

# Вычисление пределов функций

1 и 2 замечательные пределы функций, точки разрыва  
функций

# 1 замечательный предел

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1; \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = \lim_{x \rightarrow 0} \frac{\left( \frac{\sin x}{\cos x} \right)}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x} = 1; \quad \lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1$$



$$\lim_{x \rightarrow 0} \frac{\sin kx}{x} = k;$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg}(kx)}{x} = k; \dots$$

$$\lim_{x \rightarrow 0} \frac{\cos kx - \cos mx}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin \frac{k-m}{2} x \cdot \sin \frac{k+m}{2} x}{x \cdot x} = (-2) \cdot \left( \frac{k-m}{2} \right) \left( \frac{k+m}{2} \right) = \frac{m^2 - k^2}{2};$$

$$\lim_{x \rightarrow 0} \frac{\sin kx}{\sin mx} = \lim_{x \rightarrow 0} \frac{\sin kx}{\sin mx} \cdot \frac{x}{x} = \lim_{x \rightarrow 0} \frac{\sin kx}{x} \cdot \frac{x}{\sin mx} = \lim_{x \rightarrow 0} \frac{\sin kx}{x} \cdot \frac{1}{\left( \frac{\sin mx}{x} \right)} = \frac{k}{m};$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2kx}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 kx}{x^2} = 2 \lim_{x \rightarrow 0} \left( \frac{\sin kx}{x} \right)^2 = 2 \cdot k^2;$$



## Примеры:

$$\begin{aligned} 1) \lim_{x \rightarrow 0} \frac{x \cdot \operatorname{arctg} 4x}{1 - \cos 6x} &= \left( \begin{array}{l} 1 - \cos 6x = 2 \sin^2 3x \\ \frac{1 - \cos 6x}{x^2} \rightarrow 2 \cdot 3^2 = 18 \end{array} \right) = \lim_{x \rightarrow 0} \frac{x}{1 - \cos 6x} \cdot \left( \frac{x}{x} \right) \cdot \frac{\operatorname{arctg} 4x}{1} = \\ &= \lim_{x \rightarrow 0} \frac{1}{\left( \frac{1 - \cos 6x}{x^2} \right)} \cdot \frac{\operatorname{arctg} 4x}{x} = 4 \cdot \frac{1}{\lim_{x \rightarrow 0} 2 \left( \frac{\sin 3x}{x} \right)^2} = 4 \cdot \frac{1}{2 \cdot 9} = \frac{4}{18} = \frac{2}{9}; \end{aligned}$$

$$2) \lim_{x \rightarrow \pi} \frac{\sin 3x}{\pi - x} = \left( \frac{0}{0} \right) = \left( \begin{array}{l} \pi - x = t \Rightarrow x = \pi - t \\ \sin 3x = \sin(3\pi - 3t) = \sin 3t \end{array} \right) = \lim_{t \rightarrow 0} \frac{\sin 3t}{t} = 3;$$

$$3) \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\pi - 4x} = \left( \frac{\cos \frac{\pi}{2}}{0} = \frac{0}{0} \right) = \left( \begin{array}{l} t = \pi - 4x \Rightarrow x = \frac{\pi - t}{4} \\ \cos 2x = \cos \left( \frac{\pi}{2} - \frac{t}{2} \right) = \sin \frac{t}{2} \end{array} \right) = \lim_{t \rightarrow 0} \frac{\sin \frac{t}{2}}{\frac{t}{2}} = \frac{1}{2};$$

№219

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sin \pi x}{\sin 3\pi x} &= \left( \frac{0}{0} \right) = \left( \begin{array}{l} t = x - 1 \Rightarrow x = t + 1 \\ \sin \pi x = \sin \pi(t + 1) = \sin(\pi t + \pi) = -\sin \pi t \\ \sin 3\pi x = \sin 3\pi(t + 1) = \sin(3\pi t + 3\pi) = -\sin 3\pi t \end{array} \right) = \lim_{t \rightarrow 0} \frac{-\sin \pi t}{-\sin 3\pi t} \cdot \frac{t}{t} = \\ &= \lim_{t \rightarrow 0} \frac{\sin \pi t}{t} \cdot \frac{1}{\left( \frac{\sin 3\pi t}{t} \right)} = \frac{\pi}{3\pi} = \frac{1}{3}; \end{aligned}$$



4) №222

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \left( \frac{0}{0} \right) = \lim_{x \rightarrow a} \frac{2 \sin \frac{x-a}{2} \cdot \cos \frac{x+a}{2}}{x-a} = \left( \begin{array}{l} t = x - a \\ t \rightarrow 0 \Leftrightarrow x \rightarrow a \end{array} \right) =$$

$$= \lim_{t \rightarrow 0} 2 \frac{\sin \frac{t}{2}}{\frac{t}{2}} \cdot \cos \frac{t+2a}{2} = 2 \cdot \frac{1}{2} \cdot \cos a = \cos a$$

$$= \left( \begin{array}{l} t = x - a \Rightarrow x = t + a \\ \sin x = \sin(t + a) = \sin t \cos a + \cos t \sin a \end{array} \right) = \lim_{t \rightarrow 0} \frac{\sin t \cos a + \cos t \sin a - \sin a}{t} =$$

$$= \lim_{t \rightarrow 0} \left[ \left( \frac{\sin t}{t} \cdot \cos a \right) + \left( \frac{\cos t \sin a - \sin a}{t} \right) \right] = \cos a + \lim_{t \rightarrow 0} \frac{-\sin a (1 - \cos t)}{t} =$$

$$= \cos a - \sin a \cdot \lim_{t \rightarrow 0} \frac{2 \sin^2 \frac{t}{2}}{t^2} \cdot t = \cos a$$



5) №224

$$\lim_{x \rightarrow -2} \frac{\operatorname{tg} \pi x}{x+2} = \left( \frac{0}{0} \right) = \left( \frac{t = x+2 \Rightarrow x = t-2}{\operatorname{tg} \pi(t-2) = \operatorname{tg} \pi t} \right) = \lim_{t \rightarrow 0} \frac{\operatorname{tg} \pi t}{t} = \lim_{t \rightarrow 0} \left( \frac{\sin \pi t}{\cos \pi t} \right) = \lim_{t \rightarrow 0} \frac{\sin \pi t}{t} \cdot \frac{1}{\cos \pi t} = \pi \cdot \frac{1}{\cos 0} = \pi$$

6) №226

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \operatorname{tg} x} = \left( \frac{0}{0} \right) = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\left( \frac{\cos x - \sin x}{\cos x} \right)} = \lim_{x \rightarrow \frac{\pi}{4}} (-\cos x) = -\frac{1}{\sqrt{2}};$$

**ДЗ: №№ 218; 220; 223; 227;  
228;**

**230; 231; 233; 237; 238**



$$\begin{aligned}
2) \lim_{x \rightarrow \frac{\pi}{3}} \frac{2 \sin x - \sqrt{3}}{\sin 3x} &= \left( \begin{array}{l} t = x - \frac{\pi}{3} \Rightarrow \\ x = t + \frac{\pi}{3} \end{array} \right) = \lim_{t \rightarrow 0} \frac{2 \sin \left( t + \frac{\pi}{3} \right) - \sqrt{3}}{\sin 3 \left( t + \frac{\pi}{3} \right)} = \lim_{t \rightarrow 0} \frac{2 \sin t \cos \frac{\pi}{3} + 2 \cos t \sin \frac{\pi}{3} - \sqrt{3}}{\sin(3t + \pi)} = \\
&= - \lim_{t \rightarrow 0} \frac{\sin t + \sqrt{3} \cos t - \sqrt{3}}{\sin 3t} = - \lim_{t \rightarrow 0} \left( \frac{\sin t}{\sin 3t} + \sqrt{3} \frac{\cos t - 1}{\sin 3t} \right) = - \lim_{t \rightarrow 0} \left( \frac{\sin t}{t} \cdot \frac{t}{\sin 3t} \right) - \sqrt{3} \lim_{t \rightarrow 0} \frac{-2 \sin^2 \frac{t}{2}}{\sin 3t} = \\
&= - \frac{1}{3} + 2\sqrt{3} \lim_{t \rightarrow 0} \left( \frac{\sin^2 \frac{t}{2}}{t^2} \cdot \frac{t^2}{\sin 3t} \right) = - \frac{1}{3} + 2\sqrt{3} \lim_{t \rightarrow 0} \left( \left( \frac{\sin \frac{t}{2}}{t} \right)^2 \cdot \left( \frac{1}{\sin 3t} \right) \cdot t \right) = - \frac{1}{3} + 2\sqrt{3} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot 0 = - \frac{1}{3}
\end{aligned}$$

