



КОМПЛЕКСНЫЕ КОРНИ КВАДРАТНЫХ УРАВНЕНИЙ

ЦЕЛИ ОБУЧЕНИЯ

- 11.1.2.4 – решать квадратные уравнения на множестве комплексных чисел



КРИТЕРИИ ОЦЕНИВАНИЯ

- решает квадратные уравнения с комплексными корнями

если $x^2 = c$

Тогда $x = \sqrt{c}$ или $x = -\sqrt{c}$

Для любого c действительного числа

ПРИМЕР 1

$$x^2 = 40$$

$$x = \pm\sqrt{40}$$

$$x = \pm\sqrt{4 \cdot 10}$$

$$x = \pm 2\sqrt{10}$$

ПРИМЕР 2

$$5x^2 - 2 = -62$$

$$5x^2 = -60$$

$$x^2 = -12$$

$$x = \pm\sqrt{-12}$$

$$x = \pm 2\sqrt{3}i$$

$$(ax + b)^2 = c$$

ПРИМЕР 3: $(x + 3)^2 = 16$

$$x + 3 = \pm\sqrt{16}$$

$$x + 3 = \pm 4$$

$$x = -3 \pm 4 \Rightarrow x = -7 \text{ или } x = 1$$

ПРИМЕР 4:

$$7 + 25(2x + 3)^2 = 0$$

$$25(2x + 3)^2 = -7 \quad (2x + 3)^2 = \frac{-7}{25}$$

$$2x + 3 = \pm \sqrt{\frac{-7}{25}} = \pm \frac{\sqrt{7}}{5}i$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{7}}{10}i$$

Solving Equations

Solve each equation in the complex number system.

(a) $x^2 = 4$

(b) $x^2 = -9$

(a) $x^2 = 4$

$$x = \pm\sqrt{4} = \pm 2$$

The equation has two solutions, -2 and 2 . The solution set is $\{-2, 2\}$.

(b) $x^2 = -9$

$$x = \pm\sqrt{-9} = \pm\sqrt{9}i = \pm 3i$$

The equation has two solutions, $-3i$ and $3i$. The solution set is $\{-3i, 3i\}$.

EXAMPLE 1

Solving Quadratic Equations

Solve each equation.

a. $x^2 + 9 = 0$

b. $x^2 + 4x + 5 = 0$

c. $5x^2 - 4x + 1 = 0$

b. The coefficient of the x^2 -term is 1, and the coefficient of the x -term is an even number. So, solve by completing the square.

$$x^2 + 4x + 5 = 0$$

$$x^2 + 4x = -5$$

$$x^2 + 4x + 4 = -5 + 4$$

$$(x + 2)^2 = -1$$

$$x + 2 = \pm\sqrt{-1}$$

$$x = -2 \pm \sqrt{-1}$$

$$x = -2 \pm i$$

Check You can check imaginary solutions algebraically. The check for one of the imaginary solutions, $-2 + i$, is shown.

$$(-2 + i)^2 + 4(-2 + i) + 5 \stackrel{?}{=} 0$$

$$3 - 4i - 8 + 4i + 5 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

- c. The equation is not factorable, and completing the square would result in fractions. So, solve using the Quadratic Formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(1)}}{2(5)}$$

Substitute 5 for a , -4 for b , and 1 for c .

$$x = \frac{4 \pm \sqrt{-4}}{10}$$

Simplify.

$$x = \frac{4 \pm 2i}{10}$$

Write in terms of i .

$$x = \frac{2 \pm i}{5}$$

Simplify.

Check

$$\begin{aligned}f(i\sqrt{5}) &= 4(i\sqrt{5})^2 + 20 \\ &= 4(-5) + 20 \\ &= 0 \quad \checkmark\end{aligned}$$

$$\begin{aligned}f(-i\sqrt{5}) &= 4(-i\sqrt{5})^2 + 20 \\ &= 4(-5) + 20 \\ &= 0 \quad \checkmark\end{aligned}$$

EXAMPLE 2

Finding Zeros of a Quadratic Function

Find the zeros of $f(x) = 4x^2 + 20$.

SOLUTION

$$4x^2 + 20 = 0$$

Set $f(x)$ equal to 0.

$$4x^2 = -20$$

Subtract 20 from each side.

$$x^2 = -5$$

Divide each side by 4.

$$x = \pm\sqrt{-5}$$

Take the square root of each side.

$$x = \pm i\sqrt{5}$$

Write in terms of i .

► So, the zeros of f are $i\sqrt{5}$ and $-i\sqrt{5}$.

Front work

Exercise 1 Solve these equations.

1 $x^2 + 2x + 5 = 0$

3 $x^2 + 4x + 29 = 0$

5 $x^2 - 6x + 18 = 0$

7 $x^2 - 6x + 11 = 0$

9 $x^2 + 5x + 25 = 0$

2 $x^2 - 2x + 10 = 0$

4 $x^2 + 10x + 26 = 0$

6 $x^2 + 4x + 7 = 0$

8 $x^2 - 2x + 25 = 0$

ОТВЕТЫ:

1. $x = -1 \pm 2i$

2. $x = 1 \pm 3i$

3. $x = -2 \pm 5i$

4. $x = -5 \pm i$

5. $x = 3 \pm 3i$

6. $x = -2 \pm i\sqrt{3}$

7. $x = 3 \pm i\sqrt{2}$

8. $x = 1 \pm i\sqrt{6}$

9. $x = -\frac{5}{2} \pm \frac{5i\sqrt{3}}{2}$

ФРОНТАЛЬНАЯ РАБОТА (1,3,5,7,9) ДОМАШНЯЯ РАБОТА (2,4,6,8,10)

Exercise 2

Solve the following quadratics with complex numbers:

1. $2x^2 - 6x + 5 = 0$

2. $8x^2 - 4x + 5 = 0$

3. $-5x^2 + 12x - 8 = 0$

4. $-x^2 + 4x - 5 = 0$

5. $5x^2 + 8x + 5 = 0$

6. $5x^2 + 12x + 8 = 0$

7. $-6x^2 + 12x - 7 = 0$

8. $2x^2 - 6x + 7 = 0$

9. $-7x^2 + 12x = 10$

10. $5x^2 + 8x = -4$