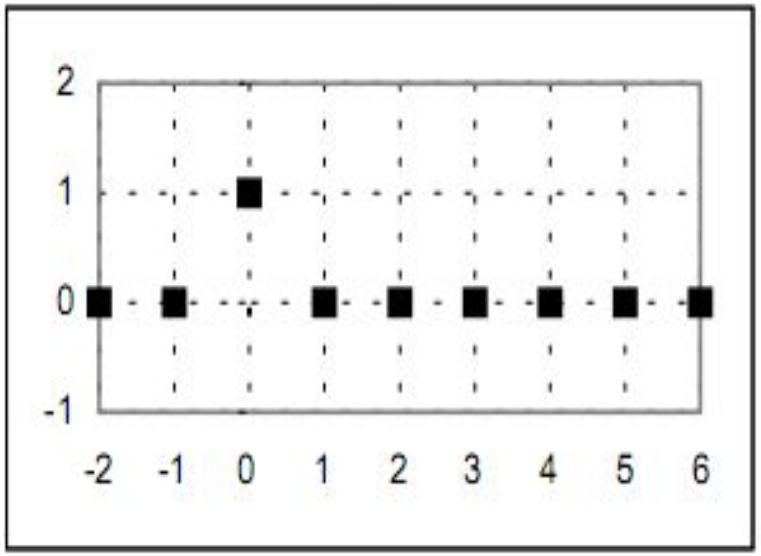
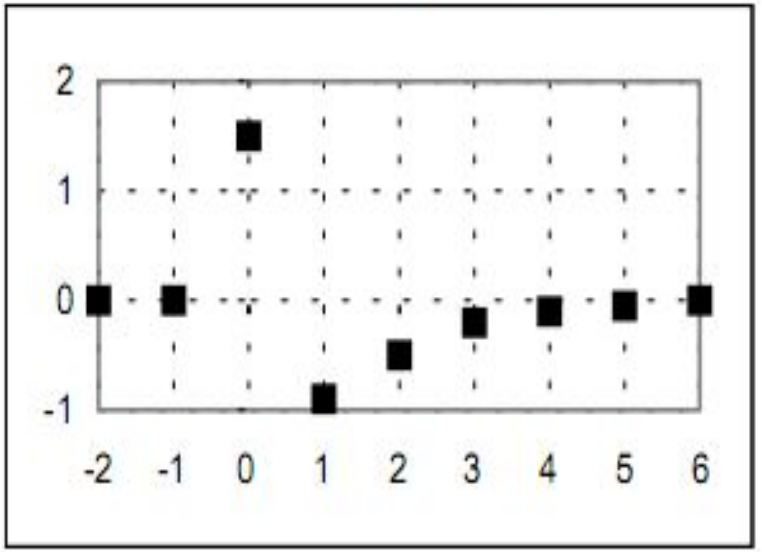
Свертка

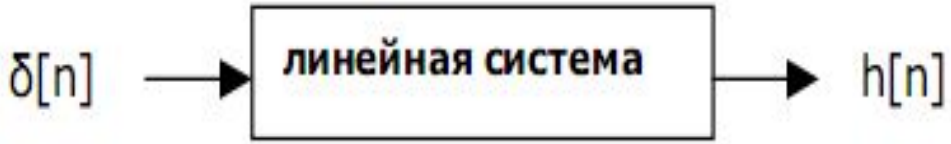
Определение дельта функции и импульсной характеристики.

дельта-функция

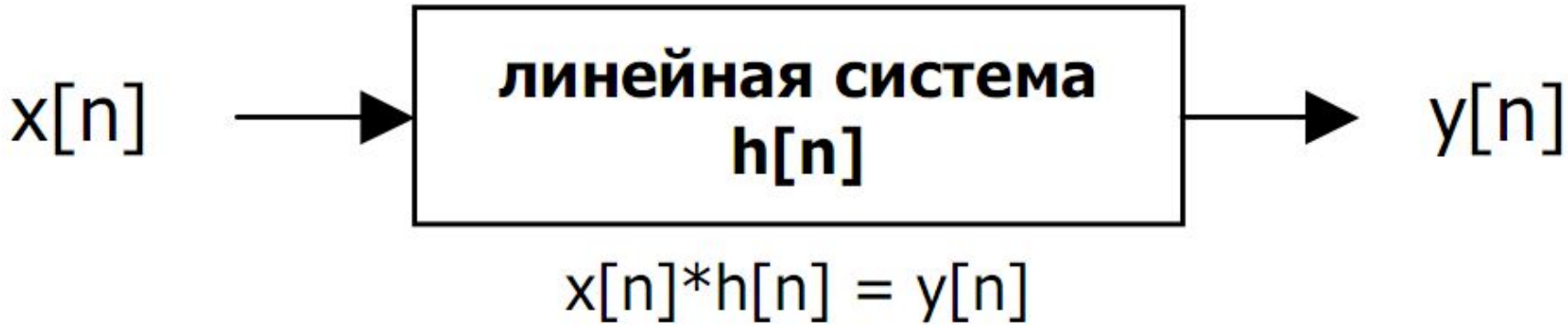


импульсная характеристика

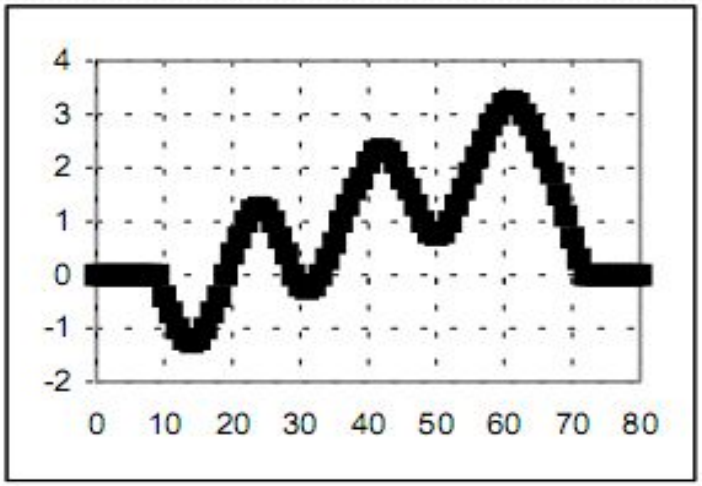




Использование операции свертки в ЦОС.



Фильтр нижних частот



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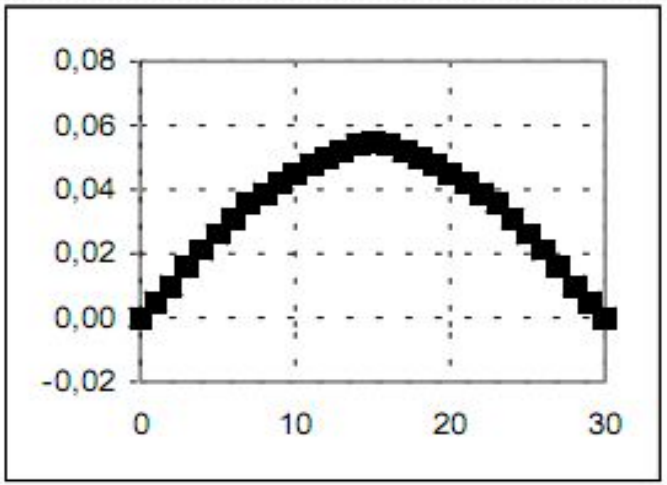

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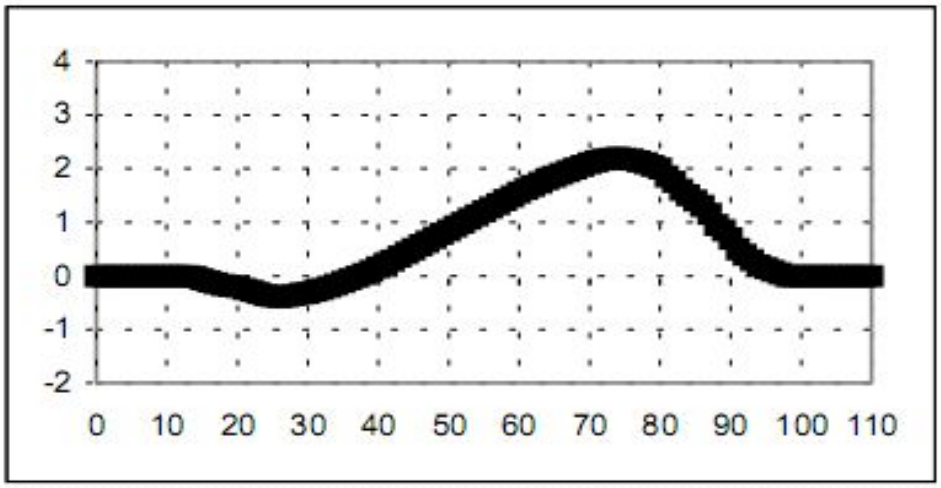
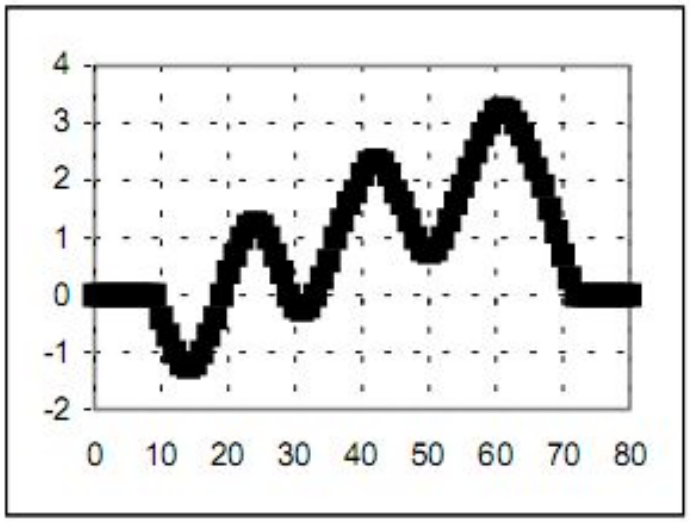


Фильтр высоких частот



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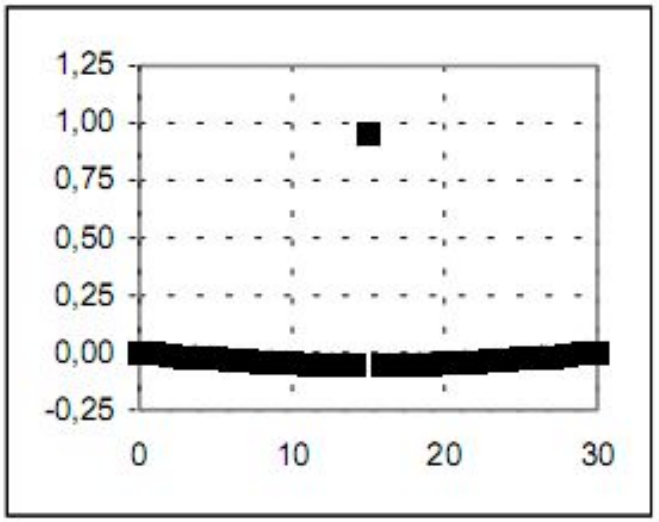

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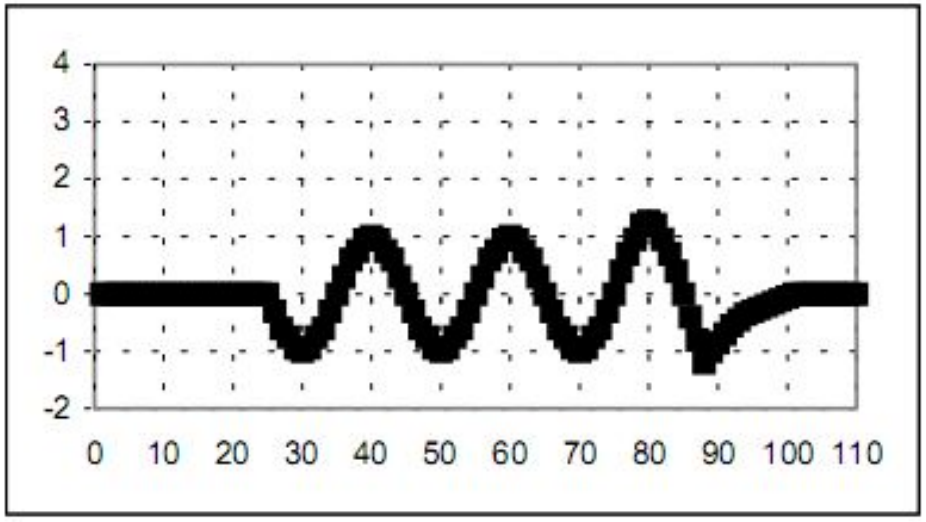
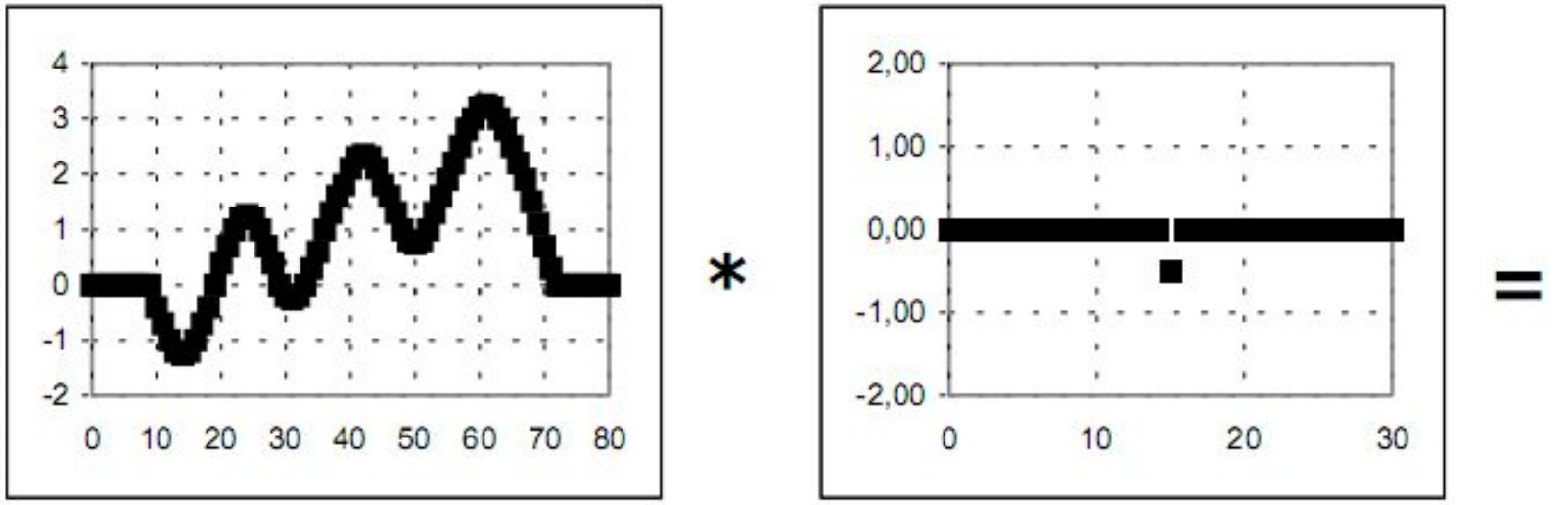
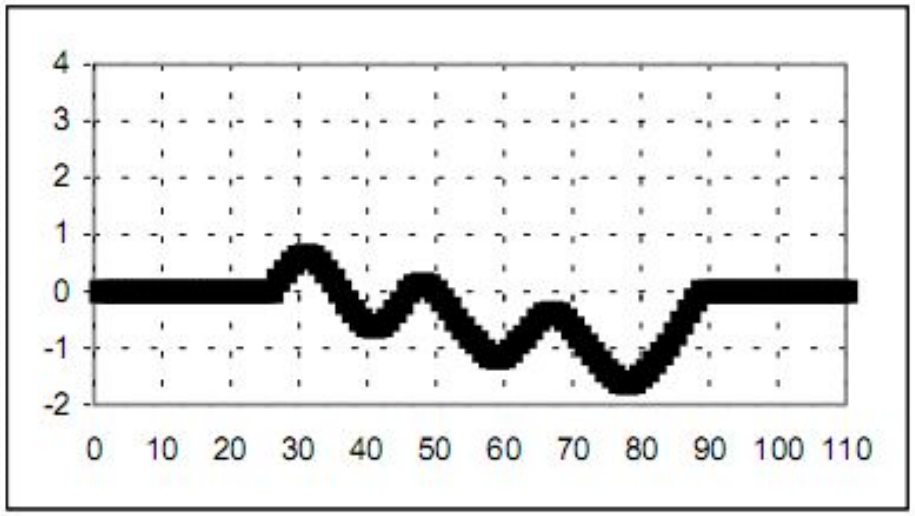
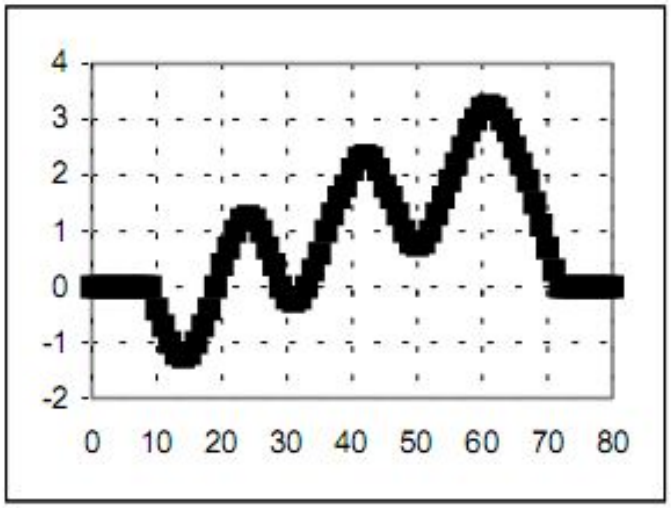


Инвертирующий аттенюатор





Дискретная производная



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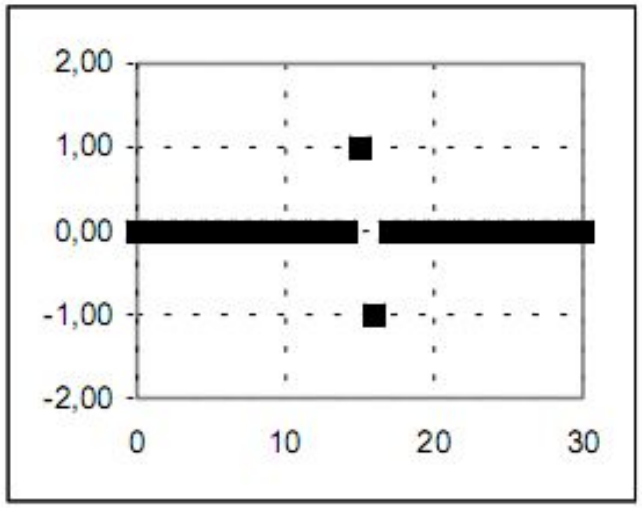

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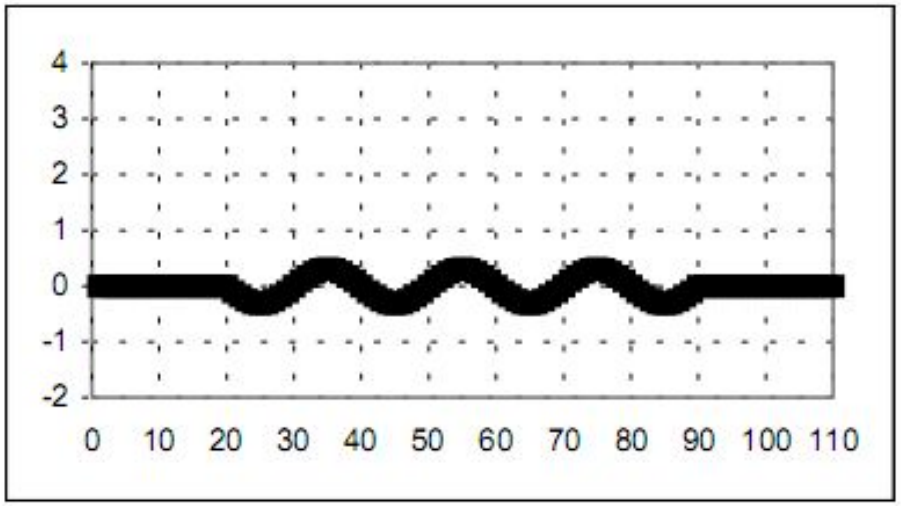
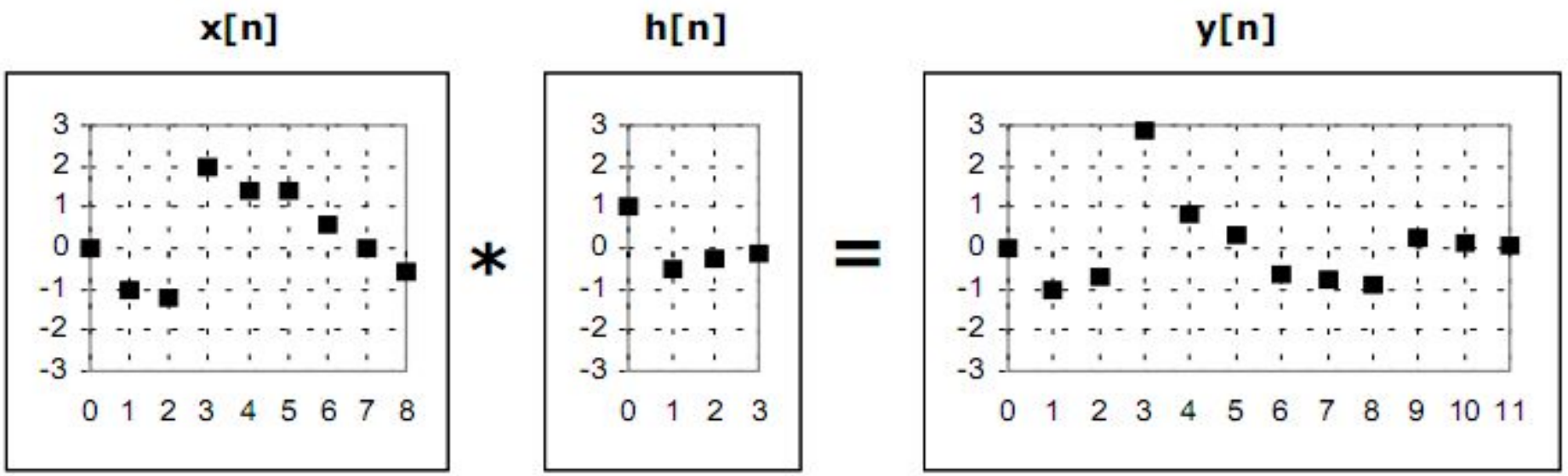
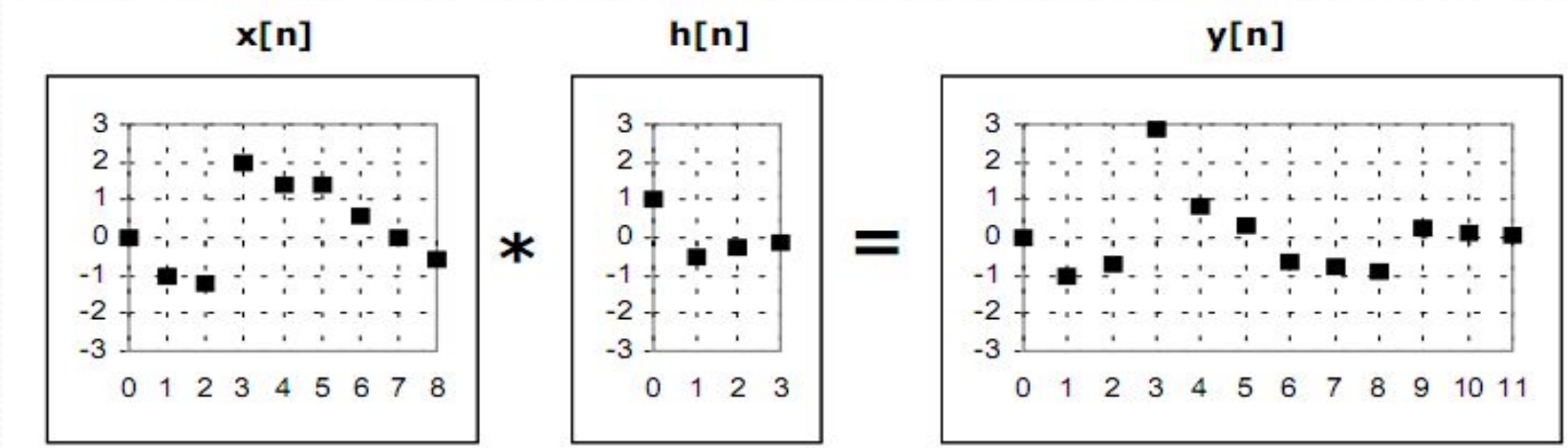
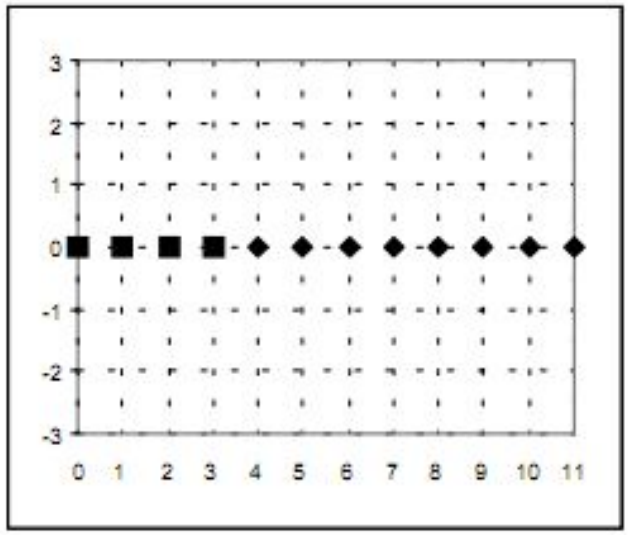


Пример операции свертки.

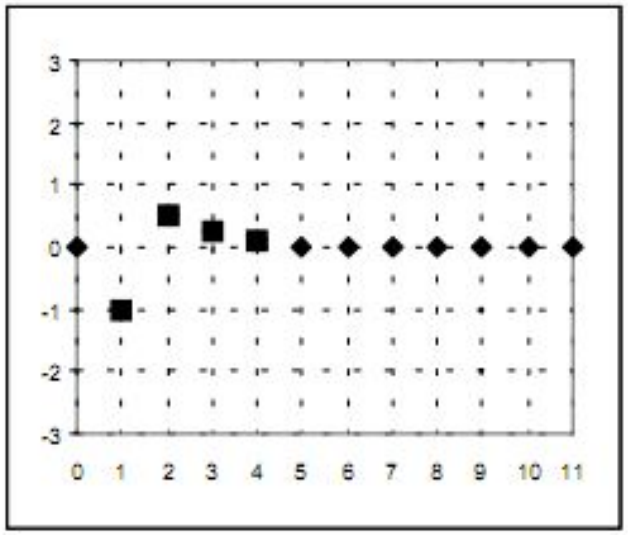


Пример операции свертки.

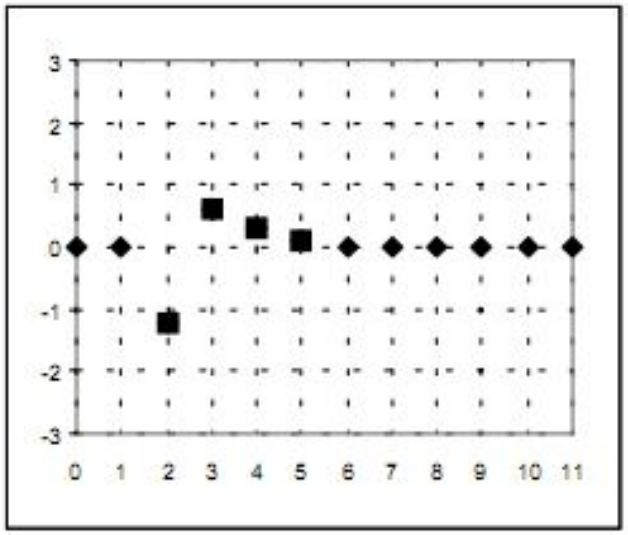




Точка 0

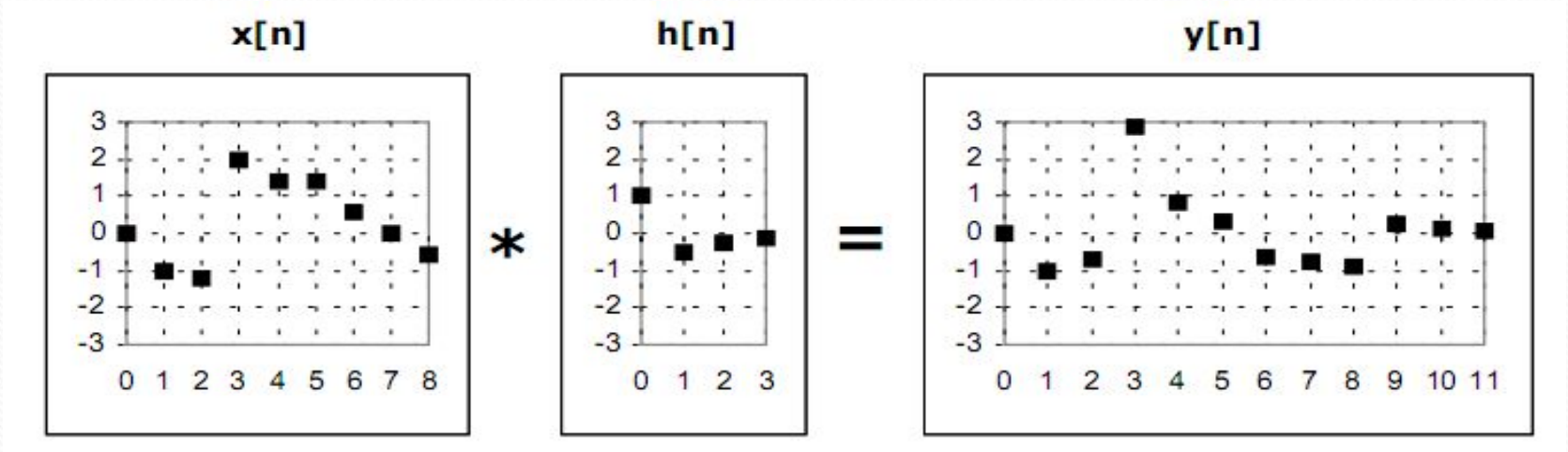


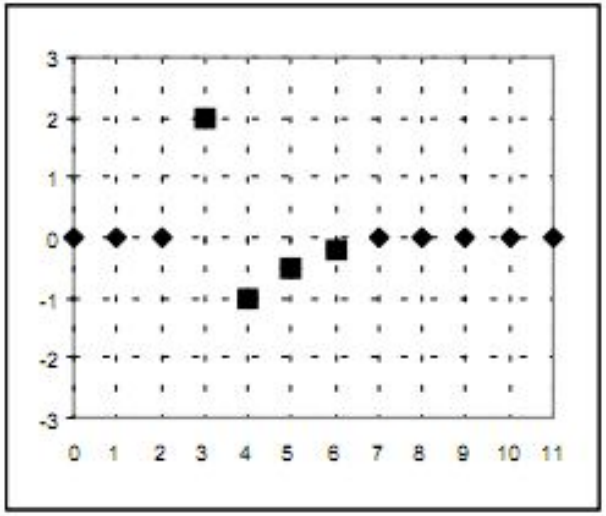
Точка 1



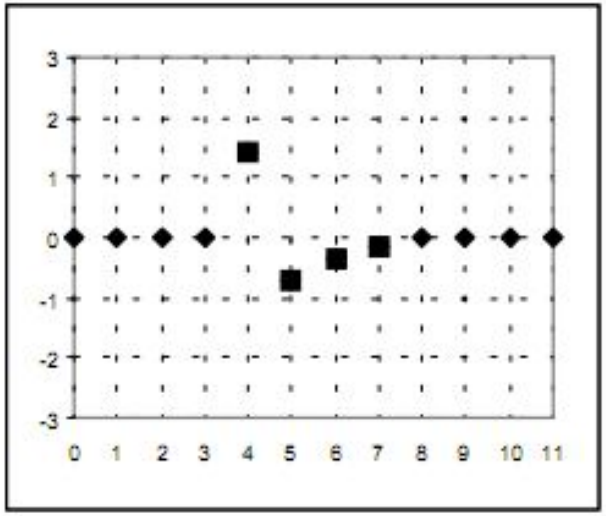
Точка 2

Пример операции свертки.

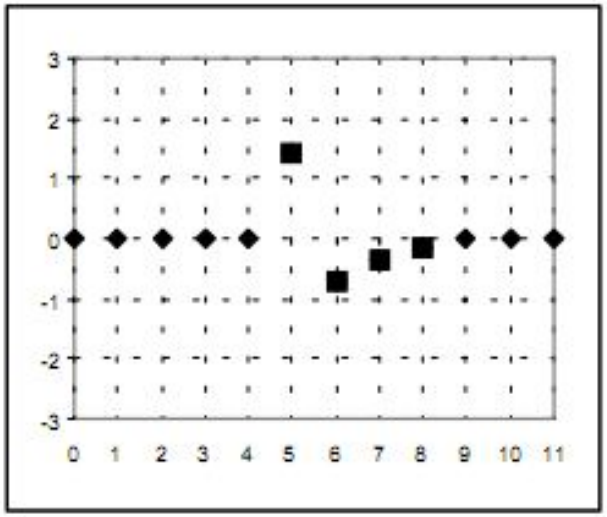




Точка 3

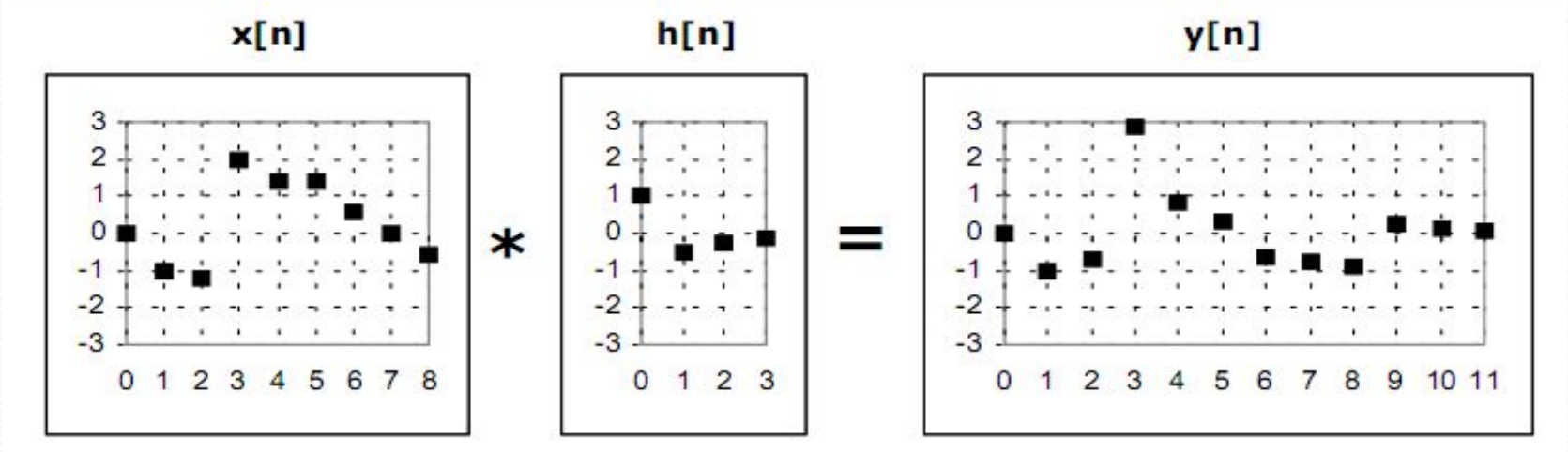


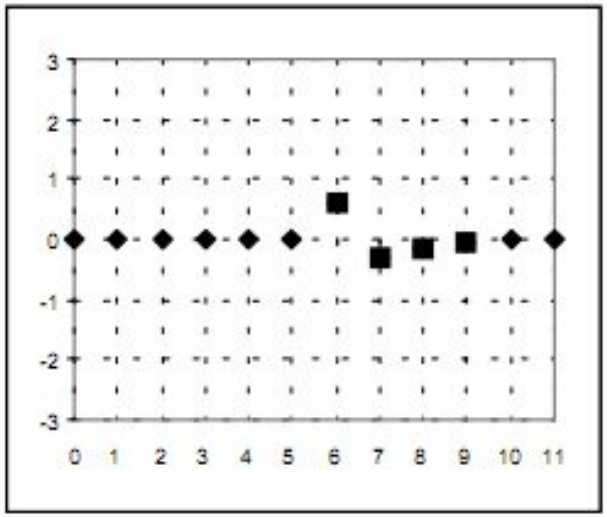
Точка 4



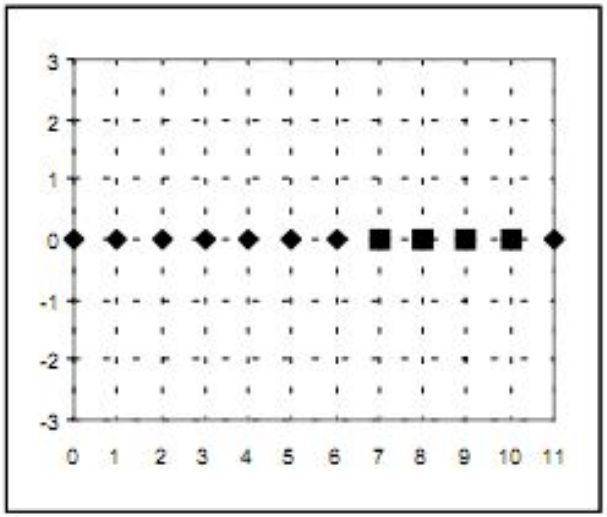
Точка 5

Пример операции свертки.

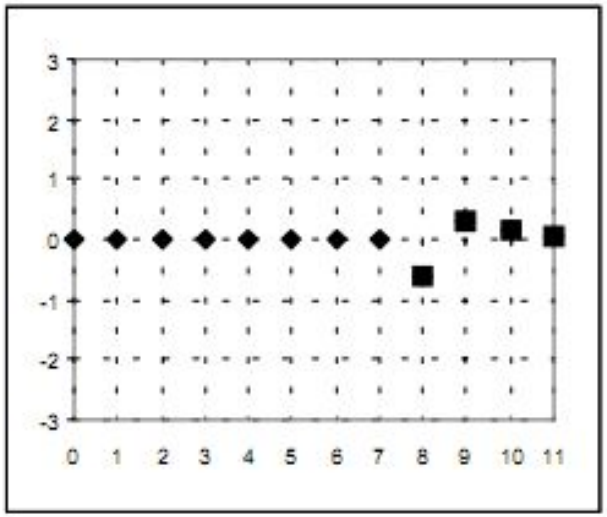




Точка 6



Точка 7

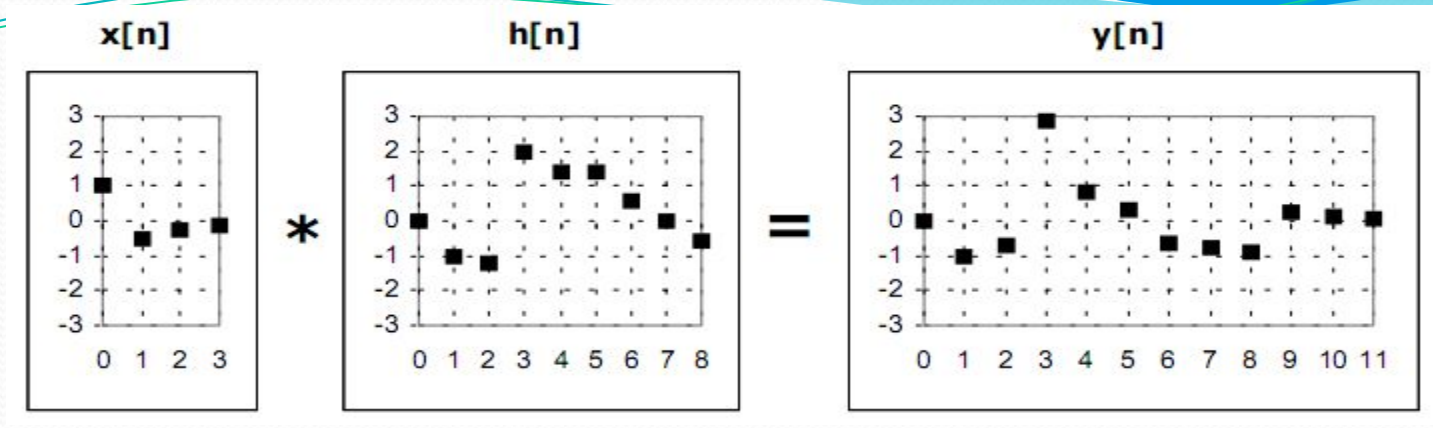


Точка 8

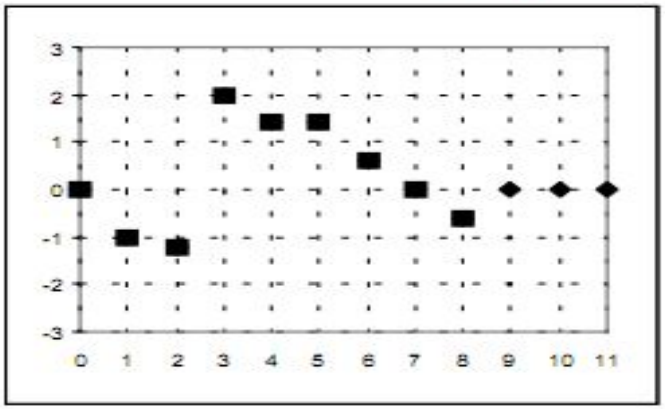
Алгоритм расчета свертки

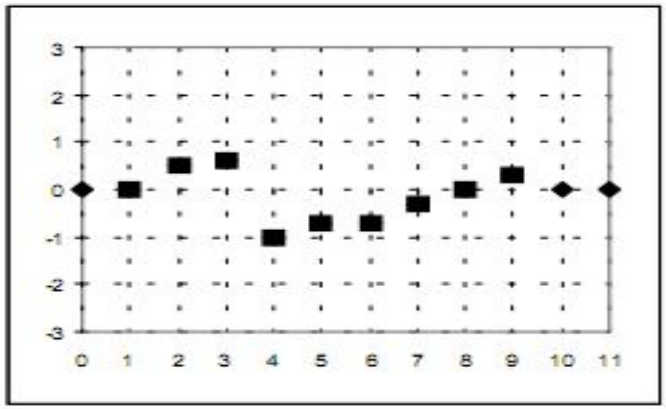
```
100 'РАСЧЕТ СВЕРТКИ, ИСПОЛЬЗУЮЩИЙ АЛГОРИТМ ВХОДНОЙ СТОРОНЫ
110 '
120 DIM X[80]                'входной сигнал, 81 точка
130 DIM H[30]                'импульсная характеристика, 31 точка
140 DIM Y[110]               'выходной сигнал, 111 точка
150 '
160 GOSUB XXXX               'фиктивная подпрограмма для загрузки X[] и H[]
170 '
180 FOR I% = 0 TO 110        'обнуление выходного массива
190     Y(I%) = 0
200 NEXT I%
210 '
220 FOR I% = 0 TO 80         'цикл для каждой точки в X[]
230     FOR J% = 0 TO 30     'цикл для каждой точки в H[]
240         Y[I%+J%] = Y[I%+J%] + X[I%] * H[J%]
250     NEXT J%
260 NEXT I% '(помните, что * это операция умножения!)
270 '
280 GOSUB XXXX               'фиктивная подпрограмма для сохранения Y[]
290 '
300 END
```

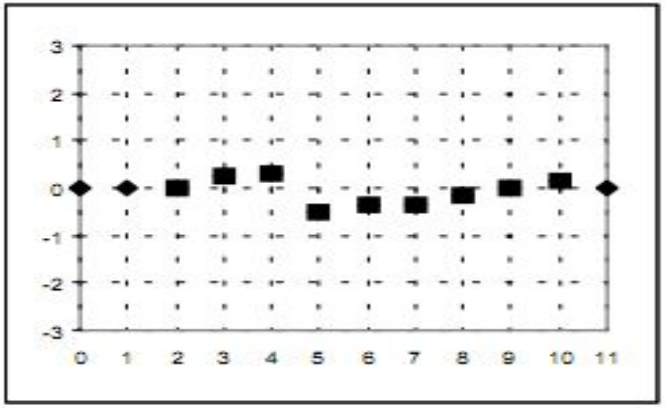
Пример операции свертки.

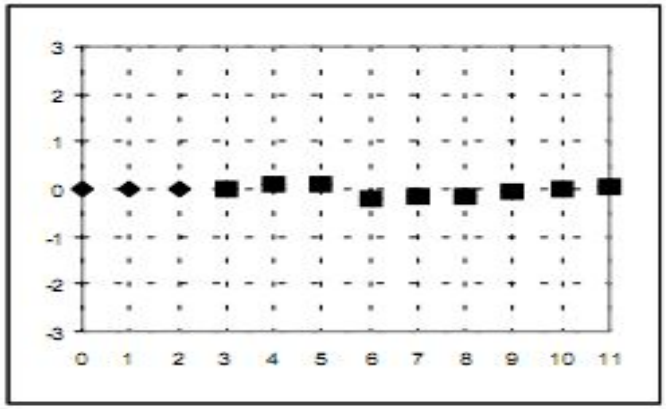


Выходные составляющие сигнала

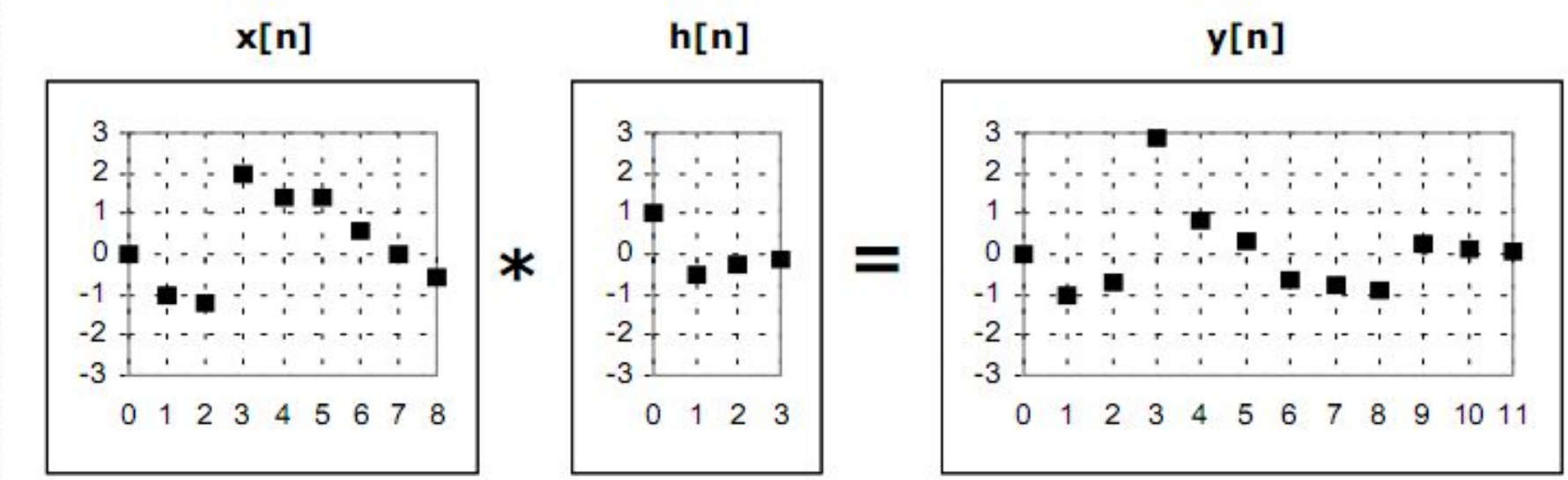


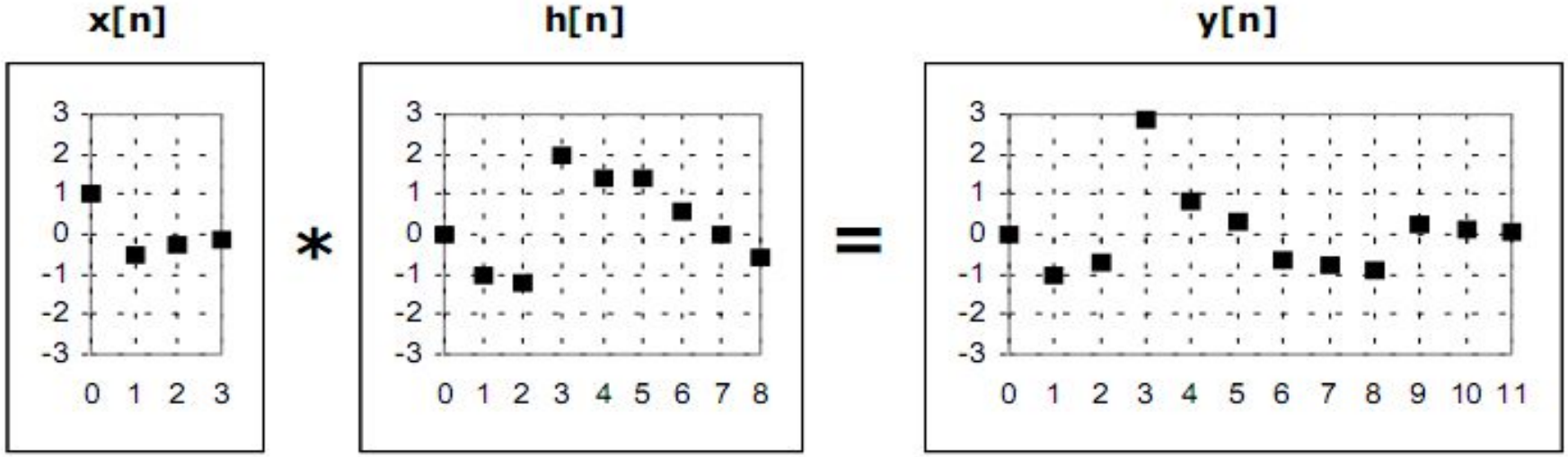


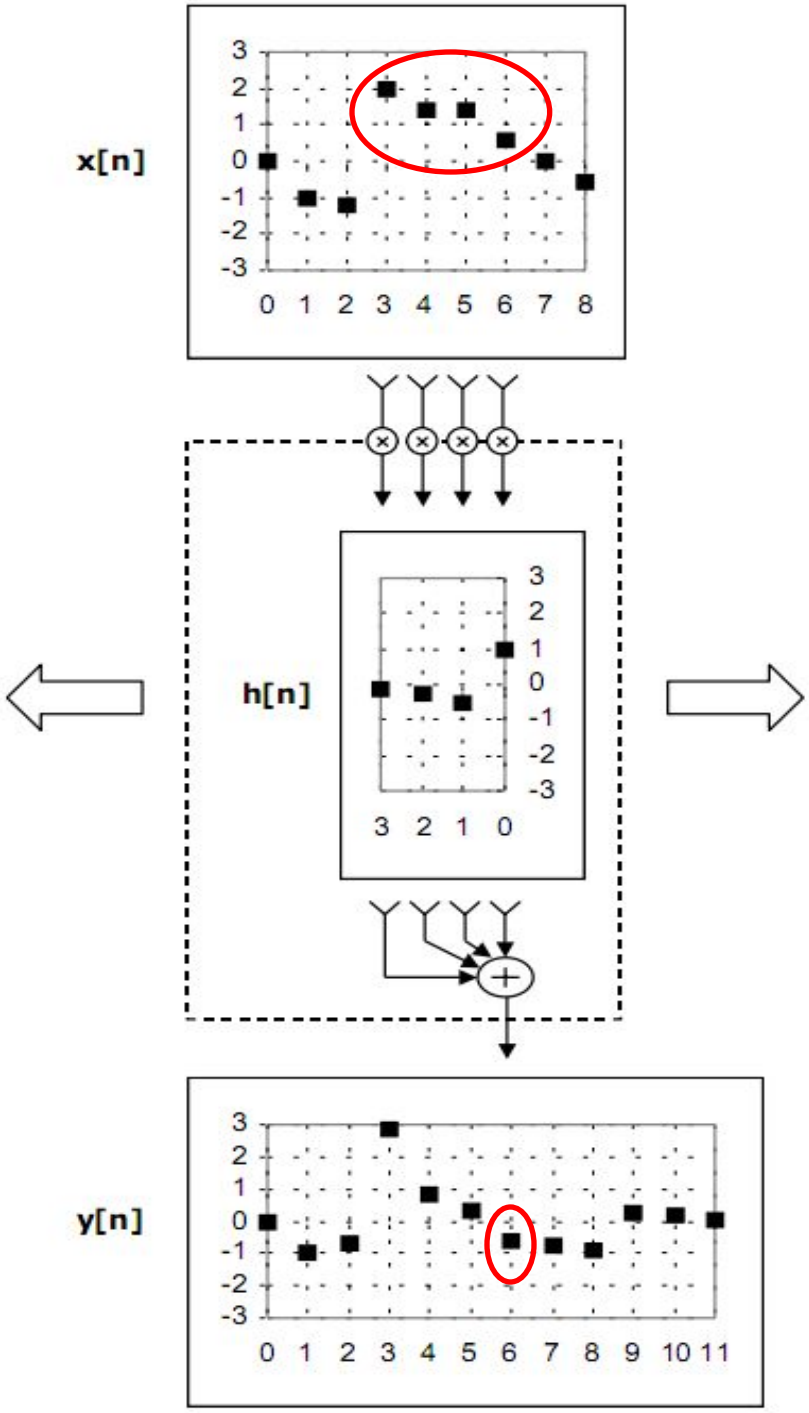


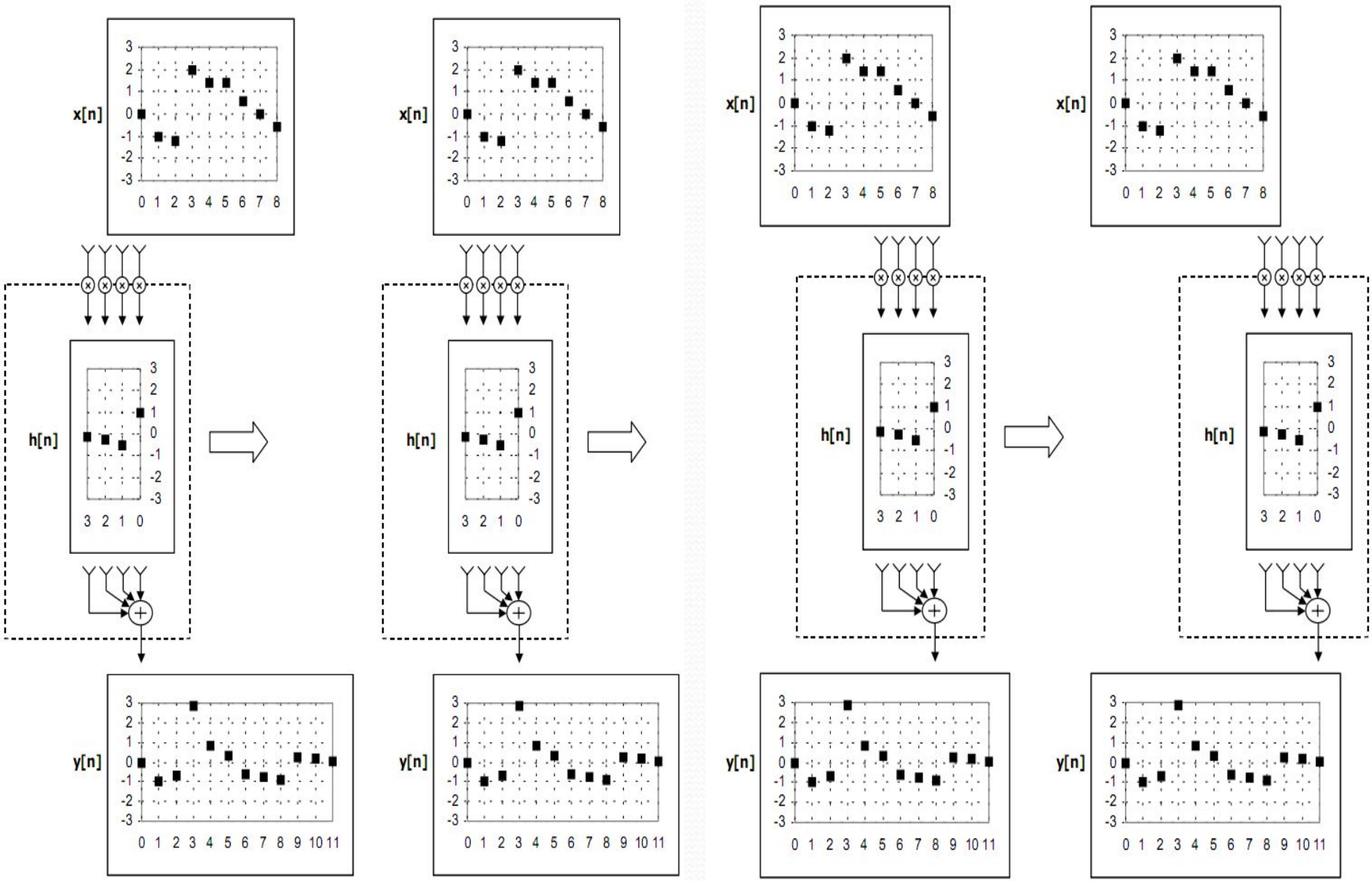


Пример операции свертки.






x[n]
h[n]
y[n]
3 2 1 0



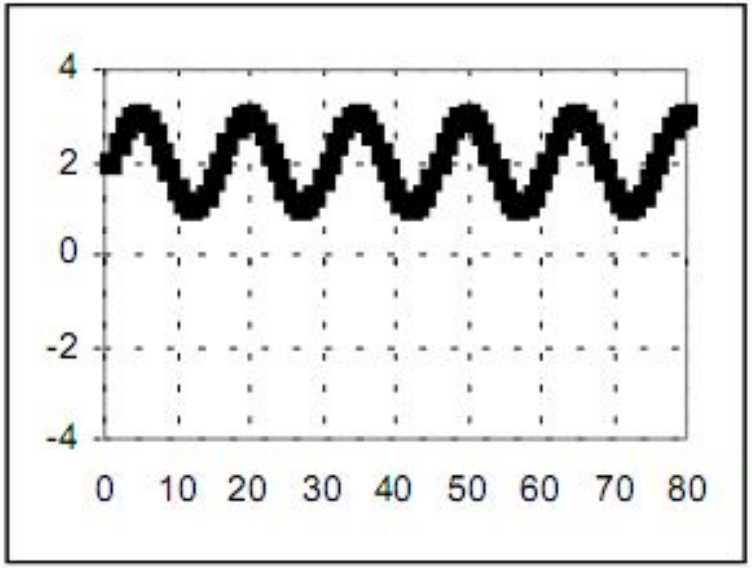
А. установлено для расчета x[0]

Б. установлено для расчета x[3]

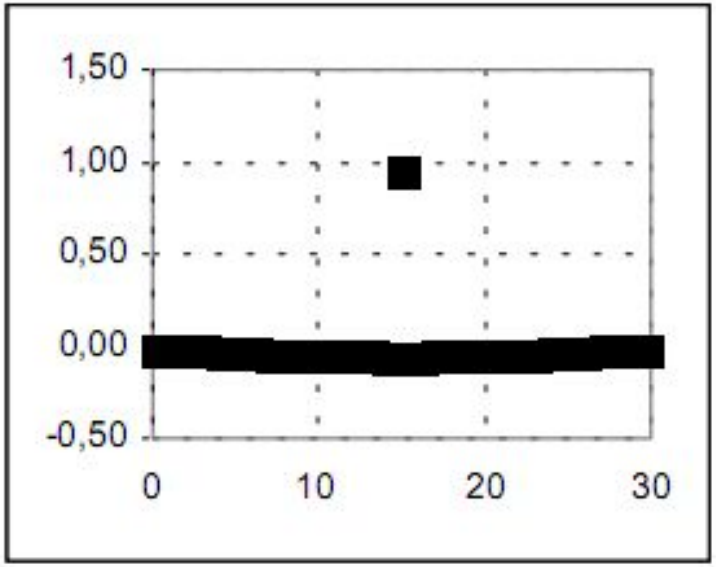
В. установлено для расчета x[8]

Г. установлено для расчета x[11]

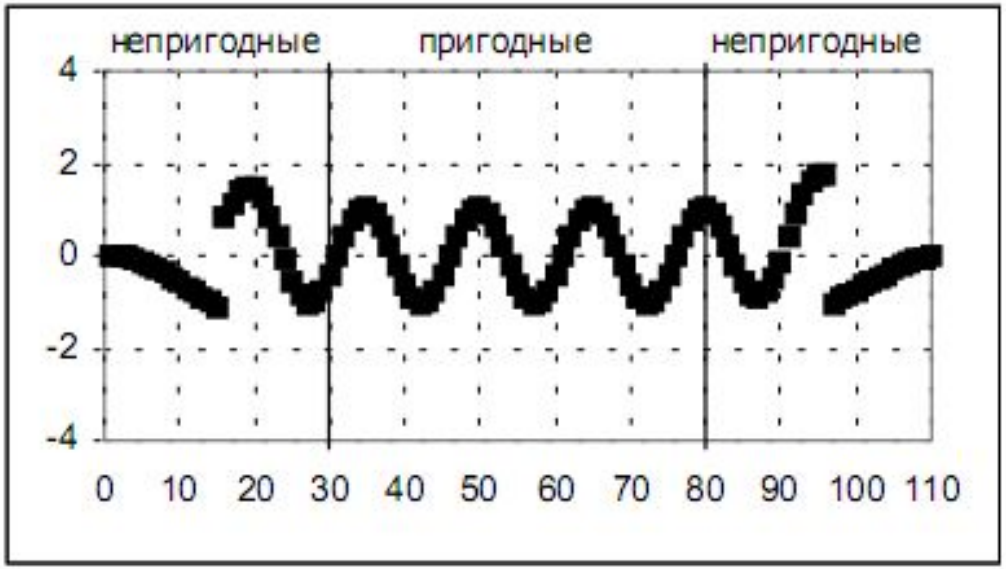
сигнала.


4
2
0
-2
-4
0 10 20 30 40 50 60 70 80

*


1,50
1,00
0,50
0,00
-0,50
0 10 20 30

=


непригодные
пригодные
непригодные
4
2
0
-2
-4
0 10 20 30 40 50 60 70 80 90 100 110

Вычисление дискретной свертки

$$y[i] = \sum_{j=0}^{M-1} h[j]\, x[i-j]$$

Где x[n] – N-точечный сигнал с индексом от 0 до N-1,
h[n] – M-точечный сигнал с индексом от 0 до M-1,

Алгоритм расчета свертки

```
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110 '
120 DIM X[80]                 'входной сигнал, 81 точка
130 DIM H[30]                 'импульсная характеристика, 31 точка
140 DIM Y[110]                'выходной сигнал, 111 точка
150 '
160 GOSUB XXXX                'фиктивная подпрограмма для загрузки X[ ] и H[ ]
170 '
180 FOR I% = 0 TO 110         'цикл по каждой выборке в Y[ ]
190     Y[I%] = 0             'обнуление значения выборки выходного массива
200     FOR J% = 0 TO 30      'цикл по каждой выборке в H[ ]
210         IF (I%-J% < 0) THEN GOTO 240
220         IF (I%-J% > 80) THEN GOTO 240
230         Y(I%) = Y(I%) + H(J%) * X(I%-J%)
240     NEXT J%
250 NEXT I%
260 '
270 GOSUB XXXX                'фиктивная подпрограмма для сохранения Y[ ]
280 '
290 END
```
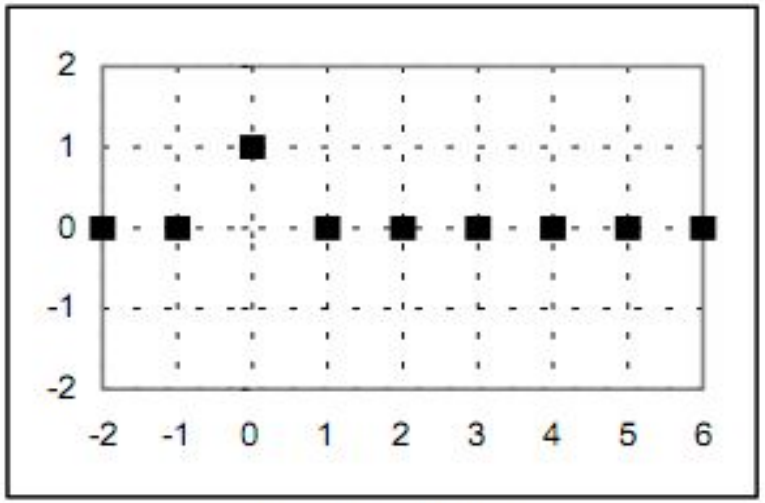
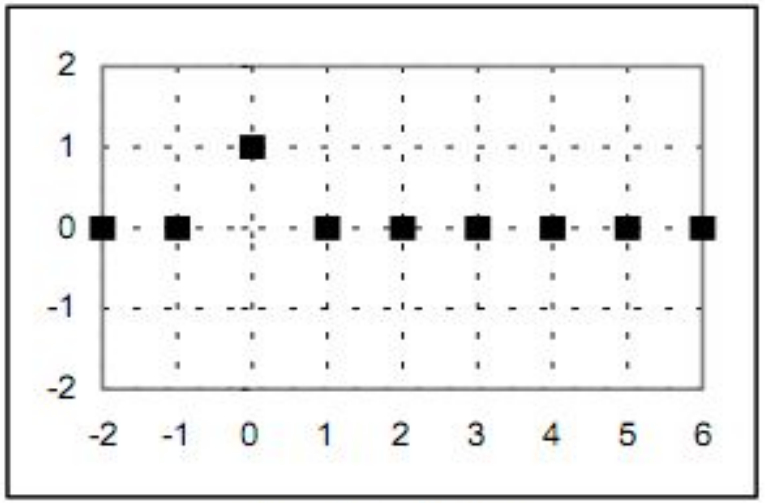
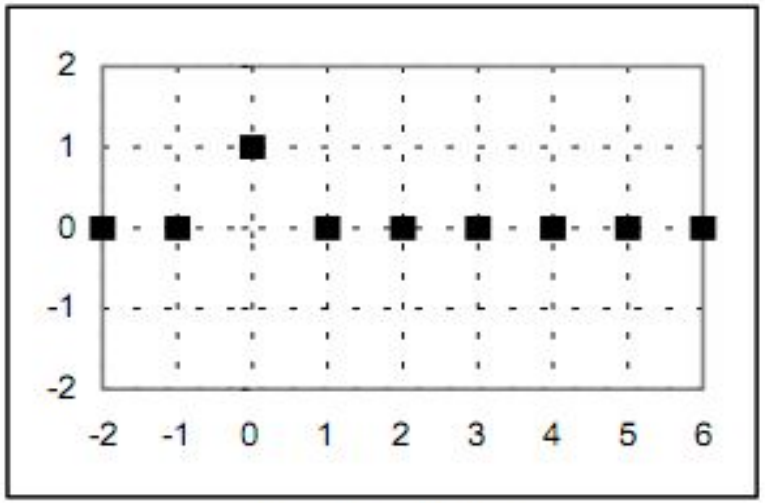
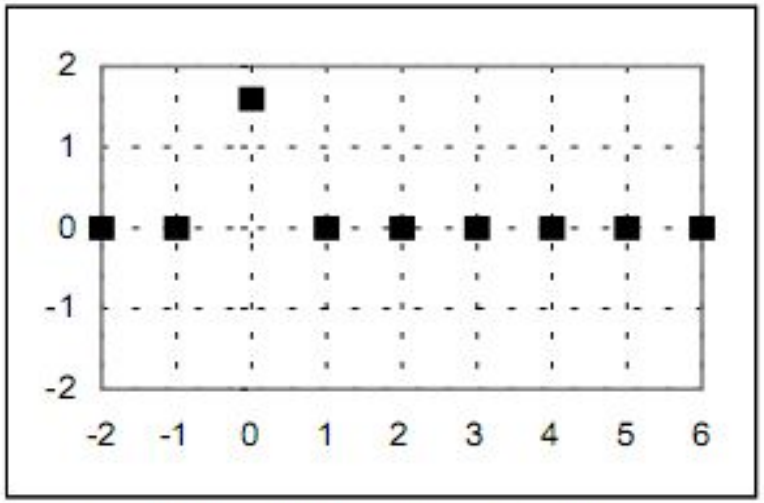

СВОЙСТВА СВЕРТКИ

Распространенные импульсные характеристики









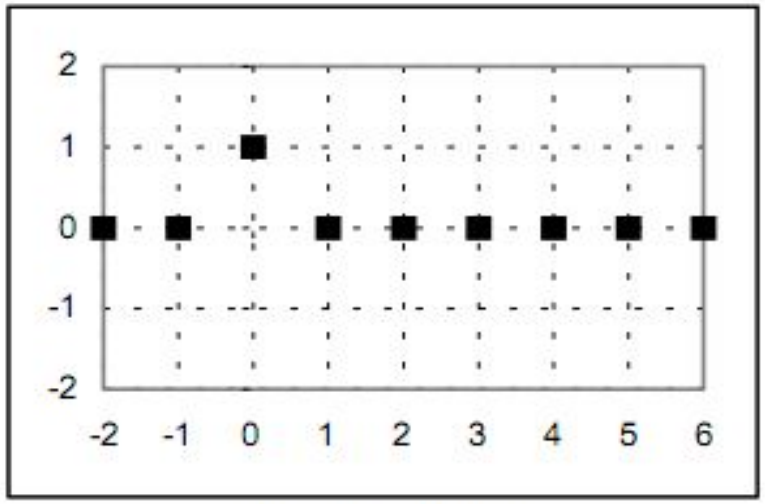
а. Тождественность

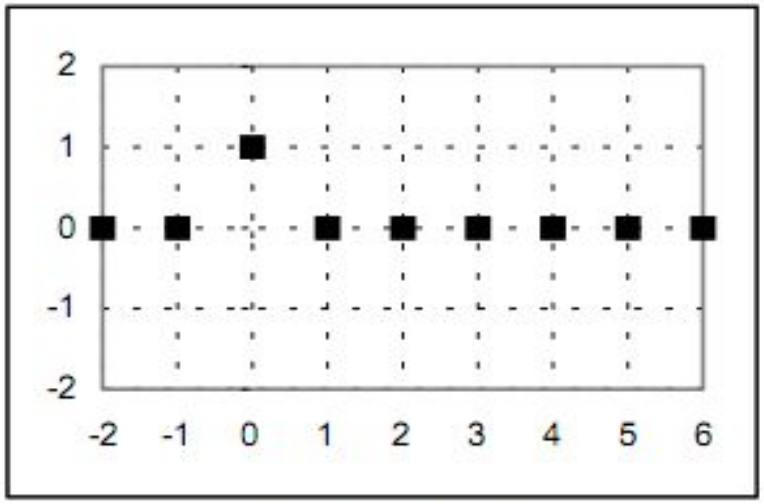
б. Усиление & Ослабление

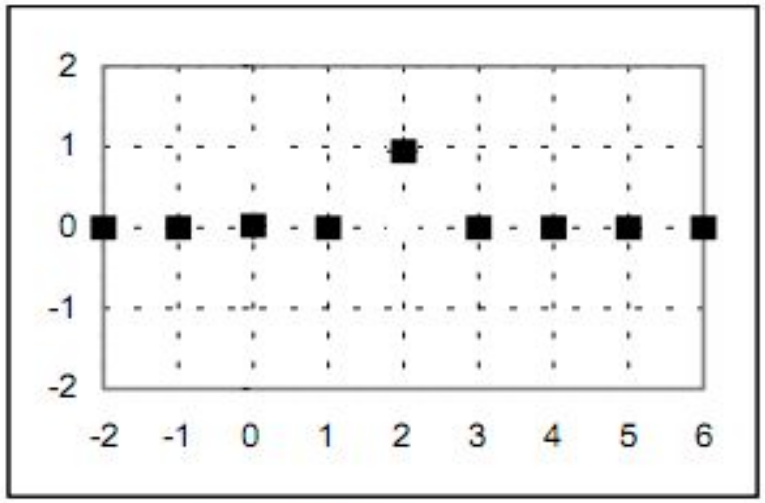
$$x[n] * \delta[n] = x[n]$$

$$x[n] * k\delta[n] = kx[n]$$

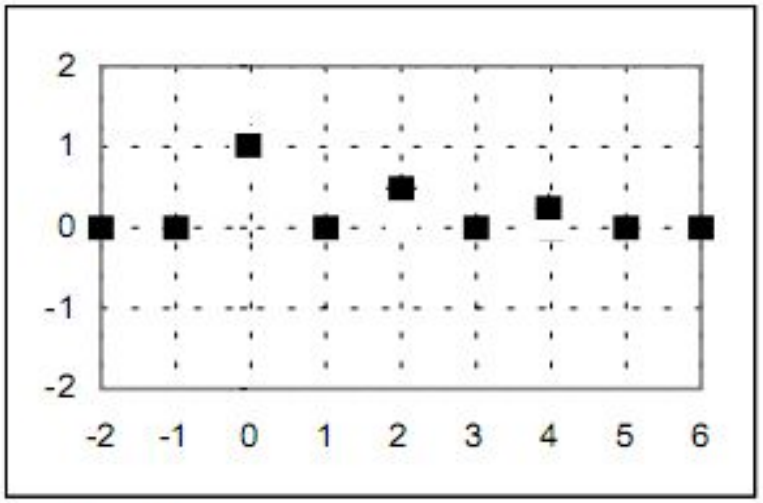
Распространенные импульсные характеристики







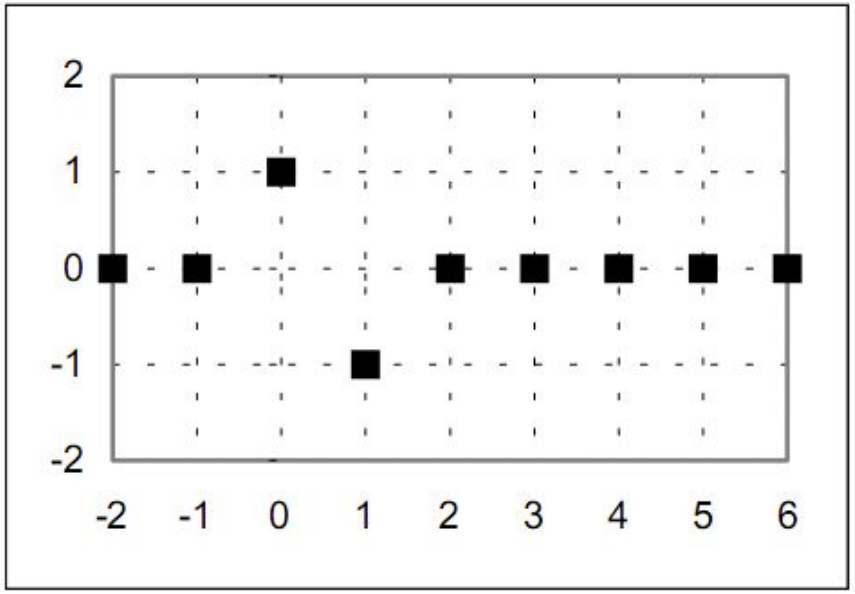
в. Сдвиг

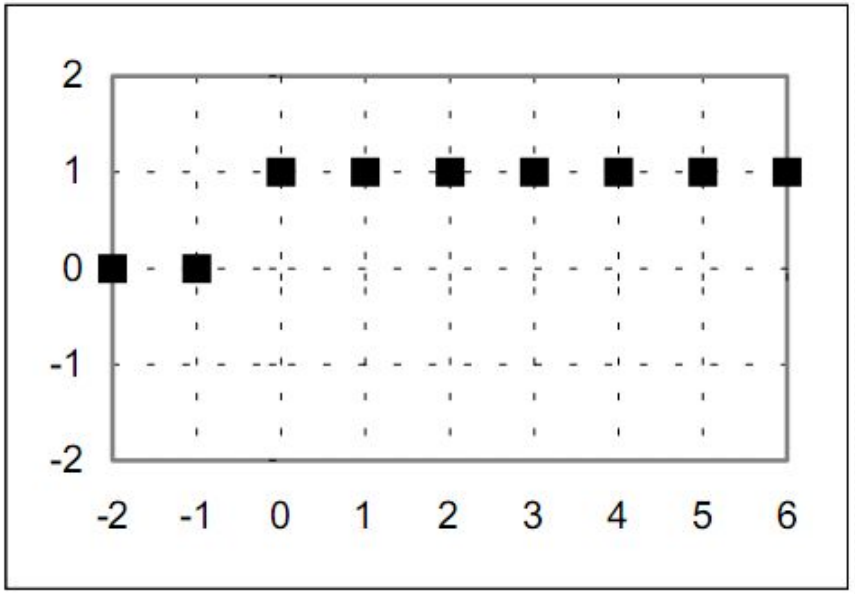


г. Эхо

$$x[n] * \delta[n+s] = x[n+s]$$

Дискретная производная и дискретный интеграл





$$y[n] = x[n] - x[n-1]$$

Дискретная производная

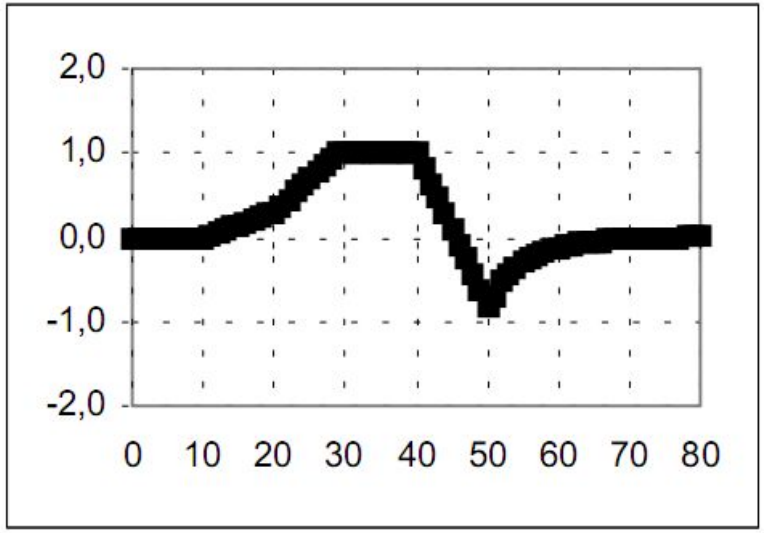
$$y[n] = x[n] + y[n-1]$$

Дискретный интеграл

Дискретная производная и дискретный интеграл

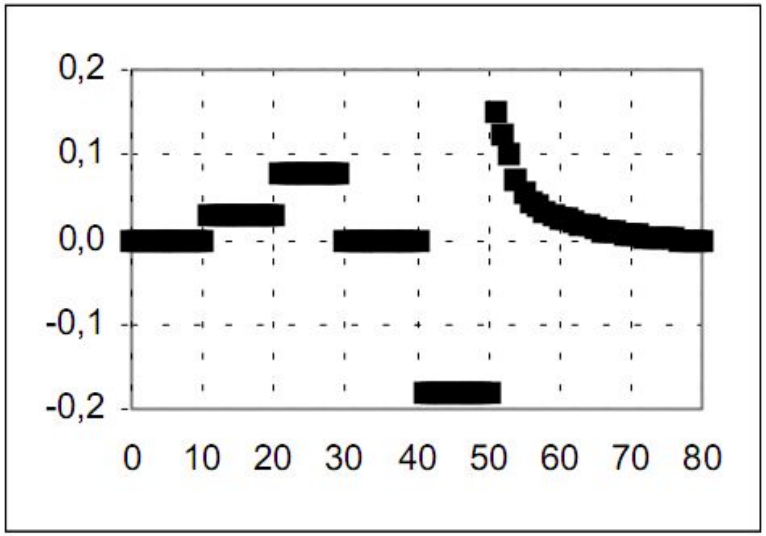
$$y[n] = x[n] + y[n-1]$$

Дискретный интеграл




```
110 Y[0] = X[0]
120 FOR I% =1 TO N%-1
130   Y[I%]=Y[I%-1]+X[I%]
140 NEXT I%
```

```
110 Y[0] = 0
120 FOR I% = 1 TO N%-1
130   Y[I%]=X[I%]-X[I%-1]
140 NEXT I%
```

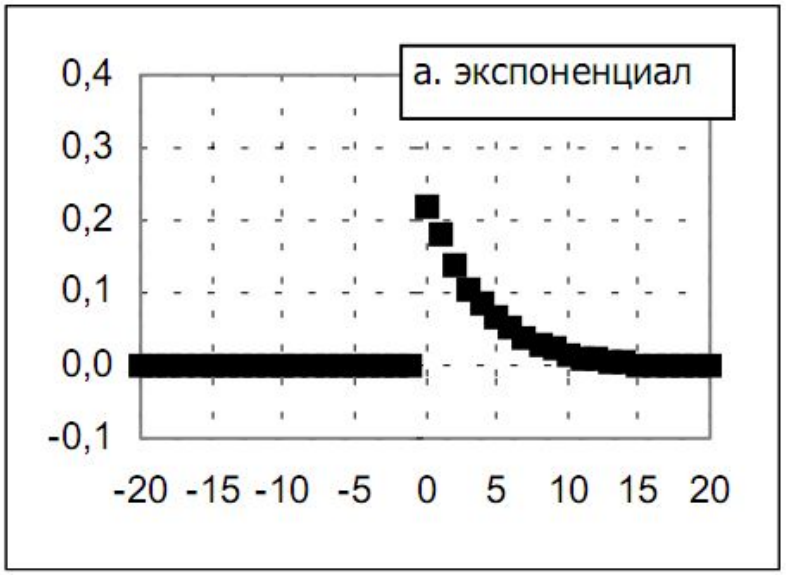


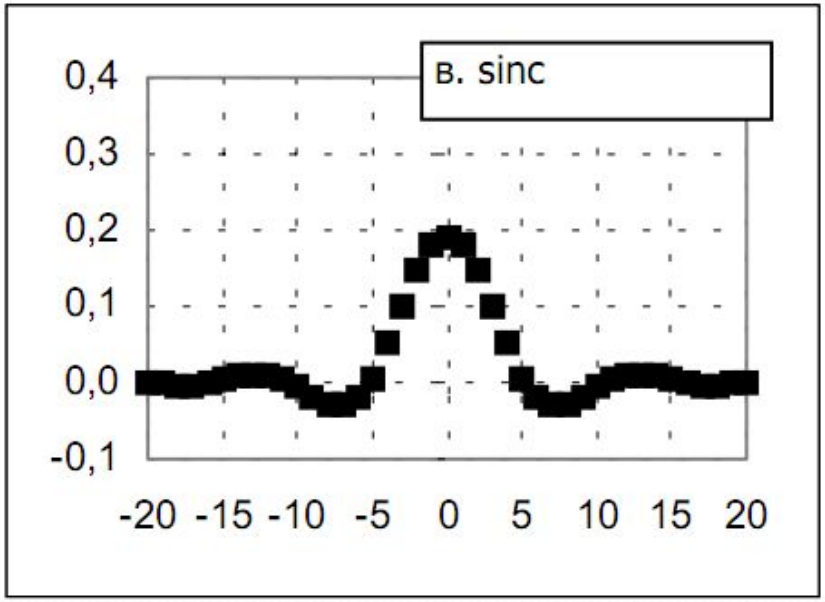
Дискретная производна

$$y[n] = x[n] - x[n-1]$$

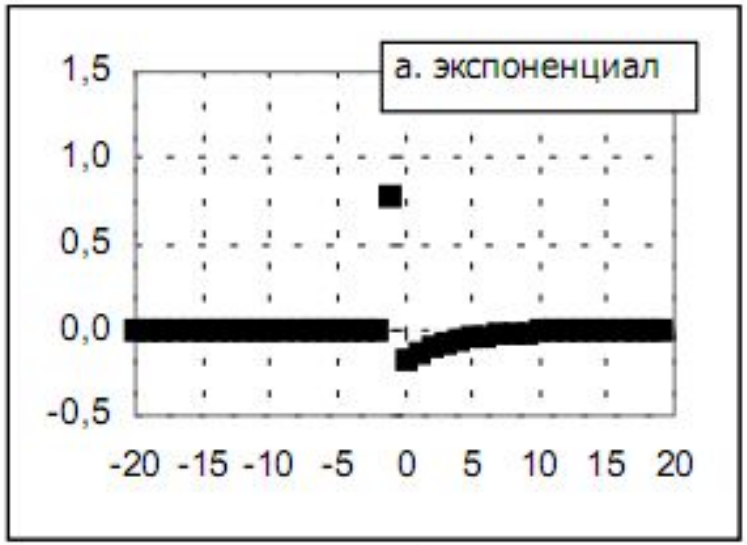
$$y[n] = (x[n-1] + x[n+1])/2.$$

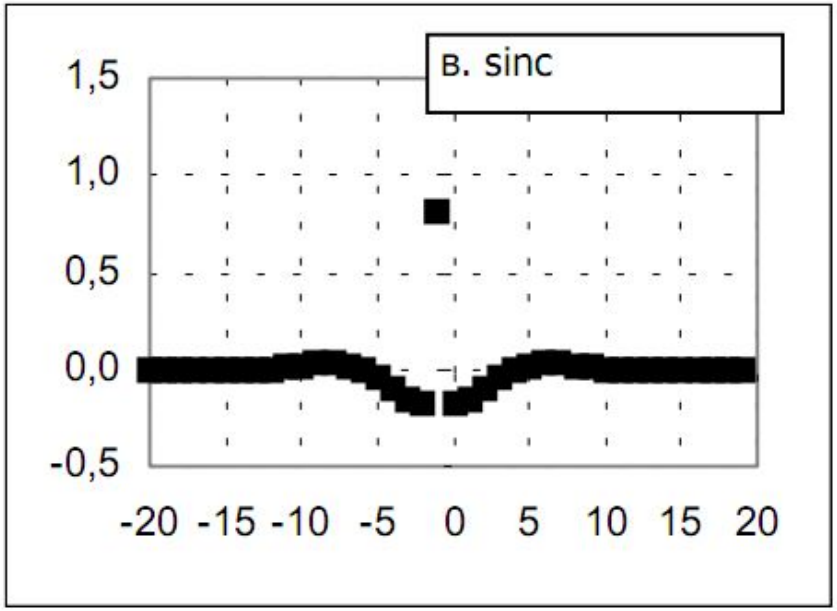
Низкочастотные фильтры



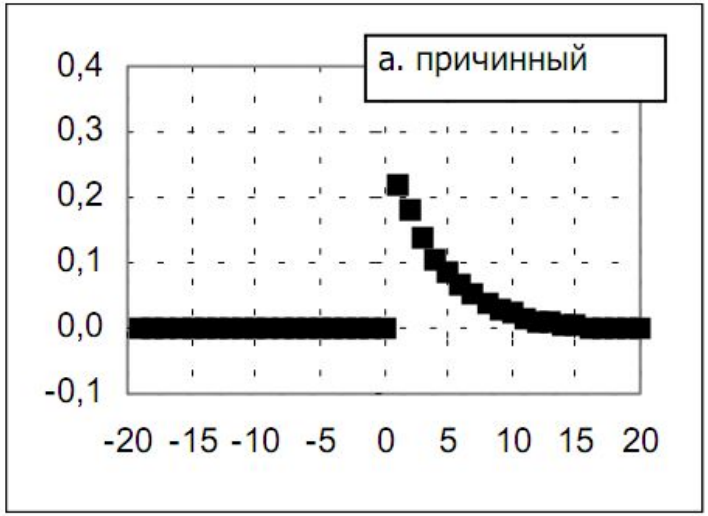


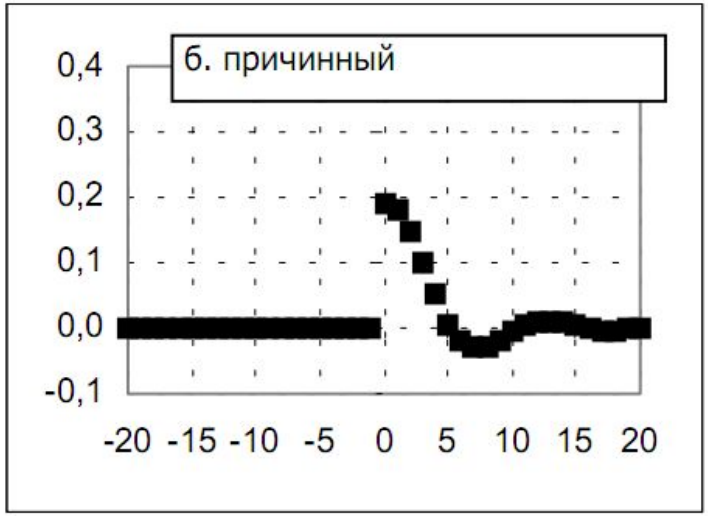
Высокочастотные фильтры

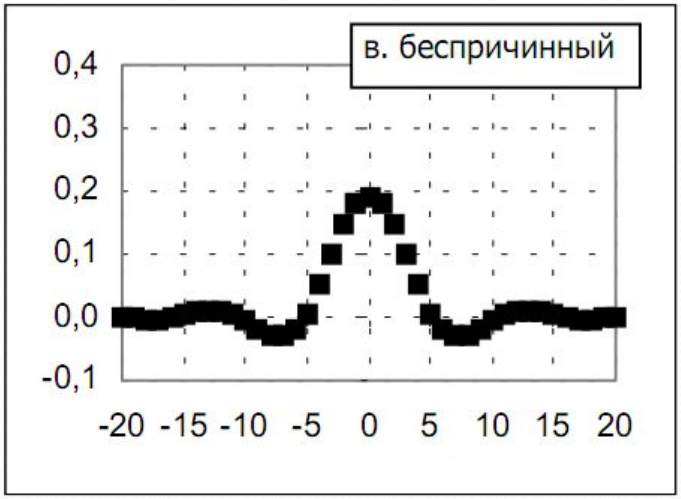




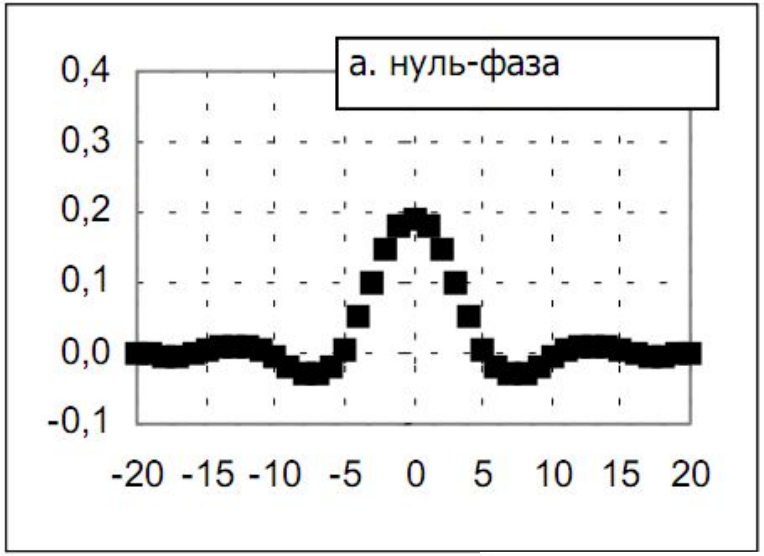
Причинные и беспричинные сигналы

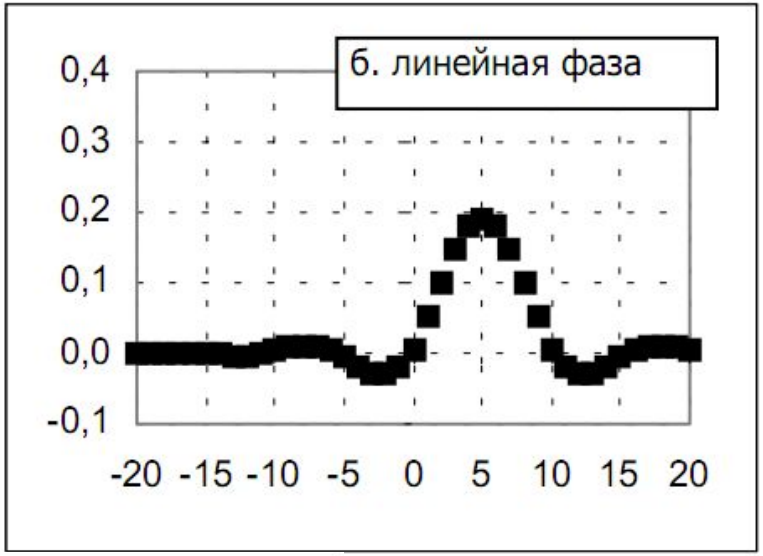


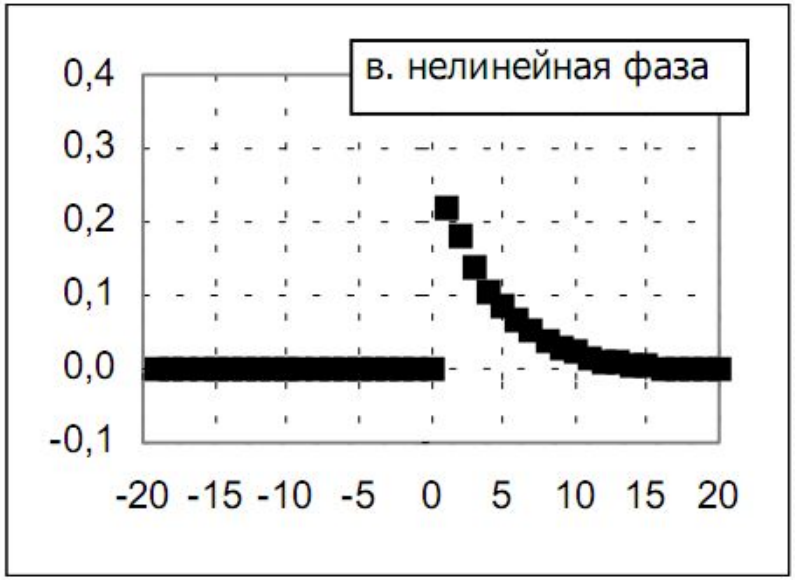




Линейная фаза и нелинейная фаза сигнала



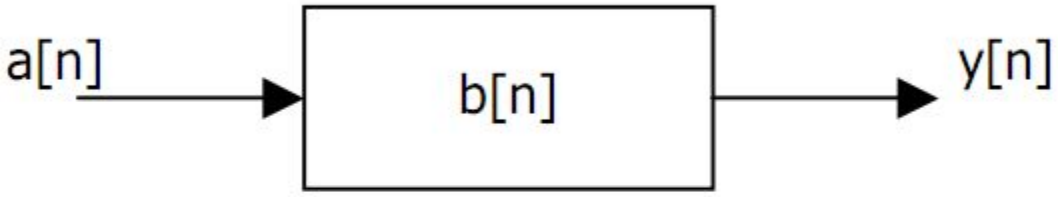




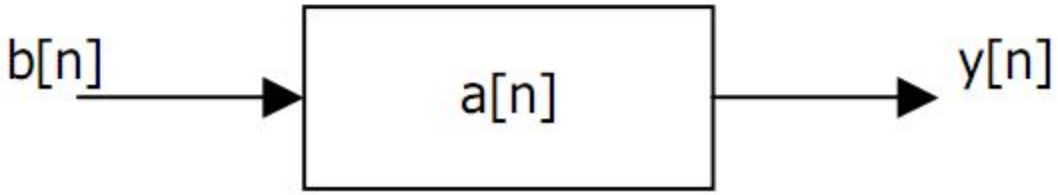
свойства

Свойство коммутативности

ЕСЛИ



ТО

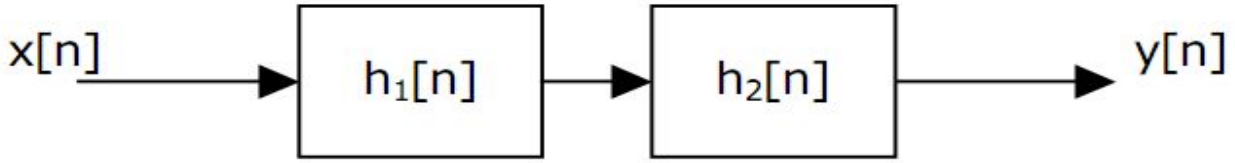


$$a[n] * b[n] = b[n] * a[n]$$

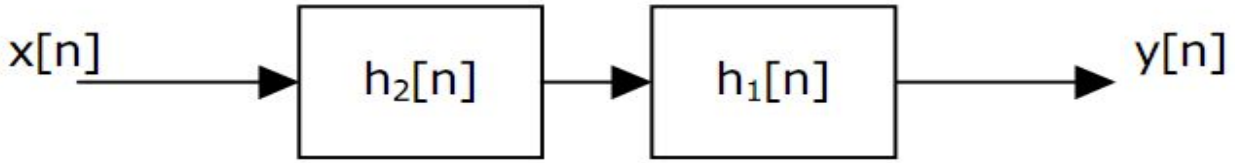
свойства

Свойство ассоциативности

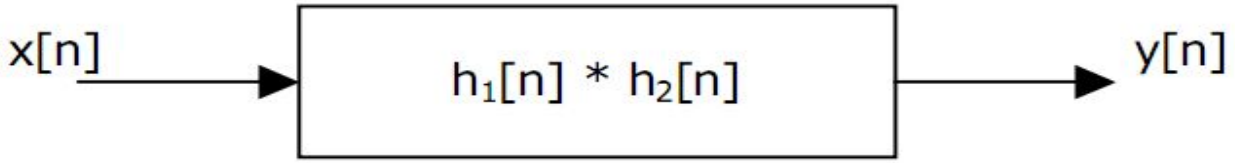
ЕСЛИ



ТО



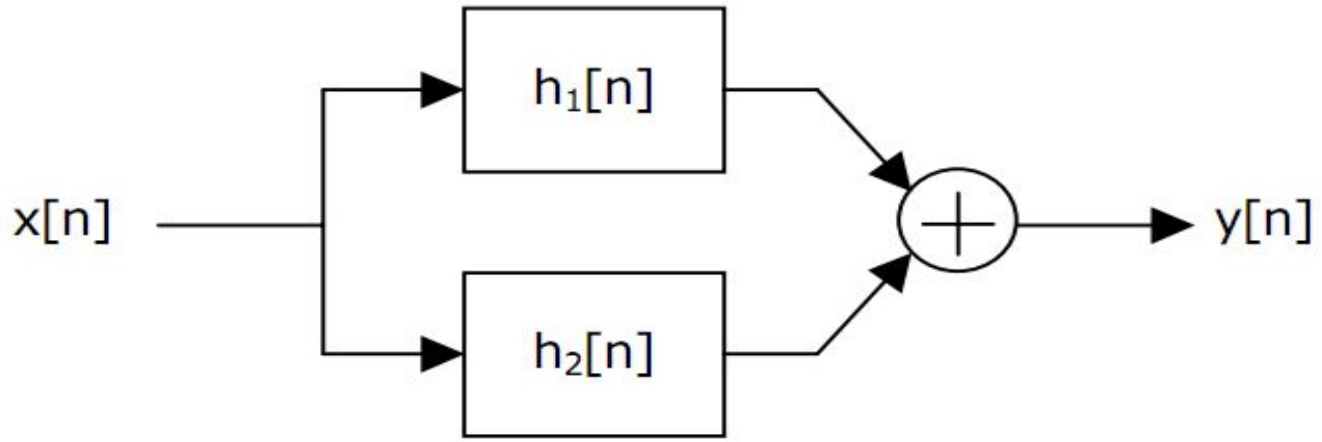
ТАКЖЕ



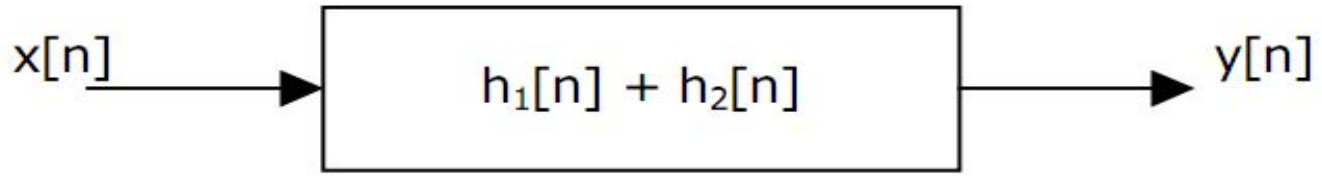
$$(a[n] * b[n]) * c[n] = a[n] * (b[n] * c[n])$$

Свойство дистрибутивности

ЕСЛИ



ТО

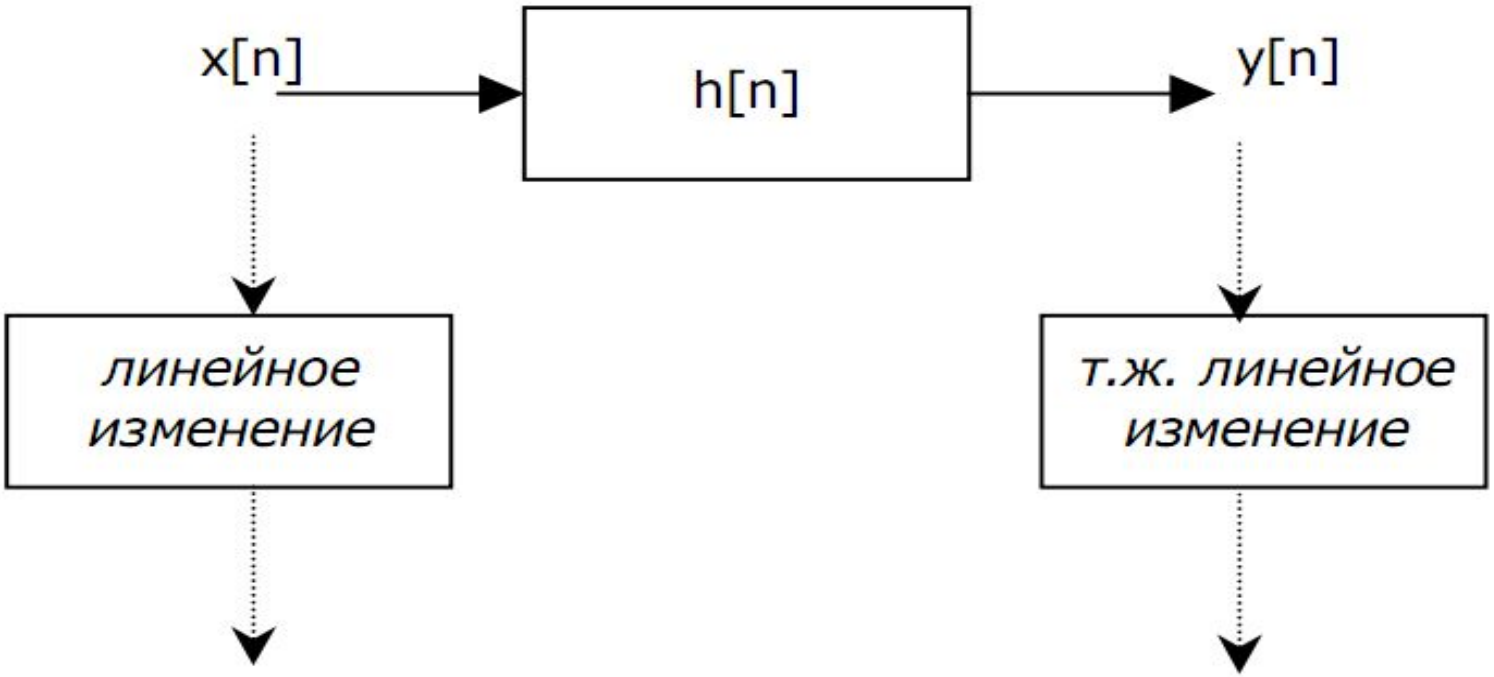


$$a[n] * b[n] + a[n] * c[n] = a[n] * (b[n] + c[n])$$

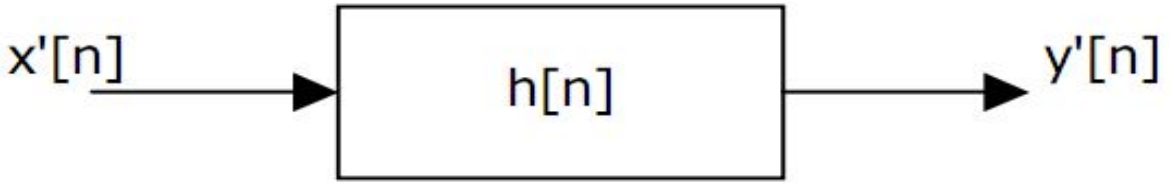
свойства

Перенос между входом и выходом

ЕСЛИ



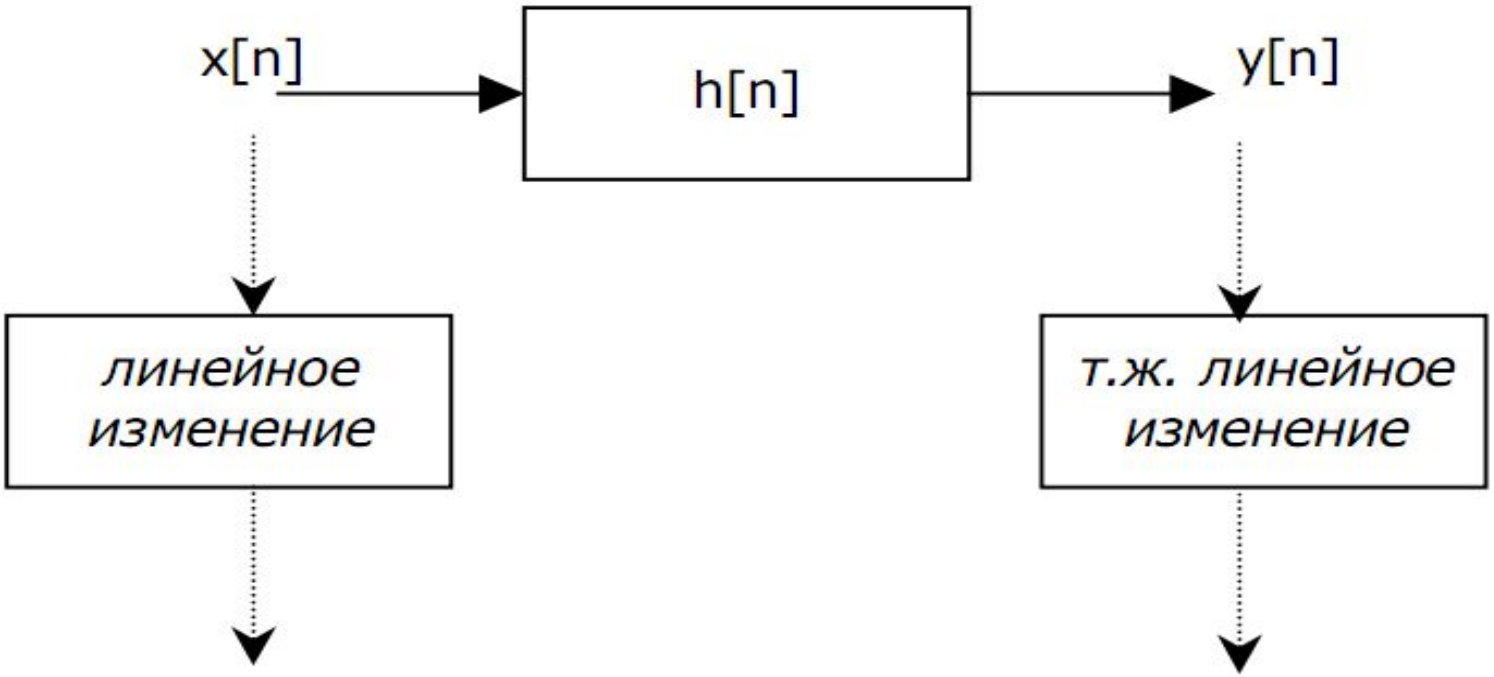
ТО



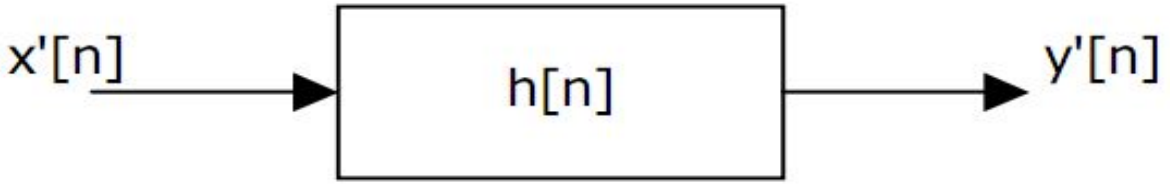
свойства

Перенос между входом и выходом

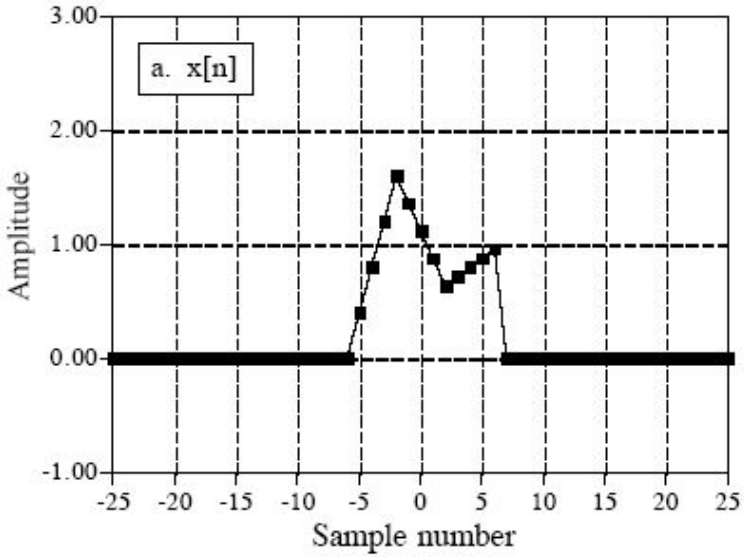
ЕСЛИ

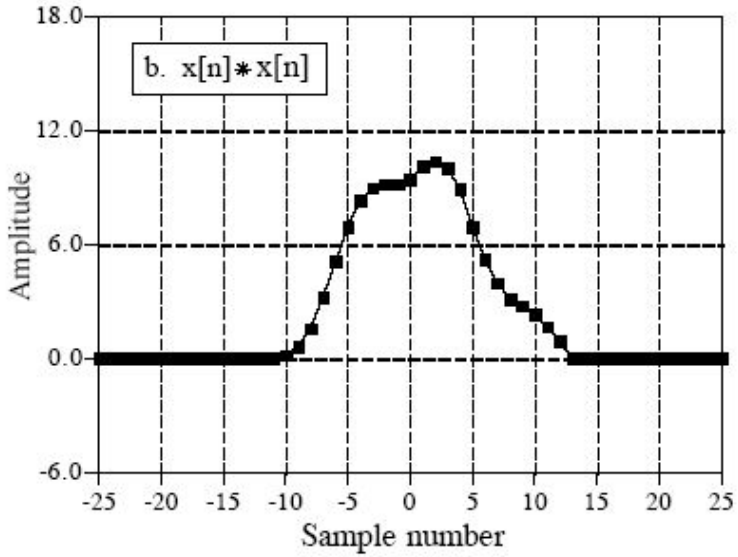


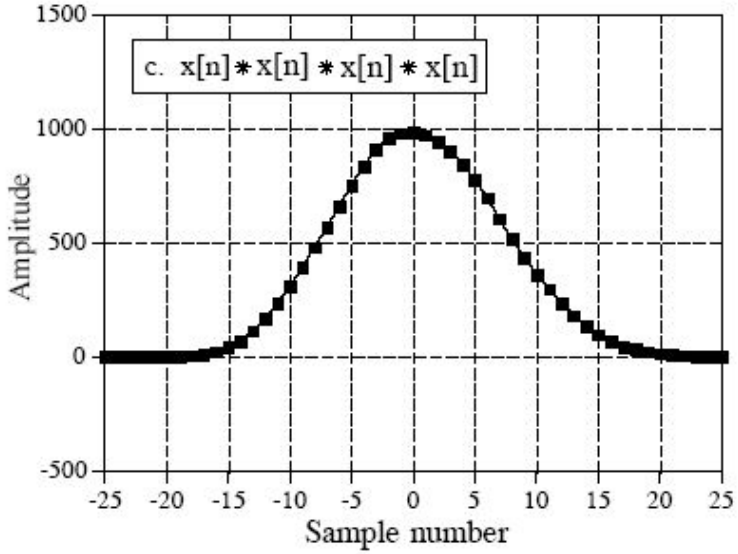
ТО



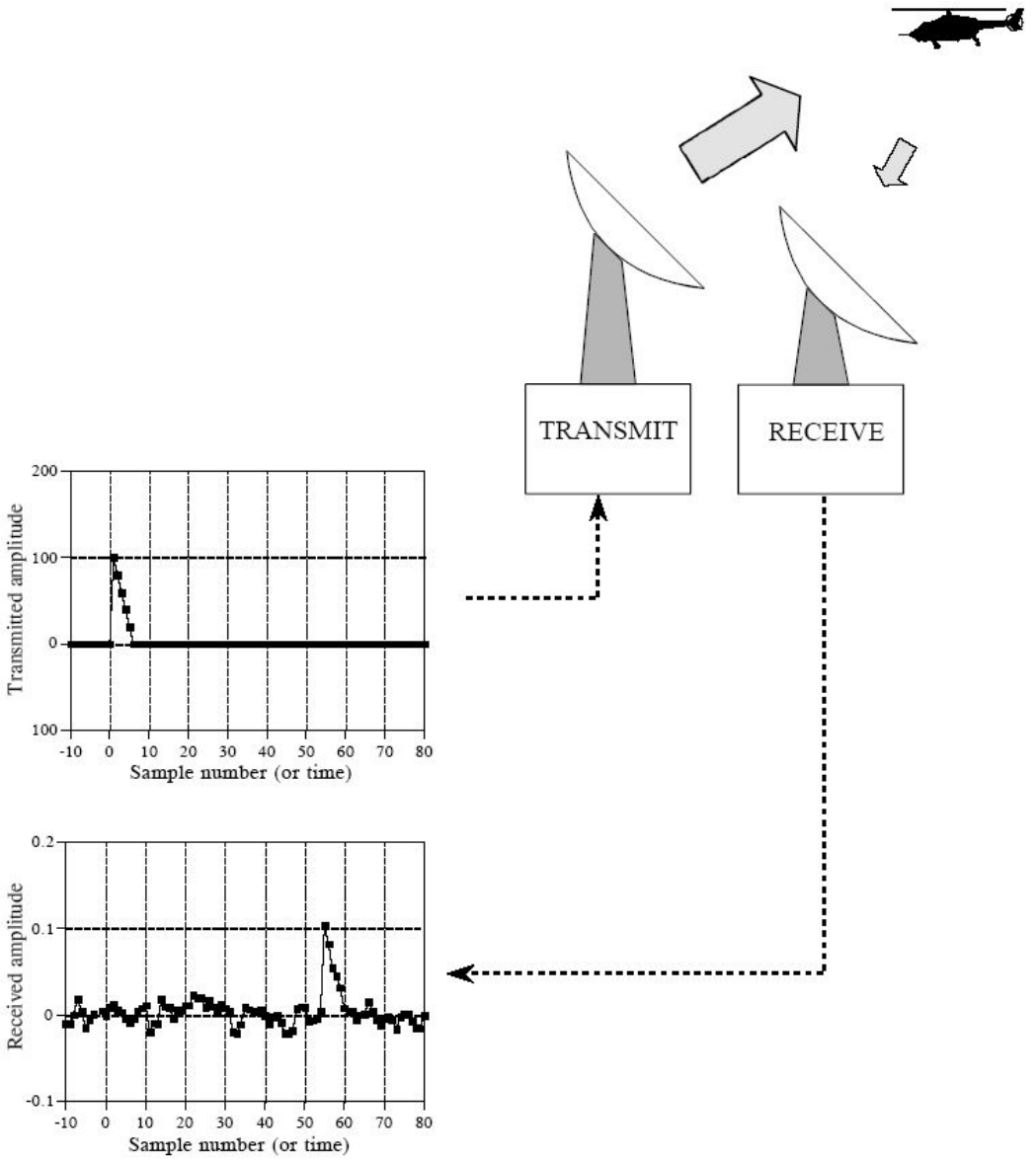
теорема

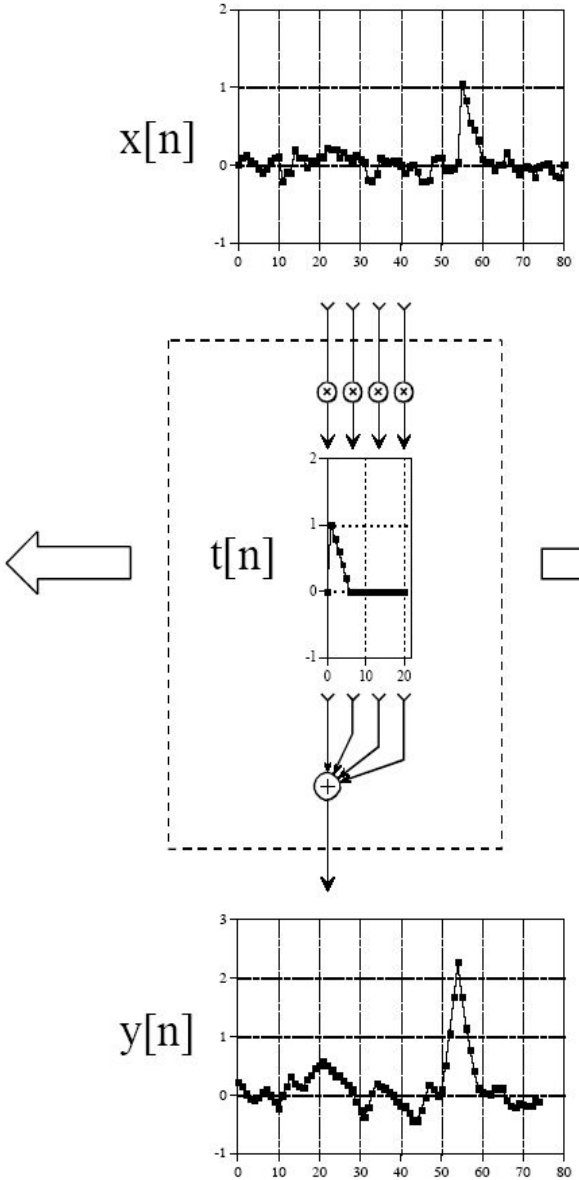

a. x[n]
Amplitude
Sample number
3.00
2.00
1.00
0.00
-1.00
-25 -20 -15 -10 -5 0 5 10 15 20 25


b. x[n]∗x[n]
Amplitude
Sample number
18.0
12.0
6.0
0.0
-6.0
-25 -20 -15 -10 -5 0 5 10 15 20 25


c. x[n]∗x[n]∗x[n]∗x[n]
Amplitude
Sample number
1500
1000
500
0
-500
-25 -20 -15 -10 -5 0 5 10 15 20 25

Корреляция


TRANSMIT
RECEIVE
Transmitted amplitude
Sample number (or time)
Received amplitude
Sample number (or time)


x[n]
t[n]
y[n]