

**CORRELATION AND SPECTRAL
MODELS IN STATIONARY
ESTIMATION PROBLEMS OF THE
DIFFERENTIAL ROTATION AND
THE LATITUDINAL DRIFT OF THE
MAGNETIC FIELDS ON THE SUN**

Yarema Zyelyk

**Space Research Institute, NAS&NSA of Ukraine,
Kyiv, 40 Academician Glushkov prospect,
building 4/1, Kyiv - 187, 03680, Ukraine**

adapt@space.is.kiev.ua

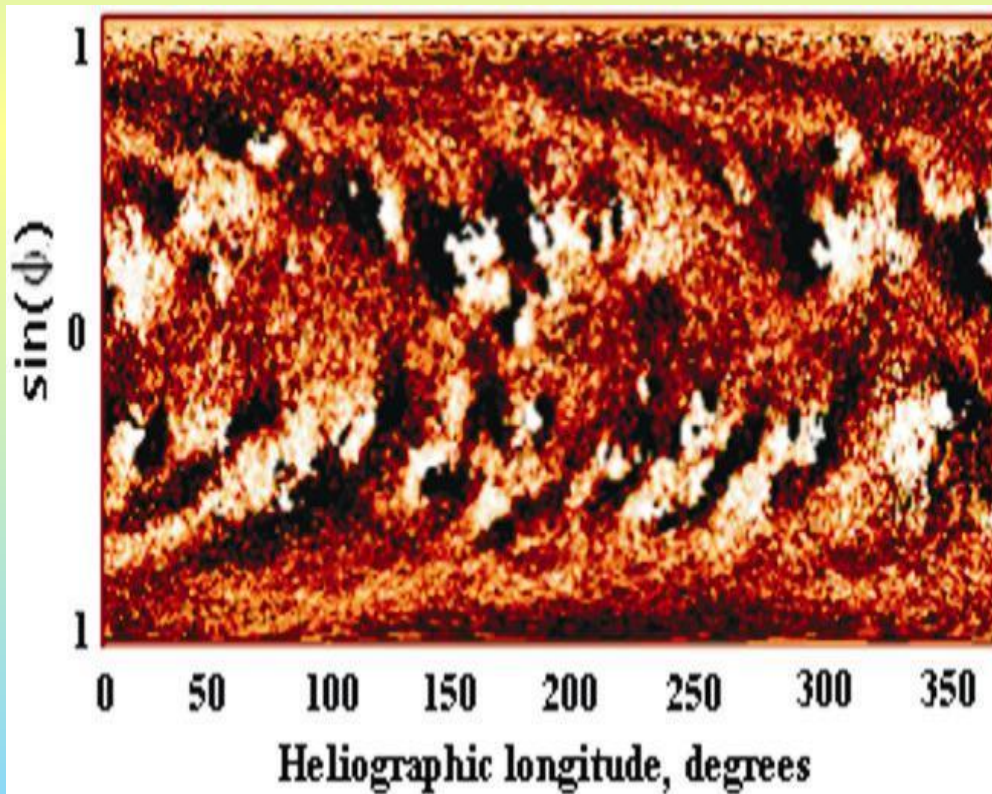
RESEARCH TASKS

- ❑ **At the research of magnetic fields on the Sun on the basis of observations measure a longitudinal magnetic field, i.e. normal component of magnetic field strength. Thus in each point of the Sun surface in some current realization of the field we observed either a field "+" (N) - northern or "-" (S) - southern polarity. The field "+" (N) - northern polarity is directed from the Sun, and a field "-" (S) - southern polarity - is directed to the Sun**
- ❑ **The imbalance of fields in each point of the Sun surface can be calculated as the vectorial sum of the N-and S-polarities field strengths. The study of the unbalance of "+" and "-" fields on the Sun is actual as the N-and S-polarities unbalance stream of magnetic fields from all Sun substantially determines a state of an interplanetary magnetic field and influences on the solar-terrestrial relationship**
- ❑ **Differential character of the magnetic field rotation is exhibited in the exclusive dependence of the rotation period (velocity) from the latitude, not stacked in the conventional mechanistic representations about the solid-state rotation (on equator rotation is faster, than at high latitudes)**
- ❑ **The purpose of the present study is the estimation of the latitudinal drift of the unbalance magnetic field stream as a whole and determination of the differential rotation and the latitudinal drift of the unbalance stream components possessing the certain significant rotation periods, and also comparison of these two drift types**

OBSERVATIONAL MATERIAL

Synoptic maps data of longitudinal vector of the magnetic field strength on the Sun on the 26 years observational interval since January, 1, 1977 till the September, 30, 2003, obtained in the NSO Kitt Peak (USA) and submitted to the author

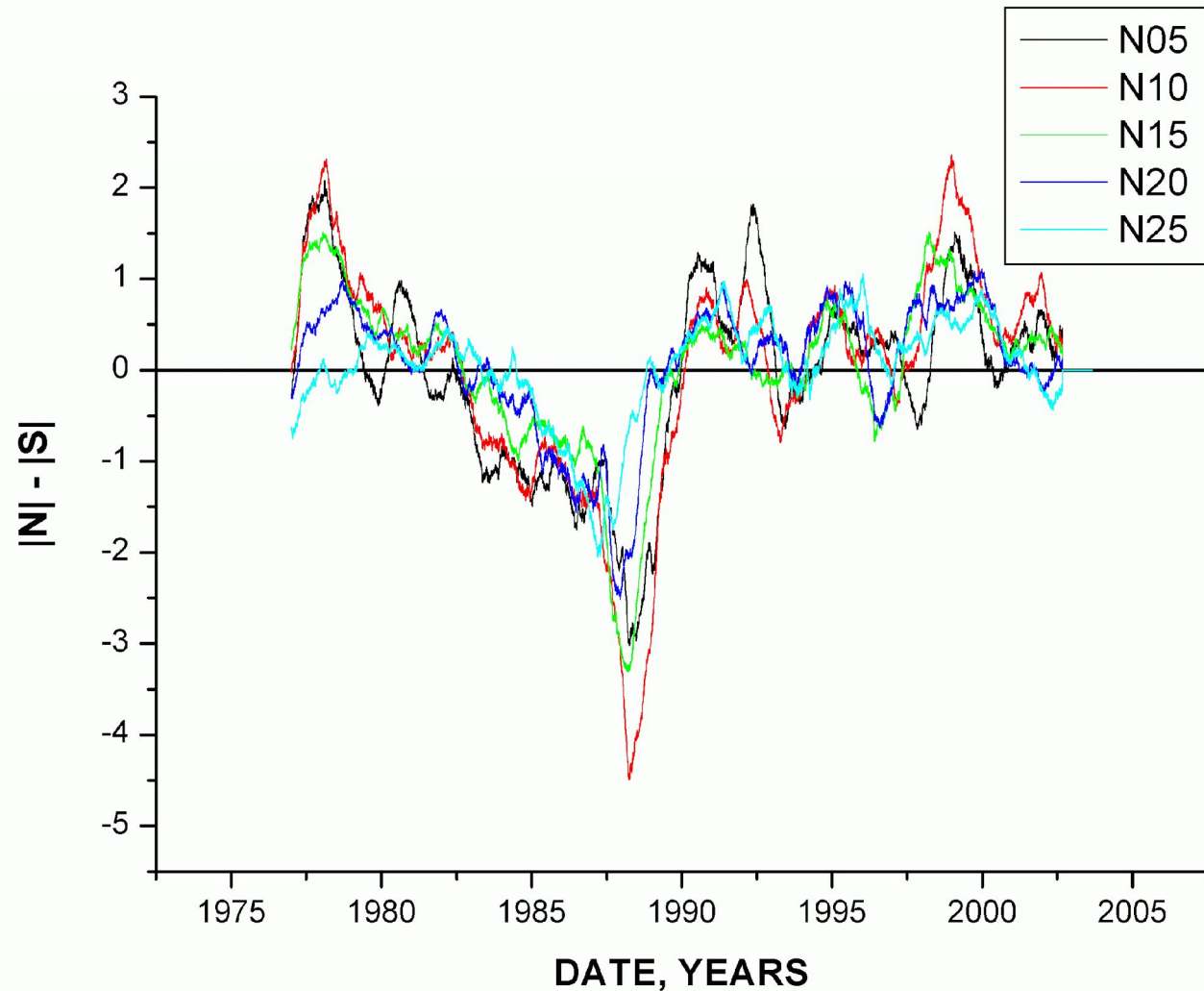
The strength module intervals of the solar magnetic fields of the N and S polarities, allocated on the basis of the analysis of the histograms of the map pixels amount dependence from the strength



Interval No.	Boundary strength modulo, Gs
1	[0; 5)
2	[5; 10)
3	[10; 20)
4	[20; 50)
5	[50; 200)
6	[200; 700)
7	[700; 1500)

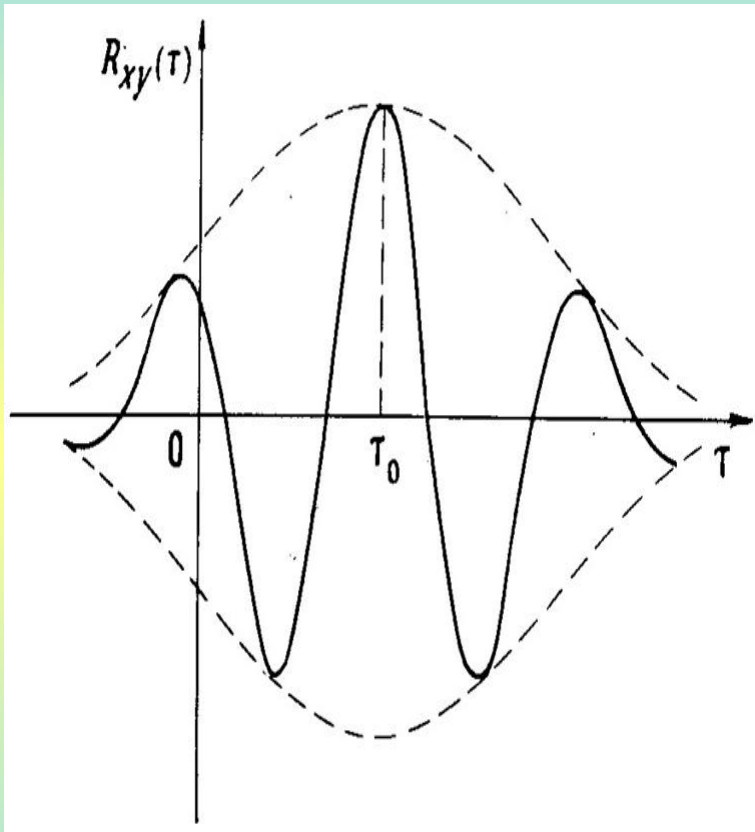
Time series constructing

- For each of the allocated 7 strength intervals in the range [0; 1500] Gs in everyone of 28 series each term of series represents the magnetic fields unbalance stream in the strength interval H_k ($k = 1, 2, \dots, 7$) on the central meridian of the Sun from the surface element in a latitude band φ_n ($n = 0, \pm 5, \pm 10, \pm 15, \dots, \pm 70$) width's 1° longitudes and height's 5° latitudes in an instant time t_l ($l = 1, 2, \dots, 128520$):
 $(|N|-|S|)(H_k, \varphi_n, t_l)$,
where $(|N|-|S|)$ – signify a difference of the absolute values of the streams of the magnetic fields «+» (N) и «-» (S) polarities.
- The sampling time is $\Delta t = 0,0757$ day (1° of the solar longitude)



Temporal variations of the magnetic fields unbalance stream in the strength interval [0; 5] Gs for five 5° latitude bands at the latitudes from the interval of [5°; 25°] of the northern Sun hemisphere

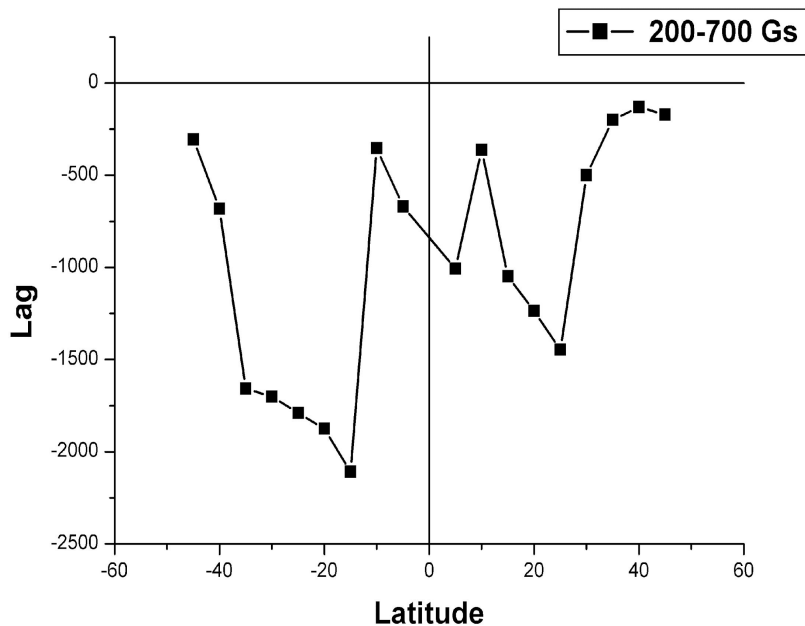
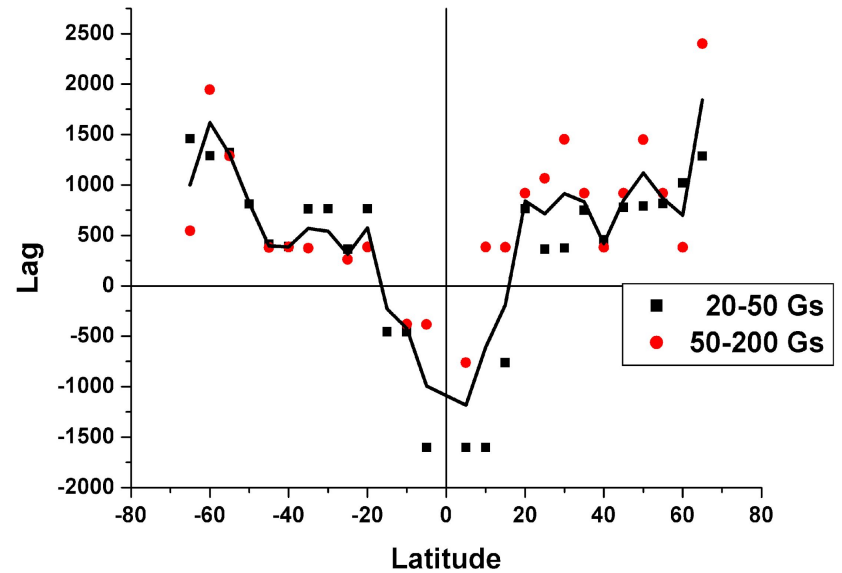
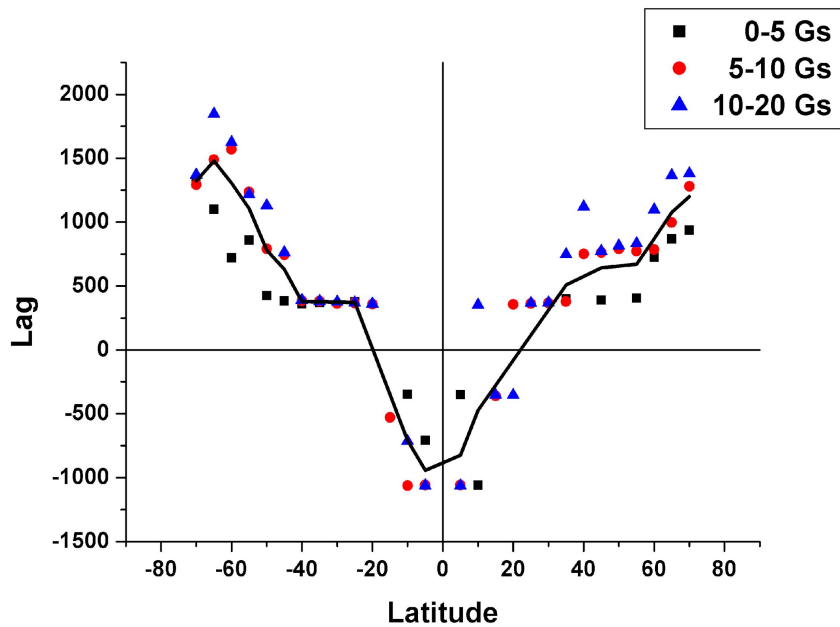
LATITUDINAL DRIFT OF THE MAGNETIC FIELDS UNBALANCE STREAM AS A WHOLE



- At the estimation of the latitudinal drift of the magnetic fields unbalance stream the method of the time delay definition of the unbalance stream as a whole as a wave train in a latitude band with a latitude $|n-1|$ concerning the unbalance stream in the next band with the greater latitude $|n|$, based on the definition of a delay of cross correlation function which corresponds to a absolute maximum of this function was used:

$$Lag = arg\{max R_{n n-1}(\tau)\}$$

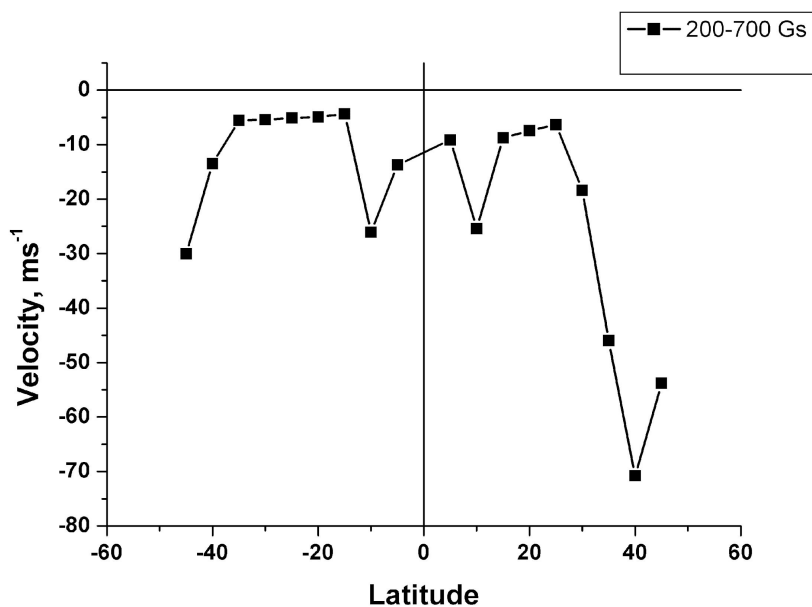
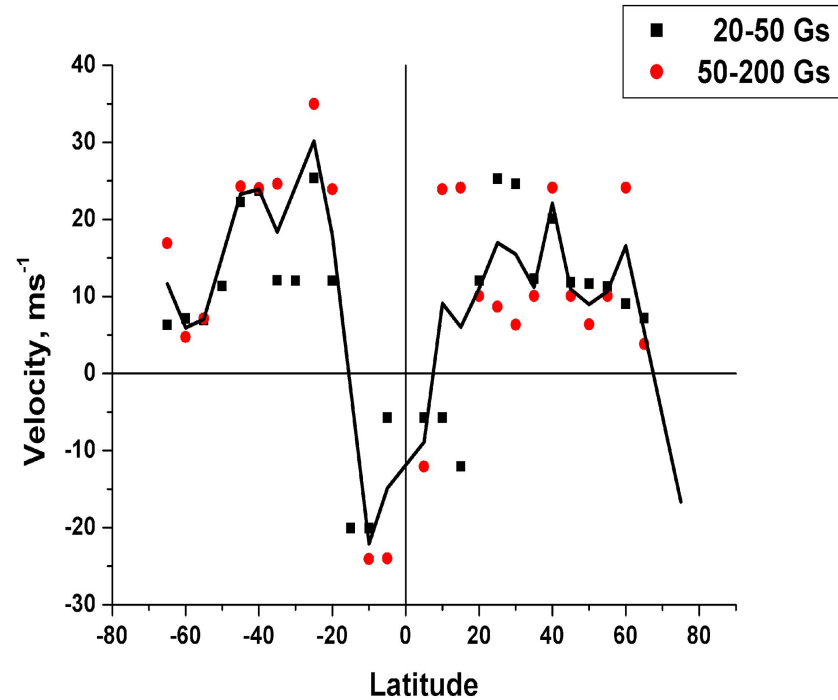
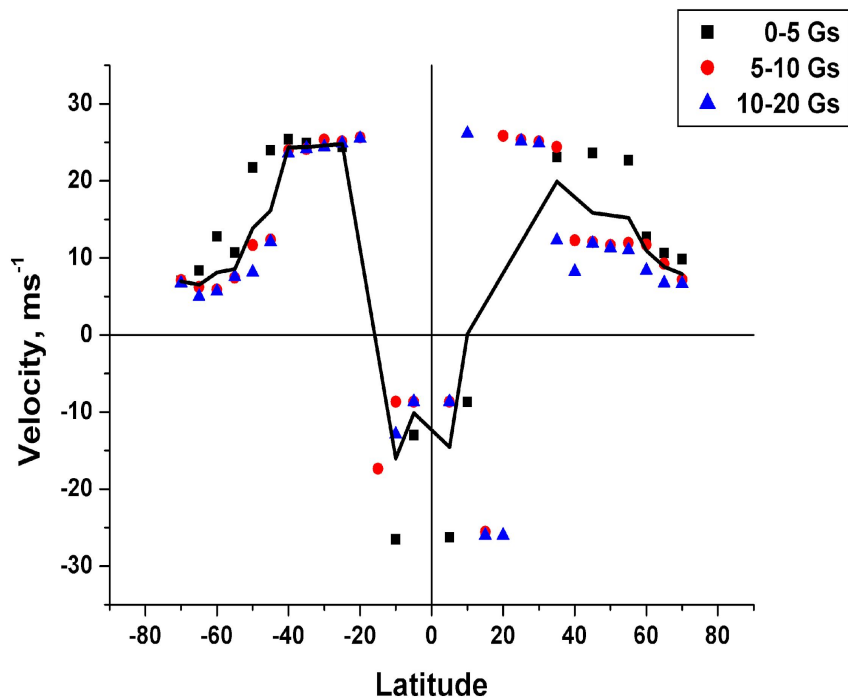
- Latitudes of the bands $|n-1|$ and $|n|$ are specially taken modulo that both in southern, and in northern hemispheres of the Sun the band $|n|$ corresponded to higher next latitude, than a band $|n-1|$



The latitudinal variations of the cross-correlation function shifts in the next latitude bands for the magnetic fields unbalance stream on the Sun in the strength range of [0; 700] Gs

Lag > 0 – Drift to poles

Lag < 0 – Drift to equator



The latitudinal dependences of the latitudinal drift velocities of the magnetic fields unbalance stream in the strength range of [0; 700] Gs

$V > 0$ – Drift to poles

$V < 0$ – Drift to equator

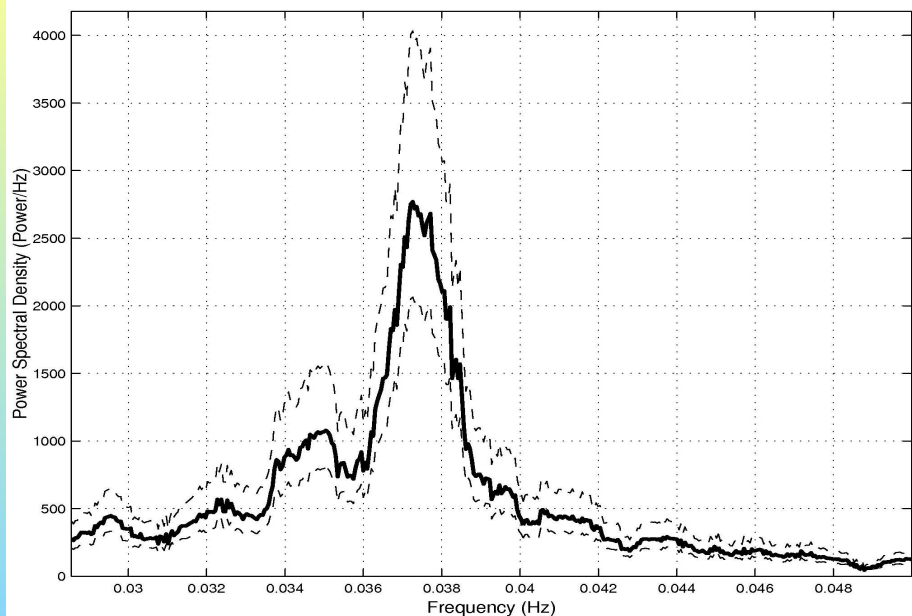
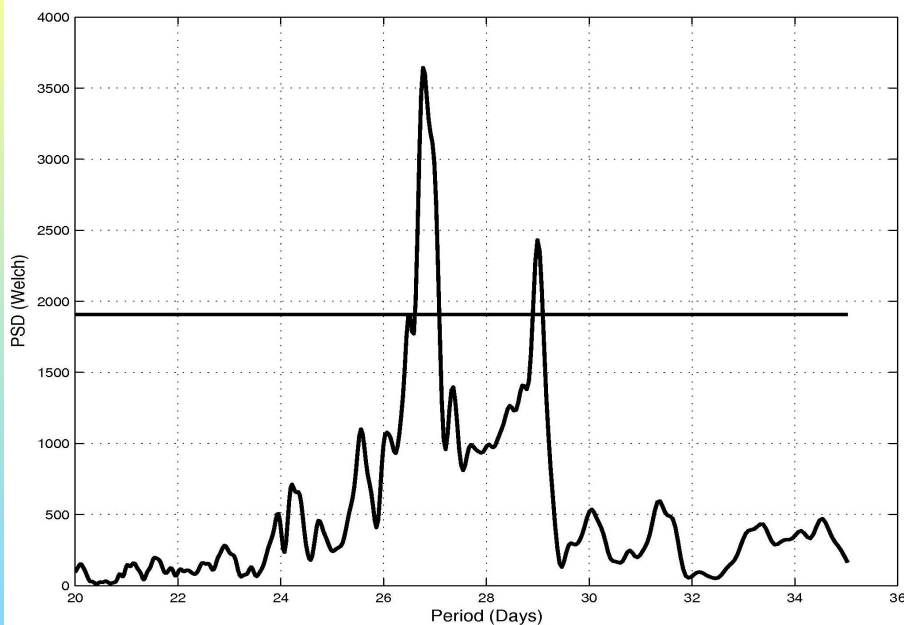
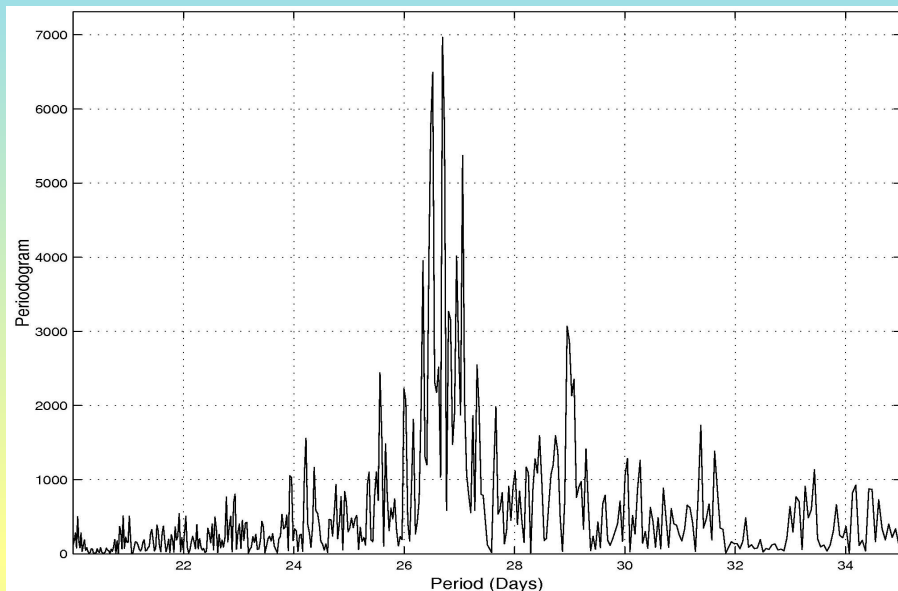
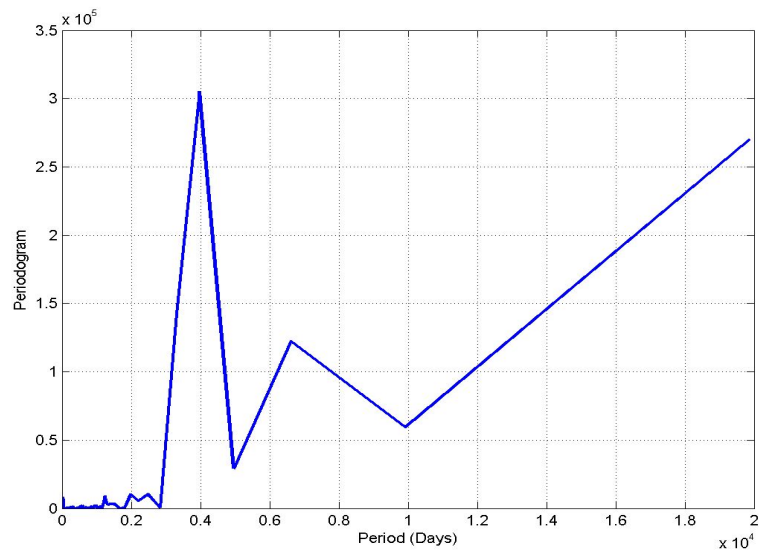
CROSS-CORRELATION DRIFT ESTIMATION RESULTS

- The analysis of the obtained latitudinal dependences of the cross-correlation function time shift and velocities of the latitudinal drift of the magnetic fields unbalance stream in the strength range of [0; 700] Gs results in following conclusion
- For the fields of strength [0; 200) Gs the pattern of the latitudinal drift velocity variation is qualitatively similar for the all strength ranges which are included in this interval. From the latitudes of $\pm (15^\circ - 20^\circ)$ the unbalance stream drift in each hemisphere of the Sun to the areas of high and low latitudes takes place. With move off this latitude interval the drift velocity decreases up to 5-10 m/s at latitudes of $\pm 60^\circ$ and on equator
- For the fields of strength [200-700) Gs the pattern is essentially another. Since at all latitudes $Lag < 0$ takes place and the latitude drift velocity is negative, the movement from high latitudes to low latitudes is observed only. The greatest velocities are observed at latitudes of $\pm(15^\circ - 20^\circ)$ and of $\pm(40^\circ - 45^\circ)$
- For the fields of all strength values there is a noticeable difference in velocities of the latitudinal drift between northern and southern hemispheres of the Sun

LATITUDINAL DRIFT OF THE SEPARATE COMPONENTS OF THE UNBALANCE STREAM, POSSESSING CERTAIN SIGNIFICANT ROTATION PERIODS

- At the estimating of the latitudinal drift of the unbalance stream separate components, possessing certain significant rotation periods the spectral analysis methods are used.
- Using Welch method the power spectral density $G_n(P)$ in each latitude band $|n|$ and the cross power spectral density (CPSD) $G_{n\ n-1}(P)$ (P - rotation period) processes in the next latitude bands $|n|$ and $|n-1|$ are estimated
- Regarding the significant peaks $G_n(P)$ the significant rotation periods P_peak are determined, and on the bases of $G_{n\ n-1}(P_peak)$ the phase angle of CPSD $\theta_{n\ n-1}(P_peak)$ is calculated
- On the bases of $\theta_{n\ n-1}(P_peak)$ the temporal delay $\tau_{n\ n-1}(P_peak)$ for harmonic component of the unbalance stream rotating with significant period P_peak is determined
- On the known distance along the meridian between medial lines of the next latitude bands and on the estimated temporal delay $\tau_{n\ n-1}(P_peak)$ the velocity of the latitudinal drift of these components to equator or to a pole is determined

Spectral methods of the rotation estimation implementation: Shuster Periodogram, Welch, Multitaper



Sampling and spectral analysis parameters

Number of samples in each time series – $N = 357 \times 360 = 128520$

Sampling time – $dt = 27,2753 / 360 = 0,07576572$ (day)

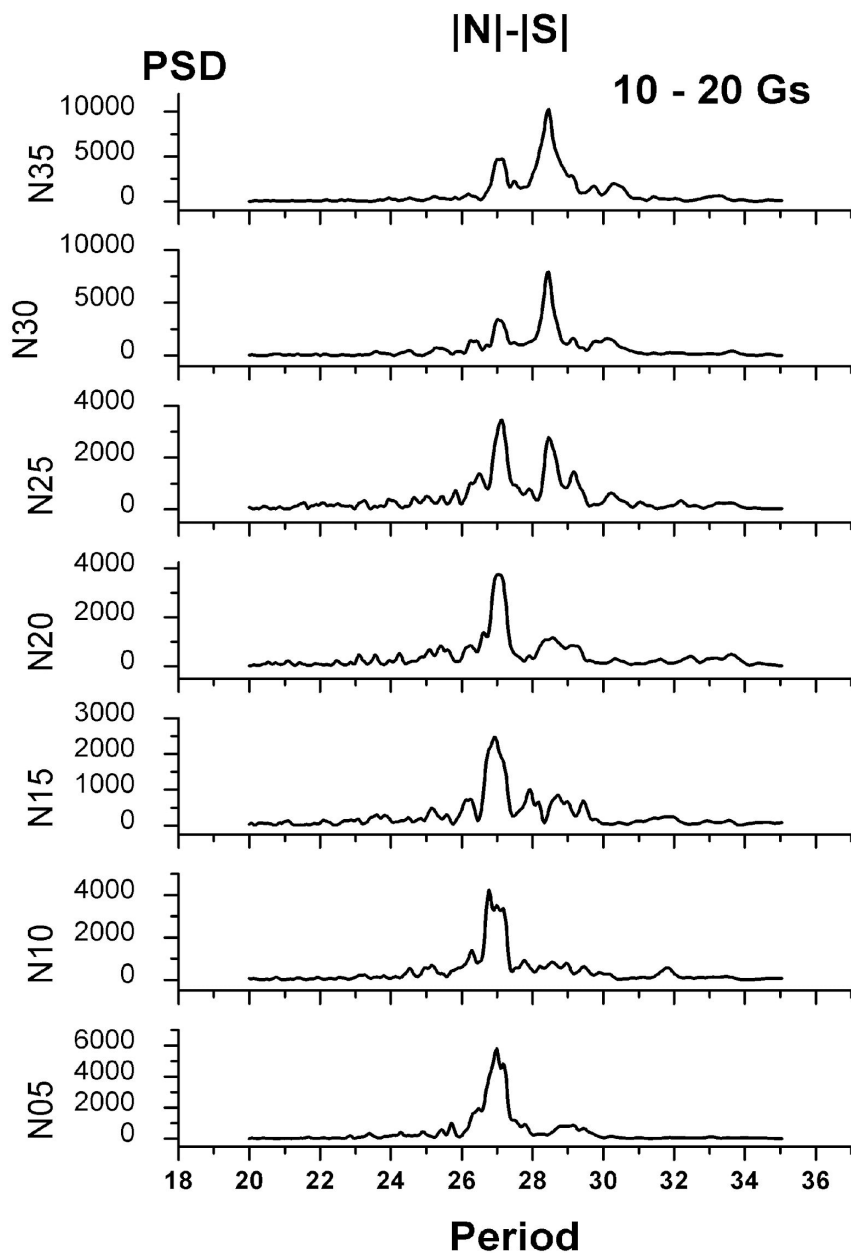
Sampling frequency – $fs = 1 / dt = 13,19875491$ (1/day)

Nyquist frequency – $nyquist = 1/2 fs = 6,599377$ (1/day)

Number of time samples subjected to FFT – $nfft = 262144$

Number of frequency samples

(Fourier coefficients) – $M = nfft / 2 + 1 = 131073$



- In the same latitude band can exist from one up to four different rotation periods corresponding to significant PSD peaks.
- These different rotation periods presented in the some current band, are spread to different on the width intervals of the next latitude bands everyone.
- An example of the power spectral density dependences from the period in the period interval of [20; 35] day for the time series of the magnetic fields unbalance stream in the strength interval of [10; 20] Gs in the latitude band interval of [5°; 35°] in the northern Sun hemisphere is represent in the figure

Latitudinal intervals (in degree) of the determined periods of significant PSD peaks in the 5 ranges of magnetic field strength

Period (days)	Magnetic field strength intervals (Gs)				
	0 -5	5-10	10-20	20-50	50-200
26,912		-25 ÷ -5	-15 ÷ +15	-20 ÷ +35	
27,717	-45 ÷ -15	-35 ÷ -15	-25 ÷ -15		
28,092	-65 ÷ -25	-65 ÷ -20	-65 ÷ -20	-40 ÷ -20	-30 ÷ -20
28,414	5 ÷ 50	20 ÷ 50	25 ÷ 40		
29,512		60 ÷ 70	-65 ÷ - 40, 65 ÷ 70		60 ÷ 70
30,323	45 ÷ 70	45 ÷ 70	45 ÷ 70	-65 ÷ -45 40 ÷ 65	
30,603	55 ÷ 70	60 ÷ 70			
31,376	-65 ÷ -50	-65 ÷ -45			

Phase angle, time shift and velocity of the latitudinal drift determination

- The phase angle $\theta_{n\ n-1}(P)$ of cross spectral density $G_{n\ n-1}(P)$ at each value of period P is determined as follows:

$$\theta_{n\ n-1}(P) = \arctg \{ -Im[G_{n\ n-1}(P)] / Re[G_{n\ n-1}(P)] \} * 180 / \pi \text{ (degree)}$$

- The positive phase angle $\theta_{n\ n-1}(P) > 0$ in a latitude band n show, on how many degrees on a longitude the harmonic component with period P of the rotation process of the magnetic fields unbalance stream in a latitude band $(n-1)$ lags behind from harmonic component with period P in a band n . At $\theta_{n\ n-1}(P) < 0$ the corresponding harmonic component with period P advancing takes place

- With the phase angle $\theta_{n\ n-1}(P)$ is unequivocally related the time shift $\tau_{n\ n-1}(P)$ at the corresponding rotation period P which is determined per phase angle as follows:

$$\tau_{n\ n-1}(P) = \theta_{n\ n-1}(P) * P / (2 \pi) \text{ (day)}$$

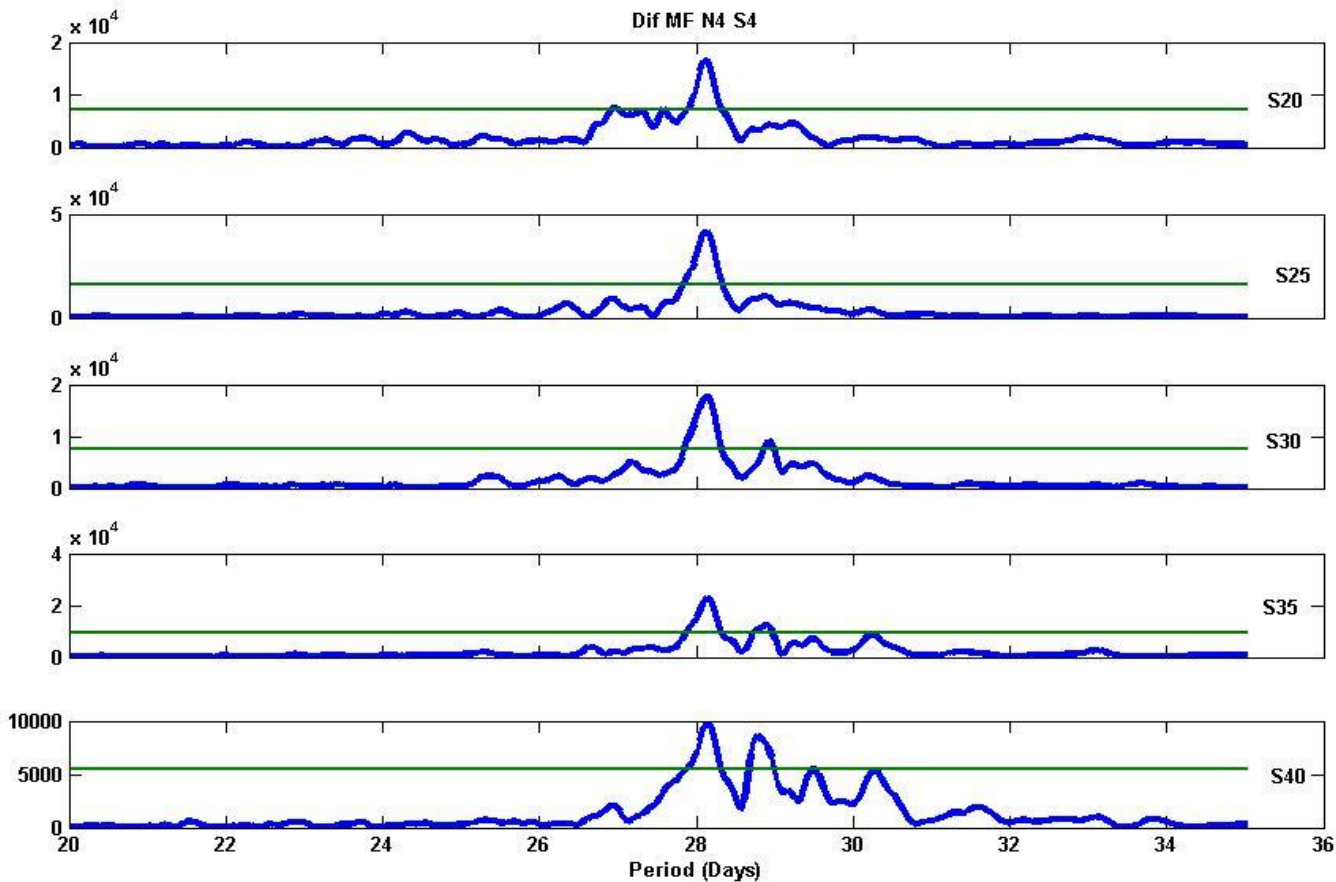
- After determination time shift $\tau_{n\ n-1}(P)$ for any value of significant rotation period P it is possible to estimate the velocity $V_{n\ n-1}(P)$ of the latitudinal drift of harmonic component with period P of rotation process of the magnetic fields unbalance stream:

$$V_{n\ n-1}(P) = d_{n\ n-1} / [\tau_{n\ n-1}(P) * 86400] \text{ (m/s)},$$

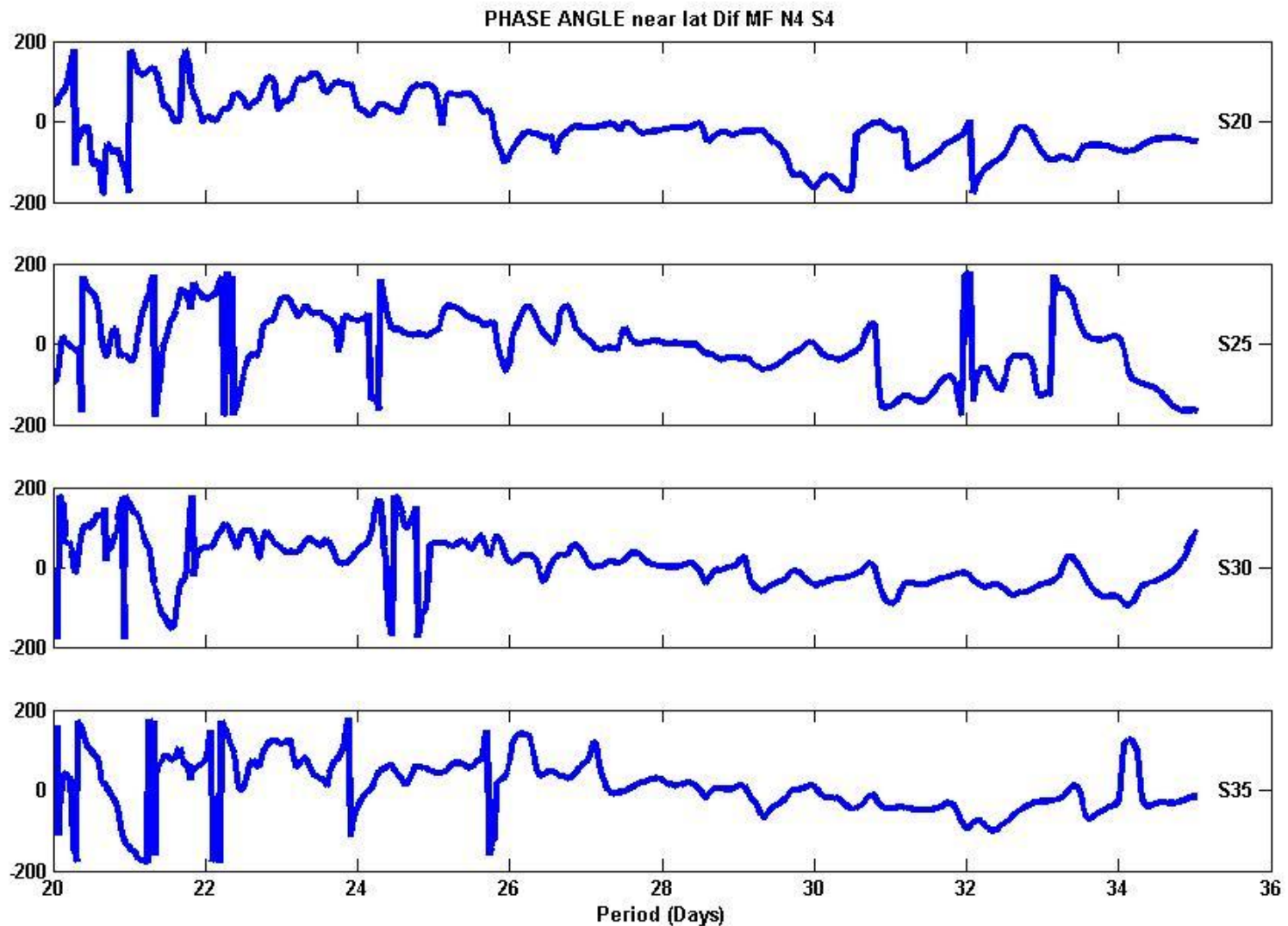
where $d_{n\ n-1} = 2 \pi * R_{Sun} * 5 / 360 \text{ (m)}$ - distance on the meridian between medial lines of the next 5° latitude bands (meter), and $R_{Sun} = 696000000 \text{ m}$ is radius of the Sun

- The sign of the latitudinal drift velocity $V_{n\ n-1}(P)$ coincides with the sign of the time shift $\tau_{n\ n-1}(P)$ and, accordingly, phase angle $\theta_{n\ n-1}(P)$

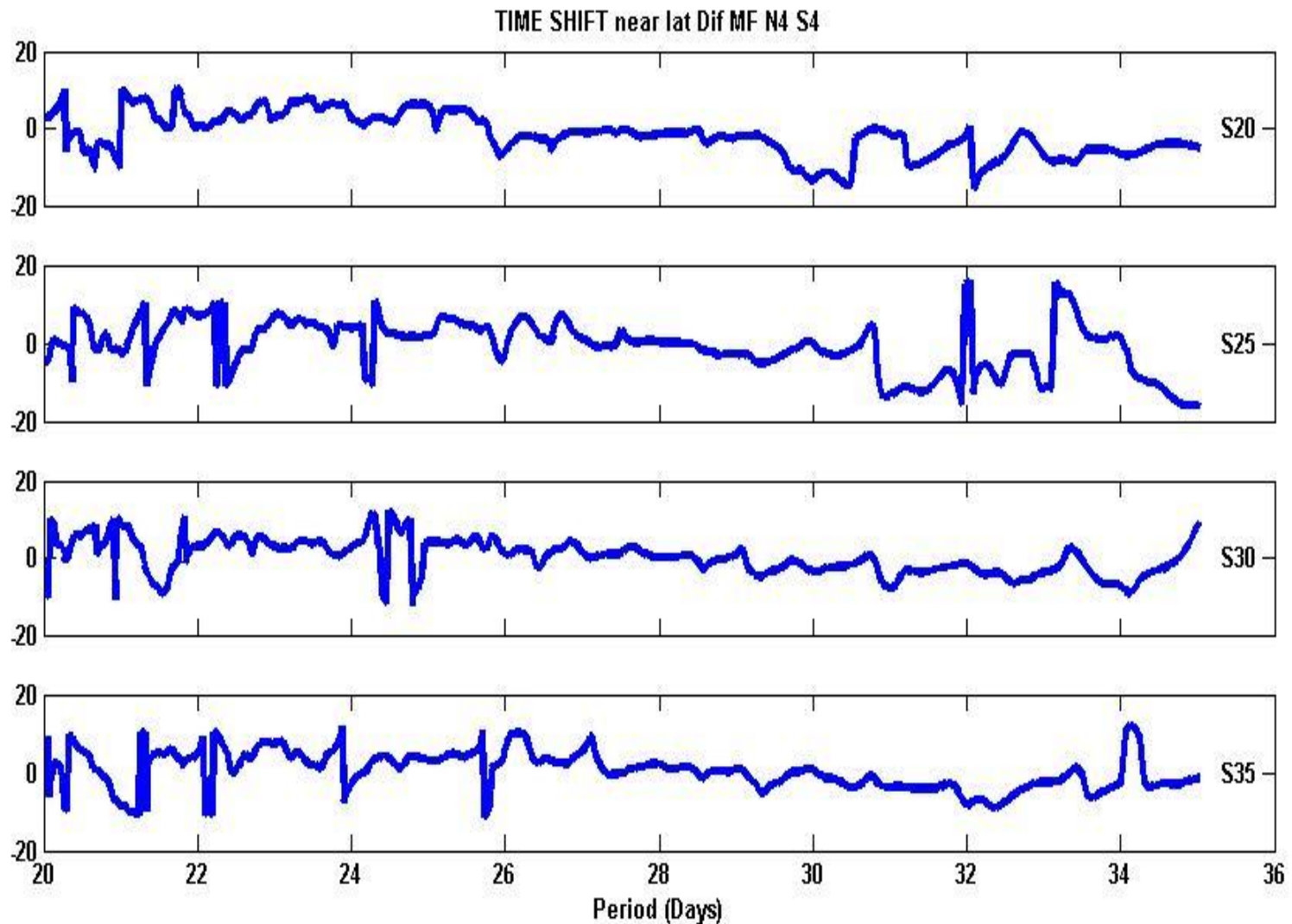
The significant peaks PSD, phase angle, time shift and latitudinal drift velocity estimation procedure



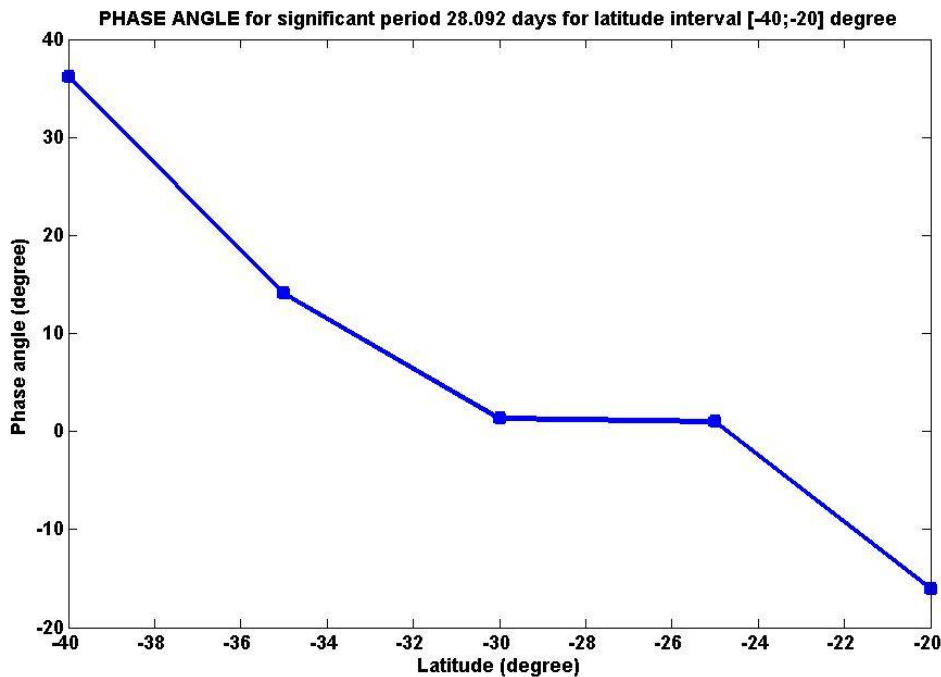
Magnetic fields unbalance stream PSD. Significant period $P = 28,1$ day in the latitudinal interval $[-40^\circ; -20^\circ]$. The magnetic field strength range is $[20; 50]$ Gs



The phase angle dependence on period and the monotonic character of its variation in neighborhood of $P=28,1$ day for the magnetic field strength range of $[20; 50]$ Gs

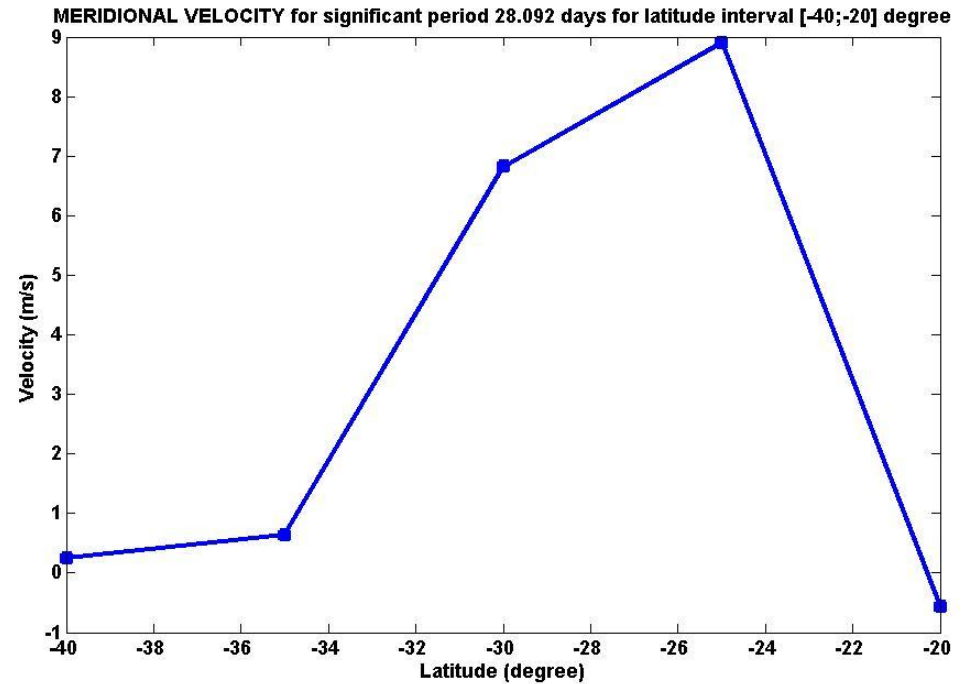


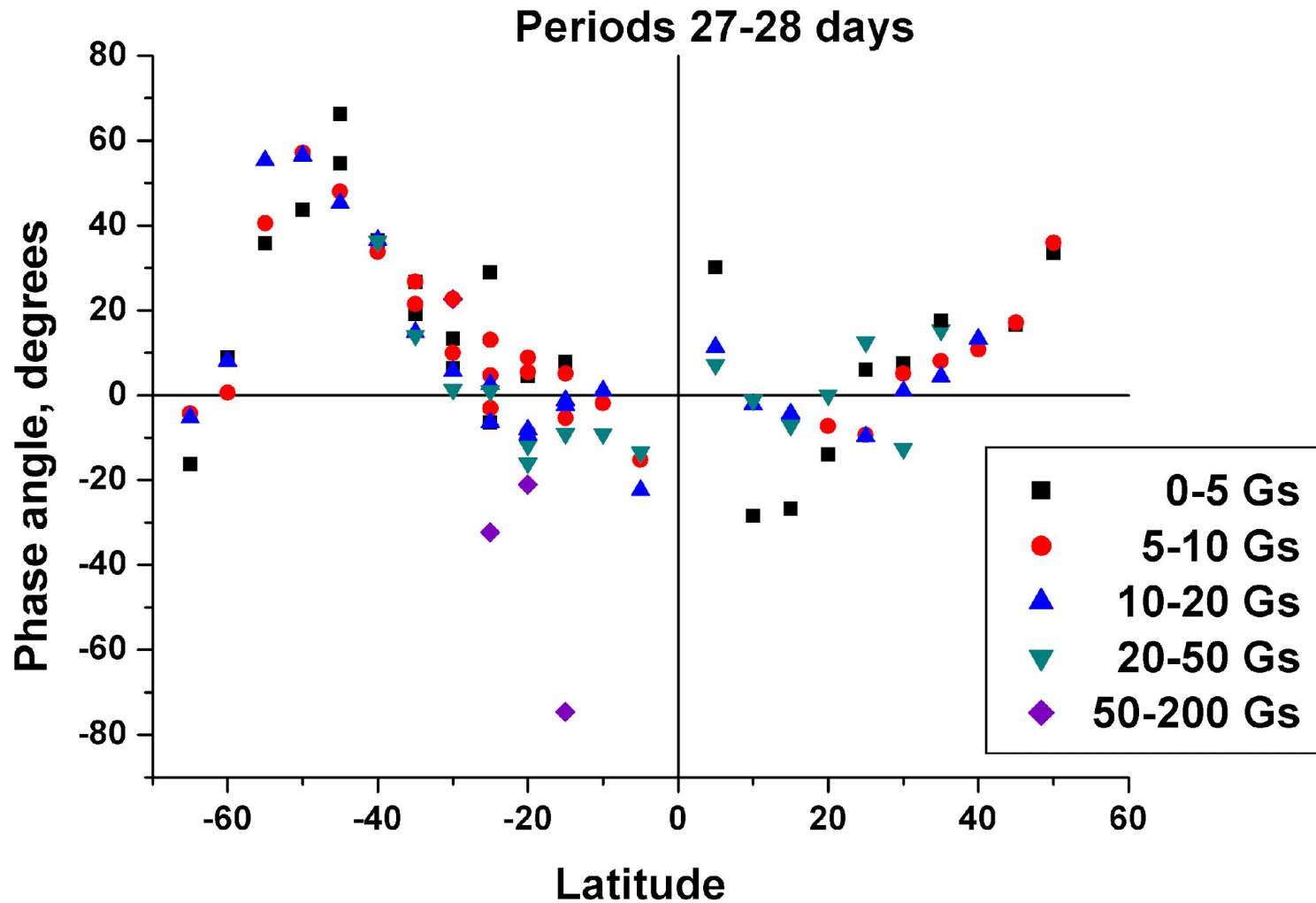
The time shift on period dependence and the monotonic character of its variation in neighborhood of $P=28,1$ day for the magnetic field strength range of $[20; 50]$ Gs



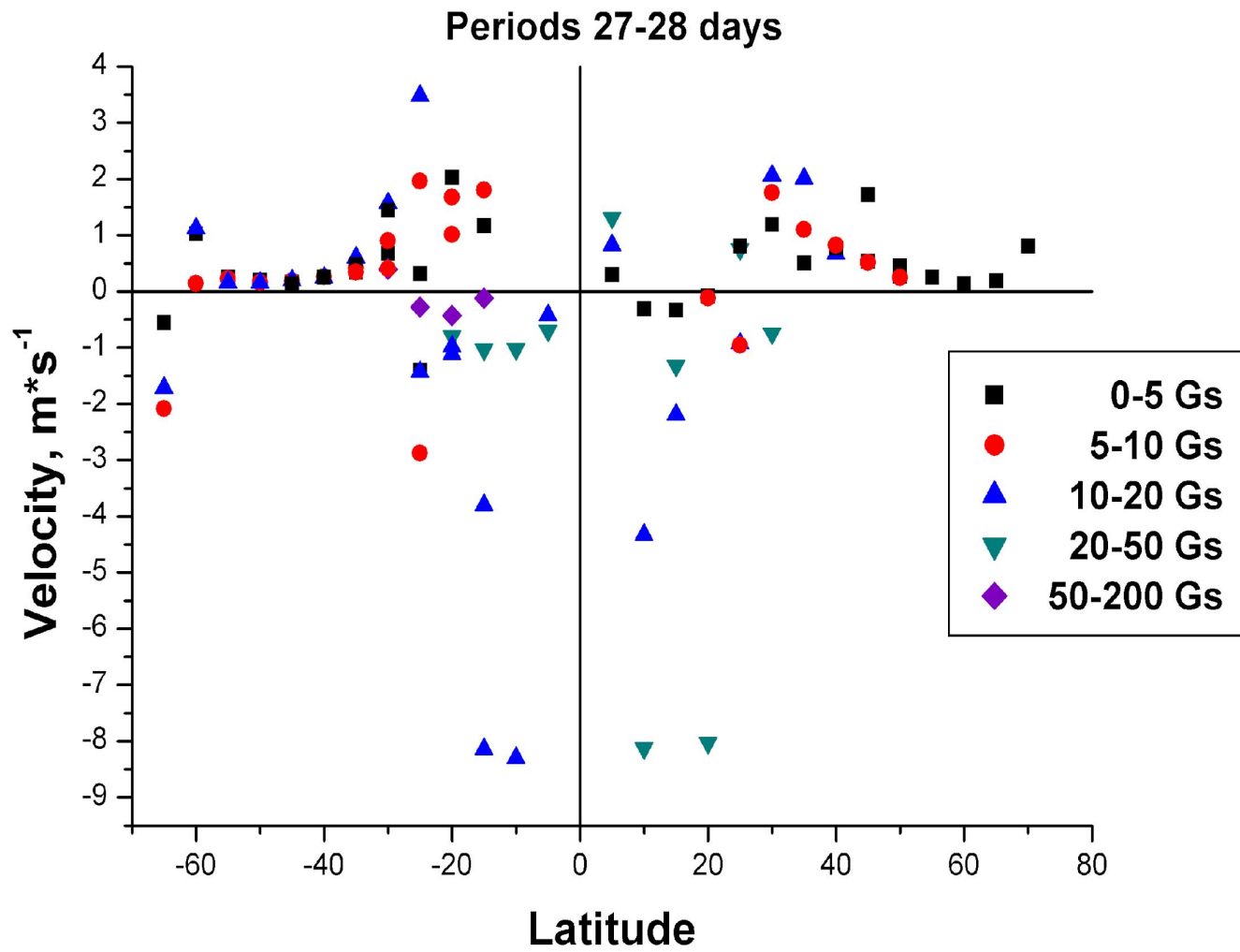
- The latitudinal drift velocity on period dependence in the latitudinal interval of [-40°;-20°] in which there is a significant rotation period of P=28,1 day
- Magnetic field strength range is [20; 50] Gs

- The phase angle on period dependence in the latitudinal interval of [-40°;-20°] in which there is a significant rotation period of P=28,1 day
- Magnetic field strength range is [20; 50] Gs



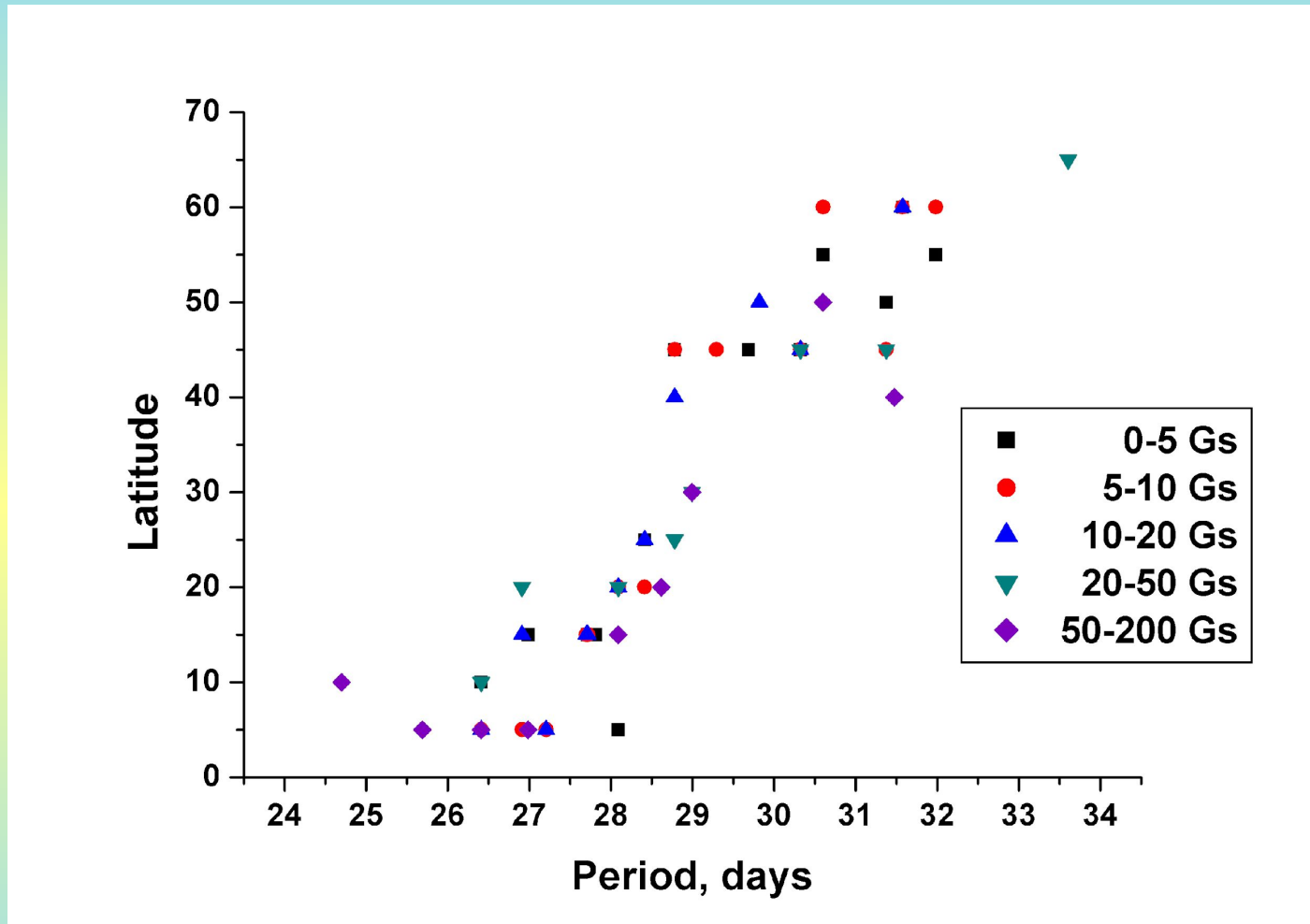


Latitudinal dependences of the phase angle of the magnetic fields unbalance stream components rotating with periods in the interval of [27; 28) day for five magnetic field strength ranges



Latitudinal dependences of the latitudinal drift velocity of the magnetic fields unbalance stream components rotating with periods in the interval of [27; 28) day for five magnetic field strength ranges

DIFFERENTIAL ROTATION AT THE LEVEL OF THE BACKGROUND FIELD SOURCE



The dependences of significant rotation periods of the unbalance stream structures from the emersion latitude for different magnetic field strength ranges

CONCLUSION

- **The correlation and spectral models of the stationary random processes describing the fields rotation in the assigned latitude bands of the Sun have been effectively applied for the estimation of stationary differential rotation and the latitudinal drift of magnetic fields on the Sun**
- **The comparison of the estimates of the cross correlation function time shifts with phase angles of the cross spectral density in the next latitude bands and the latitudinal drift velocities of the unbalance stream as a whole with the latitudinal drift velocities of the unbalance stream structures possessing the significant rotation periods has shown qualitative coincidence of the latitudinal drift of these two types**
- **Thus, at the magnetic field strength of $[0; 200)$ Gs the latitudinal drift of the unbalance stream as a whole is determined generally a latitude drift of its structures rotated with significant periods in some latitude intervals, enveloping from three and more five-degree latitude bands**

- The comparison of the latitudinal drift of the unbalance stream as a whole and the drift of its structures with significant rotation periods at the magnetic field strength of [0-200) Gs allows to represent the differential rotation of the Sun as follows:**
- The unbalance stream structures possessing the significant rotation periods (velocities), arising at the emersion latitudes (of 20° and higher - depending from period), drift sideways poles, and at latitudes lower than latitudes of emersion move to equator, maintaining their rotation periods (velocities).**
- To have been shifting to poles from the latitude of emersion approximately on 20° (or more - depending from the period), structures with some significant periods cease to exist as a unit everyone, giving up the place to other structures with other significant rotation periods (velocities).**
- The found out incremental dependences of the rotation period of the unbalance stream structures from the latitude of their emersion can be caused by the rotation of more deep stratum of the Sun which are responsible for the formation of the background magnetic fields and their unbalance**