

# *Комплексные числа*

примеры

## № 1

Решить уравнение

$$x^2 - 4x + 13 = 0$$

$$D = b^2 - 4ac = (-4)^2 - 4 \cdot 13 = -36$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

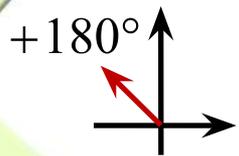
$$x_1 = \frac{4 + \sqrt{-36}}{2} = \frac{4 + \sqrt{36j^2}}{2} = \frac{4 + 6j}{2} = 2 + 3j$$

$$x_2 = \frac{4 - \sqrt{-36}}{2} = \frac{4 - 6j}{2} = 2 - 3j$$

## № 2

$$\frac{\left(\frac{-\sqrt{3}}{2} + j\frac{1}{2}\right)^{12}}{-5(\cos 30^\circ + j\sin 30^\circ)} = \frac{1e^{0^\circ j}}{5 \cdot e^{-150^\circ j}} =$$

$$1) \quad -\frac{\sqrt{3}}{2} + j\frac{1}{2} = 1e^{150^\circ j}$$



$$\rho = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{1} = 1$$

$$\alpha = \arctg\left(\frac{1}{2} : \frac{-\sqrt{3}}{2}\right) + 180^\circ = -\arctg\left(\frac{1 \cdot 2}{2 \cdot \sqrt{3}}\right) + 180^\circ = -30^\circ + 180^\circ = 150^\circ$$

$$2) \quad (1 \cdot e^{150^\circ j})^{12} = 1^{12} \cdot e^{150^\circ \cdot 12j} = 1 \cdot e^{1800^\circ j} = e^{(360^\circ \cdot 5 + 0^\circ)j} = e^{0^\circ j}$$

$$3) \quad -5 = 5e^{j180^\circ} \quad \leftarrow \text{red arrow pointing left}$$

$$4) \quad \cos 30^\circ + j\sin 30^\circ = e^{30^\circ j}$$

$$5) \quad 5 \cdot e^{180^\circ j} \cdot e^{30^\circ j} = 5 \cdot e^{(180^\circ + 30^\circ)j} = 5 \cdot e^{210^\circ j} = 5 \cdot e^{(210^\circ - 360^\circ)j} = 5 \cdot e^{-150^\circ j}$$

## Nº 2

$$\frac{\left(\frac{-\sqrt{3}}{2} + j\frac{1}{2}\right)^{12}}{-5(\cos 30^\circ + j\sin 30^\circ)} = \frac{1e^{0^\circ j}}{5 \cdot e^{-150^\circ j}} = \frac{1}{5} e^{(0^\circ - (-150^\circ))j} =$$

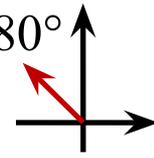
$$= \frac{1}{5} e^{150^\circ j} = \frac{1}{5} (\cos 150^\circ + j \sin 150^\circ) =$$

$$= 0,2(-0,87 + 0,5j) = -0,17 + 0,1j$$

### № 3

$$a = -4$$

$$+180^\circ$$



$$\sqrt[3]{-4 + j\sqrt{48}}$$

$$b = \sqrt{48}$$

$$\rho = \sqrt{(-4)^2 + (\sqrt{48})^2} = \sqrt{16 + 48} = \sqrt{64} = 8$$

$$\varphi = \arctg \frac{\sqrt{48}}{-4} + 180^\circ = -\arctg \frac{\sqrt{16 \cdot 3}}{4} + 180^\circ = -\arctg \sqrt{3} + 180^\circ = -60^\circ + 180^\circ = 120^\circ$$

$$-4 + j\sqrt{48} = 8e^{120^\circ j}$$

$$\sqrt[n]{z} = \sqrt[n]{\rho} \cdot e^{\frac{\varphi + 2\pi k}{n} j}$$

$$\rho = 8$$

$$\varphi = 120^\circ$$

$$n = 3$$

$$\sqrt[3]{-4 + j\sqrt{48}} = \sqrt[3]{8e^{120^\circ j}} = \sqrt[3]{8} \cdot e^{\frac{120^\circ + 360^\circ k}{3} j} = 2 \cdot e^{\frac{3(40^\circ + 120^\circ k)}{3} j} =$$

$$= 2 \cdot e^{(40^\circ + 120^\circ k) j}$$

### № 3

$$\sqrt[3]{-4 + j\sqrt{48}} = 2 \cdot e^{(40^\circ + 120^\circ k)j}$$

$$k = 0 \quad z_0 = 2 \cdot e^{(40^\circ + 120^\circ \cdot 0)j} = 2 \cdot e^{40^\circ j} =$$
$$= 2(\cos 40^\circ + j \sin 40^\circ) = 2(0,766 + j0,643) = \boxed{1,5 + 1,29j}$$

$$k = 1 \quad z_1 = 2 \cdot e^{(40^\circ + 120^\circ \cdot 1)j} = 2 \cdot e^{160^\circ j} =$$
$$= 2(\cos 160^\circ + j \sin 160^\circ) = \boxed{-1,88 + 0,68j}$$

$$k = 2 \quad z_2 = 2 \cdot e^{(40^\circ + 120^\circ \cdot 2)j} = 2 \cdot e^{280^\circ j} = 2 \cdot e^{-80^\circ j} =$$
$$= 2(\cos 80^\circ - j \sin 80^\circ) = \boxed{0,35 - 1,97j}$$

-360°

**№ 4**

$$\begin{aligned} & \frac{7,3e^{32,7^\circ} (\cos 23^\circ - j \sin 23^\circ)}{9,34e^{-53,8^\circ} 2,6(\cos 31,6^\circ + j \sin 31,6^\circ)} = \\ & = \frac{7,3e^{32,7^\circ} 1e^{-23^\circ j}}{9,34e^{-53,8^\circ} 2,6e^{31,6^\circ j}} = \\ & = \frac{7,3 \cdot 1}{9,34 \cdot 2,6} e^{(32,7^\circ + (-23^\circ) - (-53,8^\circ + 31,6^\circ))j} = \\ & = 0,3e^{31,9^\circ j} = 0,3(\cos 31,9^\circ + j \cdot \sin 31,9^\circ) = \\ & = \boxed{0,25 + 0,16j} \end{aligned}$$

$$\begin{aligned} e^{1,46j} + 3(\cos 158^{\circ} + j \sin 158^{\circ}) - e^{-123^{\circ}j} &= \\ = 0,11 + 0,99j + (-2,78 + 1,12j) - (-0,54 - 0,84j) &= \\ = \underline{0,11} + \underline{0,99j} - \underline{2,78} + \underline{1,12j} + \underline{0,54} + \underline{0,84j} &= \\ &= -2,13 + 2,95j \end{aligned}$$

$$1) e^{1,46j} = \cos 1,46 + j \sin 1,46 = 0,11 + 0,99j$$

$$2) 3(\cos 158^{\circ} + j \sin 158^{\circ}) = -2,78 + 1,12j$$

$$3) e^{-123^{\circ}j} = \cos 123^{\circ} - j \sin 123^{\circ} = -0,54 - 0,84j$$

## № 6

*Составить уравнение, если даны корни:*

$$x_1 = -4 + 3j; \quad x_2 = -4 - 3j$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$(x - (-4 + 3j))(x - (-4 - 3j)) =$$

$$= (x + 4 - 3j)(x + 4 + 3j) =$$

$$= \underline{x^2} + \underline{4x} + \cancel{3jx} + \underline{4x} + \underline{16} + \cancel{12j} - \cancel{3jx} - \cancel{12j} - \underline{9j^2} =$$

$$= x^2 + 8x + 25$$

$$x^2 + 8x + 25 = 0$$

## № 7

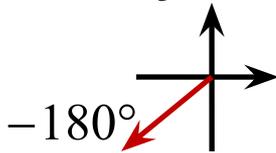
$$\text{Ln} \left( \frac{-7,3 - 5,4j}{-3(\text{Cos}15^\circ + j\text{Sin}15^\circ)} \right)$$

$$\frac{-7,3 - 5,4j}{-3(\text{Cos}15^\circ + j\text{Sin}15^\circ)} = \frac{9,08e^{-143,5^\circ j}}{3e^{j180^\circ} \cdot e^{15^\circ j}}$$

$$1) \quad -7,3 - 5,4j = 9,08e^{-j143,5^\circ}$$

$$a = -7,3$$

$$b = -5,4$$



$$\rho = \sqrt{(-7,3)^2 + (-5,4)^2} = 9,08$$

$$\varphi = \text{arctg} \frac{-5,4}{-7,3} - 180^\circ = 36,5^\circ - 180^\circ = -143,5^\circ$$

$$2) \quad -3 = 3e^{j180^\circ}$$

$$3) \quad \text{Cos}15^\circ + j\text{Sin}15^\circ = e^{15^\circ j}$$

**№ 7**

$$\text{Ln} \left( \frac{-7,3 - 5,4j}{-3(\text{Cos}15^\circ + j\text{Sin}15^\circ)} \right)$$

$$\begin{aligned} \frac{-7,3 - 5,4j}{-3(\text{Cos}15^\circ + j\text{Sin}15^\circ)} &= \frac{9,08 e^{-143,5^\circ j}}{3 e^{180^\circ j} \cdot 1 e^{15^\circ j}} = \\ &= \frac{9,08}{3 \cdot 1} e^{(-143,5^\circ - (180^\circ + 15^\circ))j} = 3,03 e^{-338,5^\circ j} = \\ &= 3,03 e^{21,5^\circ j} = 3,03 e^{0,38j} \end{aligned}$$

$$1 \text{ rad} = 57,3^\circ$$

$$\begin{aligned} \pi - 180^\circ \\ x - 21,5^\circ \end{aligned}$$

$$\text{Ln}\left(\frac{-7,3 - 5,4j}{-3(\text{Cos}15^\circ + j\text{Sin}15^\circ)}\right) =$$

$$= \ln 3,03 e^{0,38j} = \ln 3,03 + \ln e^{0,38j} =$$

$$= \ln 3,03 + 0,38j = 1,11 + 0,38j$$

# Nº 8

$$\frac{(j^{173} - j^{286}) \cdot (3 - 5j)}{j^{1891}} = \frac{(j+1) \cdot (3-5j) \cdot j}{-j \cdot j = 1}$$

$$= (-1+j) \cdot (3-5j) = -3 + 5j + 3j + 5 =$$

$$= \boxed{2 + 8j}$$

$$j^{173} = j^{\cancel{43} \cdot 4 + 1} = j^1 = j$$

$$j^{286} = j^{71 \cdot 4 + 2} = j^2 = -1$$

$$j^{1891} = j^{472 \cdot 4 + 3} = j^3 = -j$$

# ***САМОСТОЯТЕЛЬНАЯ РАБОТА***

## *Выполните действия*

1)  $(3+2j)+3(-1+3j)$

4)  $j^{101}$

2)  $j-2-(6-5j)$

5)  $\frac{3}{j}$

3)  $(1+j)(1-j)$

6) Разложите на множители в комплексных числах:

$$x^2 + 1$$

7) Изобразите комплексное число на плоскости  $z=-2+2j$

8) *Да или нет:*  $-2(\cos 90^\circ + i \sin 90^\circ)$  - является тригонометрической формой комплексного числа?

# Ответы

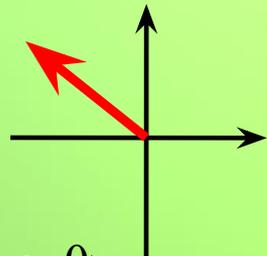
$$1) (3+2j)+3(-1+3j)=11j \quad 4) j^{101} = j$$

$$2) j-2-(6-5j)=-8+6j \quad 5) \frac{3}{j} = -3j$$

$$3) (1+j)(1-j)=2$$

$$6) x^2 + 1 = x^2 - j^2 = (x - j)(x + j)$$

$$7) z = -2 + 2j$$



8)  $-2(\cos 90^\circ + i \sin 90^\circ)$  - является тригонометрической формой комплексного числа? **Нет**