КОМПЛЕКСНЫЕ КОРНИ КВАДРАТНЫХ УРАВНЕНИЙ

ЦЕЛИ ОБУЧЕНИЯ

- 11.1.2.4 – решать квадратные уравнения на множестве комплексных чисел

КРИТЕРИИ ОЦЕНИВАНИЯ

- решает квадратные уравнения с комплексными корнями

если $x^2 = c$

Тогда $x = \sqrt{c}$ или $x = -\sqrt{c}$

Для любого c действительного числа

ПРИМЕР 1 $x^2 = 40$

$$x = \pm\sqrt{40}$$

$$x = \pm\sqrt{4 \cdot 10}$$

$$x = \pm 2\sqrt{10}$$

ПРИМЕР 2

$$5x^2 - 2 = -62$$

$$5x^2 = -60$$

$$x^2 = -12$$

$$x = \pm\sqrt{-12}$$

$$x = \pm 2\sqrt{3}i$$

$$(ax+b)^2=c$$

ПРИМЕР 3: $(x+3)^2=16$

$$x+3=\pm\sqrt{16}$$

$$x+3=\pm 4$$

$$x=-3\pm 4 \Rightarrow x=-7 \quad \text{или} \quad x=1$$

ПРИМЕР 4:

$$7+25(2x+3)^2=0$$

$$25(2x+3)^2=-7 \quad (2x+3)^2=\frac{-7}{25}$$

$$2x+3=\pm\sqrt{\frac{-7}{25}}=\pm\frac{\sqrt{7}}{5}i$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{7}}{10}i$$

Solving Equations

Solve each equation in the complex number system.

(a) $x^2 = 4$ (b) $x^2 = -9$

(a) $x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

The equation has two solutions, -2 and 2. The solution set is $\{-2, 2\}$.

(b) $x^2 = -9$

$x = \pm\sqrt{-9} = \pm\sqrt{9}\,i = \pm 3i$

The equation has two solutions, $-3i$ and $3i$. The solution set is $\{-3i, 3i\}$.

EXAMPLE 1 Solving Quadratic Equations

Solve each equation.

a. $x^2 + 9 = 0$

b. $x^2 + 4x + 5 = 0$

c. $5x^2 - 4x + 1 = 0$

b. The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

$$x^2 + 4x + 5 = 0$$

$$x^2 + 4x = -5$$

$$x^2 + 4x + 4 = -5 + 4$$

$$(x + 2)^2 = -1$$

$$x + 2 = \pm\sqrt{-1}$$

$$x = -2 \pm \sqrt{-1}$$

$$x = -2 \pm i$$

Check You can check imaginary solutions algebraically. The check for one of the imaginary solutions, $-2 + i$, is shown.

$$(-2 + i)^2 + 4(-2 + i) + 5 \stackrel{?}{=} 0$$

$$3 - 4i - 8 + 4i + 5 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

c. The equation is not factorable, and completing the square would result in fractions. So, solve using the Quadratic Formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(1)}}{2(5)}$$ Substitute 5 for a, -4 for b, and 1 for c.

$$x = \frac{4 \pm \sqrt{-4}}{10}$$ Simplify.

$$x = \frac{4 \pm 2i}{10}$$ Write in terms of i.

$$x = \frac{2 \pm i}{5}$$ Simplify.

EXAMPLE 2 Finding Zeros of a Quadratic Function

Find the zeros of $f(x) = 4x^2 + 20$.

SOLUTION

$4x^2 + 20 = 0$ — Set $f(x)$ equal to 0.

$4x^2 = -20$ — Subtract 20 from each side.

$x^2 = -5$ — Divide each side by 4.

$x = \pm\sqrt{-5}$ — Take the square root of each side.

$x = \pm i\sqrt{5}$ — Write in terms of i.

▶ So, the zeros of f are $i\sqrt{5}$ and $-i\sqrt{5}$.

Check

$$f(i\sqrt{5}) = 4(i\sqrt{5})^2 + 20$$
$$= 4(-5) + 20$$
$$= 0 \checkmark$$

$$f(-i\sqrt{5}) = 4(-i\sqrt{5})^2 + 20$$
$$= 4(-5) + 20$$
$$= 0 \checkmark$$

Front work

Exercise 1 Solve these equations.

1 $x^2 + 2x + 5 = 0$

2 $x^2 - 2x + 10 = 0$

3 $x^2 + 4x + 29 = 0$

4 $x^2 + 10x + 26 = 0$

5 $x^2 - 6x + 18 = 0$

6 $x^2 + 4x + 7 = 0$

7 $x^2 - 6x + 11 = 0$

8 $x^2 - 2x + 25 = 0$

9 $x^2 + 5x + 25 = 0$

ОТВЕТЫ:

1. $x = -1 \pm 2i$

2. $x = 1 \pm 3i$

3. $x = -2 \pm 5i$

4. $x = -5 \pm i$

5. $x = 3 \pm 3i$

6. $x = -2 \pm i\sqrt{3}$

7. $x = 3 \pm i\sqrt{2}$

8. $x = 1 \pm i\sqrt{6}$

9. $x = -\frac{5}{2} \pm \frac{5i\sqrt{3}}{2}$

ФРОНТАЛЬНАЯ РАБОТА (1,3,5,7,9)
ДОМАШНЯЯ РАБОТА (2,4,6,8,10)

Exercise 2

Solve the following quadratics with complex numbers:

1. $2x^2 - 6x + 5 = 0$
2. $8x^2 - 4x + 5 = 0$
3. $-5x^2 + 12x - 8 = 0$
4. $-x^2 + 4x - 5 = 0$
5. $5x^2 + 8x + 5 = 0$
6. $5x^2 + 12x + 8 = 0$
7. $-6x^2 + 12x - 7 = 0$
8. $2x^2 - 6x + 7 = 0$
9. $-7x^2 + 12x = 10$
10. $5x^2 + 8x = -4$