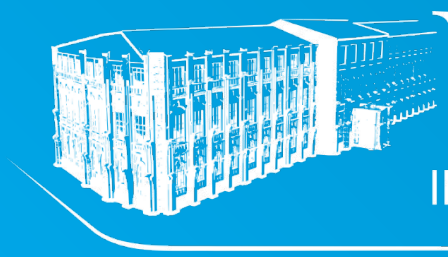


WESTMINSTER

INTERNATIONAL UNIVERSITY IN TASHKENT

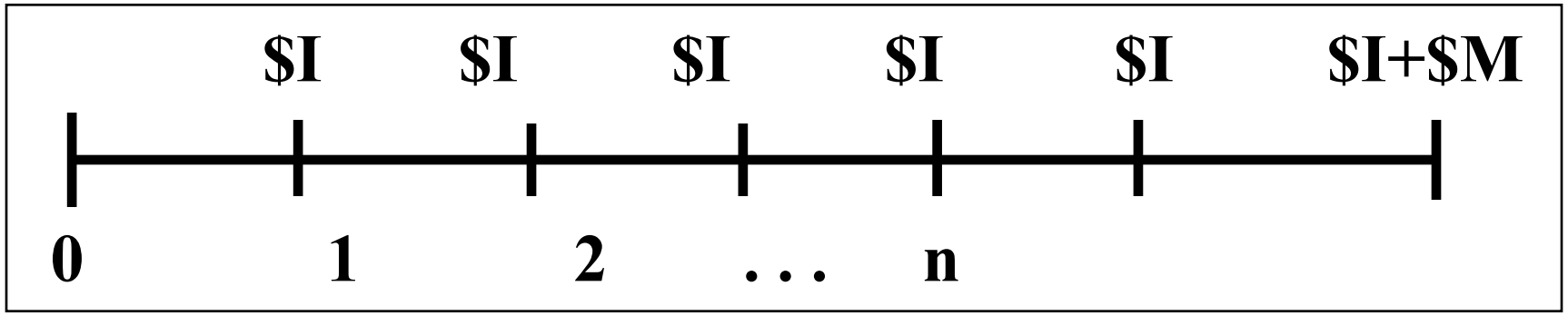
An Accredited Institution of the University of Westminster (UK)

Lecture 3. Bond Valuation

Contents :

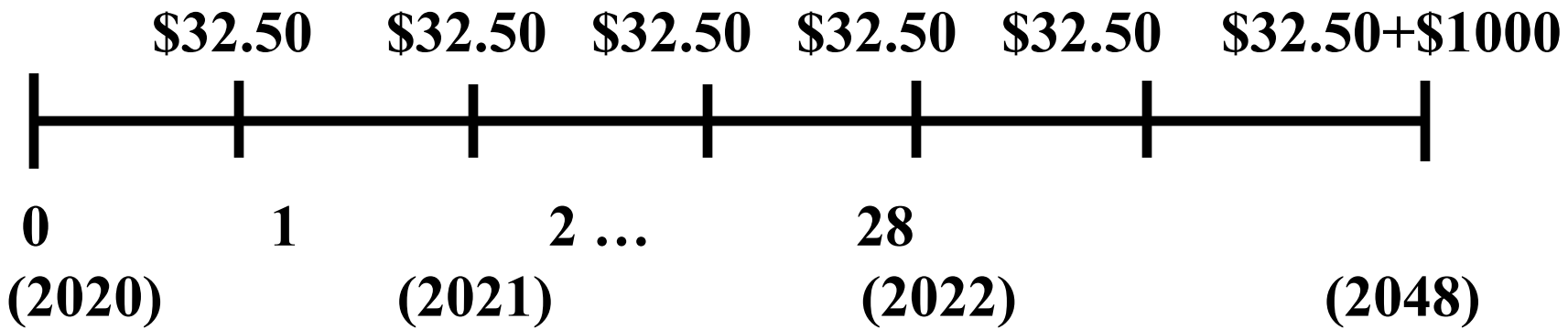
Definition of Bonds…

A type of debt (long-term promissory note) issued by the borrower, promising to pay fixed coupon (interest) payments at fixed intervals (6 months, 1 year etc) and pay the par value at maturity.



Example:

- **Par value = $1,000**
- **Coupon = 6.5% on par value per year,**
  or $65 per year ($32.50 every six months).
- **Maturity = 28 years (matures in 2048).**
- **Issued by AT&T.**



Bonds

Different Types of Bonds

- **Debentures:** Unsecured long-term debt.
- **Subordinated debentures:** Bonds that have a lower claim on assets in the event of liquidation than do other senior debt holders.
- **Mortgage bonds:** Bonds secured by a lien on specific assets of the firm, such as real estate.
- **Eurobonds:** Bonds issued in a country different from the one in whose currency the bond is denominated; for instance, a bond issued in Europe or Asia that pays interest and principal in U.S. dollars.
- **Zero and low coupon bonds:** Allow the issuing firm to issue bonds at a substantial discount from their $1,000 face value with a zero or very low coupon.

Different Types of Bonds

- **Junk Bonds**: bonds rated BB or below
- **Treasury Bonds**: Bonds issued by the federal government, sometimes referred to as government bonds.
- **Corporate Bonds:** Bonds issued by corporations.
- **Call Provision:** A provision in a bond contract that gives the issuer the right to redeem the bonds under specified terms prior to the normal maturity date.
- **Convertible Bond:** A bond that is exchangeable, at the option of the holder, for common stock of the issuing firm

Terminologies of bond

Principal

- The amount of money on which interest is paid.

Maturity date

- The date when a bond's life ends and the borrower must make the final interest payment and repay the principal.

Par/Maturity value

- The face value of a bond, which the borrower repays at maturity.

Coupon

- A fixed amount of interest that a bond promises to pay investors.

Coupon rate

- The rate derived by dividing the bond's annual coupon payment by its par value.

The Fundamental Valuation Model

$$P_0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_n}{(1+r)^n}$$

P_0 = Price of asset at time 0 (today)
CF_t = Cash flow expected at time t
r = Discount rate (reflecting asset's risk)
n = Number of discounting periods (usually years)

- This model can express the price of any asset at t = 0 mathematically.

Valuing Coupon Bonds

- A non-zero coupon-paying bond is a coupon paying bond with a finite life.

$$V = \frac{C}{(1+k_d)^1} + \frac{C}{(1+k_d)^2} + \ldots + \frac{C+MV}{(1+k_d)^n}$$

$$= \sum_{t=1}^{n} \frac{C}{(1+k_d)^t} + \frac{MV}{(1+k_d)^n}$$

$$V = C\,(PVIFA_{kd,\,n}) + MV\,(PVIF_{kd,\,n})$$

$$V = C \times \left[\frac{1}{k} - \frac{1}{k(1+k)^n}\right] + \frac{MV}{(1+k)^n}$$

Valuing Zero Coupon Bonds

- A zero coupon bond is a bond that pays no interest but sells at a deep discount from its face value; it provides compensation to investors in the form of price appreciation.

$$V = \frac{MV}{(1 + k_d)^n} = MV\ (PVIF_{kd,\ n})$$

Example. Bond Z has a $1,000 face value and a 30 year life. The appropriate discount rate is 10%. What is the value of the zero-coupon bond? 1000* 0.0573

Valuing Perpetual Bonds

- A perpetual bond is a bond that *never* matures. It has an infinite life.

$$V = \frac{C}{(1 + k_d)^1} + \frac{C}{(1 + k_d)^2} + \ldots + \frac{C}{(1 + k_d)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{C}{(1 + k_d)^t} \quad \text{or} \quad C\,(PVIFA_{kd,\infty})$$

$$V = C / k_d \quad [\textit{Reduced Form}]$$

Example. Bond P has a $1,000 face value and provides an 8% annual coupon. The appropriate discount rate is 10%. What is the value of the perpetual bond?

Non-annual Compounding

$$V = C/m \times \left[\frac{1}{k/m} - \frac{1}{k/m(1+k/m)^{n \times m}} \right] + \frac{MV}{(1+k/m)^{n \times m}}$$

- Semiannually → m=2
- Quarterly → m=4
- Monthly → m=12
- Weekly → m=52
- Daily → m=365

Semiannual Compounding

A non-zero coupon bond adjusted for semiannual compounding.

$$\text{Value} = \frac{\frac{C}{2}}{(1+\frac{k}{2})^{1}} + \frac{\frac{C}{2}}{(1+\frac{k}{2})^{2}} + \frac{\frac{C}{2}}{(1+\frac{k}{2})^{3}} + ... + \frac{\frac{C}{2}+MV}{(1+\frac{k}{2})^{2n}}$$

An example....
Value a T-Bond
Par value = $1,000
Maturity = 2 years
Coupon rate = 4%
k = 4.4% per year

$$V = \frac{\frac{\$40}{2}}{\left(1+\frac{0.044}{2}\right)^{1}} + \frac{\frac{\$40}{2}}{\left(1+\frac{0.044}{2}\right)^{2}} + \frac{\frac{\$40}{2}}{\left(1+\frac{0.044}{2}\right)^{3}} + \frac{\frac{\$40}{2}+1{,}000}{\left(1+\frac{0.044}{2}\right)^{4}}$$

Bond Premiums and Discounts

What happens to bond values if the required return is not equal to the coupon rate?

The bond's price will differ from its par value.

Coupon Interest Rate < r	→	P_0 < par value	=	DISCOUNT
Coupon Interest Rate > r	→	P_0 > par value	=	PREMIUM
Coupon Interest Rate = r	→	P_0 = par value	=	PAR

Yield to Maturity (YTM)/Expected Rate of Return

- Also called Expected Rate of Return

Estimate of return investors earn if they buy the bond at P_0 and hold it until maturity

The YTM on a bond selling at par will always equal the coupon rate.

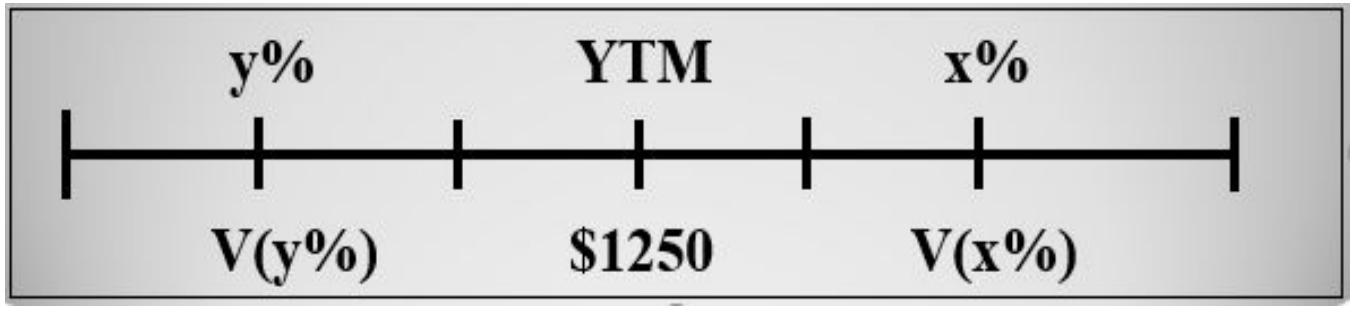
YTM is the discount rate that equates the PV of a bond's cash flows with its price.

Determining the YTM: Interpolation

Julie Miller want to determine the YTM for an issue of outstanding bonds at *Basket Wonders (BW)*. *BW* has an issue of 10% annual coupon bonds with 15 years left to maturity. The bonds have a current market value of *$1,250 (face value = $1000)*.

What is the YTM?

$$Interpolation: \frac{x\% - YTM}{x\% - y\%} = \frac{V(x\%) - V(YTM)}{V(x\%) - V(y\%)}$$



Interpolation formula

- V=$1,250 shows bond sells on premium (higher than par value 1,000)
- Therefore, Coupon rate is higher than rate of return (YTM<C), (YTM<10%)
- So we try next highest rate after 10%
- **Try 9%**

$1,250 = $100($PVIFA_{9\%,15}$) + $1,000($PVIF_{9\%,15}$)

$1,250 = $806.07 + $274.50

= $1,080.57 [*Rate is too high!*]

- **Try 8%**

$1,250 = $100($PVIFA_{8\%,15}$) + $1,000($PVIF_{8\%,15}$)

$1,250 = $855.95 + $315.2

= $1,171.15 [*Rate is still higher*]

- Value $1,171.15(k=8%) is closer to $1,250 than the value $1,080.57(k=9%), so we drop value $1,080.57(k=9%)

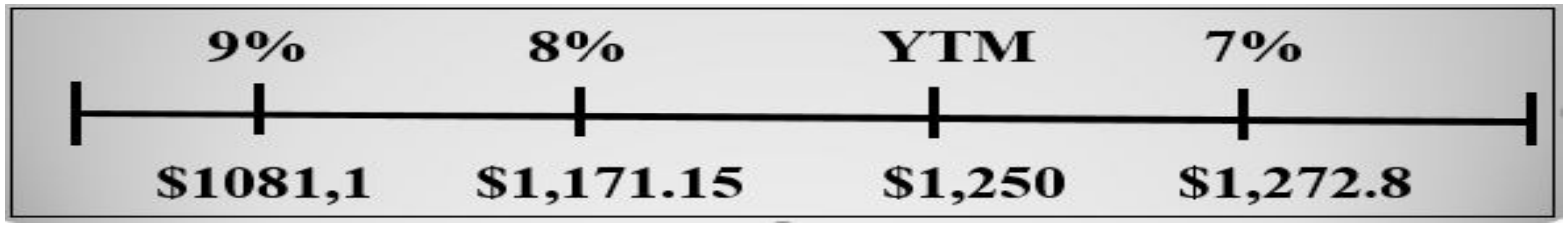
Solution

- **Try 7%**

$1,250 = $100(PVIFA$_{7\%,15}$) + $1,000(PVIF$_{7\%,\ 15}$)

$1,250 = $910.79 + $362.4

= $1,273.19 [*Rate is low!*]



$$\frac{7\% - YTM}{7\% - 8\%} = \frac{1{,}272.8 - 1250}{1{,}272.8 - 1{,}171.15}$$

- YTM = 7.22%
- **What is the meaning of V=$1250 at YTM=7.22%?**

Decision making

- With the given rate of return (7.22%), If the bond is sold at a price above the bond value ($1250) it is known as overvalued and investor should not buy the bond, or can sell existing bonds if any.
- With the given rate of return (7.22%), If the bond is sold at a price below the bond value($1250) it is known as undervalued and a wise decision is to invest in those bonds, or should not sell existing bonds if any.

Determining the YTM: Approximation Method

$$\text{Approx YTM} = \frac{C + \frac{MV - P}{n}}{\frac{MV + P}{2}}$$

- C = dollar amount of interest
- MV = face value of bond
- P = price of bond
- n = number of years to maturity
- In the previous example,

$$\text{Approx YTM} = \frac{100 + \frac{1000 - 1250}{15}}{\frac{1000 + 1250}{2}} = 7.41\%$$

FIVE IMPORTANT RELATIONSHIPS

FIRST RELATIONSHIP

- The value of the bond is inversely related to changes in the investor's required rate of return (current interest rate) k_b

- If k_d *decrease* , *the value of the bond will increase*
- If k_d *increase* , *the value of the bond will decrease*

SECOND RELATIONSHIP

- The market value (P_o) will be less than the par value (M) if the investor's required rate of return (k_b) is above the coupon rate (I), but it will be valued above the par value if the investor's required rate of return (k_b) is below the coupon rate (I),

- If $k_d = C$, then $MV = P_o$ (sold at par)
- If $k_d > C$, then $MV > P_o$ (discount bond)
- If $k_d < C$, then $MV < P_o$ (premium bond)

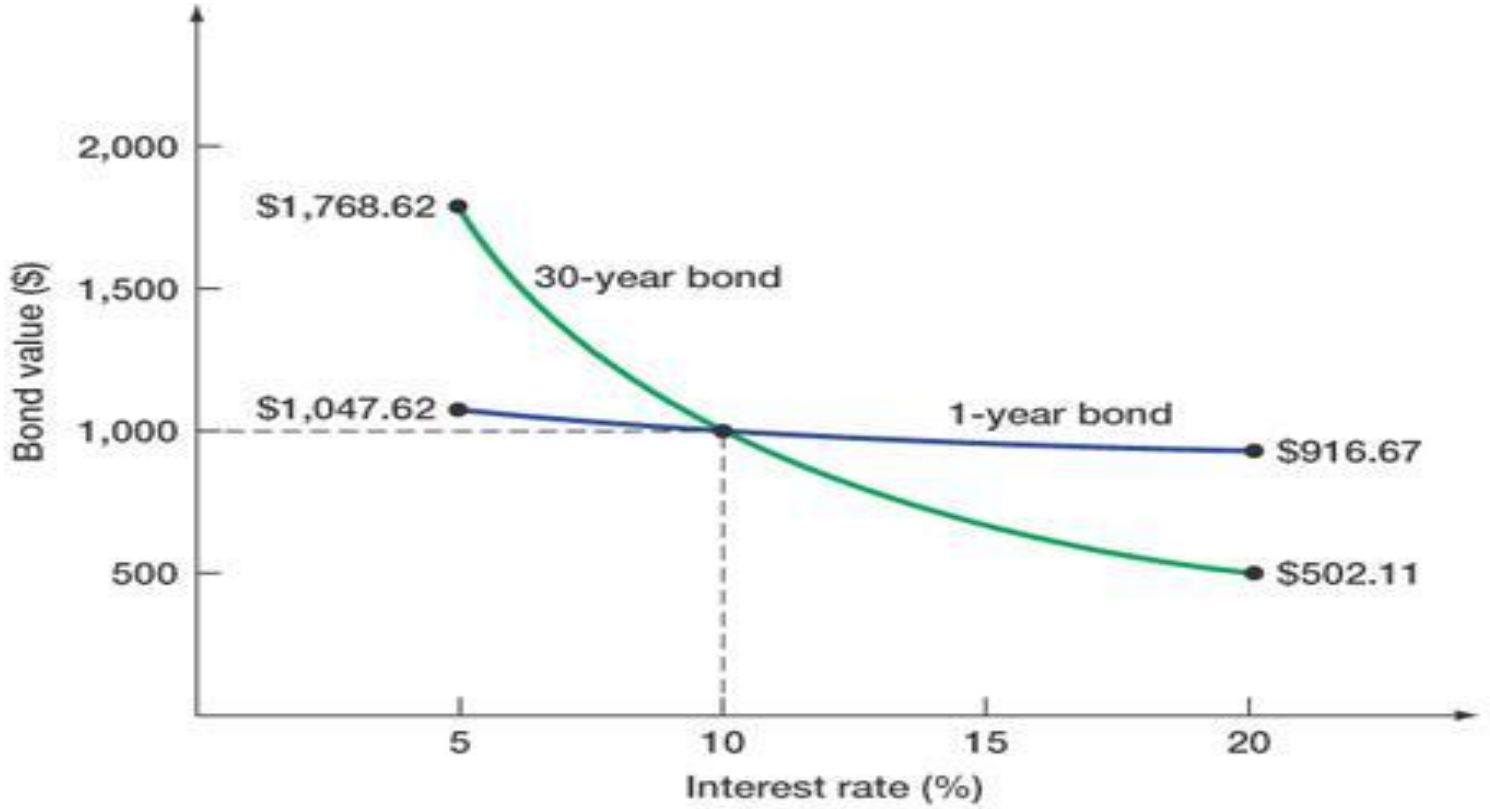
THIRD RELATIONSHIP

- As the maturity approaches, the market value of the bond approaches its par value

FOURTH RELATIONSHIP

- Change in price due to changes in interest rates
- Long term bond have greater interest rate risk than do short –term bonds
- Low coupon rate bonds have more price risk than high coupon rate bonds

Relationship between bond value & interest rate



Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

	Time to Maturity	
Interest Rate	1 Year	30 Years
5%	$1,047.62	$1,768.62
10	1,000.00	1,000.00
15	956.52	671.70
20	916.67	502.11

FIFTH RELATIONSHIP

- The sensitivity of a bond's value to changing - depends on:
  - Length of time to maturity
  - The pattern of the cash flows provided by the bond

Exercise: YTM with semiannual coupon

Suppose a bond with a 10% coupon rate and semiannual coupons, has a face value of $1000, 20 years to maturity and is selling for $1197.93.

1. Is the YTM more or less than 10%?
2. What is the semiannual coupon payment?
3. How many periods are there?
4. Calculate the YTM.

Solution

1. Because the selling price is above the par value.. This is a premium bond. For premium bond the YTM (required return) should be below the coupon rate (<10%)

2. Semi-annual coupon payment is = (10% x 1000)/2 = 50

3. n= 20 x 2 = 40

4. $YTM = \frac{\left(50+\frac{1000-1197.93}{40}\right)}{\frac{1000+1197.3}{2}} = 0.04 \text{ @ } 4\% \ (is\ this\ your\ final\ YTM?)$

5. YTM = 4% x 2 = 8% {YTM must always be in annual basis}

Reading

- R. Brealey, S. Myers and F. Allen. “Principles of Corporate Finance”, (2010) MacGraw Hill, 10th Edition (Chapter 3)
- Van Horne, J.C., Wachowicz, J.M., (2010). *Fundamentals of Financial Management*. 13th ed. Chapter 4