Комплексные числа

примеры

№ 1

Решить уравнение

$$x^2 - 4x + 13 = 0$$

$$D = b^2 - 4ac = (-4)^2 - 4 \cdot 13 = -36$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

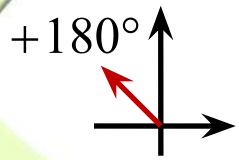
$$x_1 = \frac{4 + \sqrt{-36}}{2} = \frac{4 + \sqrt{36 j^2}}{2} = \frac{4 + 6j}{2} = 2 + 3j$$

$$x_2 = \frac{4 - \sqrt{-36}}{2} = \frac{4 - 6j}{2} = 2 - 3j$$

№ 2

$$\frac{\left(\frac{-\sqrt{3}}{2}+j\frac{1}{2}\right)^{12}}{-5(Cos30^0+jSin30^0)}=\frac{1e^{0^\circ j}}{5\cdot e^{-150^0 j}}=$$

1) $-\frac{\sqrt{3}}{2}+j\frac{1}{2}=1e^{150^0 j}$

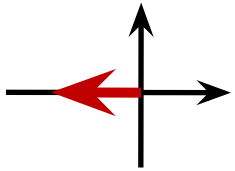


$$\rho=\sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{3}{4}+\frac{1}{4}}=\sqrt{1}=1$$

$$\alpha=arctg\left(\frac{1}{2}:\frac{-\sqrt{3}}{2}\right)+180^\circ=-arctg\left(\frac{1\cdot 2}{2\cdot\sqrt{3}}\right)+180^\circ=-30^\circ+180^\circ=150^0$$

2) $(1\cdot e^{150^0 j})^{12}=1^{12}\cdot e^{150^0\cdot 12 j}=1\cdot e^{1800^0 j}=e^{(360^0\cdot 5+0^\circ)j}=e^{0^0 j}$

3) $-5=5e^{j180^0}$



4) $Cos30^0+jSin30^0=e^{30^0 j}$

5) $5\cdot e^{180^0 j}\cdot e^{30^0 j}=5\cdot e^{(180^0+30^0)j}=5\cdot e^{210^0 j}=5\cdot e^{(210^0-360^\circ)j}=$

$=5\cdot e^{-150^0 j}$

№ 2

$$\frac{\left(\frac{-\sqrt{3}}{2}+j\frac{1}{2}\right)^{12}}{-5(Cos30^0+jSin30^0)}=\frac{1e^{0^\circ j}}{5\cdot e^{-150^0 j}}=\frac{1}{5}e^{(0^\circ-(-150^0))j}=$$

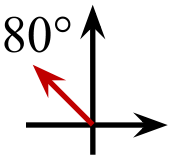
$$=\frac{1}{5}e^{150^0 j}=\frac{1}{5}(\cos 150^\circ+j\sin 150^\circ)=$$

$$=0{,}2(-0{,}87+0{,}5j)=-0{,}17+0{,}1j$$

№ 3

$$\sqrt[3]{-4+j\sqrt{48}}$$

$$a=-4 \qquad b=\sqrt{48}$$



$$\rho=\sqrt{(-4)^2+(\sqrt{48})^2}=\sqrt{16+48}=\sqrt{64}=8$$

$$\varphi=arctg\frac{\sqrt{48}}{-4}+180^\circ=-arctg\frac{\sqrt{16\cdot 3}}{4}+180^\circ=-arctg\sqrt{3}+180^\circ=-60^\circ+180^\circ=120^\circ$$

$$-4+j\sqrt{48}=8e^{120^0 j}$$

$$\sqrt[n]{z}=\sqrt[n]{\rho}\cdot e^{\frac{\varphi+2\pi k}{n}j}$$

$$\rho=8 \qquad \varphi=120^\circ \qquad n=3$$

$$\sqrt[3]{-4+j\sqrt{48}}=\sqrt[3]{8e^{120^\circ j}}=\sqrt[3]{8}\cdot e^{\frac{120^\circ+360^\circ k}{3}j}=2\cdot e^{\frac{3(40^\circ+120^\circ k)}{3}j}=$$

$$=2\cdot e^{(40^\circ+120^\circ k)j}$$

№ 3

$$\sqrt[3]{-4+j\sqrt{48}} = 2\cdot e^{(40°+120°k)j}$$

$$k=0 \qquad z_0 = 2\cdot e^{(40°+120°\cdot 0)j} = 2\cdot e^{40°j} =$$

$$= 2(\cos 40° + j\sin 40°) = 2(0{,}766 + j0{,}643) = \boxed{1{,}5 + 1{,}29j}$$

$$k=1 \qquad z_1 = 2\cdot e^{(40°+120°\cdot 1)j} = 2\cdot e^{160°j} =$$

$$= 2(\cos 160° + j\sin 160°) = \boxed{-1{,}88 + 0{,}68j}$$

-360°

$$k=2 \qquad z_2 = 2\cdot e^{(40°+120°\cdot 2)j} = 2\cdot e^{280°j} = 2\cdot e^{-80°j} =$$

$$= 2(\cos 80° - j\sin 80°) = \boxed{0{,}35 - 1{,}97j}$$

№ 4

$$\frac{7,3e^{32,7^\circ}(\cos 23^\circ - j\sin 23^\circ)}{9,34e^{-53,8^\circ}2,6(\cos 31,6^\circ + j\sin 31,6^\circ)} =$$

$$= \frac{7,3e^{32,7^\circ}1e^{-23^\circ j}}{9,34e^{-53,8^\circ}2,6e^{31,6^\circ j}} =$$

$$= \frac{7,3\cdot 1}{9,34\cdot 2,6} e^{(32,7^\circ+(-23^\circ) - (-53,8^\circ+31,6^\circ))j} =$$

$$= 0,3e^{31,9^\circ j} = 0,3(\cos 31,9^\circ + j\cdot\sin 31,9^\circ) =$$

$$= 0,25 + 0,16j$$

№ 5

$$e^{1,46\,j} + 3(\cos 158° + j \sin 158°) - e^{-123°\,j} =$$

$$= 0,11 + 0,99\,j + (-2,78 + 1,12\,j) - (-0,54 - 0,84\,j) =$$

$$= 0,11 + 0,99\,j - 2,78 + 1,12\,j + 0,54 + 0,84\,j =$$

$$= -2,13 + 2,95\,j$$

1) $e^{1,46j} = \cos 1,46 + j \sin 1,46 = 0,11 + 0,99\,j$

2) $3(\cos 158° + j \sin 158°) = -2,78 + 1,12\,j$

3) $e^{-123°\,j} = \cos 123° - j \sin 123° = -0,54 - 0,84\,j$

№ 6

Составить уравнение, если даны корни:

$$x_1 = -4 + 3j; \quad x_2 = -4 - 3j$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$(x - (-4 + 3j))(x - (-4 - 3j)) =$$

$$= (x + 4 - 3j)(x + 4 + 3j) =$$

$$= x^2 + 4x + 3jx + 4x + 16 + 12j - 3jx - 12j - 9j^2 =$$

$$= x^2 + 8x + 25$$

$$x^2 + 8x + 25 = 0$$

№ 7

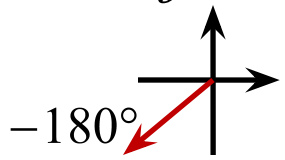
$$Ln\left(\frac{-7{,}3-5{,}4j}{-3(Cos15^\circ+jSin15^\circ)}\right)$$

$$\frac{-7{,}3-5{,}4j}{-3(Cos15^\circ+jSin15^\circ)}=\frac{9{,}08e^{-143{,}5^\circ j}}{3e^{j180^0}\cdot e^{15^0 j}}$$

1) $-7{,}3-5{,}4j=9{,}08e^{-j143{,}5^\circ}$

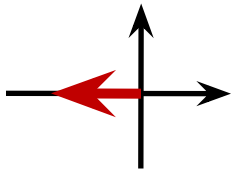
$a=-7{,}3$

$b=-5{,}4$



$$\rho=\sqrt{(-7{,}3)^2+(-5{,}4)^2}=9{,}08$$

$$\varphi=arctg\frac{-5{,}4}{-7{,}3}-180^\circ=36{,}5^\circ-180^\circ=-143{,}5^\circ$$

2) $-3=3e^{j180^0}$ 

3) $Cos15^0+jSin15^0=e^{15^0 j}$

№ 7

$$Ln\left(\frac{-7{,}3-5{,}4j}{-3(Cos15^\circ+jSin15^\circ)}\right)$$

$$\frac{-7{,}3-5{,}4j}{-3(Cos15^\circ+jSin15^\circ)}=\frac{9{,}08e^{-143{,}5^\circ j}}{3e^{180^0 j}\cdot 1e^{15^{00} j}}=$$

$$=\frac{9{,}08}{3\cdot 1}e^{(-143{,}5^\circ-(180^\circ+15^\circ))j}=3{,}03e^{-338{,}5^\circ j}=$$

$$=3{,}03e^{21{,}5^\circ j}=3{,}03e^{0{,}38j}$$

1 рад =57,3°

π - 180°

x – 21,5°

№ 7

$$Ln\left(\frac{-7{,}3-5{,}4\,j}{-3(Cos15^\circ+\,jSin15^\circ)}\right)=$$

$$=\ln 3{,}03e^{0{,}38\,j}=\ln 3{,}03+\ln e^{0{,}38\,j}=$$

$$=\ln 3{,}03+0{,}38\,j=\boxed{1{,}11+0{,}38\,j}$$

№ 8

$$\frac{(j^{173}-j^{286})\cdot(3-5j)}{j^{1891}} = \frac{(j+1)\cdot(3-5j)\cdot j}{-j\ \cdot j\ = 1}$$

$$= (-1+j)\cdot(3-5j) = -3+5j+3j+5 =$$

$$= \boxed{2+8j}$$

$$j^{173} = j^{43\cdot 4+1} = j^{1} = j$$

$$j^{286} = j^{71\cdot 4+2} = j^{2} = -1$$

$$j^{1891} = j^{472\cdot 4+3} = j^{3} = -j$$

САМОСТОЯТЕЛЬНАЯ РАБОТА

Выполните действия

1) (3+2j)+3(-1+3j)

2) j-2-(6-5j)

3) (1+j)(1-j)

4) j^{101}

5) $\frac{3}{j}$

6) Разложите на множители в комплексных числах:

$$x^2+1$$

7) Изобразите комплексное число на плоскости z=-2+2j

8) *Да или нет*: -2(cos90 ^{0}+i sin90 ^{0}) - является тригонометрической формой комплексного числа?

Ответы

1) $(3+2j)+3(-1+3j)=11j$

2) $j-2-(6-5j)=-8+6j$

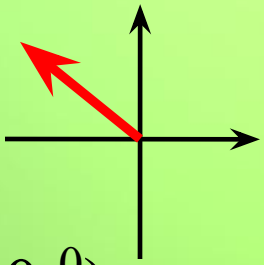
3) $(1+j)(1-j)=2$

4) $j^{101} = j$

5) $\frac{3}{j} = -3j$

6) $x^2 + 1 = x^2 - j^2 = (x - j)(x + j)$

7) $z=-2+2j$



8) $-2(\cos 90^0 + i \sin 90^0)$ - является тригонометрической формой комплексного числа? Нет