

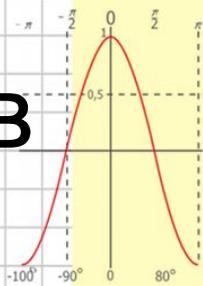
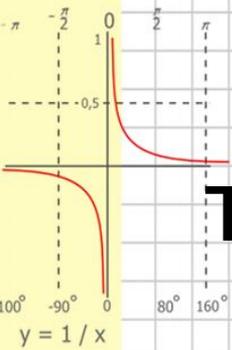
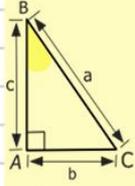
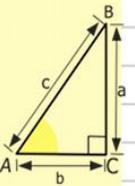
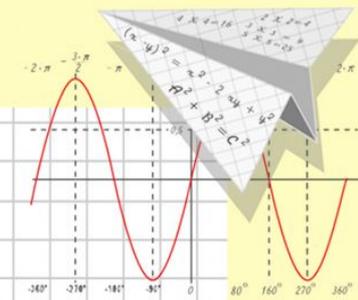
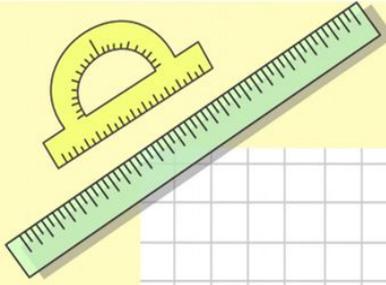
# Математик

а

## Занятие 46.

### Разложение суммы

### тригонометрических функций в произведение и наоборот



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

$$\begin{array}{l} y = \cos x \\ 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \end{array}$$

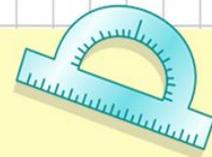


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

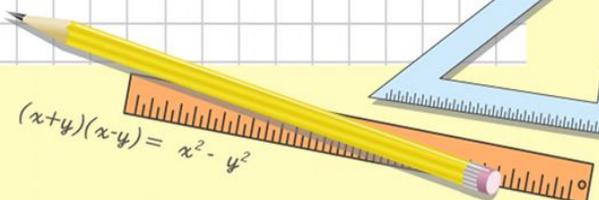
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



$$(x+y)(x-y) = x^2 - y^2$$

# Разложение суммы в произведение

$$\left. \begin{array}{l} \alpha = x + y \\ \beta = x - y \end{array} \right\} \rightarrow \left. \begin{array}{l} \alpha + \beta = x + y + x - y = 2x \\ \alpha - \beta = x + y - (x - y) = 2y \end{array} \right\} \rightarrow x = \frac{\alpha + \beta}{2} \quad y = \frac{\alpha - \beta}{2}$$

$$+ \begin{array}{l} \cos(x + y) = \cos x \cos y - \sin x \sin y \\ \cos(x - y) = \cos x \cos y + \sin x \sin y \end{array}$$

---


$$\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

Аналогично

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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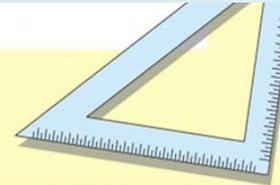
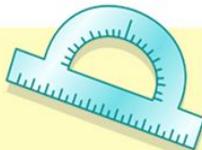
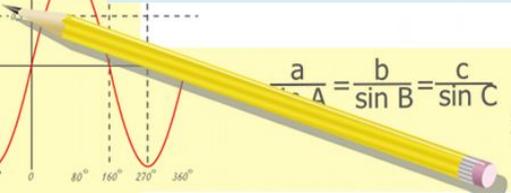
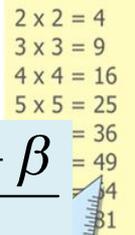
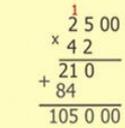
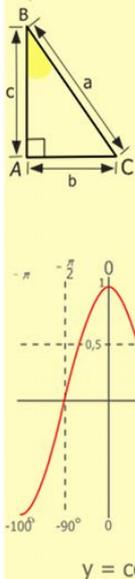
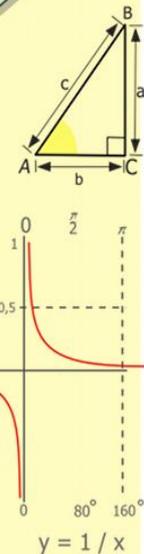
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



# Разложение суммы в произведение

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin 5x - \sin 3x = 2 \sin \frac{5x - 3x}{2} \cos \frac{5x + 3x}{2} = 2 \sin x \cos 4x$$

$$\cos 10^\circ + \cos 40^\circ = 2 \cos \frac{10^\circ + 40^\circ}{2} \cos \frac{10^\circ - 40^\circ}{2} = 2 \cos 25^\circ \cos(-15^\circ) = 2 \cos 25^\circ \cos 15^\circ$$

$$\sin(x + 18^\circ) - \sin(x - 24^\circ) = 2 \sin \frac{x + 18^\circ - x + 24^\circ}{2} \cos \frac{x + 18^\circ + x - 24^\circ}{2} =$$

$$= 2 \sin \frac{42^\circ}{2} \cos \frac{2x - 6^\circ}{2} = 2 \sin 21^\circ \cos(x - 3^\circ)$$

$$\cos \frac{7\pi}{12} - \cos \frac{5\pi}{12} = -2 \sin \frac{\frac{7\pi}{12} + \frac{5\pi}{12}}{2} \sin \frac{\frac{7\pi}{12} - \frac{5\pi}{12}}{2} = -2 \sin \frac{\pi}{2} \sin \frac{\pi}{12} =$$

$$= -2 \cdot 1 \cdot \sin \frac{\pi}{12} = -2 \cdot \sin \frac{\pi}{12}$$

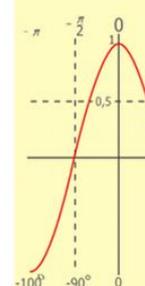
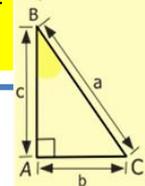
$$\sin A = \sin B = \sin C$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

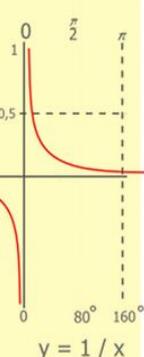
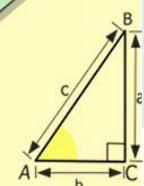
$$\sin 90^\circ = 1$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

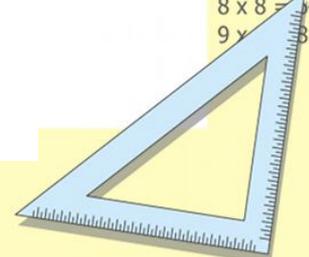
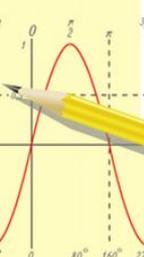
$$(x+y)(x-y) = x^2 - y^2$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
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$$\begin{array}{r} 1\ 2\ 5\ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105\ 000 \end{array}$$



# Разложение суммы в произведение

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

Упростить:

$$\begin{aligned} 1) \quad \sin 87^\circ - \sin 59^\circ - \sin 93^\circ + \sin 61^\circ &= (\sin 87^\circ - \sin 93^\circ) + (\sin 61^\circ - \sin 59^\circ) = \\ &= 2 \sin \frac{87^\circ - 93^\circ}{2} \cos \frac{87^\circ + 93^\circ}{2} + 2 \sin \frac{61^\circ - 59^\circ}{2} \cos \frac{61^\circ + 59^\circ}{2} = \\ &= 2 \sin(-3^\circ) \cos 90^\circ + 2 \sin 1^\circ \cos 60^\circ = 2 \sin(-3^\circ) \cdot 0 + 2 \sin 1^\circ \cdot \frac{1}{2} = \sin 1^\circ \end{aligned}$$

$$\begin{aligned} 2) \quad \frac{\sin 7x - \sin 3x}{\cos 7x - \cos 3x} &= \frac{2 \sin \frac{7x - 3x}{2} \cos \frac{7x + 3x}{2}}{-2 \sin \frac{7x + 3x}{2} \sin \frac{7x - 3x}{2}} = \frac{2 \sin 2x \cos 5x}{-2 \sin 5x \sin 2x} = \\ &= -\frac{\cos 5x}{\sin 5x} = -\operatorname{ctg} 5x \end{aligned}$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

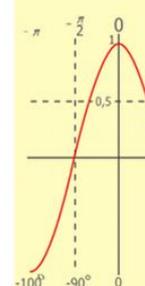
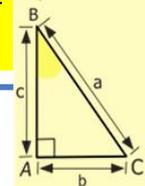
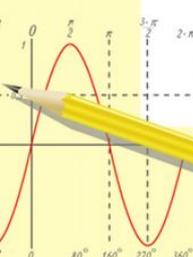
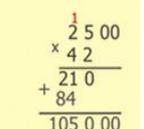
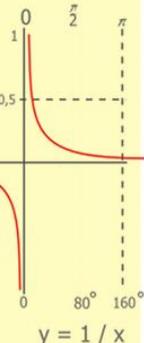
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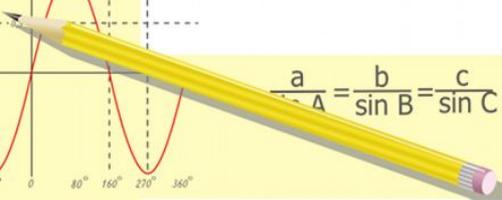
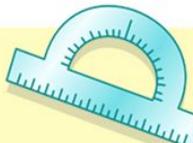
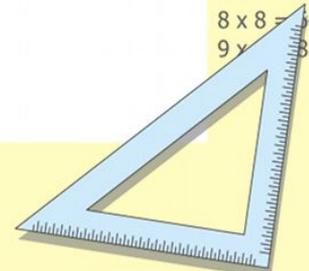
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

- 2 x 2 = 4
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# Разложение суммы в произведение

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

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Разложить на

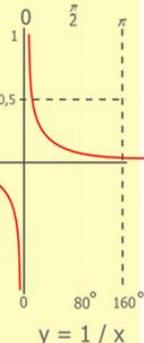
множители:

$$\cos 8x + \cos 6x + \cos 4x + \cos 2x = (\cos 8x + \cos 2x) + (\cos 6x + \cos 4x) =$$

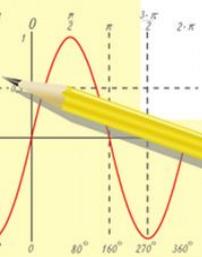
$$= 2 \cos \frac{8x + 2x}{2} \cos \frac{8x - 2x}{2} + 2 \cos \frac{6x + 4x}{2} \cos \frac{6x - 4x}{2} =$$

$$= 2 \cos 5x \cos 3x + 2 \cos 5x \cos x = 2 \cos 5x (\cos 3x + \cos x) =$$

$$= 2 \cos 5x \cdot 2 \cos \frac{3x + x}{2} \cos \frac{3x - x}{2} = 4 \cos 5x \cdot \cos 2x \cdot \cos x$$



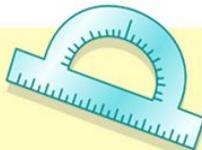
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



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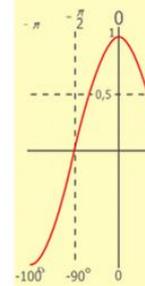
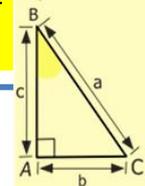
$$\sin 90^\circ = 1$$



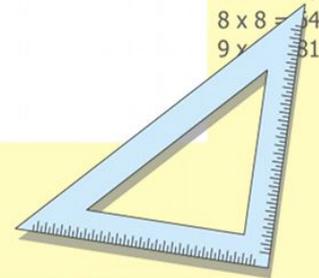
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

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- y = cos
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# Разложение произведения в сумму

$$\begin{aligned}
 & \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
 + & \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta
 \end{aligned}$$

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$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$$

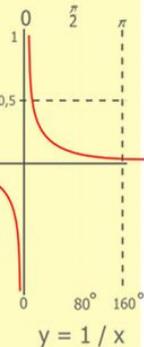
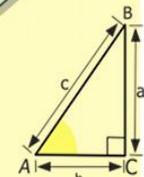
$$\frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] = \cos \alpha \cos \beta$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

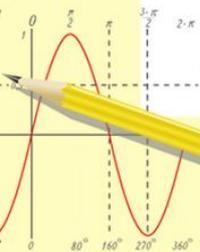
Аналогично

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$



$$\begin{array}{r}
 2500 \\
 \times 42 \\
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 \end{array}$$



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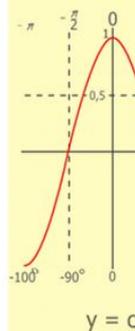
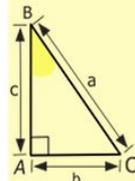
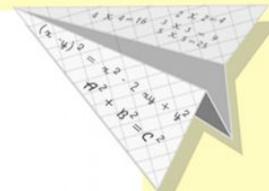
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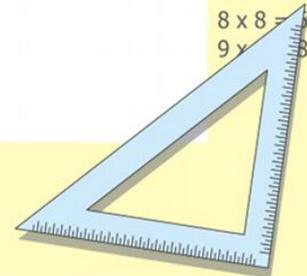
$$\begin{array}{l}
 y = 1 \\
 x = 25 + 45 \\
 \hline
 x = 70
 \end{array}$$

$$(x+y)(x-y) = x^2 - y^2$$



y = co

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 2 \times 2 = 4 \\
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 \end{array}$$



# Разложение произведения в сумму

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos 3x \cos x = \frac{1}{2} [\cos(3x + x) + \cos(3x - x)] = \frac{1}{2} (\cos 4x + \cos 2x)$$

$$\begin{aligned} \sin 50^\circ \sin 10^\circ &= \frac{1}{2} [\cos(50^\circ - 10^\circ) - \cos(50^\circ + 10^\circ)] = \\ &= \frac{1}{2} (\cos 40^\circ - \cos 60^\circ) = \frac{1}{2} \left( \cos 40^\circ - \frac{1}{2} \right) = \frac{1}{2} \cos 40^\circ - \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \sin \frac{\pi}{5} \cos \frac{3\pi}{8} &= \frac{1}{2} \left[ \sin \left( \frac{\pi}{5} + \frac{3\pi}{8} \right) + \sin \left( \frac{\pi}{5} - \frac{3\pi}{8} \right) \right] = \\ &= \frac{1}{2} \left[ \sin \frac{23\pi}{40} + \sin \frac{-7\pi}{40} \right] = \frac{1}{2} \left[ \sin \frac{23\pi}{40} - \sin \frac{7\pi}{40} \right] \end{aligned}$$

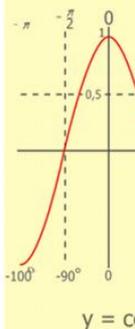
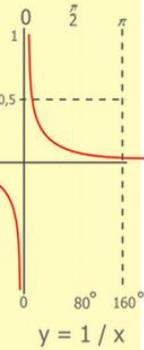
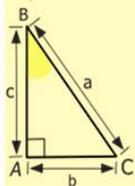
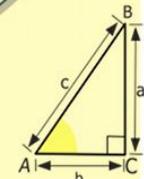
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$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

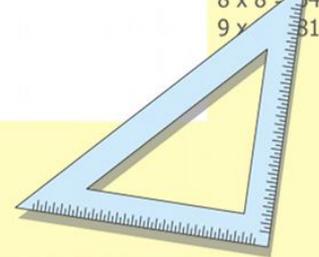
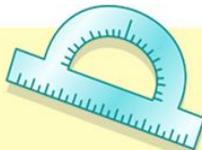
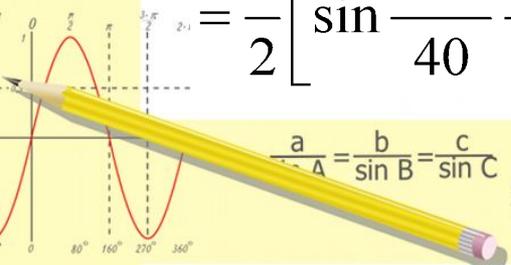
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$$(x+y)(x-y) = x^2 - y^2$$

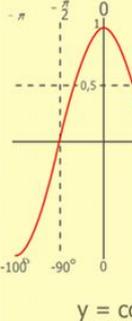
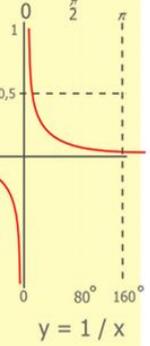
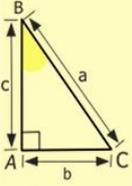
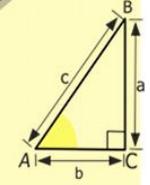
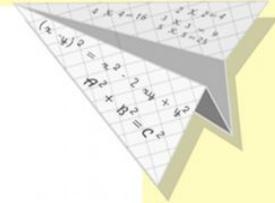
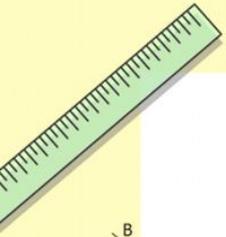


$\begin{array}{r} 12500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$

$2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$

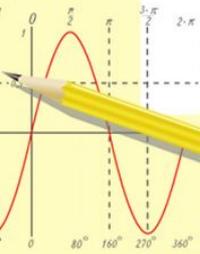


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преподаватель математики  
ГБПОУ ЗКНО  
Москва, 2020г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

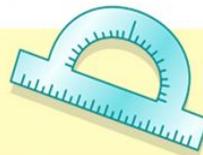
- $2 \times 2 = 4$
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- $6 \times 6 = 36$
- $7 \times 7 = 49$
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- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

