

Fractals and Chaos Theory


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Chaos Theory about disorder

- ◆ NOT denying of determinism
 - ◆ NOT denying of ordered systems
 - ◆ NOT announcement about useless of complicated systems

 - ◆ Chaos is main point of order
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What is the chaos theory?

- ◆ Learning about complicated nonlinear dynamic systems
 - ◆ Nonlinear – recursion and algorithms
 - ◆ Dynamic – variable and noncyclic
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Wrong interpretations

- ◆ Society drew attention to the chaos theory because of such movies as Jurassic Park. And because of such things people are increasing the fear of chaos theory.
- ◆ Because of it appeared a lot of wrong interpretations of chaos theory


Chaos Theory about disorder

- ◆ Truth that small changes could give huge consequences.
 - ◆ Concept: impossible to find exact prediction of condition, but it gives general condition of system
 - ◆ Task is in modeling the system based on behavior of similar systems.
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Usage of Chaos Theory

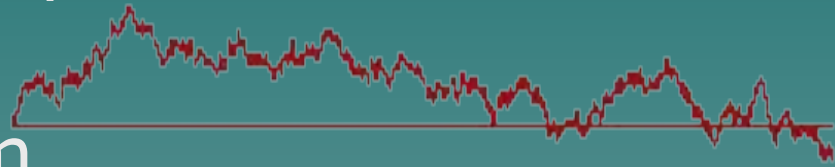
- ◆ Useful to have a look to things happening in the world different from traditional view
 - Instead of X-Y graph -> phase-spatial diagrams
 - Instead of exact position of point -> general condition of system

Usage of Chaos Theory

- ◆ Simulation of biological systems (most chaotic systems in the world)
 - ◆ Systems of dynamic equations were used for simulating everything from population growth and epidemics to arrhythmic heart beating
 - ◆ Every system could be simulated: stock exchange, even drops falling from the pipe
 - ◆ Fractal archivation claims in future coefficient of compression 600:1
 - ◆ Movie industry couldn't have realistic landscapes (clouds, rocks, shadows) without technology of fractal graphics
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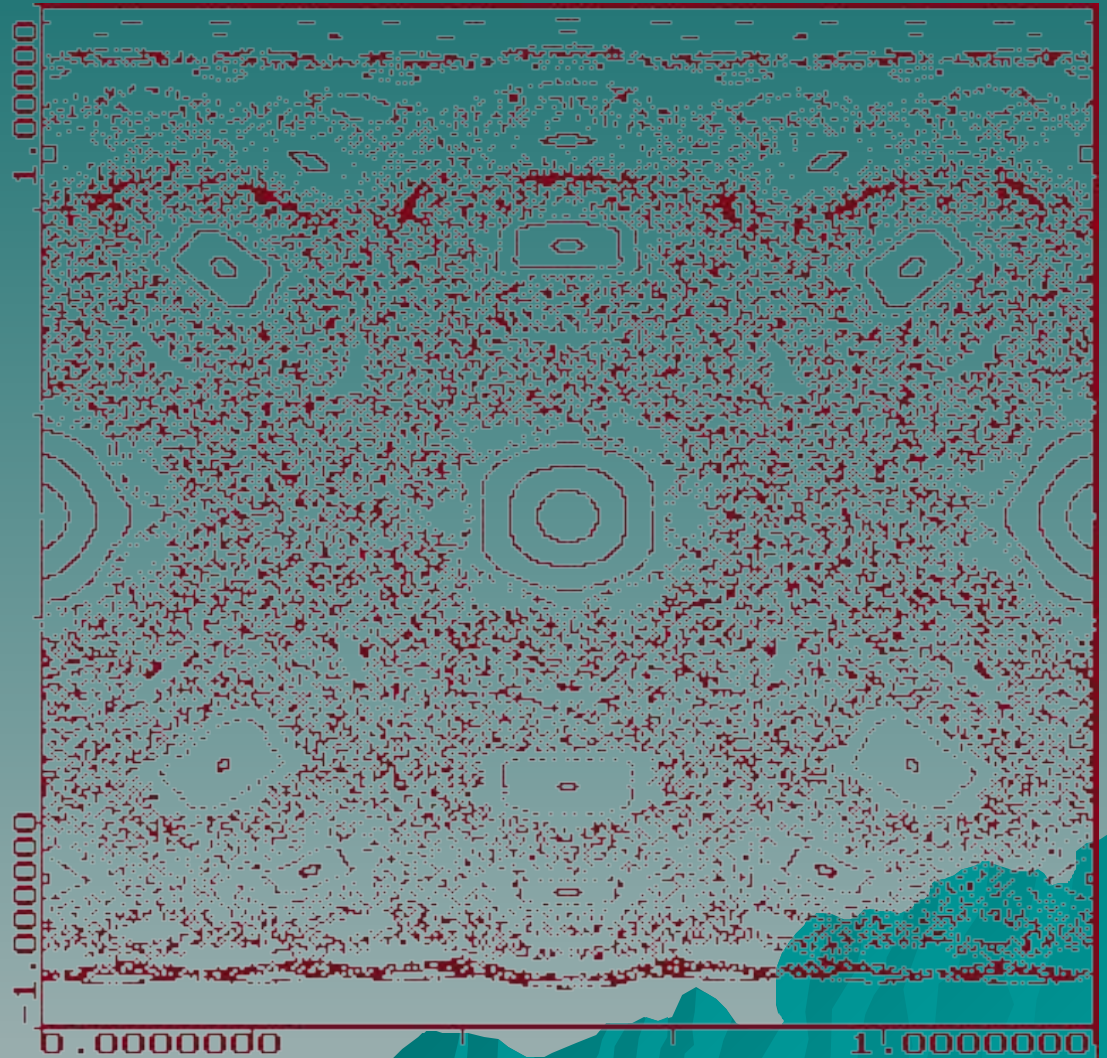
Brownian motion and it's adaptation

- ◆ Brownian motion – for example accidental and chaotic motion of dust particles, weighted in water.
- ◆ Output: frequency diagram
- ◆ Could be transformed in music
- ◆ Could be used for landscape creating



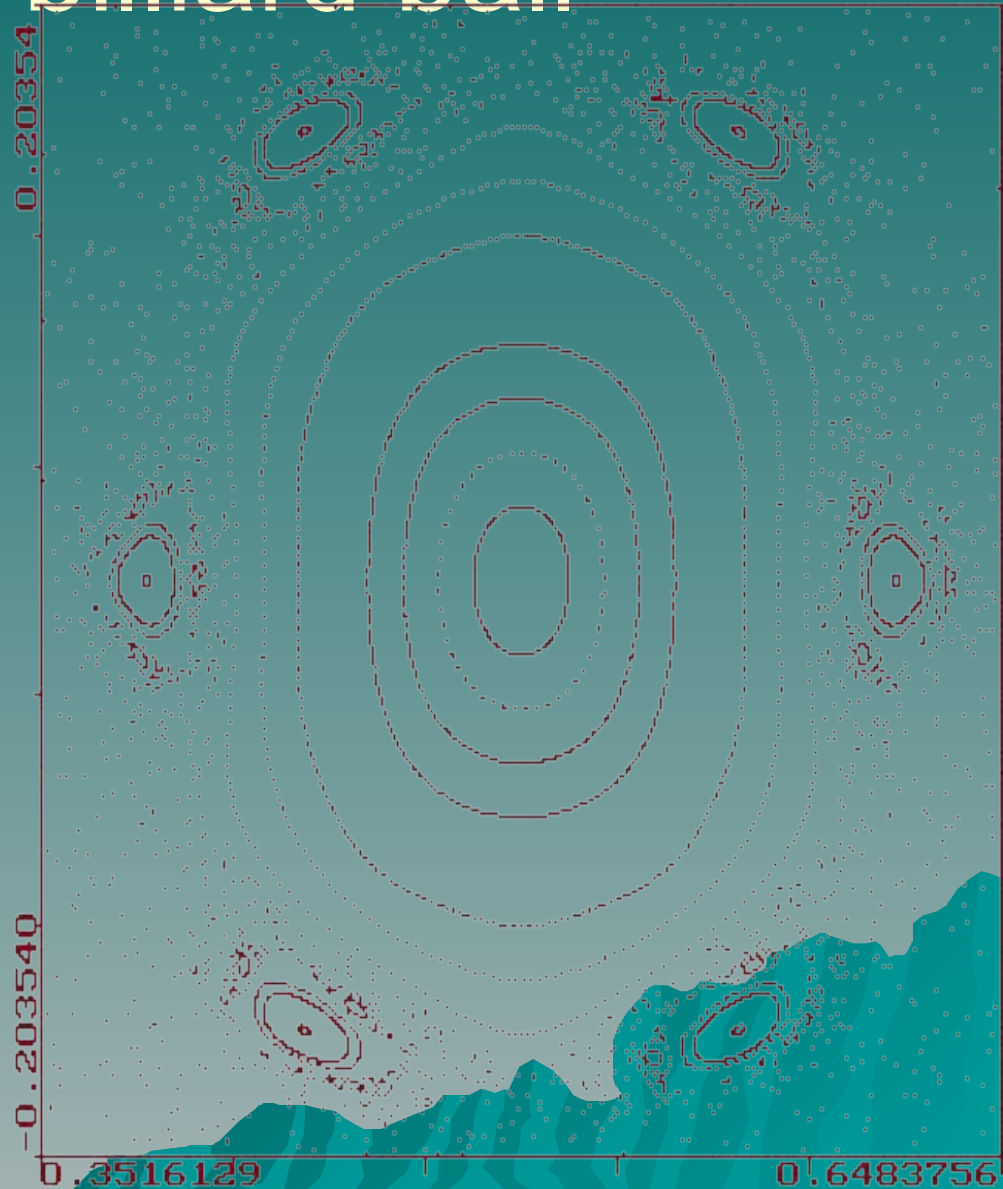
Motion of billiard ball

- ◆ The slightest mistake in angle of first kick will follow to huge disposition after few collisions.
- ◆ Impossible to predict after 6-7 hits
- ◆ Only way is to show angle and length to each hit



Motion of billiard ball

- ◆ Every single loop or dispersion area presents ball behavior
- ◆ Area of picture, where are results of one experiment is called attraction area.
- ◆ This self-similarity will last forever, if enlarge picture for long, we'll still have same forms. => this will be FRACTAL



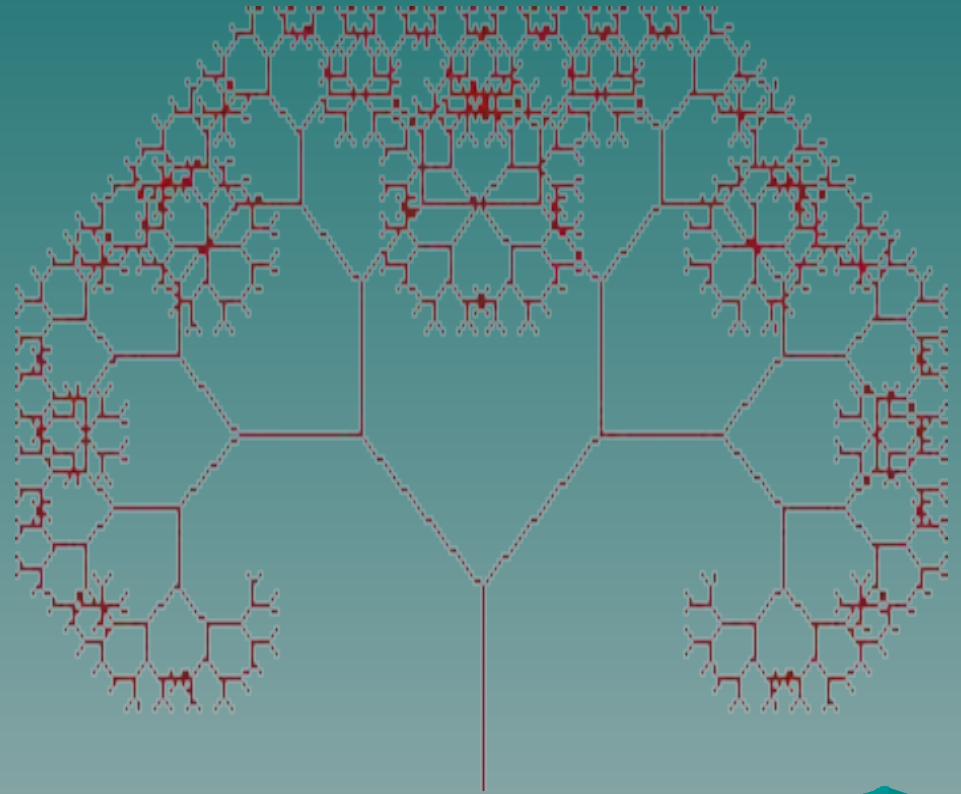
Fusion of determined fractals

- ◆ Fractals are predictable.
- ◆ Fractals are made with aim to predict systems in nature (for example migration of birds)



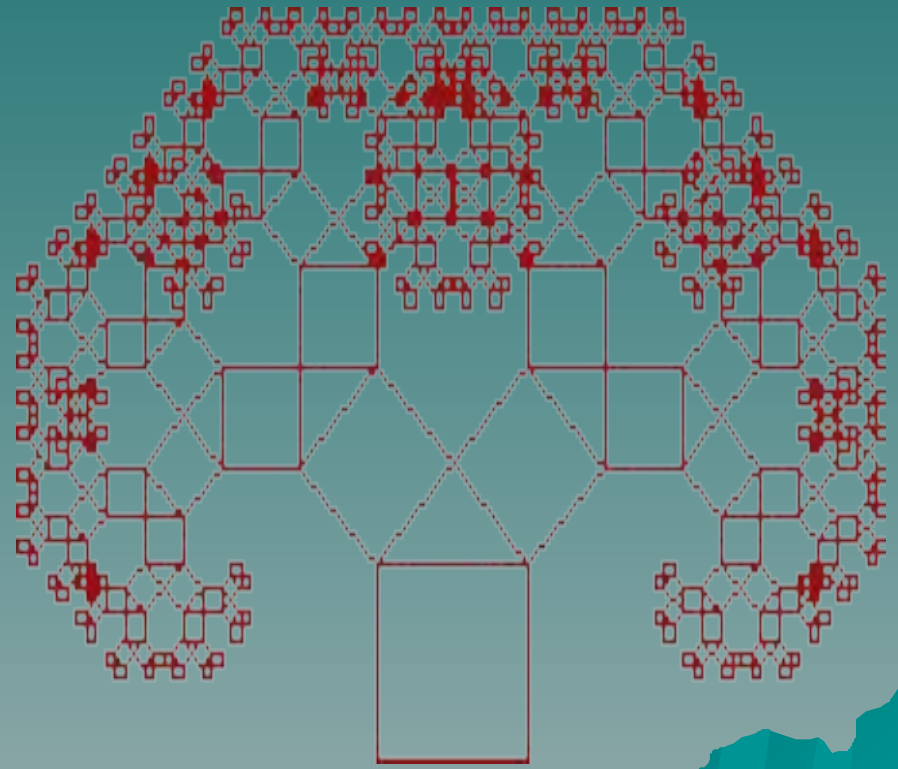
Tree simulation using Brownian motion and fractal called Pythagor Tree

- ◆ Order of leaves and branches is complicated and random, BUT can be emulated by short program of 12 rows.
- ◆ Firstly, we need to generate Pythagor Tree.



Tree simulation using Brownian motion and fractal called Pythagor Tree

- ◆ On this stage Brownian motion is not used.
- ◆ Now, every section is the centre of symmetry
- ◆ Instead of lines are rectangles.
- ◆ But it still looks like artificial



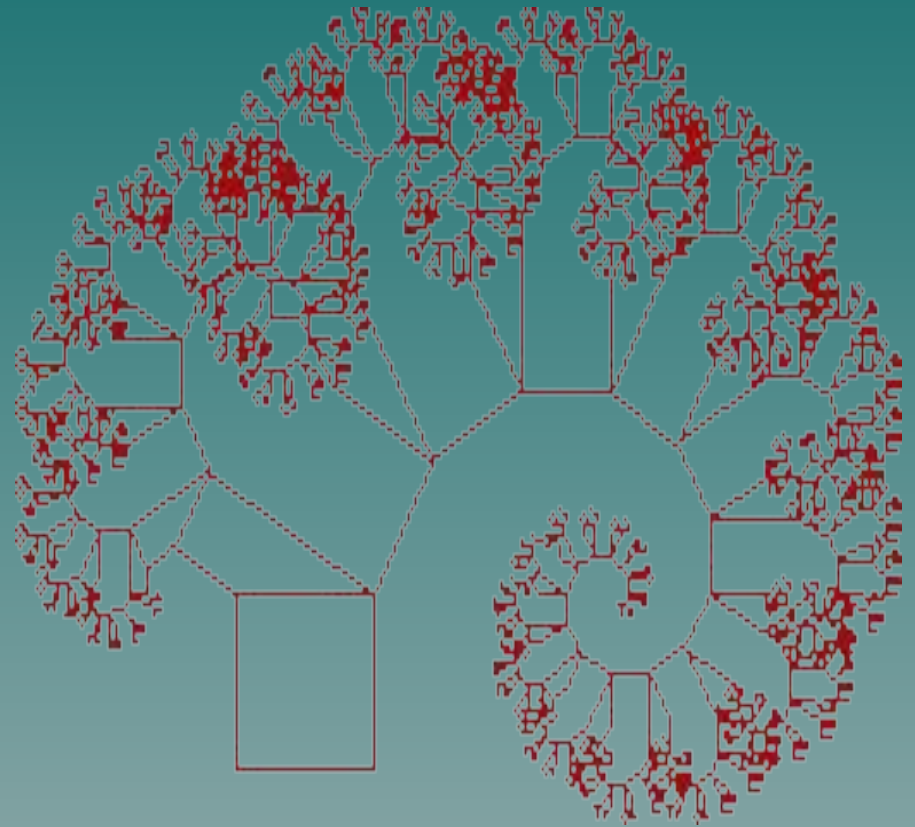
Tree simulation using Brownian motion and fractal called Pythagor Tree

- ◆ Now Brownian motion is used to make randomization
- ◆ Numbers are rounded-up to 2 rank instead of 39



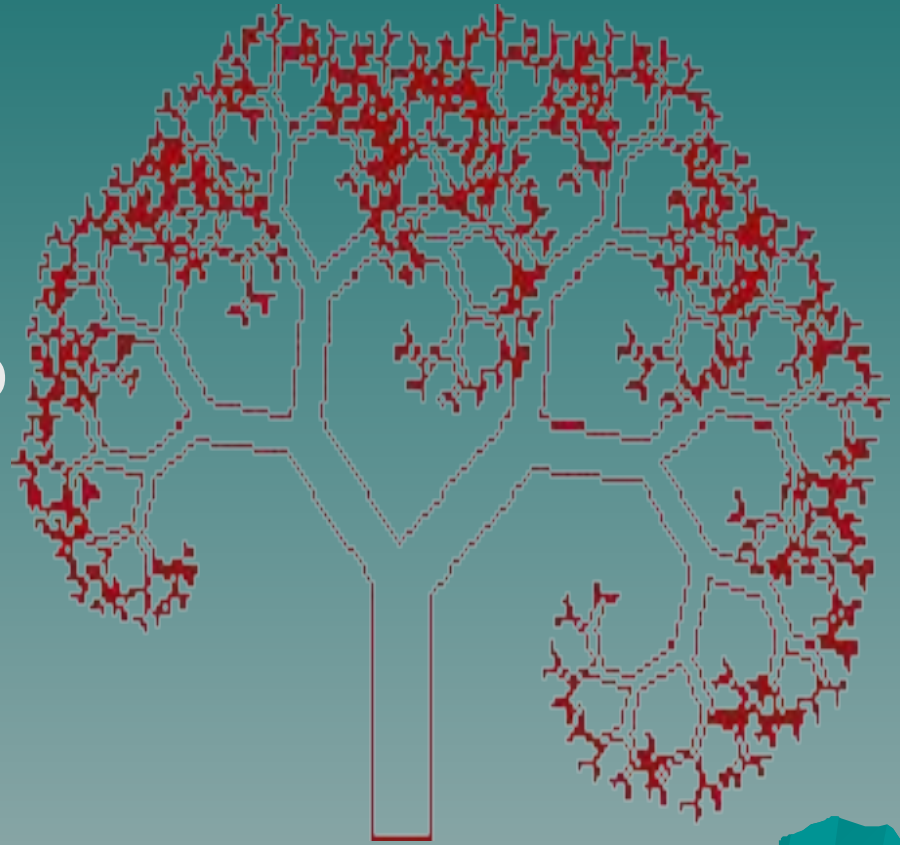
Tree simulation using Brownian motion and fractal called Pythagor Tree

- ◆ Rounded-up to 7 rank
- ◆ Now it looks like logarithmic spiral.




Tree simulation using Brownian motion and fractal called Pythagor Tree


- ◆ To avoid spiral we use Brownian motion twice to the left and only once to the right
- ◆ Now numbers are rounded-up to 24 rank




Fractals and world around

- ◆ Branching, leaves on trees, veins in hand, curving river, stock exchange – all these things are fractals.
 - ◆ Programmers and IT specialists go crazy with fractals. Because, in spite of its beauty and complexity, they can be generated with easy formulas.
 - ◆ Discovery of fractals was discovery of new art aesthetics, science and math, and also revolution in humans world perception.
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What are fractals in reality?

- ◆ Fractal – geometric figure definite part of which is repeating changing its size => principle of self-similarity.
 - ◆ There are a lot of types of fractals
 - ◆ Not just complicated figures generated by computers.
 - ◆ Almost everything which seems to be casual could be fractal, even cloud or little molecule of oxygen.
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How chaos is chaotic?

- ◆ Fractals – part of chaos theory.
 - ◆ Chaotic behaviour, so they seem disorderly and casual.
 - ◆ A lot of aspects of self-similarity inside fractal.
 - ◆ Aim of studying fractals and chaos – to predict regularity in systems, which might be absolutely chaotic.
 - ◆ All world around is fractal-like
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Geometry of 21st century

- ◆ Pioneer, father of fractals was Franco-American professor Benoit B. Mandelbrot.
- ◆ 1960 "Fractal geometry of nature"
- ◆ Purpose was to analyze not smooth and broken forms.
- ◆ Mandelbrot used word "fractal", that meant factionalism of these forms
- ◆ Now Mandelbrot, Clifford A. Pickover, James Gleick, H.O. Peitgen are trying to enlarge area of fractal geometry, so it can be used practical all over the world, from prediction of costs on stock exchange to new discoveries in theoretical physics.


Practical usage of fractals

- ◆ **Computer systems** (Fractal archivation, picture compressing without pixelization)
- ◆ **Liquid mechanics**
 - Modulating of turbulent stream
 - Modulating of tongues of flame
 - Porous material has fractal structure
- ◆ **Telecommunications** (antennas have fractal form)
- ◆ **Surface physics** (for description of surface curvature)
- ◆ **Medicine**
 - Biosensor interaction
 - Heart beating
- ◆ **Biology** (description of population model)

Fractal dimension: hidden dimensions

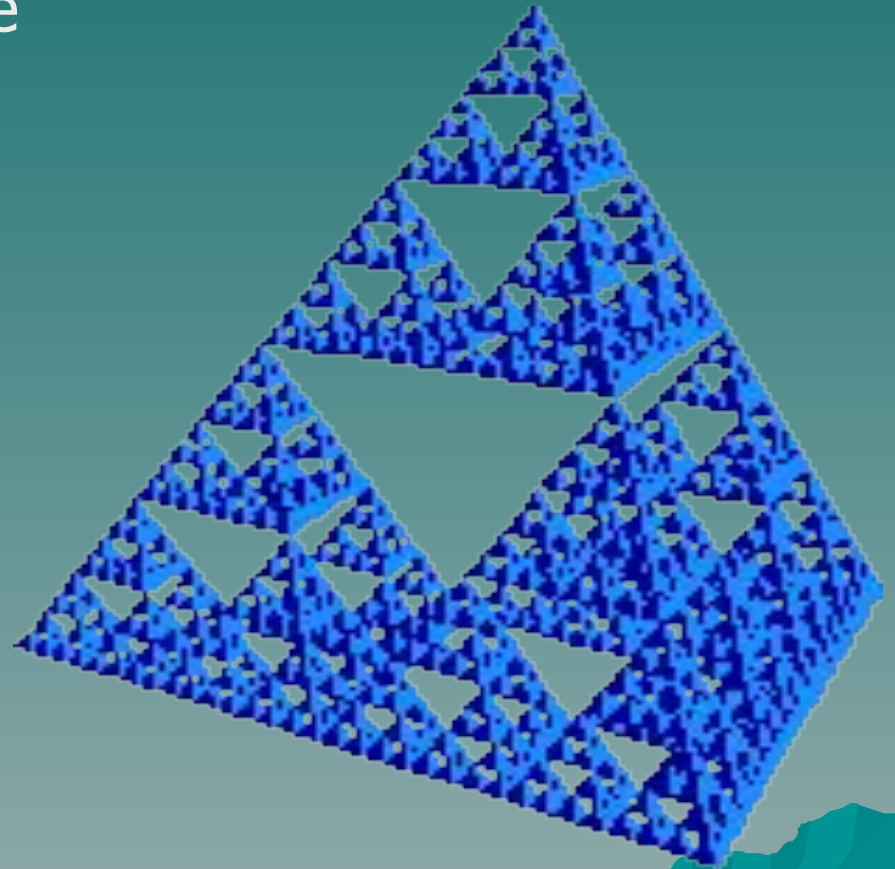
- ◆ Mandelbrot called not intact dimensions – fractal dimensions (for example 2.76)
 - ◆ Euclid geometry claims that space is straight and flat.
 - ◆ Object which has 3 dimensions correctly is impossible
 - ◆ Examples: Great Britain coastline, human body
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Deterministic fractals

- ◆ First opened fractals.
 - ◆ Self-similarity because of method of generation
 - ◆ Classic fractals, geometric fractals, linear fractals
 - ◆ Creation starts from initiator – basic picture
 - ◆ Process of iteration – adding basic picture to every result
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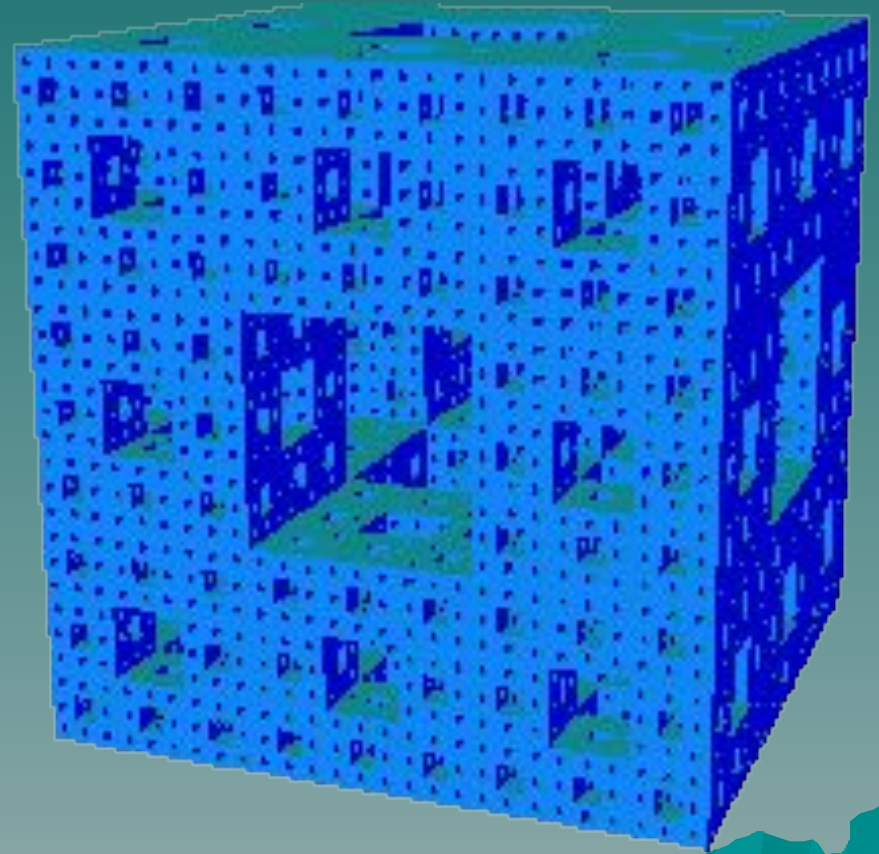
Sierpinski lattice

- ◆ Triangles made of interconnection of middle points of large triangle cut from main triangle, generating triangle with large amount of holes.
- ◆ Initiator – large triangle.
- ◆ Generator – process of cutting triangles similar to given triangle.
- ◆ Fractal dimension is 1.584962501



Sierpinski sponge

- ◆ Plane fractal cell without square, but with unlimited ties
- ◆ Would be used as building constructions



Sierpinski fractal

- ◆ Don't mix up this fractal with Sierpinski lattice.
- ◆ Initiator and generator are the same.
- ◆ Fractal dimension is 2.0



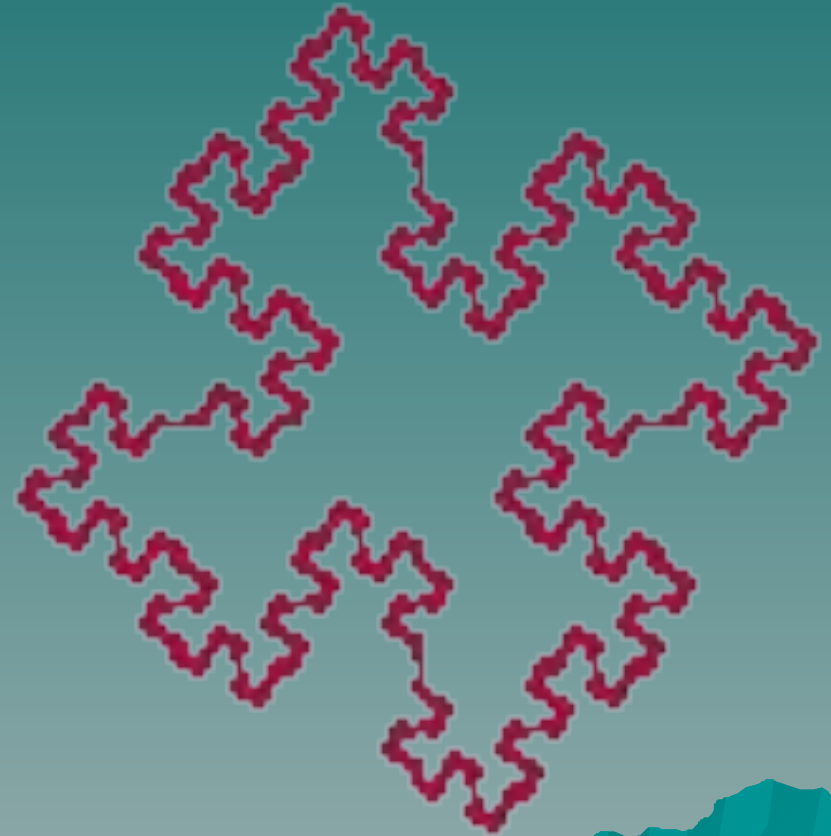
Koch Curve



- ◆ One of the most typical fractals.
- ◆ Invented by german mathematic Helge fon Koch
- ◆ Initiator – straight line. Generator – equilateral triangle.
- ◆ Mandelbrot was making experiments with Koch Curve and had as a result Koch Islands, Koch Crosses, Koch Crystals, and also Koch Curve in 3D
- ◆ Fractal dimension is 1.261859507

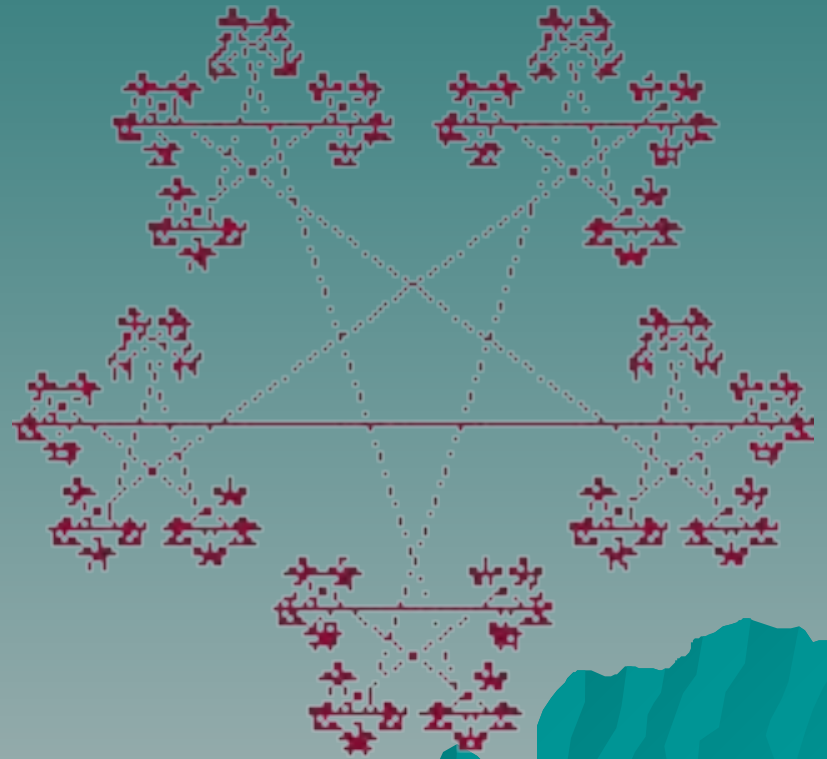
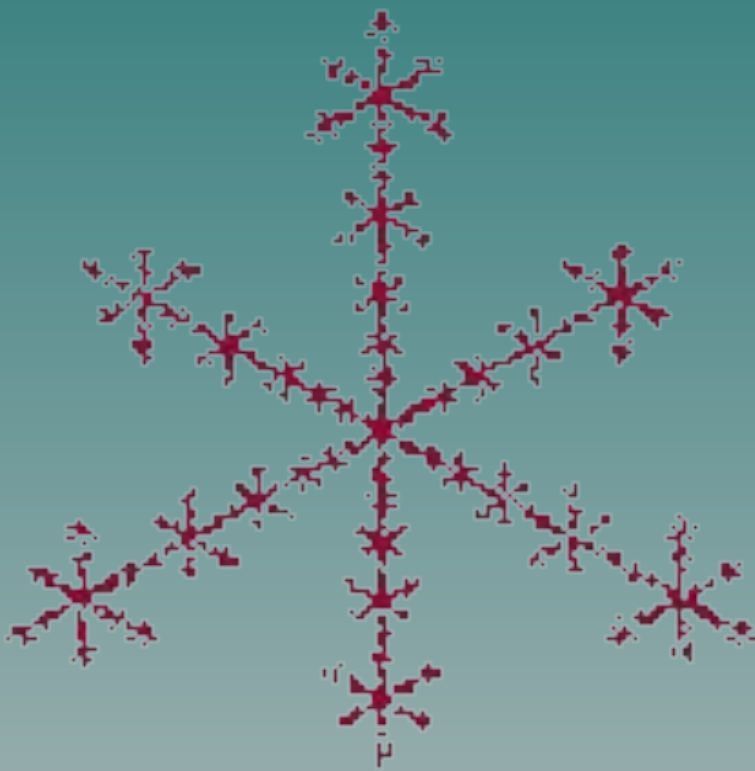
Mandelbrot fractal

- ◆ Variant of Koch Curve
- ◆ Initiator and generator are different from Koch's, but idea is still the same.
- ◆ Fractal takes half of plane.
- ◆ Fractal dimension is 1.5



Snow Crystal and Star

- ◆ These objects are classical fractals.
- ◆ Initiator and generator is one figure



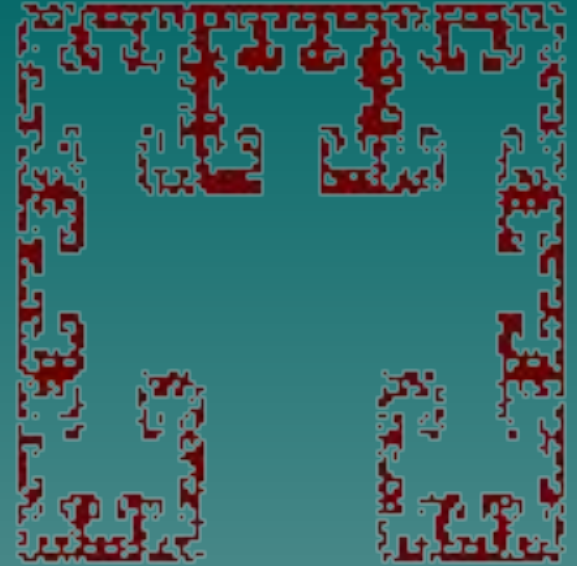
Minkovskij sausage

- ◆ Inventor is German Minkovskij.
- ◆ Initiator and generator are quite sophisticated, are made of row of straight corners and segments with different length.
- ◆ Initiator has 8 parts.
- ◆ Fractal dimension is 1.5



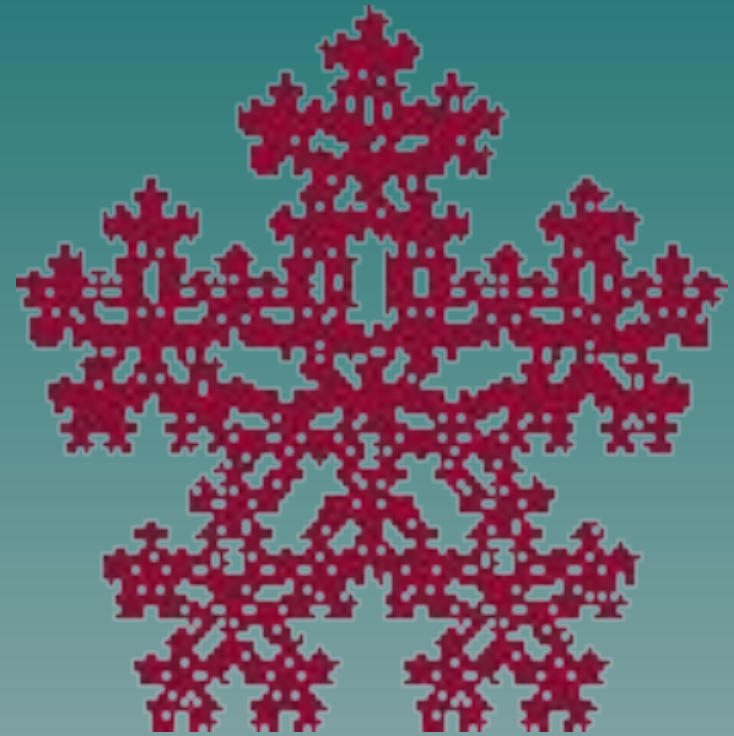
Labyrinth

- ◆ Sometimes called H-tree.
- ◆ Initiator and generator has shape of letter H
- ◆ To see it easier the H form is not painted in the picture.
- ◆ Because of changing thickness, dimension on the tip is 2.0, but elements between tips it is changing from 1.333 to 1.6667



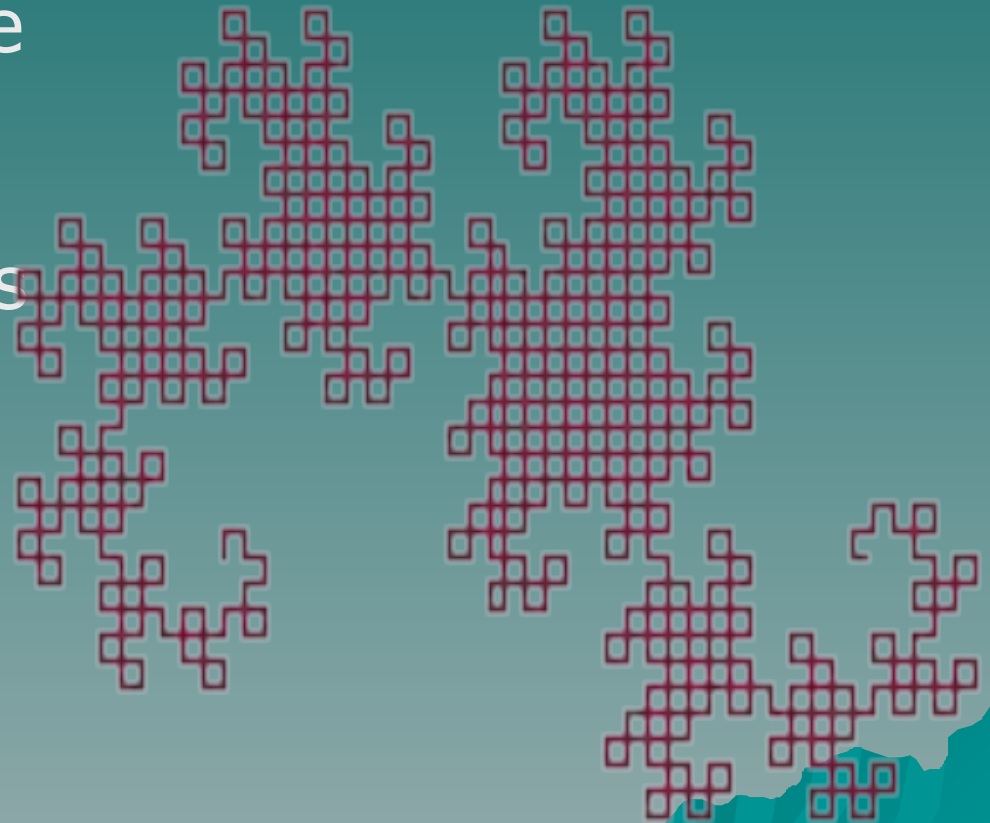
Dar'er pentagon

- ◆ Pentagon as initiator
- ◆ Isosceles triangle as generator
- ◆ Hexagon is a variant of this fractal (David Star)
- ◆ Fractal dimension is 1.86171



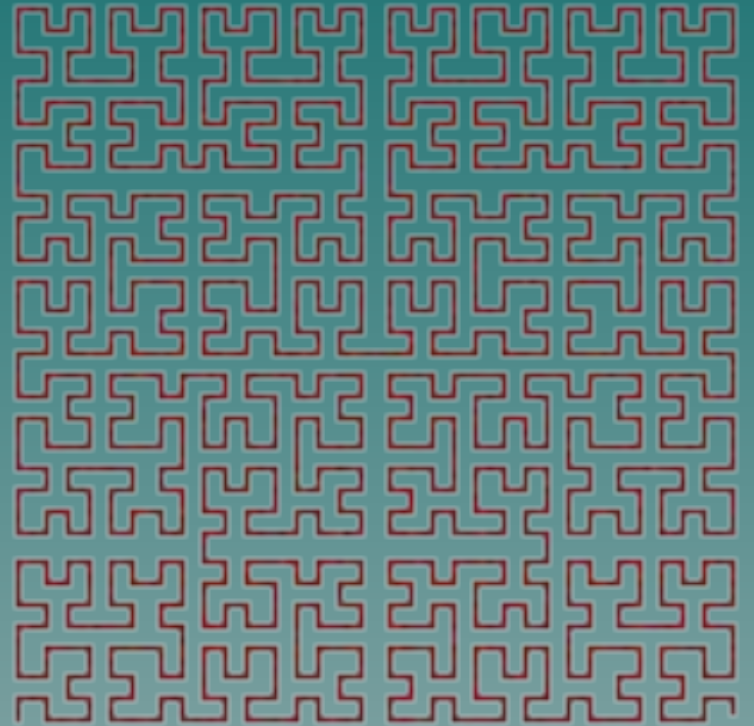
Dragon curve

- ◆ Invented by Italian mathematician Giuseppe Peano.
- ◆ Looks like Minkovskij sausage, because has the same generator and easier initiator.
- ◆ Mandelbrot called it River of Double Dragon.
- ◆ Fractal dimension is 1.5236



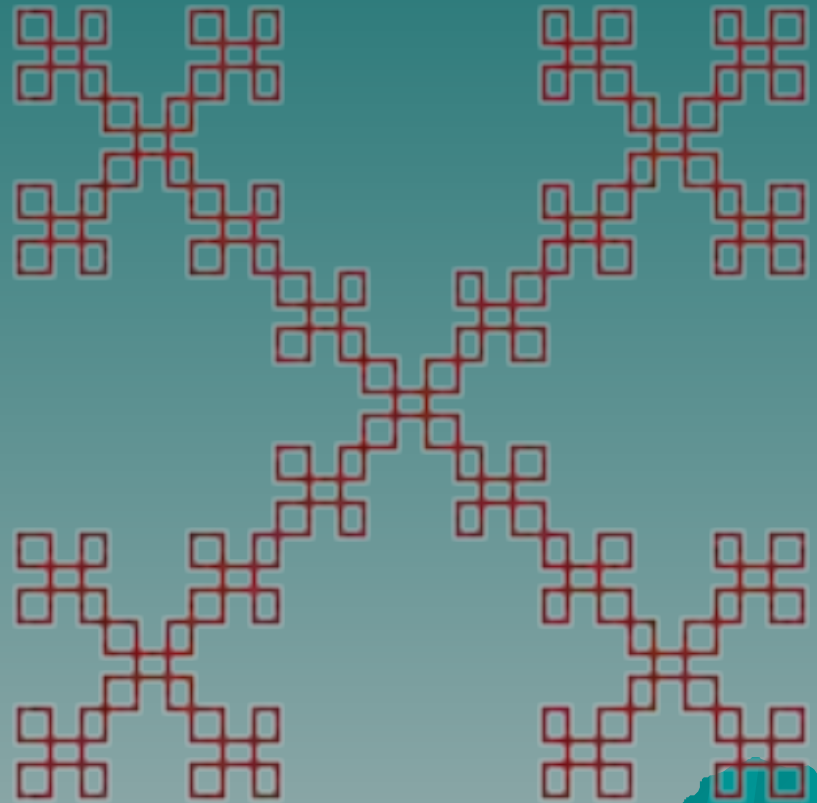
Hilbert curve

- ◆ Looks like labyrinth, but letter “U” is used and width is not changing.
- ◆ Fractal dimension is 2.0
- ◆ Endless iteration could take all plane.




Box

- ◆ Very simple fractal
- ◆ Made by adding squares to the top of other squares.
- ◆ Initiator and generator and squares.
- ◆ Fractal dimension is 1.892789261

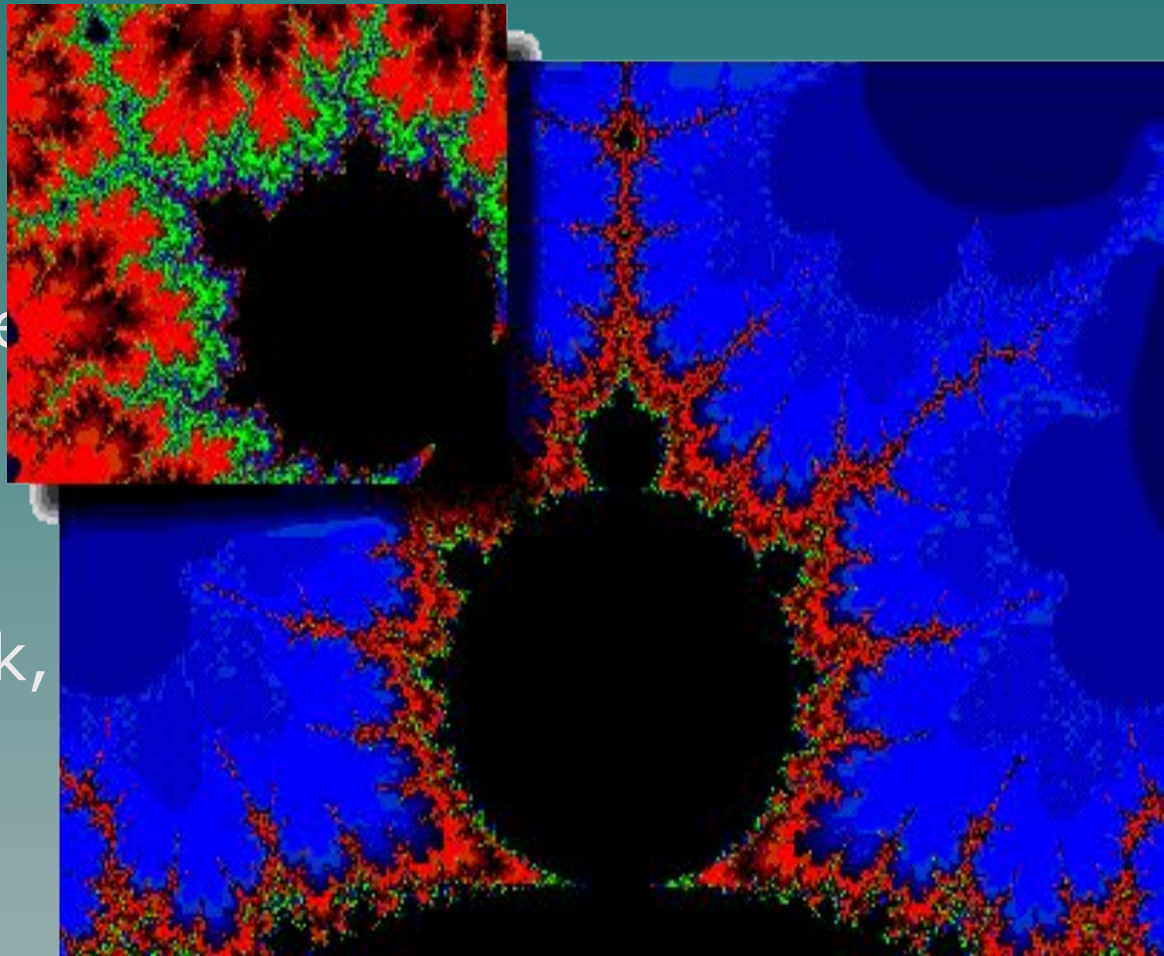


Sophisticated fractals

- ◆ Most fractals which you can meet in a real life are not deterministic.
 - ◆ Not linear and not compiled from periodic geometrical forms.
 - ◆ Practically even enlarged part of sophisticated fractal is different from initial fractal. They looks the same but not almost identical.
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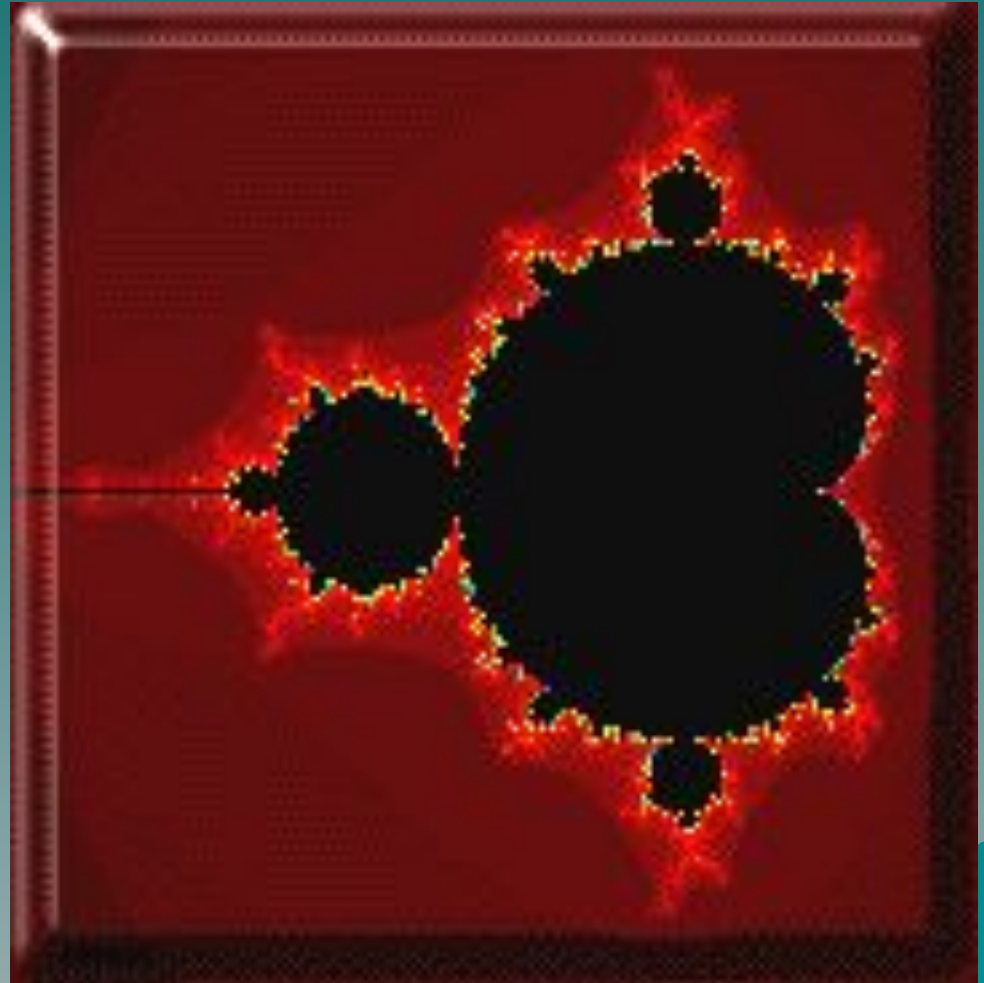
Sophisticated fractals

- ◆ Are generated by non linear algebraic equations.
- ◆ $Z_{n+1} = Z_n I + C$
- ◆ Solution involves complex and suppose numbers
- ◆ Self-similarity on different scale levels
- ◆ Stable results – black, for different speed different color



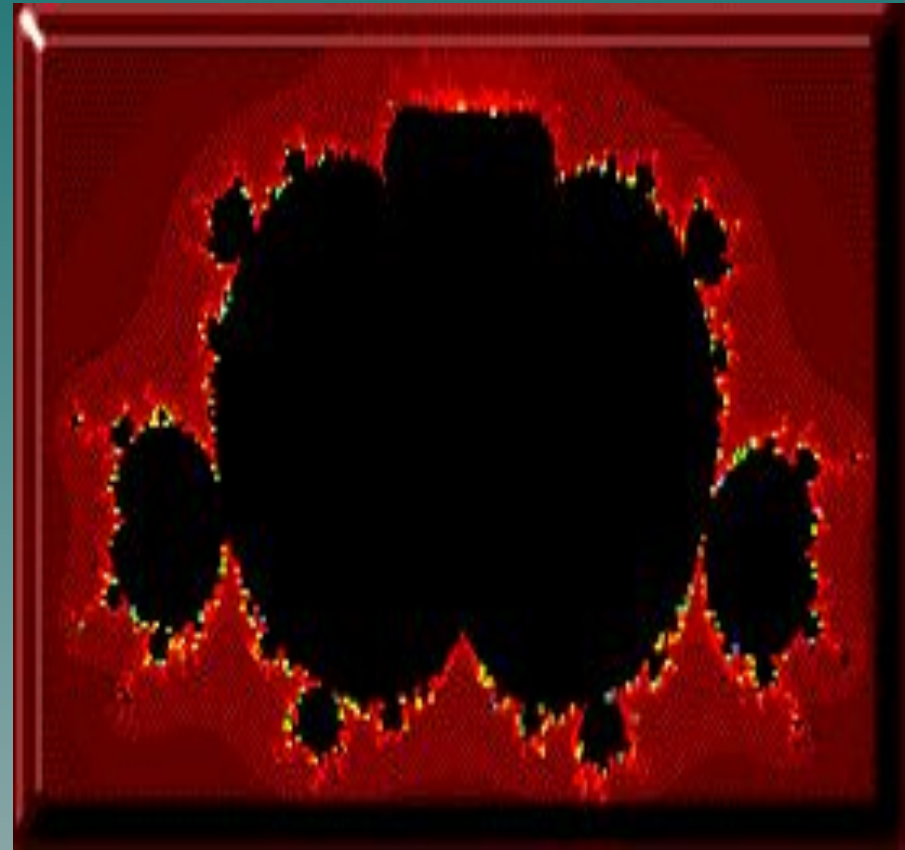
Mandelbrot multitude

- ◆ Most widespread sophisticated fractal
- ◆ $Z_{n+1} = Z_n^a + C$
- ◆ Z and C – complex numbers
- ◆ a – any positive number.



Mandelbrot multitude

- ◆ $Z = Z * \text{tg}(Z + C)$.
- ◆ Because of Tangent function it looks like Apple.
- ◆ If we switch Cosine it will look like Air Bubbles.
- ◆ So there are different properties for Mandelbrot multitude.



Zhulia multitude

- ◆ Has the same formula as Mandelbrot multitude.
- ◆ If building fractal with different initial points, we will have different pictures.
- ◆ Every dot in Mandelbrot multitude corresponds to Zhulia multitude

