Chapter 4

Operations on Bits

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OBJECTIVES

After reading this chapter, the reader should be able to:

- Apply arithmetic operations on bits when the integer is represented in two’s complement.
- Apply logical operations on bits.
- Understand the applications of logical operations using masks.
- Understand the shift operations on numbers and how a number can be multiplied or divided by powers of two using shift operations.

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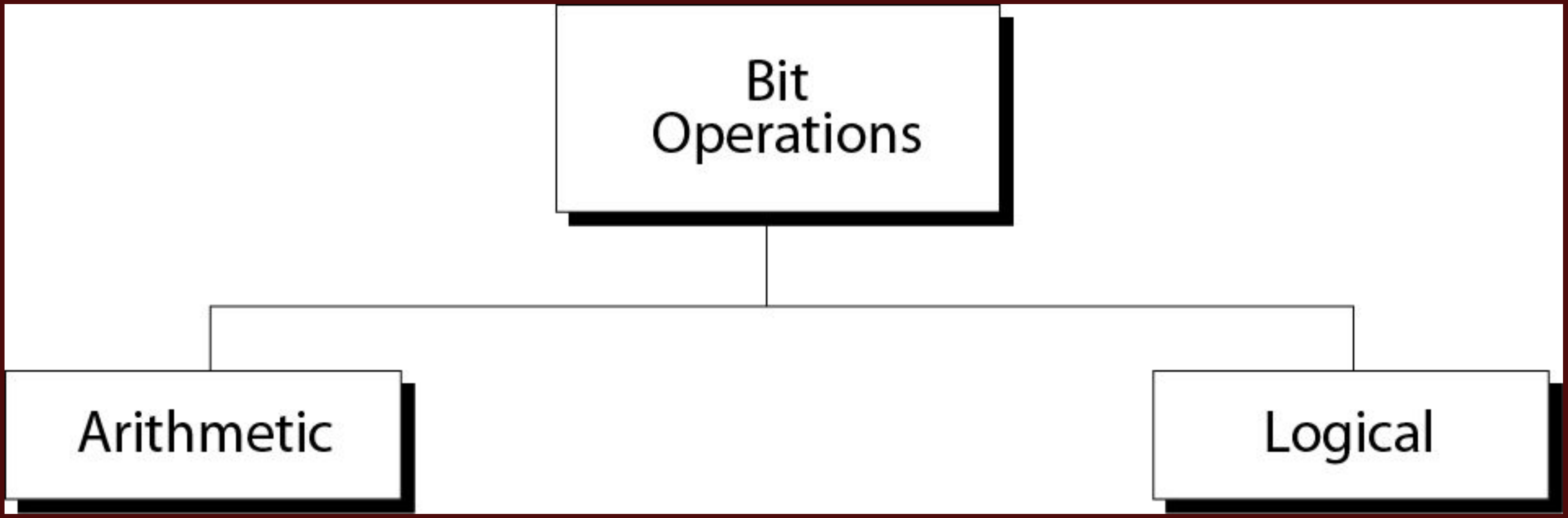
Figure 4-1

Operations on bits



4.1

ARITHMETIC OPERATIONS

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Arithmetic operations

- Arithmetic operations involve:
  - Adding (+)
  - Subtracting (--)
  - Multiplying (X)
  - Dividing (/)
  - And so on…

Addition in two's complement

Number of 1s	*Result*	*Carry*
None	0	-
One	1	-
Two	0	1
Three	1	1

Table 4.1 Adding bits

Rule of Adding Integers in Two's Complement

Add 2 bits and propagate the carry to the next column. If there is a final carry after the leftmost column addition, discard (捨棄) it.

Example 1

Add two numbers in two's complement representation: (+17) + (+22) □ (+39)

Solution

Carry			**1**						
	0	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**+**
	0	**0**	**0**	**1**	**0**	**1**	**1**	**0**	
	----------	----------	----------	----------	----------	----------	----------	----------	
Result	**0**	**0**	**1**	**0**	**0**	**1**	**1**	**1**	**□ 39**

Example 2

Add two numbers in two's complement representation: (+24) + (-17) □ (+7)

Solution

Carry		**1**	**1**	**1**	**1**	**1**				
	0	**0**	**0**	**1**	**1**	**0**	**0**	**0**	**+**	
	1	**1**	**1**	**0**	**1**	**1**	**1**	**1**		
Result	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**□**	**+7**

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Example 3

Add two numbers in two's complement representation: (-35) + (+20) □ (-15)

Solution

Carry				**1**	**1**	**1**				
	1	**1**	**0**	**1**	**1**	**1**	**0**	**1**	**+**	
	0	**0**	**0**	**1**	**0**	**1**	**0**	**0**		

Result	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**1**	□	**-15**

Example 4

Add two numbers in two's complement representation: (+127) + (+3) □ (+130)

Solution

Carry 1 1 1 1 1 1 1

0 1 1 1 1 1 1 1 +

0 0 0 0 0 0 1 1

Result 1 0 0 0 0 0 1 0 □ ***-126 (Error)***

An overflow has occurred.

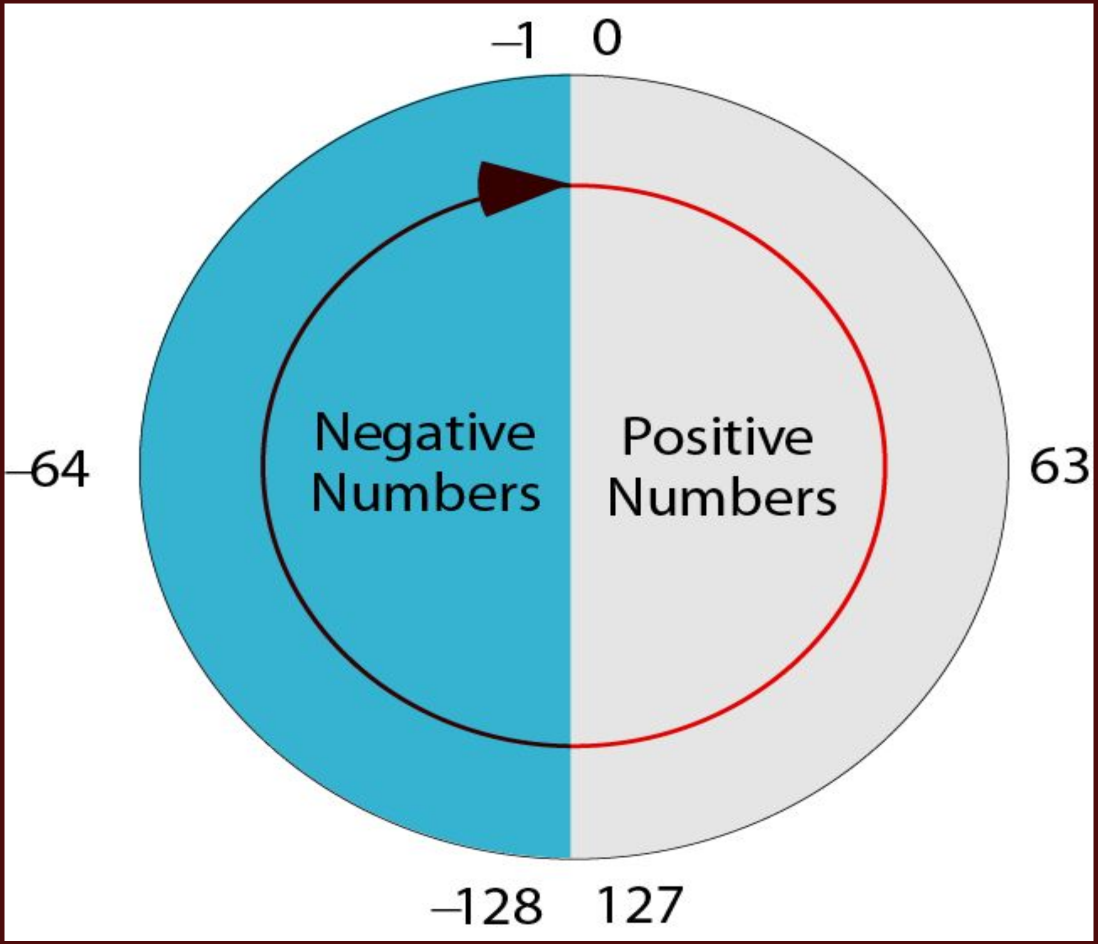
Note:

Range of numbers in two's complement representation

$- (2^{N-1})$ ---------- 0 ---------- $+(2^{N-1} -1)$

Figure 4-2

Two’s complement numbers visualization



Note:

When you do arithmetic operations on numbers in a computer, remember that each number and the result should be in the range defined by the bit allocation.

Subtraction in two's complement

Example 5

Subtract 62 from 101 in two's complement:

(+101) - (+62) (+101) + (-62)

Solution

Carry	**1**	**1**							
	0	**1**	**1**	**0**	**0**	**1**	**0**	**1**	**+**
	1	**1**	**0**	**0**	**0**	**0**	**1**	**0**	
Result	**0**	**0**	**1**	**0**	**0**	**1**	**1**	**1**	**39**

The leftmost carry is discarded.

Arithmetic operations on floating-point numbers

- Addition and subtraction for floating-point numbers are one process. (p. 54)
  - Check the sign. (a, b)
  - Move the decimal points to make the exponents the same.
  - Add or subtract the mantissas (底數).
  - Normalize the result before storing in memory.
  - Check for any overflow.

Addition

Example 6

Add two floats:

0 10000100 10110000000000000000000

0 10000010 01100000000000000000000

Solution

The exponents are 5 and 3. The numbers are:

$+2^5 \times 1.1011$ *and* $+2^3 \times 1.011$

Make the exponents the same.

$(+2^5 \times 1.1011) + (+2^5 \times 0.01011)$ □ $+2^5 \times 10.00001$

After normalization $+2^6 \times 1.000001$*, which is stored as:*

0 10000101 00000100000000000000000

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4.2

LOGICAL OPERATIONS

Logical operations

- A logical operation can accept 1 or 2 bits to create only 1 bit.
  - Unary operation (Figure4.3)
  - Binary operation (Figure4.3)

Figure 4-3

Unary and binary operations

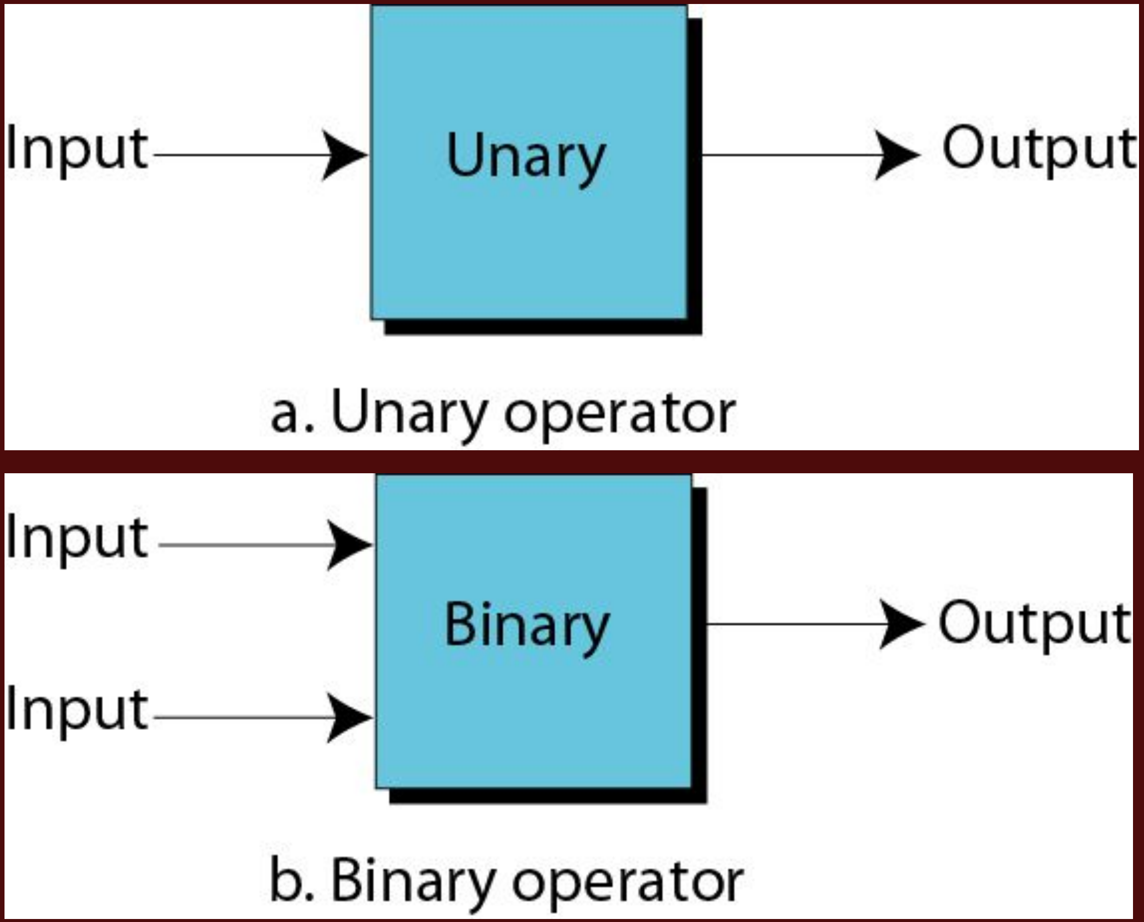


Figure 4-4

Logical operations

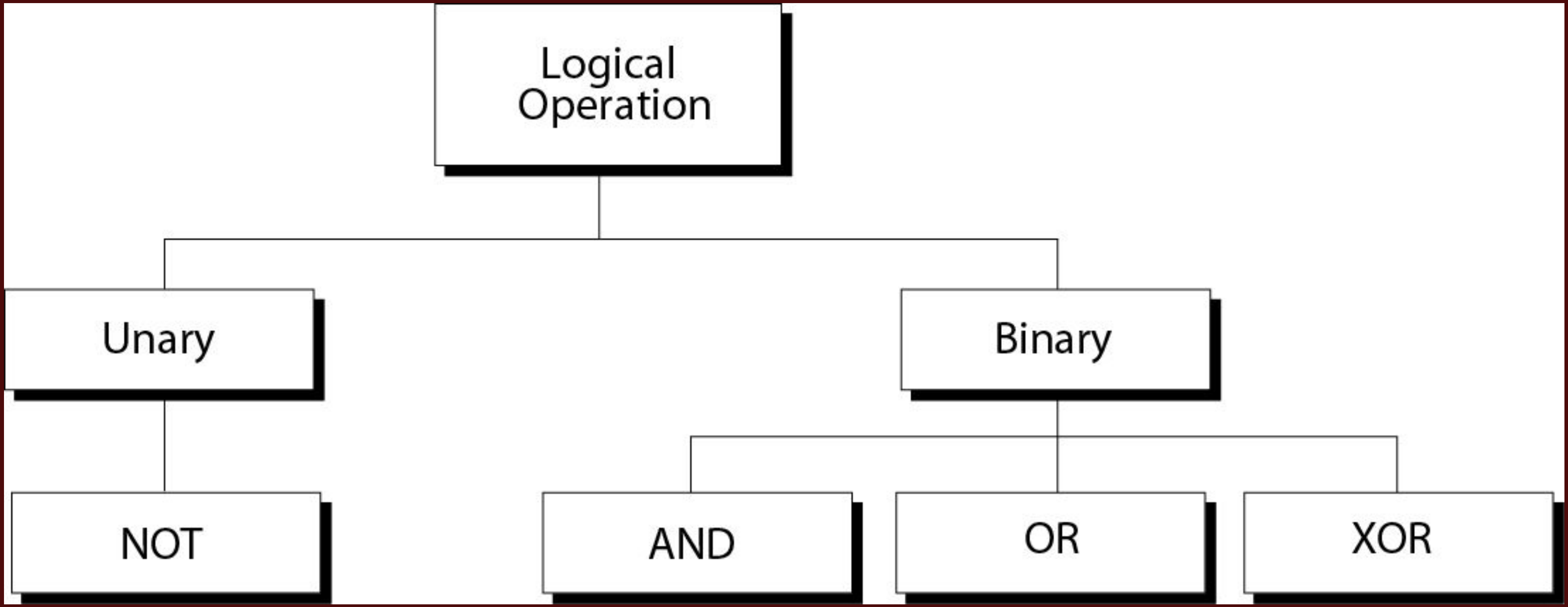


Figure 4-5

Truth tables

NOT

x	NOT x
0	1
1	0

AND

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

OR

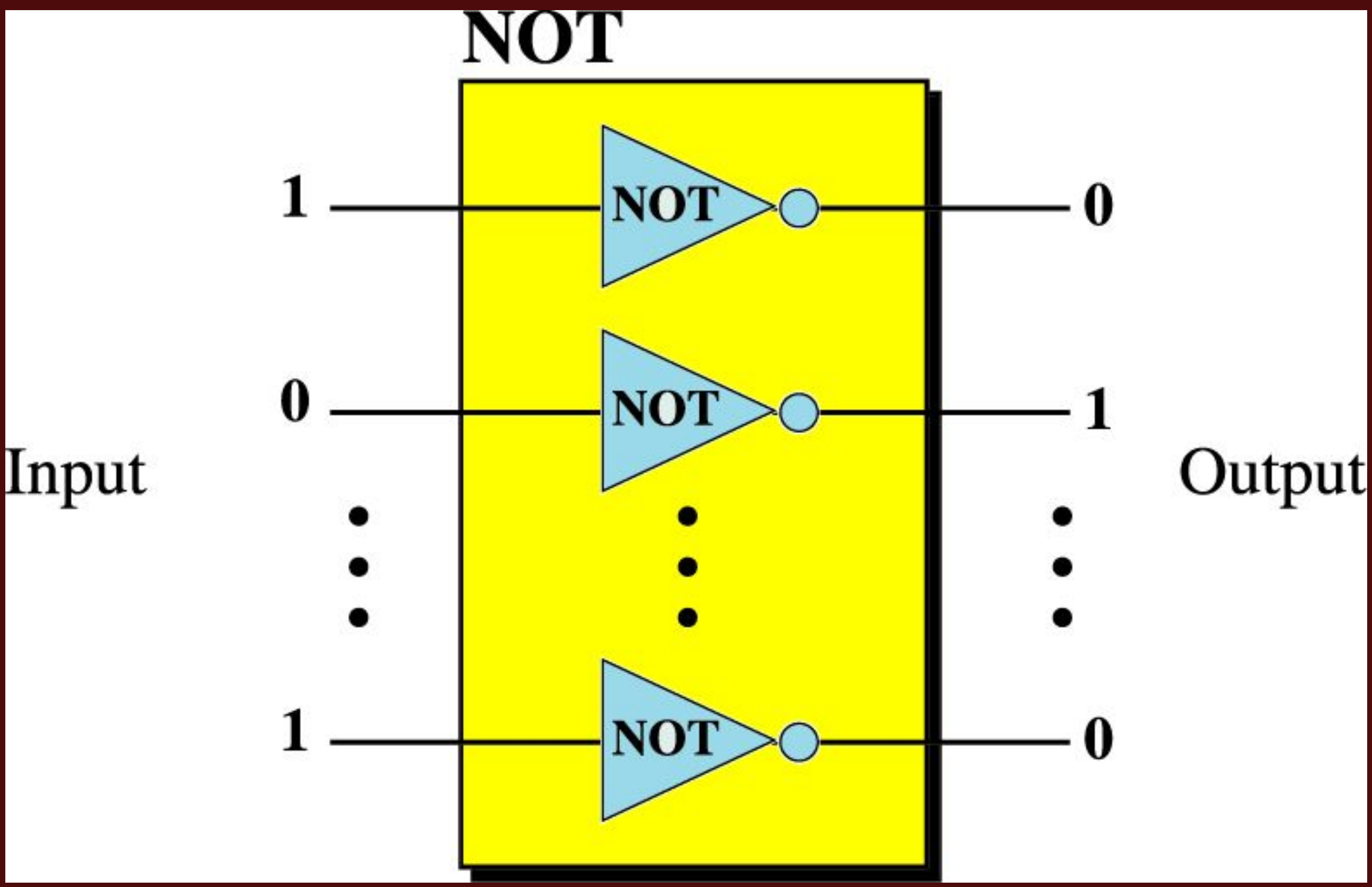
x	y	x OR y
0	0	0
0	1	1
1	0	1
1	1	1

XOR

x	y	x XOR y
0	0	0
0	1	1
1	0	1
1	1	0

Figure 4-6

Unary operator -- NOT operator



NOT operator

Example 7

Use the NOT operator on the bit pattern 10011000

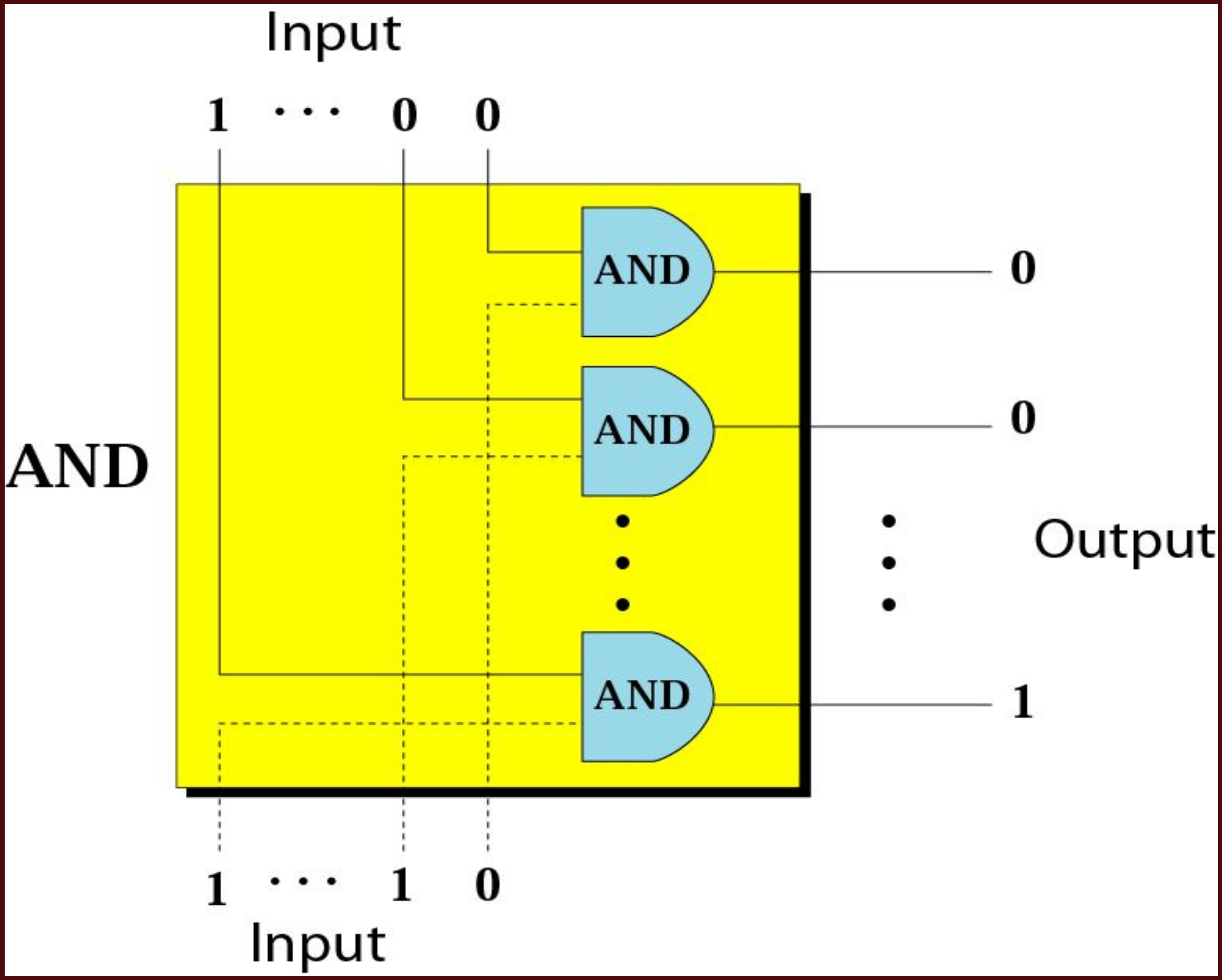
Solution

Target	**1 0 0 1 1 0 0 0**	***NOT***

Result	**0 1 1 0 0 1 1 1**	

Figure 4-7

Binary operator--AND operator



AND operator

Example 8

Use the AND operator on bit patterns 10011000 and 00110101.

Solution

Target	**1 0 0 1 1 0 0 0**	***AND***
	0 0 1 1 0 1 0 1	

Result	**0 0 0 1 0 0 0 0**	

Figure 4-8

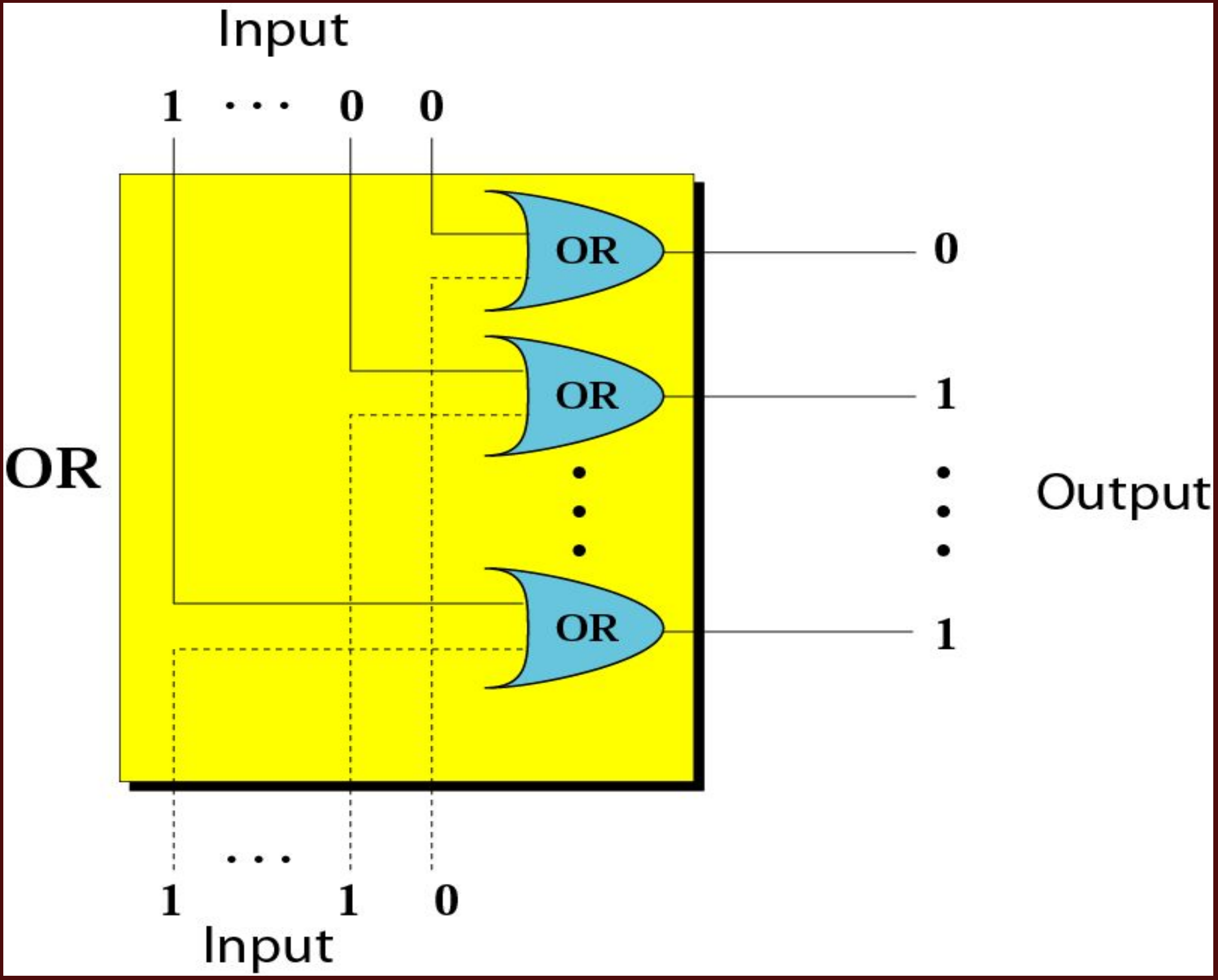
Inherent (本質的) rule of the AND operator

(0) AND (X) → (0)

(X) AND (0) → (0)

Figure 4-9

Binary operator--OR operator



OR operator

Example 9

Use the OR operator on bit patterns 10011000 and 00110101

Solution

Target	**1 0 0 1 1 0 0 0**	***OR***
	0 0 1 1 0 1 0 1	

Result	**1 0 1 1 1 1 0 1**	

Figure 4-10

Inherent rule of the OR operator

(1) OR (X) → (1)

(X) OR (1) → (1)

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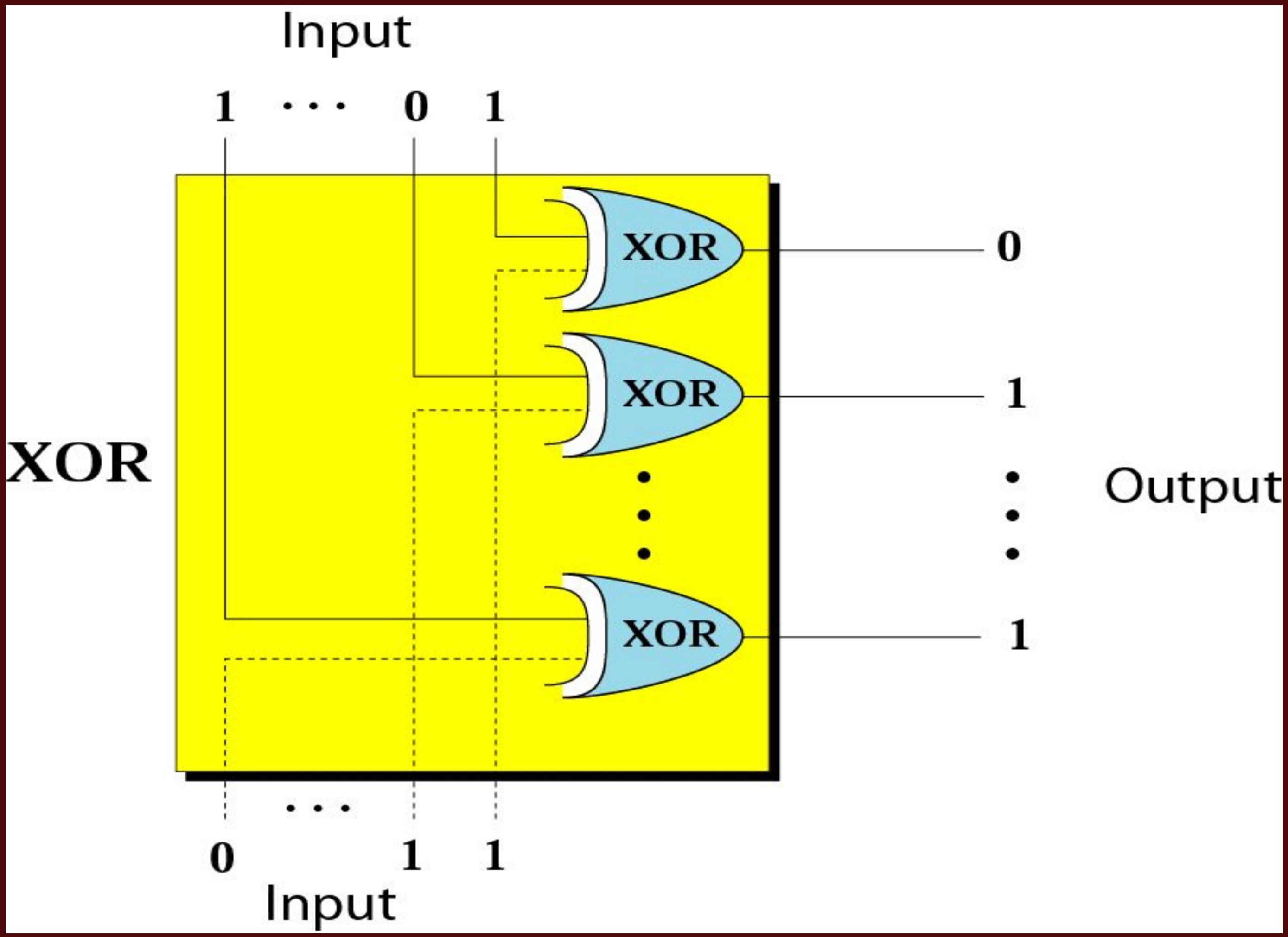
Figure 4-11

Binary operator--XOR operator



XOR operator

Example 10

Use the XOR operator on bit patterns 10011000 and 00110101.

Solution

Target	**1 0 0 1 1 0 0 0**	***XOR***
	0 0 1 1 0 1 0 1	

Result	**1 0 1 0 1 1 0 1**	

Figure 4-12

Inherent rule of the XOR operator

$$(1) \quad \text{XOR} \quad (X) \longrightarrow \text{NOT}\,(X)$$

$$(X) \quad \text{XOR} \quad (1) \longrightarrow \text{NOT}\,(X)$$

Applications

Mask (遮罩)

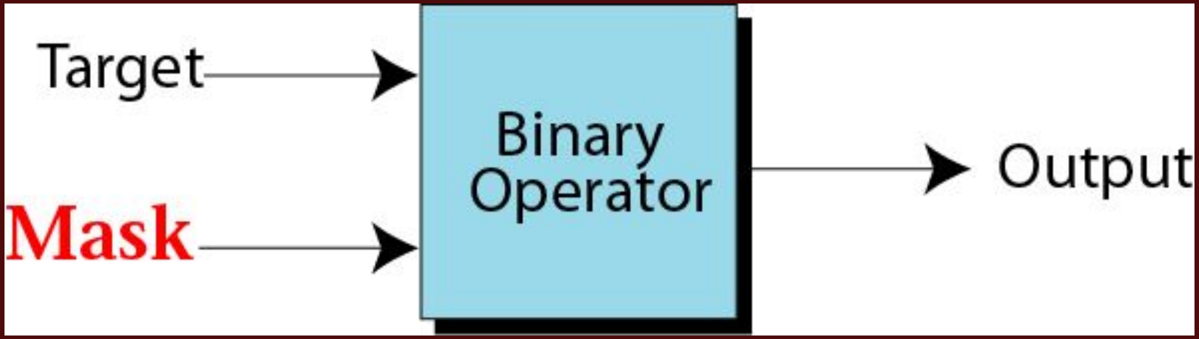
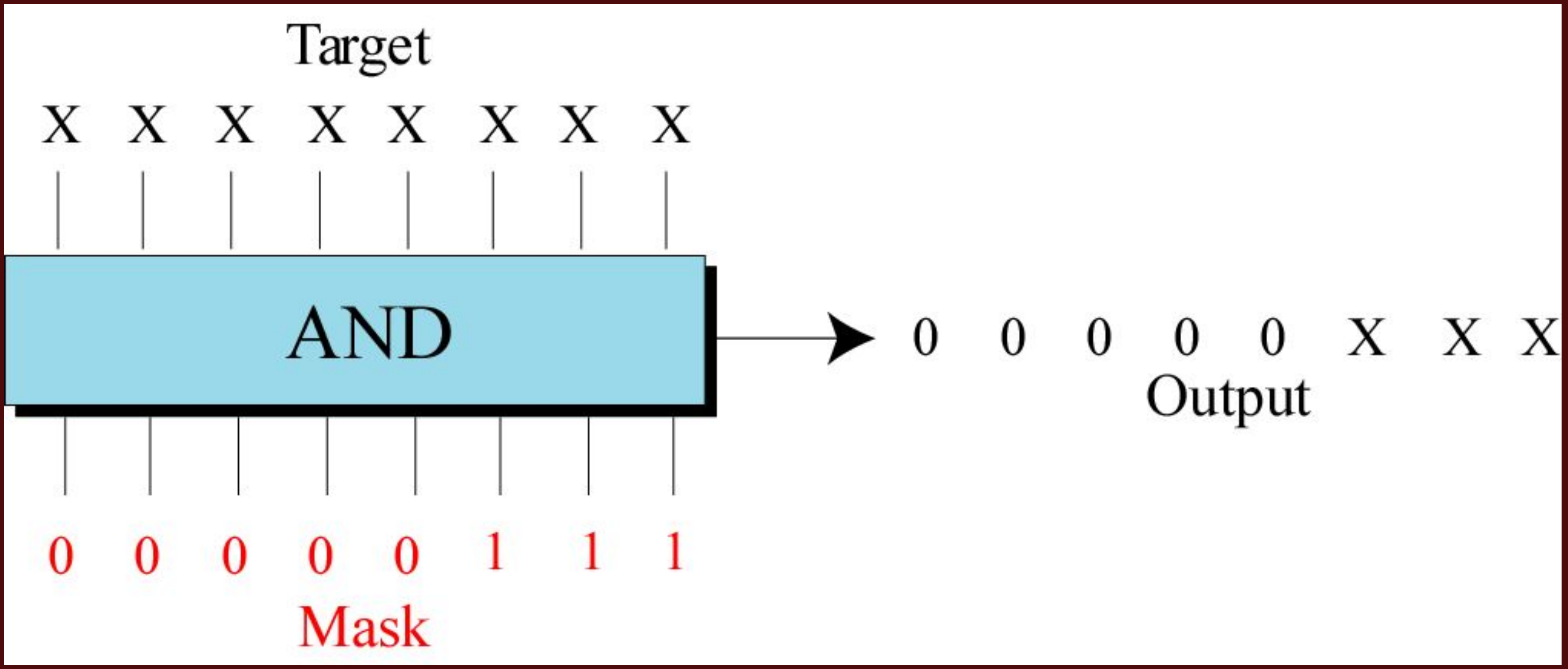


Figure 4-14

Example of unsetting specific bits



Example 11

Use a mask to unset (clear) the 5 leftmost bits of a pattern. Test the mask with the pattern 10100110.

Solution

The mask is 00000111.

Target	**1 0 1 0 0 1 1 0**	***AND***
Mask	**0 0 0 0 0 1 1 1**	

Result	**0 0 0 0 0 1 1 0**	

Example 12

Imagine a power plant (水力發電廠) that pumps water (供水) to a city using eight pumps (抽水機). The state of the pumps (on or off) can be represented by an 8-bit pattern. For example, the pattern 11000111 shows that pumps 1 to 3 (from the right), 7 and 8 are on while pumps 4, 5, and 6 are off. Now assume pump 7 shuts down. How can a mask show this situation?

Solution on the next slide.

Solution

Use the mask 10111111 to AND with the target pattern. The only 0 bit (bit 7) in the mask turns off the seventh bit in the target.

Target	**1 1 0 0 0 1 1 1**	***AND***
Mask	**1 0 1 1 1 1 1 1**	

Result	**1 0 0 0 0 1 1 1**	

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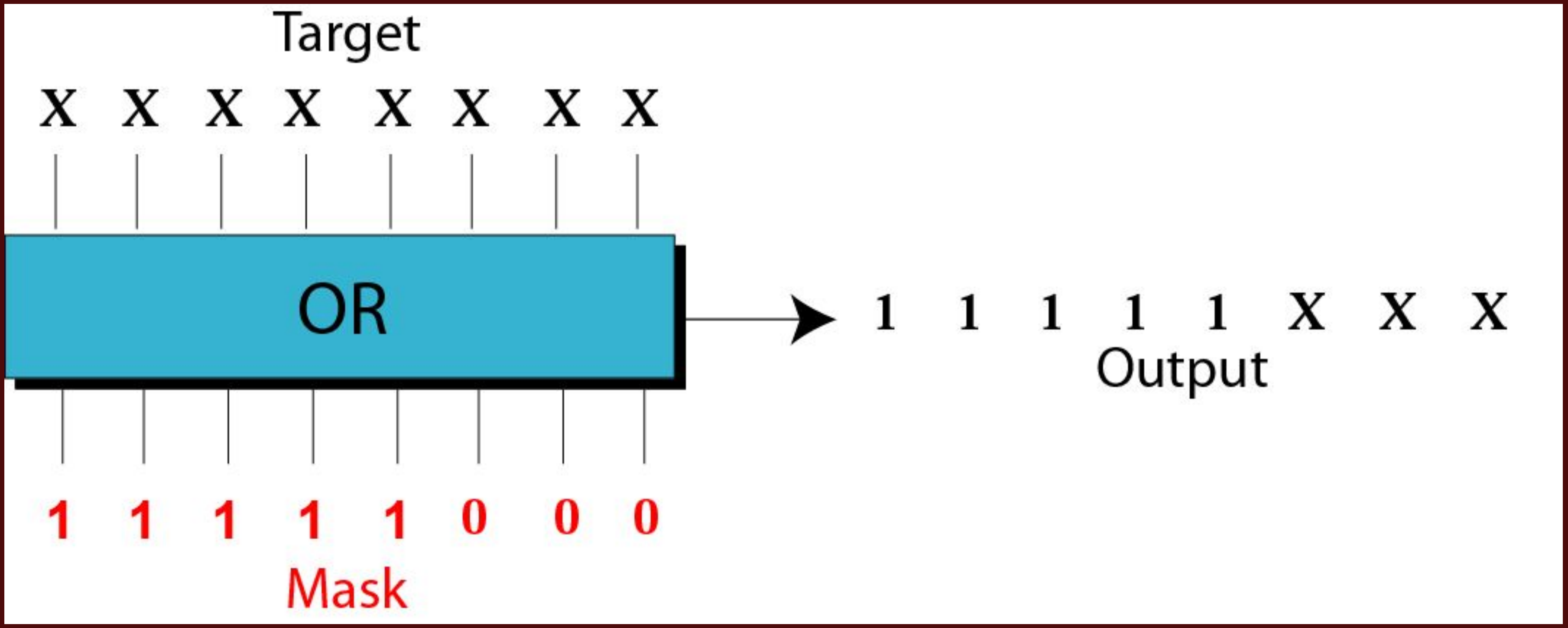
Figure 4-15

Example of setting specific bits



Example 13

Use a mask to set the 5 leftmost bits of a pattern. Test the mask with the pattern 10100110.

Solution

The mask is 11111000.

Target	**1 0 1 0 0 1 1 0**	***OR***
Mask	**1 1 1 1 1 0 0 0**	

Result	**1 1 1 1 1 1 1 0**	

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Example 14

Using the power plant example, how can you use a mask to to show that pump 6 is now turned on?

Solution

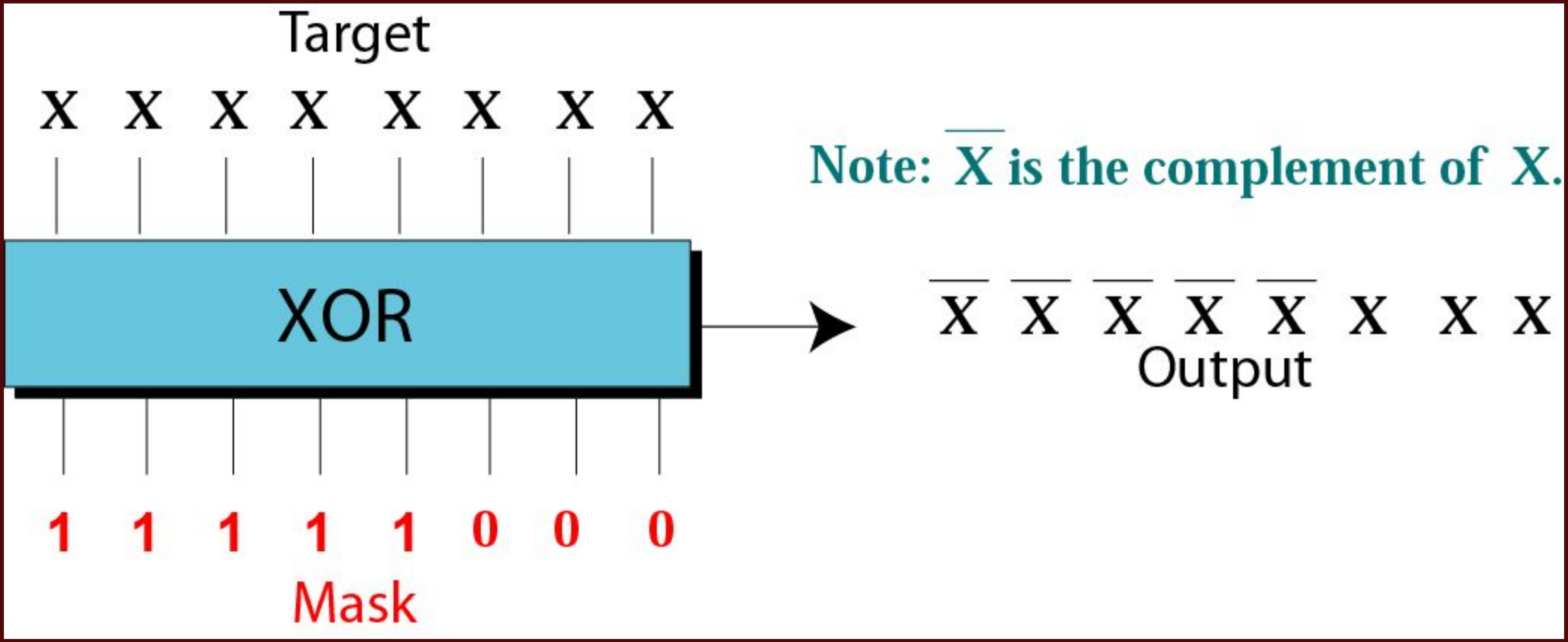
Use the mask 00100000.

Target	**1 0 0 0 0 1 1 1**	***OR***
Mask	**0 0 1 0 0 0 0 0**	

Result	**1 0 1 0 0 1 1 1**	

Figure 4-16

Example of flipping (跳動的) specific bits



Example 15

Use a mask to flip the 5 leftmost bits of a pattern. Test the mask with the pattern 10100110.

Solution

Target	**1 0 1 0 0 1 1 0**	***XOR***
Mask	**1 1 1 1 1 0 0 0**	

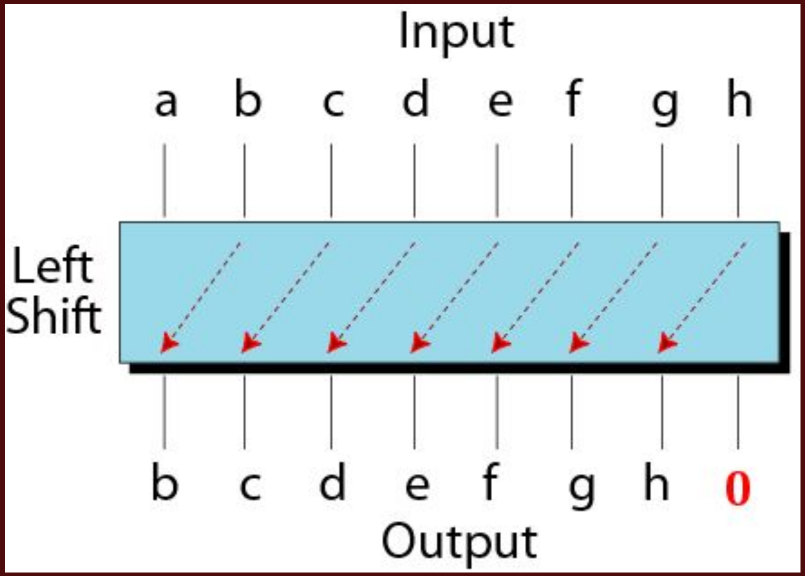
Result	**0 1 0 1 1 1 1 0**	

4.3

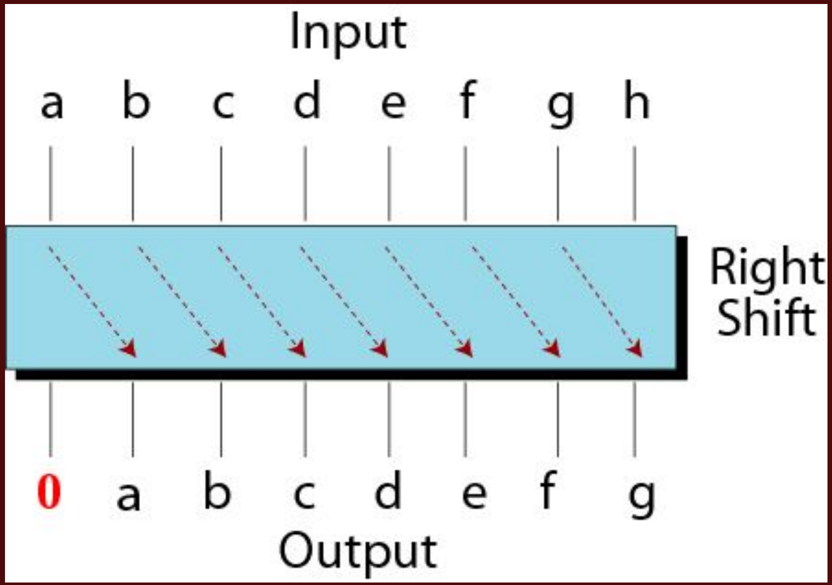
SHIFT OPERATIONS

Shift operations

Left shift



Right shift



Example 16

Show how you can divide or multiply a number by 2 using shift operations.

Solution

If a bit pattern represents an unsigned number, a right-shift operation divides the number by two. The pattern 00111011 represents 59. When you shift the number to the right, you get 00011101, which is 29. If you shift the original number to the left, you get 01110110, which is 118.

Example 17

Use a combination of logical and shift operations to find the value (0 or 1) of the fourth bit (from the right).

Solution

Use the mask 00001000 to AND with the target to keep the fourth bit and clear the rest of the bits.

Continued on the next slide

Solution (continued)

Target	**a b c d e f g h**	***AND***
Mask	**0 0 0 0 1 0 0 0**	

Result	**0 0 0 0 e 0 0 0**	

Shift the new pattern three times to the right

0000e000 □ 00000e00 □ 000000e0 □ 0000000e

Now it is easy to test the value of the new pattern as an unsigned integer. If the value is 1, the original bit was 1; otherwise the original bit was 0.

Key terms

- AND operator
- Arithmetic operation
- Binary operation
- Binary operator
- Carry
- Clear
- Flip
- Floating-point number
- Force (強迫) to 0
- Force (強迫) to 1
- Logical operation
- Mantissa
- Mask
- NOT operator
- OR operator
- Overflow
- Set
- Truth table
- Two's complement
- Unary operation
- Unary operator
- Unset
- XOR operator