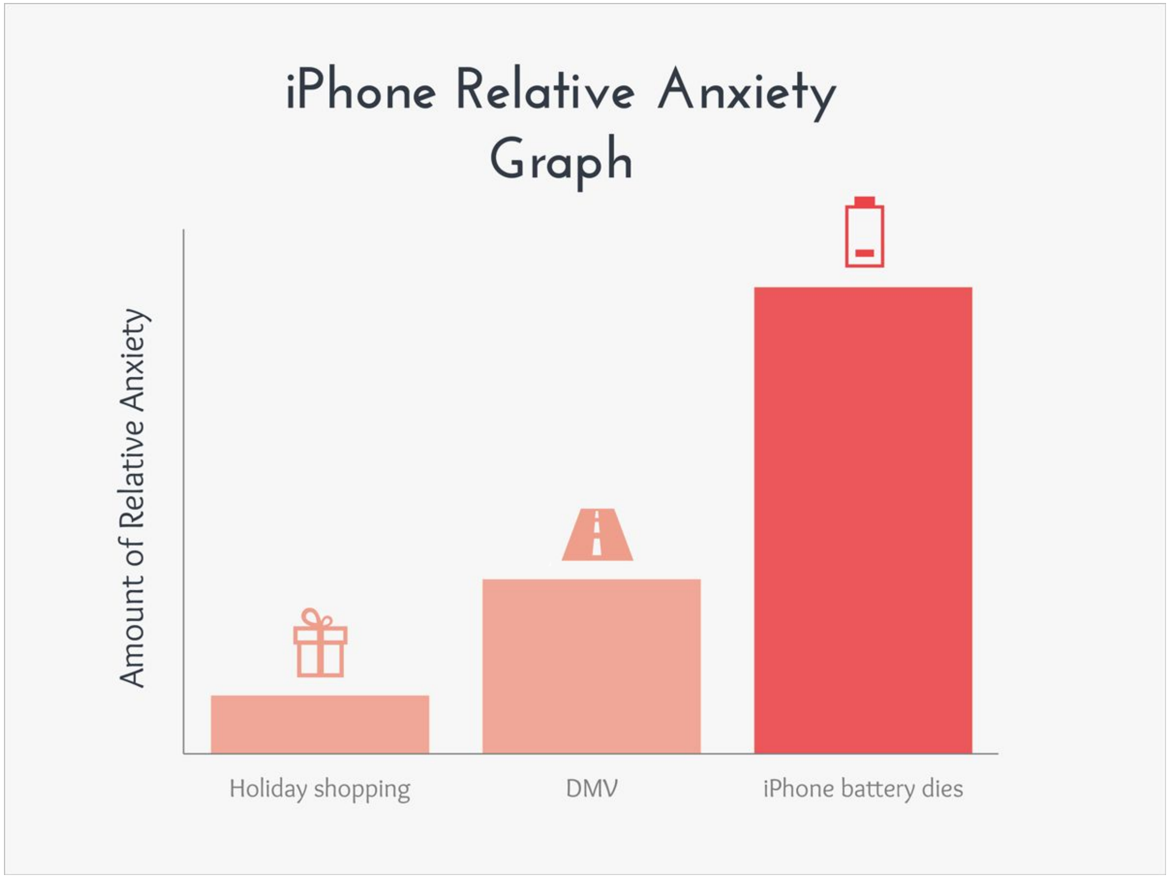
Descriptive Statistics

Graphing Techniques



Points and grades from examination

No.	Points	Grade	No.	Points	Grade	No.	Points	Grade
1	15	1	12	12	3	23	15	2
2	17	1	13	16	2	24	9	4
3	19	1	14	13	1	25	17	1
4	10	2	15	7	3	26	16	1
5	2	2	16	15	1	27	13	1
6	14	2	17	20	2	28	6	2
7	5	4	18	16	2	29	16	3
8	17	2	19	14	3	30	18	1
9	11	1	20	3	2			
10	16	2	21	15	1			
11	10	3	22	12	1			

- Sample size n=30

- Data sorting → **Frequency table**
  - **both for quantitative and qualitative data**

Exam grade

Exam grade				
	Frequency	Percent	Cumulative Frequency	Cumulative Percent
1	12	40,0	12,0	40,0
2	11	36,7	23,0	76,7
3	5	16,7	28,0	93,3
4	2	6,7	30,0	100,0
Total	30	100,0		

Notation

- Frequency … n_i
- Relative frequency … f_i

$$\boxed{f_i = \frac{n_i}{n}}$$

- Cumulative Frequency … N_i

$$\boxed{N_i = \sum_{j \le i} n_j}$$

- Cumulative Percent … F_i

$$\boxed{F_i = \sum_{j \le i} f_j}$$

Points from class test

Points from class test					
Points	**Frequency**	**Percent**	**Points**	**Frequency**	**Percent**
2	1	3,33	**13**	2	6,67
3	1	3,33	**14**	2	6,67
5	1	3,33	**15**	4	13,33
6	1	3,33	**16**	5	16,67
7	1	3,33	**17**	3	10,00
9	1	3,33	**18**	1	3,33
10	2	6,67	**19**	1	3,33
11	1	3,33	**20**	1	3,33
12	2	6,67	**Total**	**30**	**100,00**

Quantitative variables

↓

Grouping into class intervals

How to select the intervals

- Number of intervals $\rightarrow$ in order to describe the characteristics of the data

- Simple reccommendation
  - intervals of the same width

$$\boxed{k = \sqrt{n}}$$

k … number of intervals

n … sample size

…then

$$\boxed{h = \frac{R}{k}}$$

h … width of interval

R … Range=x_{max}-x_{min}

k … number of intervals

<u>Our example:</u>

n=30

R=20-2=18

$$k = \sqrt{30} = 5{,}48 \cong 6$$

$$h = \frac{18}{6} = 3$$

Points from class test

Points from class test				
Interval	Frequency	Percent	Cumulative Frequency	Cumulative Percent
5 and less	3	10,0	3	10,0
6-9	3	10,0	6	20,0
10-13	7	23,3	13	43,3
14-17	14	46,7	27	90,0
18 and more	3	10,0	30	100,0
Total	30	100,0		

Measures of Central Tendency

- Measures that represent with a proper value the tendency of most data to gather around this value
- Number of different measures of central tendency
    - *the arithmetic mean*
    - *the median*
    - *the mode*

$\overline{x}$

The arithmetic mean

Notation

arithmetic mean …… $\overline{\mathrm{x}}$

- the sum of the values of a variable divided by the number of scores (by the sample size)

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Properties of the arithmetic mean

1. it is expressed in the same unit of measure as the observed variable
2. it is the point in a distribution of measurements about which the sum of deviations are equal to zero

$$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$

Note: deviation explains the distance and direction from a reference point – here *the arithmetic mean,* it is positive when the value is greater than the mean and negative when lower than the mean

3. the mean is **very sensitive** to extreme values

Personal income (thousands CZK)

No.	x_i	$x_i - \bar{x}$	No.	x_i	$x_i - \bar{x}$
1	**13,2**	**-12,62**	**9**	**16,4**	**-9,42**
2	**13,5**	**-12,32**	**10**	**17,2**	**-8,62**
3	**14,0**	**-11,82**	**11**	**19,0**	**-6,82**
4	**14,5**	**-11,32**	**12**	**25,8**	**-0,02**
5	**14,5**	**-11,32**	**13**	**27,0**	**1,18**
6	**15,2**	**-10,62**	**14**	**35,0**	**9,18**
7	**15,6**	**-10,22**	**15**	**35,5**	**9,68**
8	**16,2**	**-9,62**	**16**	**120,5**	**94,68**
			Σ	**413,1**	**0,00**

$$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$

$$\bar{x} = \frac{13,2 + \ldots + 120,5}{16} = \frac{413,1}{16} = 25,82$$ thousands CZK

- 12 of 16 values are below the arithmetic mean, because of the highest value x_{16}=120,5 *(directors income)*
- personal income is a commonly studied variable in which other measure of central tendency is preferred

Other measures of central tendency

- **The median….** $\tilde{x}$

The value above and below which one-half of the frequencies fall

- n…odd number

  median case number=(n+1)/2

- n…even number

  the arithmetic mean of the two middle values

Properties: Insensitive to extreme values

Other measures of central tendency

- **The mode....** $\hat{x}$

The value that occurs with greatest frequency

- for qualitative (nominal and ordinal) and quantitative discrete data
- from a statistical perspective it is also the most probable value

Personal income (thousands CZK)

n=16... even number

No.	x_i	**No.**	x_i
1	**13,2**	**9**	**16,4**
2	**13,5**	**10**	**17,2**
3	**14,0**	**11**	**19,0**
4	**14,5**	**12**	**25,8**
5	**14,5**	**13**	**27,0**
6	**15,2**	**14**	**35,0**
7	**15,6**	**15**	**35,5**
8	**16,2**	**16**	**120,5**

the median **the mode**

Personal income (thousands CZK)

n=16… even number

No.	x_i	No.	x_i
1	**13,2**	**9**	**16,4**
2	**13,5**	**10**	**17,2**
3	**14,0**	**11**	**19,0**
4	**14,5**	**12**	**25,8**
5	**14,5**	**13**	**27,0**
6	**15,2**	**14**	**35,0**
7	**15,6**	**15**	**35,5**
8	**16,2**	**16**	**120,5**

the median

$$\tilde{x} = \frac{x_8 + x_9}{2} = \frac{16,2 + 16,4}{2} = 16,3$$

the mode

$$\hat{x} = 14,5$$

Use of mean, median and mode

The arithmetic mean

- member of mathematical system in advanced statistical analysis
- preferred measure of central tendency if the distribution is not skewed

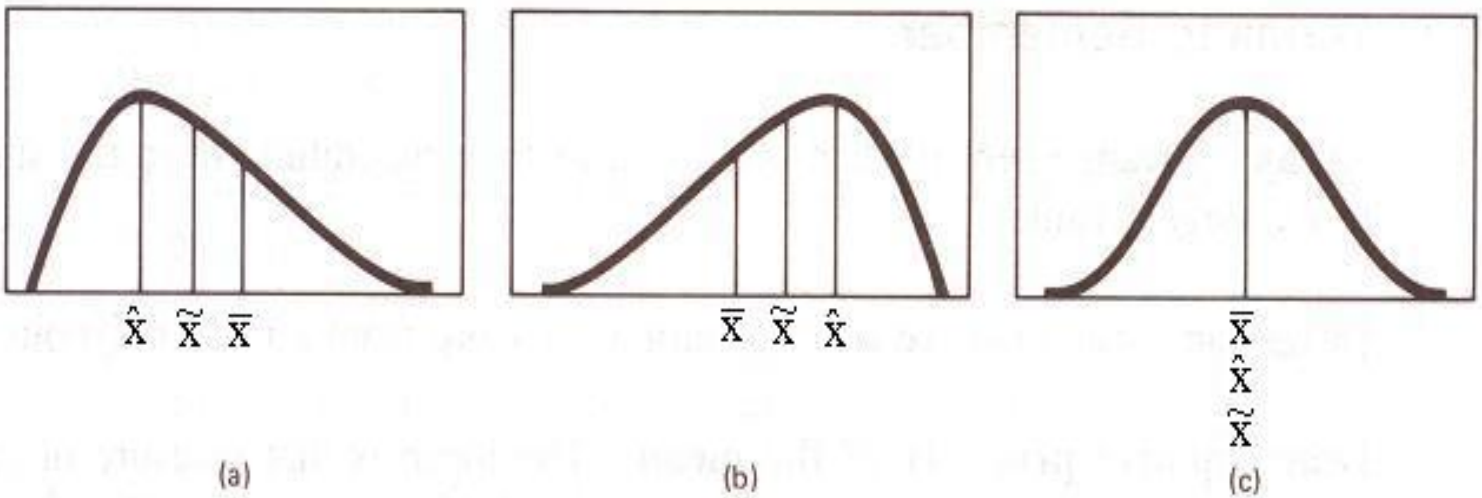
The median

- when the distribution is skewed

The mode

- whenever a quick, rough estimate of central tendency is desired

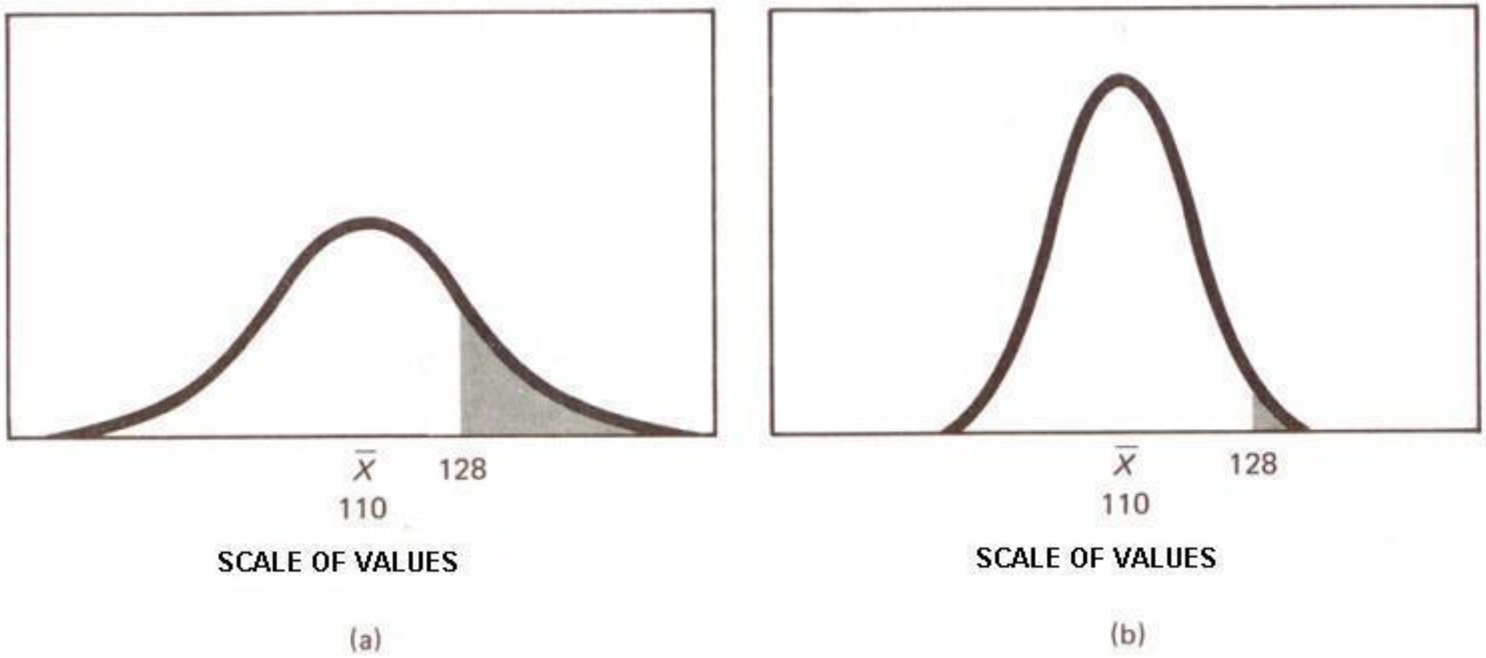
The mean, median, mode and skewness



The relationship among the mean, median, and mode in (a) positively skewed, (b) negatively skewed, and (c) symmetrical distributions.

Measures of Dispersion

- to describe the spread of the data, its variation around a central value
- we want to express the distance along the scale of values



Two frequency curves with identical means but differing in dispersion or variability.

The Range….R

- it is the distance between the largest and the smallest value

$R = x_{max} - x_{min}$

  - it does not explain the variability inside the range !
  - very simple and straightforward measure of dispersion

The Variance…s^2

- it is an average squared deviation of each value from the mean

  → it is the sum of the squared deviations from the mean divided by n

- when computing the variation based on **sample** we correct the calculation

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Working formulas

- For easier computation

Formula 1

$$\mathbf{s}^{2} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{x_i}^{2} - \bar{x}\sum_{i=1}^{n} x_i}{\mathbf{n - 1}}$$

Formula 2

$$\mathbf{s}^{2} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{x_i}^{2} - n\bar{x}^{2}}{\mathbf{n - 1}}$$

- **the variance** explains both
  - the variability of the values around the arithmetic mean
  - the variability among the values

- difficult interpretation

  (it is expressed in the squares of the unit of measure)

The Standard Deviation…s

- it is the square root of variance
  - when computing the variation based on **sample**

$$\mathbf{s} = \sqrt{\mathbf{s}^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}}$$

Properties of the standard deviation

- it is expressed in the same unit of measure as the observed variable
- the size of the standard deviation is related to the variability in the values
  - the more homogeneous values, the smaller SD
  - the heterogeneous values, the larger SD
- member of mathematical system in advanced statistical analysis (like the arthmetic mean)

Two data sets with the same arithmetic mean and different SD

Array A

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
4	0	0
4	0	0
4	0	0
4	0	0
4	0	0

$$\sum_{i=1}^{n} x_i = 20 \qquad \sum_{i=1}^{n} (x_i - \bar{x}) = 0 \qquad \sum_{i=1}^{n} (x_i - \bar{x})^2 = 0$$

$$\bar{x} = 4 \qquad n = 5$$

$$s = \sqrt{\frac{0}{5}} = 0$$

Array B

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
2	−2	4
2	−2	4
3	−1	1
4	0	0
9	+5	25

$$\sum_{i=1}^{n} x_i = 20 \qquad \sum_{i=1}^{n} (x_i - \bar{x}) = 0 \qquad \sum_{i=1}^{n} (x_i - \bar{x})^2 = 34$$

$$\bar{x} = 4 \qquad n = 5$$

$$s = \sqrt{\frac{34}{5}} = 2{,}6$$

Example – Personal income (thousands CZK)

No.	x_i	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$
1	**13,2**	**-12,62**	**159,2644**
2	**13,5**	**-12,32**	**151,7824**
…	**…**	**…**	**…**
16	**120,5**	**94,68**	**8 964,3024**
		Σ	**10 370,04**

$$s^2 = \frac{10370{,}04}{16-1} = 691{,}3363$$

$$s = \sqrt{s^2} = \sqrt{691{,}3363} = 26{,}2938$$ thousands CZK

Coefficient of Variation…V

- the ratio of the standard deviation to the mean

$$V = \frac{s}{\bar{x}}$$

- often reported as a percentage (%) by multiplying by 100

- it is a relative measure of dispersion
- used when comparing two data sets with different units or widely different means
- values higher than 50% indicate large variability

Example – Personal income (thousands CZK)

No.	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
1	**13,2**	**-12,62**	**-159,2644**
2	**13,5**	**-12,32**	**-151,7824**
…	**…**	**…**	**…**
16	**120,5**	**94,68**	**8 964,3024**
		Σ	**10 370,04**

$$s = 26{,}2938 \qquad \bar{x} = 25{,}82$$

$$V = \frac{s}{\bar{x}} = \frac{26{,}2938}{25{,}82} = 1{,}01835$$

$$V = 1{,}01835 * 100 = 101{,}835\%$$

Percentiles (Centiles)

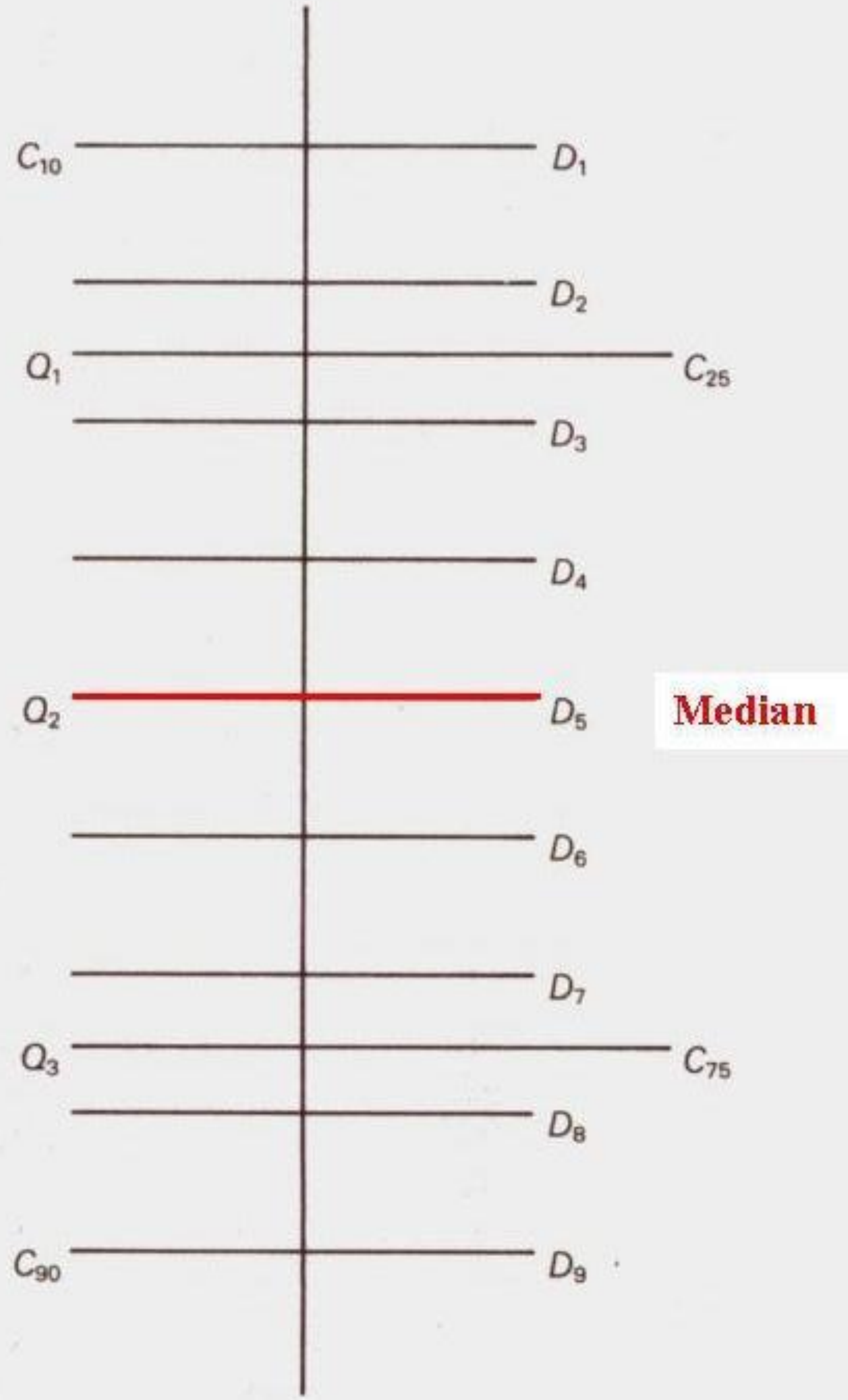
- value below which a certain percent of observations fall
- scale of percentile ranks is comprised of 100 units
- insensitive to extreme values

Deciles

- divides a distribution into 10 equal parts
- there are 9 deciles
- D_1 – 1st decile
  - 10 percent of values fall below it
- D_9 – 9th decile
  - 90 percent of values fall below it

Quartiles

- divides a distribution into 4 equal parts
  - Q_1 - 25 percent of values fall below it
    - 25th centile
  - Q_2 - 50 percent of values fall below it
    - 50th centile
  - Q_3 – 75 percent fall below it
    - 75th centile


C10
D1
D2
Q1
C25
D3
D4
Q2
D5
Median
D6
D7
Q3
C75
D8
C90
D9

Graphing Techniques

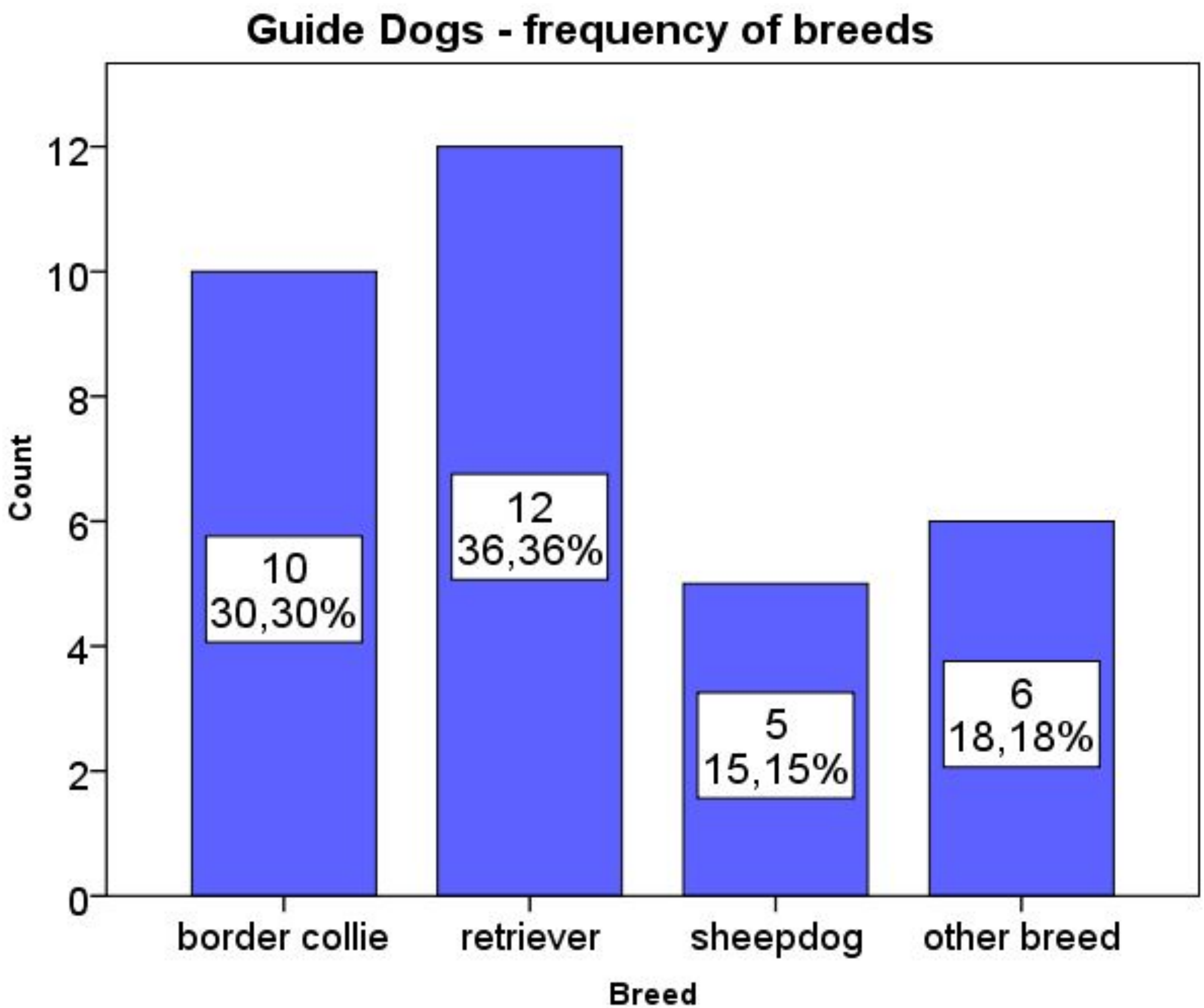
Constructing graphs – **Bar graph**

- x – axis: labels of categories
- y – axis: frequency (relative frequency)

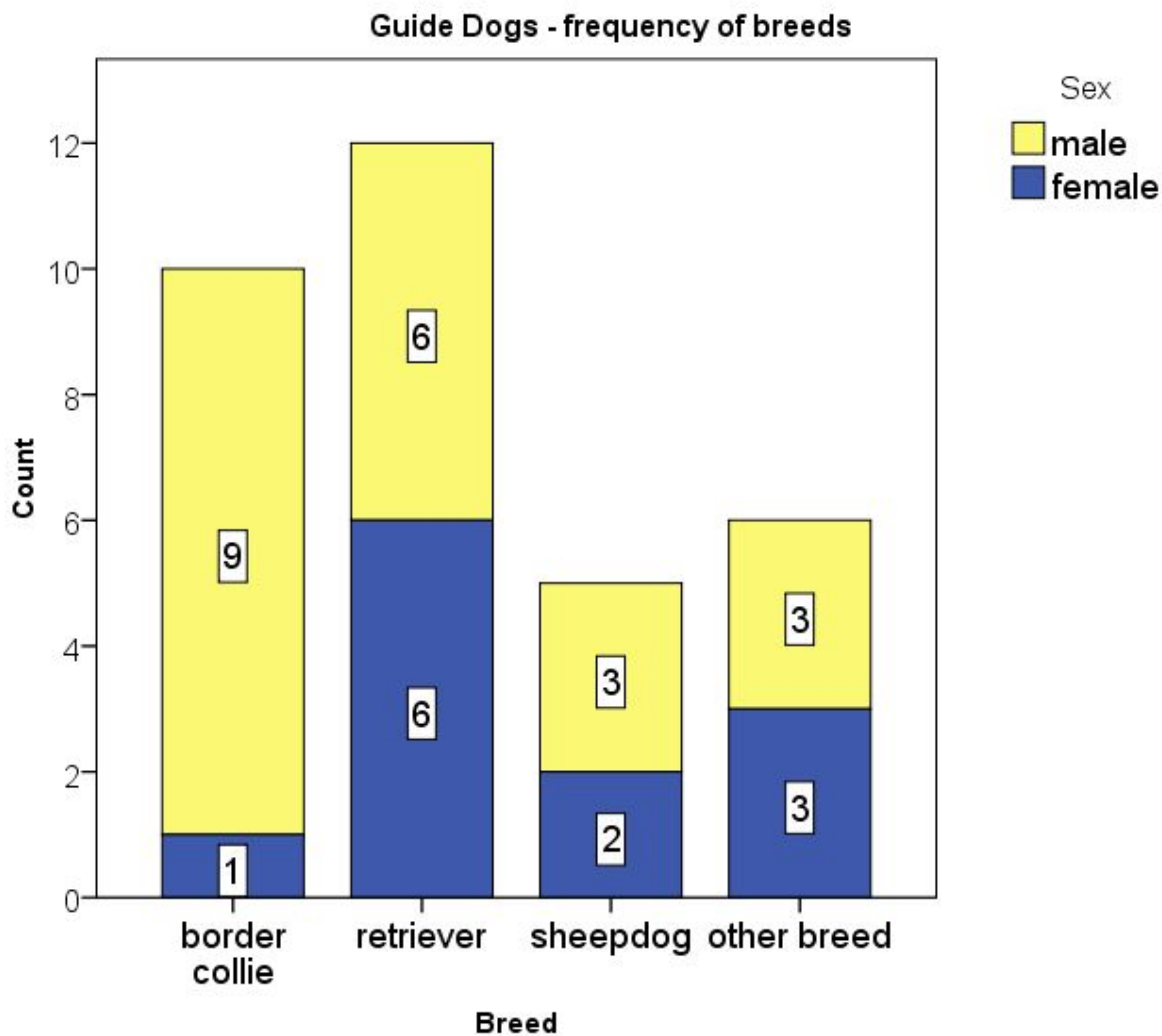
The height of each rectangle is the category`s frequency or relative frequency.

Arranging the graph

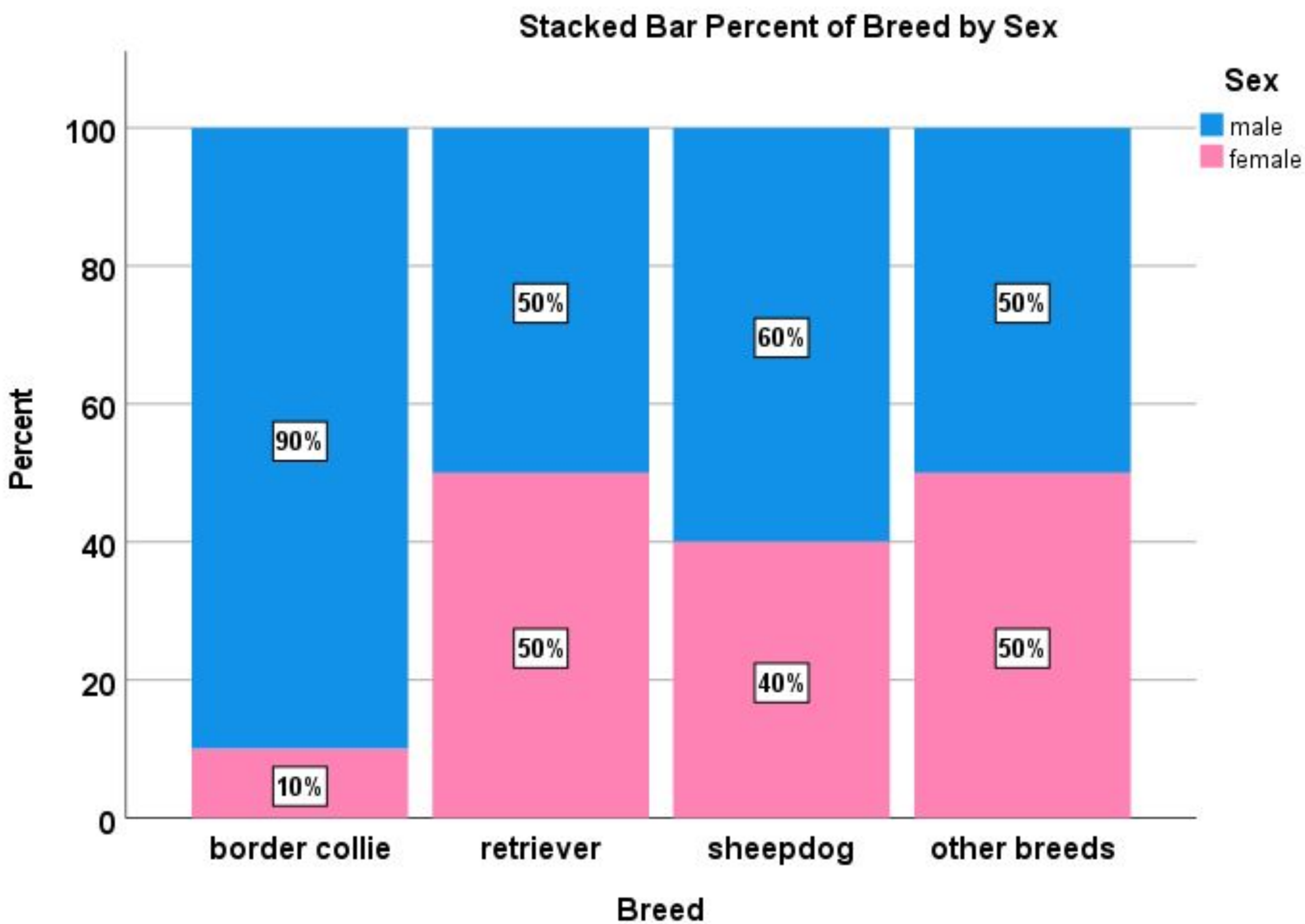
- **nominal variables** – we can arrange the categories in any order:alphabetically, decreasing/increasing order of frequency
- **ordinal variables** – the categories should be placed in their naturally occuring order



Guide Dogs - frequency of breeds
Count
12
10
8
6
4
2
0
10
30,30%
12
36,36%
5
15,15%
6
18,18%
border collie
retriever
sheepdog
other breed
Breed



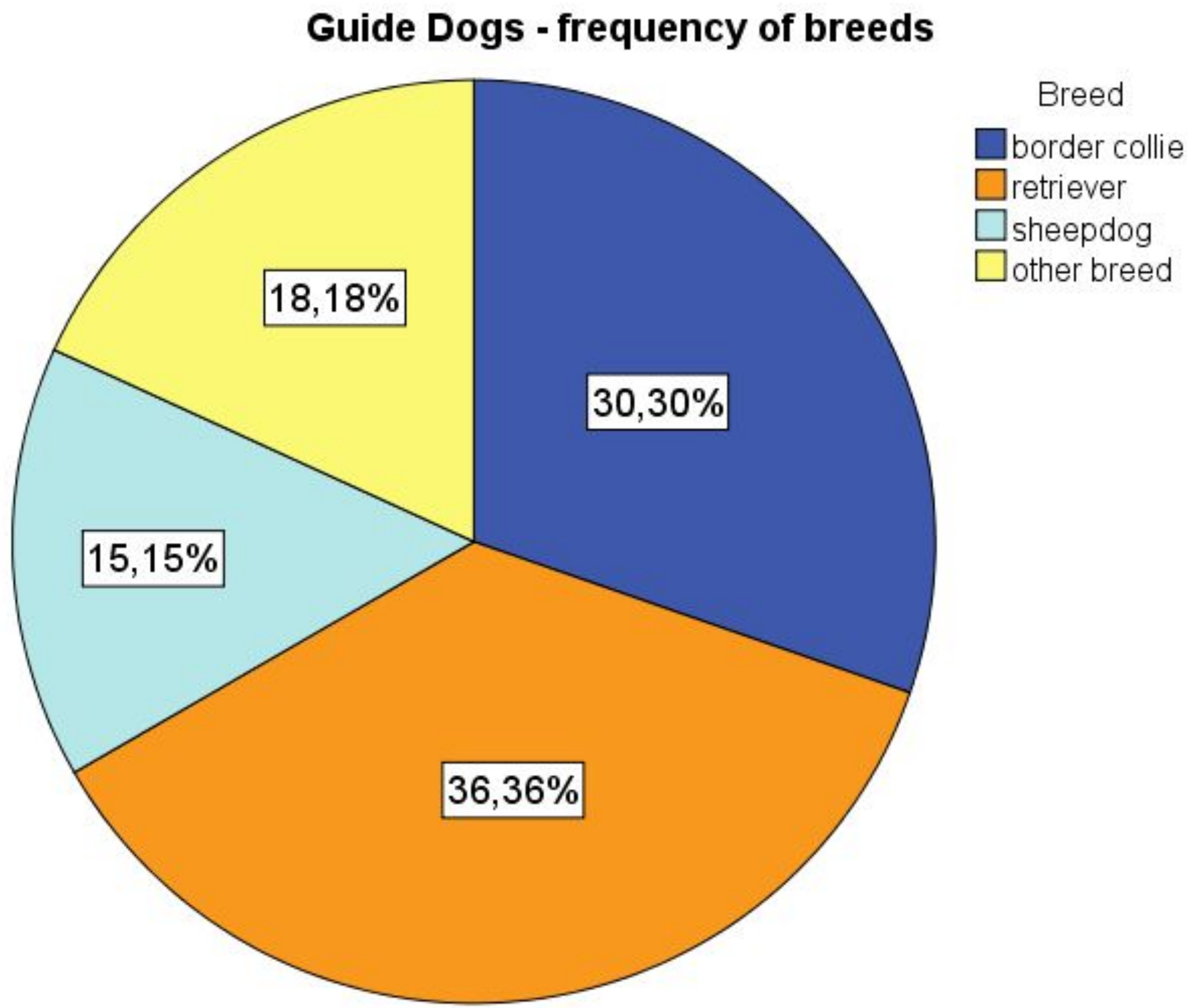
Guide Dogs - frequency of breeds
Sex
male
female
Count
0
2
4
6
8
10
12
1
9
6
6
2
3
3
3
border collie
retriever
sheepdog
other breed
Breed



Stacked Bar Percent of Breed by Sex
Sex
male
female
100
80
60
40
20
0
Percent
90%
10%
50%
50%
60%
40%
50%
50%
border collie
retriever
sheepdog
other breeds
Breed

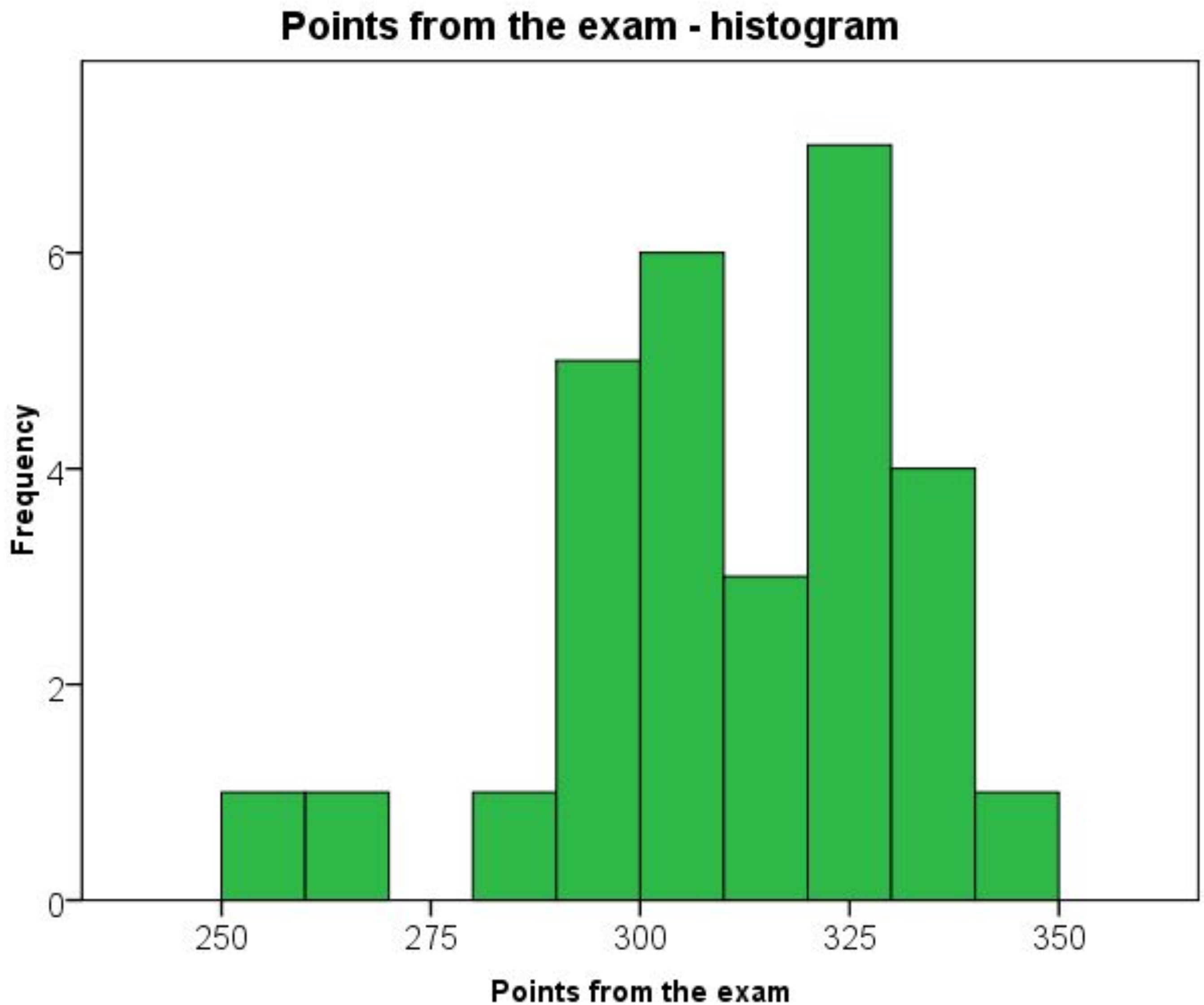
Constructing graphs – **Pie graph**

- **Pie chart** – a circle divided into sectors
  - each sector represents a category of data
  - the area of each sector is proportional to the frequency of the category


Guide Dogs - frequency of breeds
Breed
border collie
retriever
sheepdog
other breed
18,18%
30,30%
15,15%
36,36%

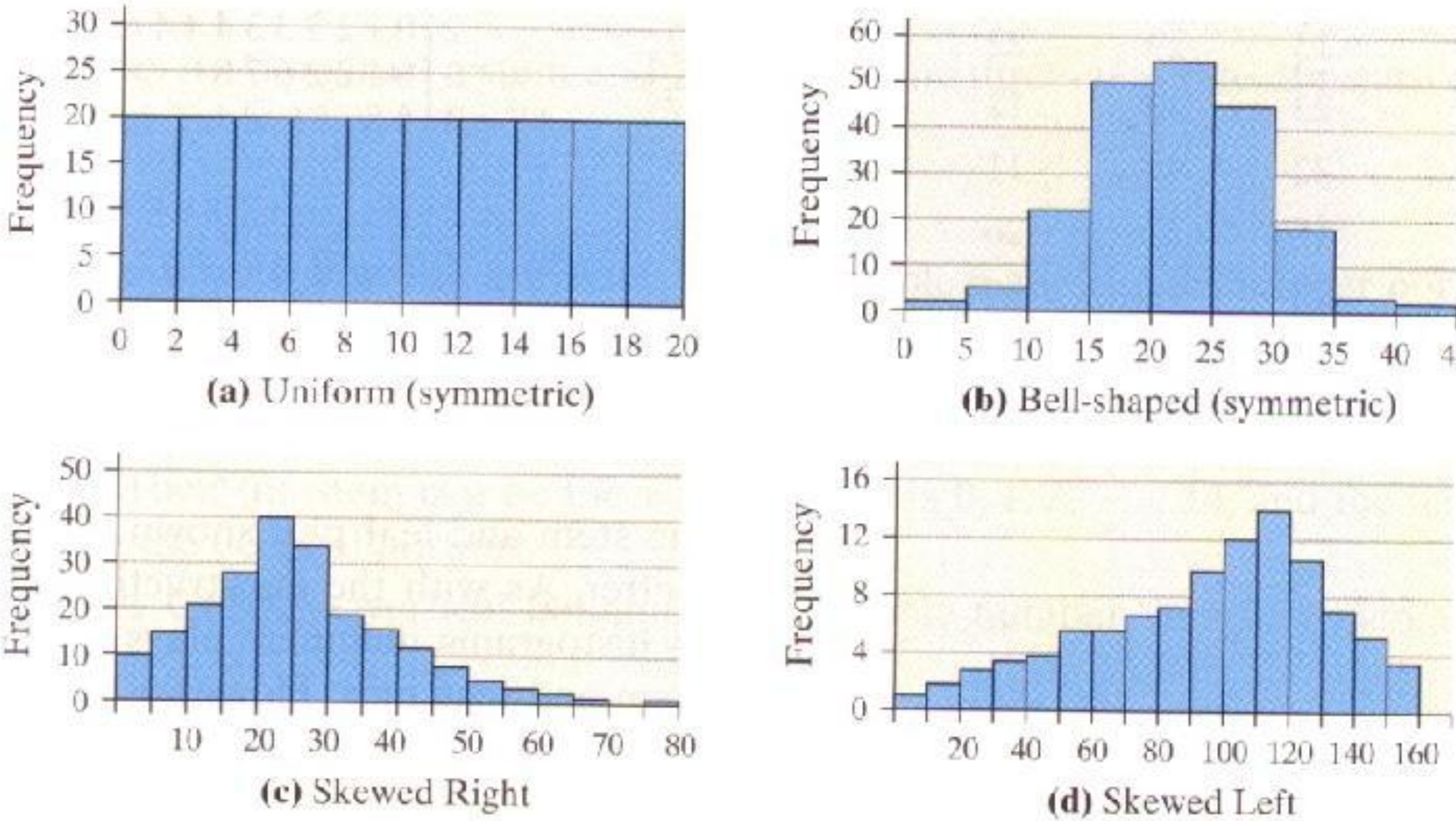
Constructing graphs – **Histogram**

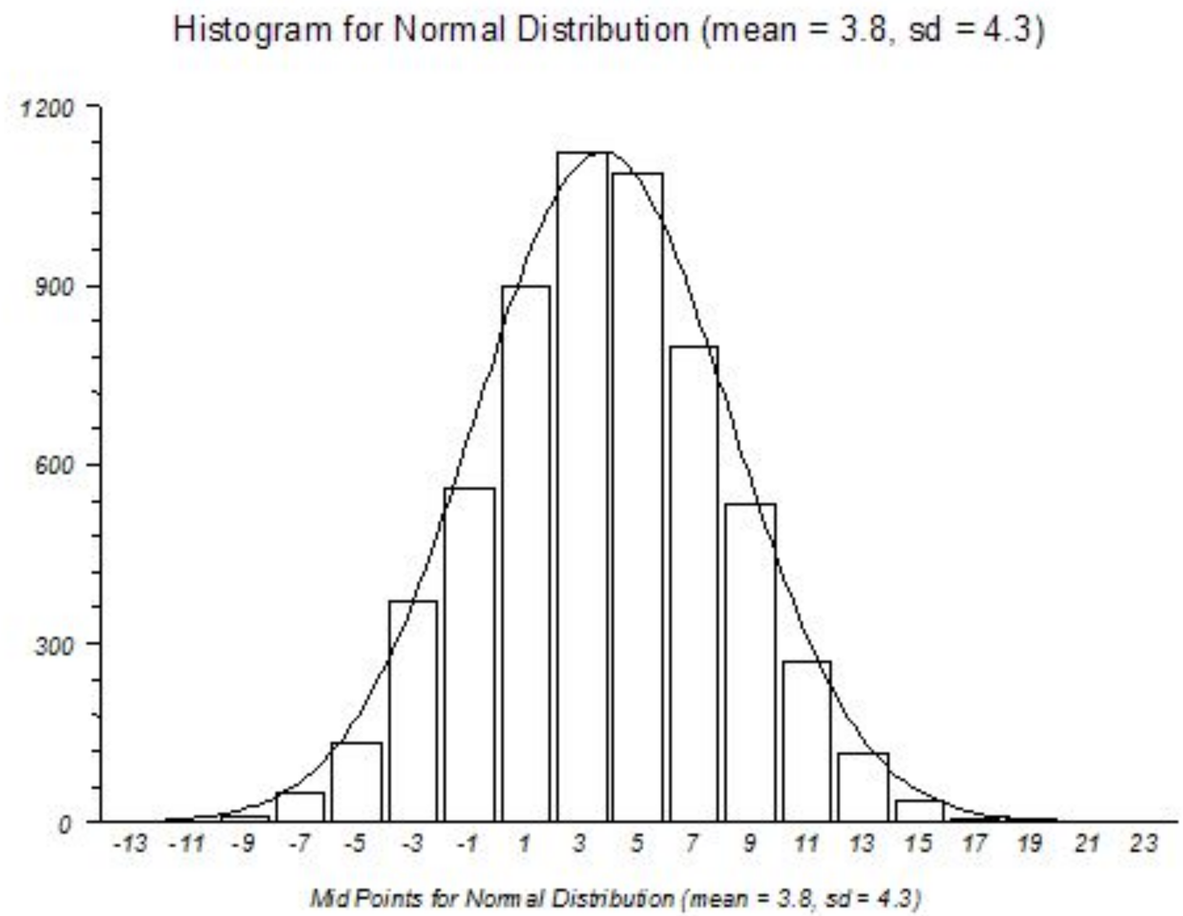
- bar graph for **quantitative** data
- values are grouped into intervals (classes)
- constructed by drawing rectangles for each class of data
- the height of each rectangle is the frequency of the class
- the width of each rectangle is the same


Points from the exam - histogram
Frequency
0
2
4
6
250
275
300
325
350
Points from the exam

Histogram

Figure 15



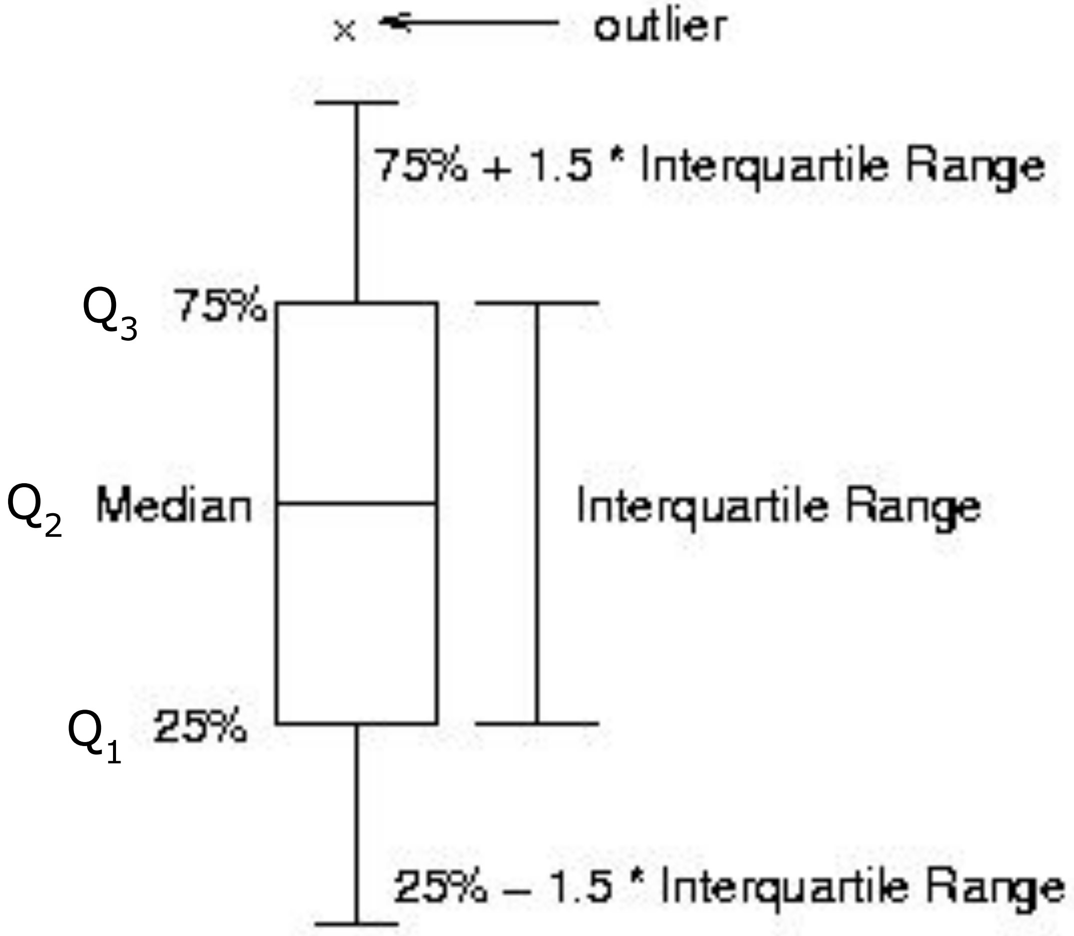


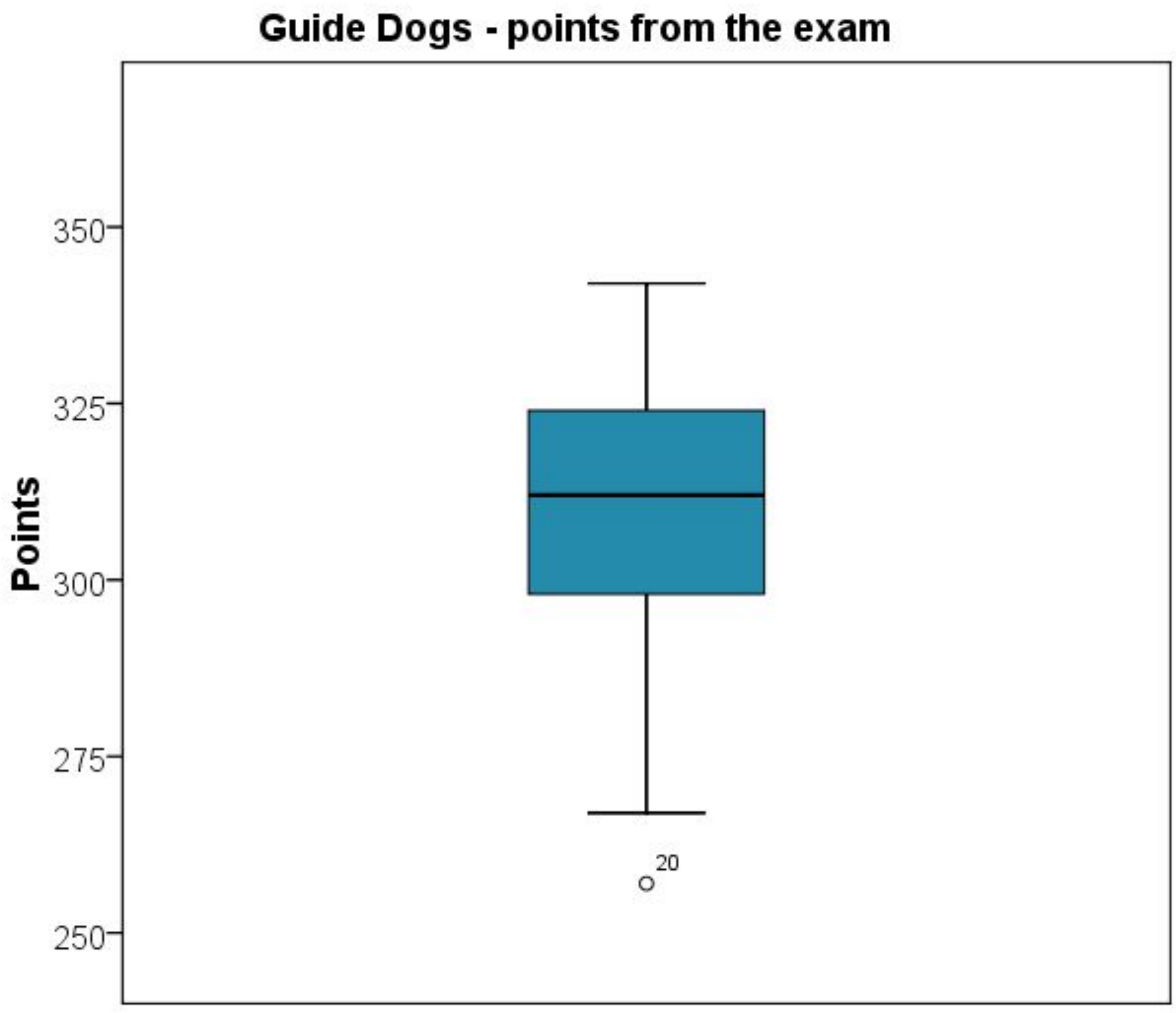
Histogram for Normal Distribution (mean = 3.8, sd = 4.3)
1200
900
600
300
0
-13 -11 -9 -7 -5 -3 -1 1 3 5 7 9 11 13 15 17 19 21 23
Mid Points for Normal Distribution (mean = 3.8, sd = 4.3)

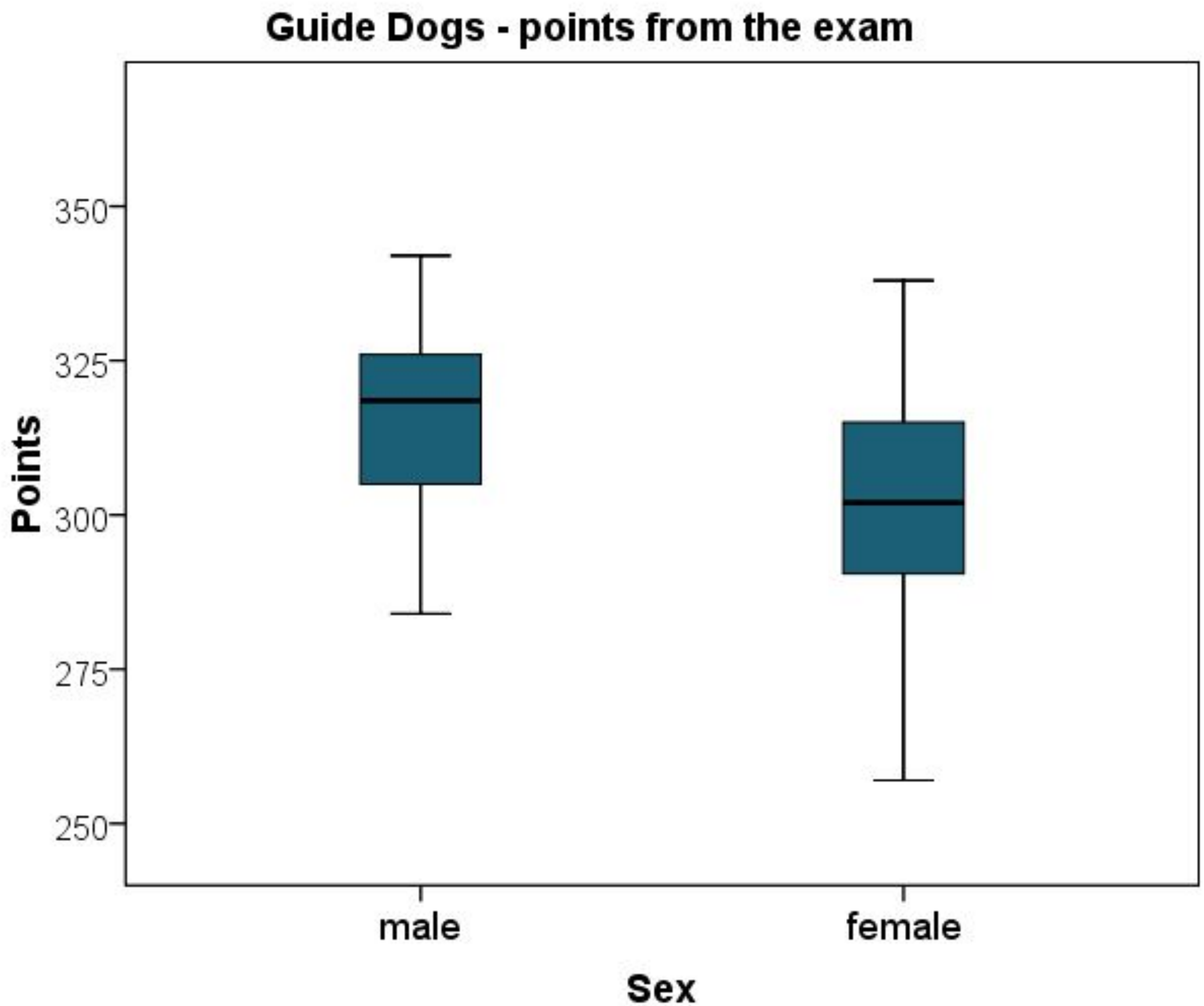
Constructing graphs – **Boxplot**

- box-and-whisker diagram
- five number summary

Boxplot


outlier
75% + 1.5 * Interquartile Range
Q3 75%
Q2 Median
Interquartile Range
Q1 25%
25% – 1.5 * Interquartile Range


Guide Dogs - points from the exam
Points
350
325
300
275
250
20


Guide Dogs - points from the exam
Points
350
325
300
275
250
male
female
Sex