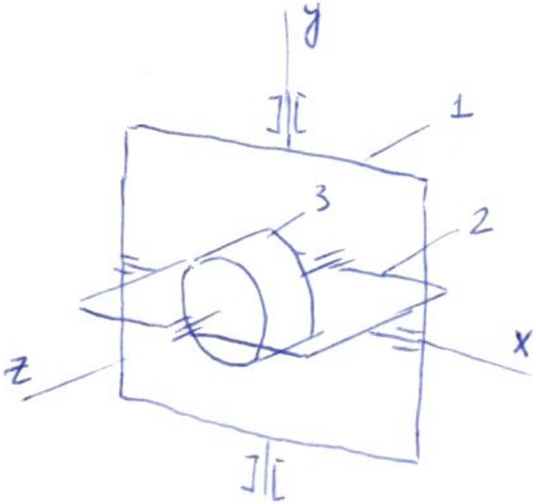
Лекція 6

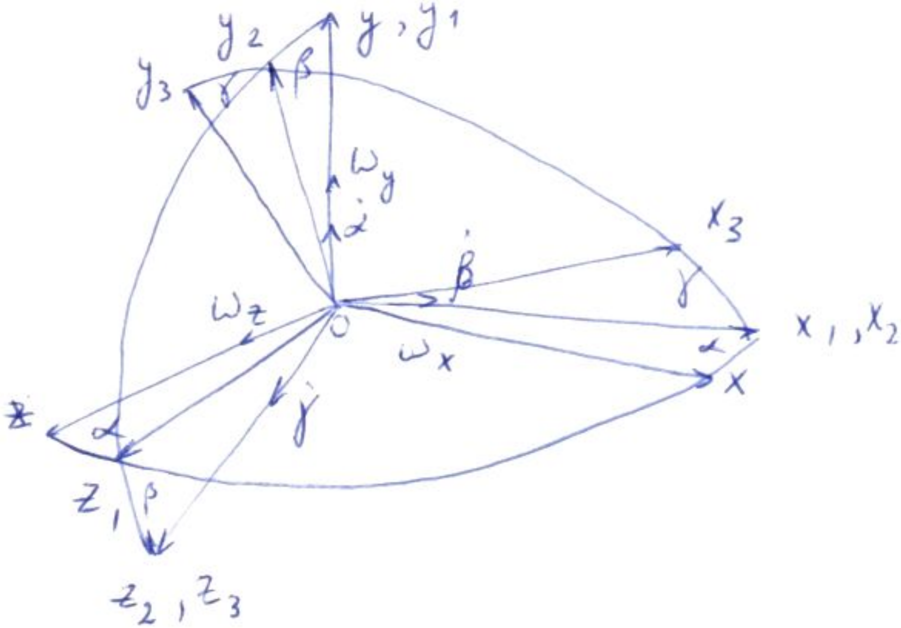
Рівняння руху гіроскопу в кардановому підвісі



$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j}-\frac{\partial T}{\partial q_j}=Q_j$$

$$Oxyz$$

$$Ox_iy_iz_i \qquad i=1 \quad i=2 \qquad i=3$$



$$\alpha, \beta, \gamma$$

$$\omega_x, \omega_y, \omega_z$$

$\mathbf{A}_1$	x	y	z
x_1	$\cos\alpha$	0	$-\sin\alpha$
y_1	0	1	0
z_1	$\sin\alpha$	0	$\cos\alpha$

$$\omega_{x_1} = \omega_x \cos\alpha - \omega_z \sin\alpha,$$

$$\omega_{y_1} = \dot{\alpha} + \omega_y,$$

$$\omega_{z_1} = \omega_x \sin\alpha + \omega_z \cos\alpha$$

$\mathbf{A}_2$	x_1	y_1	z_1
x_2	1	0	0
y_2	0	$\cos\beta$	$\sin\beta$
z_2	0	$-\sin\beta$	$\cos\beta$

$$\omega_{x2} = \dot{\beta} + \omega_{x1},$$

$$\omega_{y2} = \omega_{y1}\cos\beta + \omega_{z1}\sin\beta$$

$$\omega_{z2} = -\omega_{y1}\sin\beta + \omega_{z1}\cos\beta$$

$\mathbf{A}_3$	x_2	y_2	z_2
x_3	$\cos\gamma$	$\sin\gamma$	0
y_3	$-\sin\gamma$	$\cos\gamma$	0
z_3	0	0	1

$$\omega_{x3} = \omega_{x2}\cos\gamma + \omega_{y2}\sin\gamma,$$

$$\omega_{y3} = -\omega_{x2}\sin\gamma + \omega_{y2}\cos\gamma$$

$$\omega_{z3} = \dot{\gamma} + \omega_{z2}$$

$$T = \frac{1}{2}\sum_{i=1}^{3}\left(I_{xi}\omega_{xi}^2 + I_{yi}\omega_{yi}^2 + I_{zi}\omega_{zi}^2\right).$$

$$I_{x3} = I_{y3} = I \qquad\qquad I_{z3} = I$$

$$T = \frac{1}{2}I_{x1}\omega_{x1}^2 + \frac{1}{2}I_{y1}\omega_{y1}^2 + \frac{1}{2}I_{z1}\omega_{z1}^2 + \frac{1}{2}I_{x2}\omega_{x2}^2 + \frac{1}{2}I_{y2}\omega_{y2}^2 + \frac{1}{2}I_{z2}\omega_{z2}^2 +$$

$$+\frac{1}{2}I_{x3}\omega_{x3}^2 + \frac{1}{2}I_{y3}\omega_{y3}^2 + \frac{1}{2}I_{z3}\omega_{z3}^2$$

$$T = \frac{1}{2}I_{x1}\omega_{x1}^2 + \frac{1}{2}I_{y1}\omega_{y1}^2 + \frac{1}{2}I_{z1}\omega_{z1}^2 + \frac{1}{2}I_{x2}\omega_{x2}^2 + \frac{1}{2}I_{y2}\omega_{y2}^2 + \frac{1}{2}I_{z2}\omega_{z2}^2 +$$

$$+\frac{1}{2}I_{э}\left(\omega_{x3}^2 + \omega_{y3}^2\right) + \frac{1}{2}I\omega_{z3}^2.$$

$$\omega_{x3}^2 + \omega_{y3}^2$$

$$\omega_{x3}^2 + \omega_{y3}^2 = \left(\omega_{x2}\cos\gamma + \omega_{y2}\sin\gamma\right)^2 + \left(-\omega_{x2}\sin\gamma + \omega_{y2}\cos\gamma\right)^2 = \omega_{x2}^2 + \omega_{y2}^2.$$

$$T = \frac{1}{2}I_{x1}\omega_{x1}^2 + \frac{1}{2}I_{y1}\omega_{y1}^2 + \frac{1}{2}I_{z1}\omega_{z1}^2 + \frac{1}{2}I_{x2}\omega_{x2}^2 + \frac{1}{2}I_{y2}\omega_{y2}^2 + \frac{1}{2}I_{z2}\omega_{z2}^2 +$$
$$+\frac{1}{2}I_э\left(\omega_{x2}^2 + \omega_{y2}^2\right) + \frac{1}{2}I\omega_{z3}^2$$

$$T = \frac{1}{2}I_{x1}\omega_{x1}^2 + \frac{1}{2}I_{y1}\omega_{y1}^2 + \frac{1}{2}I_{z1}\omega_{z1}^2 +$$
$$+\frac{1}{2}\left(I_{x2} + I_э\right)\omega_{x2}^2 + \frac{1}{2}\left(I_{y2} + I_э\right)\omega_{y2}^2 + \frac{1}{2}I_{z2}\omega_{z2}^2 + \frac{1}{2}I\omega_{z3}^2$$

$$\alpha, \beta, \gamma$$

1)
ротор

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\gamma}} - \frac{\partial T}{\partial \gamma} = M_{z3}$$

$$\frac{\partial T}{\partial \gamma} = 0;$$

$$\frac{\partial T}{\partial \dot{\gamma}} = \frac{1}{2} I \cdot 2\omega_{z3} \frac{\partial}{\partial \dot{\gamma}} \omega_{z3} = I\omega_{z3} \quad = H$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\gamma}} = \dot{H}$$

$$\boxed{\dot{H} = M_{z3}}$$

2) внутрішня рамка

$$\frac{d}{dt}\frac{\partial T}{\partial \dot\beta} - \frac{\partial T}{\partial \beta} = M_{x2}$$

$$\frac{\partial T}{\partial \beta} = \left(I_{y2} + I_y\right)\omega_{y2}\frac{\partial \omega_{y2}}{\partial \beta} + I_{z2}\omega_{z2}\frac{\partial \omega_{z2}}{\partial \beta} + I\omega_{z3}\frac{\partial \omega_{z3}}{\partial \beta}$$

$$\frac{\partial \omega_{y2}}{\partial \beta} = -\omega_{y1}\sin\beta + \omega_{z1}\cos\beta = \omega_{z2}$$

$$\frac{\partial \omega_{z2}}{\partial \beta} = -\omega_{y1}\cos\beta - \omega_{z1}\sin\beta = -\omega_{y2}$$

$$\frac{\partial \omega_{z3}}{\partial \beta} = \frac{\partial \omega_{z2}}{\partial \beta} = -\omega_{y2}$$

$$\frac{\partial T}{\partial \beta} = \left(I_{y2} + I_y\right)\omega_{y2}\omega_{z2} - I_{z2}\omega_{y2}\omega_{z2} - H\omega_{y2}$$

$$\frac{\partial T}{\partial \dot\beta} = \left(I_{x2} + I_x\right)\omega_{x2}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot\beta} = \left(I_{x2} + I_x\right)\dot\omega_{x2}$$

$$\boxed{\left(I_{x2} + I_x\right)\dot\omega_{x2} + \left(I_{y2} - I_{э2} - I_y\right)\omega_{y2}\omega_{z2} + H\omega_{y2} = M_{x2}}$$

3) зовнішня рамка

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = M_{y1}$$

$$\frac{\partial T}{\partial \alpha} = I_{x1}\omega_{x1}\frac{\partial \omega_{x1}}{\partial \alpha} + I_{z1}\omega_{z1}\frac{\partial \omega_{z1}}{\partial \alpha} + \left(I_{x2} + I\ \right)\omega_{\ 2}\frac{\partial \omega_{x2}}{\partial \alpha} + \left(I_{\ 2} + I\ \right)\omega_{\ 2}\frac{\partial \omega_{y2}}{\partial \alpha} + I_{\ 2}\omega_{\ 2}\frac{\partial \omega_{z2}}{\partial \alpha} + I\omega_{\ 3}\frac{\partial \omega_{z3}}{\partial \alpha}.$$

$$\frac{\partial \omega_{x1}}{\partial \alpha} = -\omega_x \sin\alpha - \omega_z \cos\alpha \quad = -\omega_{z_1};$$

$$\frac{\partial \omega_{z1}}{\partial \alpha} = \omega_x \cos\alpha - \omega_z \sin\alpha \quad = \omega_{x1};$$

$$\frac{\partial \omega_{x2}}{\partial \alpha} = \frac{\partial \omega_{x1}}{\partial \alpha} = -\omega_{z_1}$$

$$\frac{\partial \omega_{y2}}{\partial \alpha} = \frac{\partial \omega_{z1}}{\partial \alpha}\sin\beta = \omega_{x1}\sin\beta$$

$$\frac{\partial \omega_{z2}}{\partial \alpha} = \frac{\partial \omega_{z1}}{\partial \alpha}\cos\beta = \omega_{x1}\cos\beta$$

$$\frac{\partial \omega_{z3}}{\partial \alpha} = \frac{\partial \omega_{z2}}{\partial \alpha} = \omega_{x1}\cos\beta$$

$$\frac{\partial T}{\partial \alpha} = -I_{x1}\omega_{x1}\omega_{z1} + I_{z1}\omega_{x1}\omega_{z1} - \left(I_{y2} + I_x\right)\omega_{y2}\omega_{x1} + \left(I_{z2} + I_x\right)\omega_{z2}\omega_{x1}\sin\beta + I_{z2}\omega_{y2}\omega_{x1}\cos\beta + H\omega_{x1}\cos\beta$$

$$= -\left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} - \left(I_{y2} + I_y\right)\omega_{y2}\omega_{x1} + \left(I_{z2} + I_x\right)\omega_{z2}\omega_{x1}\sin\beta + I_{z2}\omega_{y2}\omega_{x1}\cos\beta + H\omega_{x1}\cos\beta.$$

$$\frac{\partial T}{\partial \dot\alpha} = I_{y1}\omega_{y1}\frac{\partial \omega_{y1}}{\partial \dot\alpha} + \left(I_{y2} + I_z\right)\omega_{y2}\frac{\partial \omega_{y2}}{\partial \dot\alpha} + I_{z2}\omega_{z2}\frac{\partial \omega_{z2}}{\partial \dot\alpha} + I\omega_{z3}\frac{\partial \omega_{z3}}{\partial \dot\alpha}.$$

$$\frac{\partial \omega_{y2}}{\partial \dot\alpha} = \frac{\partial \omega_{y1}}{\partial \dot\alpha}\cos\beta = \cos\beta$$

$$\frac{\partial \omega_{z2}}{\partial \dot\alpha} = -\frac{\partial \omega_{y1}}{\partial \dot\alpha}\sin\beta = -\sin\beta$$

$$\frac{\partial \omega_{z3}}{\partial \dot\alpha} = \frac{\partial \omega_{z2}}{\partial \dot\alpha} = -\sin\beta$$

$$\frac{\partial T}{\partial \dot{\alpha}} = I_{y1}\omega_{y1} + \left(I_{y2} + I_{z}\right)\omega_{z2}\cos\beta - I_{\,2}\omega_{\,2}\sin\beta - H\sin\beta.$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} = I_{y1}\dot{\omega}_{y1} + \left(I_{y2} + I_{y}\right)\dot{\omega}_{\,2}\cos\beta - \left(I_{\,2} + I_{y}\right)\omega_{\,2}\dot{\beta}\sin\beta -$$
$$-I_{z2}\dot{\omega}_{z2}\sin\beta - I_{z2}\omega_{z2}\dot{\beta}\cos\beta - \dot{H}\sin\beta - H\dot{\beta}\cos\beta.$$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{y2} + I_{y}\right)\dot{\omega}_{\,2}\cos\beta - \left(I_{\,2} + I_{y}\right)\omega_{\,2}\dot{\beta}\sin\beta -$$
$$-I_{z2}\dot{\omega}_{z2}\sin\beta - I_{z2}\omega_{z2}\dot{\beta}\cos\beta - \dot{H}\sin\beta - H\dot{\beta}\cos\beta + \left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} +$$
$$+\left(I_{x2} + I_{x}\right)\omega_{z2}\omega_{y1} - \left(I_{э2} + I_{y}\right)\omega_{x2}\omega_{\,1}\sin\beta - I_{z2}\omega_{x2}\omega_{\,1}\cos\beta - H\omega_{x1}\cos\beta = M_{y1}.$$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} + \left(I_{y2} + I_{y}\right)\dot{\omega}_{\,2}\cos\beta - \left(I_{\,2} + I_{y}\right)\omega_{\,2}\left(\dot{\beta} + \omega_{\,1}\right)\sin\beta - \dot{H}\sin\beta -$$
$$-I_{z2}\dot{\omega}_{z2}\sin\beta - I_{z2}\omega_{z2}\left(\dot{\beta} + \omega_{x1}\right)\cos\beta - H\left(\dot{\beta} + \omega_{x1}\right)\cos\beta + \left(I_{x2} + I_{x}\right)\omega_{\,2}\omega_{\,1} = M_{\,1}.$$

$\mathbf{A}_2$: $\omega_{z_1} = \omega_{y2}\sin\beta + \omega_{z2}\cos\beta$ $\qquad \dot{\beta} + \omega_{x1} = \omega_{x2}$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} + \left(I_{y2} + I_{y}\right)\dot{\omega}_{y2}\cos\beta - \left(I_{\ 2} + I\ \right)\omega_{\ 2}\omega_{\ 2}\sin\beta - \dot{H}\sin\beta -$$

$$-I_{z2}\dot{\omega}_{z2}\sin\beta - I_{z2}\omega_{z2}\omega_{x2}\cos\beta - H\omega_{x2}\cos\beta + \left(I_{x2} + I\ \right)\omega_{\ 2}\left(\omega_{\ 2}\sin\beta + \omega_{\ 2}\cos\beta\right) = M_{\ 1}.$$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} + \left(I_{y2} + I_{y}\right)\dot{\omega}_{y2}\cos\beta - \left(I_{\ 2} + I\ \right)\omega_{\ 2}\omega_{\ 2}\sin\beta -$$

$$-I_{z2}\dot{\omega}_{z2}\sin\beta - I_{z2}\omega_{z2}\omega_{x2}\cos\beta - H\omega_{x2}\cos\beta - \dot{H}\sin\beta +$$

$$+\left(I_{x2} + I_{x}\right)\omega_{y2}\omega_{x2}\sin\beta + \left(I_{э2} + I_{x}\right)\omega_{z2}\omega_{x2}\cos\beta$$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{x1} - I_{z1}\right)\omega_{x1}\omega_{z1} +$$

$$+\left[\left(I_{y2} + I_{y}\right)\dot{\omega}_{y2} + \left(I_{э2} + I_{z} - I_{x2}\right)\omega_{z2}\omega_{x2} - H\omega_{x2}\right]\cos\beta -$$

$$-\left[I_{z2}\dot{\omega}_{z2} + \left(I_{y2} - I_{x2}\right)\omega_{x2}\omega_{y2} + \dot{H}\right]\sin\beta = M_{y1}.$$

$$I_{y1}\dot{\omega}_{y1} + \left(I_{x1} - I_{z1э}\right)\omega_{x1}\omega_{z1} + \left[\left(I_{zy2} + I_{x}\right)\dot{\omega}_{2} + \left(I_{2} + I - I_{2}\right)\omega_{2}\omega_{2} - H\omega_{2}\right]\cos\beta -$$

$$-\left[I_{z2}\dot{\omega}_{z2} + \left(I_{y2} - I_{x2}\right)\omega_{x2}\omega_{y2} + \dot{H}\right]\sin\beta = M_{y1};$$

$$\left(I_{x2} + I_{x}\right)\dot{\omega}_{z2} + \left(I_{y2} - I_{э2} - I_{y}\right)\omega_{z2}\omega_{2} + H\omega_{2x} = M_{2}$$

$$\dot{H} = M_{z3}$$