Спектральный анализ периодических процессов

$$\mathbf{C}_k = \left|\mathbf{C}_k\right| e^{j\varphi_k}$$

$$\varphi_k = \arg(\mathbf{C}_k)$$

$$\left|\mathbf{C}_k\right| = \frac{1}{2}\sqrt{a_k^2 + b_k^2},$$

$$\varphi_k = -arctg\left(\frac{b_k}{a_k}\right)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t} = \sum_{k=-\infty}^{\infty} \left|\mathbf{C}_k\right| e^{j(k\omega_1 t + \varphi_k)}$$

$$g(t) = \mathbf{C}_0 + \sum_{k=1}^{\infty}\left(\mathbf{C}_k e^{jk\omega_1 t} + \mathbf{C}_{-k} e^{-jk\omega_1 t}\right) =$$

$$= \mathbf{C}_0 + 2\sum_{k=1}^{\infty} \left|\mathbf{C}_k\right| \cos\left(k\omega_1 t + \varphi_k\right)$$

$$\mathbf{C}_k = \frac{2}{T}\frac{1}{\omega_1 k}\sin(\omega_1 k) = \frac{1}{\pi k}\sin\left(\frac{2\pi k}{T}\right), \quad k = \pm 1, \pm 2, \ldots$$

$$\mathbf{C}_0 = \frac{1}{T}\int_{-1}^{1} 1 \cdot dt = \frac{2}{T}$$

$$\mathbf{C}'_k = a\mathbf{C}_k$$

$$\mathbf{C}'_k = \frac{1}{T}\int_{-T/2}^{T/2} a g(t) e^{-j\omega_1 kt} dt = a\frac{1}{T}\int_{-T/2}^{T/2} g(t) e^{-j\omega_1 kt} dt =$$
$$= a\mathbf{C}_k, \quad k = 0, \pm 1, \pm 2, \ldots$$

$$\mathbf{C}'_k = \mathbf{C}_{1k} + \mathbf{C}_{2k}$$

$$\mathbf{C}'_k = \frac{1}{T}\int_{-T/2}^{T/2} \left[g_1(t) + g_2(t)\right] e^{-j\omega_1 kt} dt = \mathbf{C}_{1k} + \mathbf{C}_{2k}, \quad k = 0, \pm 1, \pm 2, \ldots$$

$$a_1 g_1(t) + a_2 g_2(t) \xrightarrow{\text{Разложение в ряд Фурье}} a_1\mathbf{C}_{1k} + a_2\mathbf{C}_{2k}$$

$$\mathbf{C}'_k = e^{-j\omega_1 k\tau}\mathbf{C}_k$$

$$g(-\tau) \xrightarrow{\text{Разложение в ряд Фурье}} e^{-j\omega_1 k\tau}\mathbf{C}_k$$

$$g(t-\tau) \qquad \mathbf{C}'_k = \frac{1}{T}\int_{-T/2}^{T/2} g(t-\tau)e^{-j\omega_1 kt}dt$$

$$u = (t-\tau)$$

$$dt = du, \quad -\frac{T}{2}-\tau < u < \frac{T}{2}-\tau$$

$$\mathbf{C}'_k = \frac{1}{T}\int_{(-T/2)-\tau}^{(T/2)-\tau} g(u)e^{-j\omega_1 k(u+\tau)}du =$$

$$= e^{-j\omega_1 k\tau}\frac{1}{T}\int_{(-T/2)-\tau}^{(T/2)-\tau} g(u)e^{-j\omega_1 ku}du = e^{-j\omega_1 k\tau}\mathbf{C}_k$$

$$\left\langle g^2(t) \right\rangle = P_{cp} = \frac{1}{T}\int_0^T g^2(t)dt$$

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k \exp\left(\frac{2\pi jkt}{T}\right)$$

$$P_{cp} = \frac{1}{T}\int_0^{\infty} g^2(t)dt =$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_m \int_0^{\infty} \exp\left(\frac{2\pi j(k+m)t}{T}\right)dt$$

$$P_{cp} = \sum_{k=-\infty}^{\infty} \left|\mathbf{C}_k\right|^2$$

$$A_k = 2|\mathbf{C}_k|$$

$$\langle g^2(t) \rangle = P_{cp} = \frac{1}{4} \sum_{k=-\infty}^{\infty} A_k^2$$

g(t)=i(t)

$$P_{cp} = \langle r i^2(t) \rangle = r\left[I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2} + ... \right]$$

$|\mathbf{C}_k|^2$ *k=0*

g(t) [-T/2, T/2]

$$g(t) = \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t}$$

g(t) $\mathbf{C}_k$

$$\left\| g(t) \right\|^2 = \sum_{k=-\infty}^{\infty} \left| \mathbf{C}_k \right|^2$$

$$\left\langle g(t), g(t) \right\rangle = \left\| g(t) \right\|^2$$

$$\left\langle \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t}, g(t) \right\rangle = \left\langle \sum_{k=-\infty}^{\infty} \mathbf{C}_k e^{jk\omega_1 t}, \sum_{l=-\infty}^{\infty} \mathbf{C}_l e^{jl\omega_1 t} \right\rangle =$$

$$= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \left\langle \mathbf{C}_k e^{jk\omega_1 t}, \mathbf{C}_l e^{jl\omega_1 t} \right\rangle = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_l^* \left\langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \right\rangle$$

$$\left\langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \right\rangle = \delta_{lk}$$

$$\sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_l^* \left\langle e^{jk\omega_1 t}, e^{jl\omega_1 t} \right\rangle =$$

$$= \sum_{k=-\infty}^{\infty} \mathbf{C}_k \mathbf{C}_k^* = \sum_{k=-\infty}^{\infty} \left|\mathbf{C}_k\right|^2$$

$$-T/2 \le t \le T/2$$

$$\left\| g(t) \right\|^2 = \frac{1}{T} \int_{-T/2}^{T/2} \left| g(t) \right|^2 dt$$

$$\frac{1}{T} \int_{-T/2}^{T/2} \left| g(t) \right|^2 dt = \sum_{k=-\infty}^{\infty} \left|\mathbf{C}_k\right|^2$$