

# Ряды Фурье

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(k\omega_1 t) + b_k \sin(k\omega_1 t))$$

$$\omega_1 = 2\pi / T$$

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos(k\omega_1 t) dt$$

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \sin(k\omega_1 t) dt$$

$$\frac{a_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} g(t) dt$$

$$\begin{aligned}
 a_k &= \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos\left(k \frac{2\pi}{T} t\right) dt = \frac{1}{\pi} \int_{-T/2}^{T/2} g(t) \cos\left(k \frac{2\pi}{T} t\right) d\left(\frac{2\pi}{T} t\right) = \\
 &= \frac{1}{\pi} \int_{-T'}^{T'} g(t) \cos\left(k \frac{2\pi}{T} t\right) d\left(\frac{2\pi}{T} t\right)
 \end{aligned}$$

$$t' = \frac{2\pi}{T} t, \quad T' = \pi$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t) \cos(kt') d(t')$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t) \cos(kt) dt, \quad k = 0, 1, 2, \dots$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t) \sin(kt) dt \quad k = 0, 1, 2, \dots$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kt) + b_k \sin(kt))$$

$$a_k = \frac{2}{\pi} \int_0^{\pi} g(t) \cos(kt) dt, \quad k = 0, 1, 2, \dots$$

$$b_k = \frac{2}{\pi} \int_0^{\pi} g(t) \sin(kt) dt, \quad k = 0, 1, 2, \dots$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_1 t + \varphi_k)$$

$$A_k = \sqrt{a_k^2 + b_k^2}, \quad \cos \varphi_k = a_k / A_k, \quad -\sin \varphi_k = b_k / A_k,$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kt + b_k \sin kt)$$

$$g(t) = \frac{a_0}{2} + 2 \sum_{k=1}^{\infty} \tilde{A}_k \cos(kt + \varphi_k)$$

$$\tilde{A}_k = \frac{1}{2} \sqrt{a_k^2 + b_k^2},$$

$$\cos \varphi_k = a_k / 2A_k, \quad -\sin \varphi_k = b_k / 2A_k$$

$$\tilde{A}_k = A_k / 2$$

$$e^{jx} = \cos x + j \sin x \quad (3.15)$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx}) \quad (3.16)$$

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{A_k}{2} (\exp(jk\omega_1 t + j\varphi_k) + \exp(-jk\omega_1 t - j\varphi_k)) \quad (3.17)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t} \quad (3.18)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k \exp\left(\frac{2\pi jkt}{T}\right) \quad (3.19)$$

$$\mathbf{C}_k = \frac{1}{2} A_k e^{j\varphi_k}$$

$$A_k = 2|\mathbf{C}_k|, \quad \varphi_k = \arg(\mathbf{C}_k) \quad (3.20)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jkt} \quad (3.21)$$

$$\mathbf{C}_k = \tilde{A}_k e^{j\varphi_k} \quad (3.22)$$

$$\tilde{A}_k = A_k / 2$$

$$g(t) = \sum_{k=-\infty}^{\infty} \frac{1}{2} A_k e^{j(k\omega_1 t + \varphi_k)} \quad (3.23)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \tilde{A}_k e^{j(k t + \varphi_k)} \quad (3.24)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \tilde{A}_k e^{jk(t + \tau_k)} \quad (3.25) \quad k\tau_k = \varphi_k$$

$$\mathbf{C}_k = \frac{a_k}{2} - j \frac{b_k}{2} \quad (3.26)$$

$$a_k = 2 \operatorname{Re}(\mathbf{C}_k), \quad b_k = -2 \operatorname{Im}(\mathbf{C}_k)$$

$$\varphi_k = -\operatorname{arctg}(b_k / a_k) \quad (3.27)$$

$$\mathbf{C}_{-k} = \mathbf{C}_k^* \quad (3.29)$$

$$\mathbf{C}_k = \frac{1}{T} \int_{-T/2}^{T/2} g(t) e^{-jk\omega_1 t} dt \quad (3.30)$$

$$\mathbf{C}_k = \frac{1}{T} \int_{-T/2}^{T/2} g(t) \exp\left(\frac{-2\pi jkt}{T}\right) dt \quad (3.31)$$