

# Спектральный анализ периодических процессов

$$\mathbf{C}_k = |\mathbf{C}_k| e^{j\varphi_k}$$

$$\varphi_k = \arg(\mathbf{C}_k)$$

$$|\mathbf{C}_k| = \frac{1}{2} \sqrt{a_k^2 + b_k^2},$$

$$\varphi_k = -\operatorname{arctg}\left(\frac{b_k}{a_k}\right)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t} = \sum_{k=-\infty}^{\infty} |\mathbf{C}_k| e^{j(k\omega_1 t + \varphi_k)}$$

$$g(t) = \mathbf{C}_0 + \sum_{k=1}^{\infty} \left( \mathbf{C}_k e^{jk\omega_1 t} + \mathbf{C}_{-k} e^{-jk\omega_1 t} \right) =$$

$$= \mathbf{C}_0 + 2 \sum_{k=1}^{\infty} |\mathbf{C}_k| \cos(k\omega_1 t + \varphi_k)$$

$$\mathbf{C}_k = \frac{2}{T} \frac{1}{\omega_1 k} \sin(\omega_1 k) = \frac{1}{\pi k} \sin\left(\frac{2\pi k}{T}\right), \quad k = \pm 1, \pm 2, \dots$$

$$\mathbf{C}_0 = \frac{1}{T} \int_{-1}^1 1 \cdot dt = \frac{2}{T}$$

$$\mathbf{C}'_k = a\mathbf{C}_k$$

$$\begin{aligned}\mathbf{C}'_k &= \frac{1}{T} \int_{-T/2}^{T/2} ag(t)e^{-j\omega_1 kt} dt = a \frac{1}{T} \int_{-T/2}^{T/2} g(t)e^{-j\omega_1 kt} dt = \\ &= a\mathbf{C}_k, \quad k = 0, \pm 1, \pm 2, \dots\end{aligned}$$

$$\mathbf{C}'_k = \mathbf{C}_{1k} + \mathbf{C}_{2k}$$

$$\mathbf{C}'_k = \frac{1}{T} \int_{-T/2}^{T/2} [g_1(t) + g_2(t)]e^{-j\omega_1 kt} dt = \mathbf{C}_{1k} + \mathbf{C}_{2k}, \quad k = 0, \pm 1, \pm 2, \dots$$

Разложение в ряд Фурье

$$a_1 g_1(t) + a_2 g_2(t) \quad \rightarrow \quad a_1 \mathbf{C}_{1k} + a_2 \mathbf{C}_{2k}$$

$$\mathbf{C}'_k = e^{-j\omega_1 k \tau} \mathbf{C}_k$$

Разложение в ряд Фурье

$$g(t - \tau) \rightarrow e^{-j\omega_1 k \tau} \mathbf{C}_k$$

$$g(t - \tau) \quad \mathbf{C}'_k = \frac{1}{T} \int_{-T/2}^{T/2} g(t - \tau) e^{-j\omega_1 k t} dt$$

$$u = (t - \tau)$$

$$dt = du, \quad -\frac{T}{2} - \tau < u < \frac{T}{2} - \tau$$

$$\begin{aligned} \mathbf{C}'_k &= \frac{1}{T} \int_{(-T/2) - \tau}^{(T/2) - \tau} g(u) e^{-j\omega_1 k (u + \tau)} du = \\ &= e^{-j\omega_1 k \tau} \frac{1}{T} \int_{(-T/2) - \tau}^{(T/2) - \tau} g(u) e^{-j\omega_1 k u} du = e^{-j\omega_1 k \tau} \mathbf{C}_k \end{aligned}$$

$$\langle g^2(t) \rangle = P_{cp} = \frac{1}{T} \int_0^T g^2(t) dt$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k \exp\left(\frac{2\pi j k t}{T}\right)$$

$$P_{cp} = \frac{1}{T} \int_0^T g^2(t) dt =$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_m \int_0^T \exp\left(\frac{2\pi j (k+m)t}{T}\right) dt$$

$$P_{cp} = \sum_{k=-\infty}^{\infty} |\mathbf{C}_k|^2$$

$$A_k = 2|C_k|$$

$$\langle g^2(t) \rangle = P_{cp} = \frac{1}{4} \sum_{k=-\infty}^{\infty} A_k^2$$

$$g(t) = i(t)$$

$$P_{cp} = \langle ri^2(t) \rangle = r \left[ I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2} + \dots \right]$$

$$|C_k|^2 \quad k=0$$

$$g(t) \quad [-T/2, T/2]$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t} \quad g(t) \quad \mathbf{C}_k$$

$$\|g(t)\|^2 = \sum_{k=-\infty}^{\infty} |\mathbf{C}_k|^2$$

$$\langle g(t), g(t) \rangle = \|g(t)\|^2$$

$$\begin{aligned} \left\langle \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t}, g(t) \right\rangle &= \left\langle \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t}, \sum_{l=-\infty}^{\infty} \mathbf{C}_l e^{jl\omega_1 t} \right\rangle = \\ &= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \langle \mathbf{C}_k e^{jk\omega_1 t}, \mathbf{C}_l e^{jl\omega_1 t} \rangle = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_l^* \langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \rangle \end{aligned}$$



$$\langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \rangle = \delta_{lk}$$

$$\sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_l^* \langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \rangle =$$

$$= \sum_{k=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_k^* = \sum_{k=-\infty}^{\infty} |\mathbf{C}_k|^2$$

$$-T/2 \leq t \leq T/2 \quad \|g(t)\|^2 = \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

$$\frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \sum_{k=-\infty}^{\infty} |\mathbf{C}_k|^2$$