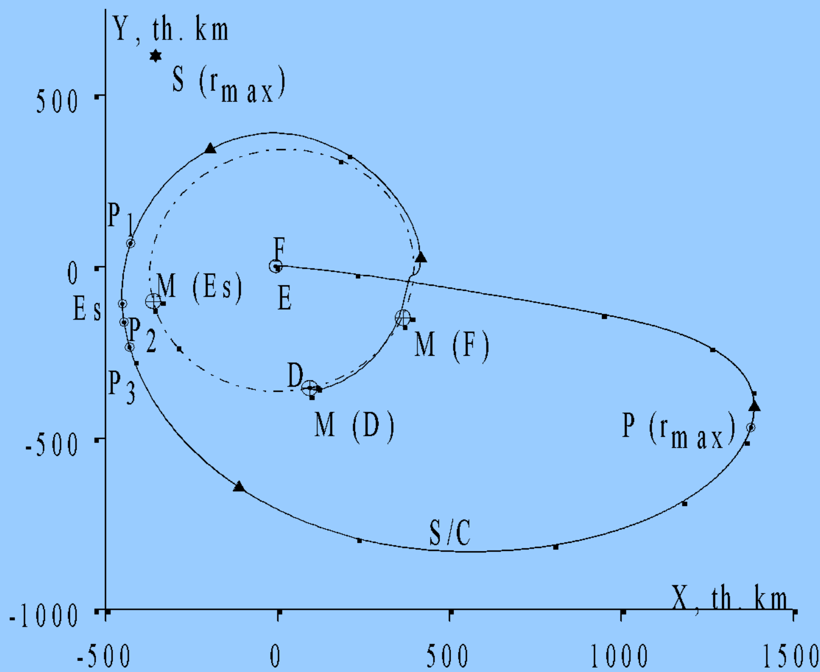


Figure 7. The XY view of the geocentric trajectory for detour type: D-departure (11.05.2001), Es – escape (V_∞=0), r_{max}≈1.47·10^6 km, F-final point (H_π =50 km, Δt ≈ 113 days), M - Moon, E – Earth

Scheme of Detour Moon-Earth flight

These Moon-to-Earth flights in frame of the Earth-Moon-Sun-particle system use first flight from to the Moon orbit and Earth behind the Earth gravity influence sphere and then flight to the Earth. We shall call them by “detour” flights. From dynamical point of view they differ from the Sternfeld bi-elliptical flights: ***flight from the Moon is performed along an elliptical orbit due to the Earth effect* and *descending the perigee is performed by the Sun gravity but not by the impulse*.**

Algorithm of calculations

The trajectories are defined by integration [Stepan’yants et al] of the particle motion equations in Cartesian nonrotating geocentric-equatorial coordinate system OXYZ. There are taken into account the Earth gravity with its main harmonic c_{20}, the Moon gravity, and the Sun one.

Some Numerical Results. A family of detour trajectories for space flight to the Earth from elliptic orbits of the lunar satellite are found. These trajectories correspond to the spacecraft start from both the Moon surface and the low-Moon elliptic orbit for several positions of the Moon on its orbit.

Figure 7 gives a typical “detour” trajectory.

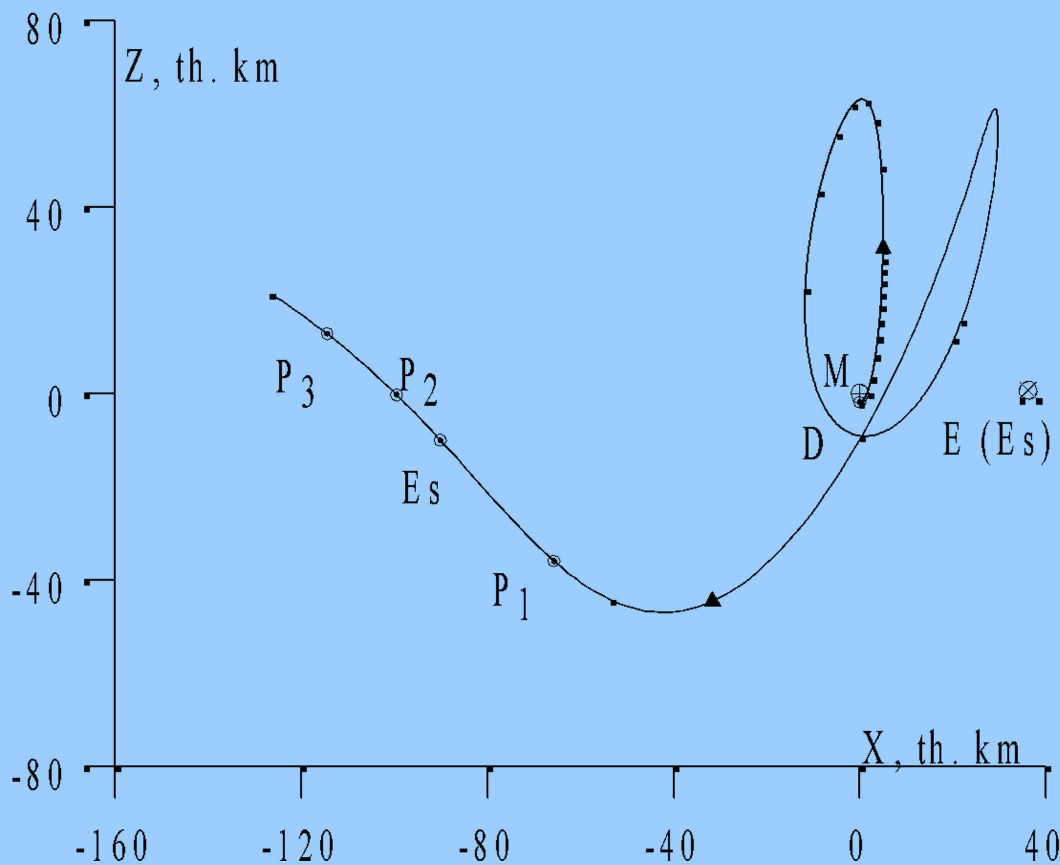


Figure 8. The XZ view for the Moon-to-Earth seleno-centric trajectory of detour type at initial part of the flight

Figure 8 gives the particle **selenocentric** motion for initial part of the trajectory**.**

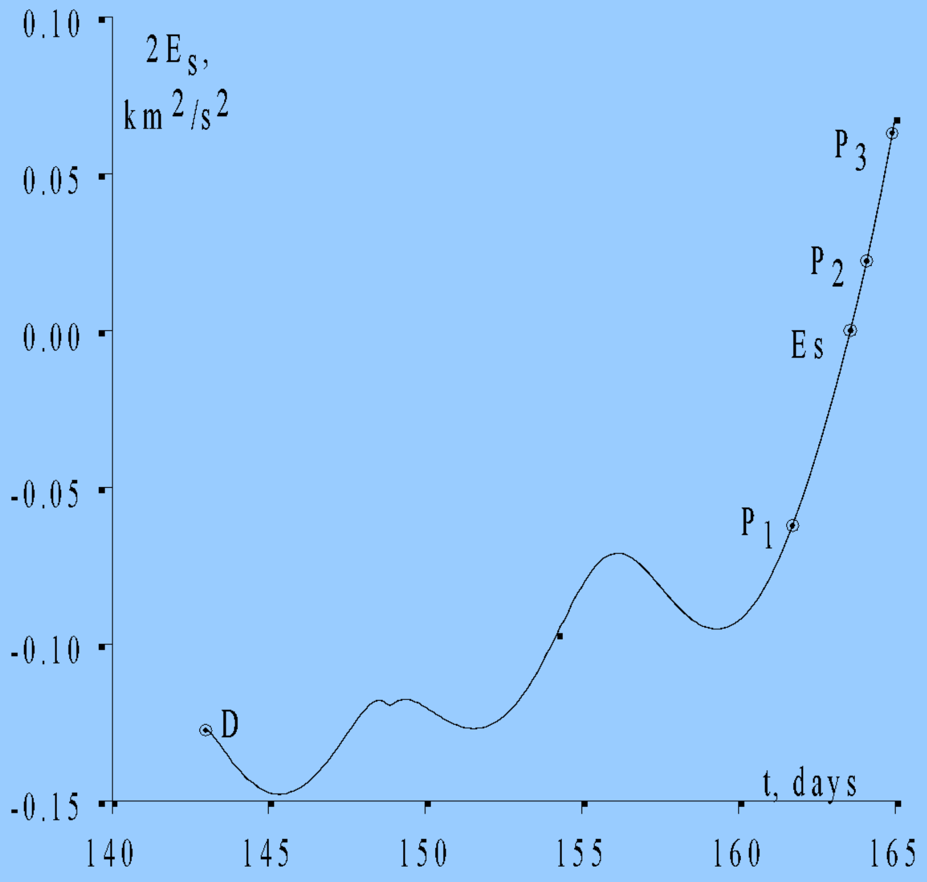
At the point **D**, on May 11, 2001, for the position of the Moon near its orbit apogee, the spacecraft flies away from the perilune of an **initial *elliptic*** orbit with the perilune altitude $H_{\pi 0}$ = 100 km, initial selenocentric semimajor axis a_0 = 38 455 km, and apolune distance r_α ~75 $\cdot 10^3$ km.

Arc **D P$_1$ Es** gives ***elliptic*** motion**.** At the point P_1 in the flight time $\Delta t \approx 19$ days**,** $a_S \approx 79 \cdot 10^3$ km, and distance $\rho \approx 76 \cdot 10^3$ km.

Es is the ***escape*** point. Here, in $\Delta t \approx 20{,}6$ days, there is zero selenocentric energy, $E_S = 0$, $\rho \approx 92 \cdot 10^3$ km, E (Es) gives direction to the Earth. So, there is the escape near translunar libration point **L$_2$**,

Arc **Es P$_2$ P$_3$** gives ***hyperbolic*** motion. At the point P_2, for $\Delta t \approx 21.1$ days: $\rho \approx 101 \cdot 10^3$ km, $V_\infty = 0.15$ km s^{-1}. At the point P_3, for $\Delta t \approx 21.9$ days: $\rho \approx 120.2 \cdot 10^3$ km, $V_\infty = 0.25$ km s^{-1}. Then, the spacecraft flies away from both the lunar orbit and the Earth.

2. MOON-EARTH “DETOUR” FLIGHT IN THE EARTH-MOON-SUN SYSTEM-c



Figue 9. Selenocentric energy versus the time for initial part of the Moon-to-Earth detour flight

Figure 9 gives the selenocentric energy constant

$$h=2E_S=V^2-2\mu_M/\rho$$

versus the time for the initial part of the motion. Here V and ρ are the selenocentric velocity of the particle and its distance from the Moon.

For leaving a 100 km-circular lunar-satellite orbit with a high thrust, the velocity increment is $\Delta V_0 \approx 649$ m/s, that is at about 161 m/s less than for the optimal case of usual direct flight.

For a case when spacecraft leaves Moon's surface, the “detour” trajectory (with $a_0 = 38455$ km again) has approximately the same characteristics as for the indicated case of the start from the lunar satellite orbit. The decrease in the velocity increment is equal to about 156 m/s in this case.

If initial semimajor axis a_0 is less, the decreasing in energy will be more, as it is shown at Figure 10.

Decreasing of the velocity impulse

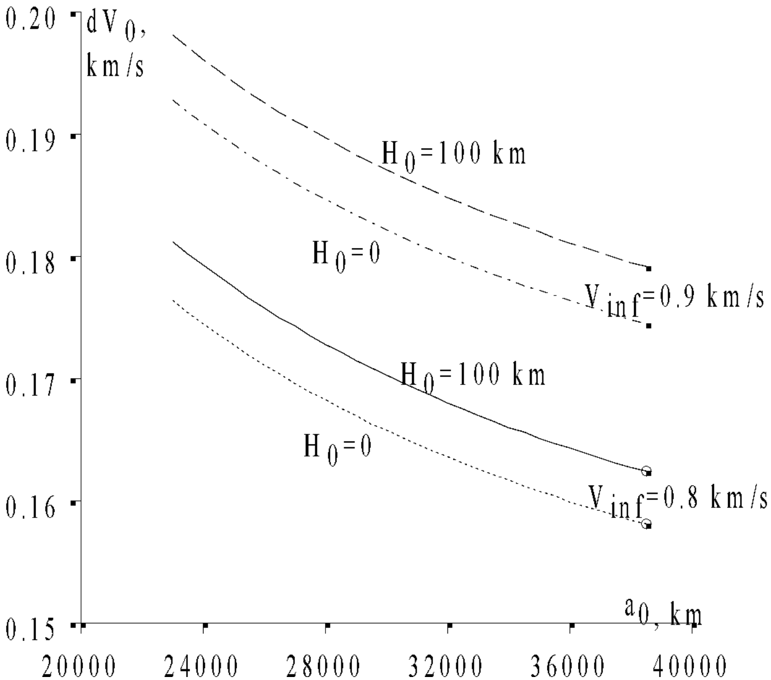


Figure 10. Decreasing of the velocity impulse for the Moon-Earth detour flight relative to the direct flight depending on the initial semimajor axis

Lines H_0=100 km correspond to the spacecraft start from the satellite orbit perilune with altitude H_0=100 km. Lines H_0=0 correspond to the spacecraft start from the Moon surface.

Value V_{inf} is velocity at “infinity” V_∞ for direct flight: approximately, V_∞=0.8 km/s corresponds to optimal direct flight from the Moon apogee and V_∞=0.9 - to optimal direct flight from the Moon perigee.

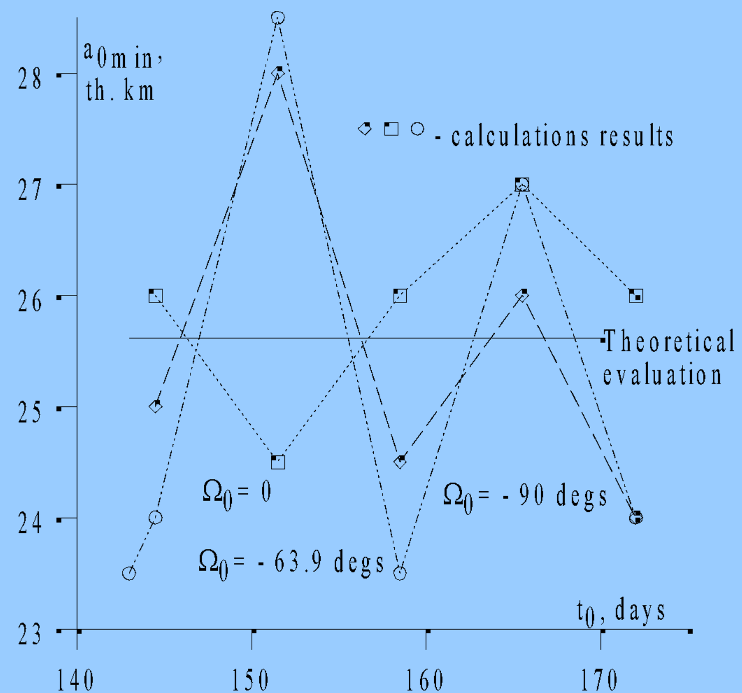
3. THEORETICAL ANALYSIS OF "DETOUR"FLIGHT - a

3.1. EARTH GRAVITY EFFECT ON PARTICLE'S ESCAPE

First, we shall evaluate possibility to have energy increasing $\Delta E_S = -E_0$ for the particle selenocentric motion from initial energy $E_0 < 0$ to zero using the evolution theory (M.L. Lidov 1961, 1962). Suppose eccentricity e_S is ~ 1, middle energy E_s is - $\Delta E_s/2$. Then

$$\Delta E_S \approx \text{sign}\,\beta\,((15/2)\,\pi\,\mu_E\,(\mu_M / a_M)^3\, n_M\,|\beta|)^{2/9} > 0. \tag{3.1}$$

Here n_M is angular velocity of the Moon orbital motion, a_M is semi-major axis of Moon's orbit, $\beta = \cos^2\gamma \sin 2\alpha > 0$, γ, α are angles of the Moon-Earth vector orientation relative to the particle orbit plane, $|\beta| \le 1$. Let β be 0.5. Then $\Delta E_S \approx 0.096$ km^2/s^2, $a_0 \approx 25{,}600$ km. This estimates minimal value of semimajor axis a_0 for initial elliptic selenocentric orbit in the Moon-to-Earth detour trajectory.



This fits numerical data (see Fig.11, where time t is counted off from the Julian date 2451898.5, that is 20.12.2000.0).

Hence, ***the Earth gravity allows increasing the particle energy from initial negative value for elliptical orbit to zero and escape from the Moon attraction.***

Figure 11. Minimal value of initial semimajor axis depending on the time of start from near-Moon elliptic selenocentric orbit for the Moon-to-Earth detour trajectories

3. THEORETICAL ANALYSIS OF "DETOUR"FLIGHT - b
3.2. EARTH GRAVITY EFFECT ON PARTICLE'S ACCELERATION TO HYPERBOLIC MOTION

Now we approximately analyze the acceleration of the particle motion with respect to the Moon from the zero energy to a positive one for a hyperbolic motion with velocity at "infinity" $V_\infty \approx 0.15 - 0.25$ km/s on the following short arc Es $P_2 P_3$. We use here an approximate linear model, see Figure 12.

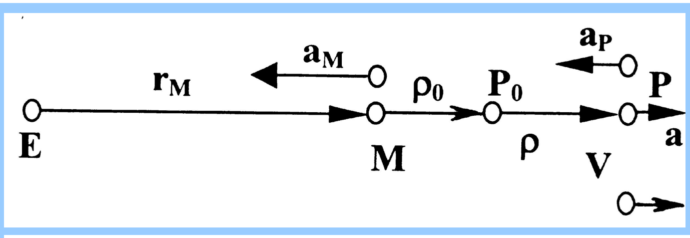


Figure 12. A model for the particle selenocentric hyperbolic motion from the Moon (arc **Es P_2 P_3*)***

The Earth perturbation is

$$\boldsymbol{a} = \boldsymbol{a}_P - \boldsymbol{a}_M = -(\mu_E / (r_M + \rho)^2)((\boldsymbol{r}_M + \boldsymbol{\rho}) / (r_M + \rho)) + (\mu_E / r_M^2)(\boldsymbol{r}_M / r_M). \tag{3.2}$$

It increases the particle selenocentric energy. Let the Earth-Moon distance r_M be constant. Then the energy E_S is defined by the Moon-particle distance ρ and back:

$$E_S(\rho) - E_0 = (\mu_E / r_M^2)(\rho - \rho_0) + \mu_E/(r_M + \rho) - \mu_E/(r_M + \rho_0),\ E_S(\rho_0) = E_0; \tag{3.3}$$

$$\rho(E_S) = B/2 + (B^2/4 + r_M B)^{1/2},\ B = (E_S - E_0) r_M^2 / \mu_E + \rho_0^2/(r_M + \rho_0). \tag{3.4}$$

Example. Let for the trajectory above in the escape point the energy E_S be E_0=0, distance ρ be ρ_0=91850 km. Then the model (3.2-3.4) gives:
ρ= 102.5 $\cdot 10^3$ km *for* V_∞=0.15 km/s (point P_2, with exact numerical distance ρ_n=101 $\cdot 10^3$ km);
ρ= 120.4 $\cdot 10^3$ km for V_∞=0. 25 km/s (point P_3, with exact numerical distance ρ_n=120.2 $\cdot 10^3$ km).

So, ***near the translunar libration point*** **L_2*, the particle can be accelerated by Earth's gravity from parabolic selenocentric orbit in the escape point Es to the hyporbolic one and move from the Earth.***

3. THEORETICAL ANALYSIS OF "DETOUR"FLIGHT - c
3.3. SUN GRAVITY EFFECT ON DECREASE OF THE PARTICLE ORBIT PERIGEE DISTANCE

Next, we estimate approximately the Sun gravity effect on the variation Δr_π of the particle orbit perigee distance r_π on the final arc P_3 F of the space flight as the orbit revolution.
Suppose that eccentricity $e \approx 1$, $r_{\pi f} \approx 0$, middle value $r_\pi \approx - \Delta r_\pi / 2$. Then, using the evolution theory [Lidov 1961, 1962] for the Earth-Sun fixed direction, we have:

$$\Delta r_\pi \approx \operatorname{sign}\beta\, ((15/2)\, \pi\, (\mu_S/\mu_E)\, \beta)^2\, a^7 / a_E^{\,6} < 0. \qquad (3.5)$$

Here μ_E, μ_S are the Earth and Sun gravitational parameters; a_E is a distance to the Sun; a is semi-major axis of the particle orbit; $\beta = \cos^2\gamma \sin 2\alpha < 0$, γ, α are angles of the Earth-Sun vector orientation relative to the particle orbit plane, $|\beta| \le 1$. Semi-major axis a that leads to the perigee change Δr_π is

$$a \approx [\,|\Delta r_\pi|\; a_E^{\,6} / ((15/2)\, \pi\, (\mu_S/\mu_E)\beta)^2]^{1/7}. \qquad (3.6)$$

To evaluate necessary value of semi-major axis a, suppose Δr_π = - 500,000 km, β = - 0,5. Then $a \approx$ 870,000 km, apogee distance $r_\alpha \approx$ *1.5 million km.*

Hence, if the Sun orientation is suitable and the apogee distance is large enough, the perigee distance is decreased to ~ zero, that gives possibility to approach passively the Earth. Numerical calculations confirm this result.

4. CONCLUSIONS

Numerical and theoretical studies prove existence of "detour" trajectories

for the Earth-to-Moon passive flight to a lunar satellite orbit with spacecraft's gravitational capture and

for the Moon-to-Earth passive flight from a lunar satellite orbit with spacecraft's gravitational escape from lunar attraction.

They require less fuel consumption, although have a long enough flight time and need more exact navigation support.

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