МАТЕМАТИКА

кафедра *«Математика»УГНТУ*

math.rusoil.net

ЛИНЕЙНАЯ АЛГЕБРА

Матрица

$$A_{mxn} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$$A_{4x3} = \begin{pmatrix} 2 & -2 & -4 \\ -10 & 3 & 6 \\ 1 & -7 & 0 \\ 5 & -9 & 8 \end{pmatrix}$$

КВАДРАТНАЯ МАТРИЦА

$$A_{nxn} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{(n-1)1} & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2(n-1)} & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{(n-1)1} & a_{(n-1)2} & \cdots & a_{(n-1)(n-1)} & a_{(n-1)n} \\ a_{n1} & a_{n2} & \cdots & a_{n(n-1)} & a_{nn} \end{pmatrix}$$

$$A_{4x4} = \begin{pmatrix} -6 & -9 & 2 & 9 \\ 2 & 4 & -5 & -2 \\ -3 & -10 & 1 & 6 \\ 0 & -1 & -4 & 8 \end{pmatrix}$$

ДИАГОНАЛЬНАЯ МАТРИЦА

$$A_{n\times n}=\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$$

$$A=\begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$

$$A=\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$A_{6x6}=\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

ЕДИНИЧНАЯ МАТРИЦА

$$\mathrm{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$A_{1xn} = (a_{11}\ a_{12} \ldots a_{1n})$

$(6\ {-8}\ 0\ 7)$

$$A_{mx1} = \begin{pmatrix} a_{11} \\ a_{22} \\ \ldots \\ a_{m1} \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -9 \\ 4 \\ -8 \\ 1 \end{pmatrix}$$

ТРАНСПОНИРОВАННАЯ МАТРИЦА

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}$$

СУММА МАТРИЦ

$$A_{mxn} + B_{mxn} =$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix} =$$

$$= C_{mxn}$$

Разность матриц

$$A_{mxn} - B_{mxn} =$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{pmatrix} =$$

$$= C_{mxn}$$

$$\begin{pmatrix} 3 & -2 & -4 & -8 & 8 \\ -9 & 7 & -6 & -10 & -7 \end{pmatrix} + \begin{pmatrix} 2 & -5 & 10 & 7 & 4 \\ 10 & 8 & -6 & 6 & -3 \end{pmatrix} =$$

$$= \begin{pmatrix} 3+2 & -2+(-5) & -4+10 & -8+7 & 8+4 \\ -9+10 & 7+8 & -6+(-6) & -10+6 & -7+(-3) \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & -7 & 6 & -1 & 12 \\ 1 & 15 & -12 & -4 & -10 \end{pmatrix}$$

$$\begin{pmatrix} 9 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \\ -6 \end{pmatrix} =$$

$$= \begin{pmatrix} 9-(-3) \\ 0-6 \\ 8-(-6) \end{pmatrix} = \begin{pmatrix} 12 \\ -6 \\ 14 \end{pmatrix}$$

СВОЙСТВА СУММЫ МАТРИЦ

- $A+B=B+A$;
- $(A+B)+C=A+(B+C)$;
- $A+0=A$;
- $A+(-A)=0$.

УМНОЖЕНИЕ МАТРИЦЫ НА ЧИСЛО

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \cdot \lambda = \begin{pmatrix} a_{11}\cdot\lambda & a_{12}\cdot\lambda & \cdots & a_{1n}\cdot\lambda \\ a_{21}\cdot\lambda & a_{22}\cdot\lambda & \cdots & a_{2n}\cdot\lambda \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}\cdot\lambda & a_{m2}\cdot\lambda & \cdots & a_{mn}\cdot\lambda \end{pmatrix}$$

$$\begin{pmatrix} 9 & -3 & -1 \\ 5 & 1 & -9 \end{pmatrix} \cdot (-2) = \begin{pmatrix} 9\cdot(-2) & -3\cdot(-2) & -1\cdot(-2) \\ 5\cdot(-2) & 1\cdot(-2) & -9\cdot(-2) \end{pmatrix} = \begin{pmatrix} -18 & 6 & 2 \\ -10 & -2 & 18 \end{pmatrix}$$

СВОЙСТВА ПРОИЗВЕДЕНИЯ МАТРИЦЫ НА ЧИСЛО

- $1 \cdot A = A$
- $(\alpha\beta)A = \alpha(\beta A)$
- $(\alpha+\beta)A = \alpha A + \beta A$
- $\alpha(A+B) = \alpha A + \alpha B$

ПРОИЗВЕДЕНИЕ МАТРИЦ

$$A_{mxn} \cdot B_{nxp} = \begin{pmatrix} \boxed{a_{11} \; a_{12} \cdots a_{1n}} \\ \boxed{a_{21} \; a_{22} \cdots a_{2n}} \\ \cdots \; \cdots \; \cdots \; \cdots \\ \boxed{a_{m1} \; a_{m2} \cdots a_{mn}} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \cdots + a_{1n} \cdot b_{n1} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + \cdots + a_{1n} \cdot b_{n2} & \cdots & a_{11} \cdot b_{1p} + a_{12} \cdot b_{2p} + \cdots + a_{1n} \cdot b_{np} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + \cdots + a_{2n} \cdot b_{n1} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + \cdots + a_{2n} \cdot b_{n2} & \cdots & a_{21} \cdot b_{1p} + a_{22} \cdot b_{2p} + \cdots + a_{2n} \cdot b_{np} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} \cdot b_{11} + a_{m2} \cdot b_{21} + \cdots + a_{mn} \cdot b_{n1} & a_{m1} \cdot b_{12} + a_{m2} \cdot b_{22} + \cdots + a_{mn} \cdot b_{n2} & \cdots & a_{m1} \cdot b_{1p} + a_{m2} \cdot b_{2p} + \cdots + a_{mn} \cdot b_{np} \end{pmatrix}$$

$$= C_{mxp}$$

$$A_{2x2} \cdot B_{2x2} =$$

$$= \begin{pmatrix} 8 & 3 \\ -1 & -5 \end{pmatrix} \cdot \begin{pmatrix} 2 & -2 \\ 1 & 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 8 \cdot 2 + 3 \cdot 1 & 8 \cdot (-2) + 3 \cdot 5 \\ -1 \cdot 2 + (-5) \cdot 1 & -1 \cdot (-2) + (-5) \cdot 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 19 & -1 \\ -7 & -23 \end{pmatrix} =$$

$$= C_{2x2}$$

$$A_{4x2} \cdot B_{2x1} =$$

$$= \begin{pmatrix} 3 & -5 \\ 9 & -3 \\ -10 & -9 \\ 8 & -1 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 8 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 \cdot 7 + (-5) \cdot 8 \\ 9 \cdot 7 + (-3) \cdot 8 \\ -10 \cdot 7 + (-9) \cdot 8 \\ 8 \cdot 7 + (-1) \cdot 8 \end{pmatrix} =$$

$$= \begin{pmatrix} -19 \\ 39 \\ -142 \\ 48 \end{pmatrix} =$$

$$= C_{4x1}$$

СВОЙСТВА ПРОИЗВЕДЕНИЯ МАТРИЦ

- $(AB)C=A(BC)$
- $AB \neq BA$
- $AE=EA$
- $(A+B)C=AC+BC$; $A(B+C)=AB+AC$
- $(kA)B=k(AB)=A(kB)$

ОПРЕДЕЛИТЕЛЬ

$$A_{2x2} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$A_{2x2} = \begin{pmatrix} -5 & -9 \\ 1 & -3 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} -5 & -9 \\ 1 & -3 \end{vmatrix} = -5\cdot(-3) - (-9)\cdot 1 = 15 + 9 = 24$$

ПРАВИЛО ТРЕУГОЛЬНИКА ДЛЯ ВЫЧИСЛЕНИЯ ОПРЕДЕЛИТЕЛЯ МАТРИЦЫ ТРЕТЬЕГО ПОРЯДКА

$$A_{3x3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} -$$

$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{33}a_{12}a_{21}$$

$$A_{3x3} = \begin{pmatrix} 5 & 1 & -4 \\ -3 & 7 & -10 \\ -7 & -9 & -2 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 5 & 1 & -4 \\ -3 & 7 & -10 \\ -7 & -9 & -2 \end{vmatrix} = 5 \cdot 7 \cdot (-2) + 1 \cdot (-10) \cdot (-7) + (-3) \cdot (-9) \cdot (-4) -$$

$$-(-4) \cdot 7 \cdot (-7) - 5 \cdot (-10) \cdot (-9) - (-2) \cdot 1 \cdot (-3) =$$

$$= -70 + 70 - 108 - 196 - 450 - 6 = -760$$

ПРАВИЛО САРРЮСА

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} =$$

$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$

$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

СВОЙСТВА ОПРЕДЕЛИТЕЛЯ

$$\begin{vmatrix} \boxed{a_{11} & a_{12}} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 3 & -8 \\ -4 & 10 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ -8 & 10 \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = -\begin{vmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 9 & -9 & 3 \\ 8 & -10 & -3 \\ 2 & 4 & 10 \end{vmatrix} = -\begin{vmatrix} 9 & 3 & -9 \\ 8 & -3 & -10 \\ 2 & 10 & 4 \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{vmatrix} = 0$$

$$\begin{vmatrix} 6 & 1 & 0 \\ 8 & 4 & 2 \\ 6 & 1 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{vmatrix} \cdot k = \begin{vmatrix} ka_{11} & ka_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\begin{vmatrix} 8 & 9 & 6 \\ 0 & -4 & -3 \\ -9 & -6 & 1 \end{vmatrix} \cdot 4 = \begin{vmatrix} 8 & 9\cdot 4 & 6 \\ 0 & -4\cdot 4 & -3 \\ -9 & -6\cdot 4 & 1 \end{vmatrix} = \begin{vmatrix} 8 & 36 & 6 \\ 0 & -16 & -3 \\ -9 & -24 & 1 \end{vmatrix}$$

$$\begin{vmatrix} ka_{21} & ka_{22} \\ a_{21} & a_{22} \end{vmatrix} = 0$$

$$\begin{vmatrix} -4 & 2 & 1 \\ 8 & -4 & 0 \\ -6 & 3 & 5 \end{vmatrix} = \begin{vmatrix} 2\cdot(-2) & 2 & 1 \\ -4\cdot(-2) & -4 & 0 \\ 3\cdot(-2) & 3 & 5 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} + ka_{11} \\ a_{21} & a_{22} + ka_{21} \end{vmatrix}$$

$$\begin{vmatrix} -3 & 6 & -2 \\ 9 & 7 & 5 \\ -8 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -3 + 9\cdot 3 & 6 + 7\cdot 3 & -2 + 5\cdot 3 \\ 9 & 7 & 5 \\ -8 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 24 & 27 & 13 \\ 9 & 7 & 5 \\ -8 & 2 & 1 \end{vmatrix}$$

МИНОР
M_{ij}

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$M_{21} = a_{12}$$

$$\begin{pmatrix} -3 & 6 \\ 9 & 7 \end{pmatrix}$$

$$M_{11} = 7$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\begin{pmatrix} 2 & -8 & 5 \\ 4 & 7 & -1 \\ 9 & -6 & 3 \end{pmatrix}$$

$$M_{13} = \begin{vmatrix} 4 & 7 \\ 9 & -6 \end{vmatrix} = -87$$

АЛГЕБРАИЧЕСКОЕ ДОПОЛНЕНИЕ

A_{ij}

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$

$$\begin{pmatrix} 8 & -5 & 10 & -2 \\ -4 & -3 & -1 & -10 \\ 0 & 4 & 2 & 5 \\ -9 & 6 & 9 & 1 \end{pmatrix}$$

$$A_{24} = (-1)^{2+4} \cdot M_{24} = (-1)^6 \cdot \begin{vmatrix} 8 & -5 & 10 \\ 0 & 4 & 2 \\ -9 & 6 & 9 \end{vmatrix} = +642.$$

Разложение определителя по строке или столбцу

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum_{j=1}^{n} a_{ij} A_{ij}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum_{i=1}^{n} a_{ij} A_{ij}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} =$$

$$a_{11} \cdot (-1)^{1+1} \cdot M_{11} + a_{21} \cdot (-1)^{2+1} \cdot M_{21} + a_{31} \cdot (-1)^{3+1} \cdot M_{31} =$$

$$a_{11} \cdot (-1)^2 \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} \cdot (-1)^3 \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \cdot (-1)^4 \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} =$$

$$a_{11} \cdot (+1) \cdot (a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) + a_{21} \cdot (-1) \cdot (a_{12} \cdot a_{33} - a_{13} \cdot a_{32}) + a_{31} \cdot (+1) \cdot (a_{12} \cdot a_{23} - a_{13} \cdot a_{22}) =$$

$$(a_{11} \cdot a_{22} \cdot a_{33} - a_{11} \cdot a_{23} \cdot a_{32}) - (a_{21} \cdot a_{12} \cdot a_{33} - a_{21} \cdot a_{13} \cdot a_{32}) + (a_{31} \cdot a_{12} \cdot a_{23} - a_{31} \cdot a_{13} \cdot a_{22})$$

$$\Delta = \begin{vmatrix} -7 & 0 & 1 \\ 7 & -3 & -6 \\ -8 & -5 & -8 \end{vmatrix} =$$

$$7 \cdot A_{21} + (-3) \cdot A_{22} + (-6) \cdot A_{23} = 7 \cdot (-1)^{2+1} \cdot M_{21} + (-3) \cdot (-1)^{2+2} \cdot M_{22} + (-6) \cdot (-1)^{2+3} \cdot M_{23} =$$

$$7 \cdot (-1)^3 \cdot \begin{vmatrix} 0 & 1 \\ -5 & -8 \end{vmatrix} + (-3) \cdot (-1)^4 \cdot \begin{vmatrix} -7 & 1 \\ -8 & -8 \end{vmatrix} + (-6) \cdot (-1)^5 \cdot \begin{vmatrix} -7 & 0 \\ -8 & -5 \end{vmatrix} =$$

$$7 \cdot (-1) \cdot 5 + (-3) \cdot (+1) \cdot 64 + (-6) \cdot (-1) \cdot 35 = -17$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} =$$

$$a_{13} \cdot A_{13} + a_{23} \cdot A_{23} + a_{33} \cdot A_{33} + a_{43} \cdot A_{43} =$$

$$a_{13} \cdot (-1)^{1+3} \cdot M_{13} + a_{23} \cdot (-1)^{2+3} \cdot M_{23} + a_{33} \cdot (-1)^{3+3} \cdot M_{33} + a_{43} \cdot (-1)^{4+3} \cdot M_{43} =$$

$$a_{13} \cdot (-1)^{4} \cdot \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} + a_{23} \cdot (-1)^{2+3} \cdot \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} + a_{33} \cdot (-1)^{3+3} \cdot \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{vmatrix}$$

$$+ a_{43} \cdot (-1)^{4+3} \cdot \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \end{vmatrix}$$

$$\Delta=\begin{vmatrix}3 & 2 & 5 & -1\\ -5 & 4 & 7 & -8\\ 9 & -6 & -3 & -4\\ 1 & 0 & -2 & 0\end{vmatrix}=$$

$$1\cdot A_{41}+0\cdot A_{42}+(-2)\cdot A_{43}+0\cdot A_{44}=$$

$$1\cdot(-1)^{4+1}\cdot M_{41}+0+(-2)\cdot(-1)^{4+3}\cdot M_{43}+0=$$

$$-\begin{vmatrix}2 & 5 & -1\\ 4 & 7 & -8\\ -6 & -3 & -4\end{vmatrix}+2\cdot\begin{vmatrix}3 & 2 & -1\\ -5 & 4 & -8\\ 9 & -6 & -4\end{vmatrix}=$$

$$-186+2\cdot(-370)=-926$$

РАНГ МАТРИЦЫ А (RANGA)

$$A_{4\times 6} = \begin{pmatrix} -1 & -2 & 6 & 0 & 12 & 0 \\ -5 & -10 & -5 & 5 & -10 & 1{,}25 \\ -4 & -8 & -7 & -4 & -14 & -1 \\ -10 & 8 & -7 & -4 & -14 & -1 \end{pmatrix}$$

$$M_3 = \begin{vmatrix} -2 & 6 & 0 \\ -10 & -5 & 5 \\ -8 & -7 & -4 \end{vmatrix} = -590 \neq 0$$

rangA=3

$$\begin{pmatrix} 0 & 9 & -2 & -4 \\ 4 & -5 & 7 & 14 \\ 3 & -1 & -6 & -12 \\ 1 & -10 & 8 & 16 \end{pmatrix} \quad I \leftrightarrow VI \quad \begin{pmatrix} 1 & -10 & 8 & 16 \\ 4 & -5 & 7 & 14 \\ 3 & -1 & -6 & -12 \\ 0 & 9 & -2 & -4 \end{pmatrix} \quad \begin{matrix} I\cdot 4 - II \to II \\ I\cdot 3 - III \to III \end{matrix}$$

$$\begin{pmatrix} 1 & -10 & 8 & 16 \\ 0 & -35 & 25 & 50 \\ 0 & -29 & 30 & 60 \\ 0 & 9 & -2 & -4 \end{pmatrix} \quad \begin{matrix} II\cdot(-29) - III\cdot(-35) \to III \\ II\cdot(9) - IV\cdot(-35) \to IV \end{matrix} \quad \begin{pmatrix} 1 & -10 & 8 & 16 \\ 0 & -35 & 25 & 50 \\ 0 & 0 & 325 & 650 \\ 0 & 0 & 155 & 310 \end{pmatrix} \quad III\cdot 155 - IV\cdot 325 \to IV$$

$$\begin{pmatrix} 1 & -10 & 8 & 16 \\ 0 & -35 & 25 & 50 \\ 0 & 0 & 325 & 650 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

нулевую строку отбрасываем

$$\begin{pmatrix} 1 & -10 & 8 & 16 \\ 0 & -35 & 25 & 50 \\ 0 & 0 & 325 & 650 \end{pmatrix}$$

$$\begin{vmatrix} -10 & 8 & 16 \\ -35 & 25 & 50 \\ 0 & 325 & 650 \end{vmatrix} \neq 0$$

rangA=3