

## Прохождение случайной волны через отверстие в экране. Теорема Ван-Циттерта-Цернике

$$v(\underline{\rho}) \quad u(\underline{r}) \quad \underline{r} = (\underline{\rho}, z) \quad v(\underline{\rho}) = u(\underline{r})_{z=0} = u(\underline{\rho}) \quad l_v$$

$$u_\varepsilon(\underline{\rho}) = \begin{cases} v(\underline{\rho}), & \underline{\rho} \in S, \\ 0, & \underline{\rho} \notin S. \end{cases} \quad M(\underline{\rho}) \equiv E[S] \quad u_\varepsilon(\underline{\rho}) = M(\underline{\rho})v(\underline{\rho})$$

$$u(\underline{r}) = \frac{ke^{ikz}}{2\pi iz} \int_{-\infty}^{\infty} M(\underline{\rho}') v(\underline{\rho}') \exp\left[\frac{ik}{2z} (\underline{r} - \underline{\rho}')^2\right] d\underline{\rho}' \quad \text{К.а. поля за отверстием в приближении Френеля}$$

$$\langle u(\underline{r}) \rangle = 0$$

$$\Gamma_{\perp}(\underline{\rho}_1, \underline{\rho}_2, z) = \langle u(\underline{\rho}_1, z) u^*(\underline{\rho}_2, z) \rangle = \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\underline{\rho}') M(\underline{\rho}'') *$$

$$* \Gamma_v(\underline{\rho}' - \underline{\rho}'') \exp\left\{\frac{ik}{2z} [(\underline{\rho}_1 - \underline{\rho}')^2 - (\underline{\rho}_2 - \underline{\rho}'')^2]\right\} d\underline{\rho}' d\underline{\rho}'' \quad a \ll \lambda$$

$a \ll l_v$  Малое отверстие  $\Gamma_v(0) = \langle I_v \rangle$

$$\Gamma_{\perp}(\rho_1, \rho_2, z) = \langle I_v \rangle u_M(\rho_1, z) u_M^*(\rho_2, z)$$

$$u_M(\rho, z) = \frac{ke^{ikz}}{2\pi iz} \int_{-\infty}^{\infty} M(\rho') \exp\left[\frac{ik}{2z}(\rho - \rho')^2\right] d\rho'$$

$$K_{\perp}(\rho_1, \rho_2, z) = \frac{\Gamma_{\perp}(\rho_1, \rho_2, z)}{\sqrt{\Gamma_{\perp}(\rho_1, \rho_1, z) \Gamma_{\perp}(\rho_2, \rho_2, z)}} = \frac{u_M(\rho_1, z) u_M^*(\rho_2, z)}{|u_M(\rho_1, z)| |u_M(\rho_2, z)|} \quad |K_{\perp}| = 1$$

$\sqrt{\lambda L}$

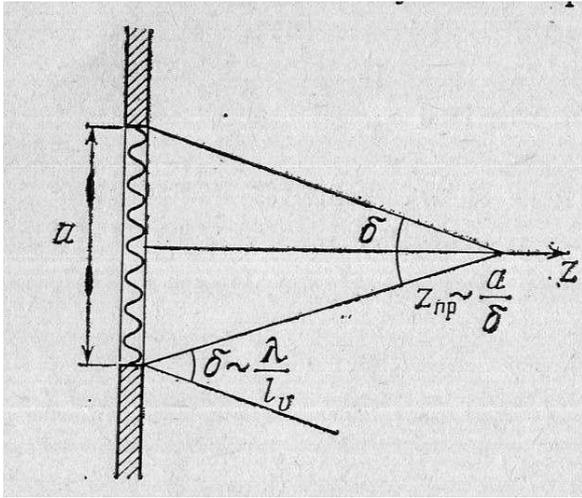
$a \ll l_v$  Большое отверстие

Длина дифракции

$$D = \frac{ab}{\lambda L} \quad a = b = l_v, \quad D = 1, \quad L = L \quad \frac{l_v^2}{\lambda L_{\delta}} = 1$$

$$L_{\delta} = \frac{l_v^2}{\lambda} \ll kl_v^2$$

$$\sin \delta = \chi/k \quad \delta \ll \chi_v/k$$



$$v(\rho) \quad F_v(\chi) \quad \chi_v \approx 1/l_v \quad \delta \approx \chi_v/k \approx \lambda/l_v$$

$$z_{np} \approx \frac{a}{\delta} \approx \frac{al_v}{\lambda} \approx kal_v$$

$$\frac{z_{np}}{kl_v^2} \approx \frac{a}{l_v} \approx 1 \quad \frac{z_{np}}{ka^2} \approx \frac{l_v}{a} \approx 1$$

$z_{np}$  -дальняя зона по отношению к неоднородности и ближняя по отношению к отверстию

Поле нормализуется еще в ближней зоне апертуры

$$z \approx z_{np} \quad \xi = \rho' - \rho'' \quad \eta = (\rho' + \rho'')/2 \quad \rho_+ \equiv (\rho_1 + \rho_2)/2 \quad \rho \equiv \rho_1 - \rho_2$$

$$\Gamma_{\perp}(\rho, \rho_+, z) = \left(\frac{k}{2\pi z}\right)^2 \exp\left(\frac{ik\rho\rho_+}{z}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\eta + \frac{\xi}{2}) M(\eta - \frac{\xi}{2}) \Gamma_v(\xi) \exp\left\{\frac{ik}{2z}(\xi\eta - \eta\rho - \xi\rho_+)\right\} d\xi d\eta$$

$$\xi \leq l_v \approx a, \quad \eta \leq a, \Rightarrow M(\eta + \frac{\xi}{2}) M(\eta - \frac{\xi}{2}) \approx M^2(\eta) = M(\eta)$$

$$\frac{k\xi\eta}{z} \leq \frac{kal_v}{z} \approx \frac{z_{np}}{z} \approx 1$$

$$z \approx z_{np}$$

$$z \approx L_{\delta} = ka^2$$

$$\Gamma_{\perp}(\rho, \rho_+, z) = \left(\frac{k}{2\pi z}\right)^2 \exp\left(\frac{ik\rho\rho_+}{z}\right) \int_{-\infty}^{\infty} M(\eta) \exp\left\{\frac{-ik\eta\rho}{z}\right\} d\eta \int_{-\infty}^{\infty} \Gamma_v(\xi) \exp\left\{\frac{-ik\xi\rho_+}{z}\right\} d\xi$$

$$\exp\left(\frac{ik\rho\rho_+}{z}\right) = \exp\left(\frac{ik}{2z}(|\rho_1|^2 - |\rho_2|^2)\right) \approx \exp[ik(r_1 - r_2)]$$

$$a \sim \lambda/a \sim \Delta\rho \sim \lambda z/a \Rightarrow l_{\perp} \sim \lambda z/a \sim \lambda/\gamma \sim \gamma \sim a/z$$

$$\langle I_u(\rho_+, z) \rangle = \frac{k^2 S}{(2\pi z)^2} \int_{-\infty}^{\infty} \Gamma_v(\xi) \exp\left\{\frac{-ik\xi\rho_+}{z}\right\} d\xi$$

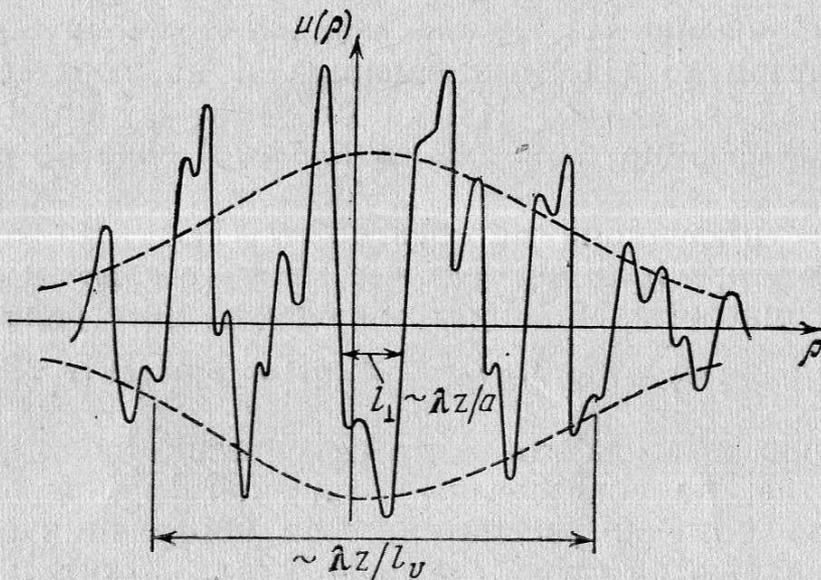
$$z = \text{const.} \quad \rho_+ \sim \lambda z/l_v \quad (\rho_+/z \sim \lambda/l_v \sim 1/kl_v) \quad \frac{\lambda z}{l_v} \sim l_{\perp} \sim \frac{\lambda z}{a}$$

$$\sqrt{\langle I_u(\rho) \rangle} \quad F_M(0) = 1$$

$$F_M(\chi) = \frac{1}{S} \int_{-\infty}^{\infty} M(\eta) \exp(-i\chi\eta) d\eta$$

$$K_{\perp}(\rho) = \exp\left(\frac{ik\rho\rho_+}{z}\right) F_M\left(\frac{k\rho}{z}\right)$$

$$M(\rho) = 1, \rho < a; \quad F_M(\chi) = 2J_1(\chi a)/\chi a$$



$$l_v \lambda \Rightarrow \int_{-\infty}^{\infty} \Gamma_v(\xi) \exp\left\{\frac{-ik\xi\rho_+}{z}\right\} d\xi \approx \langle I_v \rangle S_v$$

$$S_v = \frac{1}{\langle I_v \rangle} \int_{-\infty}^{\infty} \Gamma_v(\xi) d\xi = \int_{-\infty}^{\infty} K_v(\xi) d\xi \quad S_v \approx l_v^2$$

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$$\Gamma_{\perp}(\rho, \rho_+, z) = \frac{1}{z^2} \exp\left(\frac{ik\rho\rho_+}{z}\right) \int_{-\infty}^{\infty} J(\rho') \exp\left\{\frac{-ik\rho\rho'}{z}\right\} d\rho'$$

$$J(\rho') = \frac{k^2}{4\pi^2} \langle I_v \rangle S_v M(\rho')$$