

# Potential Flow Theory

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*Only Mathematics Available for Invention.....*

# Elementary fascination Functions

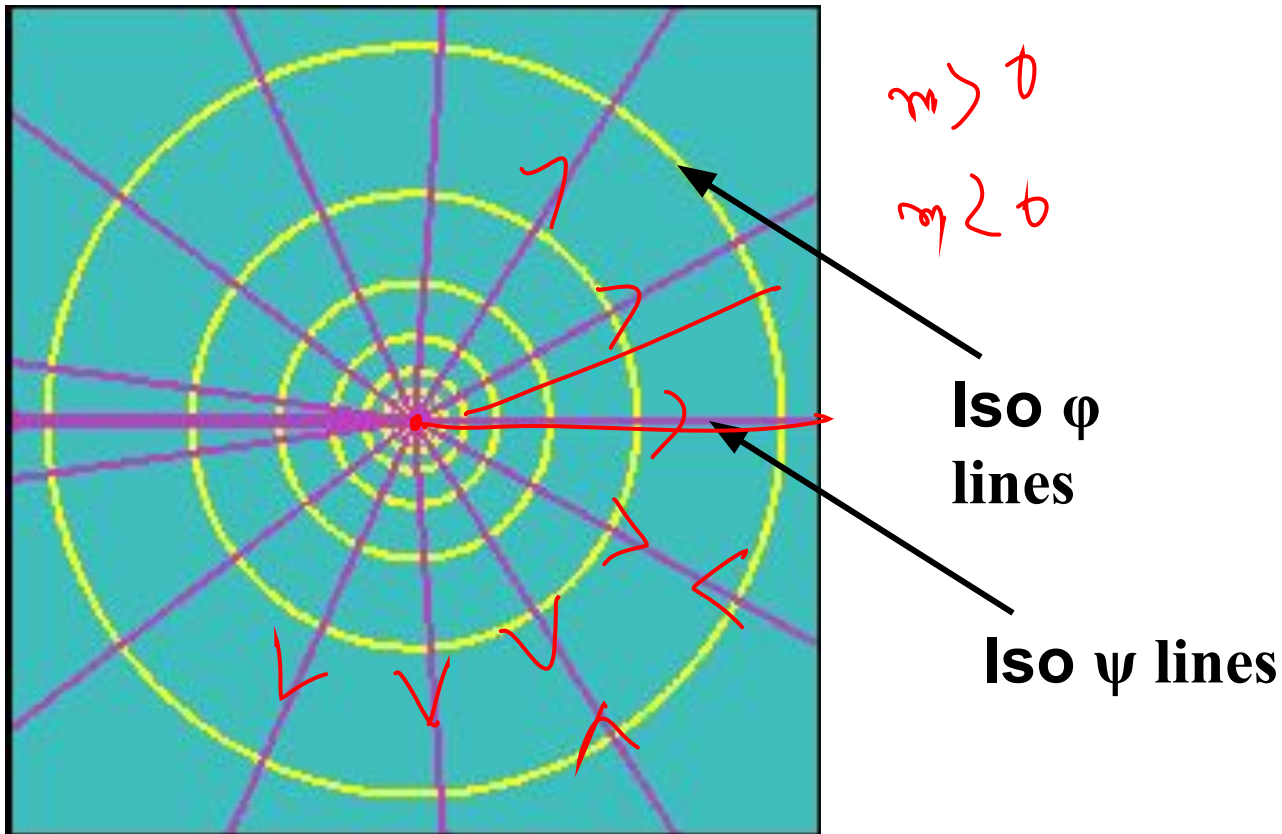
- To Create **IRROTATIONAL PLANE FLOWS**
- The uniform flow
- The source and the sink
- The vortex

# THE SOURCE OR SINK

- source (or sink), the complex potential of which is

$$W = \phi + i\psi = \frac{m}{2\pi} \ln z \quad / \quad \begin{array}{l} -\frac{m}{2\pi} \ln(z) \\ \uparrow \\ \text{sink} \end{array}$$

- This is a pure radial flow, in which all the streamlines converge at the origin, where there is a singularity due to the fact that continuity can not be satisfied.
- At the origin there is a source,  $m > 0$  or sink,  $m < 0$  of fluid.
- Traversing any closed line that does not include the origin, the mass flux (and then the discharge) is always zero.
- On the contrary, following any closed line that includes the origin the discharge is always nonzero and equal to  $m$ .

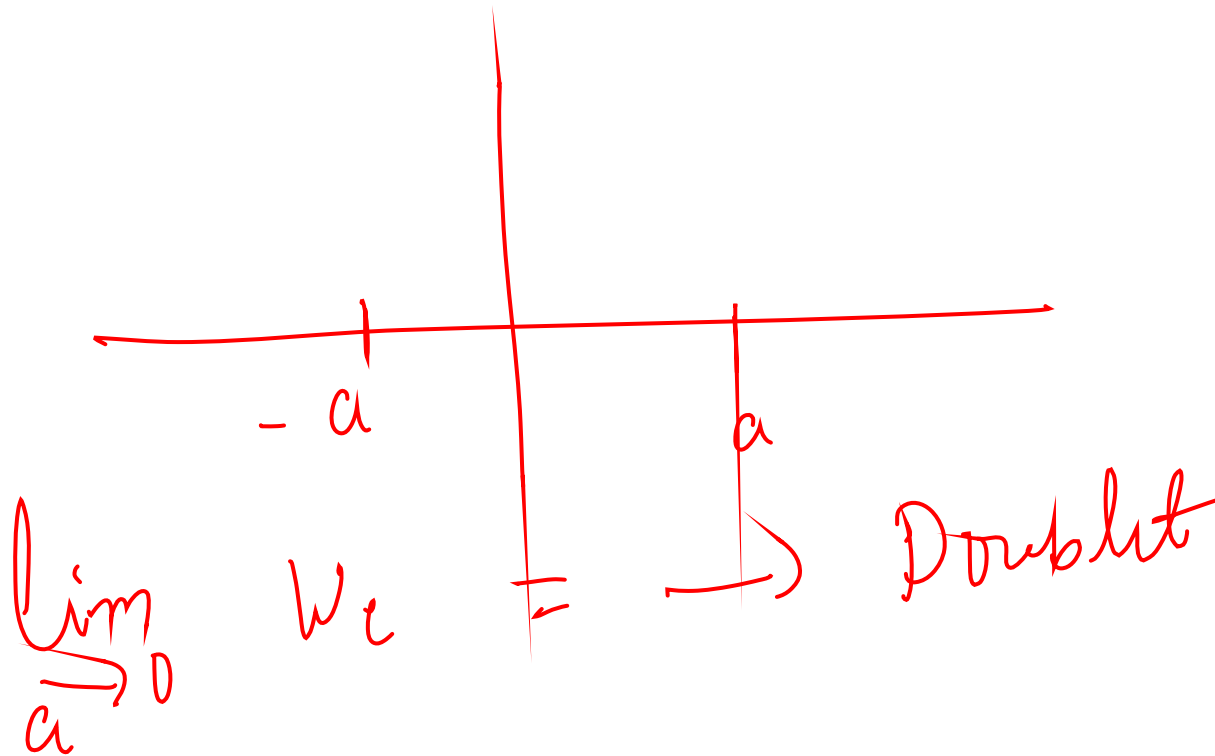


❖ The flow field is uniquely determined upon deriving the complex potential  $W$  with respect to  $z$ .

$$W = \phi + i\psi = \frac{m}{2\pi} \ln z$$

# A Combination of Source & Sink

$$W_c = \frac{m}{4\pi} \left( \ln(z+a) - \ln(z-a) \right)$$

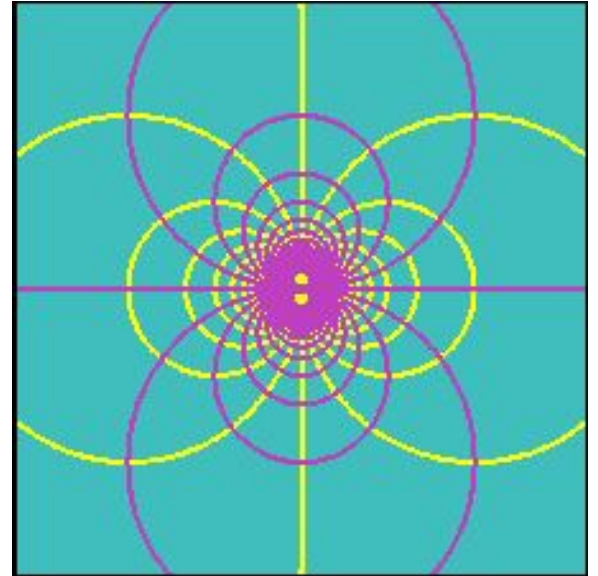


# THE DOUBLET

- The complex potential of a doublet

$$W = \frac{\mu}{2\pi z}$$

$$\mu = 2m\alpha$$



# Uniform Flow Past A Doublet

- The superposition of a doublet and a uniform flow gives the complex potential

$$W = Uz + \frac{\mu}{2\pi z}$$

$$W = \frac{2\pi Uz^2 + \mu}{2\pi z}$$

$$W = \frac{2\pi U(x + iy)^2 + \mu}{2\pi(x + iy)}$$

$$W = \frac{2\pi U(x^3 + xy^2) + \mu x}{2\pi(x^2 + y^2)} + i \frac{[2\pi U(x^2 y + y^3) - \mu y]}{2\pi(x^2 + y^2)} = \phi + i\psi$$

$$\phi = \frac{2\pi U(x^3 + xy^2) + \mu x}{2\pi(x^2 + y^2)} \quad \& \quad \psi = \frac{[2\pi U(x^2 y + y^3) - \mu y]}{2\pi(x^2 + y^2)}$$

$$\psi = Uy - \frac{\mu y}{2\pi(x^2 + y^2)}$$

Find out a stream line corresponding to a value of stream function is zero

$$0 = Uy - \frac{\mu y}{2\pi(x^2 + y^2)}$$



$$0 = Uy - \frac{\mu y}{2\pi(x^2 + y^2)}$$

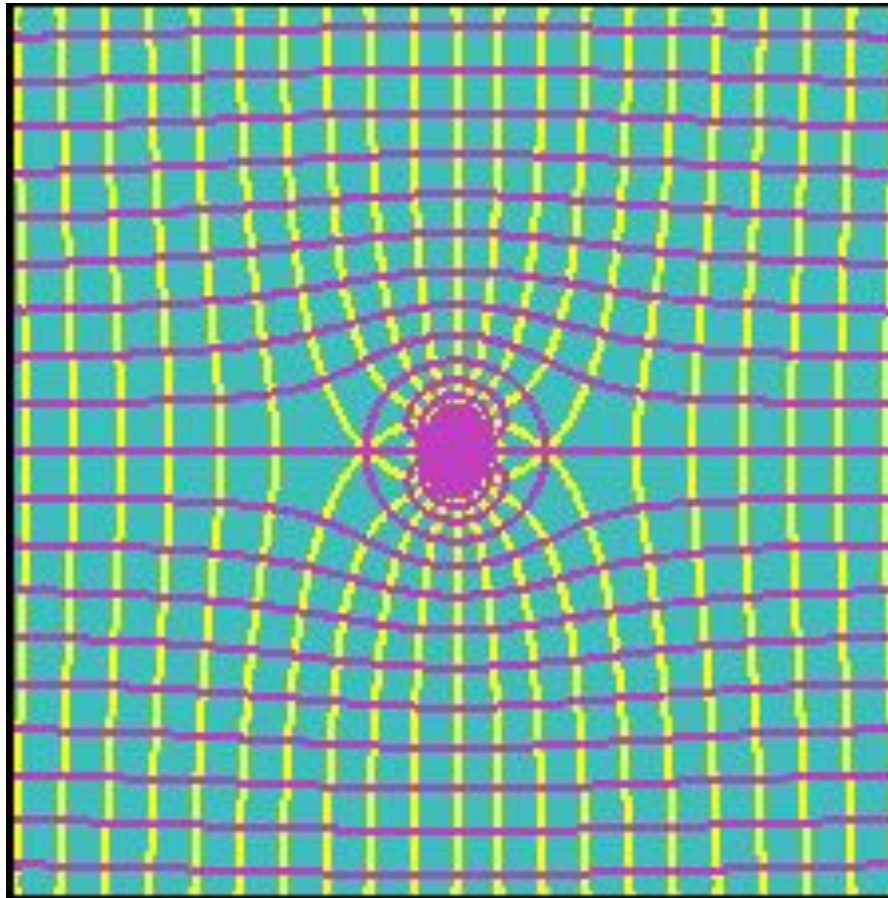
$$0 = 2\pi Uy(x^2 + y^2) - \mu y$$

$$0 = 2\pi U(x^2 + y^2) - \mu$$

$$x^2 + y^2 = \frac{\mu}{2\pi U}$$

$$x^2 + y^2 = \frac{\mu}{2\pi U} = R^2$$

- There exist a circular stream line of radius R, on which value of stream function is zero.
- Any stream function of zero value is an impermeable solid wall.
- Plot shapes of iso-streamlines.



Note that one of the streamlines is closed and surrounds the origin at a constant distance equal to

$$R = \sqrt{\frac{\mu}{2\pi U}}$$

Recalling the fact that, by definition, a streamline cannot be crossed by the fluid, this complex potential represents the irrotational flow around a cylinder of radius  $R$  approached by a uniform flow with velocity  $U$ .

Moving away from the body, the effect of the doublet decreases so that far from the cylinder we find, as expected, the undisturbed uniform flow.

$$W = Uz + \frac{\mu}{2\pi z} \quad \lim_{z \rightarrow \infty} W = U_{\infty} z : \textit{Uniform Flow}$$

In the two intersections of the  $x$ -axis with the cylinder, the velocity will be found to be zero.

These two points are thus called stagnation points.

To obtain the velocity field, calculate  $dW/dz$ .  $W = Uz + \frac{\mu}{2\pi z}$

$$\frac{dW}{dz} = U - \frac{\mu}{2\pi z^2}$$

$$\frac{dW}{dz} = U - \frac{\mu}{2\pi} \left\{ \frac{x^2 - y^2 - 2ixy}{(x^2 - y^2)^2 - 4x^2 y^2} \right\}$$

$$\frac{dW}{dz} = U - \frac{\mu}{2\pi} \left\{ \frac{x^2 - y^2}{(x^2 - y^2)^2 - 4x^2 y^2} \right\} + 2i \frac{\mu}{2\pi} \left\{ \frac{xy}{(x^2 - y^2)^2 - 4x^2 y^2} \right\}$$

$$\frac{dW}{dz} = u - iv$$

$$u = U - \frac{\mu}{2\pi} \left\{ \frac{x^2 - y^2}{(x^2 - y^2)^2 - 4x^2y^2} \right\} \quad v = -\frac{\mu}{\pi} \left\{ \frac{xy}{(x^2 - y^2)^2 - 4x^2y^2} \right\}$$

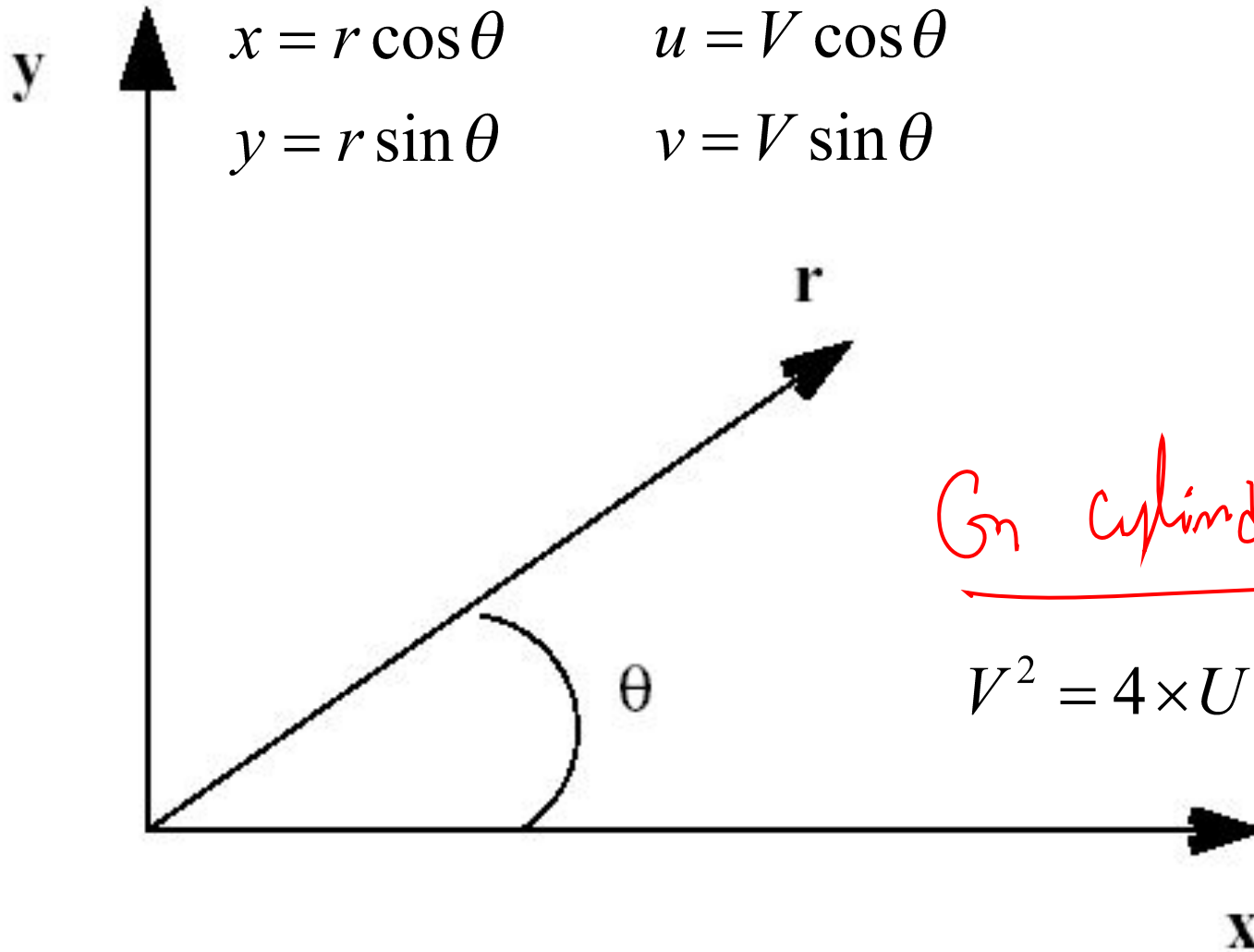
$$V^2 = u^2 + v^2$$

$$V^2 = \left[ U - \frac{\mu}{2\pi} \left\{ \frac{x^2 - y^2}{(x^2 - y^2)^2 - 4x^2y^2} \right\} \right]^2 + \left[ -\frac{\mu}{\pi} \left\{ \frac{xy}{(x^2 - y^2)^2 - 4x^2y^2} \right\} \right]^2$$

Equation of zero stream line:

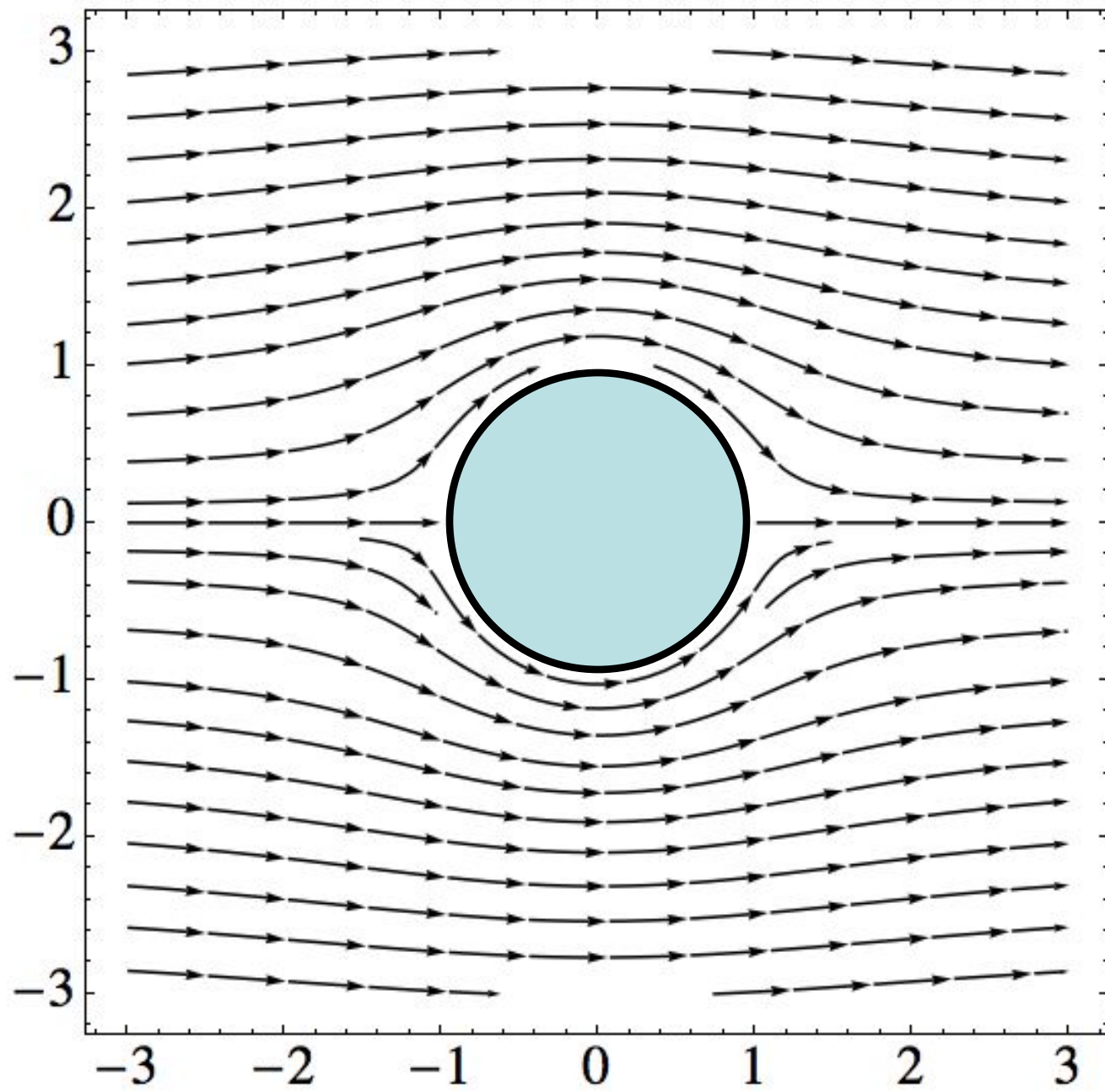
$$R^2 = x^2 + y^2 \quad \text{with} \quad R = \sqrt{\frac{\mu}{2\pi U}}$$

# Cartesian and polar coordinate system



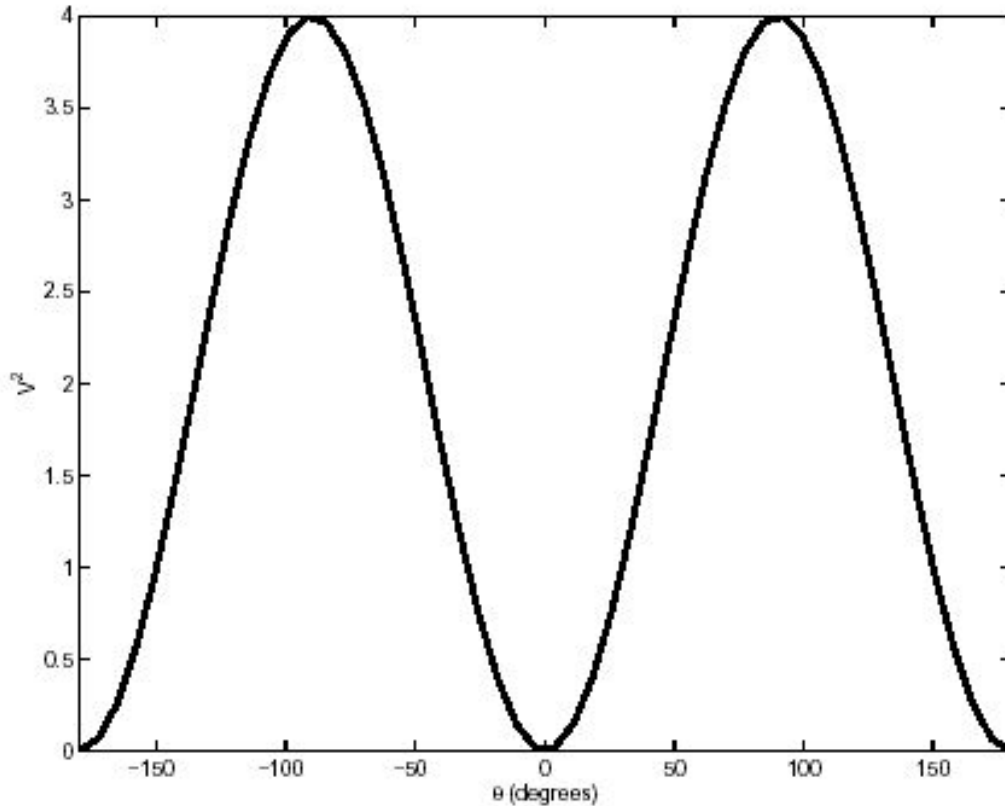
On cylinder wall

$$V^2 = 4 \times U^2 \sin^2 \theta$$



# $V^2$ Distribution of flow over a circular cylinder

$$V^2 = U^2 \left\{ 1 - 2 \frac{R^2}{r^2} \cos(2\theta) + \frac{R^4}{r^4} \right\}$$



The velocity of the fluid is zero at  $\theta = 0^\circ$  and  $\theta = 180^\circ$ . Maximum velocity occur on the sides of the cylinder at  $\theta = 90^\circ$  and  $\theta = -90^\circ$ .



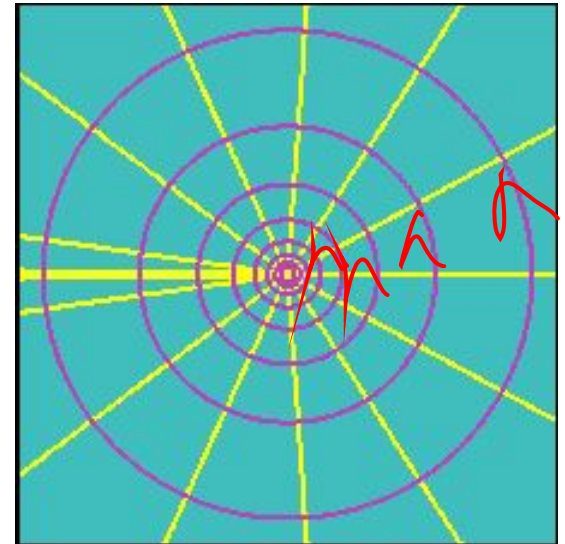
# THE VORTEX

- In the case of a vortex, the flow field is purely tangential.

The picture is similar to that of a source but streamlines and equipotential lines are reversed.

The complex potential is

$$W = \phi + i\psi = i \frac{\gamma}{2\pi} \ln z$$



There is again a singularity at the origin, this time associated to the fact that the circulation along any closed curve including the origin is nonzero and equal to  $\gamma$ .

If the closed curve does not include the origin, the circulation will be zero.

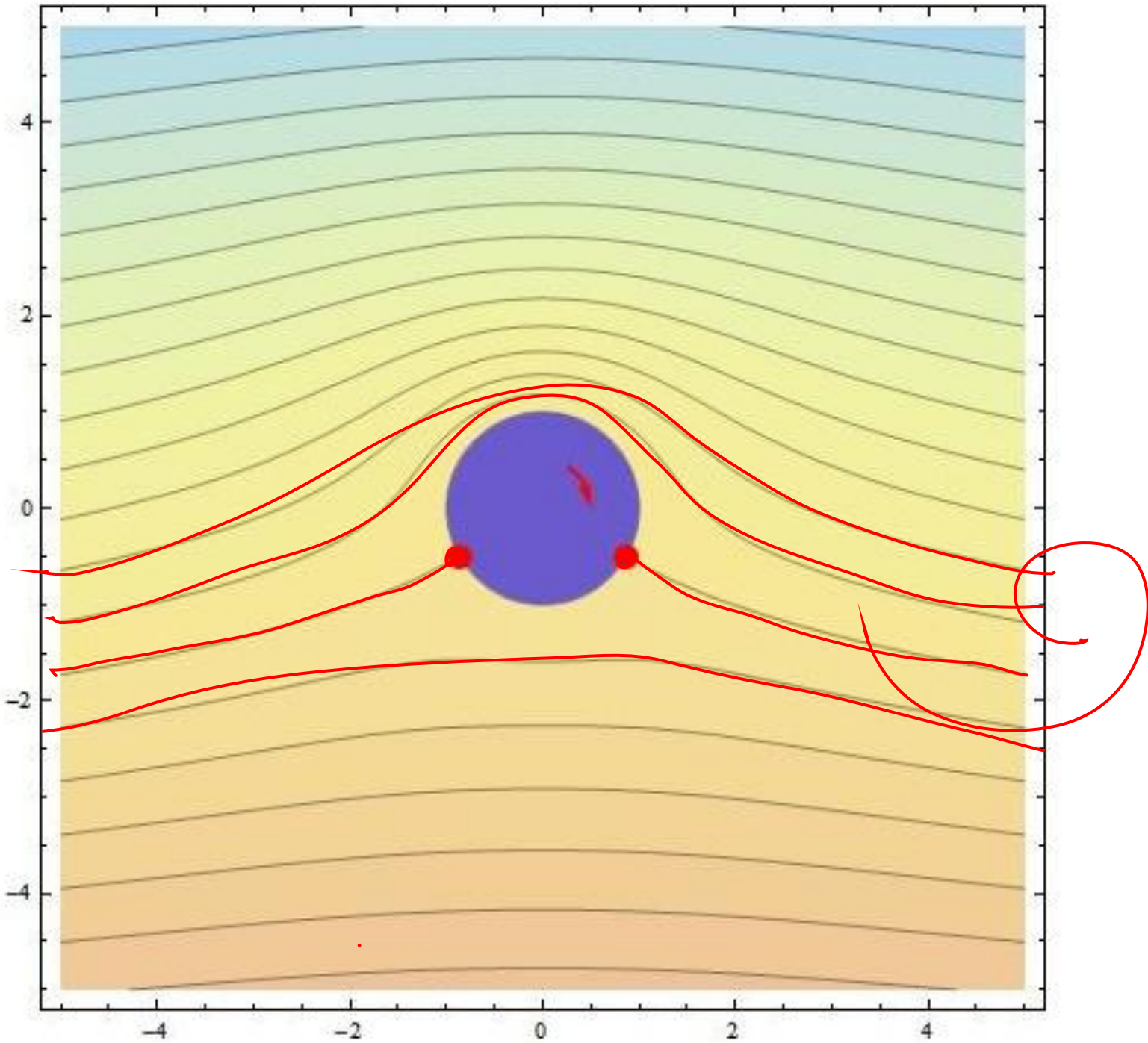
# Uniform Flow Past A Doublet with Vortex

- The superposition of a doublet and a uniform flow gives the complex potential

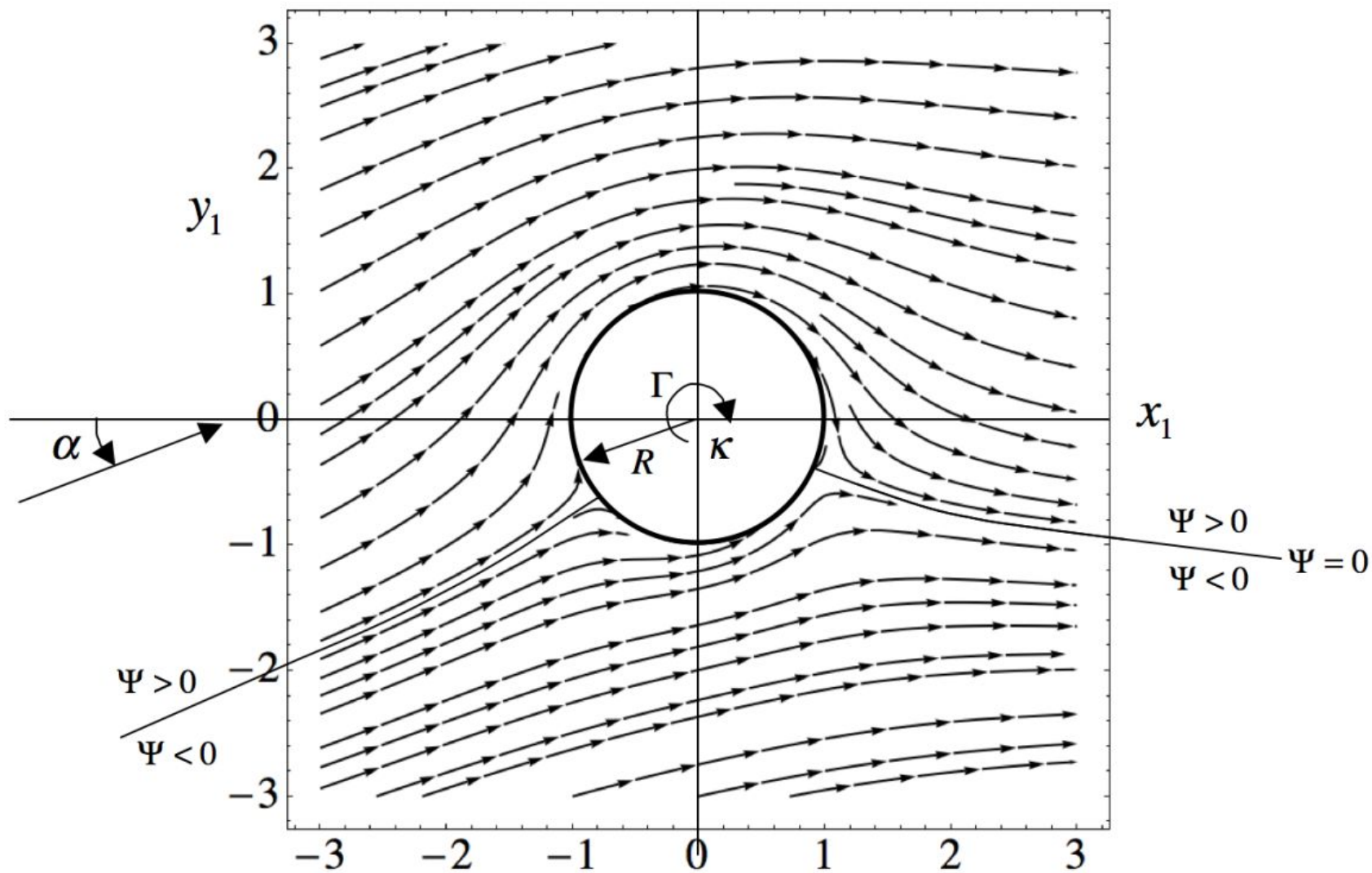
$$W = Uz + \frac{\mu}{2\pi z} + i \frac{\gamma}{2\pi} \ln z$$

$$W = \frac{2\pi Uz^2 + \mu + iz\gamma \ln z}{2\pi z}$$

$$W = \frac{2\pi U(x + iy)^2 + \mu + i\gamma(x + iy) \times \ln(x + iy)}{2\pi(x + iy)}$$

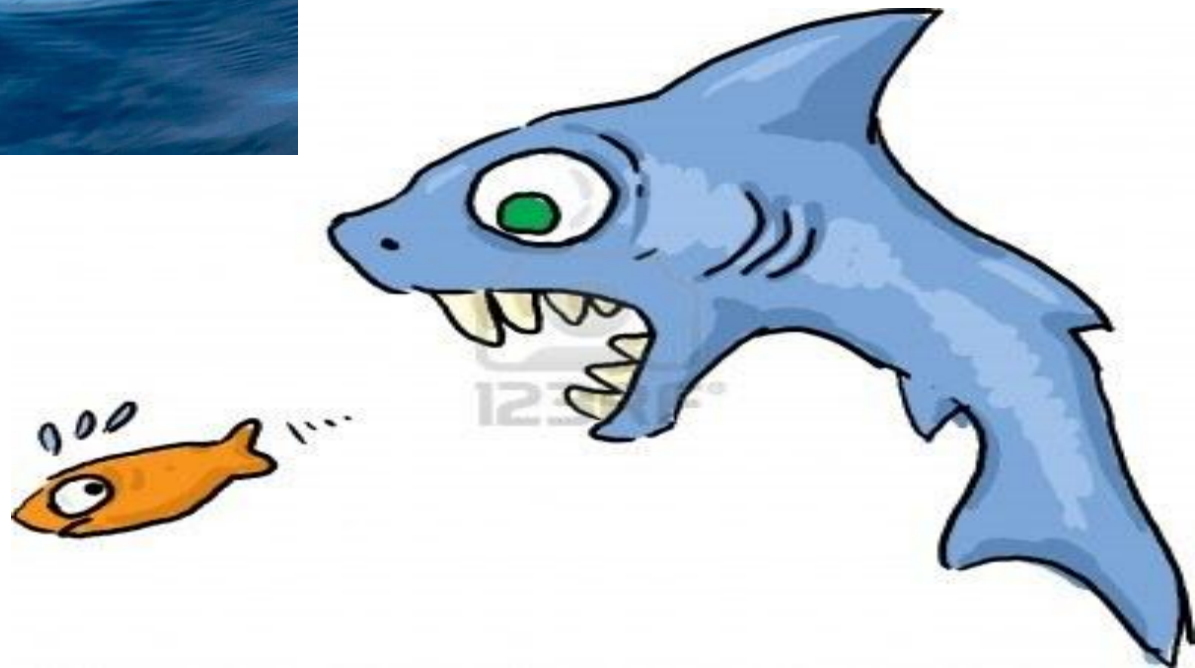


# Angle of Attack



The Natural Genius  
&  
The Art of Generating Lift

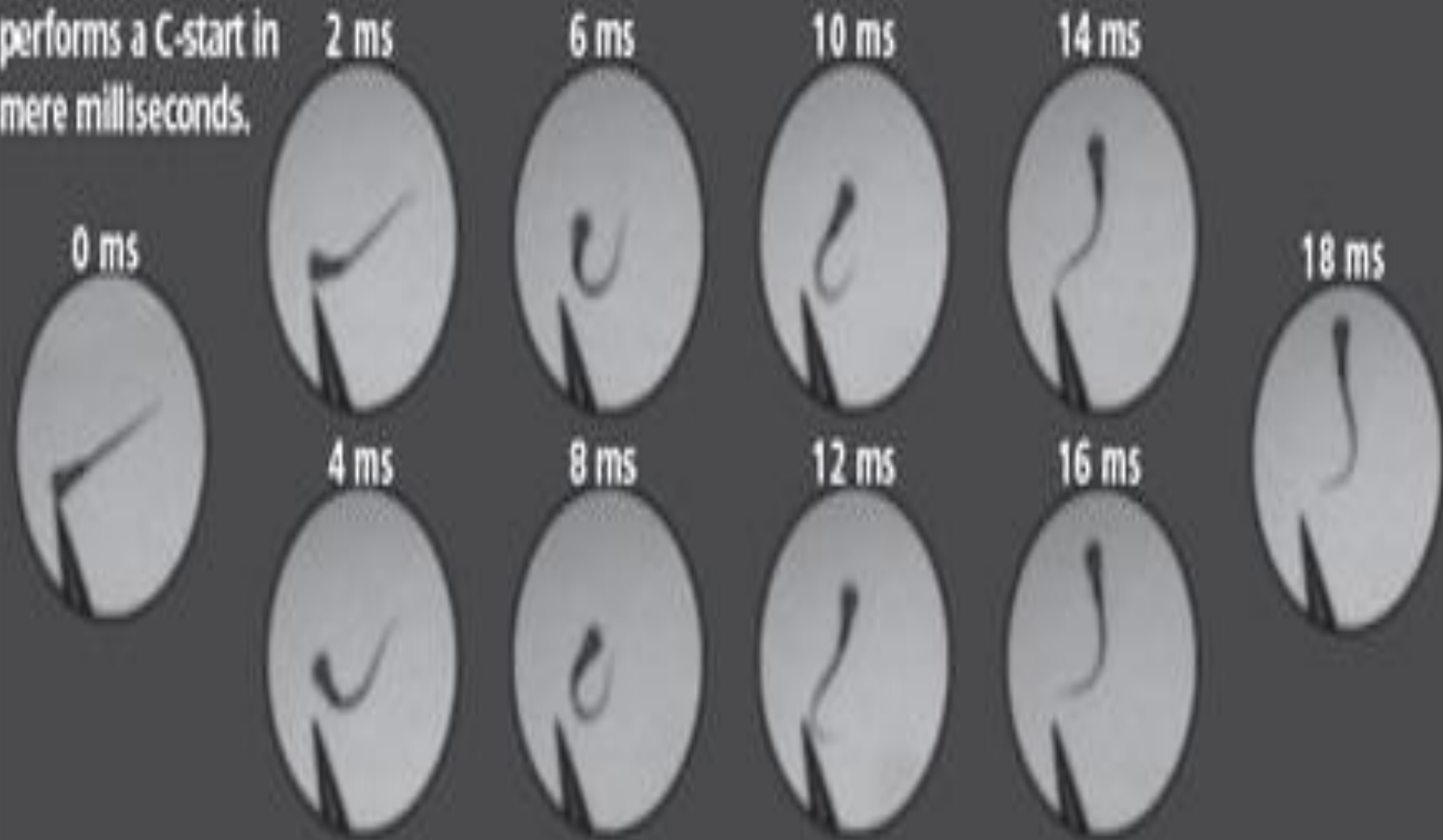
# Hydrodynamics of Prey & Predators



# The Art of C-Start



A prodded zebrafish performs a C-start in mere milliseconds.





# The Art of Complex Swimming



**1**



**2**



**3**

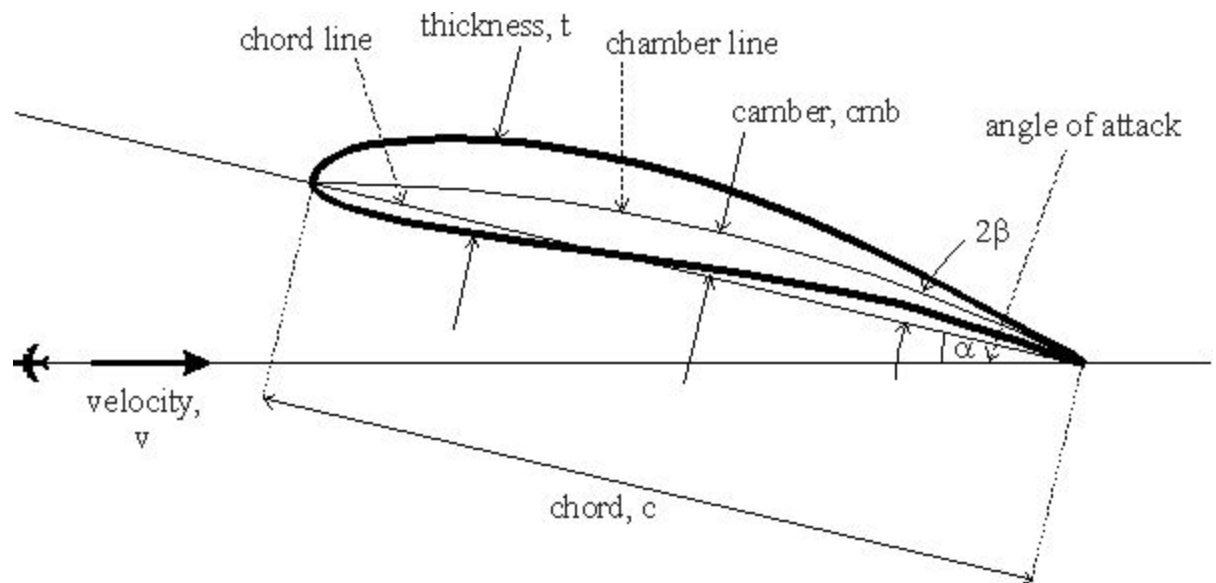


**4**

# Development of an Ultimate Fluid machine



Wright 1908



# The Art of Transformation

- Our goal is to map the flow past a cylinder to flow around a device which can generate an Upwash on existing Fluid.
- There are several free parameters that can be used to choose the shape of the new device.
- First we will itemize the steps in the mapping:

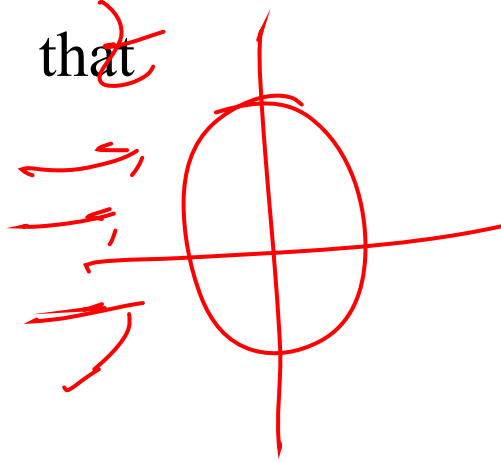
# Transformation for Inventing a Machine

- A large amount of airfoil theory has been developed by distorting flow around a cylinder to flow around an airfoil.
- The essential feature of the distortion is that the potential flow being distorted ends up also as potential flow.
- The most common Conformal transformation is the Jowkowski transformation which is given by

$$f(z) = z + \frac{c^2}{z}$$

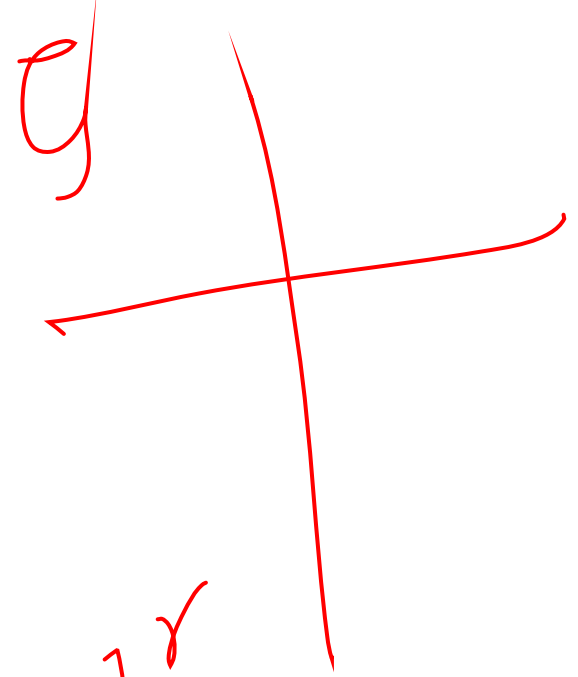
To see how this transformation changes flow pattern in the  $z$  (or  $x - y$ ) plane, substitute  $z = x + iy$  into the expression above to get

This means  
that

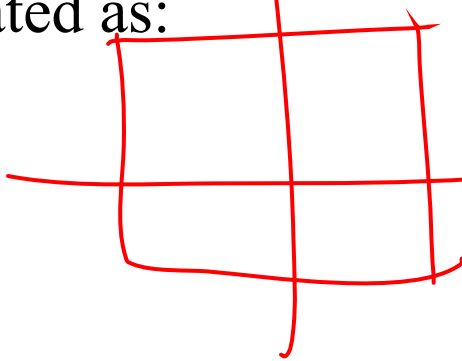


$$\xi = x \left( 1 + \frac{c^2}{x^2 + y^2} \right)$$

$$\eta = y \left( 1 - \frac{c^2}{x^2 + y^2} \right)$$



For a circle of radius  $r$  in  $Z$  plane  $x$  and  $y$  are  
related as:



$$x^2 + y^2 = r^2$$

