Введение в комбинаторную логику

Combinatory Logic

Логика

ЛОГИКА

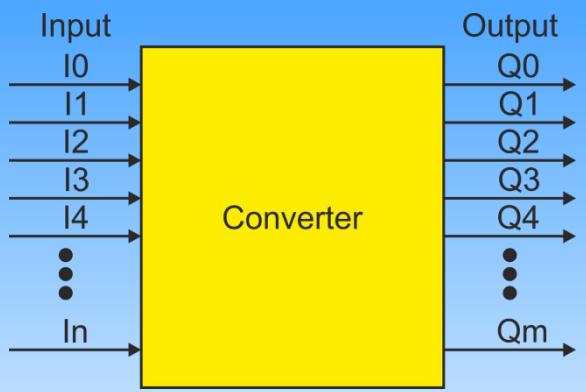
Комбинаторная

Последовательностная

Преобразование кодов

Логика с памятью

Преобразование кодов



Стандартная часть. Входы

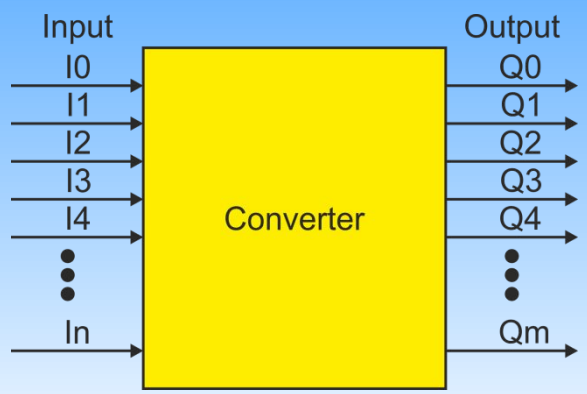
Оригинальная часть. Выходы

Таблица истинности

2^n строк

In	•••	I2	I1	I0	Qm	•••	Q2	Q1	Q0
0		0	0	0					
0		0	0	1					
0		0	1	0					
0		0	1	1					
0		1	0	0					
		•••					•••		
1		1	1	0					
1		1	1	1					

Преобразование кодов



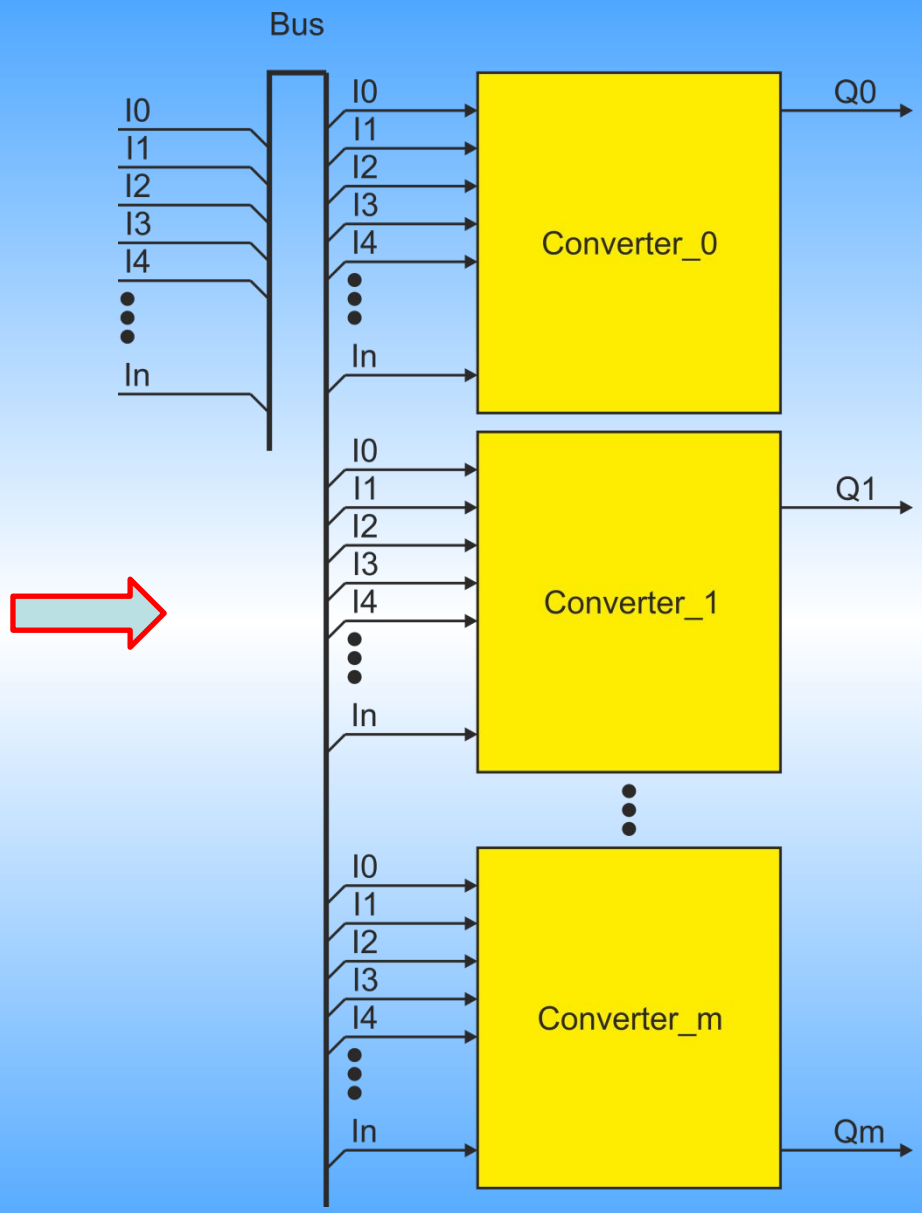
$$Q0 = f_0(I0, I1, I2 \dots In)$$

$$Q1 = f_1(I0, I1, I2 \dots In)$$

$$Q2 = f_2(I0, I1, I2 \dots In)$$

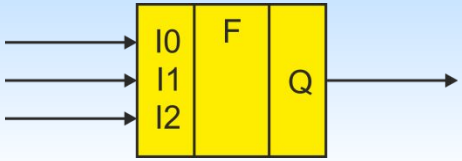
$$\vdots$$

$$Qm = f_m(I0, I1, I2 \dots In)$$



Таблицы истинности

Пример: преобразователь для 3 переменных



I2	I1	I0	Q
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Все комбинации входных сигналов.
Для n входов 2^n строк.

Определение функции

Функции одной переменной

$$Q = a \times x + b$$

$$Q = \sin x$$

$$Q = x^2$$

$$Q = \sqrt[n]{x}$$

$$Q = e^{-i\omega x}$$

$$Q = \tan x$$

$$Q = \sqrt[2]{x}$$

$$Q = \int f(x)dx$$

$$Q = \cos x$$

$$Q = x^n$$

$$Q =$$

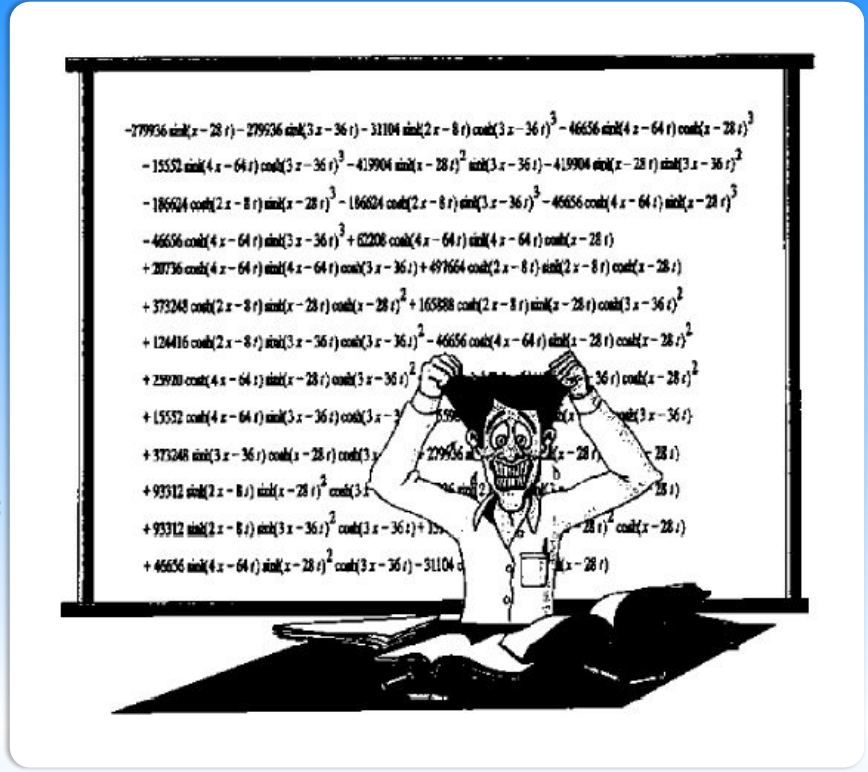


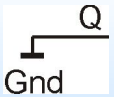
Рисунок из книги

Enns, Richard H.
Nonlinear physics with Mathematica for scientists and engineers / Richard H. Enns and George C. McGuire.

Логические функции одной переменной

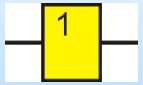
I	Q
0	
1	

I	Q
0	0
1	0

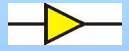


I	Q
0	0
1	1

Буфер



Другое обозначение



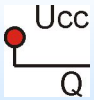
Другое обозначение

I	Q
0	1
1	0

Инвертор

Другое обозначение

I	Q
0	1
1	1

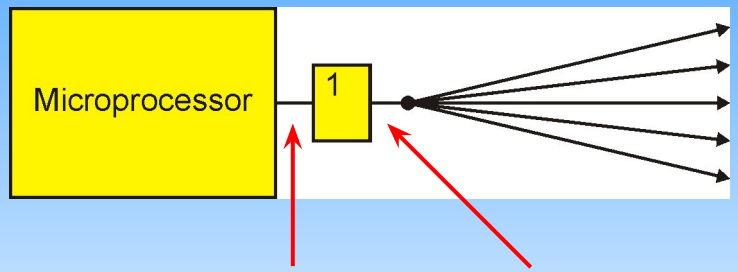


Буферы и преобразователи уровней

Буфер
Buffer

I	Q
0	0
1	1

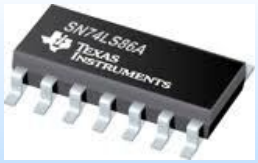
Увеличение нагрузочной способности



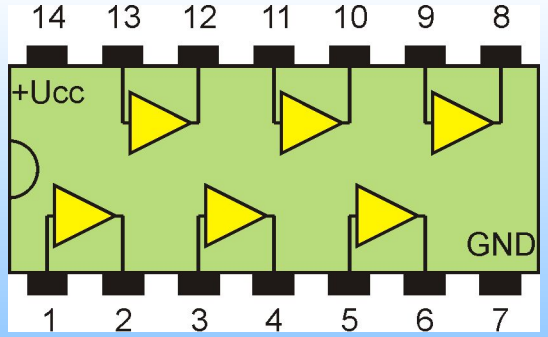
$I_{max} = 5mA$ $I_{max} = 40mA$

DIP14

Пример: 7407



SO14

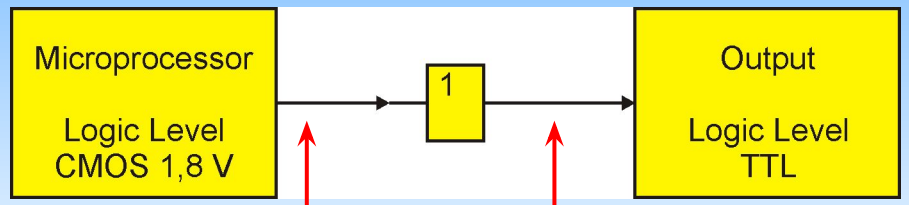


6 элементов в одном корпусе

Буферы и преобразователи уровней

Преобразователи уровней
Level shifter

I	Q
0	0
1	1



Логические уровни	
Out_0	0 ÷ 0,1 В
Out_1	1,7 ÷ 1,8 В

Логические уровни	
Out_0	0 ÷ 0,4 В
Out_1	2,4 ÷ 5 В

Пример: MC14504B

Инвертор

Единственная нетривиальная логическая функция от одной переменной

Таблица истинности
(определение функции)

I	Q
0	1
1	0

NOT

Графическое
обозначение

или

Символ отрицания

Алгебраическое обозначение

$$Q = \bar{I}$$

Штрих Шеффера

$$Q = -I$$

$$Q = !\,I$$

$$Q = \sim I$$

$$Q = \neg I$$

Функции двух переменных

I1	I0	Q
0	0	
0	1	
1	0	
1	1	

4

Всего 2^4=16 функций.
Нетривиальных и симметричных → 6.

I1	I0	Q
0	0	0
0	1	0
1	0	0
1	1	1

2AND

I1	I0	Q
0	0	1
0	1	1
1	0	1
1	1	0

2NAND

$$NAND = \overline{AND}$$

I1	I0	Q
0	0	0
0	1	1
1	0	1
1	1	1

2OR

I1	I0	Q
0	0	1
0	1	0
1	0	0
1	1	0

2NOR

$$NOR = \overline{OR}$$

I1	I0	Q
0	0	0
0	1	1
1	0	1
1	1	0

XOR

I1	I0	Q
0	0	1
0	1	0
1	0	0
1	1	1

NXOR

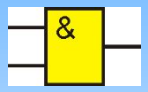
$$NXOR = \overline{XOR}$$

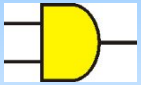
Логическое умножение (конъюнкция)

B	A	Q
0	0	0
0	1	0
1	0	0
1	1	1

2И
2AND

Графическое обозначение





Алгебраическое обозначение

$$Q = A \times B$$

$$Q = A \,\&\, B$$

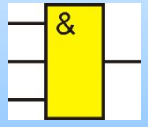
$$Q = A \wedge B$$

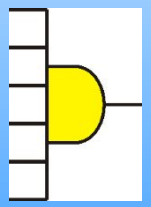
Элементы с количеством входов больше двух

C	B	A	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

3И
3AND

Графическое обозначение





Алгебраическое обозначение

$$Q = A \times B \times C$$

$$Q = A \,\&\, B \,\&\, C$$

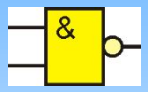
$$Q = A \wedge B \wedge C$$

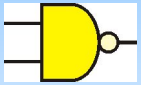
Логическое умножение с отрицанием

B	A	Q
0	0	1
0	1	1
1	0	1
1	1	0

2И-НЕ
2NAND

Графическое обозначение





Алгебраическое обозначение

$$Q = \overline{A \times B}$$

$$Q = \overline{A \,\&\, B}$$

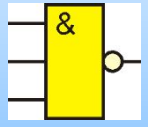
$$Q = \neg(A \wedge B)$$

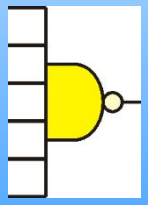
Элементы с количеством входов больше двух

C	B	A	Q
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

3И-НЕ
3NAND

Графическое обозначение





Алгебраическое обозначение

$$Q = \overline{A \times B \times C}$$

$$Q = \overline{A \,\&\, B \,\&\, C}$$

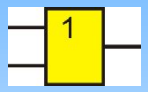
$$Q = \neg(A \wedge B \wedge C)$$

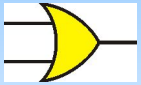
Логическое сложение (дизъюнкция)

B	A	Q
0	0	0
0	1	1
1	0	1
1	1	1

2ИЛИ
2OR

Графическое обозначение





Алгебраическое обозначение

$$Q = A + B$$

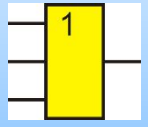
$$Q = A \vee B$$

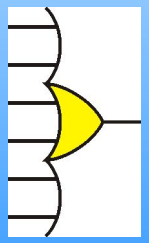
Элементы с количеством входов больше двух

C	B	A	Q
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

3ИЛИ
3NOR

Графическое обозначение





Алгебраическое обозначение

$$Q = A + B + C$$

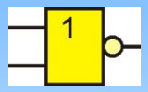
$$Q = A \vee B \vee C$$

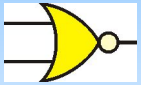
Логическое сложение с отрицанием

B	A	Q
0	0	1
0	1	1
1	0	1
1	1	0

2ИЛИ-НЕ
2NOR

Графическое обозначение





Алгебраическое обозначение

$$Q = \overline{A + B}$$

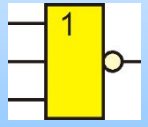
$$Q = \neg(A \vee B)$$

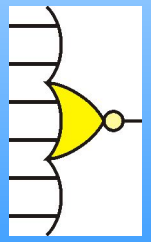
Элементы с количеством входов больше двух

C	B	A	Q
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

3ИЛИ-НЕ
3NOR

Графическое обозначение





Алгебраическое обозначение

$$Q = \overline{A + B + C}$$

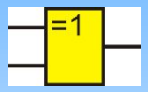
$$Q = \neg(A \vee B \vee C)$$

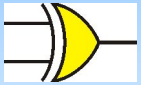
Исключающее ИЛИ (сложение по модулю 2)

B	A	Q
0	0	0
0	1	1
1	0	1
1	1	0

2XOR

Графическое обозначение

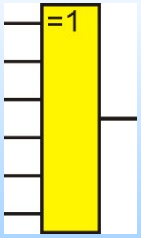




Алгебраическое обозначение

$$Q = A \oplus B$$

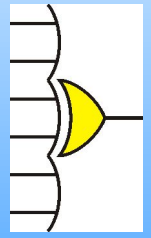
Элементы с количеством входов больше двух



$$Q = A \oplus B \oplus C \oplus D \oplus E \oplus F$$

C	B	A	Q
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

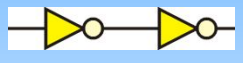
3XOR



Четное кол-во 1 → Q=0
Нечетное кол-во 1 → Q=1

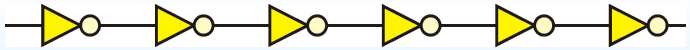
Свойства логических функций. (Постулаты Булевой алгебры)

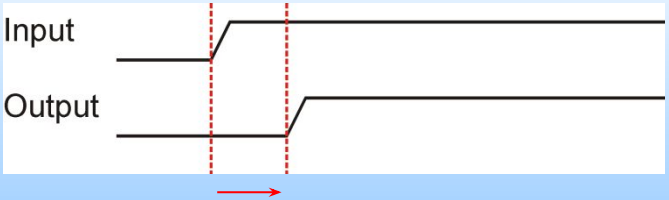
$$\overline{(\overline{A})} = A$$



Джордж Буль
1815÷1864

Четное количество



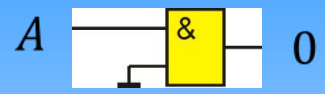


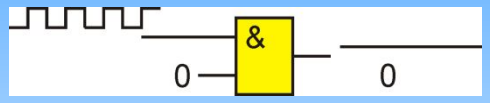
$$t = n \times \tau$$

Линия задержки

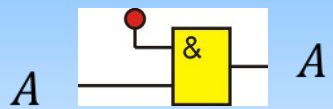
Свойства логических функций. (Постулаты Булевой алгебры)

$$A \times 0 = 0$$

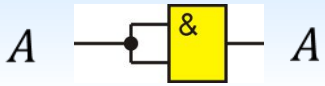




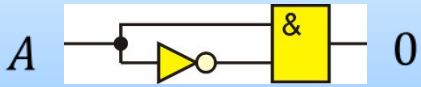
$$A \times 1 = A$$



$$A \times A = A$$

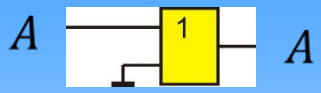


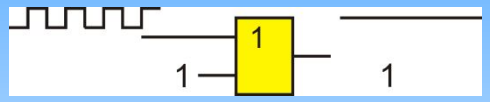
$$A \times \overline{A} = 0$$



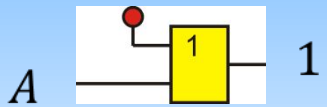
Свойства логических функций. (Постулаты Булевой алгебры)

$$A + 0 = A$$

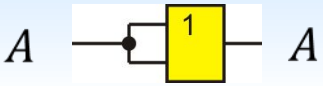




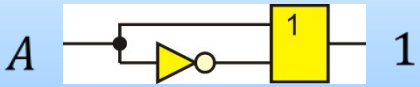
$$A + 1 = 1$$



$$A + A = A$$

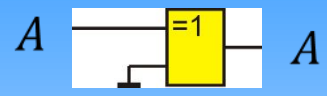


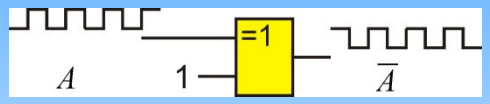
$$A + \overline{A} = 1$$



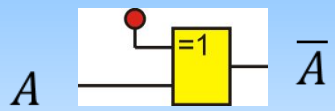
Свойства логических функций. (Постулаты Булевой алгебры)

$A \oplus 0 = A$

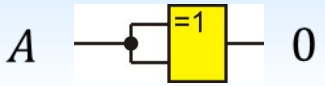




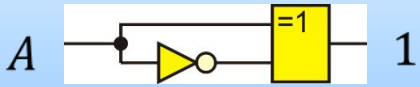
$A \oplus 1 = \overline{A}$



$A \oplus A = 0$



$A \oplus \overline{A} = 1$



Законы Булевой алгебры

Коммутативный

$$A \times B = B \times A$$

$$A \oplus B = B \oplus A$$

$$A + B = B + A$$

Законы Булевой алгебры

Ассоциативный

$$A \times B \times C = (A \times B) \times C = A \times (B \times C)$$

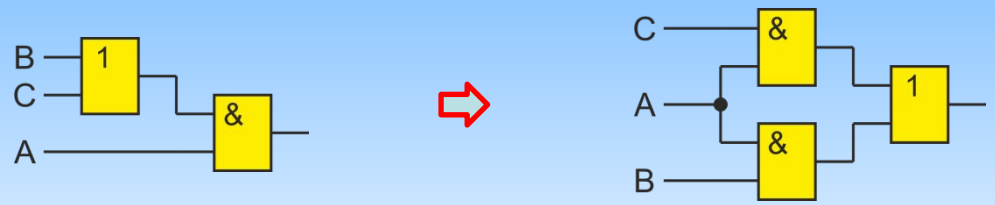
$$A + B + C = (A + B) + C = A + (B + C)$$

$$A \oplus B \oplus C = (A \oplus B) \oplus C = A \oplus (B \oplus C)$$

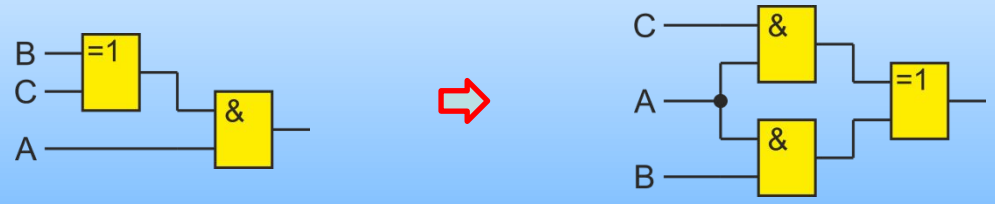
Законы Булевой алгебры

Дистрибутивный

$$A \times (B + \mathrm{C}) = A \times B + A \times \mathrm{C}$$



$$A \times (B \oplus \mathrm{C}) = (A \times B) \oplus (A \times \mathrm{C})$$



Свойства логических функций. (Постулаты Булевой алгебры)

Поглощение

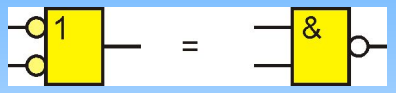
$$A \times (A + B) = A$$

$$A \times (A + B) = A + A \times B = A \times (1 + B) = A$$

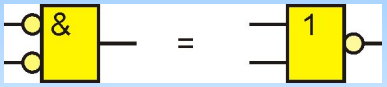
Свойства логических функций. (Постулаты Булевой алгебры)

Теорема Де Моргана

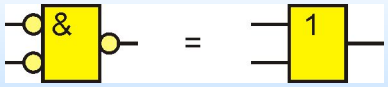
$$\overline{A} + \overline{B} = \overline{A \times B}$$

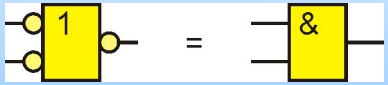


$$\overline{A} \times \overline{B} = \overline{A + B}$$



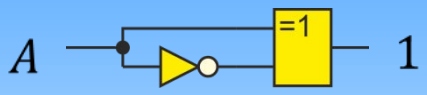
Следствие теоремы Де Моргана



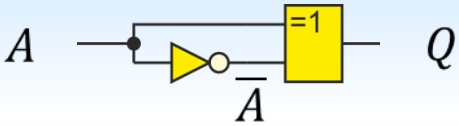


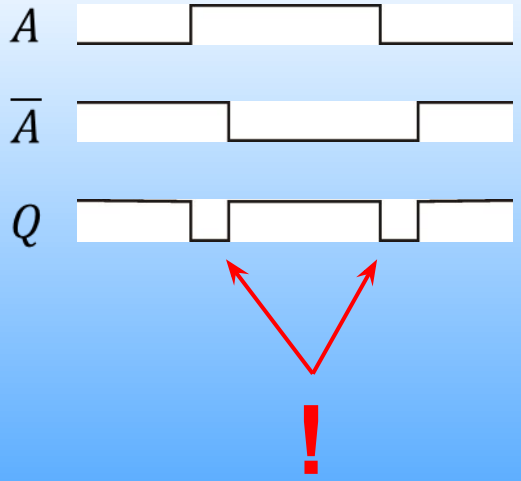
Важность учета задержек

$$A \oplus \overline{A} = 1$$



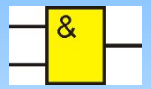
В реальных устройствах не совсем так!

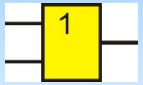




Базис

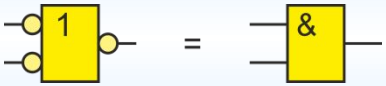
Сколь угодно сложное логическое выражение может быть представлено с использованием только основных логических элементов.





Используя теорему де-Моргана

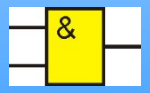
$$\overline{\overline{A} + \overline{B}} = A \times B$$



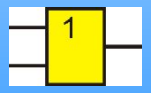
$$\overline{\overline{A} \times \overline{B}} = A + B$$

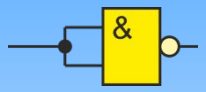
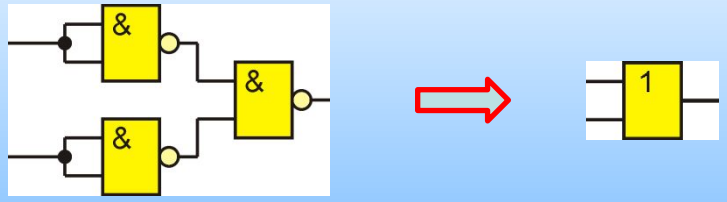
Можно обойтись только двумя функциями:

Базис 1



Базис 2



NAND

Из элементов 2И-НЕ можно сделать все!