Прохождение случайной волны через отверстие в экране. Теорема Ван-Циттерта-Цернике

$$v(\vec{\rho}) \qquad u(\vec{r}) \qquad \vec{r} = (\vec{\rho}, z) \qquad v(\vec{\rho}) = u(\vec{r})_{z=0} = u(\vec{\rho}) \qquad l_v$$

$$u_в(\vec{\rho}) = \begin{cases} v(\vec{\rho}), & \vec{\rho} \in S, \\ 0, & \vec{\rho} \notin S. \end{cases} \qquad M(\vec{\rho}) \equiv E[S] \qquad u_в(\vec{\rho}) = M(\vec{\rho}) v(\vec{\rho})$$

$$u(\vec{r}) = \frac{ke^{ikz}}{2\pi iz} \int_{-\infty}^{\infty} M(\vec{\rho}') v(\vec{\rho}') \exp[\frac{ik}{2z}(\vec{r} - \vec{\rho})^2] d\vec{\rho}'$$

К.а. поля за отверстием в приближении Френеля

$$< u(\vec{r}) >= 0$$

$$\Gamma_{\perp}(\vec{\rho}_1, \vec{\rho}_2, z) =< u(\vec{\rho}_1, z) u^*(\vec{\rho}_2, z) >= \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\vec{\rho}') M(\vec{\rho}'') *$$

$$* \Gamma_v(\vec{\rho}' - \vec{\rho}'') \exp\left\{\frac{ik}{2z}[(\vec{\rho}_1 - \vec{\rho}')^2 - (\vec{\rho}_2 - \vec{\rho}'')^2]^2\right\} d\vec{\rho}' d\vec{\rho}''$$

$$a \gg \lambda$$

$a \ll l_v$ Малое отверстие $\Gamma_v(0) = \langle I_v \rangle$

$$\Gamma_\perp(\vec{\rho}_1, \vec{\rho}_2, z) = \langle I_v \rangle u_M(\vec{\rho}_1, z) u_M^*(\vec{\rho}_2, z)$$

$$u_M(\vec{\rho}, z) = \frac{ke^{ikz}}{2\pi iz} \int\limits_{-\infty}^{\infty} M(\vec{\rho}\,') \exp[\frac{ik}{2z}(\vec{\rho} - \vec{\rho}\,')^2] d\vec{\rho}\,'$$

$$K_\perp(\vec{\rho}_1, \vec{\rho}_2, z) = \frac{\Gamma_\perp(\vec{\rho}_1, \vec{\rho}_2, z)}{\sqrt{\Gamma_\perp(\vec{\rho}_1, \vec{\rho}_1, z)\Gamma_\perp(\vec{\rho}_2, \vec{\rho}_2, z)}} = \frac{u_M(\vec{\rho}_1, z) u_M^*(\vec{\rho}_2, z)}{|u_M(\vec{\rho}_1, z)||u_M(\vec{\rho}_2, z)|}$$

$\boxed{|K_\perp| = 1}$

$\sqrt{\lambda L}$

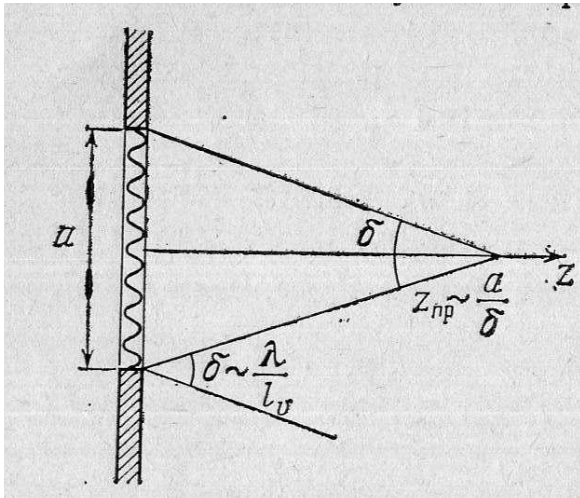
$a \gg l_v$ Большое отверстие

Длина дифракции

$$D = \frac{ab}{\lambda L} \quad a = b = l_\partial, \quad D = 1, \quad L = L \qquad \frac{l_v^2}{\lambda L_\partial} = 1$$

$$\boxed{L_\partial = \frac{l_v^2}{\lambda} \sim k l_v^2}$$

$$\sin\delta = \chi / k \qquad \delta \sim \chi_v / k$$



$v(\vec{\rho}) \qquad F_v(\chi) \qquad \chi_v \sim 1/l_v \quad \delta \sim \chi_v/k \sim \lambda/l_v$

$$z_{np} \sim \frac{a}{\delta} \sim \frac{al_v}{\lambda} \sim kal_v$$

$$\frac{z_{np}}{kl_v^2} \sim \frac{a}{l_v} \gg 1 \qquad \frac{z_{np}}{ka^2} \sim \frac{l_v}{a} \ll 1$$

z_{np} -дальняя зона по отношению к неоднородности и ближняя по отношению к отверстию

Поле нормализуется еще в ближней зоне апертуры

$$z \gg z_{np} \quad \vec{\xi} = \vec{\rho}' - \vec{\rho}'' \quad \vec{\eta} = (\vec{\rho}' + \vec{\rho}'')/2 \qquad \vec{\rho}_+ \equiv (\vec{\rho}_1 + \vec{\rho}_2)/2 \quad \vec{\rho} \equiv \vec{\rho}_1 - \vec{\rho}_2$$

$$\Gamma_\perp(\vec{\rho}, \vec{\rho}_+, z) = \left(\frac{k}{2\pi z}\right)^2 \exp\left(\frac{ik\vec{\rho}\vec{\rho}_+}{z}\right) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M(\vec{\eta} + \frac{\vec{\xi}}{2}) M(\vec{\eta} - \frac{\vec{\xi}}{2}) \Gamma_v(\vec{\xi}) \exp\left\{\frac{ik}{2z}(\vec{\xi}\vec{\eta} - \vec{\eta}\vec{\rho} - \vec{\xi}\vec{\rho}_+)\right\} d\vec{\xi}\, d\vec{\eta}$$

$$\xi \le l_v \ll a, \quad \eta \le a, \quad \Rightarrow M(\vec{\eta} + \frac{\vec{\xi}}{2}) M(\vec{\eta} - \frac{\vec{\xi}}{2}) \approx M^2(\vec{\eta}) = M(\vec{\eta})$$

$$\frac{k\vec{\xi}\vec{\eta}}{z} \le \frac{kal_v}{z} \sim \frac{z_{np}}{z} \ll 1$$

$$z \gg z_{np} \qquad z \ll L_д = ka^2$$

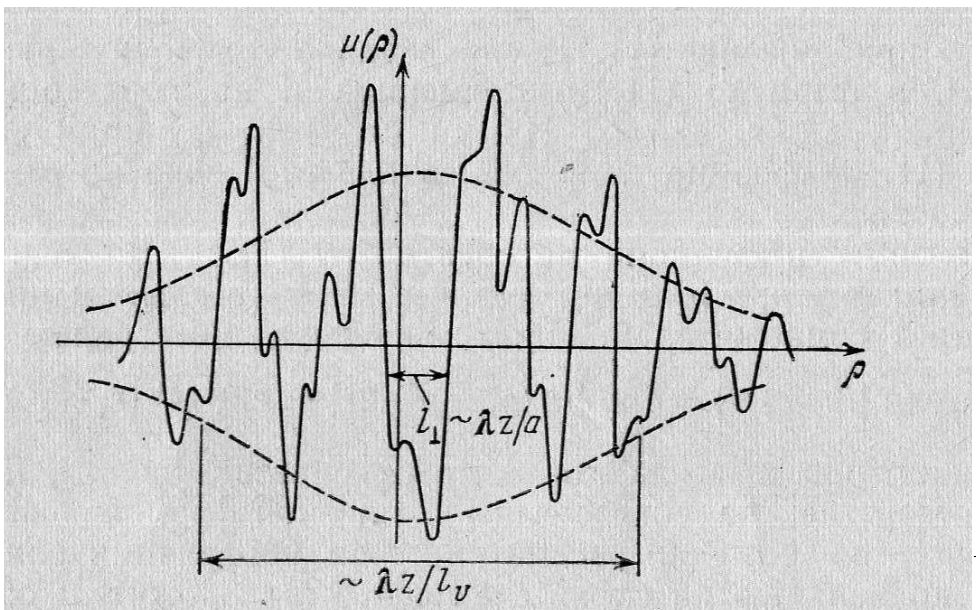
$$\Gamma_\perp(\vec{\rho}, \vec{\rho}_+, z) = \left(\frac{k}{2\pi z}\right)^2 \exp\left(\frac{ik\vec{\rho}\vec{\rho}_+}{z}\right) \int_{-\infty}^{\infty} M(\vec{\eta}) \exp\left\{\frac{-ik\vec{\eta}\vec{\rho}}{z}\right\} d\vec{\eta} \int_{-\infty}^{\infty} \Gamma_v(\vec{\xi}) \exp\left\{\frac{-ik\vec{\xi}\vec{\rho}_+}{z}\right\} d\vec{\xi}$$

$$\exp\left(\frac{ik\vec{\rho}\vec{\rho}_+}{z}\right) = \exp\left(\frac{ik}{2z}\left(|\vec{\rho}_1|^2 - |\vec{\rho}_2|^2\right)\right) \approx \exp[ik(\vec{r}_1 - \vec{r}_2)]$$

$$a \quad \lambda/a \quad \Delta\rho \sim \lambda z/a \quad \Rightarrow \quad l_\perp \sim \lambda z/a \sim \lambda/\gamma \quad \gamma \sim a/z$$

$$< I_u(\vec{\rho}_+, z) > = \frac{k^2 S}{(2\pi z)^2} \int_{-\infty}^{\infty} \Gamma_v(\vec{\xi}) \exp\left\{\frac{-ik\vec{\xi}\vec{\rho}_+}{z}\right\} d\vec{\xi}$$

$$z = \text{const.} \quad \rho_+ \sim \lambda z/l_v \quad (\rho_+/z \sim \lambda/l_v \sim 1/kl_v) \qquad \frac{\lambda z}{l_v} \gg l_\perp \sim \frac{\lambda z}{a}$$



$$\sqrt{< I_u(\vec{\rho}) >} \qquad F_M(0) = 1$$

$$F_M(\vec{\chi}) = \frac{1}{S} \int_{-\infty}^{\infty} M(\vec{\eta}) \exp(-i\vec{\chi}\vec{\eta}) d\vec{\eta}$$

$$K_\perp(\vec{\rho}) = \exp\left(\frac{ik\vec{\rho}\vec{\rho}_+}{z}\right) F_M\left(\frac{k\vec{\rho}}{z}\right)$$

$$M(\vec{\rho}) = 1, \; \rho < a; \; F_M(\chi) = 2J_1(\chi a)/\chi a$$

$$l_v \sim \lambda \;\Rightarrow\; \int\limits_{-\infty}^{\infty} \Gamma_v(\vec{\xi}) \exp\left\{ \frac{-ik\vec{\xi}\vec{\rho}_+}{z} \right\} d\vec{\xi} \approx < I_v > S_v$$

$$S_v = \frac{1}{< I_v >} \int\limits_{-\infty}^{\infty} \Gamma_v(\vec{\xi})\, d\vec{\xi} = \int\limits_{-\infty}^{\infty} K_v(\vec{\xi})\, d\vec{\xi} \qquad S_v \sim l_v^2$$

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$$\Gamma_\perp(\vec{\rho}, \vec{\rho}_+, z) = \frac{1}{z^2} \exp\left(\frac{ik\vec{\rho}\vec{\rho}_+}{z} \right) \int\limits_{-\infty}^{\infty} J(\vec{\rho}\,') \exp\left\{ \frac{-ik\vec{\rho}\vec{\rho}\,'}{z} \right\} d\vec{\rho}\,'$$

$$J(\vec{\rho}\,') = \frac{k^2}{4\pi^2} < I_v > S_v M(\vec{\rho}\,')$$