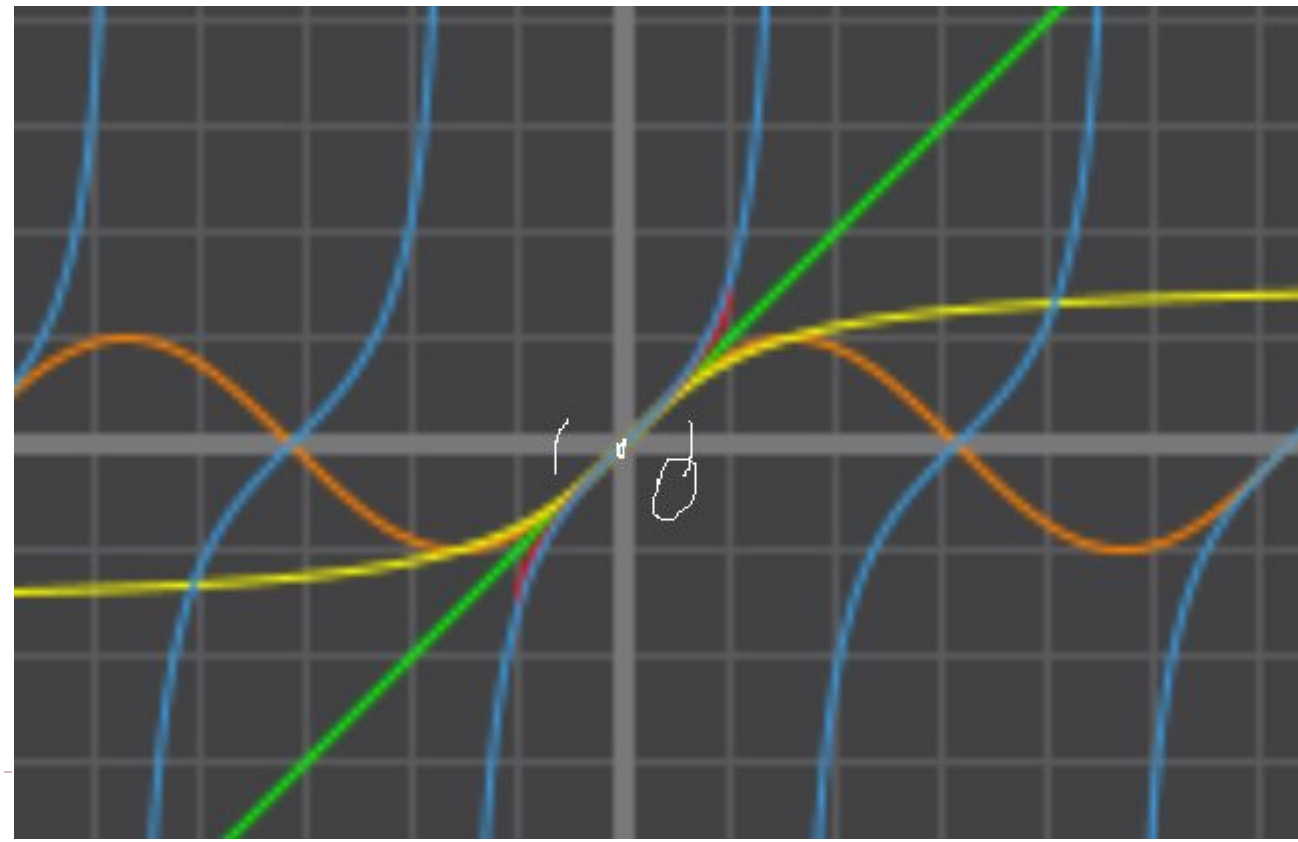
Вычисление пределов функций

1 и 2 замечательные пределы функций, точки разрыва функций

1 замечательный предел

$$\lim_{x\to 0}\frac{\sin x}{x}=1;\quad \lim_{x\to 0}\frac{tgx}{x}=\lim_{x\to 0}\frac{\left(\dfrac{\sin x}{\cos x}\right)}{x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}=1$$

$$\lim_{x\to 0}\frac{arctgx}{x}=1;\ \lim_{x\to 0}\frac{\arcsin x}{x}=1$$



$$\lim_{x\to 0}\frac{\sin kx}{x}=k; \qquad \lim_{x\to 0}\frac{arctg(kx)}{x}=k;\ldots$$

$$\lim_{x\to 0}\frac{\cos kx-\cos mx}{x^2}=\lim_{x\to 0}\frac{-2\sin\frac{k-m}{2}x\cdot\sin\frac{k+m}{2}x}{x\cdot x}=(-2)\cdot\left(\frac{k-m}{2}\right)\left(\frac{k+m}{2}\right)=\frac{m^2-k^2}{2};$$

$$\lim_{x\to 0}\frac{\sin kx}{\sin mx}=\lim_{x\to 0}\frac{\sin kx}{\sin mx}\cdot\frac{x}{x}=\lim_{x\to 0}\frac{\sin kx}{x}\cdot\frac{x}{\sin mx}=\lim_{x\to 0}\frac{\sin kx}{x}\cdot\frac{1}{\left(\frac{\sin mx}{x}\right)}=\frac{k}{m};$$

$$\lim_{x\to 0}\frac{1-\cos 2kx}{x^2}=\lim_{x\to 0}\frac{2\sin^2 kx}{x^2}=2\lim_{x\to 0}\left(\frac{\sin kx}{x}\right)^2=2\cdot k^2;$$

Примеры:

$$1)\ \lim_{x\to 0}\frac{x\cdot arctg\,4x}{1-\cos 6x}=\left(\begin{array}{l}1-\cos 6x=2\sin^2 3x\\ \dfrac{1-\cos 6x}{x^2}\to 2\cdot 3^2=18\end{array}\right)=\lim_{x\to 0}\frac{x}{1-\cos 6x}\cdot\left(\frac{x}{x}\right)\cdot\frac{arctg\,4x}{1}=$$

$$=\lim_{x\to 0}\frac{1}{\left(\dfrac{1-\cos 6x}{x^2}\right)}\cdot\frac{arctg\,4x}{x}=4\cdot\frac{1}{\lim\limits_{x\to 0}2\left(\dfrac{\sin 3x}{x}\right)^2}=4\cdot\frac{1}{2\cdot 9}=\frac{4}{18}=\frac{2}{9};$$

$$2)\ \lim_{x\to\pi}\frac{\sin 3x}{\pi-x}=\left(\frac{0}{0}\right)=\left(\begin{array}{l}\pi-x=t\Rightarrow x=\pi-t\\ \sin 3x=\sin(3\pi-3t)=\sin 3t\end{array}\right)=\lim_{t\to 0}\frac{\sin 3t}{t}=3;$$

$$3)\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\pi-4x}=\left(\frac{\cos\frac{\pi}{2}}{0}=\frac{0}{0}\right)=\left(\begin{array}{l}t=\pi-4x\Rightarrow x=\dfrac{\pi-t}{4}\\ \cos 2x=\cos\left(\dfrac{\pi}{2}-\dfrac{t}{2}\right)=\sin\dfrac{t}{2}\end{array}\right)=\lim_{t\to 0}\frac{\sin\frac{t}{2}}{t}=\frac{1}{2};$$

№219

$$\lim_{x\to 1}\frac{\sin \pi x}{\sin 3\pi x}=\left(\frac{0}{0}\right)=\begin{pmatrix} t=x-1\Rightarrow x=t+1 \\ \sin \pi x=\sin \pi(t+1)=\sin(\pi t+\pi)=-\sin \pi t \\ \sin 3\pi x=\sin 3\pi(t+1)=\sin(3\pi t+3\pi)=-\sin 3\pi t \end{pmatrix}=\lim_{t\to 0}\frac{-\sin \pi t}{-\sin 3\pi t}\cdot\frac{t}{t}=$$

$$=\lim_{t\to 0}\frac{\sin \pi t}{t}\cdot\frac{1}{\left(\dfrac{\sin 3\pi t}{t}\right)}=\frac{\pi}{3\pi}=\frac{1}{3};$$

4) №222

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\left(\frac{0}{0}\right)=\lim_{x\to a}\frac{2\sin\frac{x-a}{2}\cdot\cos\frac{x+a}{2}}{x-a}=\begin{pmatrix} t=x-a \\ t\to 0\Leftrightarrow x\to a\end{pmatrix}=$$

$$=\lim_{t\to 0}2\frac{\sin\frac{t}{2}}{t}\cdot\cos\frac{t+2a}{2}=2\cdot\frac{1}{2}\cdot\cos a=\cos a$$

$$=\begin{pmatrix} t=x-a\Rightarrow x=t+a \\ \sin x=\sin(t+a)=\sin t\cos a+\cos t\sin a\end{pmatrix}=\lim_{t\to 0}\frac{\sin t\cos a+\cos t\sin a-\sin a}{t}=$$

$$=\lim_{t\to 0}\left[\left(\frac{\sin t}{t}\cdot\cos a\right)+\left(\frac{\cos t\sin a-\sin a}{t}\right)\right]=\cos a+\lim_{t\to 0}\frac{-\sin a(1-\cos t)}{t}=$$

$$=\cos a-\sin a\cdot\lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t^2}\cdot t=\cos a$$

5) №224

$$\lim_{x\to -2}\frac{tg\pi x}{x+2}=\left(\frac{0}{0}\right)=\left(\begin{array}{l} t=x+2\Rightarrow x=t-2 \\ tg\pi(t-2)=tg\pi t \end{array}\right)=\lim_{t\to 0}\frac{tg\,\pi t}{t}=\lim\frac{\left(\dfrac{\sin\pi t}{\cos\pi t}\right)}{t}=\lim_{t\to 0}\frac{\sin\pi t}{t}\cdot\frac{1}{\cos\pi t}=\pi\cdot\frac{1}{\cos 0}=\pi$$

6) №226

$$\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-tgx}=\left(\frac{0}{0}\right)=\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\left(\dfrac{\cos x-\sin x}{\cos x}\right)}=\lim_{x\to\frac{\pi}{4}}(-\cos x)=-\frac{1}{\sqrt{2}};$$

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230; 231; 233; 237; 238

$$2)\ \lim_{x\to\frac{\pi}{3}}\frac{2\sin x-\sqrt{3}}{\sin 3x}=\left(\begin{array}{l} t=x-\dfrac{\pi}{3}\Rightarrow \\ x=t+\dfrac{\pi}{3}\end{array}\right)=\lim_{t\to 0}\frac{2\sin\left(t+\dfrac{\pi}{3}\right)-\sqrt{3}}{\sin 3\left(t+\dfrac{\pi}{3}\right)}=\lim_{t\to 0}\frac{2\sin t\cos\dfrac{\pi}{3}+2\cos t\sin\dfrac{\pi}{3}-\sqrt{3}}{\sin(3t+\pi)}=$$

$$=-\lim_{t\to 0}\frac{\sin t+\sqrt{3}\cos t-\sqrt{3}}{\sin 3t}=-\lim\left(\frac{\sin t}{\sin 3t}+\sqrt{3}\frac{\cos t-1}{\sin 3t}\right)=-\lim_{t\to 0}\left(\frac{\sin t}{t}\cdot\frac{t}{\sin 3t}\right)-\sqrt{3}\lim_{t\to 0}\frac{-2\sin^2\dfrac{t}{2}}{\sin 3t}=$$

$$=-\frac{1}{3}+2\sqrt{3}\lim_{t\to 0}\left(\frac{\sin^2\dfrac{t}{2}}{t^2}\cdot\frac{t^2}{\sin 3t}\right)=-\frac{1}{3}+2\sqrt{3}\lim_{t\to 0}\left(\left(\frac{\sin\dfrac{t}{2}}{t}\right)^2\cdot\left(\frac{1}{\dfrac{\sin 3t}{t}}\right)\cdot t\right)=-\frac{1}{3}+2\sqrt{3}\cdot\frac{1}{4}\cdot\frac{1}{3}\cdot 0=-\frac{1}{3}$$