

Решение тригонометрических уравнений

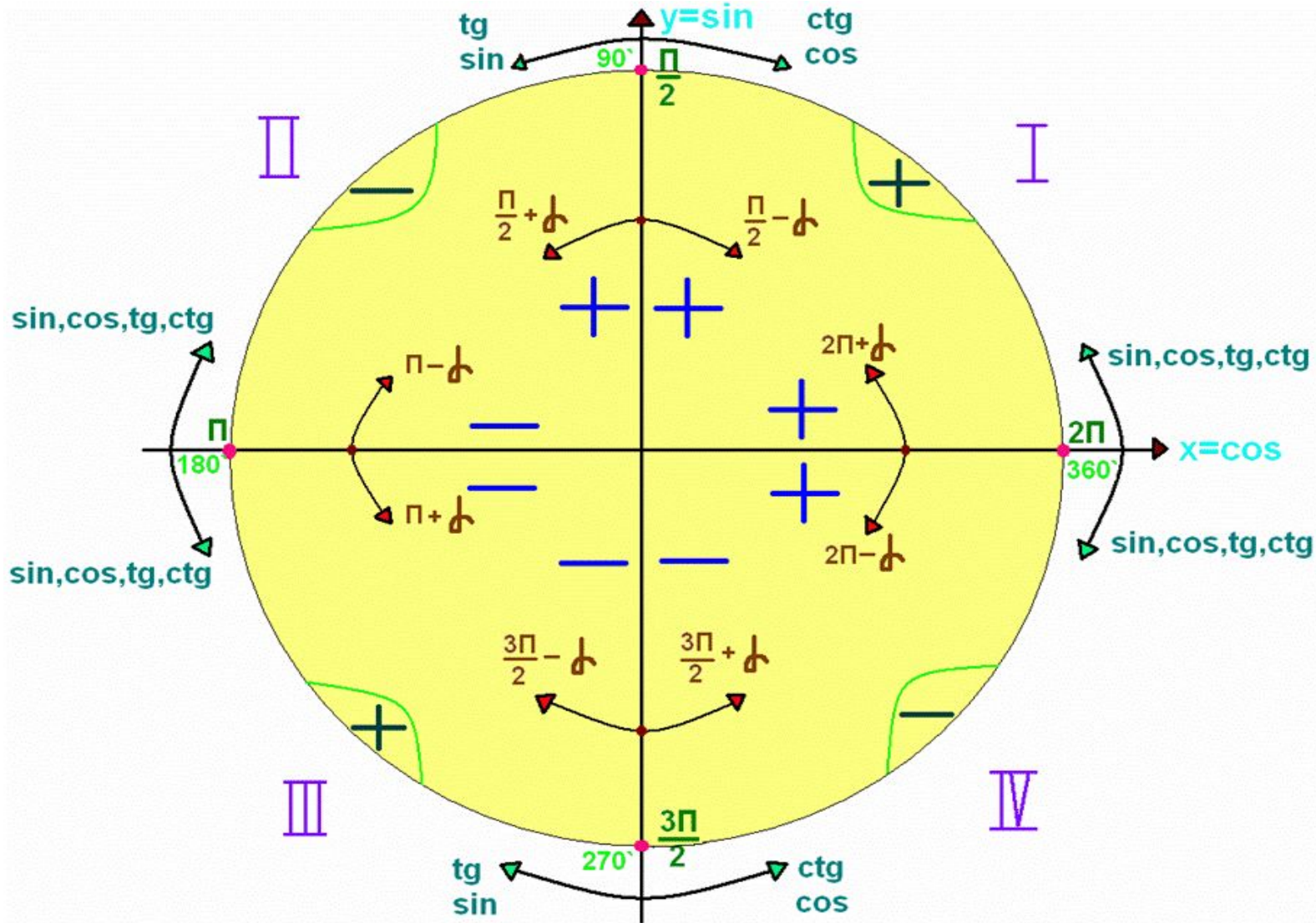
C1

Балагурова-Шемота Н.Ю
лицей №90

До экзамена осталось

• 160 дней

Повторим



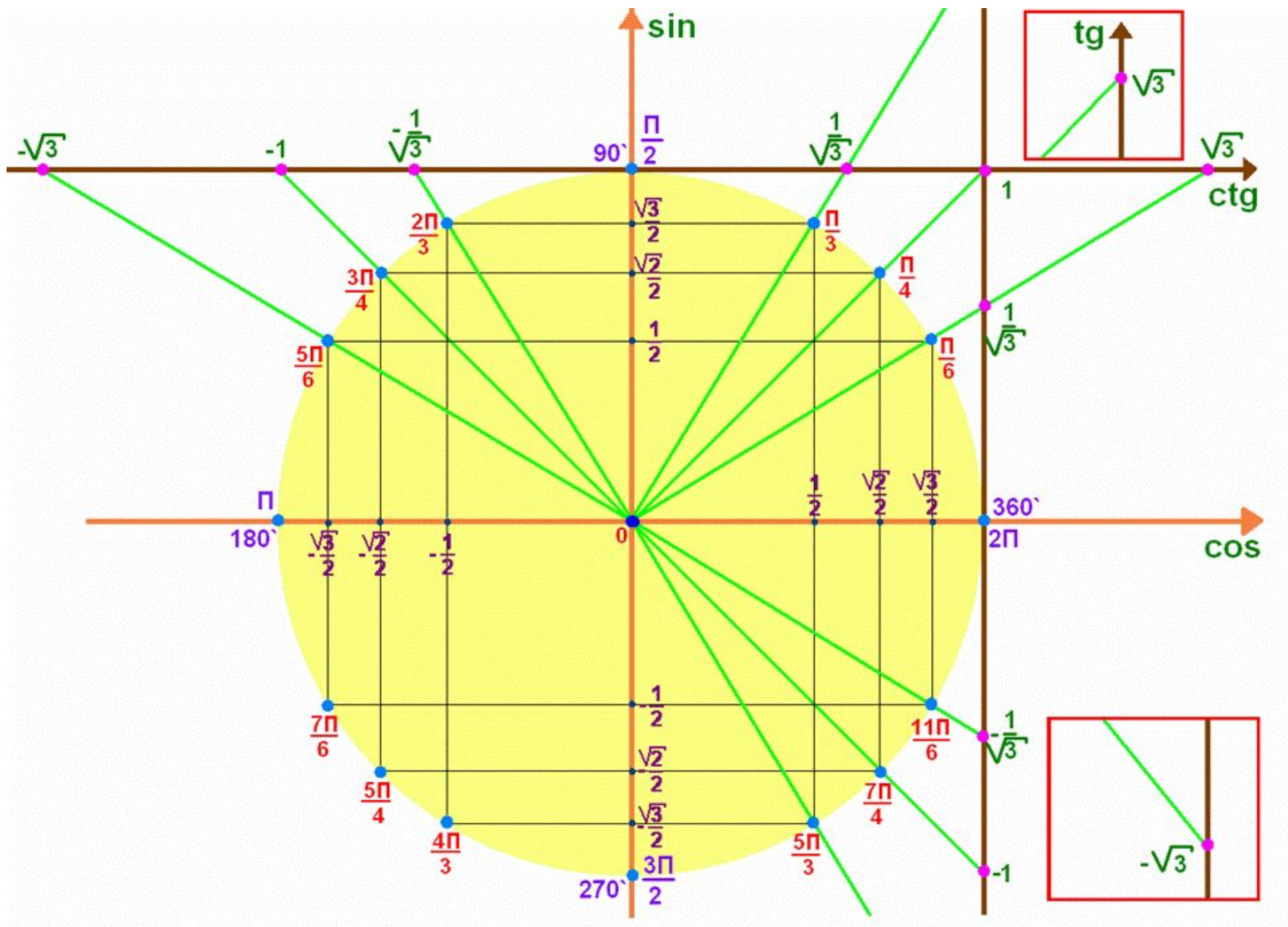
Повторим

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

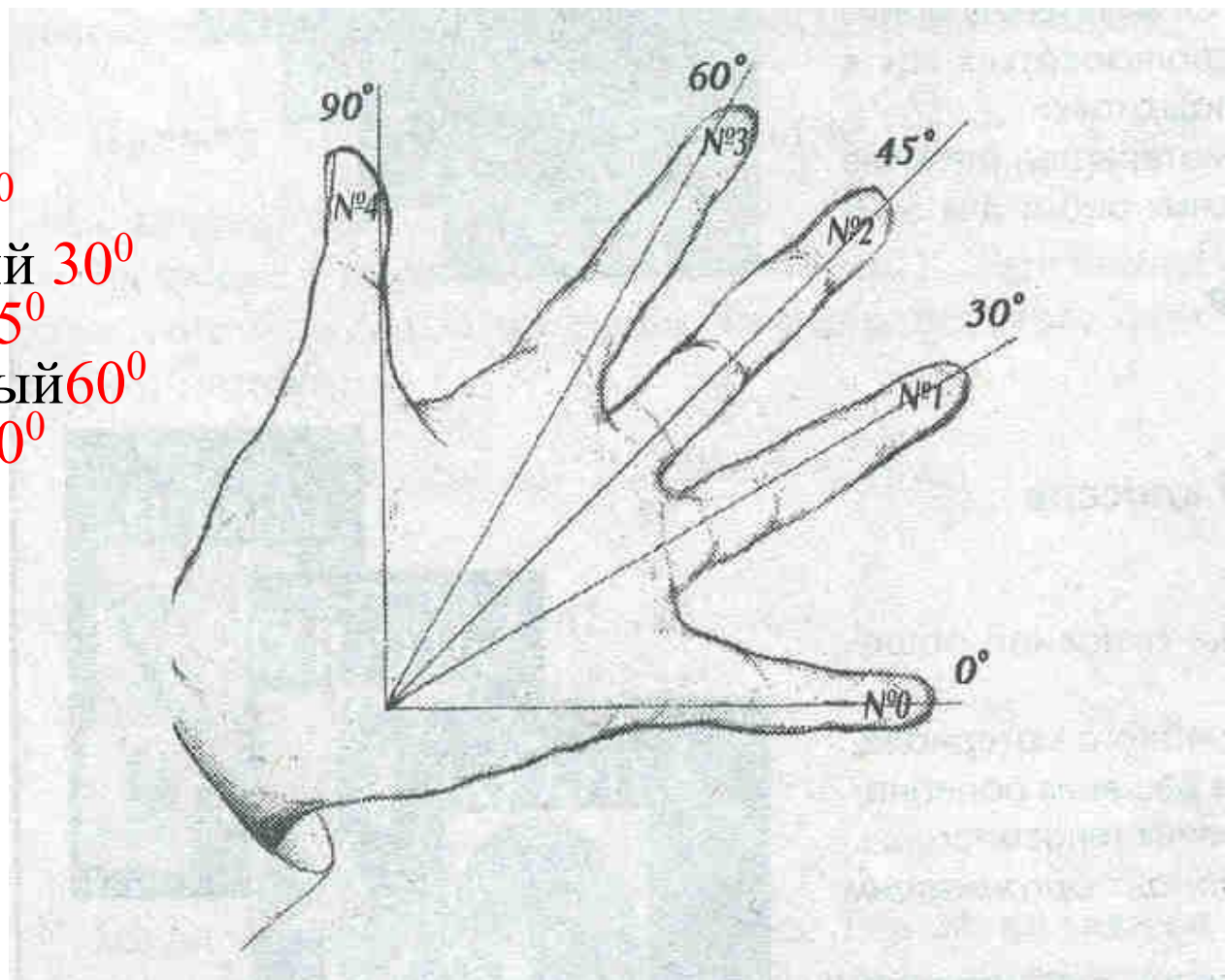
$$\cos^2\alpha = \frac{1 + \cos(2\alpha)}{2}$$

$$\sin^2\alpha = \frac{1 - \cos(2\alpha)}{2}$$

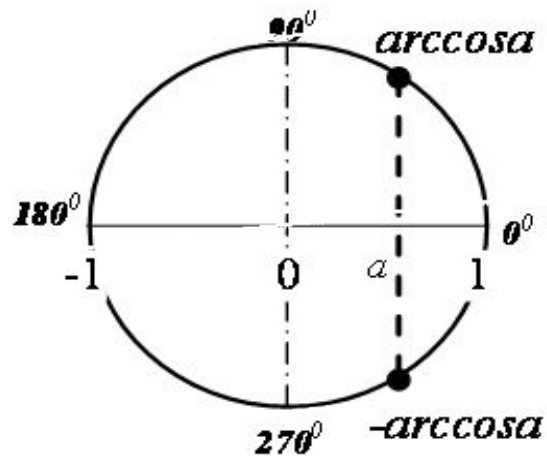


- $\sin \alpha = \frac{\sqrt{n}}{2}$

- №0 Мизинец 0°
- №1 Безымянный 30°
- №2 Средний 45°
- №3 Указательный 60°
- №4 Большой 90°



лицей №90 Балагурова-Шемота Н.Ю.

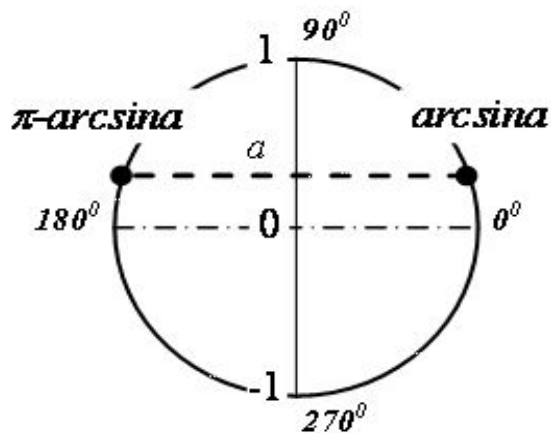


$$\cos x = a \Rightarrow x = \pm \arccos a + 2\pi n, n \in \mathbb{Z}$$

$$\cos x = -1 \Rightarrow x = \pi + 2\pi n$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + \pi n$$

$$\cos x = 1 \Rightarrow x = 2\pi n$$



$$\sin x = a \Rightarrow x = (-1)^n \arcsin a + \pi n, n \in \mathbb{Z}$$

$$\sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi n$$

$$\sin x = 0 \Rightarrow x = \pi n$$

$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi n$$

$$\cos\left(\frac{3\pi}{2} + 2x\right) = \cos x$$

а) Решите уравнение.

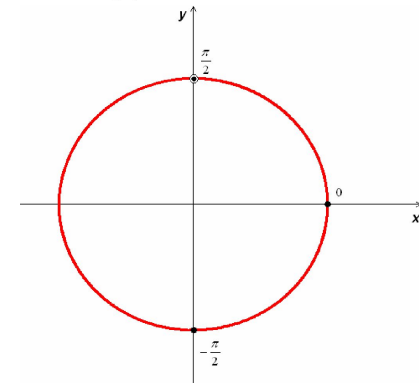
б) Укажите корни уравнения, принадлежащие отрезку $\left[\frac{5\pi}{2}; 4\pi\right]$

$$\sin 2x = \cos x; \quad 2\sin x \cos x - \cos x = 0;$$

$$\cos x \cdot (2\sin x - 1) = 0;$$

$$\cos x = 0 \quad \text{или} \quad 2\sin x - 1 = 0,$$

$$x = \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}, \quad x = \frac{\pi}{6} + 2\pi n, \quad x = \frac{5\pi}{6} + 2\pi n, \quad n \in \mathbb{Z}$$



Отбор корней

$$\frac{5\pi}{2} \leq \frac{\pi}{2} + \pi n \leq 4\pi; \quad \frac{5\pi}{2} \leq \frac{5\pi}{6} + 2\pi n \leq 4\pi \quad \frac{5\pi}{2} \leq \frac{\pi}{6} + 2\pi n \leq 4\pi$$
$$2 \leq n \leq \frac{7}{2}, \quad n \in \mathbb{Z}. \quad \frac{5}{6} \leq n \leq \frac{19}{12}, \quad n \in \mathbb{Z} \quad \frac{7}{6} \leq n \leq \frac{23}{12}, \quad n \in \mathbb{Z}.$$

$$x = \frac{5\pi}{2}; \quad x = \frac{7\pi}{2}.$$

$$x = \frac{17\pi}{6}.$$

нет корней.

Ответ: а) $\frac{\pi}{2} + \pi n, \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, n \in \mathbb{Z}$; б) $\frac{5\pi}{2}, \frac{17\pi}{6}, \frac{7\pi}{2}$.

Найти ошибку

$$\cos\left(\frac{3\pi}{2} + 2x\right) = \cos x$$

$$\left[\frac{5\pi}{2}; 4\pi\right]$$

a) $\sin 2x = \cos x$

$$2 \sin x \cos x = \cos x$$

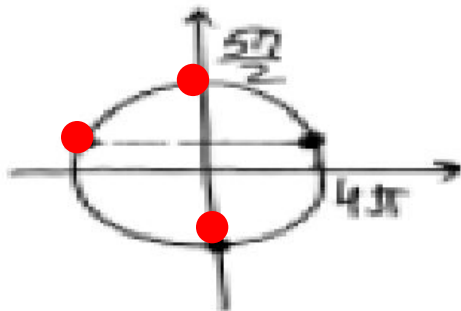
$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \text{или} \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2} + \pi k$$

$$x = (-1)^n \cdot \frac{\pi}{6} + \pi n$$

b) $\left[\frac{5\pi}{2}; 4\pi\right]$



$$\left\{ \frac{7\pi}{2}; \frac{5\pi}{6}; \frac{5\pi}{2} \right\}$$

$$\boxed{C1} \quad \cos\left(\frac{3\pi}{2} + 2x\right) = \cos x \quad \left[\frac{5\pi}{2}; 4\pi\right]$$

Решение:

$$\begin{aligned} \text{a) } -\sin 2x &= \cos x \\ -2 \sin x \cos x &= \cos x \\ \cos x + 2 \sin x \cos x &= 0 \\ \cos x (1 + 2 \sin x) &= 0 \end{aligned}$$

$$\begin{aligned} \cos x &= 0 \\ x &= \frac{\pi}{2} + \pi n \end{aligned}$$

или

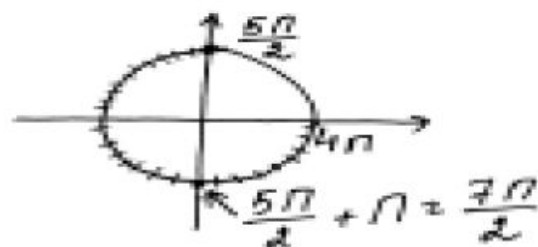
$$\sin x = -\frac{1}{2}$$

$$x = (-1)^k \arcsin\left(-\frac{1}{2}\right) + \pi k$$

$$x = (-1)^k \left(-\frac{\pi}{6}\right) + \pi k$$

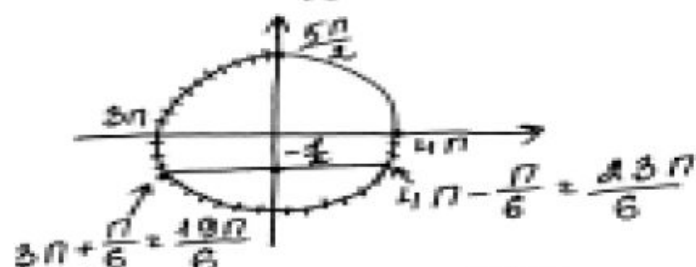
$$x = (-1)^{k+1} \frac{\pi}{6} + \pi k$$

$$\text{б) } \cos x = 0$$



$$x_1 = \frac{5\pi}{2} \quad x_2 = \frac{7\pi}{2}$$

$$\sin x = -\frac{1}{2}$$



$$x_3 = \frac{19\pi}{6} \quad x_4 = \frac{23\pi}{6}$$

Ответ: а) $\frac{\pi}{2} + \pi n$, $(-1)^{k+1} \cdot \frac{\pi}{6} + \pi k$, где $n \in \mathbb{Z}$, $k \in \mathbb{Z}$

$$\text{б) } \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$\sin 2x - \cos\left(\frac{3\pi}{2} - x\right) + 2\sqrt{2} \sin^2 \frac{x}{2} = \frac{3\sqrt{2}}{2} \quad \left[-\frac{3\pi}{2}; \pi\right].$$

$$2 \sin x \cos x + \sin x + \sqrt{2}(1 - \cos x) - \frac{3\sqrt{2}}{2} = 0$$

$$\sin x(2 \cos x + 1) - \frac{\sqrt{2}}{2}(2 \cos x + 1) = 0$$

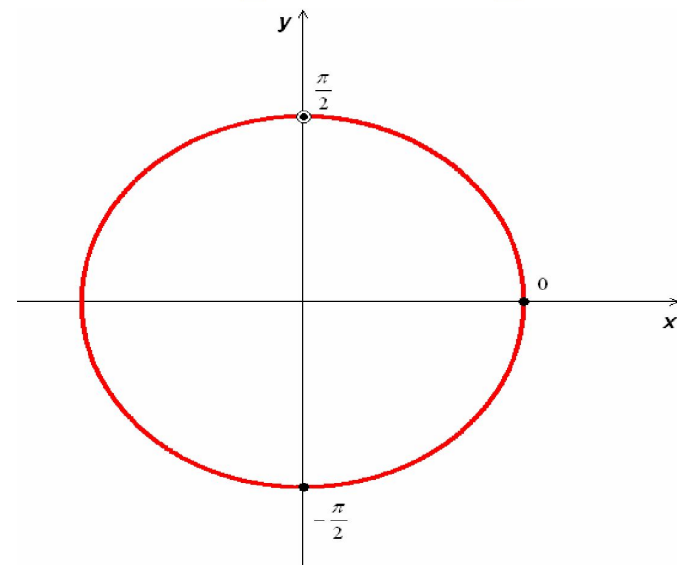
$$(2 \cos x + 1)\left(\sin x - \frac{\sqrt{2}}{2}\right) = 0$$

$$\cos x = -\frac{1}{2}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \pm \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

$$x = (-1)^k \frac{\pi}{4} + \pi k, k \in \mathbb{Z}$$



$$\left[\begin{array}{l} x = \frac{\pi}{4} + 2\pi m \\ x = \frac{3\pi}{4} + 2\pi m \end{array} \right. , m \in \mathbb{Z}$$

$$-\frac{3\pi}{2} \leq \frac{2\pi}{3} + 2\pi n \leq \pi$$

$$-\frac{3}{2} \leq \frac{2}{3} + 2n \leq 1$$

$$-\frac{13}{6} \leq 2n \leq \frac{1}{3}$$

$$-\frac{13}{12} \leq n \leq \frac{1}{6} \Rightarrow n = -1; 0$$

$$-\frac{3\pi}{2} \leq -\frac{2\pi}{3} + 2\pi n \leq \pi$$

$$-\frac{3}{2} \leq -\frac{2}{3} + 2n \leq 1$$

$$-\frac{5}{6} \leq 2n \leq \frac{5}{3}$$

$$-\frac{5}{12} \leq n \leq \frac{5}{6} \Rightarrow n = 0$$

$$-\frac{3\pi}{2} \leq \frac{3\pi}{4} + 2\pi m \leq \pi$$

$$-\frac{3}{2} \leq \frac{3}{4} + 2m \leq 1$$

$$-\frac{9}{4} \leq 2m \leq \frac{1}{4}$$

$$-\frac{9}{8} \leq m \leq \frac{1}{8} \Rightarrow m = -1; 0$$

$$x_1 = -\frac{4\pi}{3}; x_2 = \frac{2\pi}{3}$$

$$x_3 = -\frac{2\pi}{3}$$

$$x_5 = -\frac{5\pi}{4}; x_6 = \frac{3\pi}{4}$$

$$-\frac{3\pi}{2} \leq \frac{\pi}{4} + 2\pi m \leq \pi$$

$$-\frac{3}{2} \leq \frac{1}{4} + 2m \leq 1$$

$$-\frac{7}{4} \leq 2m \leq \frac{3}{4}$$

$$-\frac{7}{8} \leq m \leq \frac{3}{8} \Rightarrow m = 0$$

$$x_4 = \frac{\pi}{4}$$

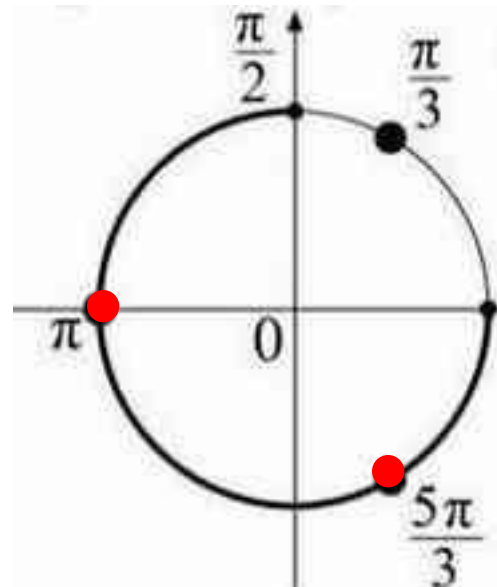
ОТВЕТ:

$$\text{а) } x = \pm \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z} ; x = (-1)^k \frac{\pi}{4} + \pi k, k \in \mathbb{Z}$$

$$\text{б) } x \in \left\{ -\frac{4\pi}{3}; -\frac{5\pi}{4}; -\frac{2\pi}{3}; \frac{\pi}{4}; \frac{2\pi}{3}; \frac{3\pi}{4} \right\}.$$

а) Решите уравнение $\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} = \cos 2x$

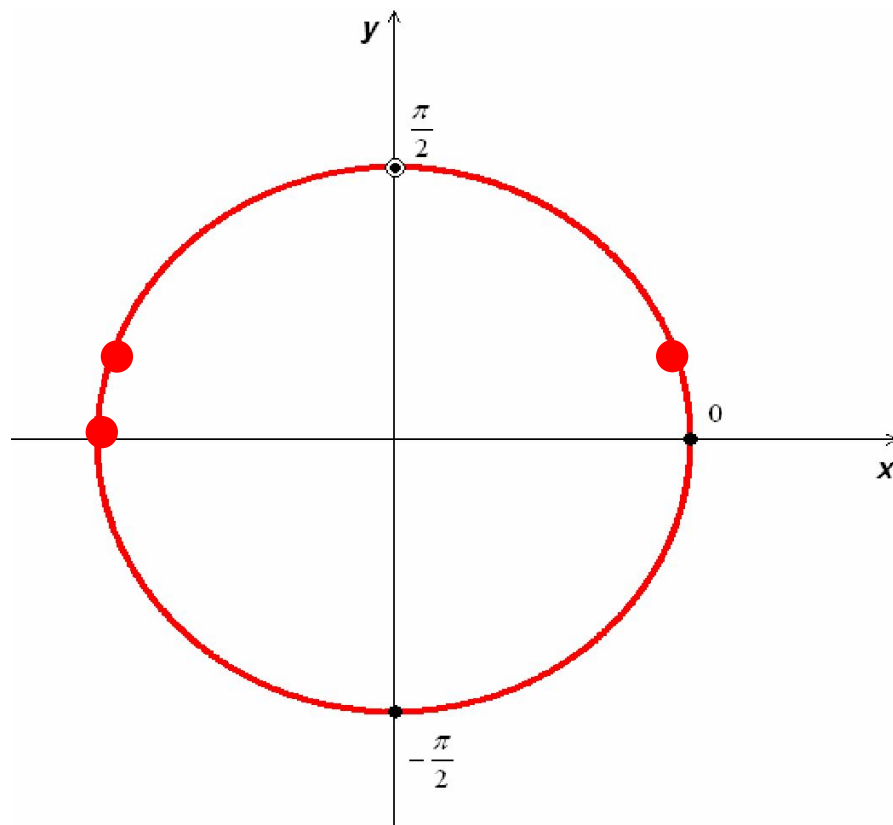
б) Найдите корни этого уравнения, принадлежащие промежутку $\left[\frac{\pi}{2}; 2\pi \right]$



Решите уравнение $\cos x + 1 = 2 \sin 2x + 4 \sin x$

Укажите корни, принадлежащие отрезку $\left[-\frac{7\pi}{2}, -\frac{3\pi}{2}\right]$

$$\pi + 2n\pi, \quad (-1)^k \arcsin \frac{1}{4} + k\pi$$



$$x_1 = -3\pi$$

$$x_2 = -3\pi - \arcsin \frac{1}{4}$$

$$x_3 = -2\pi + \arcsin \frac{1}{4}$$

$$\sqrt{3} \cdot \sin x - \operatorname{tg}x + \operatorname{tg}x \cdot \sin x = \sqrt{3}$$

$$(\sin x - 1)(\sqrt{3} + \operatorname{tg}x) = 0$$

$$\left\{ \begin{array}{l} \cos x \neq 0 \\ \sin x = 1 \\ \operatorname{tg}x = -\sqrt{3} \end{array} \right.$$

$$\operatorname{tg}x = -\sqrt{3}$$

$$x = -\frac{\pi}{3} + k\pi \quad k \in \mathbb{Z}$$

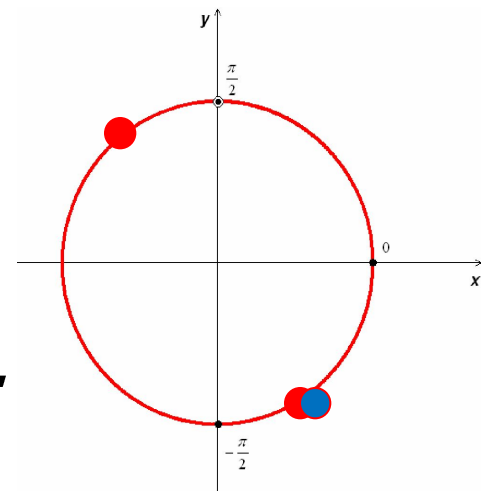
$$\left[-\frac{\pi}{2}, \frac{7\pi}{4} \right]$$

Ответ:

$$\frac{\pi}{3}$$

$$\frac{2\pi}{3}$$

$$\frac{5\pi}{3}$$



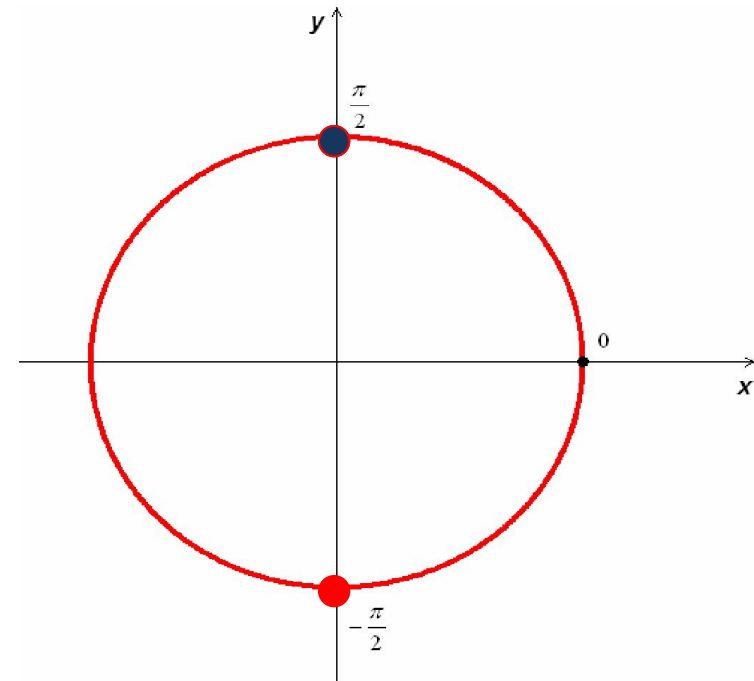
Решите уравнение $\frac{1}{\cos x - 1} + \frac{1}{\cos x + 1} = 2\operatorname{ctg}^2 x$

Укажите корни, принадлежащие промежутку $\left(-\frac{\pi}{2}, 8\right]$

$$\frac{\pi}{2} + k\pi.$$

$$x_1 = \frac{\pi}{2}$$

$$x_2 = \frac{3\pi}{2}, \quad x_3 = \frac{5\pi}{2}$$



$$a) 2 \sin^2\left(\frac{3\pi}{2} - x\right) = \cos x \quad b) \left[-\frac{3\pi}{2}; 0\right]$$

$$\textcircled{-} 2 \cos^2 x = \cos x$$

$$2 \cos^2 x + \cos x = 0$$

$$\cos x (2 \cos x + 1) = 0$$

$$1) \cos x = 0$$

$$\times \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

$$2) \cos x = -\frac{1}{2}$$

$$x = \textcircled{\frac{2\pi}{3}} + 2\pi n, n \in \mathbb{Z}$$

$$b) \left[-\frac{3\pi}{2}; 0\right]$$

$$1) n = -1$$

$$\frac{\pi}{2} - \pi = -\frac{\pi}{2} \text{ — не в } \mathbb{Z}$$

$$n = -2$$

$$\frac{\pi}{2} - 2\pi = -\frac{3\pi}{2} \text{ — не в } \mathbb{Z}$$

$$2) n = -1$$

$$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3} \text{ — не в } \mathbb{Z}$$

$$n = -2$$

$$\frac{2\pi}{3} - 4\pi = -\frac{10\pi}{3} \text{ — не в } \mathbb{Z}$$

$$\text{Ответ: a) } \frac{\pi}{2} + \pi n; \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

$$b) -\frac{\pi}{2}; -\frac{3\pi}{2}; -\frac{4\pi}{3}$$

$$C_1 \quad a) \quad 2\sin^4 x + 3\cos 2x + 1 = 0$$

$$2\sin^4 x - 6\sin^2 x + 4 = 0 \quad /: 2$$

$$\sin^4 x - 3\sin^2 x + 2 = 0$$

Пусть $\sin^2 x = t$, $|t| \geq 1$

$$t^2 - 3t + 2 = 0$$

$$t_1 = 1$$

$$t_2 = 2$$

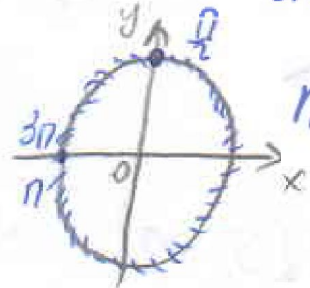
не ур уса ($|t| \geq 1$)

Вернемся к замене

$$\sin^2 x = 1$$
$$\sin x = \pm 1$$

$$x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

б)



$$n=1, x = \frac{5\pi}{2}$$

Ответ: а) $\frac{\pi}{2} + 2\pi n$,

б) $\frac{5\pi}{2}$

НАЙТИ ОШИБКУ

р1.

$$a) \cos 2x + 0,5 = \cos^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x - \sin^2 x + 0,5 = \cos^2 x$$

$$-\sin^2 x + 0,5 = 0$$

$$\sin^2 x = 0,5$$

$$\begin{cases} \sin x = \frac{\sqrt{2}}{2} \\ \sin x = -\frac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{4} + \pi k, k \in \mathbb{Z} \\ x = -\frac{\pi}{4} + \pi k, k \in \mathbb{Z} \end{cases} \quad x = \frac{\pi}{4} + \frac{\pi m}{2}, m \in \mathbb{Z}$$

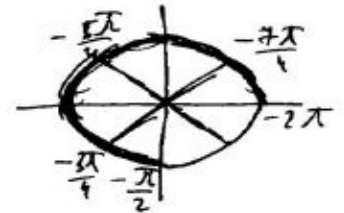
б) на $[-2\pi; -\frac{\pi}{2}]$

$$\begin{cases} x = -\frac{3\pi}{4} \\ x = -\frac{5\pi}{4} \\ x = -\frac{7\pi}{4} \end{cases}$$

Ответ:

$$a) x = \frac{\pi}{4} + \frac{\pi m}{2}, m \in \mathbb{Z}$$

$$б) \text{ на отрезке } [-2\pi; -\frac{\pi}{2}] \quad \begin{cases} x = -\frac{3\pi}{4} \\ x = -\frac{5\pi}{4} \\ x = -\frac{7\pi}{4} \end{cases}$$



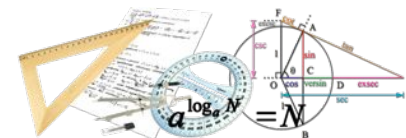
- <http://mathege.ru/or/ege/Main> Открытый банк заданий
- <http://uztest.ru/exam?idexam=30>
- http://egeurok.ru/generators/ege_matem_2014/generator_variantov_ege_matem_2014.html Генератор вариантов
- <http://alexlarin.net/ege14.html> Ларин Александр Александрович
- <http://reshuege.ru/>



РЕШУ ЕГЭ

Образовательный портал для подготовки к экзаменам

МАТЕМАТИКА



Повтори

$$\sin(2\alpha) = 2\sin\alpha \cos\alpha$$

$$\operatorname{tg}^2\alpha = \frac{1 - \cos(2\alpha)}{1 + \cos(2\alpha)}$$

$$\sin(2\alpha) = \frac{2\operatorname{tg}\alpha}{1 + \operatorname{tg}^2\alpha} = \frac{2\operatorname{ctg}\alpha}{1 + \operatorname{ctg}^2\alpha} = \frac{2}{\operatorname{tg}\alpha + \operatorname{ctg}\alpha}$$

$$\operatorname{ctg}^2\alpha = \frac{1 + \cos(2\alpha)}{1 - \cos(2\alpha)}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\cos^2\alpha = \frac{1 + \cos(2\alpha)}{2}$$

$$\cos(2\alpha) = \frac{1 - \operatorname{tg}^2\alpha}{1 + \operatorname{tg}^2\alpha} = \frac{\operatorname{ctg}^2\alpha - 1}{\operatorname{ctg}^2\alpha + 1} = \frac{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}{\operatorname{ctg}\alpha + \operatorname{tg}\alpha}$$

$$\sin^2\alpha = \frac{1 - \cos(2\alpha)}{2}$$

$$\operatorname{tg}(2\alpha) = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha} = \frac{2\operatorname{ctg}\alpha}{\operatorname{ctg}^2\alpha - 1} = \frac{2}{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}, \quad 0 \leq \alpha \leq 2\pi$$

$$\operatorname{ctg}(2\alpha) = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha} = \frac{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}{2}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}, \quad -\pi \leq \alpha \leq \pi$$

$$\operatorname{ctg}\frac{\alpha}{2} = \frac{\sin\alpha}{1 - \cos\alpha} = \frac{1 + \cos\alpha}{\sin\alpha}$$

$$\operatorname{tg}\frac{\alpha}{2} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$