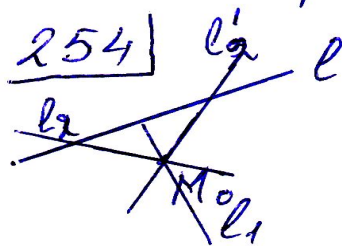


Семинар 5

доцент Волков Н.П.

Занятие 5

Прямая на плоскости.



Дано $l: 2x + 3y + 4 = 0$

$M_0(2; 1)$

Найти $l_2: (l, l_2) = 45^\circ, l_2 \ni M_0$

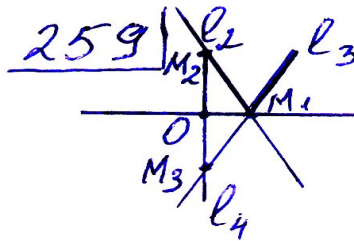
Решение: Построим $l_1: l_1 \perp l$ и $l_1 \ni M_0$

$$\vec{n}_1 = \vec{n} = \{2; 3\} \Rightarrow l_1: \frac{x-2}{2} = \frac{y-1}{3} \Rightarrow l_1: 3x - 2y - 4 = 0$$

$$\vec{n}_1 = \{3; -2\}. \text{ Найдем } \vec{n}_2 = \vec{n} + \vec{n}_1 = \{5; 1\}$$

$$\Rightarrow l_2: \frac{x-2}{5} = \frac{y-1}{1} \Rightarrow \boxed{l_2: x - 5y + 3 = 0}$$

$$l_2' \perp l_2 \Rightarrow l_2': \frac{x-2}{1} = \frac{y-1}{-5} \Rightarrow \boxed{l_2': 5x + y - 11 = 0}$$



$$l_2: x - 2y + 5 = 0$$

$l_3 - ?$

$$l_1: 3x - 2y + 7 = 0$$

Возьмем точку $M_2 \in l_2$, например,

$M_2(-5; 0)$ и найдем точку M_3 -

симметричную точке M_2 относительно прямой l_1 .

Построим $l_4: l_4 \ni M_2, l_4 \perp l_1$

$$\vec{n}_4 = \vec{n}_1 = \{3; -2\} \Rightarrow l_4: \frac{x+5}{3} = \frac{y}{-2}$$

$$l_4: 2x + 3y + 10 = 0$$

$$0 = l_1 \cap l_4 \Rightarrow 0: \begin{cases} 3x - 2y + 7 = 0 \\ 2x + 3y + 10 = 0 \end{cases} \begin{matrix} \cdot 3 \\ \cdot 2 \end{matrix} \Rightarrow 13x = -41$$

$$\Rightarrow x = -\frac{41}{13}, y = -\frac{16}{13} \Rightarrow O\left(-\frac{41}{13}; -\frac{16}{13}\right)$$

$$\begin{cases} x_3 = 2x_0 - x_2 = -\frac{82}{13} + 5 = -\frac{17}{13} \\ y_3 = 2y_0 - y_2 = -\frac{32}{13} + 0 = -\frac{32}{13} \end{cases} \Rightarrow M_3\left(-\frac{17}{13}; -\frac{32}{13}\right)$$

$$M_1? M_1 = l_1 \cap l_2 \Rightarrow M_1: \begin{cases} 3x - 2y + 7 = 0 \\ x - 2y + 5 = 0 \end{cases} \Rightarrow$$

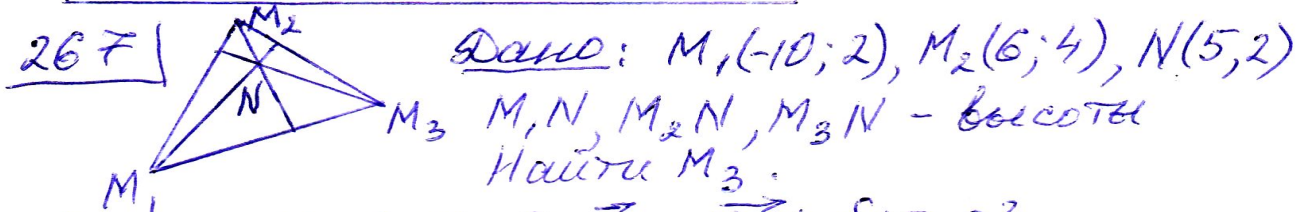
$$\Rightarrow 2x = -2 \Rightarrow x = -1, y = 2 \Rightarrow M_1(-1; 2)$$

$$\vec{l}_3 = \overline{M_1 M_3} = \left\{ -\frac{4}{13}; -\frac{58}{13} \right\}$$

$$\Rightarrow l_3: l_3 \ni M_1, l_3 \ni M_3$$

$$l_3: \frac{x+1}{-\frac{4}{13}} = \frac{y-2}{-\frac{58}{13}} \Leftrightarrow \frac{x+1}{2} = \frac{y-2}{29}$$

$$l_3: 29x - 2y + 33 = 0$$



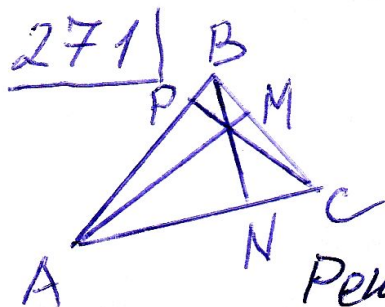
$$\text{Решение: } M_2 M_3? \vec{n}_1 = \overline{M_1 N} = \{15; 0\}$$

$$M_2 M_3: 15(x-6) + 0(y-4) = 0 \Rightarrow M_2 M_3: x - 6 = 0$$

$$M_1 M_3? \vec{n}_2 = \overline{N M_2} = \{1; 2\} \quad M_1 M_3: (x+10) + 2(y-2) = 0$$

$$\Rightarrow M_1 M_3: x + 2y + 6 = 0$$

$$M_3: \begin{cases} x - 6 = 0 \\ x + 2y + 6 = 0 \end{cases} \Rightarrow \begin{matrix} x = 6 \\ y = -6 \end{matrix} \Rightarrow M_3(6; -6)$$



Дано: $B(-4; -5)$

$AM: 5x + 3y - 4 = 0, CP: 3x + 8y + 13 = 0$

Найти AB, BC, AC

Решение:

1) $AB - ? \vec{\tau}_1 = \vec{n}_{CP} = \{3; 8\} \Rightarrow AB: \frac{x+4}{3} = \frac{y+5}{8}$

$AB: 8x - 3y + 17 = 0$

2) $BC - ? \vec{\tau}_2 = \vec{n}_{AM} = \{5; 3\} \Rightarrow BC: \frac{x+4}{5} = \frac{y+5}{3}$

$BC: 3x - 5y - 13 = 0$

3) $A - ? A: \begin{cases} 8x - 3y + 17 = 0 \\ 5x + 3y - 4 = 0 \end{cases} + \Rightarrow 13x + 13 = 0$

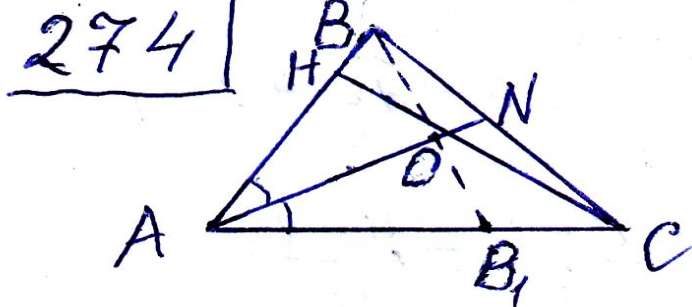
$x = -1 \Rightarrow -8 - 3y + 17 = 0 \Rightarrow y = 3 \Rightarrow A(-1; 3)$

$C - ? C = CP \cap BC \Rightarrow C: \begin{cases} 3x + 8y + 13 = 0 \\ 3x - 5y - 13 = 0 \end{cases} - \Rightarrow$

$13y + 26 = 0 \Rightarrow y = -2 \Rightarrow x = 1 \Rightarrow C(1; -2)$

$\vec{\tau}_{AC} = \vec{AC} = \{2; -5\} \quad AC: \frac{x+1}{2} = \frac{y-3}{-5}$

$\Rightarrow AC: 5x + 2y - 1 = 0$



Дано: $B(2; -1)$

CH: $3x - 4y + 27 = 0$

высота

AN: $x + 2y - 5 = 0$

биссектриса

Найти: AB, BC, AC .

Решение:

1) $AB - ?$ $\vec{r}_{AB} = \vec{n}_{CH} = \{3; -4\} \Rightarrow AB: \frac{x-2}{3} = \frac{y+1}{-4}$

$AB: 4x + 3y - 5 = 0$

2) $AC - ?$ $A = AN \cap AB \Rightarrow A: \begin{cases} x + 2y - 5 = 0 \\ 4x + 3y - 5 = 0 \end{cases} \cdot (-4)$

$\Rightarrow -5y + 15 = 0 \Rightarrow y = 3, x = -1 \Rightarrow A(-1; 3)$

B_1 - точка симметричная точке B относительно прямой AN и $B_1 \in AC$, т.к. AN - биссектриса

$\vec{r}_{BB_1} = \vec{n}_{AN} = \{1; 2\} \Rightarrow BB_1: \frac{x-2}{1} = \frac{y+1}{2}$

$\Rightarrow BB_1: 2x - y - 5 = 0$

$$O = AN \cap BB_1 \quad O: \begin{cases} x + 2y - 5 = 0 \\ 2x - y - 5 = 0 \end{cases} \cdot 2^+ \Rightarrow 5x = 15$$

$$x = 3, y = 1 \Rightarrow O(3; 1)$$

$$\begin{cases} x_{B_1} = 2x_0 - x_B \\ y_{B_1} = 2y_0 - y_B \end{cases} \Rightarrow \begin{cases} x_{B_1} = 4 \\ y_{B_1} = 3 \end{cases} \Rightarrow B_1(4; 3)$$

$$\vec{AC} = \vec{AB_1} = \{5; 0\} \Rightarrow AC: \frac{x+1}{5} = \frac{y-3}{0}$$

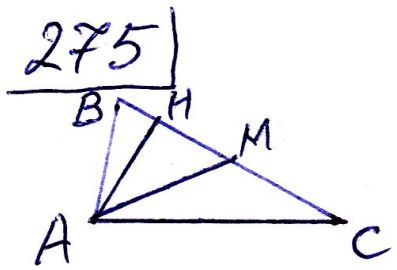
$$\boxed{AC: y - 3 = 0}$$

$$3) BC - ? \quad C = AC \cap CH \Rightarrow C: \begin{cases} y - 3 = 0 \\ 3x - 4y + 27 = 0 \end{cases}$$

$$y = 3, x = -5 \Rightarrow C(-5; 3)$$

$$\vec{BC} = \vec{BC} = \{-7; 4\} \quad BC: \frac{x-2}{-7} = \frac{y+1}{4}$$

$$\boxed{BC: 4x + 7y - 1 = 0}$$



Дано: $C(4; -1)$

$$AH: 2x - 3y + 12 = 0$$

$$AM: 2x + 3y = 0$$

Найти AB, BC, AC .

$$1) BC - ? \quad \vec{r}_{BC} = \vec{n}_{AH} = \{2; -3\} \Rightarrow BC: \frac{x-4}{2} = \frac{y+1}{-3}$$

$$\boxed{BC: 3x + 2y - 10 = 0}$$

$$2) AC - ? \quad A = AH \cap AM \Rightarrow A: \begin{cases} 2x - 3y + 12 = 0 \\ 2x + 3y = 0 \end{cases} +$$

$$\Rightarrow 4x + 12 = 0 \Rightarrow x = -3 \quad y = 2 \Rightarrow A(-3; 2)$$

$$\vec{r}_{AC} = \vec{AC} = \{7; -3\} \Rightarrow AC: \frac{x+3}{7} = \frac{y-2}{-3}$$

$$\boxed{AC: 3x + 7y - 5 = 0}$$

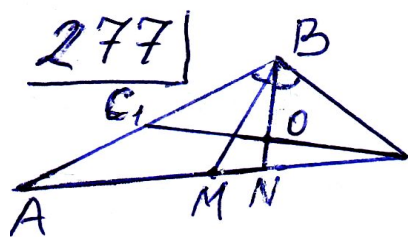
$$3) AB - ? \quad M = AM \cap BC \Rightarrow M: \begin{cases} 2x + 3y = 0 \\ 3x + 2y - 10 = 0 \end{cases} \begin{matrix} \cdot 2 \\ \cdot (-3) \end{matrix} \Rightarrow$$

$$\Rightarrow -5x + 30 = 0 \Rightarrow x = 6, \quad y = -4 \Rightarrow M(6; -4)$$

$$\begin{cases} x_B = 2x_M - x_C = 8 \\ y_B = 2y_M - y_C = -7 \end{cases} \Rightarrow B(8; -7)$$

$$\vec{r}_{AB} = \vec{AB} = \{11; -9\} \quad AB: \frac{x+3}{11} = \frac{y-2}{-9}$$

$$\boxed{AB: 9x + 11y + 5 = 0}$$



Дано: $C(4; 3)$

$BN: x + 2y - 5 = 0$ - биссектриса

$CM: 4x + 13y - 10 = 0$ - медиана

Найти AB, BC, AC .

Решение:

1) $BC - ?$ $B = BM \cap BN$ $B: \begin{cases} x + 2y - 5 = 0 \\ 4x + 13y - 10 = 0 \end{cases} \begin{matrix} \cdot (-4) \\ + \end{matrix}$

$\Rightarrow 5y + 10 = 0 \Rightarrow y = -2, x = 9 \Rightarrow B(9; -2)$

$\vec{r}_{BC} = \vec{BC} = \{-5; 5\} \Rightarrow BC: \frac{x-9}{-5} = \frac{y+2}{5}$

$\Rightarrow \boxed{BC: x + y - 7 = 0}$

2) $AB - ?$ C_1 - точка симметричная точке C относительно BN

$C_1 \in AB$, т.к. BN - биссектриса.

При этом $CC_1 \perp BN$

$\vec{r}_{CC_1} = \vec{n}_{BN} = \{1, 2\}$ $CC_1: \frac{x-4}{1} = \frac{y-3}{2}$

$CC_1: 2x - y - 5 = 0$

$O = BN \cap CC_1 \Rightarrow O: \begin{cases} x + 2y - 5 = 0 \\ 2x - y - 5 = 0 \end{cases} \cdot 2 \Rightarrow 5x = 15$

$x = 3, y = 1 \Rightarrow O(3; 1)$

$\begin{cases} x_{C_1} = 2x_O - x_C = 2 \\ y_{C_1} = 2y_O - y_C = -1 \end{cases} \Rightarrow C_1(2; -1)$

$\vec{r}_{AB} = \vec{C_1B} = \{7; -1\} \Rightarrow AB: \frac{x-9}{7} = \frac{y+2}{-1}$

$\Rightarrow \boxed{AB: x + 7y + 5 = 0}$

$$3) AC - ? \quad \begin{cases} x_M = \frac{x_A + x_C}{2} \\ y_M = \frac{y_A + y_C}{2} \\ x_A + 7y_A + 5 = 0 \\ 4x_M + 13y_M - 10 = 0 \end{cases} \Rightarrow \begin{cases} x_A + 7y_A + 5 = 0 \\ 2(x_A + 4) + \frac{13}{2}(y_A + 3) - 10 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x_A + 7y_A + 5 = 0 \\ 4x_A + 13y_A + 35 = 0 \end{cases} \cdot (-4) \Rightarrow -15y_A + 15 = 0$$

$$\Rightarrow y_A = 1, \quad x_A = -12 \Rightarrow A(-12; 1)$$

$$\vec{r}_{AC} = \vec{AC} = \{16, 2\} \Rightarrow AC: \frac{x-4}{16} = \frac{y-3}{2}$$

$$\boxed{AC: x - 8y + 20 = 0}$$

$$298 \mid l_1: 2x - y + 3 = 0, \quad l_2: x + y + 3 = 0,$$

$$l_3: 2x + y - 13 = 0$$

При каких α $l_1 \cap l_2 \cap l_3 = M_0$

Решение:

$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 3 \\ \alpha & 1 & -13 \end{vmatrix} = 0 \Rightarrow \alpha(-3-3) - 1(6-3) - 13(2+1) = 0$$

$$-6\alpha - 3 - 39 = 0 \Rightarrow \boxed{\alpha = -7}$$

Дома: К. 270, 272, 276, 278, 280, 285, 301.