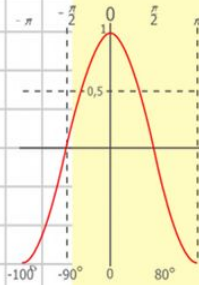
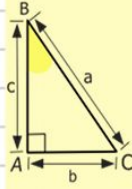
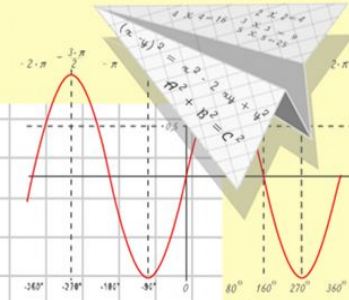
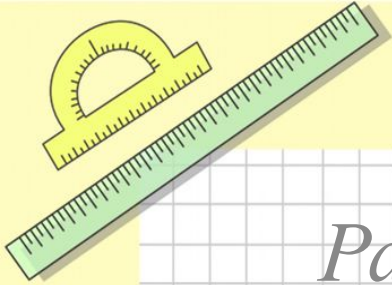


Математик

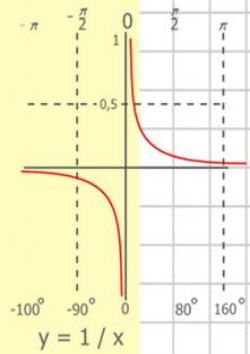
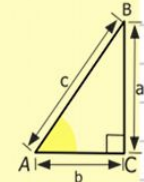
Раздел 7. ^аФункции и графики

Занятие 70. Схема исследования функции



$y = \cos x$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

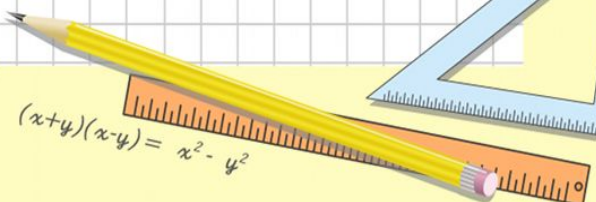
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$$\sin 90^\circ = 1$$



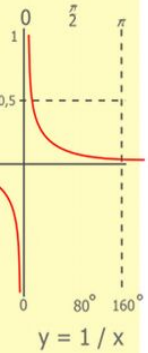
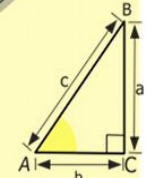
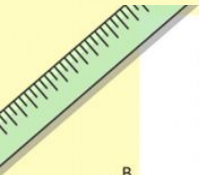
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



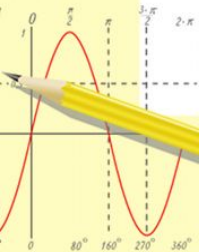
$$(x+y)(x-y) = x^2 - y^2$$

Свойства функций

1. Область определения $D(f)$
2. Область значений $E(f)$
3. Нули функции
4. Промежутки знакопостоянства
5. Промежутки монотонности
6. Экстремумы
7. Четность/нечетность
8. Периодичность (если есть)
9. Уравнения асимптот (если есть)



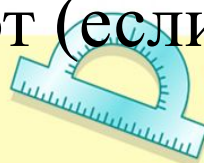
$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

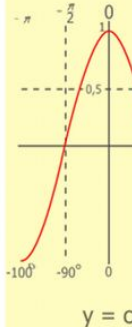
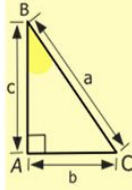
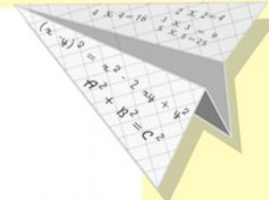
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

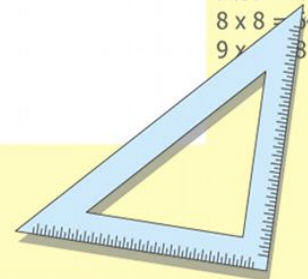


$$\begin{cases} y = \sin 90^\circ \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

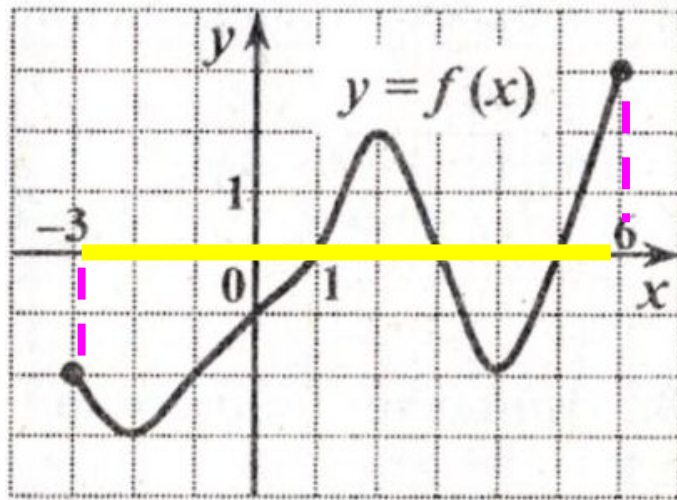


Область определения

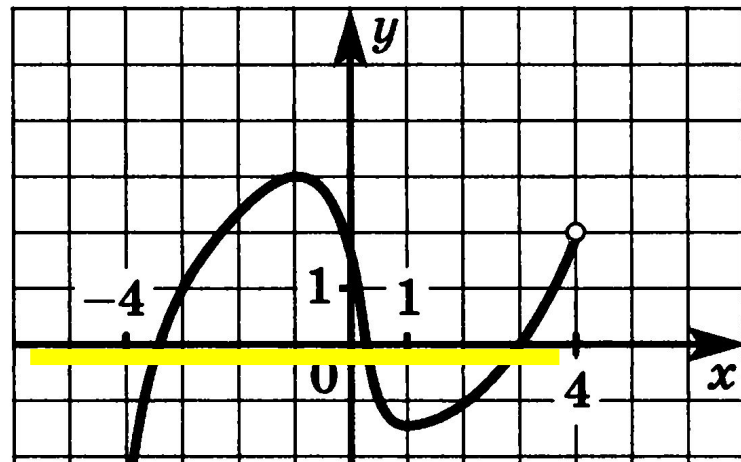
Область определения функции – это множество всех возможных значений переменной x .

$$D(f)$$

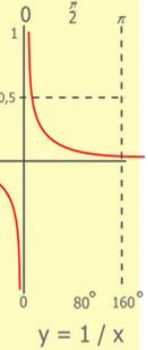
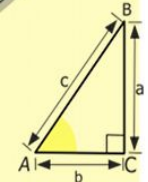
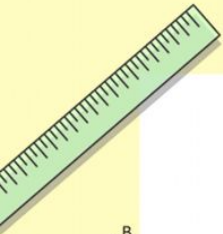
Если имеется график функции, то его область определения – это проекция графика на ось Ox .



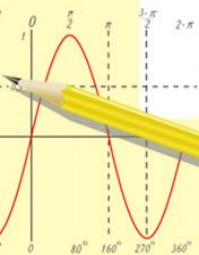
$$D(f) = [-3; 6]$$



$$D(f) = (-\infty; 4)$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

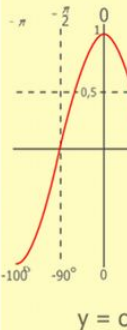
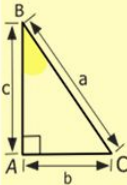
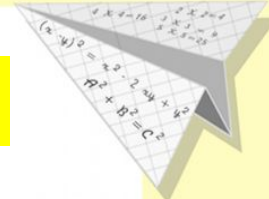
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

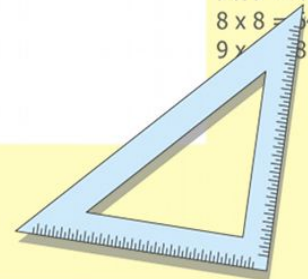
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

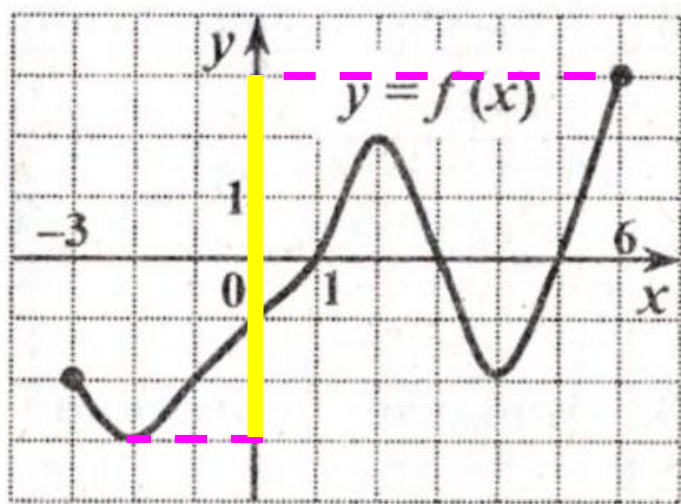


Область значений

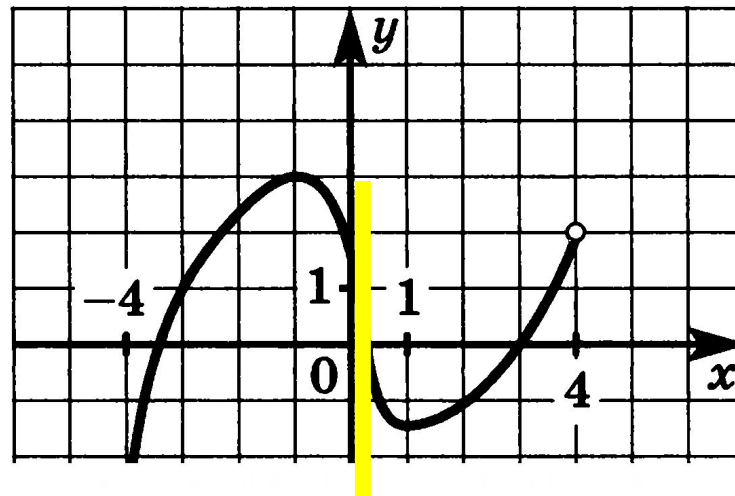
Область значений функции – это множество всех возможных значений переменной y .

$$E(f)$$

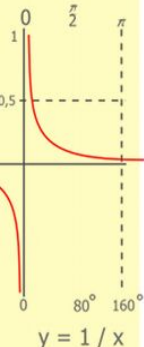
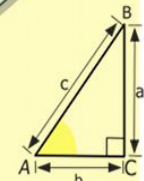
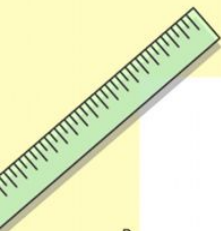
Если имеется график функции, то его область определения – это проекция графика на ось Ox .



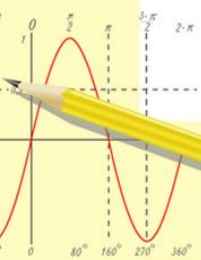
$$E(f) = [-3; 3]$$



$$E(f) = (-\infty; 3]$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 210 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

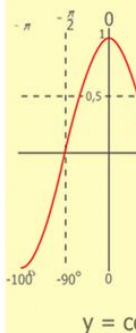
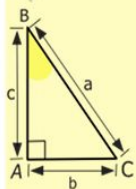
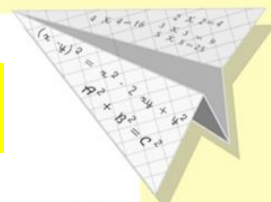
$$\sin 90^\circ = 1$$



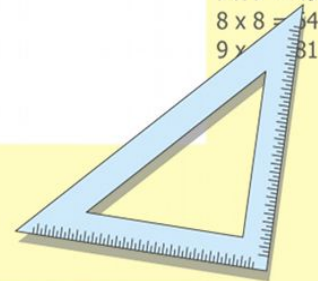
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

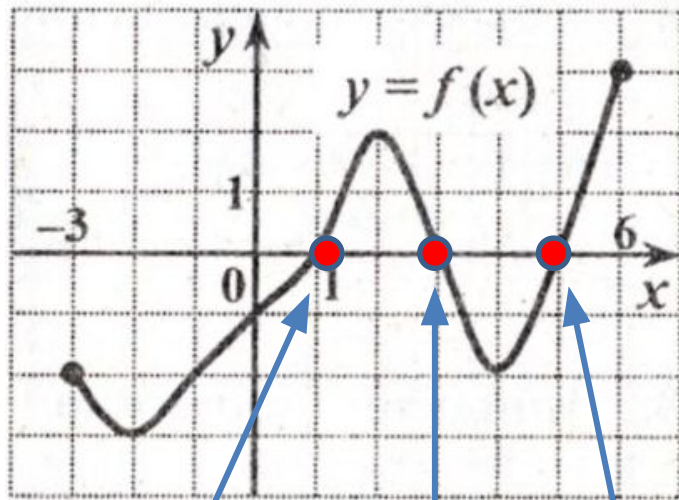


Нули функции

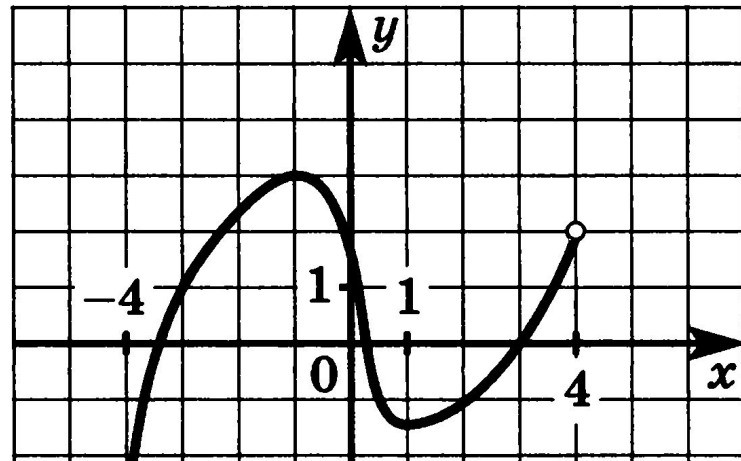
Нули функции – это значения переменной x , при которых переменная y становится равной 0, т.е. выполняется условие

$$f(x) = 0$$

Если имеется график функции, то нули функции – это точки пересечения графика с осью Ox .



$$x_1 = 1 \quad x_2 = 3 \quad x_3 = 5$$



$$x_1 \approx -3,5 \quad x_2 \approx 0,3 \quad x_3 = 3$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

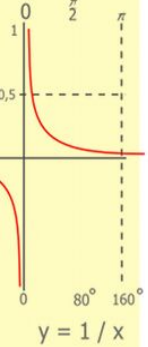
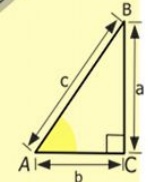
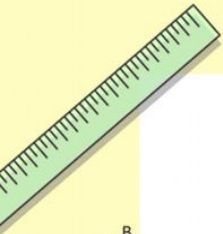
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

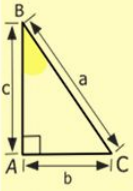
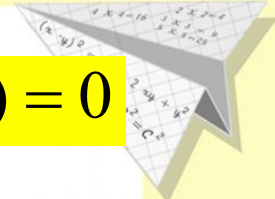
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

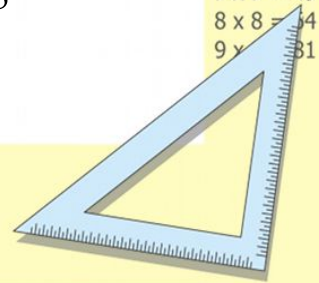
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Промежутки знакопостоянства

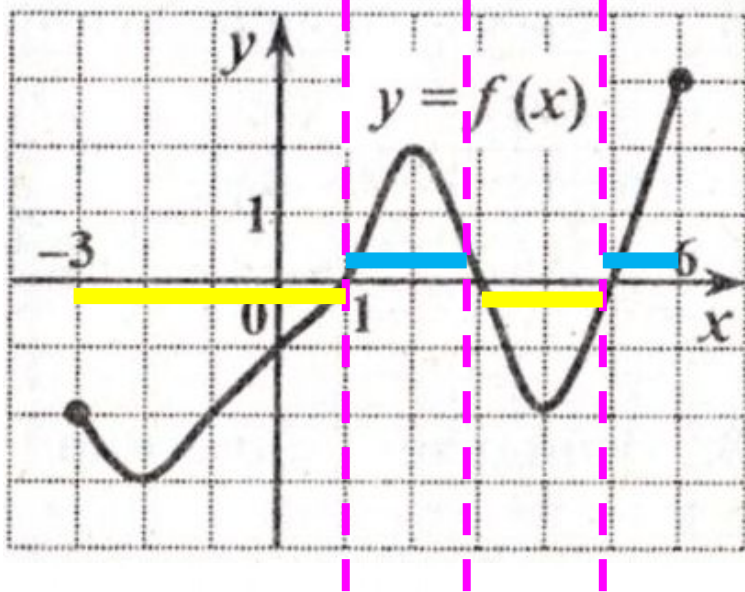
Промежутки знакопостоянства – это значения переменной x , при которых значения функции (переменная y) сохраняет постоянный знак, т.е. выполняются условия:

Если имеется график функции, то промежутки знакопостоянства – это промежутки на оси Ox , где часть графика находится целиком над осью Ox , либо под осью Ox .

$$f(x) > 0$$

либо

$$f(x) < 0$$



$$y > 0 \text{ при } x \in (1; 3) \cup (5; 6]$$

$$y < 0 \text{ при } x \in [-3; 1) \cup (3; 5)$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

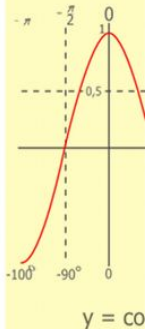
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

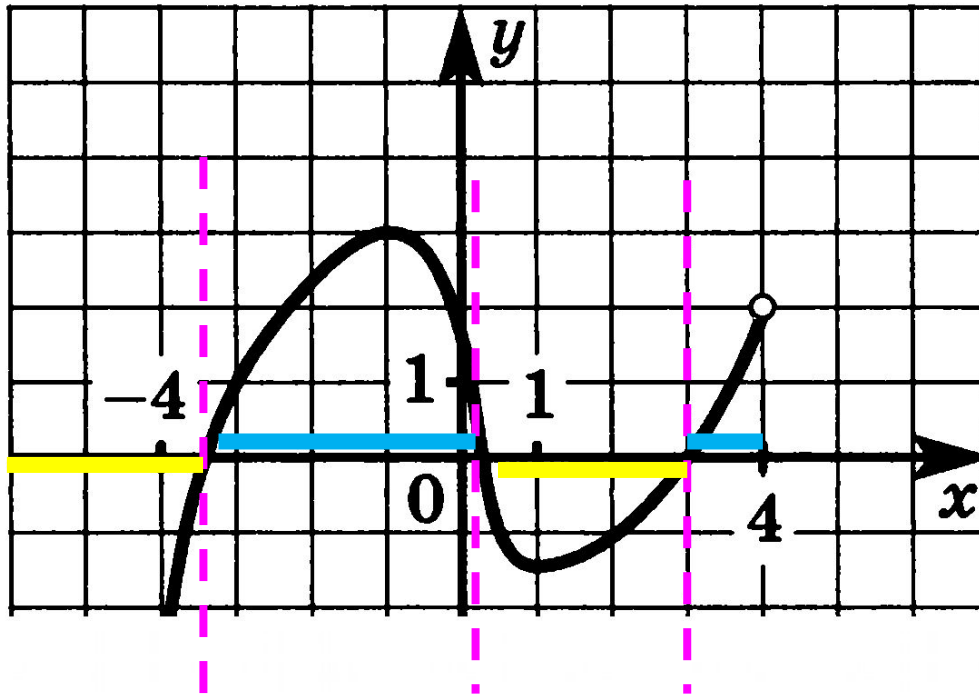
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



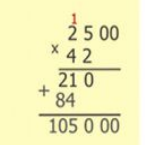
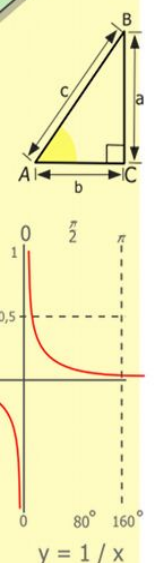
- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81

Промежутки знакопостоянства



$$y > 0 \text{ при } x \in (-3,5; 0,3) \boxtimes (3; 4)$$

$$y < 0 \text{ при } x \in (-\infty; -3,5) \boxtimes (0,3; 3)$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

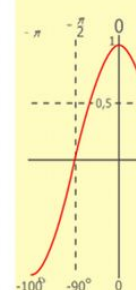
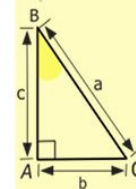
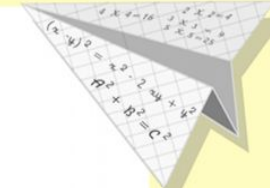


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

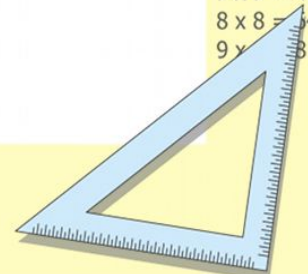
$$\frac{x}{70}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



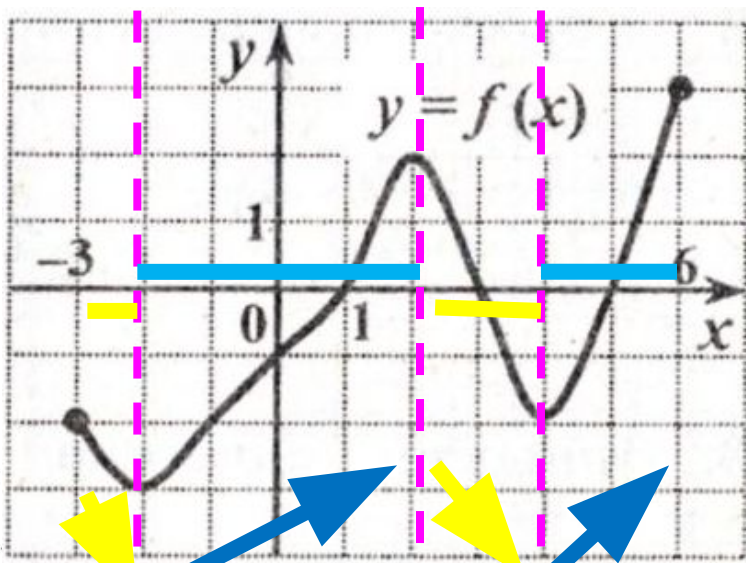
Промежутки монотонности

Промежуток возрастания функции – это промежуток значений переменной x , в котором выполняется условие: большему x

$$x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$$

Промежуток убывания функции – это промежуток значений переменной x , в котором выполняется условие: большему x

$$x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$$



$$f \uparrow (-2; 2), (4; 6)$$

$$f \downarrow (-3; -2), (2; 4)$$

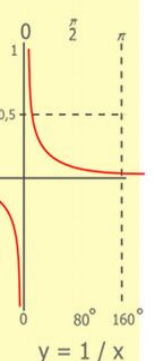
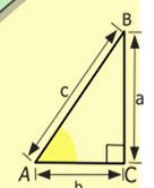
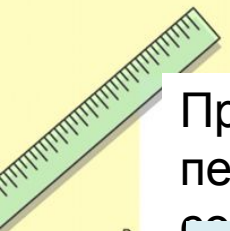
$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

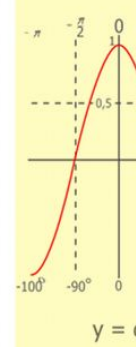
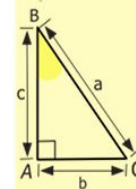
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

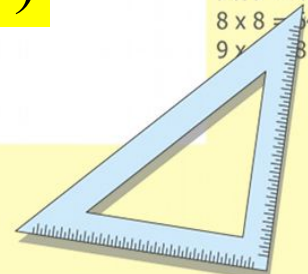
$$(x+y)(x-y) = x^2 - y^2$$



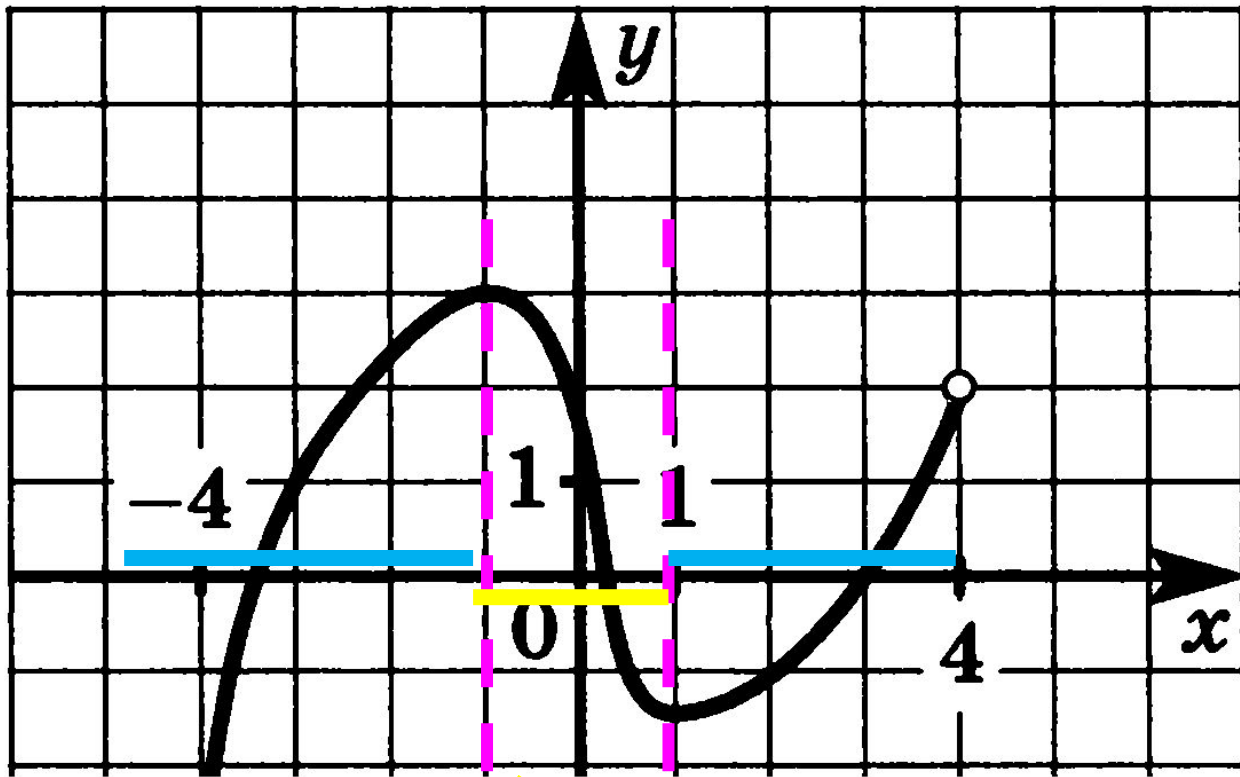
$$\begin{array}{r} 1\ 5\ 00 \\ \times 42 \\ \hline 21\ 0 \\ + 84 \\ \hline 105\ 0\ 00 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

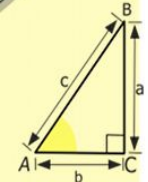
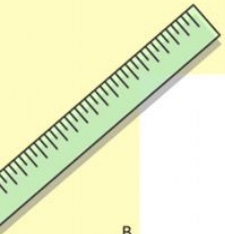


Промежутки монотонности

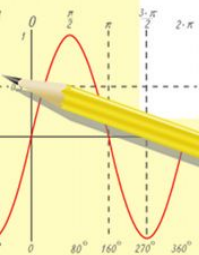


$$f \uparrow (-\infty; -1), (1; 4)$$

$$f \downarrow (-1; 1)$$



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 4\ 2 \\ \hline 21\ 0 \\ + 84 \\ \hline 105\ 0\ 00 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

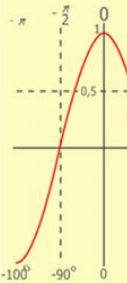
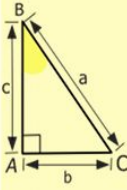
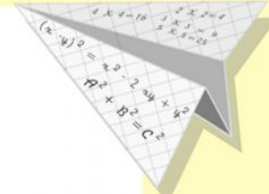


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

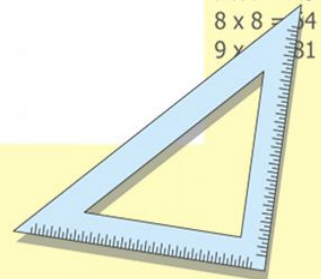
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

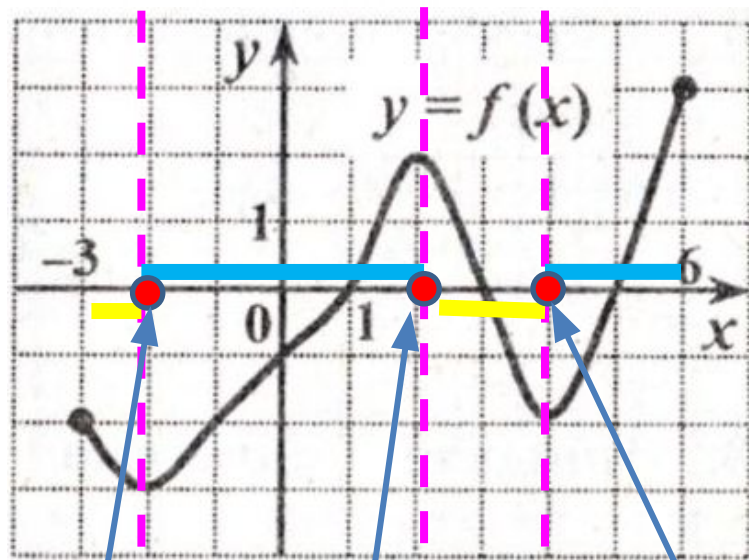
$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Экстремумы

Экстремумы – это значения x , в которых меняется монотонность функции

Если возрастание сменяется на убывание, то это точка максимума, если же убывание сменяется на возрастание – точка минимума.



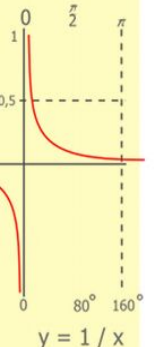
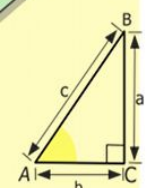
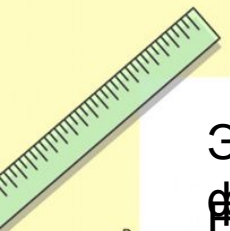
$$f \uparrow (-2; 2), (4; 6)$$

$$f \downarrow (-3; -2), (2; 4)$$

$$x_{\min} = -2$$

$$x_{\max} = 2$$

$$x_{\min} = 4$$



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

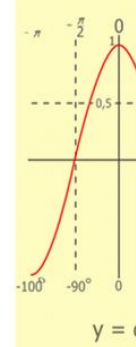
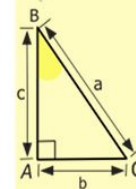
$$\sin 90^\circ = 1$$



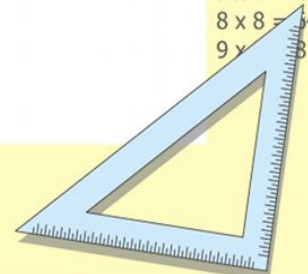
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

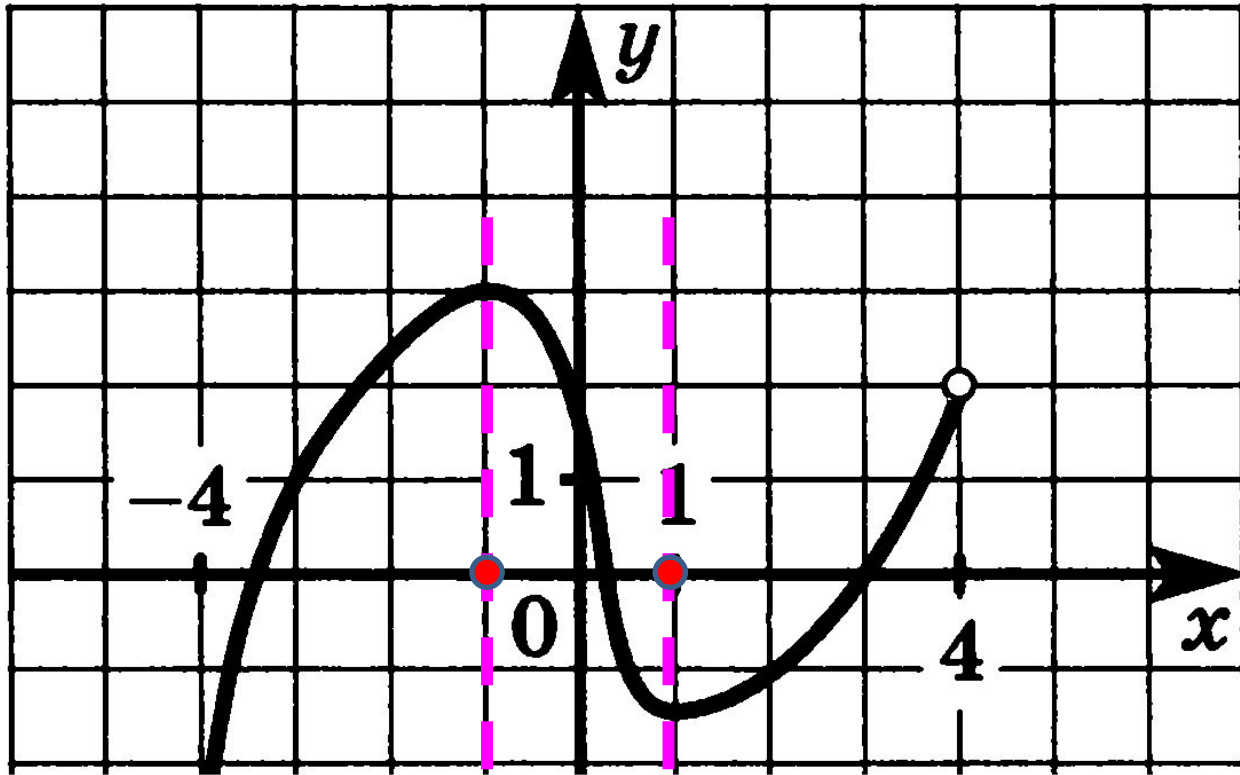
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

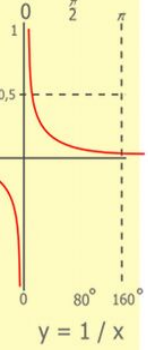
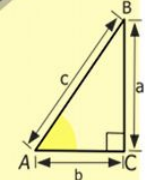
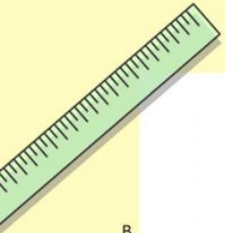


Экстремумы

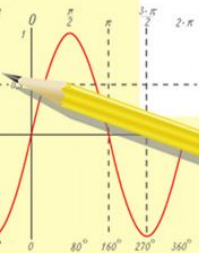


$$x_{\max} = -1$$

$$x_{\min} = 1$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

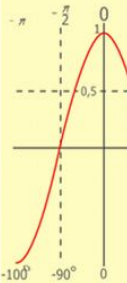
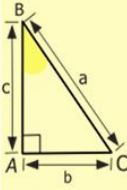
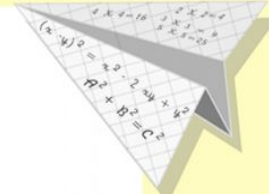


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

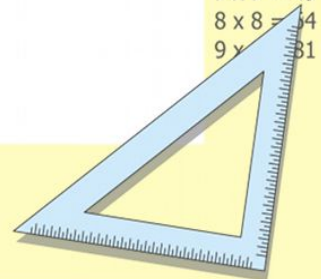
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



Исследовать функцию

$$D(f) = \mathbb{R}$$

$$E(f) = \mathbb{R}$$

$$x_1 = -3,5 \quad x_2 = 0 \quad x_3 = 3,5$$

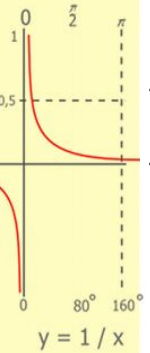
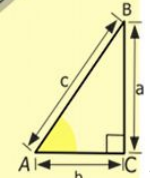
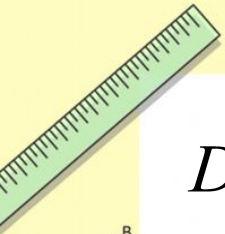
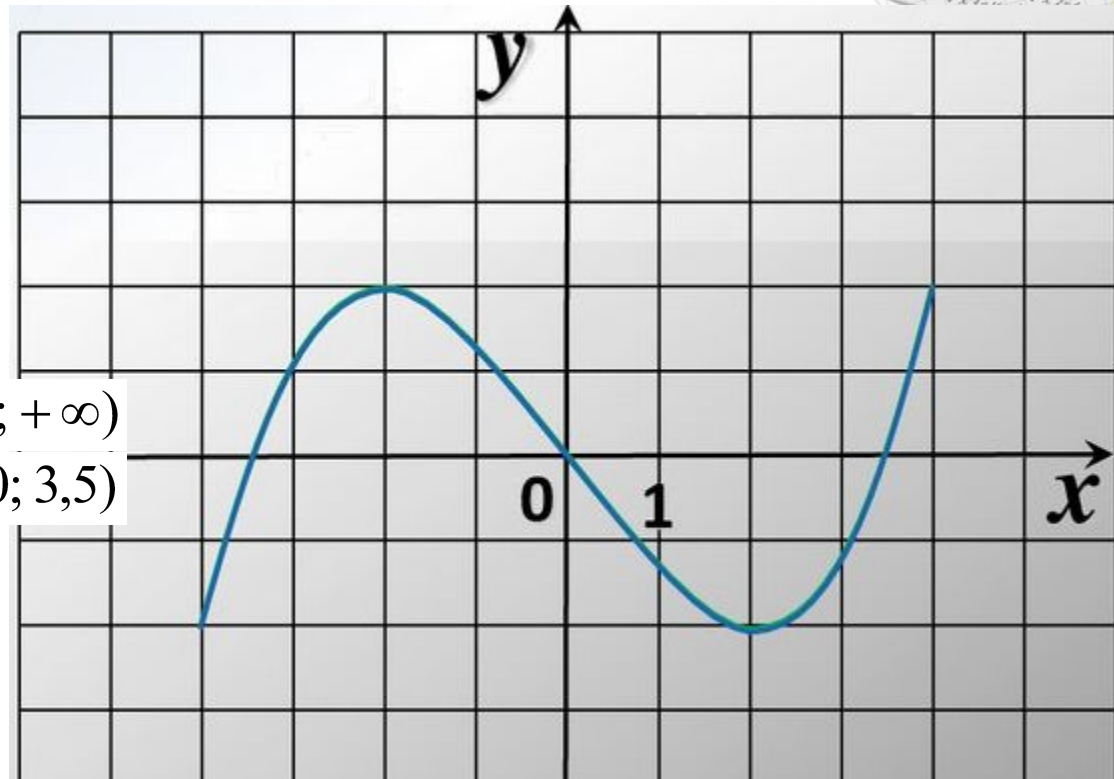
$$y > 0 \text{ при } x \in (-3,5; 0) \cup (3,5; +\infty)$$

$$y < 0 \text{ при } x \in (-\infty; -3,5) \cup (0; 3,5)$$

$$f \uparrow (-\infty; -2), (2; +\infty)$$

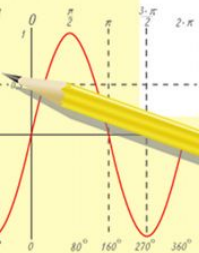
$$f \downarrow (-2; 2)$$

$$x_{\max} = -2 \quad x_{\min} = 2$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

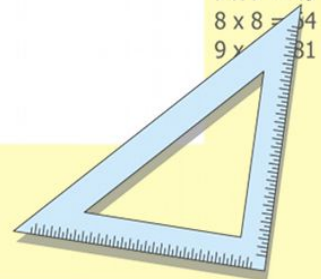


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

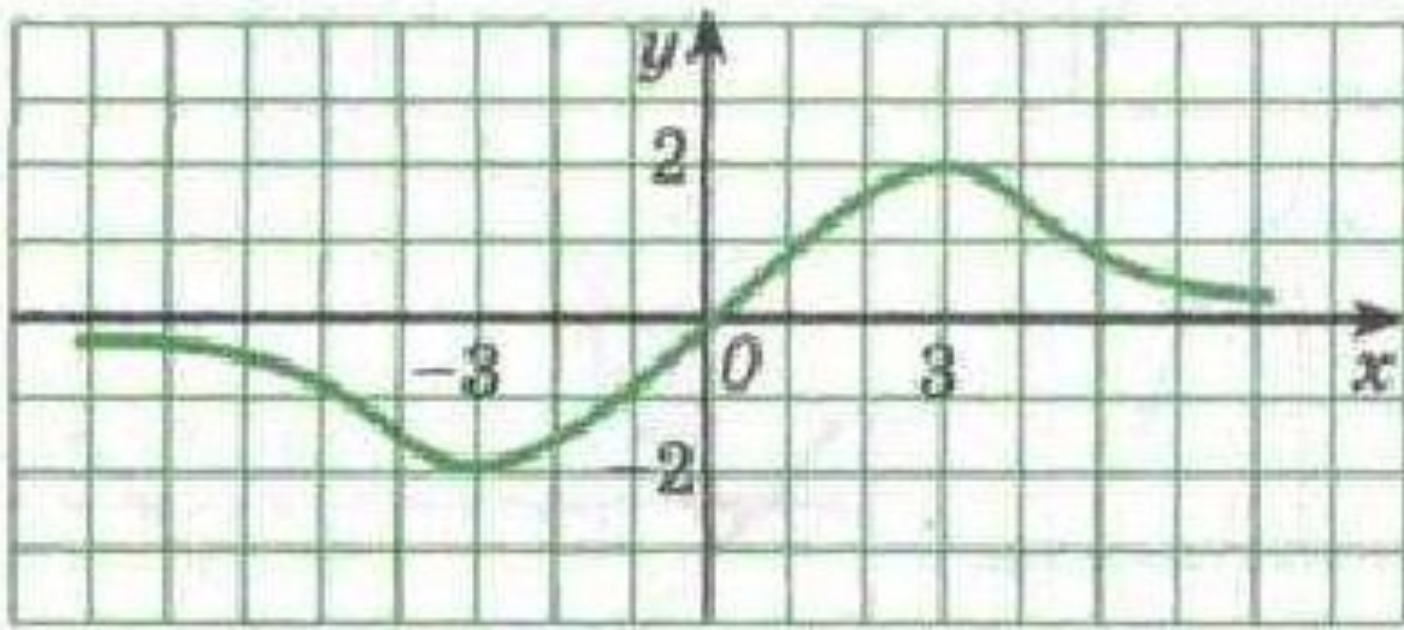
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



Исследовать функцию



$$D(f) = \mathbb{R}$$

$$E(f) = [-2; 2]$$

$$x = 0$$

$$y > 0 \text{ при } x \in (0; +\infty)$$

$$y < 0 \text{ при } x \in (-\infty; 0)$$

$$f \uparrow (-3; 3)$$

$$f \downarrow (-\infty; -3), (3; +\infty)$$

$$x_{\max} = 3 \quad x_{\min} = -3$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

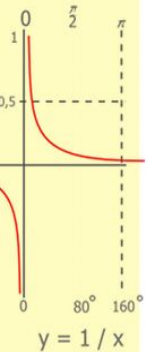
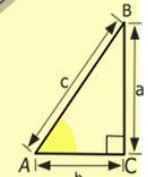
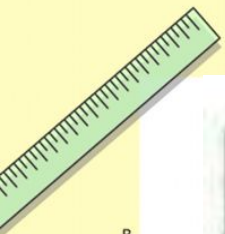
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

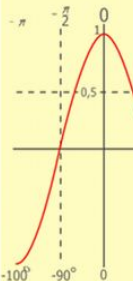
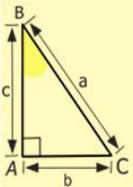
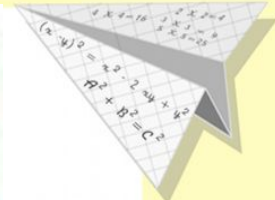
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



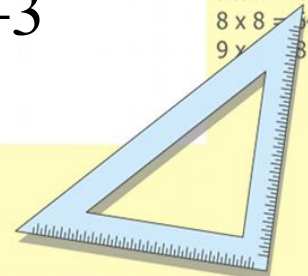
$$y = 1/x$$

$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

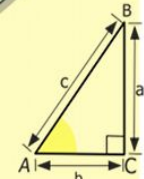
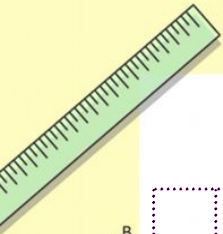


$$y = \cos$$

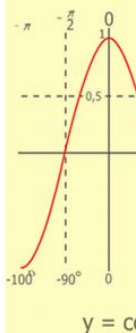
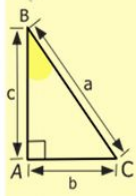
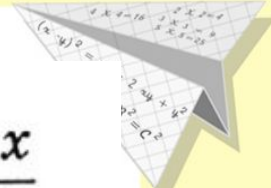
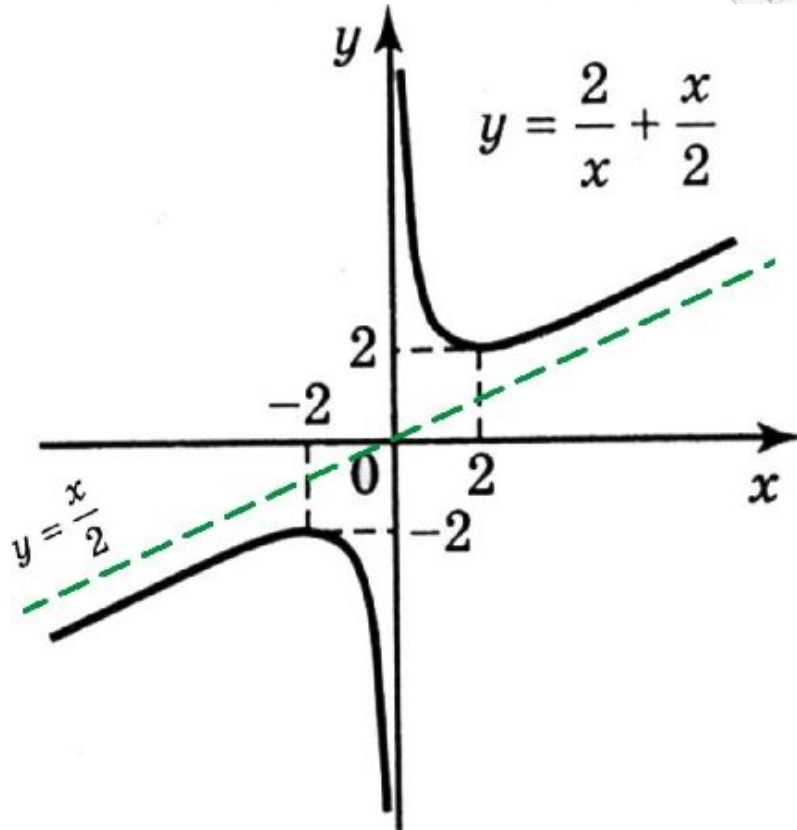
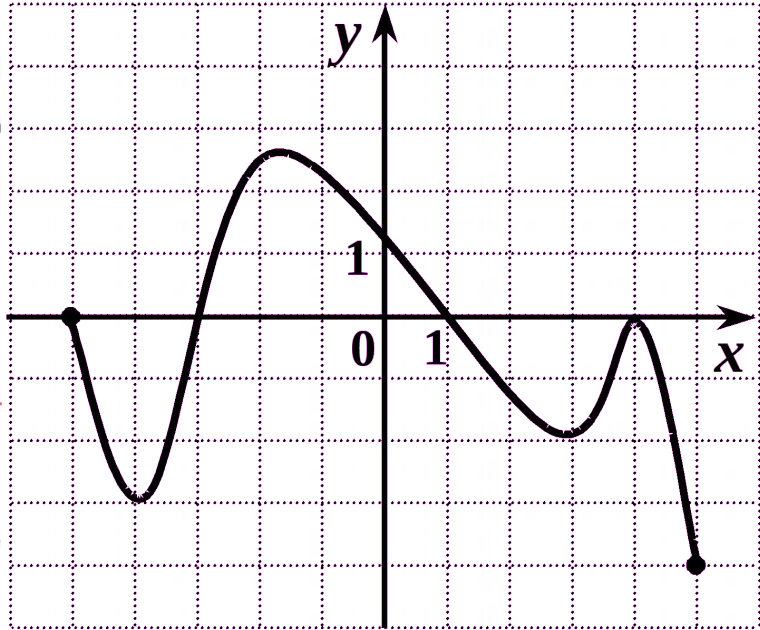
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Исследовать функцию



$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105\ 000 \end{array}$$



- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
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- $7 \times 7 = 49$
- $8 \times 8 = 64$
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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

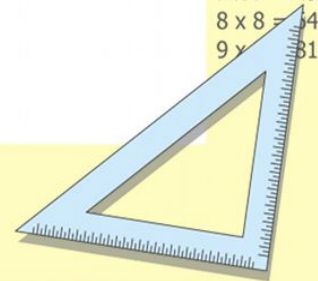
$\sin 90^\circ = 1$



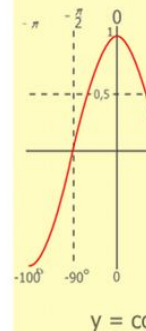
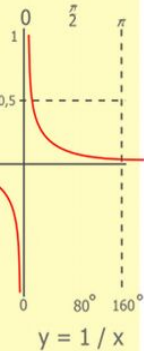
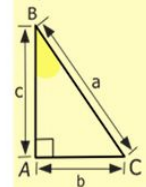
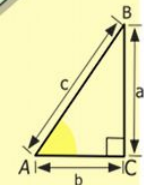
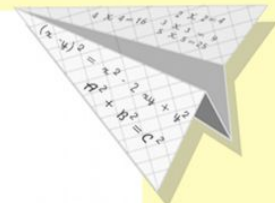
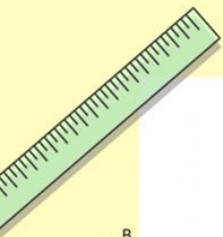
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

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$(x+y)(x-y) = x^2 - y^2$



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Юрьевич
преподаватель математики
ГБПОУ ЗКНО
Москва, 2021г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

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$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

