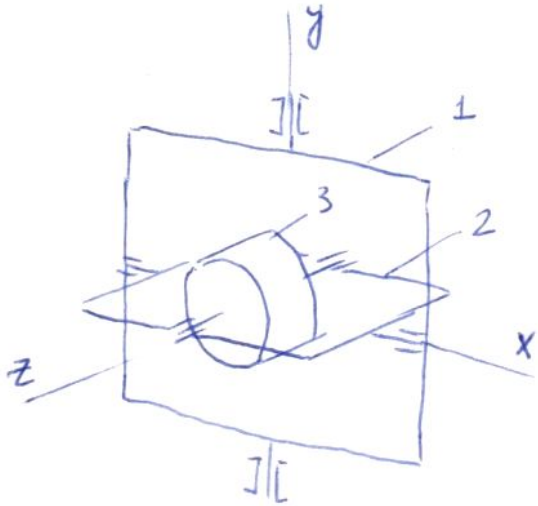


Лекція 6

Рівняння руху гіроскопу в кардановому підвісі



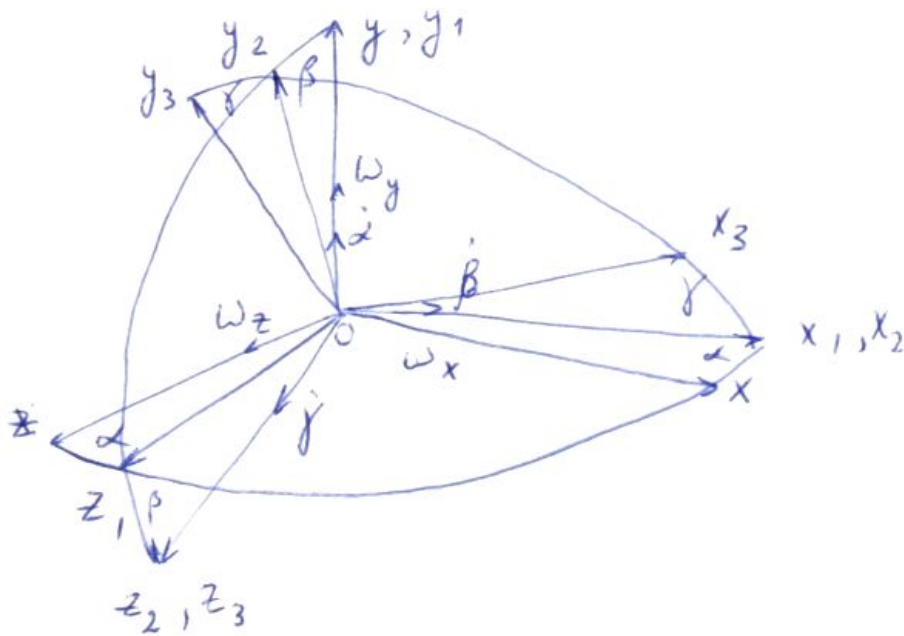
$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j$$

Oxyz

Ox_iy_iz_i i = 1 i = 2 i = 3

α, β, γ

$\omega_x, \omega_y, \omega_z$



$$\omega_{x_1} = \omega_x \cos \alpha - \omega_z \sin \alpha,$$

$$\omega_{y_1} = \dot{\alpha} + \omega_y,$$

$$\omega_{z_1} = \omega_x \sin \alpha + \omega_z \cos \alpha$$

\mathbf{A}_1	x	y	z
x_1	$\cos \alpha$	0	$-\sin \alpha$
y_1	0	1	0
z_1	$\sin \alpha$	0	$\cos \alpha$

\mathbf{A}_2	x_1	y_1	z_1
x_2	1	0	0
y_2	0	$\cos \beta$	$\sin \beta$
z_2	0	$-\sin \beta$	$\cos \beta$

$$\omega_{x_2} = \dot{\beta} + \omega_{x_1},$$

$$\omega_{y_2} = \omega_{y_1} \cos \beta + \omega_{z_1} \sin \beta$$

$$\omega_{z_2} = -\omega_{y_1} \sin \beta + \omega_{z_1} \cos \beta$$

\mathbf{A}_3	x_2	y_2	z_2
x_3	$\cos \gamma$	$\sin \gamma$	0
y_3	$-\sin \gamma$	$\cos \gamma$	0
z_3	0	0	1

$$\omega_{x3} = \omega_{x2} \cos \gamma + \omega_{y2} \sin \gamma,$$

$$\omega_{y3} = -\omega_{x2} \sin \gamma + \omega_{y2} \cos \gamma$$

$$\omega_{z3} = \dot{\gamma} + \omega_{z2}$$

$$T = \frac{1}{2} \sum_{i=1}^3 \left(I_{xi} \omega_{xi}^2 + I_{yi} \omega_{yi}^2 + I_{zi} \omega_{zi}^2 \right).$$

$$I_{x3} = I_{y3} = I$$

$$I_{z3} = I$$

$$T = \frac{1}{2} I_{x1} \omega_{x1}^2 + \frac{1}{2} I_{y1} \omega_{y1}^2 + \frac{1}{2} I_{z1} \omega_{z1}^2 + \frac{1}{2} I_{x2} \omega_{x2}^2 + \frac{1}{2} I_{y2} \omega_{y2}^2 + \frac{1}{2} I_{z2} \omega_{z2}^2 +$$

$$+ \frac{1}{2} I_{x3} \omega_{x3}^2 + \frac{1}{2} I_{y3} \omega_{y3}^2 + \frac{1}{2} I_{z3} \omega_{z3}^2$$

$$T = \frac{1}{2} I_{x1} \omega_{x1}^2 + \frac{1}{2} I_{y1} \omega_{y1}^2 + \frac{1}{2} I_{z1} \omega_{z1}^2 + \frac{1}{2} I_{x2} \omega_{x2}^2 + \frac{1}{2} I_{y2} \omega_{y2}^2 + \frac{1}{2} I_{z2} \omega_{z2}^2 +$$

$$+ \frac{1}{2} I_3 \left(\omega_{x3}^2 + \omega_{y3}^2 \right) + \frac{1}{2} I \omega_{z3}^2.$$

$$\omega_{x3}^2 + \omega_{y3}^2$$

$$\omega_{x3}^2 + \omega_{y3}^2 = \left(\omega_{x2} \cos \gamma + \omega_{y2} \sin \gamma \right)^2 + \left(-\omega_{x2} \sin \gamma + \omega_{y2} \cos \gamma \right)^2 = \omega_{x2}^2 + \omega_{y2}^2.$$

$$T = \frac{1}{2} I_{x1} \omega_{x1}^2 + \frac{1}{2} I_{y1} \omega_{y1}^2 + \frac{1}{2} I_{z1} \omega_{z1}^2 + \frac{1}{2} I_{x2} \omega_{x2}^2 + \frac{1}{2} I_{y2} \omega_{y2}^2 + \frac{1}{2} I_{z2} \omega_{z2}^2 +$$

$$+ \frac{1}{2} I_{\vartheta} \left(\omega_{x2}^2 + \omega_{y2}^2 \right) + \frac{1}{2} I \omega_{z3}^2$$

$$T = \frac{1}{2} I_{x1} \omega_{x1}^2 + \frac{1}{2} I_{y1} \omega_{y1}^2 + \frac{1}{2} I_{z1} \omega_{z1}^2 +$$

$$+ \frac{1}{2} \left(I_{x2} + I_x \right) \omega_{x2}^2 + \frac{1}{2} \left(I_{y2} + I_y \right) \omega_{y2}^2 + \frac{1}{2} I_{z2} \omega_{z2}^2 + \frac{1}{2} I \omega_{z3}^2$$

α, β, γ

1)
ротор

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\gamma}} - \frac{\partial T}{\partial \gamma} = M_{z3}$$

$$\frac{\partial T}{\partial \gamma} = 0;$$

$$\frac{\partial T}{\partial \dot{\gamma}} = \frac{1}{2} I \cdot 2\omega_{z3} \frac{\partial}{\partial \dot{\gamma}} \omega_{z3} = I\omega_{z3} = H$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\gamma}} = \dot{H}$$

$$\dot{H} = M_{z3}$$

2) внутрішня
рамка

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} - \frac{\partial T}{\partial \beta} = M_{x2}$$

$$\frac{\partial T}{\partial \beta} = (I_{y2} + I_x) \omega_2 \frac{\partial \omega_{y2}}{\partial \beta} + I_{z2} \omega_2 \frac{\partial \omega_{z2}}{\partial \beta} + I \omega_3 \frac{\partial \omega_{z3}}{\partial \beta}$$

$$\frac{\partial \omega_{y2}}{\partial \beta} = -\omega_{y1} \sin \beta + \omega_{z1} \cos \beta = \omega_{z2}$$

$$\frac{\partial \omega_{z2}}{\partial \beta} = -\omega_{y1} \cos \beta - \omega_{z1} \sin \beta = -\omega_{y2}$$

$$\frac{\partial \omega_{z3}}{\partial \beta} = \frac{\partial \omega_{z2}}{\partial \beta} = -\omega_{y2}$$

$$\frac{\partial T}{\partial \beta} = (I_{y2} + I_x) \omega_2 \omega_{z2} - I_{y2} \omega_2 \omega_{z2} - I_y H \omega_2$$

$$\frac{\partial T}{\partial \dot{\beta}} = (I_{y2} + I_x) \omega_2$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} = (I_{y2} + I_x) \dot{\omega}_2$$

$$(I_{y2} + I_x) \dot{\omega}_2 + (I_{y2} - I_{y2} - I_y) \omega_2 \omega_{z2} + H \omega_{2x} = M_{x2}$$

3) зовнішня
рамка

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = M_{y1}$$

$$\frac{\partial T}{\partial \alpha} = I_{x1} \omega_{x1} \frac{\partial \omega_{x1}}{\partial \alpha} + I_{z1} \omega_{z1} \frac{\partial \omega_{z1}}{\partial \alpha} + (I_{x2} + I_z) \omega_{z2} \frac{\partial \omega_{x2}}{\partial \alpha} + (I_{z2} + I) \omega_{z2} \frac{\partial \omega_{y2}}{\partial \alpha} + I_{z2} \omega_{z2} \frac{\partial \omega_{z2}}{\partial \alpha} + I \omega_{z3} \frac{\partial \omega_{z3}}{\partial \alpha}.$$

$$\frac{\partial \omega_{x1}}{\partial \alpha} = -\omega_x \sin \alpha - \omega_z \cos \alpha = -\omega_{z1};$$

$$\frac{\partial \omega_{z1}}{\partial \alpha} = \omega_x \cos \alpha - \omega_z \sin \alpha = \omega_{x1};$$

$$\frac{\partial \omega_{x2}}{\partial \alpha} = \frac{\partial \omega_{x1}}{\partial \alpha} = -\omega_{z1}$$

$$\frac{\partial \omega_{y2}}{\partial \alpha} = \frac{\partial \omega_{z1}}{\partial \alpha} \sin \beta = \omega_{x1} \sin \beta$$

$$\frac{\partial \omega_{z2}}{\partial \alpha} = \frac{\partial \omega_{z1}}{\partial \alpha} \cos \beta = \omega_{x1} \cos \beta$$

$$\frac{\partial \omega_{z3}}{\partial \alpha} = \frac{\partial \omega_{z2}}{\partial \alpha} = \omega_{x1} \cos \beta$$

$$\frac{\partial T}{\partial \alpha} = -I_{y1} \omega_{x1} \omega_{z1} + I_{z1} \omega_{y1} \omega_{z1} - (I_{yx2} + I_x) \omega_2 \omega_1 + (I_x + I) \omega_2 \omega_1 \sin \beta + I_2 \omega_2 \omega_1 \cos \beta + H \omega_1 \cos \beta$$

$$= -(I_{y1} - I_{z1}) \omega_{x1} \omega_{z1} - (I_{yx2} + I_x) \omega_2 \omega_1 + (I_x + I) \omega_2 \omega_1 \sin \beta + I_2 \omega_2 \omega_1 \cos \beta + H \omega_1 \cos \beta.$$

$$\frac{\partial T}{\partial \alpha} = I_{y1} \omega_{y1} \frac{\partial \omega_{y1}}{\partial \alpha} + (I_{yx2} + I_x) \omega_2 \frac{\partial \omega_{y2}}{\partial \alpha} + I_2 \omega_2 \frac{\partial \omega_{z2}}{\partial \alpha} + I \omega_3 \frac{\partial \omega_{z3}}{\partial \alpha}.$$

$$\frac{\partial \omega_{y2}}{\partial \alpha} = \frac{\partial \omega_{y1}}{\partial \alpha} \cos \beta = \cos \beta$$

$$\frac{\partial \omega_{z2}}{\partial \alpha} = -\frac{\partial \omega_{y1}}{\partial \alpha} \sin \beta = -\sin \beta$$

$$\frac{\partial \omega_{z3}}{\partial \alpha} = \frac{\partial \omega_{z2}}{\partial \alpha} = -\sin \beta$$

$$\frac{\partial T}{\partial \alpha} = I_{y1} \omega_{y1} + (I_{y2} + I_z) \omega_{z2} \cos \beta - I_{z2} \omega_{z2} \sin \beta - H \sin \beta.$$

$$\frac{d}{dt} \frac{\partial T}{\partial \alpha} = I_{y1} \dot{\omega}_{y1} + (I_{y2} + I_z) \dot{\omega}_{z2} \cos \beta - (I_{z2} + I_z) \omega_{z2} \dot{\beta} \sin \beta -$$

$$- I_{z2} \dot{\omega}_{z2} \sin \beta - I_{z2} \omega_{z2} \dot{\beta} \cos \beta - \dot{H} \sin \beta - H \dot{\beta} \cos \beta.$$

$$I_{y1} \dot{\omega}_{y1} + (I_{y2} + I_z) \dot{\omega}_{z2} \cos \beta - (I_{z2} + I_z) \omega_{z2} \dot{\beta} \sin \beta -$$

$$- I_{z2} \dot{\omega}_{z2} \sin \beta - I_{z2} \omega_{z2} \dot{\beta} \cos \beta - \dot{H} \sin \beta - H \dot{\beta} \cos \beta + (I_{x1} - I_{z1}) \omega_{x1} \omega_{z1} +$$

$$+ (I_{y2} + I_z) \omega_{z2} \omega_{y1} - (I_{y2} + I_z) \omega_{z2} \omega_{x1} \sin \beta - I_{z2} \omega_{z2} \omega_{x1} \cos \beta - H \omega_{x1} \cos \beta = M_{x1}.$$

$$I_{y1} \dot{\omega}_{y1} + (I_{x1} - I_{z1}) \omega_{x1} \dot{\omega}_{x1} + (I_{y2} + I_z) \dot{\omega}_{z2} \cos \beta - (I_{z2} + I_z) \omega_{z2} (\dot{\beta} + \omega_{x1}) \sin \beta - \dot{H} \sin \beta -$$

$$- I_{z2} \dot{\omega}_{z2} \sin \beta - I_{z2} \omega_{z2} (\dot{\beta} + \omega_{x1}) \cos \beta - H (\dot{\beta} + \omega_{x1}) \cos \beta + (I_{x2} + I_z) \omega_{z2} \omega_{x1} = M_{x1}.$$

$$\mathbf{A}_2 : \quad \omega_{z1} = \omega_{y2} \sin \beta + \omega_{z2} \cos \beta \quad \dot{\beta} + \omega_{x1} = \omega_{x2}$$

$$I_{y1} \dot{\omega}_{y1} + (I_{x1} - I_{z1}) \omega_{x1} \omega_{z1} + (I_{y2} + I_z) \dot{\omega}_2 \cos \beta - (I_{x2} + I_z) \omega_2 \omega_2 \sin \beta - \dot{H} \sin \beta - \\ - I_{z2} \dot{\omega}_{z2} \sin \beta - I_{z2} \omega_{z2} \omega_{x2} \cos \beta - H \omega_{x2} \cos \beta + (I_{x2} + I_z) \omega_2 (\omega_2 \sin \beta + \omega_2 \cos \beta) = M_{y1}.$$

$$I_{y1} \dot{\omega}_{y1} + (I_{x1} - I_{z1}) \omega_{x1} \omega_{z1} + (I_{y2} + I_z) \dot{\omega}_2 \cos \beta - (I_{x2} + I_z) \omega_2 \omega_2 \sin \beta - \\ - I_{z2} \dot{\omega}_{z2} \sin \beta - I_{z2} \omega_{z2} \omega_{x2} \cos \beta - H \omega_{x2} \cos \beta - \dot{H} \sin \beta + \\ + (I_{x2} + I_z) \omega_2 \omega_2 \sin \beta + (I_{z2} + I_x) \omega_2 \omega_2 \cos \beta$$

$$I_{y1} \dot{\omega}_{y1} + (I_{x1} - I_{z1}) \omega_{x1} \omega_{z1} + \\ + \left[(I_{y2} + I_z) \dot{\omega}_2 + (I_{z2} + I_x - I_{x2}) \omega_2 \omega_2 - H \omega_2 \right] \cos \beta - \\ - \left[I_{z2} \dot{\omega}_{z2} + (I_{y2} - I_{x2}) \omega_{x2} \omega_{y2} + \dot{H} \right] \sin \beta = M_{y1}.$$

$$I_{y1} \dot{\omega}_{y1} + (I_{x1} - I_{z1}) \omega_{x1} \omega_{z1} + \left[(I_{y2} + I_x) \dot{\omega}_{z2} + (I_{z2} + I_x - I_{y2}) \omega_{z2} \omega_{y2} - H \omega_{z2} \right] \cos \beta - \left[I_{z2} \dot{\omega}_{z2} + (I_{y2} - I_{x2}) \omega_{x2} \omega_{y2} + \dot{H} \right] \sin \beta = M_{y1};$$

$$(I_{y2} + I_x) \dot{\omega}_{z2} + (I_{y2} - I_{z2} - I_y) \omega_{z2} \omega_{y2} + H \omega_{z2} = M_{z2}$$

$$\dot{H} = M_{z3}$$