Ряды Фурье

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_1 t) + b_k \sin(k\omega_1 t) \right)$$

$$\omega_1 = 2\pi / T$$

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos(k\omega_1 t) dt$$

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin(k\omega_1 t) dt$$

$$\frac{a_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} g(t) dt$$

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t)\cos(k\frac{2\pi}{T}t)dt = \frac{1}{\pi} \int_{-T/2}^{T/2} g(t)\cos(k\frac{2\pi}{T}t)d(t\frac{2\pi}{T}) =$$

$$= \frac{1}{\pi} \int_{-T'}^{T'} g(t)\cos(k\frac{2\pi}{T}t)d(\frac{2\pi}{T}t)$$

$$t' = \frac{2\pi}{T}t, \quad T' = \pi$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t)\cos(kt')d(t')$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t)\cos(kt)dt, \quad k = 0,1,2,\ldots$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t)\sin(kt)dt \quad k = 0,1,2,\ldots$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(kt) + b_k \sin(kt)\right)$$

$$a_k = \frac{2}{\pi}\int_0^{\pi} g(t)\cos(kt)\,dt, \quad k = 0,1,2,...$$

$$b_k = \frac{2}{\pi}\int_0^{\pi} g(t)\sin(kt)\,dt, \quad k = 0,1,2,...$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_1 t + \varphi_k)$$

$$A_{k=}\sqrt{a_k^2 + b_k^2}, \quad \cos\varphi_k = a_k / A_k, \quad -\sin\varphi_k = b_k / A_k,$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos kt + b_k \sin kt\right)$$

$$g(t) = \frac{a_0}{2} + 2\sum_{k=1}^{\infty} \tilde{A}_k \cos(kt + \varphi_k)$$

$$\tilde{A}_{k=}\frac{1}{2}\sqrt{a_k^2 + b_k^2},$$

$$\cos\varphi_k = a_k / 2A_k, \quad -\sin\varphi_k = b_k / 2A_k$$

$$\tilde{A}_{k=} A_k / 2$$

$$e^{jx} = \cos x + j \sin x \qquad (3.15)$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx}) \qquad (3.16)$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{A_k}{2} \left(\exp(jk\omega_1 t + j\varphi_k) + \exp(-jk\omega_1 t - j\varphi_k) \right) \qquad (3.17)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t} \qquad (3.18)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k \exp\left(\frac{2\pi jkt}{T} \right) \qquad (3.19)$$

$$\mathbf{C}_k = \frac{1}{2} A_k e^{j\varphi_k}$$

$$A_k = 2\left|\mathbf{C}_k\right|, \quad \varphi_k = \arg(\mathbf{C}_k) \qquad (3.20)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jkt} \qquad (3.21) \qquad\qquad \mathbf{C}_k = \widetilde{A}_k e^{j\varphi_k} \qquad (3.22)$$

$$\widetilde{A}_{k=} A_k / 2$$

$$g(t) = \sum_{k=-\infty}^{\infty} \frac{1}{2} A_k e^{j(k\omega_1 t + \varphi_k)} \qquad (3.23)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \widetilde{A}_k e^{j(kt + \varphi_k)} \qquad (3.24)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \widetilde{A}_k e^{jk(t + \tau_k)} \qquad (3.25) \qquad\qquad k\tau_k = \varphi_k$$

$$\mathbf{C}_k = \frac{a_k}{2} - j\frac{b_k}{2} \tag{3.26}$$

$$a_k = 2\,\mathrm{Re}(\mathbf{C}_k), \quad b_k = -2\,\mathrm{Im}(\mathbf{C}_k)$$

$$\varphi_k = -arctg(b_k / a_k) \tag{3.27}$$

$$\mathbf{C}_{-k} = \mathbf{C}_k^* \tag{3.29}$$

$$\mathbf{C}_k = \frac{1}{T}\int_{-T/2}^{T/2} g(t)e^{-jk\omega_1 t}\,dt \tag{3.30}$$

$$\mathbf{C}_k = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\exp\left(\frac{-2\pi jkt}{T}\right)dt \tag{3.31}$$