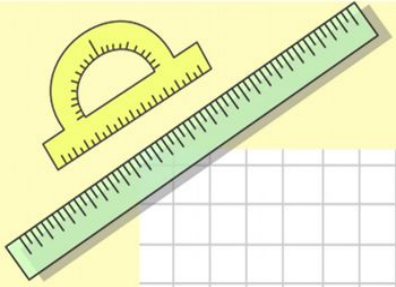
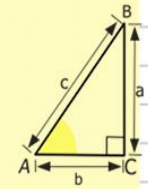
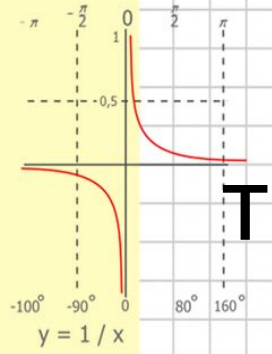
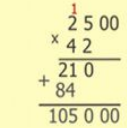
Математика

Занятие 46. Разложение суммы тригонометрических функций в произведение и наоборот









$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

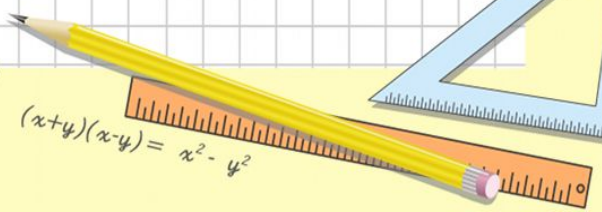
$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

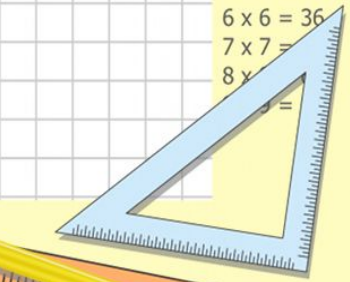
$$\begin{cases} y=\sin 90 \\ x=25y+45 \end{cases}$$

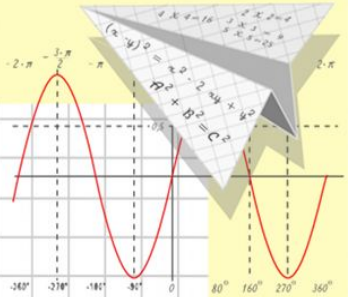
$$\begin{cases} y=1 \\ x=25+45 \end{cases}$$

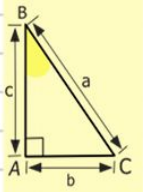
$$x=70$$

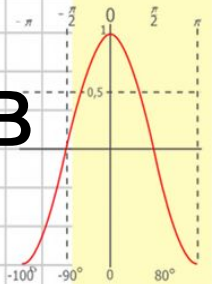


$$(x+y)(x-y)=x^2-y^2$$









2 x 2 = 4
3 x 3 = 9
4 x 4 = 16
5 x 5 = 25
6 x 6 = 36
7 x 7 =
8 x

Разложение суммы в произведение

$$\begin{cases} \alpha = x + y \\ \beta = x - y \end{cases} \Rightarrow \begin{cases} \alpha + \beta = x + y + x - y = 2x \\ \alpha - \beta = x + y - (x - y) = 2y \end{cases} \Rightarrow x = \frac{\alpha + \beta}{2} \quad y = \frac{\alpha - \beta}{2}$$

$$+ \begin{array}{l} \cos(x + y) = \cos x \cos y - \sin x \sin y \\ \cos(x - y) = \cos x \cos y + \sin x \sin y \end{array}$$

$$\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

Аналогично

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2}\cos\frac{\alpha + \beta}{2}$$

Разложение суммы в произведение

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\sin 5x-\sin 3x=2\sin\frac{5x-3x}{2}\cos\frac{5x+3x}{2}=2\sin x\cos 4x$$

$$\cos 10^\circ+\cos 40^\circ=2\cos\frac{10^\circ+40^\circ}{2}\cos\frac{10^\circ-40^\circ}{2}=2\cos 25^\circ\cos(-15^\circ)=2\cos 25^\circ\cos 15^\circ$$

$$\sin(x+18^\circ)-\sin(x-24^\circ)=2\sin\frac{x+18^\circ-x+24^\circ}{2}\cos\frac{x+18^\circ+x-24^\circ}{2}=$$

$$=2\sin\frac{42^\circ}{2}\cos\frac{2x-6^\circ}{2}=2\sin 21^\circ\cos(x-3^\circ)$$

$$\cos\frac{7\pi}{12}-\cos\frac{5\pi}{12}=-2\sin\frac{\frac{7\pi}{12}+\frac{5\pi}{12}}{2}\sin\frac{\frac{7\pi}{12}-\frac{5\pi}{12}}{2}=-2\sin\frac{\pi}{2}\sin\frac{\pi}{12}=$$

$$=-2\cdot 1\cdot\sin\frac{\pi}{12}=-2\cdot\sin\frac{\pi}{12}$$

Разложение суммы в произведение

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Упростить:

1) $\sin 87^\circ-\sin 59^\circ-\sin 93^\circ+\sin 61^\circ=(\sin 87^\circ-\sin 93^\circ)+(\sin 61^\circ-\sin 59^\circ)=$

$$=2\sin\frac{87^\circ-93^\circ}{2}\cos\frac{87^\circ+93^\circ}{2}+2\sin\frac{61^\circ-59^\circ}{2}\cos\frac{61^\circ+59^\circ}{2}=$$

$$=2\sin(-3^\circ)\cos 90^\circ+2\sin 1^\circ\cos 60^\circ=2\sin(-3^\circ)\cdot 0+2\sin 1^\circ\cdot\frac{1}{2}=\sin 1^\circ$$

2) $$\frac{\sin 7x-\sin 3x}{\cos 7x-\cos 3x}=\frac{2\sin\frac{7x-3x}{2}\cos\frac{7x+3x}{2}}{-2\sin\frac{7x+3x}{2}\sin\frac{7x-3x}{2}}=\frac{2\sin 2x\cos 5x}{-2\sin 5x\sin 2x}=$$

$$=-\frac{\cos 5x}{\sin 5x}=-ctg5x$$

Разложение суммы в произведение

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Разложить на множители:

$$\cos 8x+\cos 6x+\cos 4x+\cos 2x=(\cos 8x+\cos 2x)+(\cos 6x+\cos 4x)=$$

$$=2\cos\frac{8x+2x}{2}\cos\frac{8x-2x}{2}+2\cos\frac{6x+4x}{2}\cos\frac{6x-4x}{2}=$$

$$=2\cos 5x\cos 3x+2\cos 5x\cos x=2\cos 5x(\cos 3x+\cos x)=$$

$$=2\cos 5x\cdot 2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}=\quad 4\cos 5x\cdot\cos 2x\cdot\cos x$$

Разложение произведения в сумму

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$+\quad\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$$

$$\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]=\cos\alpha\cos\beta$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

Аналогично

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

Разложение произведения в сумму

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

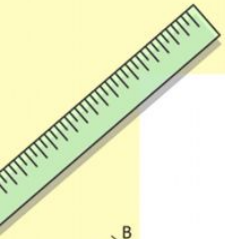
$$\cos 3x\cos x=\frac{1}{2}\left[\cos(3x+x)+\cos(3x-x)\right]=\frac{1}{2}(\cos 4x+\cos 2x)$$

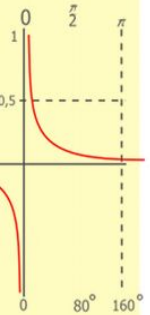
$$\sin 50^\circ\sin 10^\circ=\frac{1}{2}\left[\cos(50^\circ-10^\circ)-\cos(50^\circ+10^\circ)\right]=$$

$$=\frac{1}{2}(\cos 40^\circ-\cos 60^\circ)=\frac{1}{2}\left(\cos 40^\circ-\frac{1}{2}\right)=\frac{1}{2}\cos 40^\circ-\frac{1}{4}$$

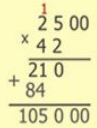
$$\sin\frac{\pi}{5}\cos\frac{3\pi}{8}=\frac{1}{2}\left[\sin\left(\frac{\pi}{5}+\frac{3\pi}{8}\right)+\sin\left(\frac{\pi}{5}-\frac{3\pi}{8}\right)\right]=$$

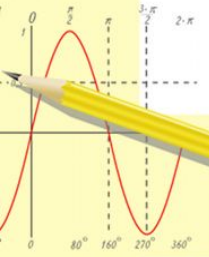
$$=\frac{1}{2}\left[\sin\frac{23\pi}{40}+\sin\frac{-7\pi}{40}\right]=\frac{1}{2}\left[\sin\frac{23\pi}{40}-\sin\frac{7\pi}{40}\right]$$





y = 1 / x





$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

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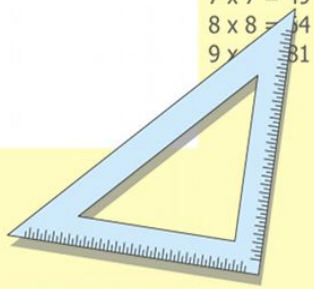
$\sin 90° = 1$

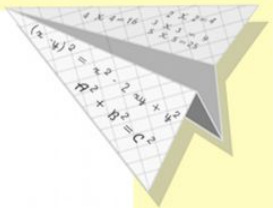
$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$

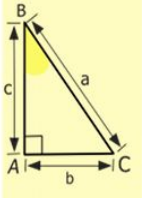
$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$

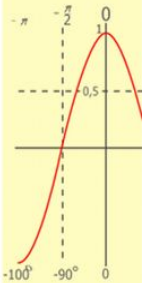
$x = 70$

$(x+y)(x-y) = x^2 - y^2$









y = co

2 x 2 = 4
3 x 3 = 9
4 x 4 = 16
5 x 5 = 25
6 x 6 = 36
7 x 7 = 49
8 x 8 = 64
9 x 9 = 81

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Москва, 2020г.