Решение тригонометрических уравнений

С1

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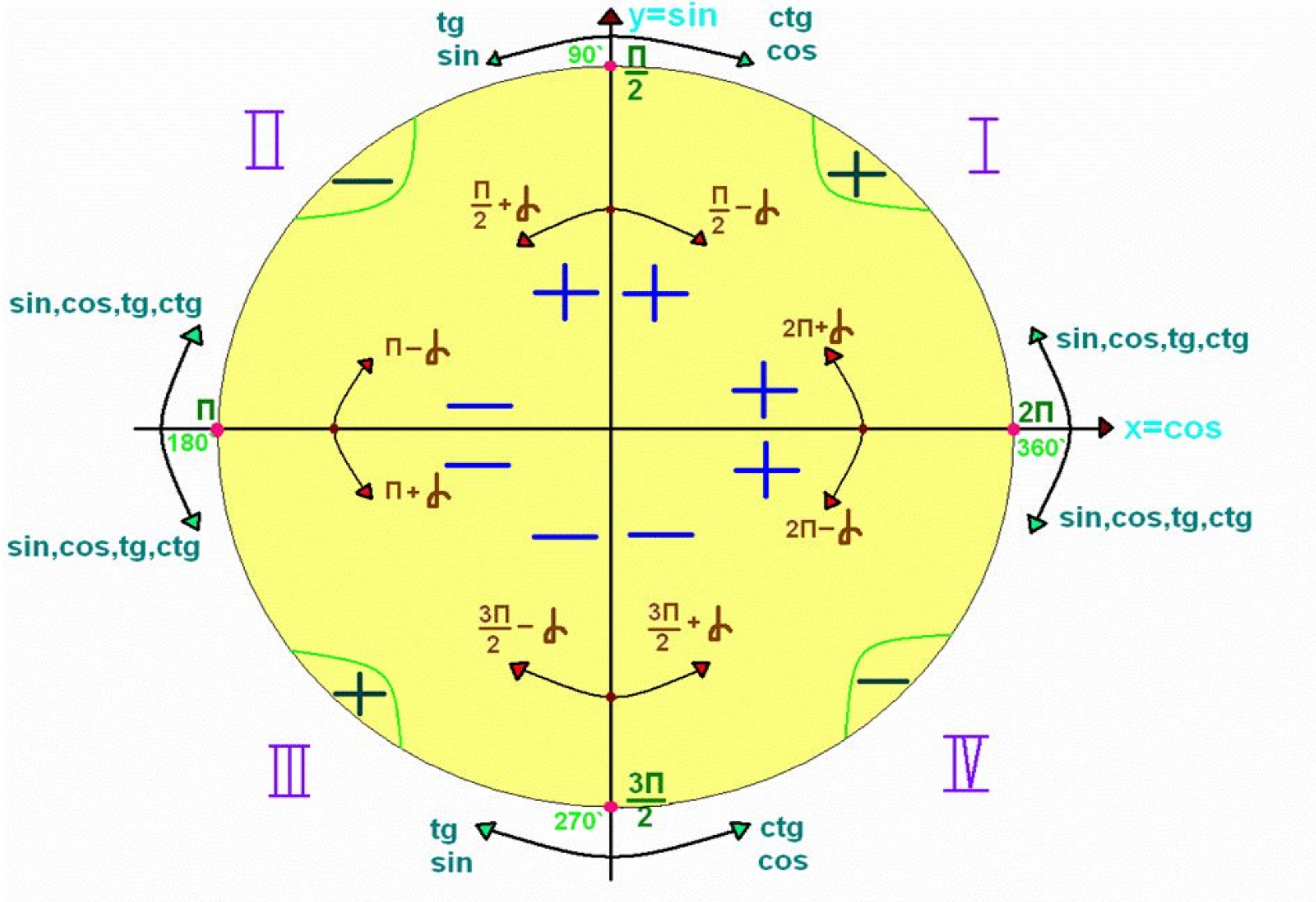
лицей №90

До экзамена осталось

- 160 дней

Повторим


y=sin
tg
sin
ctg
cos
90`
П
2
II
I
П/2 + α
П/2 − α
sin,cos,tg,ctg
sin,cos,tg,ctg
2П+α
П−α
П
180`
2П
360`
x=cos
П+α
2П−α
sin,cos,tg,ctg
sin,cos,tg,ctg
3П/2 − α
3П/2 + α
III
IV
3П
2
270`
tg
sin
ctg
cos

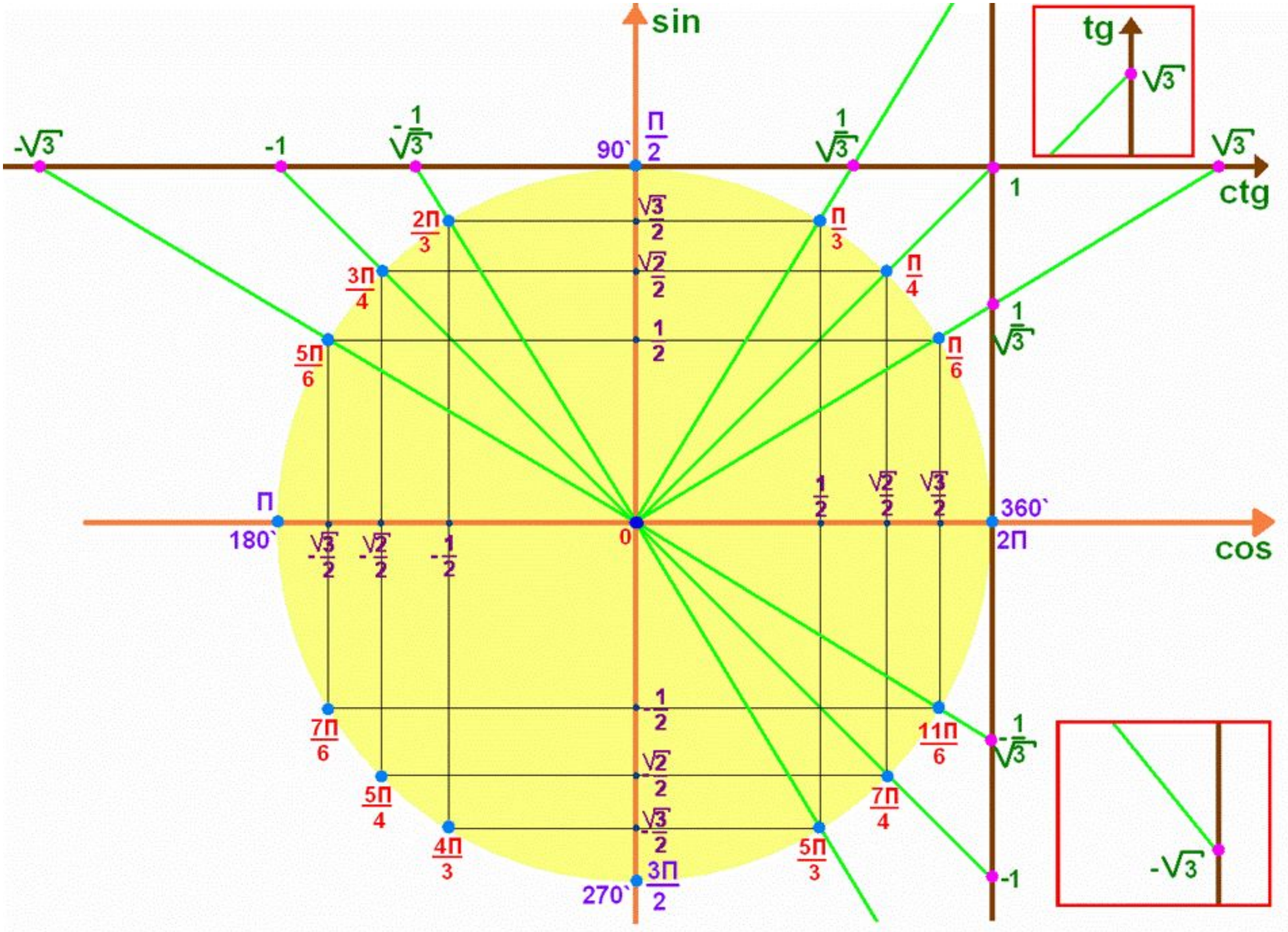
Повторим

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \ cos^2\alpha - sin^2\alpha = 2cos^2\alpha - 1 = 1 - 2sin^2\alpha$$

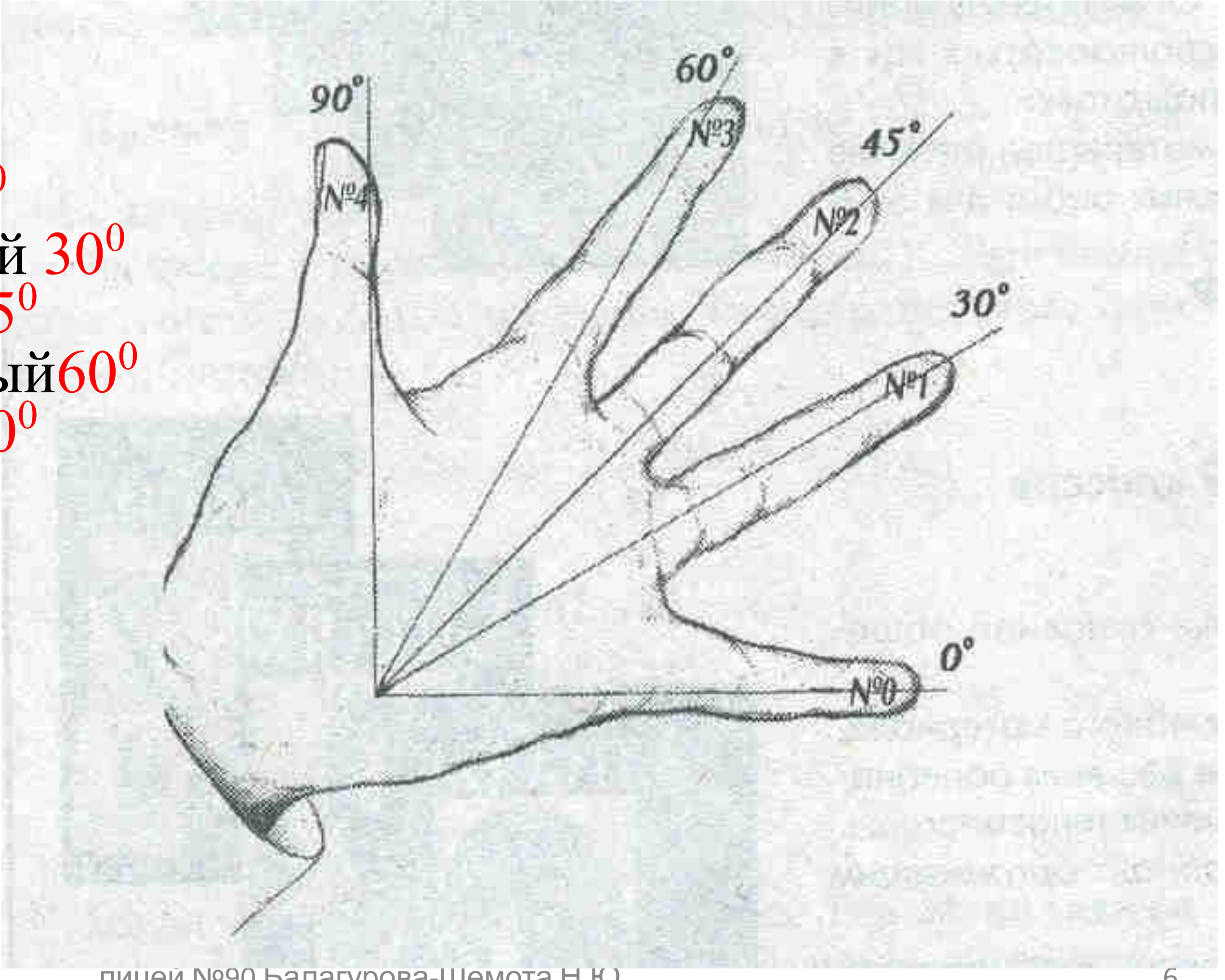
$$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$$

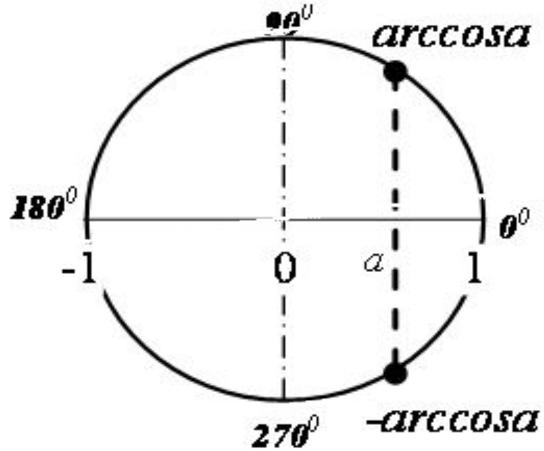
$$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$$


sin
tg
√3
ctg
cos
-√3
-1
-1/√3
90`
П/2
1/√3
1
√3/2
√2/2
1/2
2П/3
П/3
3П/4
П/4
5П/6
П/6
П
180`
0
360`
2П
-√3/2
-√2/2
-1/2
7П/6
11П/6
5П/4
7П/4
4П/3
5П/3
270`
3П/2

- $\sin\alpha = \frac{\sqrt{n}}{2}$

№0 Мизинец 0^0
№1 Безымянный 30^0
№2 Средний 45^0
№3 Указательный 60^0
№4 Большой 90^0



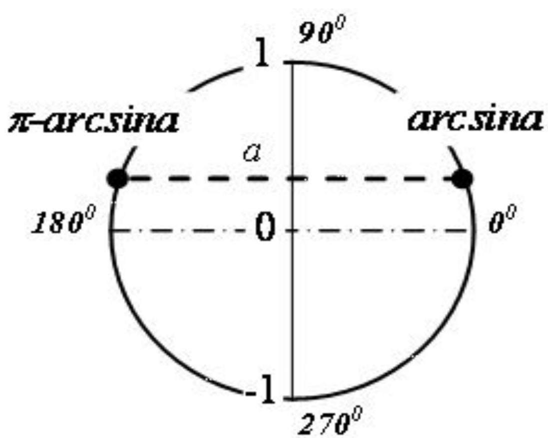


$$\cos x = a \Rightarrow x = \pm \arccos a + 2\pi n,\ n \in Z$$

$$\cos x = -1 \Rightarrow x = \pi + 2\pi n$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + \pi n$$

$$\cos x = 1 \Rightarrow x = 2\pi n$$



$$\sin x = a \Rightarrow x = (-1)^n \arcsin a + \pi n,\ n \in Z$$

$$\sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi n$$

$$\sin x = 0 \Rightarrow x = \pi n$$

$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi n$$

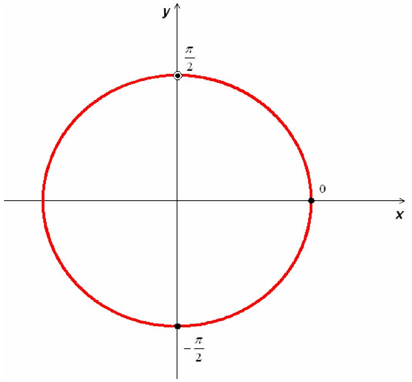
$$\cos\left(\frac{3\pi}{2}+2x\right)=\cos x$$

а) Решите уравнение.

б) Укажите корни уравнения, принадлежащие отрезку $\left[\frac{5\pi}{2};\,4\pi\right]$

$$\sin 2x=\cos x\,; \qquad 2\sin x\cos x-\cos x=0\,;$$

$$\cos x\cdot(2\sin x-1)=0\,;$$



$$\cos x=0 \quad \text{или} \quad 2\sin x-1=0\,,$$

$$x=\frac{\pi}{2}+\pi n\,,\;\; n\in Z\,, \qquad x=\frac{\pi}{6}+2\pi n\,,\;\; x=\frac{5\pi}{6}+2\pi n\,,\;\; n\in Z$$

Отбор корней

$$\frac{5\pi}{2} \le \frac{\pi}{2} + \pi n \le 4\pi;$$

$$2 \le n \le \frac{7}{2}, \quad n \in Z.$$

$$\boxed{x = \frac{5\pi}{2};\ x = \frac{7\pi}{2}.}$$

$$\frac{5\pi}{2} \le \frac{5\pi}{6} + 2\pi n \le 4\pi$$

$$\frac{5}{6} \le n \le \frac{19}{12}, \quad n \in Z$$

$$\boxed{x = \frac{17\pi}{6}}.$$

$$\frac{5\pi}{2} \le \frac{\pi}{6} + 2\pi n \le 4\pi$$

$$\frac{7}{6} \le n \le \frac{23}{12}, \quad n \in Z.$$

нет корней.

Ответ: а) $\frac{\pi}{2} + \pi n$, $\frac{\pi}{6} + 2\pi n$, $\frac{5\pi}{6} + 2\pi n$, $n \in Z$; б) $\frac{5\pi}{2}$, $\frac{17\pi}{6}$, $\frac{7\pi}{2}$.

Найти ошибку

$\cos\left(\frac{3\pi}{2}+2x\right)=\cos x$ $\left[\frac{5\pi}{2};4\pi\right]$

а) $\sin 2x=\cos x$

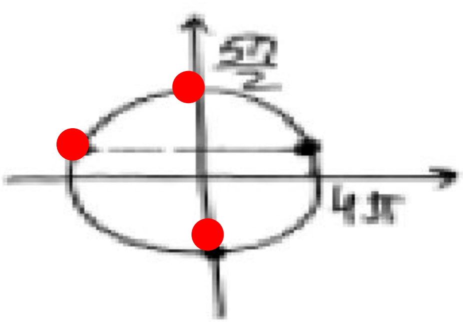
$2\sin x\cos x=\cos x$

$\cos x(2\sin x-1)=0$

$\cos x=0$ или $\sin x=\frac{1}{2}$

$x=\frac{\pi}{2}+\pi k$ $x=(-1)^n\cdot\frac{\pi}{6}+\pi n$

б) $\left[\frac{5\pi}{2};4\pi\right]$



$\left\{\frac{7\pi}{2};\cancel{\frac{5\pi}{6}};\frac{5\pi}{2}\right\}$

$\boxed{C1}$ $\cos\left(\frac{3\pi}{2}+2x\right)=\cos x$ $\left[\frac{5\pi}{2};4\pi\right]$

Решение:

а) $-\sin 2x=\cos x$

$-2\sin x\cos x=\cos x$

$\cos x+2\sin x\cos x=0$

$\cos x(1+2\sin x)=0$

$\cos x=0$ или $\sin x=-\frac{1}{2}$

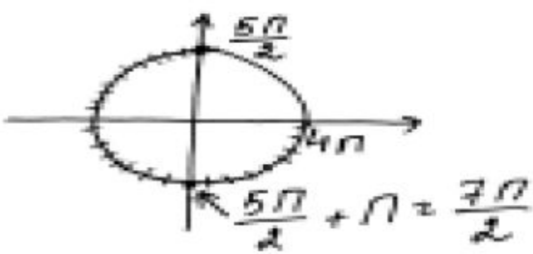
$x=\frac{\pi}{2}+\pi n$

$x=(-1)^k\arcsin\left(-\frac{1}{2}\right)+\pi k$

$x=(-1)^k\left(-\frac{\pi}{6}\right)+\pi k$

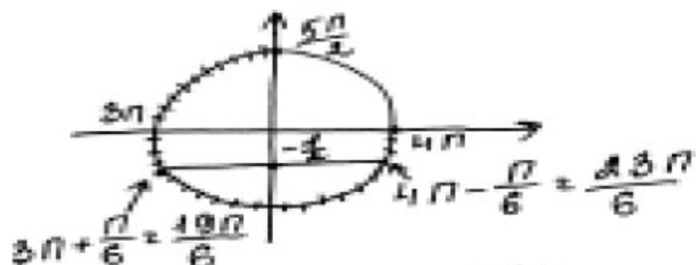
$x=(-1)^{k+1}\frac{\pi}{6}+\pi k$

б) $\cos x=0$



$x_1=\frac{5\pi}{2}$ $x_2=\frac{7\pi}{2}$

$\sin x=-\frac{1}{2}$



$x_3=\frac{19\pi}{6}$ $x_4=\frac{23\pi}{6}$

Ответ: а) $\frac{\pi}{2}+\pi n$, $(-1)^{k+1}\cdot\frac{\pi}{6}+\pi k$, где $n\in Z$, $k\in Z$

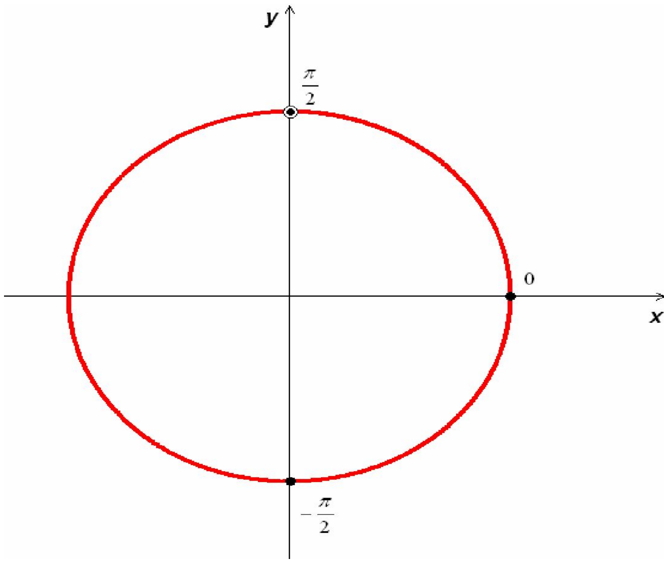
б) $\frac{5\pi}{2}$, $\frac{7\pi}{2}$, $\frac{19\pi}{6}$, $\frac{23\pi}{6}$

$$\sin 2x - \cos\left(\frac{3\pi}{2} - x\right) + 2\sqrt{2}\sin^2\frac{x}{2} = \frac{3\sqrt{2}}{2} \qquad \left[-\frac{3\pi}{2};\pi\right].$$

$$2\sin x\cos x + \sin x + \sqrt{2}(1-\cos x) - \frac{3\sqrt{2}}{2} = 0$$

$$\sin x(2\cos x + 1) - \frac{\sqrt{2}}{2}(2\cos x + 1) = 0$$

$$(2\cos x + 1)(\sin x - \frac{\sqrt{2}}{2}) = 0$$



$$\cos x = -\frac{1}{2} \qquad \sin x = \frac{\sqrt{2}}{2}$$

$$x = \pm\frac{2\pi}{3} + 2\pi n, n \in Z \qquad x = (-1)^k\frac{\pi}{4} + \pi k, k \in Z \qquad \left[\begin{array}{l} x = \frac{\pi}{4} + 2\pi m \\ x = \frac{3\pi}{4} + 2\pi m \end{array}\right., m \in Z$$

$$-\frac{3\pi}{2} \le \frac{2\pi}{3} + 2\pi n \le \pi$$

$$-\frac{3}{2} \le \frac{2}{3} + 2n \le 1$$

$$-\frac{13}{6} \le 2n \le \frac{1}{3}$$

$$-\frac{13}{12} \le n \le \frac{1}{6} \Rightarrow n = -1; 0$$

$$x_1 = -\frac{4\pi}{3}; x_2 = \frac{2\pi}{3}$$

$$-\frac{3\pi}{2} \le \frac{\pi}{4} + 2\pi m \le \pi$$

$$-\frac{3}{2} \le \frac{1}{4} + 2m \le 1$$

$$-\frac{7}{4} \le 2m \le \frac{3}{4}$$

$$-\frac{7}{8} \le m \le \frac{3}{8} \Rightarrow m = 0$$

$$-\frac{3\pi}{2} \le -\frac{2\pi}{3} + 2\pi n \le \pi$$

$$-\frac{3}{2} \le -\frac{2}{3} + 2n \le 1$$

$$-\frac{5}{6} \le 2n \le \frac{5}{3}$$

$$-\frac{5}{12} \le n \le \frac{5}{6} \Rightarrow n = 0$$

$$x_3 = -\frac{2\pi}{3}$$

$$x_4 = \frac{\pi}{4}$$

$$-\frac{3\pi}{2} \le \frac{3\pi}{4} + 2\pi m \le \pi$$

$$-\frac{3}{2} \le \frac{3}{4} + 2m \le 1$$

$$-\frac{9}{4} \le 2m \le \frac{1}{4}$$

$$-\frac{9}{8} \le m \le \frac{1}{8} \Rightarrow m = -1; 0$$

$$x_5 = -\frac{5\pi}{4}; x_6 = \frac{3\pi}{4}$$

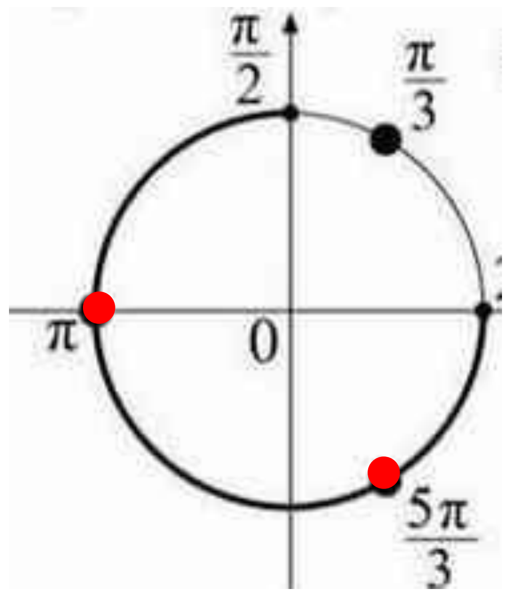
Ответ:

а) $x = \pm\frac{2\pi}{3} + 2\pi n, n \in Z$; $x = (-1)^k \frac{\pi}{4} + \pi k, k \in Z$

б) $x \in \{-\frac{4\pi}{3}; -\frac{5\pi}{4}; -\frac{2\pi}{3}; \frac{\pi}{4}; \frac{2\pi}{3}; \frac{3\pi}{4}\}.$

а) Решите уравнение $\sin^2\frac{x}{2}-\cos^2\frac{x}{2}=\cos 2x$

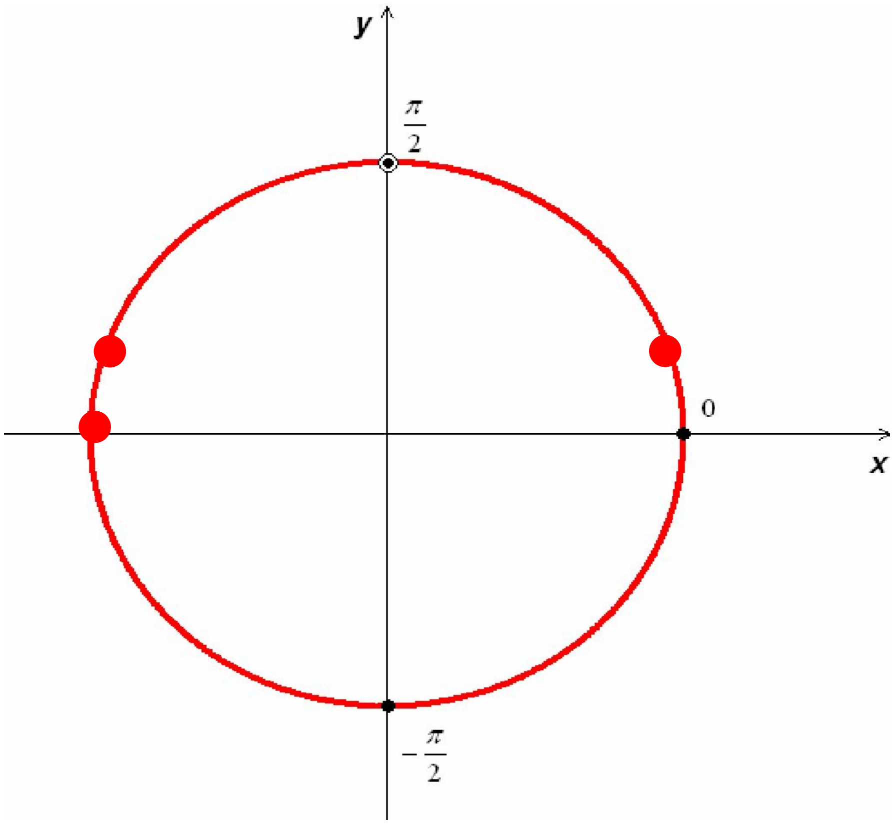
б) Найдите корни этого уравнения, принадлежащие промежутку $\left[\frac{\pi}{2};2\pi\right]$



Решите уравнение $\cos x + 1 = 2\sin 2x + 4\sin x$

Укажите корни, принадлежащие отрезку $\left[-\frac{7\pi}{2},\ -\frac{3\pi}{2}\right]$

$\boxed{\pi + 2n\pi \quad , \quad (-1)^k \arcsin\frac{1}{4} + k\pi}$



$x_1 = -3\pi$

$x_2 = -3\pi - \arcsin\frac{1}{4}$

$x_3 = -2\pi + \arcsin\frac{1}{4}$

$$\sqrt{3}\cdot\sin x - tgx + tgx\cdot\sin x = \sqrt{3}$$

$$(\sin x - 1)\left(\sqrt{3} + tgx\right) = 0$$

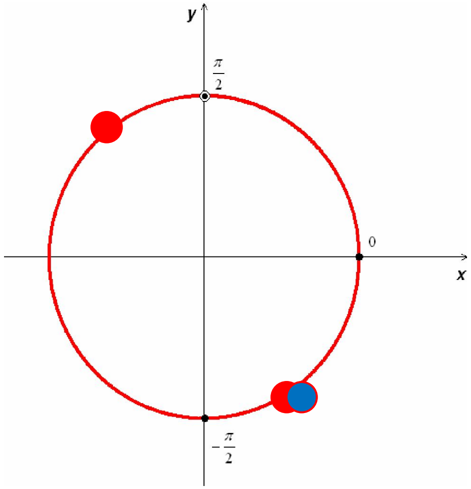
$$\left[-\frac{\pi}{2}, \frac{7\pi}{4}\right]$$

$$\begin{cases} \cos x \neq 0 \\ \left[\begin{array}{l} \sin x = 1 \\ tgx = -\sqrt{3} \end{array}\right. \end{cases}$$

$$tgx = -\sqrt{3}$$

$$x = -\frac{\pi}{3} + k\pi \qquad k \in Z$$

Ответ:



$$-\frac{\pi}{3} \qquad \frac{2\pi}{3} \qquad \frac{5\pi}{3}$$

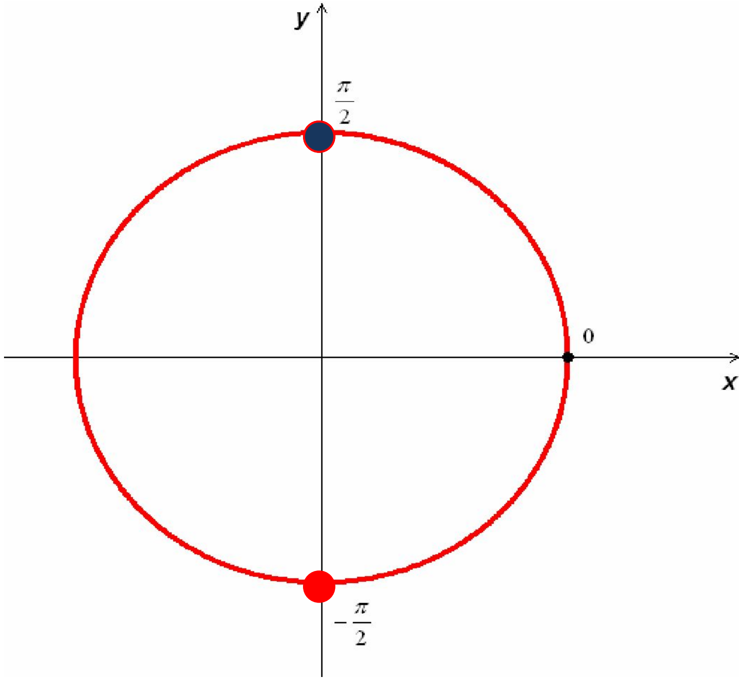
Решите уравнение $\frac{1}{\cos x-1}+\frac{1}{\cos x+1}=2\operatorname{ctg}^2 x$

Укажите корни, принадлежащие промежутку $\left(-\frac{\pi}{2},\ 8\right]$

$\boxed{\frac{\pi}{2}+k\pi}.$

$\boxed{x_1=\frac{\pi}{2}}$

$\boxed{x_2=\frac{3\pi}{2},\quad x_3=\frac{5\pi}{2}}$



а) $2\sin^2\left(\frac{3\pi}{2}-x\right)=\cos x$ б) $\left[-\frac{3\pi}{2};0\right]$

$-2\cos^2 x=\cos x$

$2\cos^2 x+\cos x=0$

$\cos x\,(2\cos x+1)=0$

1) $\cos x=0$

$\frac{\pi}{2}+\pi n,\ n\in Z$

2) $\cos x=-\frac{1}{2}$

$x=\pm\frac{2\pi}{3}+2\pi n,\ n\in Z$

б) $\left[-\frac{3\pi}{2};0\right]$

1) $n=-1$

$\frac{\pi}{2}-\pi=-\frac{\pi}{2}$ - уд

$n=-2$

$\frac{\pi}{2}-2\pi=-\frac{3\pi}{2}$ - уд

2) $n=-1$

$\frac{2\pi}{3}-2\pi=-\frac{4\pi}{3}$ - уд

$n=-2$

$\frac{2\pi}{3}-4\pi=-\frac{10\pi}{3}$ - не уд.

Ответ: а) $\frac{\pi}{2}+\pi n$; $\frac{2\pi}{3}+2\pi n,\ n\in Z$ б) $-\frac{\pi}{2}$; $-\frac{3\pi}{2}$; $-\frac{4\pi}{3}$

C1 а) $2\sin^4 x + 3\cos 2x + 1 = 0$

$2\sin^4 x - 6\sin^2 x + 4 = 0 \;\;|:2$

$\sin^4 x - 3\sin^2 x + 2 = 0$

Пусть $\sin^2 x = t$, $|t| \geq 1$

$t^2 - 3t + 2 = 0$

$t_1 = 1$ $\qquad$ $t_2 = 2$

не уд усл $|t| \geq 1$,

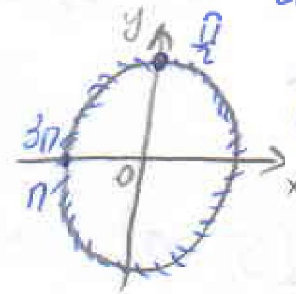
Вернемся к замене

$\sin^2 x = 1$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n,\ n \in$

б)



$n = 1,\ x = \frac{5\pi}{2}$

Ответ: а) $\frac{\pi}{2} + 2\pi n$,

б) $\frac{5\pi}{2}$

НАЙТИ ОШИБКУ

С1.

а) $\cos 2x + 0{,}5 = \cos^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

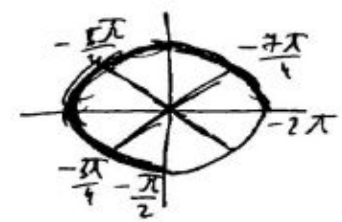
$\cos^2 x - \sin^2 x + 0{,}5 = \cos^2 x$

$-\sin^2 x + 0{,}5 = 0$

$\sin^2 x = 0{,}5$

$\left[\begin{array}{l} \sin x = \frac{\sqrt{2}}{2} \\ \sin x = -\frac{\sqrt{2}}{2} \end{array}\right.$ $\quad \left[\begin{array}{l} x = \frac{\pi}{4} + \pi n,\ n \in \mathbb{Z} \\ x = -\frac{\pi}{4} + \pi k,\ k \in \mathbb{Z} \end{array}\right.$ $\quad x = \frac{\pi}{4} + \frac{\pi m}{2},\ m \in \mathbb{Z}$

б) на $[-2\pi; -\frac{\pi}{2}]$



$\left[\begin{array}{l} x = -\frac{3\pi}{4} \\ x = -\frac{5\pi}{4} \\ x = -\frac{7\pi}{4} \end{array}\right.$

Ответ: а) $x = \frac{\pi}{4} + \frac{\pi m}{2},\ m \in \mathbb{Z}$

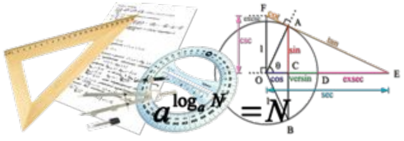
б) на отрезке $[-2\pi; \frac{\pi}{2}]$ $\left[\begin{array}{l} x = -\frac{3\pi}{4} \\ x = -\frac{5\pi}{4} \\ x = -\frac{7\pi}{4} \end{array}\right.$

- http://mathege.ru/or/ege/Main Открытый банк заданий
- http://uztest.ru/exam?idexam=30
- http://egeurok.ru/generators/ege_matem_2014/generator_variantov_ege_matem_2014.html Генератор вариантов
- http://alexlarin.net/ege14.html Ларин Александр Александрович
- http://reshuege.ru/

РЕШУ ЕГЭ

Образовательный портал для подготовки к экзаменам

МАТЕМАТИКА



Повтори

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$tg^2\alpha = \frac{1-\cos(2\alpha)}{1+\cos(2\alpha)}$$

$$ctg^2\alpha = \frac{1+\cos(2\alpha)}{1-\cos(2\alpha)}$$

$$\sin(2\alpha) = \frac{2tg\alpha}{1+tg^2\alpha} = \frac{2ctg\alpha}{1+ctg^2\alpha} = \frac{2}{tg\alpha + ctg\alpha}$$

$$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$$

$$\cos(2\alpha) = \frac{1-tg^2\alpha}{1+tg^2\alpha} = \frac{ctg^2\alpha - 1}{ctg^2\alpha + 1} = \frac{ctg\alpha - tg\alpha}{ctg\alpha + tg\alpha}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}, \quad 0 \le \alpha \le 2\pi$$

$$tg(2\alpha) = \frac{2tg\alpha}{1-tg^2\alpha} = \frac{2ctg\alpha}{ctg^2\alpha - 1} = \frac{2}{ctg\alpha - tg\alpha}$$

$$ctg(2\alpha) = \frac{ctg^2\alpha - 1}{2ctg\alpha} = \frac{ctg\alpha - tg\alpha}{2}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}, \quad -\pi \le \alpha \le \pi$$

$$ctg\frac{\alpha}{2} = \frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$$

$$tg\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$