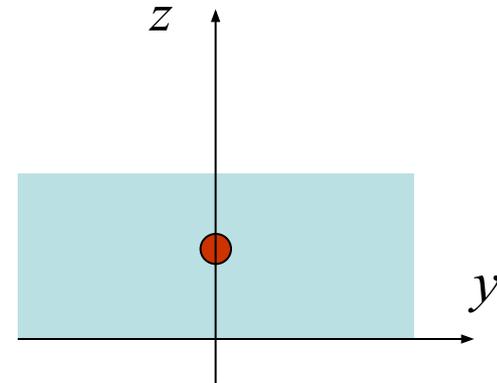
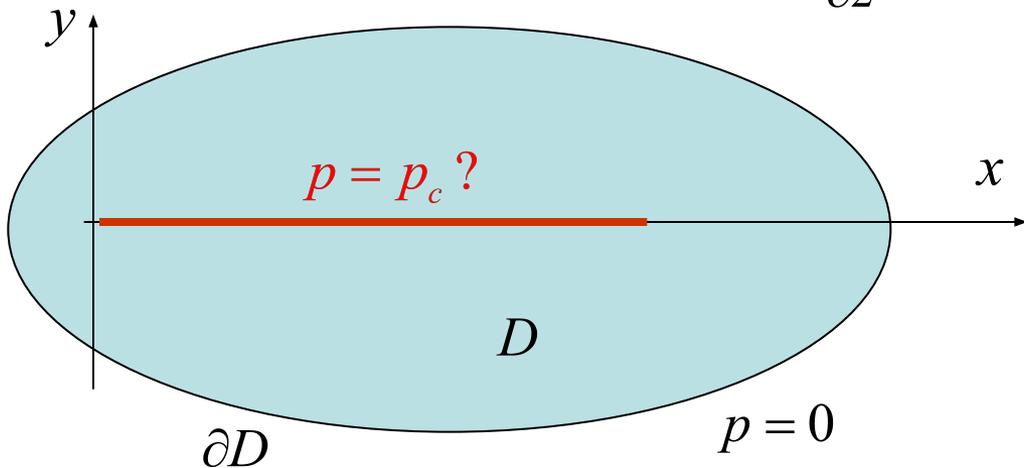
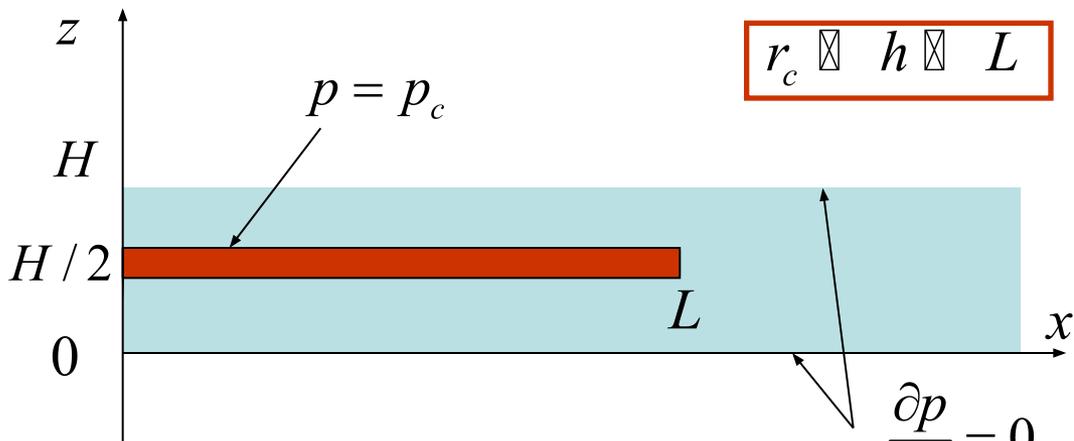


Введение в асимптотические методы.

Лекция 3: продолжение

Граничные условия на
горизонтальной скважине

1. ПОСТАНОВКА ПРОБЛЕМЫ



$$\Delta P_3 = 0, \quad (x, y, z) \in D \times [0, H] \setminus S$$

$$P_3 = 0, \quad (x, y, z) \in \partial D \times [0, H]$$

$$\partial P_3 / \partial z = 0, \quad z = 0, \quad z = H$$

$$P_3 = P_c, \quad (x, y, z) \in \partial S$$

$P_3 \rightarrow P_2 ??$

2. ТЕХНИКА

$$\begin{cases} P_2(x, y) = \int_0^L G_2(x, y; \tilde{x}, 0) q(\tilde{x}) d\tilde{x} \\ \Delta G_2 = \frac{1}{H} \delta(x - \tilde{x}) \delta(y - \tilde{y}) \\ G_2 = 0, \quad (x, y) \in \partial D \end{cases}$$

$$\begin{cases} P_3(r) = \int_0^L G_3(x, y, z; \tilde{x}, 0, H/2) q(\tilde{x}) d\tilde{x} \\ \Delta G_3 = \delta(x - \tilde{x}) \delta(y - \tilde{y}) \delta(z - \tilde{z}) \\ G_3 = 0, \quad (x, y, z) \in \partial D \times [0, H] \\ \partial G_3 / \partial z = 0, \quad z = 0, \quad z = H \end{cases}$$

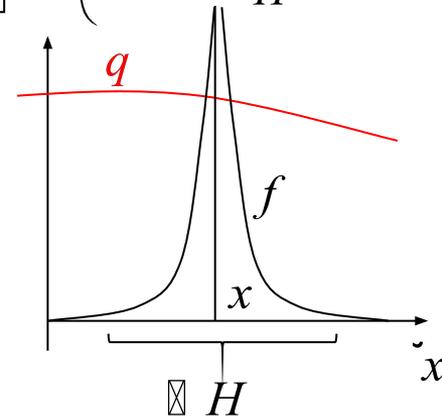
$$P_3 - P_2 = \int_0^L R(x, y, z; \tilde{x}, 0, H/2) q(\tilde{x}) d\tilde{x} \quad R = G_3 - G_2$$

$$\begin{cases} \Delta R = \delta(x - \tilde{x}) \delta(y) [\delta(z - H/2) - 1/H] \\ \partial R / \partial z = 0, \quad z = 0, \quad z = H \\ R \rightarrow 0, \quad \sqrt{y^2 + (x - \tilde{x})^2} \rightarrow \infty \end{cases} \xrightarrow{\text{Преобразование Фурье}} R = \frac{1}{\pi H} \sum_{n=1}^{\infty} \cos\left[\frac{2\pi n}{H}(z - H/2)\right] K_0\left(\frac{2\pi n \sqrt{y^2 + (x - \tilde{x})^2}}{H}\right)$$

$$P_c - P_2 = \int_0^L R(x, 0, r_c + H/2; \tilde{x}, 0, H/2) q(\tilde{x}) d\tilde{x} \approx q(x) \int_{-\infty}^{\infty} f(|x|) d\tilde{x}$$

$$f(|x - \tilde{x}|) = \frac{1}{\pi H} \sum_{n=1}^{\infty} \cos(2\pi n(r_c/H)) K_0(2\pi n|x - \tilde{x}|/H)$$

$$\frac{1}{2\pi} \ln\left(\frac{H}{2\pi r_c}\right)$$



3. РЕЗУЛЬТАТ

$$\Delta_2 P_2 = 0, \quad (x, y) \in D$$

$$P_2 = 0, \quad (x, y) \in \partial D$$

$$[\partial P_2 / \partial n] = H^{-1} q(x), \quad y = 0, \quad 0 < x < L$$



$$P_c - P_2 = \alpha \left[\frac{\partial P_2}{\partial n} \right]$$

$$\alpha = \frac{H}{2\pi} \ln \left(\frac{H}{2\pi r_c} \right)$$

Пример:

$$M_1 = L_2 = 50$$

$$H = 10$$

$$m_c = 0.1$$

Δ	q_0	q_1	q_2	$\delta_1, \%$	$\delta_2, \%$
100	17.54	17.37	21.03	1.01	19.8
50	20.54	20.30	25.56	1.19	24.4
10	28.41	27.91	40.29	1.78	41.81
2	34.13	33.27	58.69	2.49	72
0.5	36.44	35.22	74.53	3.35	104

