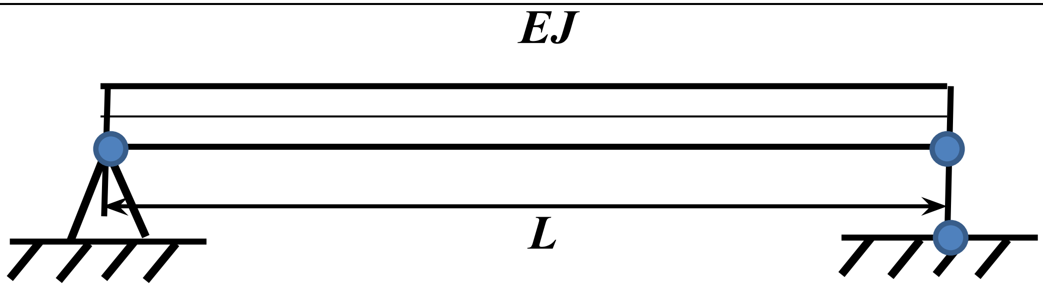
Семинар 15. Изгибные колебания стержня (самостоятельная)

Определить собственные частоты и формы изгибных колебаний стержня



1. Записать уравнение в частных производных.

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho F}{EJ}\frac{\partial^2 w}{\partial t^2} = 0 \quad (10.2)$$

2. Записать решение, разделяя переменные по времени и координате

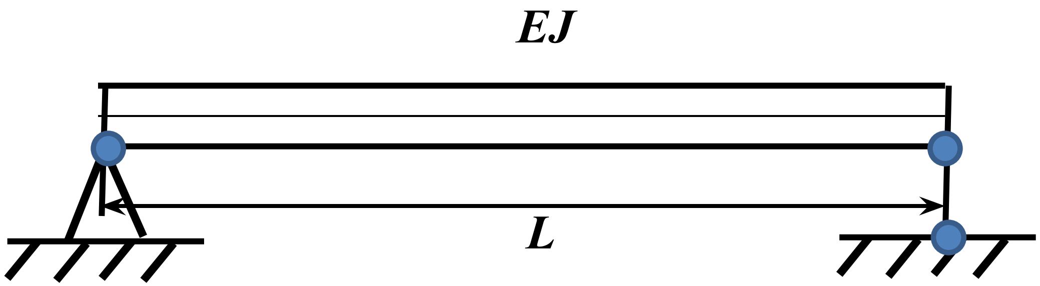
$$w(x,t) = W(x)\sin\omega t \quad (10.3)$$

3. Подстановкой привести к обыкновенному дифференциальному уравнению относительно форм колебаний

$$W^{IV} - \beta^4 W = 0 \quad (10.4)$$

4. Записать решение для форм колебаний

$$W(x) = C_1 \sin\beta x + C_2 \cos\beta x + C_3 sh\beta x + C_4 ch\beta x \quad (10.5)$$



5. Записать граничные условия относительно форм колебаний

$$w(0)=0, \quad EJ\frac{\partial^2 w}{\partial x^2}(0)=0, \quad w(L)=0, \quad EJ\frac{\partial^2 w}{\partial x^2}(L)=0$$

6. Составить условие ненулевого решения для определения собственных частот

$$W(x)=C_1\sin\beta x+C_2\cos\beta x+C_3 sh\beta x+C_4 ch\beta x$$

$$W''(x)=-C_1\beta^2\sin\beta x-C_2\beta^2\cos\beta x+C_3\beta^2 sh\beta x+C_4\beta^2 ch\beta x$$

6. Составить условие ненулевого решения для определения собственных частот

$$W(x) = C_1 \sin \beta x + C_2 \cos \beta x + C_3 sh\beta x + C_4 ch\beta x$$

$$W''(x) = -C_1\beta^2 \sin \beta x - C_2\beta^2 \cos \beta x + C_3\beta^2 sh\beta x + C_4\beta^2 ch\beta x$$

$$W(0) = 0 + C_2 * 1 + 0 + C_4 * 1 = 0$$

$$W''(0) = 0 - C_2 * 1 + 0 + C_4 * 1 = 0$$

$$C_2 + C_4 = 0 \qquad C_2 = 0 \quad C_4 = 0$$

$$-C_2 + C_4 = 0$$

$$W(x) = C_1 \sin \beta x + C_3 sh\beta x$$

$$W''(x) = -C_1\beta^2 \sin \beta x + C_3\beta^2 sh\beta x$$

7. Записать выражение для определения собственных частот

$$W(L) = C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$W''(L) = -C_1 \sin \beta L + C_3 sh\beta L = 0$$

7. Записать выражение для определения собственных частот

$$C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$-C_1 \sin \beta L + C_3 sh\beta L = 0$$

Преобразования

$$C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$+$$

$$-C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$+$$

$$-C_1 \sin \beta L + C_3 sh\beta L = 0$$

$$C_3 = 0$$

$$\sin \beta L = 0$$

$$\beta L = n\pi \quad n = 1,2,\ldots.. \quad \beta^4 = \frac{\rho F \omega^2}{EJ}$$

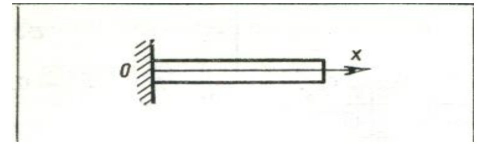
$$\omega_n = \frac{n^2 \pi^2}{L^2}\sqrt{\frac{EJ}{\rho F}}$$

8. Записать выражение для собственных форм колебаний

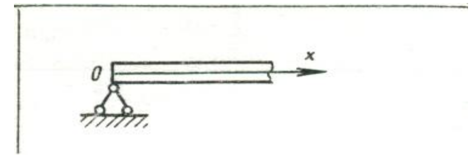
$$W(x) = C_1 \sin \beta x$$

Основные типы краевых условий для изгибных колебаний стержней

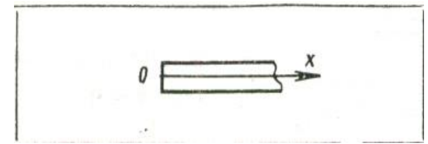
$$1.\ w = 0, \quad \frac{\partial w}{\partial x} = 0$$



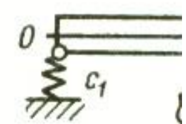
$$2.\ w = 0, \quad EJ\frac{\partial^2 w}{\partial x^2} = 0$$



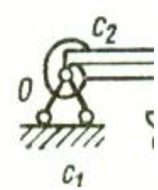
$$3.\ \frac{\partial}{\partial x}\left(EJ\frac{\partial^2 w}{\partial x^2}\right) = 0, \quad EJ\frac{\partial^2 w}{\partial x^2} = 0$$



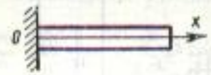
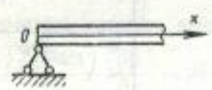
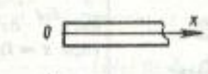
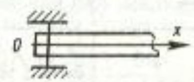
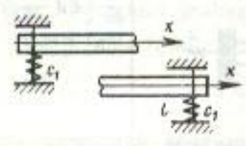
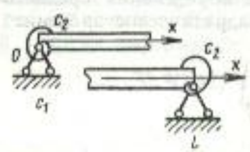
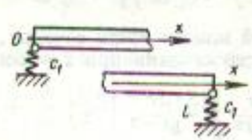
$$4.\ \frac{\partial}{\partial x}\left(EJ\frac{\partial^2 w}{\partial x^2}\right) - c_1 w = 0, \quad EJ\frac{\partial^2 w}{\partial x^2} = 0$$

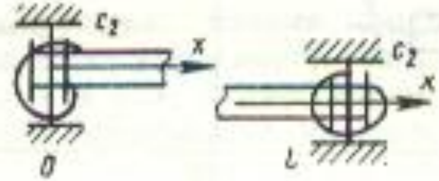
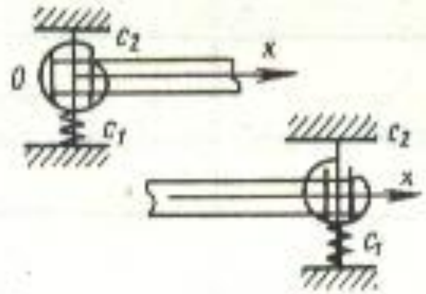
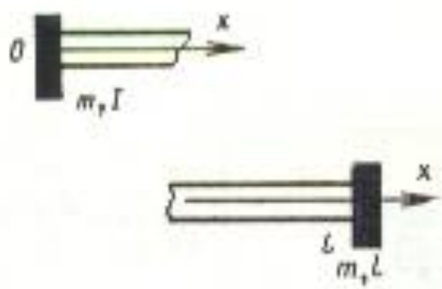


$$5.\ w = 0, \quad EJ\frac{\partial^2 w}{\partial x^2} - c_2\frac{\partial w}{\partial x} = 0$$



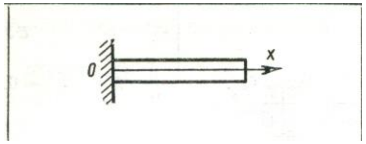
6. Основные типы краевых условий для изгибных колебаний стержней

Вид закрепления	Схема	Условия при $x = 0$
Заделка		$w = 0, \quad \dfrac{\partial w}{\partial x} = 0$
Свободное опирание		$w = 0, \quad EJ\dfrac{\partial^2 w}{\partial x^2} = 0$
Свободный конец		$\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) = 0, \quad EJ\dfrac{\partial^2 w}{\partial x^2} = 0$
Плавающая заделка		$\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) = 0, \quad \dfrac{\partial w}{\partial x} = 0$
		$\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) - c_1 w = 0,$ $\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) + c_1 w = 0,$ (при $x = l$) $\dfrac{\partial w}{\partial x} = 0$
Упругое закрепление		$w = 0, \quad EJ\dfrac{\partial^2 w}{\partial x^2} - c_2\dfrac{\partial w}{\partial x} = 0,$ $EJ\dfrac{\partial^2 w}{\partial x^2} + c_2\dfrac{\partial w}{\partial x} = 0$ (при $x = l$)
		$\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) - c_1 w = 0,$ $\dfrac{\partial}{\partial x}\left(EJ\dfrac{\partial^2 w}{\partial x^2}\right) + c_1 w = 0,$ (при $x = l$) $EJ\dfrac{\partial^2 w}{\partial x^2} = 0$

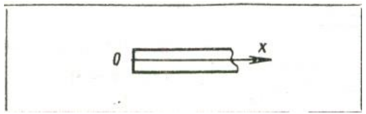
Вид закрепления	Схема	Условия при $x = 0$
Упругое закрепление		$EJ\,\dfrac{\partial^2 w}{\partial x^2} - c_2\,\dfrac{\partial w}{\partial x} = 0,$ $\dfrac{\partial}{\partial x}\left(EJ\,\dfrac{\partial^2 w}{\partial x^2}\right) = 0,$ $EJ\,\dfrac{\partial^2 w}{\partial x^2} + c_2\,\dfrac{\partial w}{\partial x} = 0$ (при $x = l$)
		$\dfrac{\partial}{\partial x}\left(EJ\,\dfrac{\partial^2 w}{\partial x^2}\right) - c_1 w = 0,$ $EJ\,\dfrac{\partial^2 w}{\partial x^2} - c_2\,\dfrac{\partial w}{\partial x} = 0$ $\dfrac{\partial}{\partial x}\left(EJ\,\dfrac{\partial^2 w}{\partial x^2}\right) + c_1 w = 0,$ (при $x = l$) $EJ\,\dfrac{\partial^2 w}{\partial x^2} + c_2\,\dfrac{\partial w}{\partial x} = 0$ (при $x = l$)
Сосредоточенный инерционный элемент на конце		$\dfrac{\partial}{\partial x}\left(EJ\,\dfrac{\partial^2 w}{\partial x^2}\right) = m\,\dfrac{\partial^2 w}{\partial t^2}$ $EJ\,\dfrac{\partial^2 w}{\partial x^2} = I\,\dfrac{\partial^3 w}{\partial x \partial t^2}$ $\dfrac{\partial}{\partial x}\left(EJ\,\dfrac{\partial^2 w}{\partial x^2}\right) = -\,m\,\dfrac{\partial^2 w}{\partial t^2}$ (при $x = l$) $EJ\,\dfrac{\partial^2 w}{\partial x^2} = -\,I\,\dfrac{\partial^3 w}{\partial x \partial t^2}$ (при $x = l$)

Основные типы краевых условий для продольных колебаний стержней

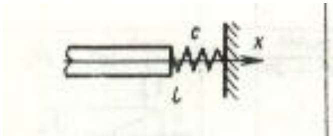
1. $u = 0 \quad при \quad x=0$



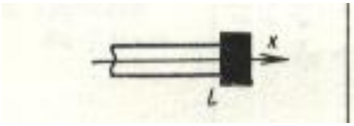
$$2.\ EF\left(\frac{\partial u}{\partial x}\right) = 0 \quad при\ x = 0$$



$$4.2.\ EF\left(\frac{\partial u}{\partial x}\right) + cu = 0 \quad при\ x = L$$

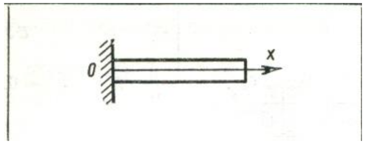


$$5.2.\ EF\left(\frac{\partial u}{\partial x}\right) = -M\frac{\partial^2 u}{\partial t^2} \quad при\ x = L$$

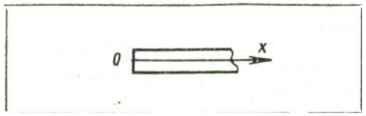


Основные типы краевых условий для крутильных колебаний стержней

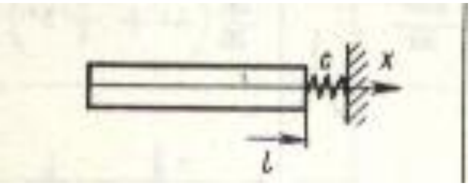
1. $\theta = 0 \quad при\ x = 0$



2. $GJ_к \left(\dfrac{\partial \theta}{\partial x} \right) = 0 \quad при\ x = 0$



4.2. $GJ_к \left(\dfrac{\partial \theta}{\partial x} \right) + c\theta = 0 \quad при\ x = L$



5.2. $GJ_к \left(\dfrac{\partial \theta}{\partial x} \right) = -I_x \dfrac{\partial^2 \theta}{\partial t^2} \quad при\ x = L$