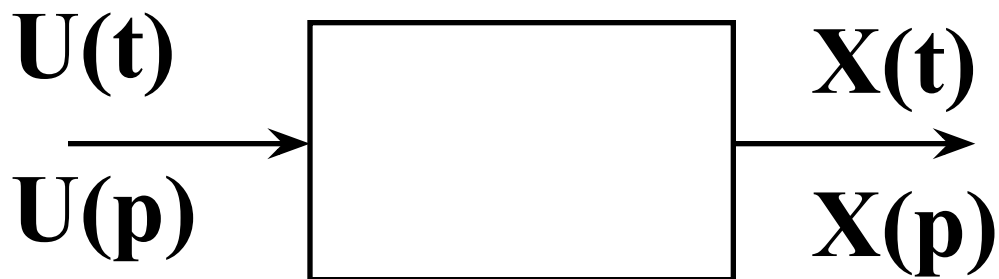


# ЗАКОНЫ РЕГУЛИРОВАНИЯ:



# 1. Пропорциональный (П):

$$1.1 \quad x(t) = ku(t)$$

$$1.2 \quad X(p) = kU(p)$$

$$1.3 \quad W(p) = k$$

## 2. Интегральный (И):

$$2.1 \quad x(t) = k \int_0^t u(t) dt = \frac{1}{T_u} \int_0^t u(t) dt$$

$$2.2 \quad X(p) = \frac{1}{T_u p} U(p)$$

$$2.3 \quad W(p) = \frac{1}{T_u p}$$

### 3. Дифференциальный (Д):

$$3.1 \quad x(t) = T_{\partial} \frac{du}{dt}$$

$$3.2 \quad X(p) = T_{\partial} p \cdot U(p)$$

$$3.3 \quad W(p) = T_D p$$

$$3.4 \quad W(p) = \frac{T_D p}{\tau_\delta p + 1},$$

$$\text{где } \tau_\delta = (0,01 \div 0,1) T_D$$

## 4. ПИ:

$$4.1 \quad x(t) = k(u(t) + \frac{1}{T_u} \int_0^t u(t) dt)$$

$$4.2 \quad X(p) = k(1 + \frac{1}{T_u p})U(p)$$

$$4.3 \quad W(p) = k(1 + \frac{1}{T_u p})$$

## 5. ПИД:

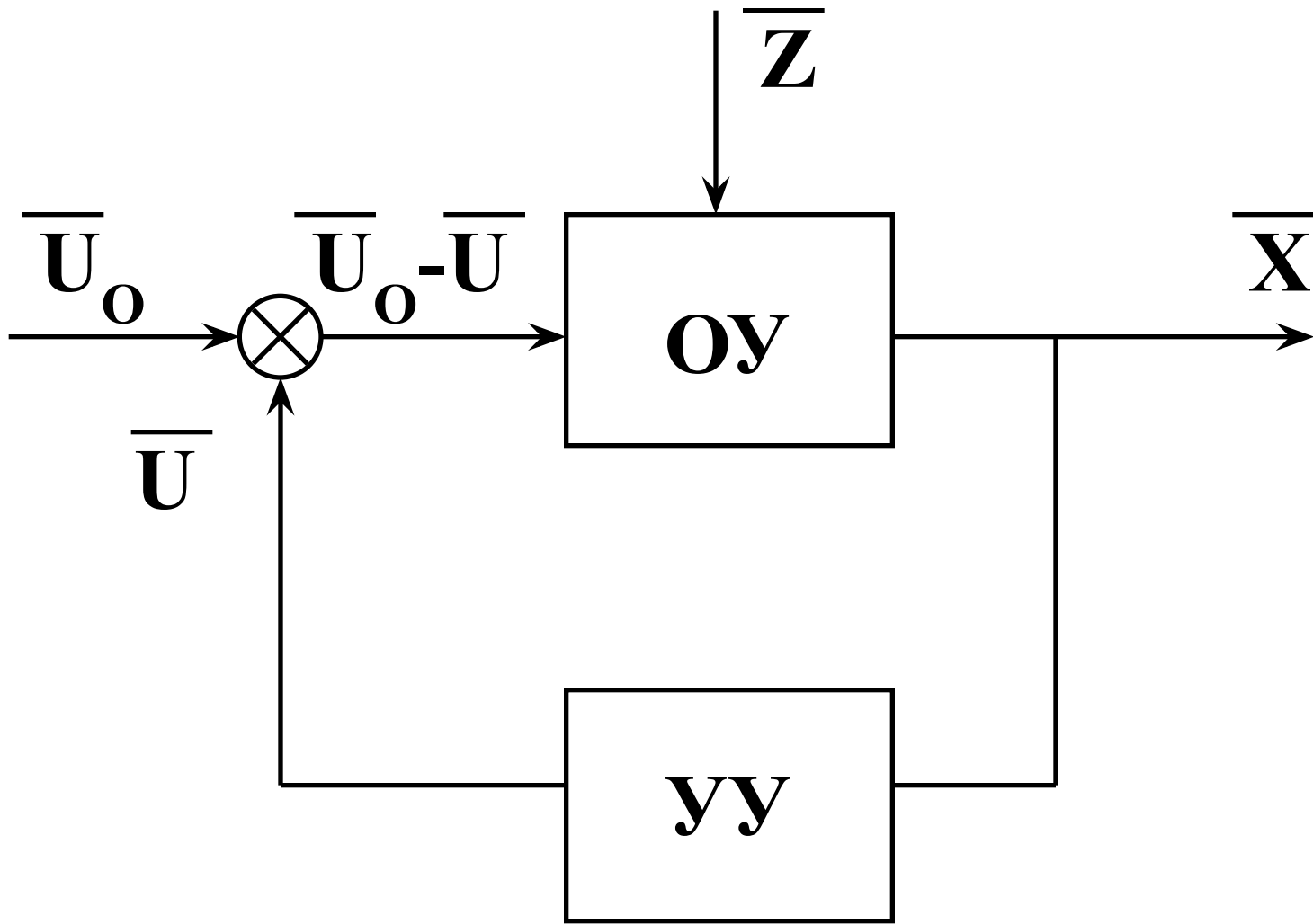
$$5.1 \quad x(t) = k(u(t) + \frac{1}{T_u} \int_0^t u(t) dt + T_\delta \frac{du}{dt})$$

$$5.2 \quad X(p) = k(1 + \frac{1}{T_u p} + T_\delta p) \cdot U(p)$$

$$5.3 \quad W(p) = k\left(1 + \frac{1}{T_u p} + T_\delta p\right)$$

$$5.4 \quad W(p) = k\left(1 + \frac{1}{T_u p} + \frac{T_\delta p}{\tau_\delta p + 1}\right)$$





$$\begin{aligned} & a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_n x = \\ & = b_0 \frac{d^m u}{dt^m} + b_1 \frac{d^{m-1} u}{dt^{m-1}} + \dots + b_m u + \\ & + c_0 \frac{d^k z}{dt^k} + c_1 \frac{d^{k-1} z}{dt^{k-1}} + \dots + c_k z \Rightarrow \end{aligned}$$

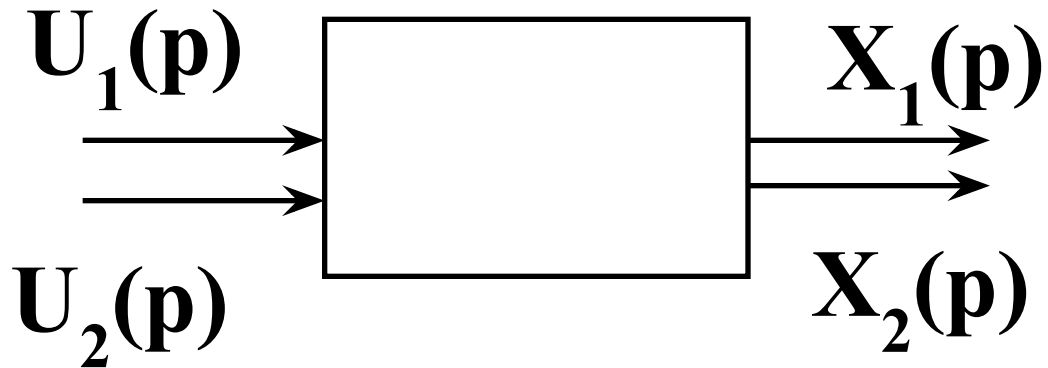
$$\bar{A}x(t) = B\bar{u}(t) + C\bar{z}(t) \Rightarrow$$

$$A(p)X(p) = B(p)U(p) + C(p)Z(p)$$

$$X(p) = \int_0^{\infty} x(t)e^{-pt} dt$$



$$W(p) = \frac{X(p)}{U(p)}$$



$$1) \quad U_1(p) = 0 \quad U_2(p) \rightarrow X_{12}(p) \\ \rightarrow X_{22}(p)$$

$$W_{12}(p) = \frac{X_{12}(p)}{U_2(p)}; \quad W_{22}(p) = \frac{X_{22}(p)}{U_2(p)}$$

$$2) \quad U_2(p) = 0 \quad \begin{array}{l} U_1(p) \rightarrow X_{11}(p) \\ \phantom{U_1(p)} \rightarrow X_{21}(p) \end{array}$$

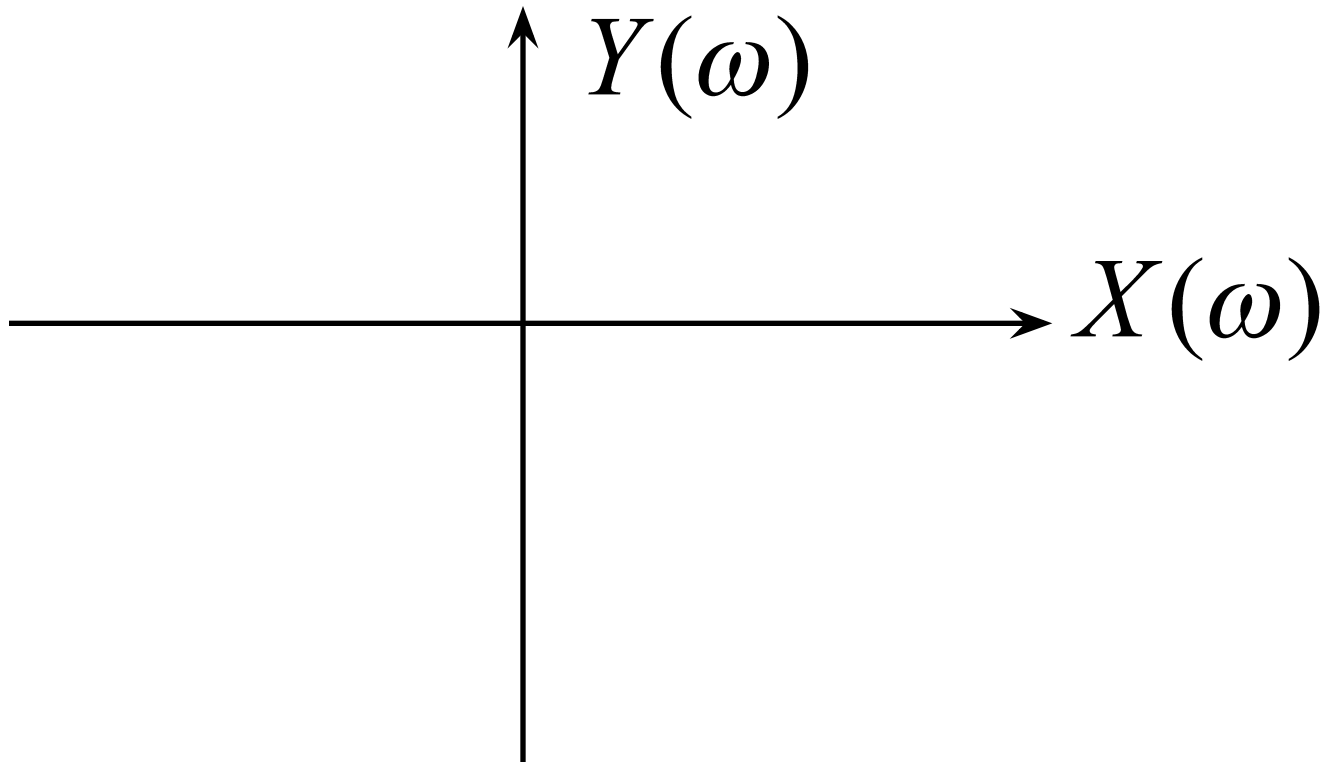
$$W_{11}(p) = \frac{X_{11}(p)}{U_1(p)}; \quad W_{21}(p) = \frac{X_{21}(p)}{U_1(p)}$$

$$3) \quad X_i(p) = X_{1i}(p) + X_{2i}(p)$$

$$W_{1i}(p) = \frac{B_{1i}(p)}{A_{1i}(p)}$$

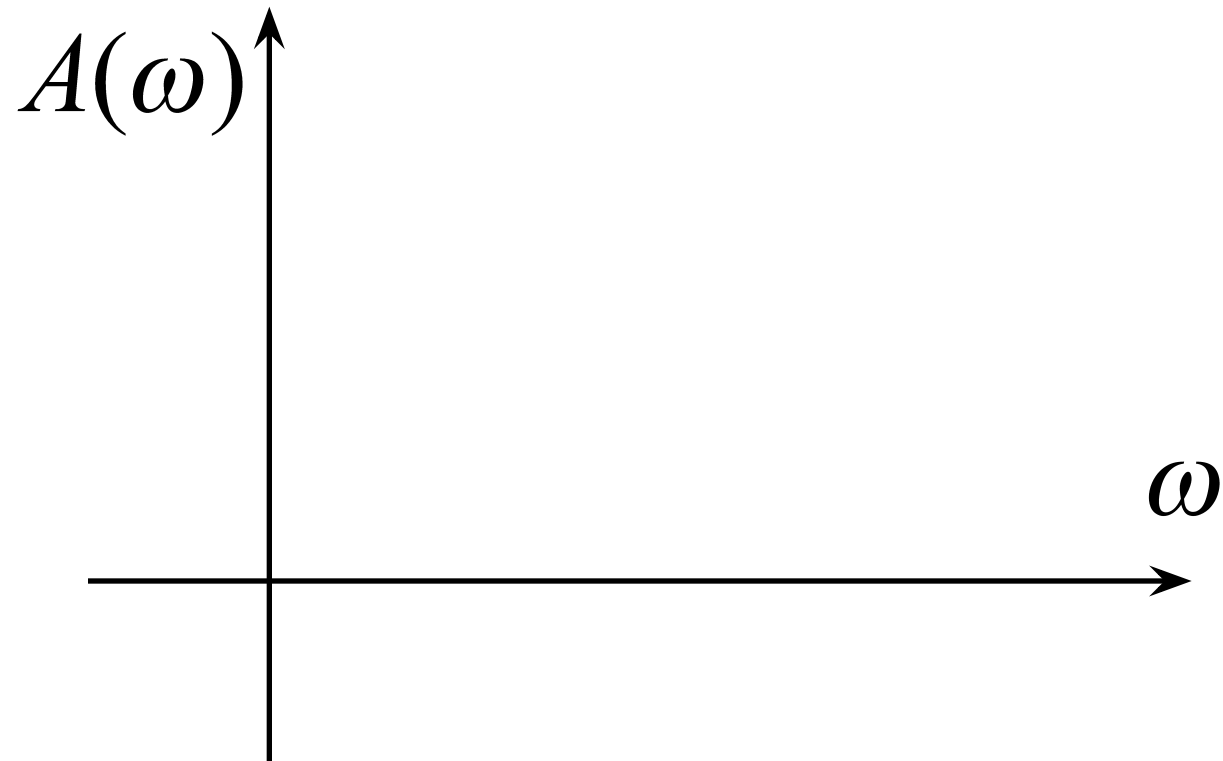
$$W_{2i}(p) = \frac{C_{2i}(p)}{A_{2i}(p)}$$

$$W(j\omega) = X(\omega) + jY(\omega)$$

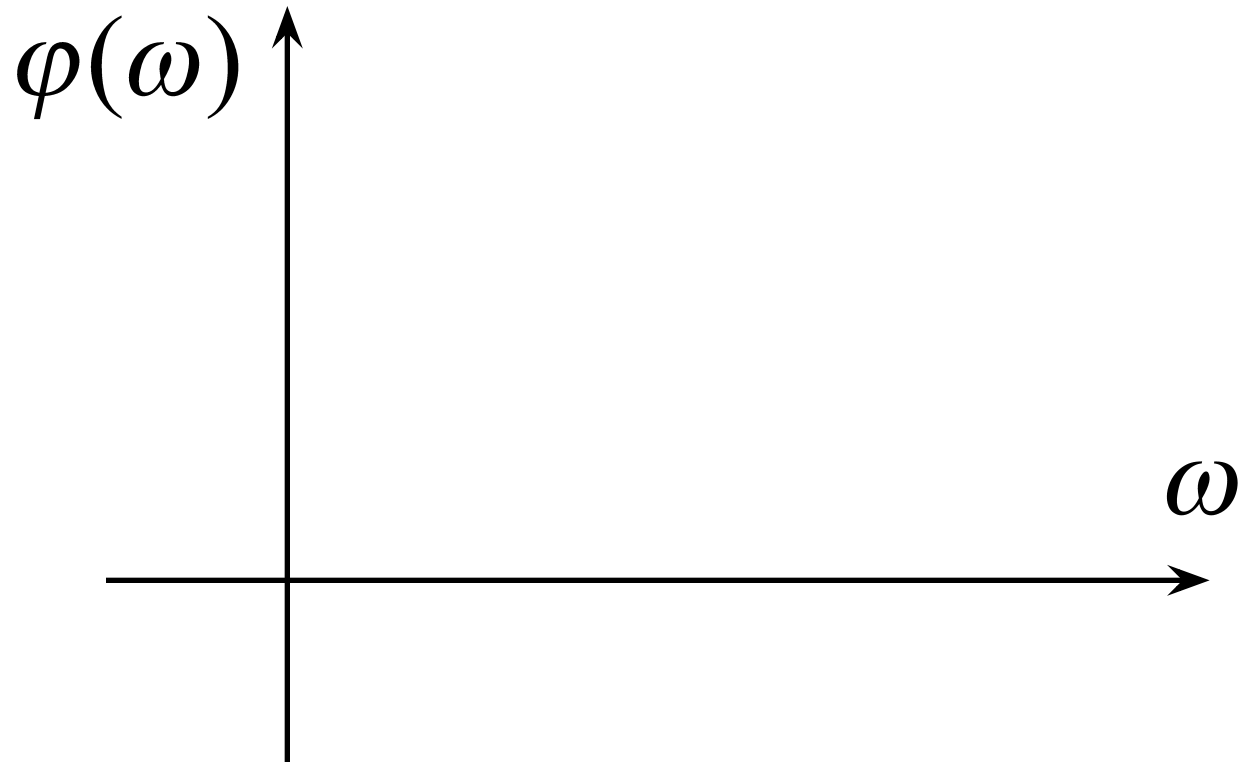




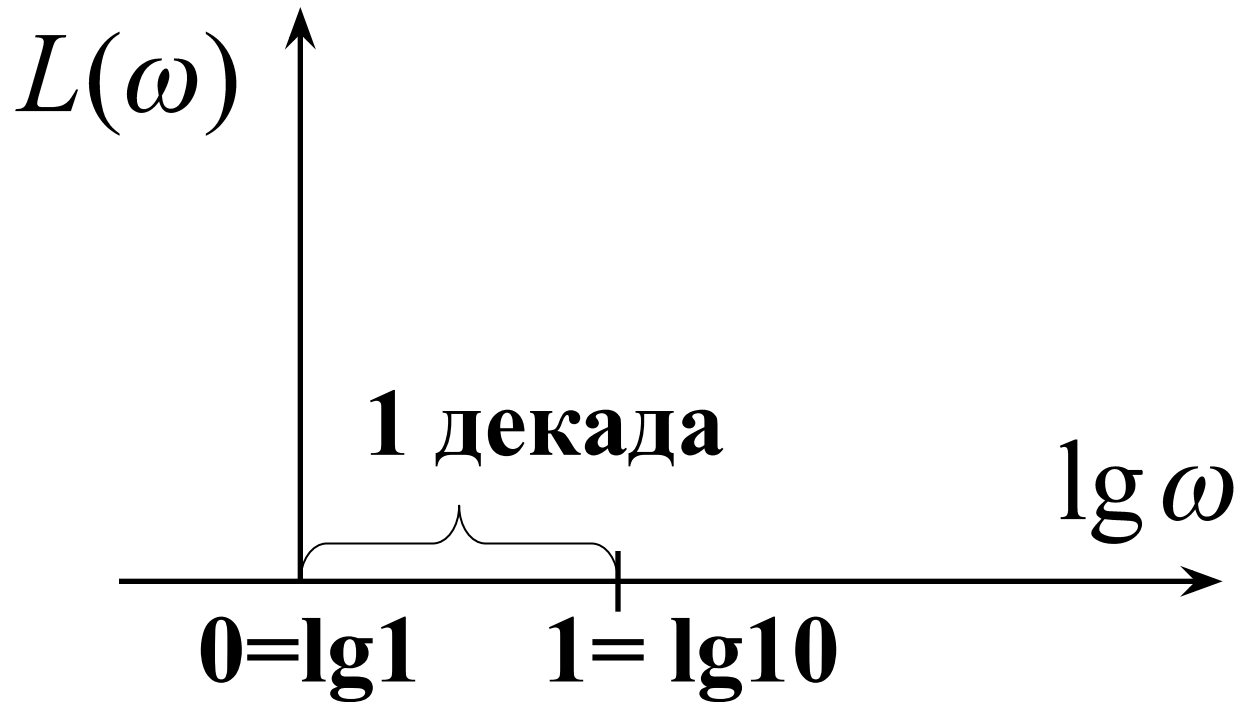
$$A(\omega) = |W(j\omega)| = \sqrt{X^2(\omega) + Y^2(\omega)}$$



$$\varphi(\omega) = \operatorname{arctg} \frac{Y(\omega)}{X(\omega)}$$

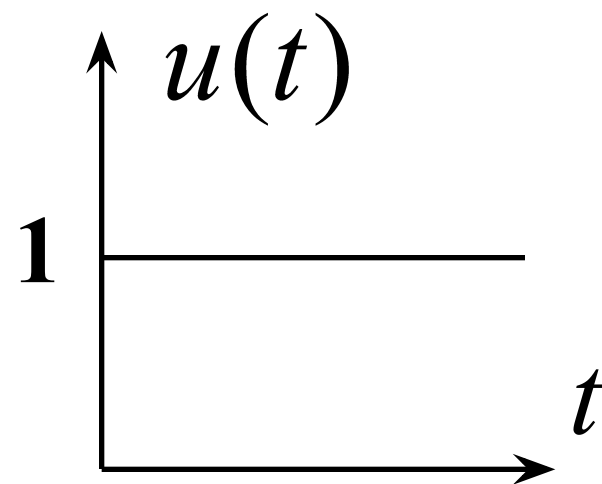


$$L(\omega) = 20 \lg A(\omega)$$

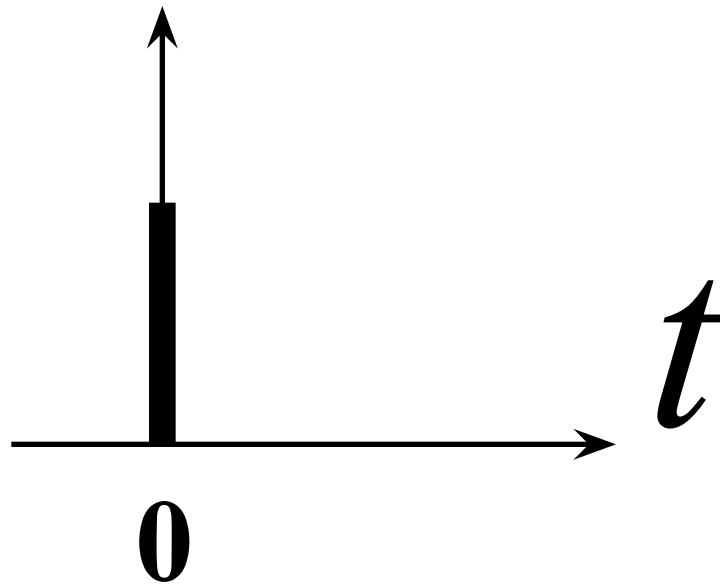


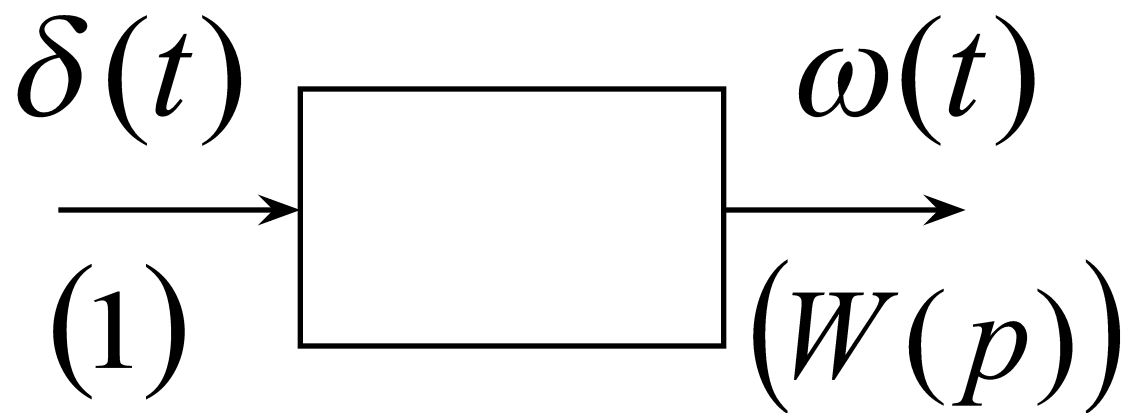
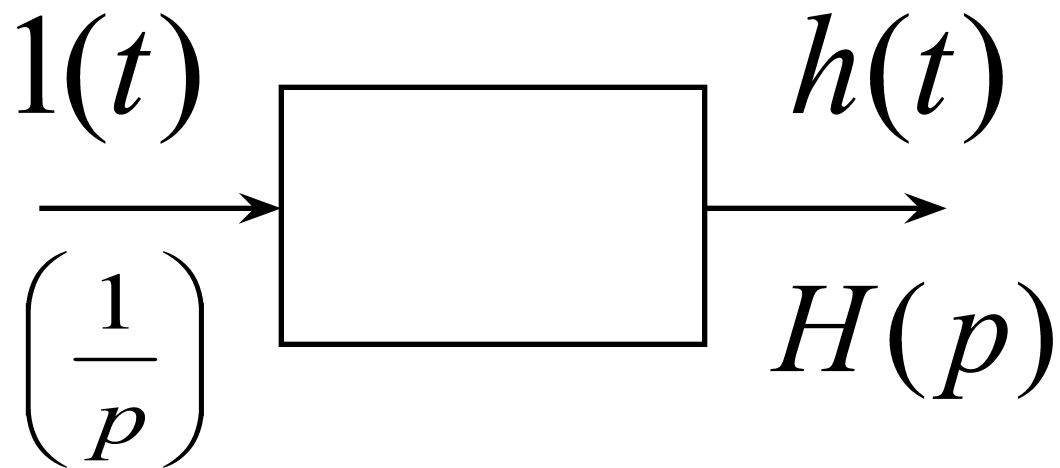


1)  $u(t) = 1(t)$



$$2) \quad \delta(t) = (1(t))'$$





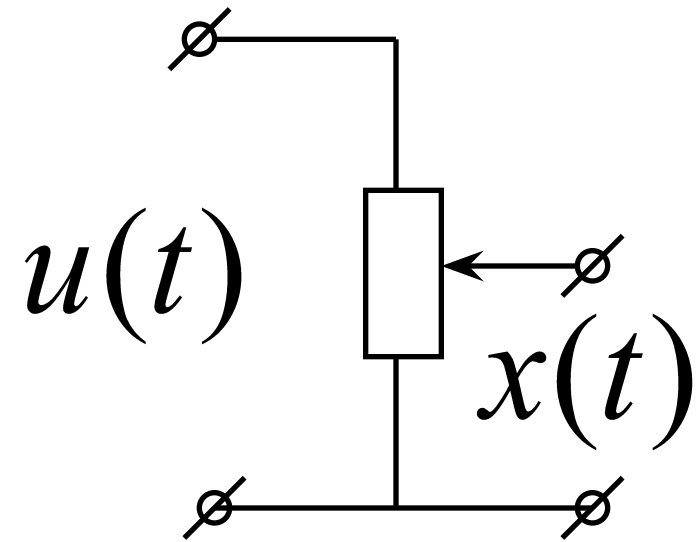
$$h(t) = L^{-1} \left( \frac{W(p)}{p} \right)$$

$$\omega(t) = L^{-1} (W(p))$$

# Пропорциональное звено

1.  $x(t) = ku(t)$

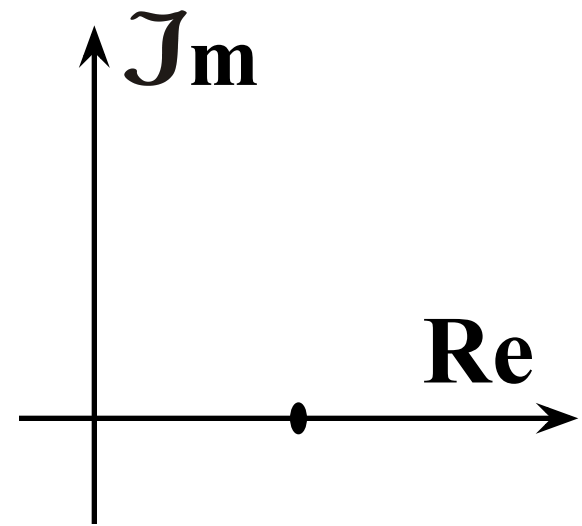
2.  $X(p) = kU(p)$



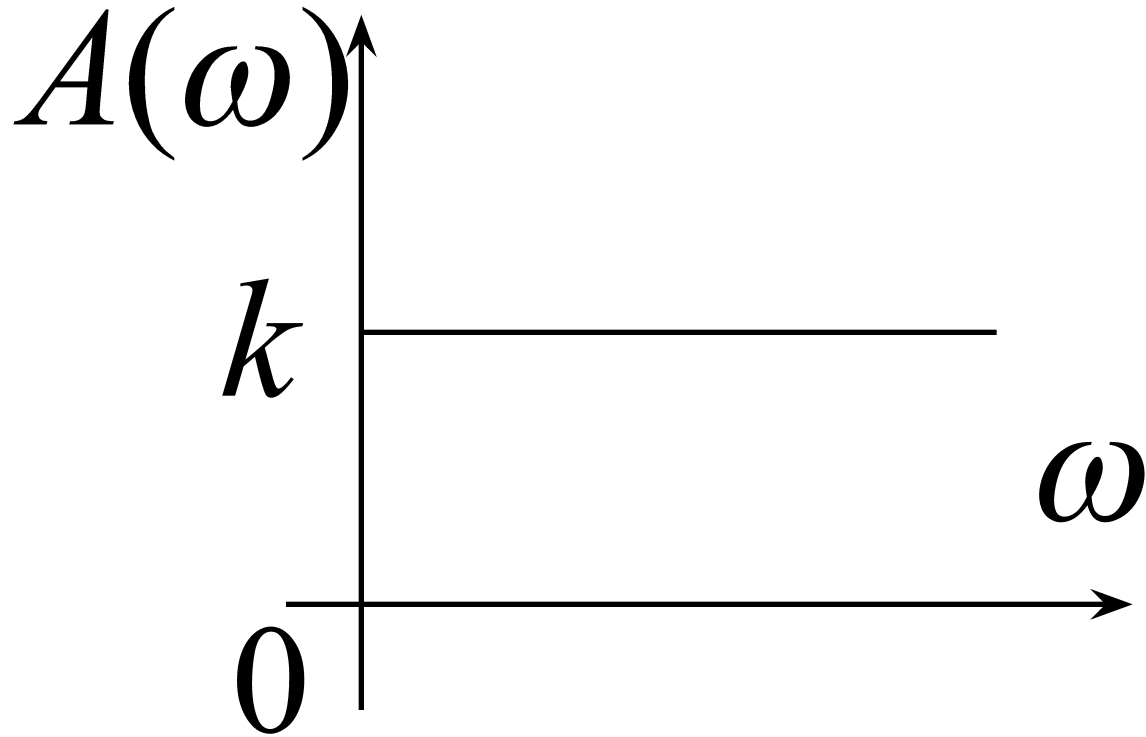


$$3 \quad W(p) = k$$

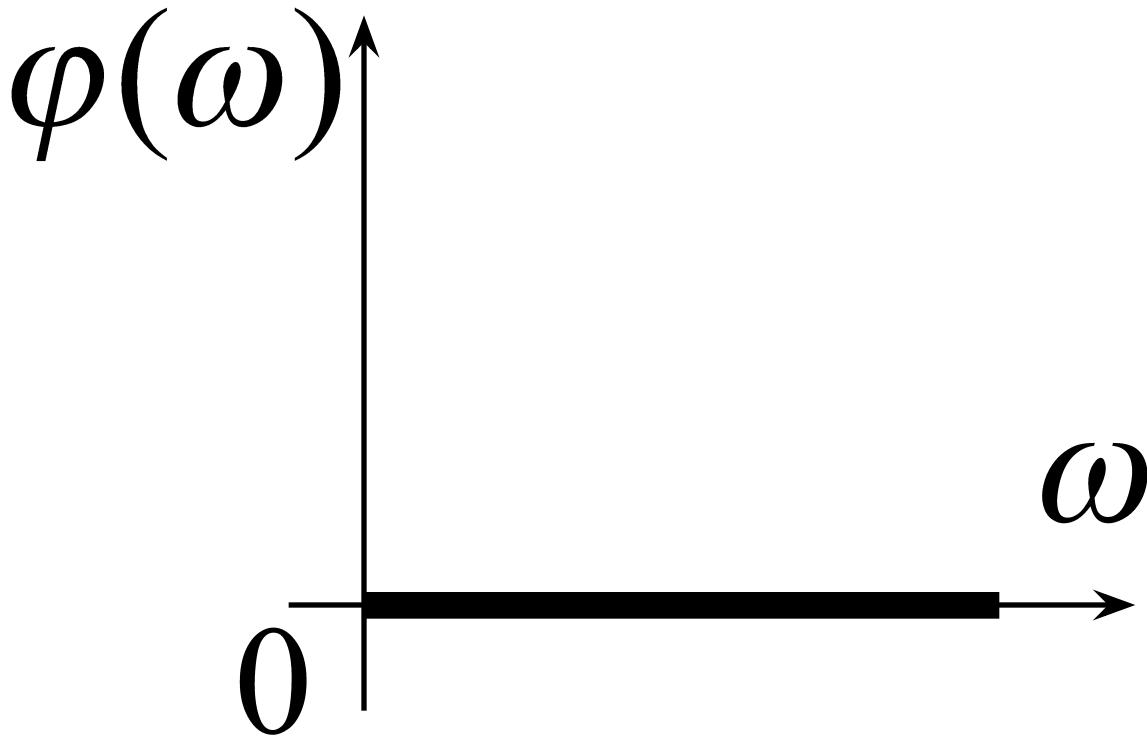
$$4.1 \quad W(j\omega) = k$$



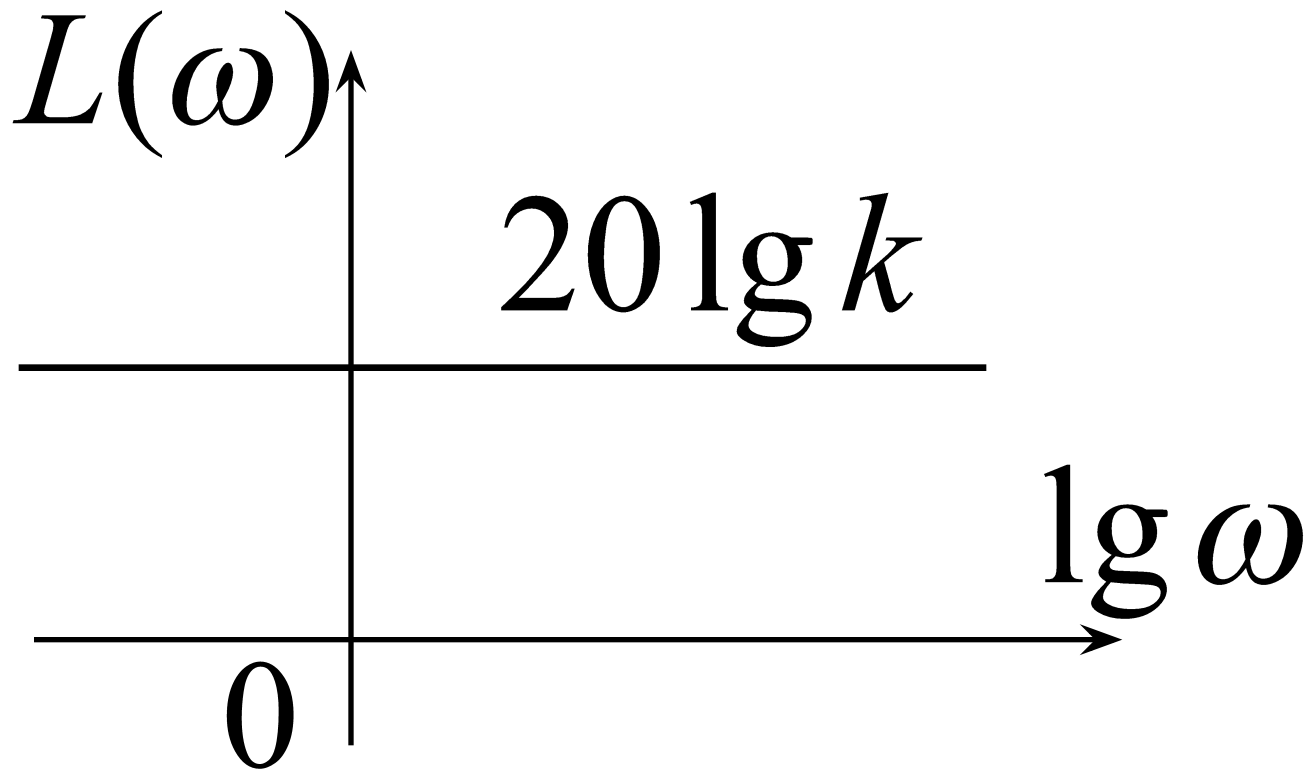
$$4.2 \quad A(\omega) = k$$



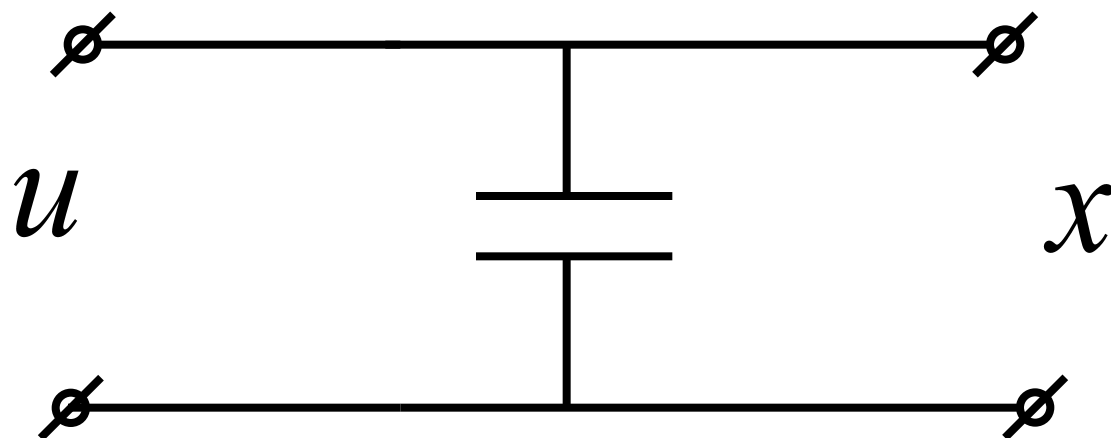
$$4.3 \quad \varphi(\omega) = \operatorname{arctg}\left(\frac{0}{k}\right) = 0$$



$$4.4 \quad L(\omega) = 20 \lg k$$



# Интегрирующее звено

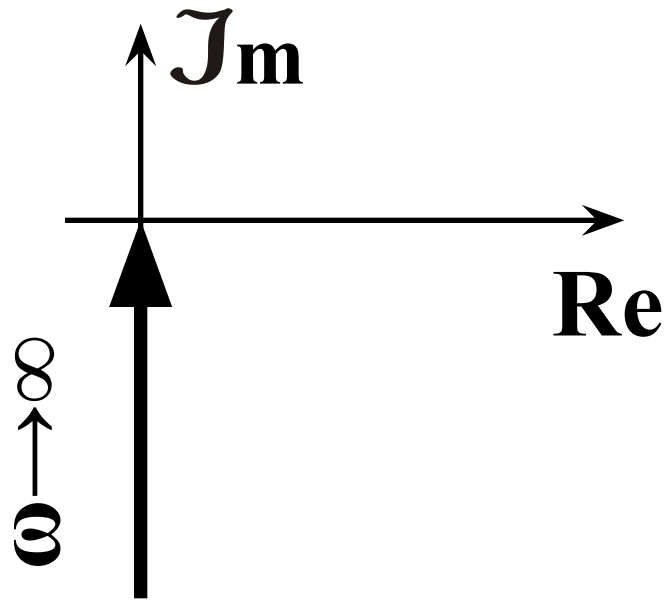


$$1. \quad X(t) = k \int_0^t u(t) dt$$

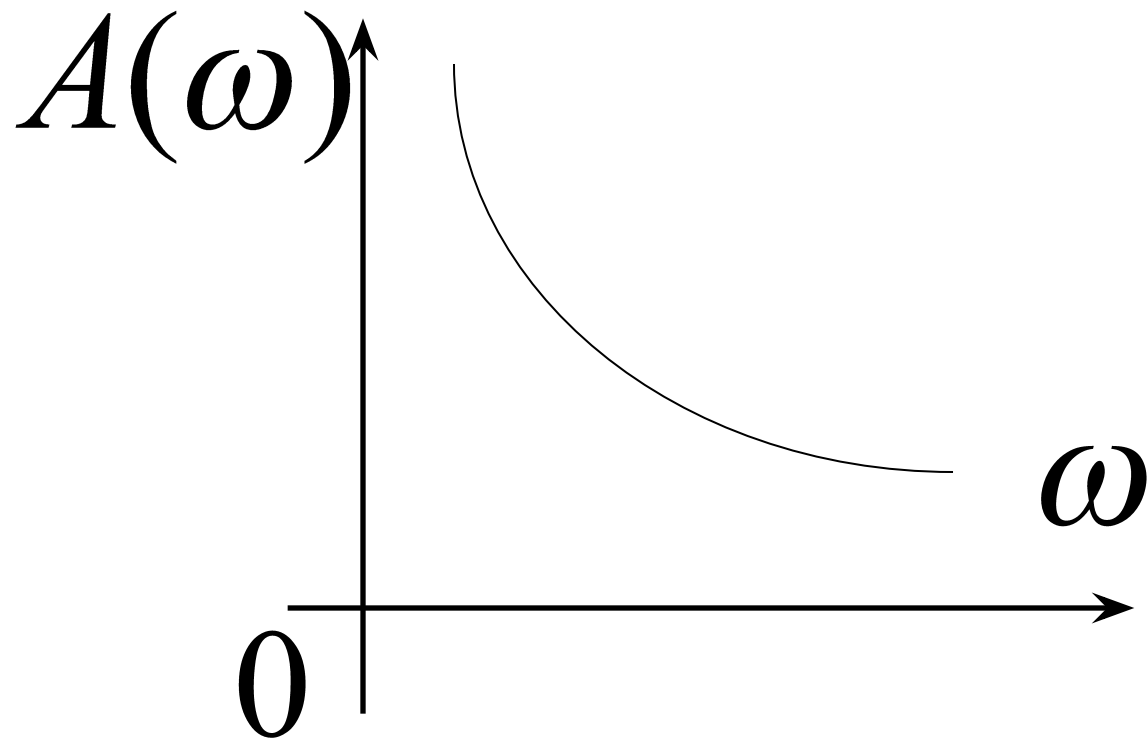
$$2. \quad X(p) = \frac{k}{p} U(p)$$

$$3. \quad W(p) = \frac{k}{p}$$

$$4.1 \quad W(j\omega) = \frac{k}{j\omega} = -j \frac{k}{\omega}$$

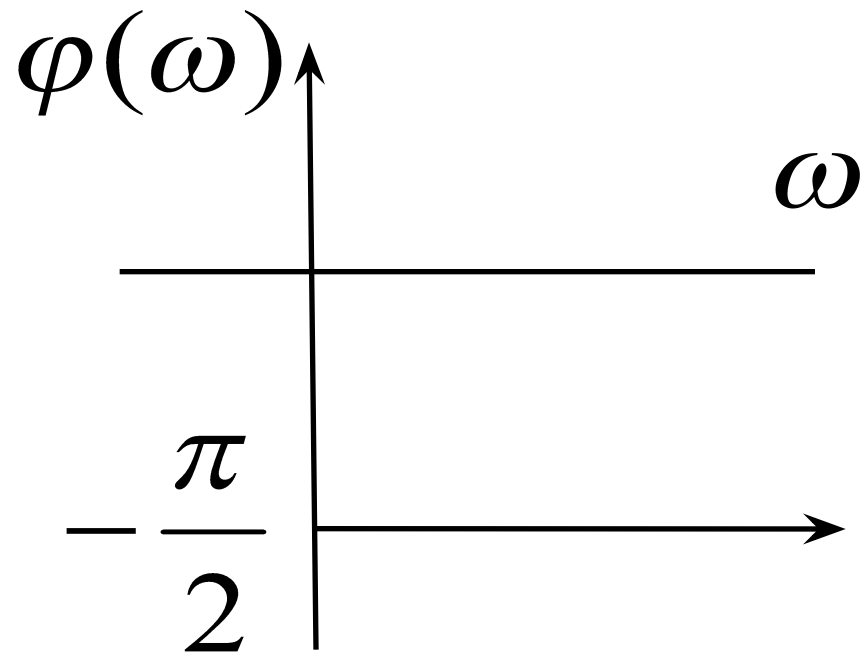


$$4.2 \quad A(\omega) = \left| -j \frac{k}{\omega} \right| = \frac{k}{\omega}$$

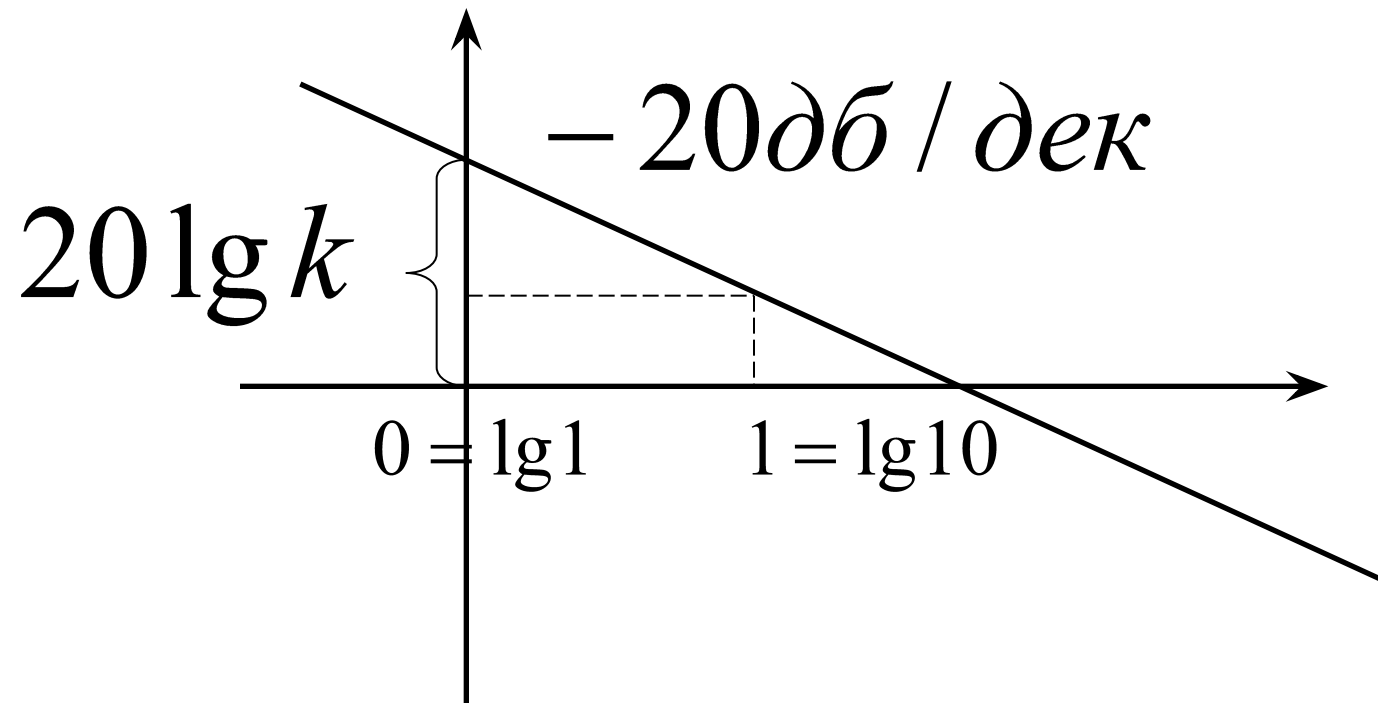




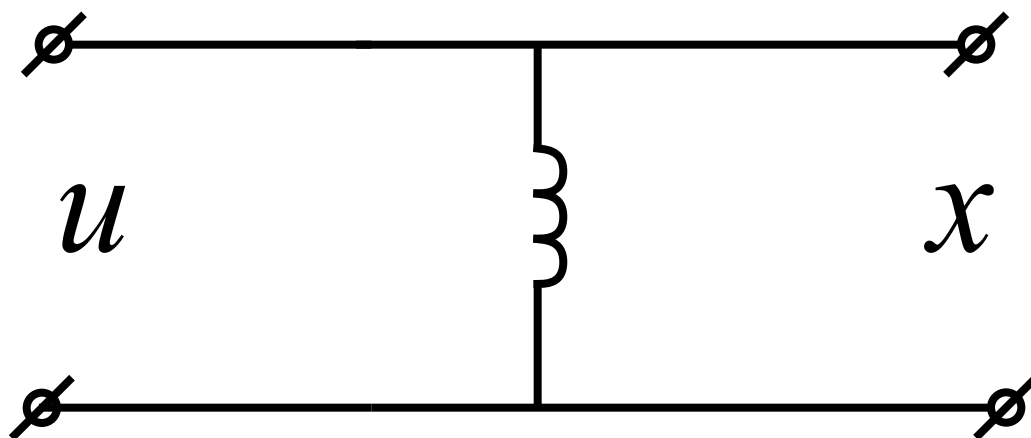
$$4.3 \quad \varphi(\omega) = \operatorname{arctg} \left( \frac{-k/\omega}{0} \right) = -\frac{\pi}{2}$$



$$4.4 \quad L(\omega) = 20 \lg \left( \frac{k}{\omega} \right) = 20 \lg k - 20 \lg \omega$$



# Дифференцирующее звено

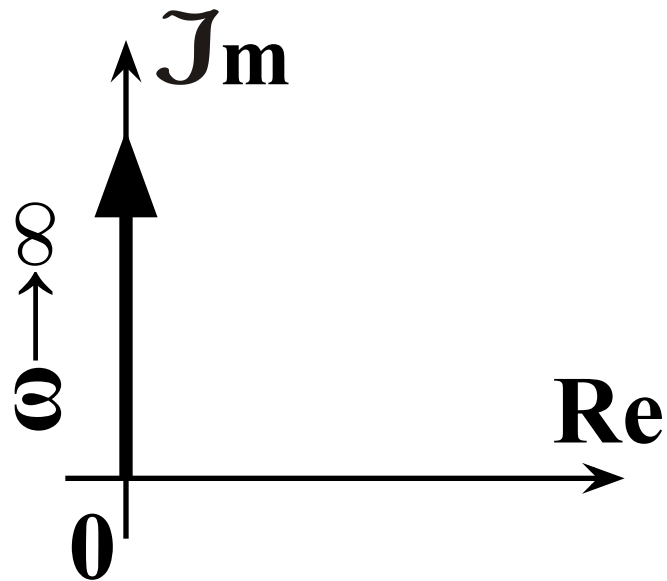


$$1. \quad x(t) = T_{\delta} \frac{du}{dt}$$

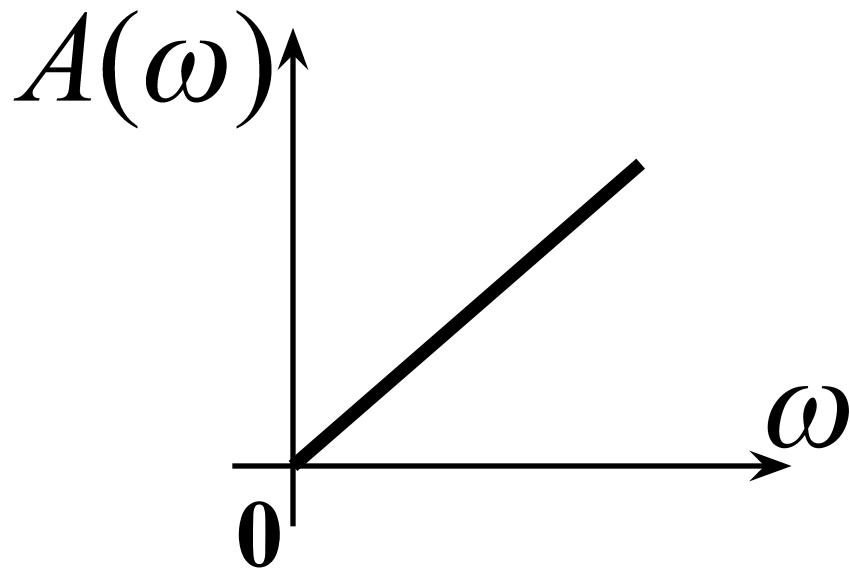
$$2. \quad X(p) = T_{\delta} p \cdot U(p)$$

$$3. \quad W(p) = T_{\delta} p$$

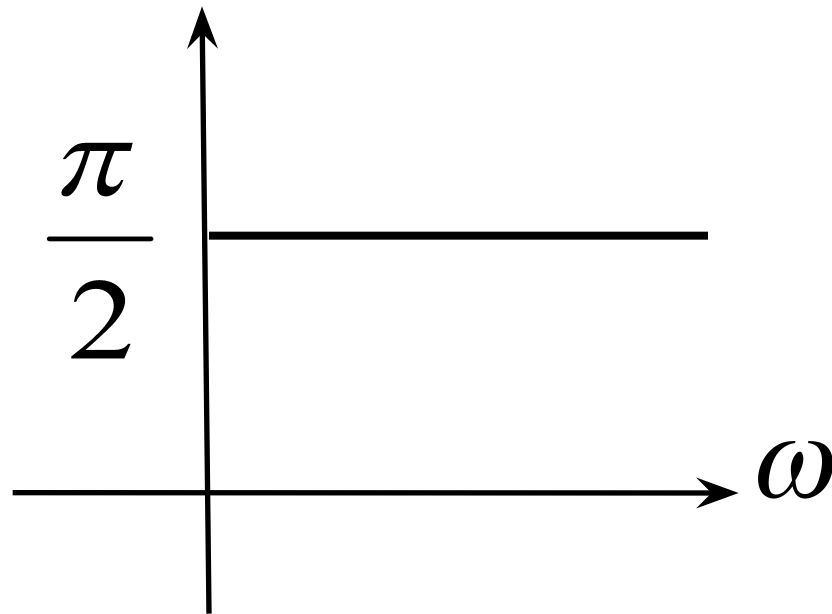
$$4.1 \quad W(j\omega) = T_\delta(j\omega)$$



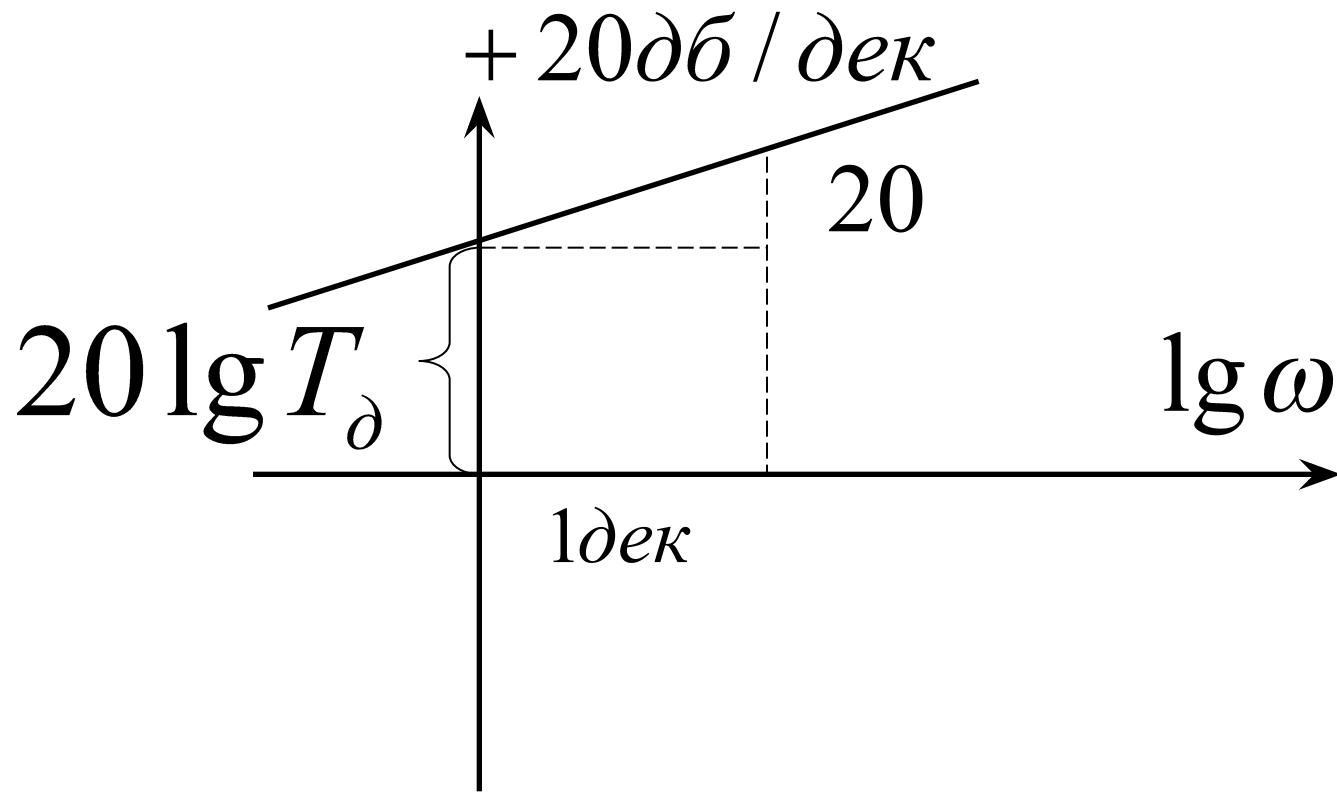
$$4.2 \quad A(\omega) = T_{\delta} \omega$$



$$4.3 \quad \phi(\omega) = \operatorname{arctg} \left( \frac{T\omega}{0} \right) = \frac{\pi}{2}$$

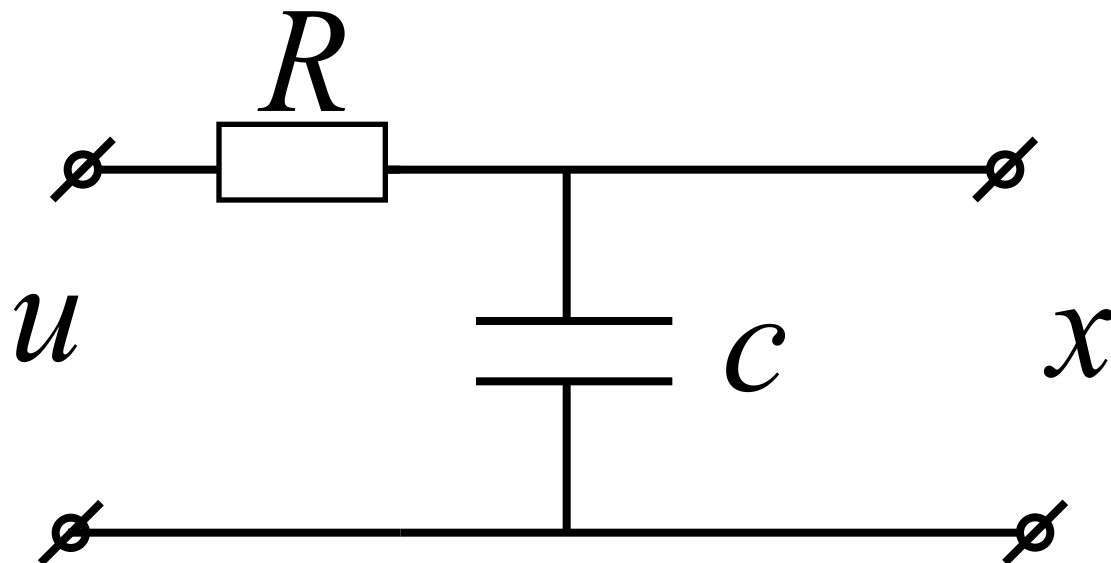


$$4.4 \quad L(\omega) = 20 \lg(T_{\delta} \omega) = 20 \lg T_{\delta} + 20 \lg \omega$$





# Апериодическое звено



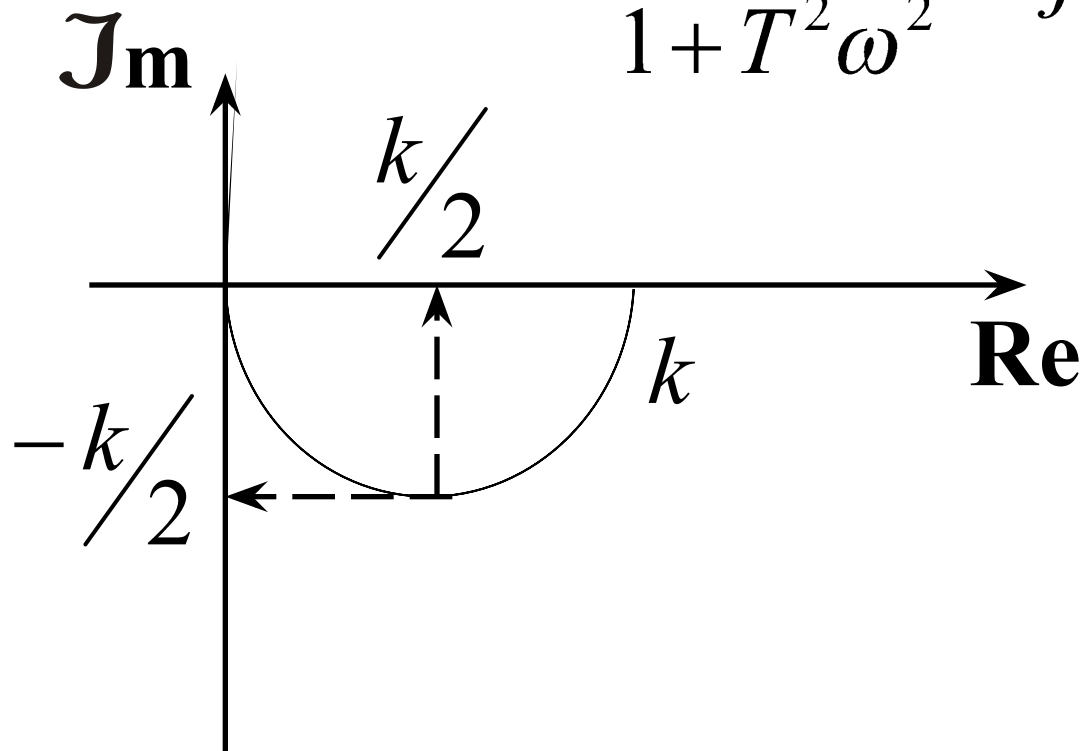
$$1. \quad T \frac{dx}{dt} + x = ku$$

$$2. \quad (Tp + 1)X(p) = kU(p)$$

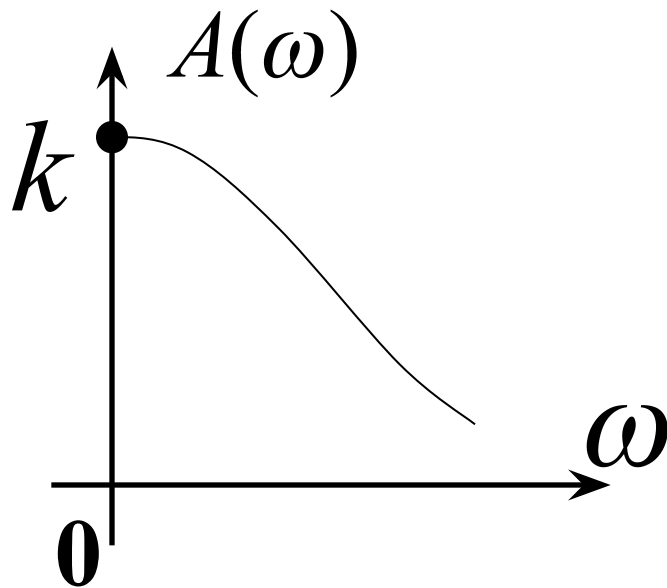
$$3. \quad W(p) = \frac{k}{Tp + 1}$$

$$4.1 \quad W(j\omega) = \frac{k}{1+T(j\omega)} = \frac{k(1-jT\omega)}{1+T^2\omega^2} =$$

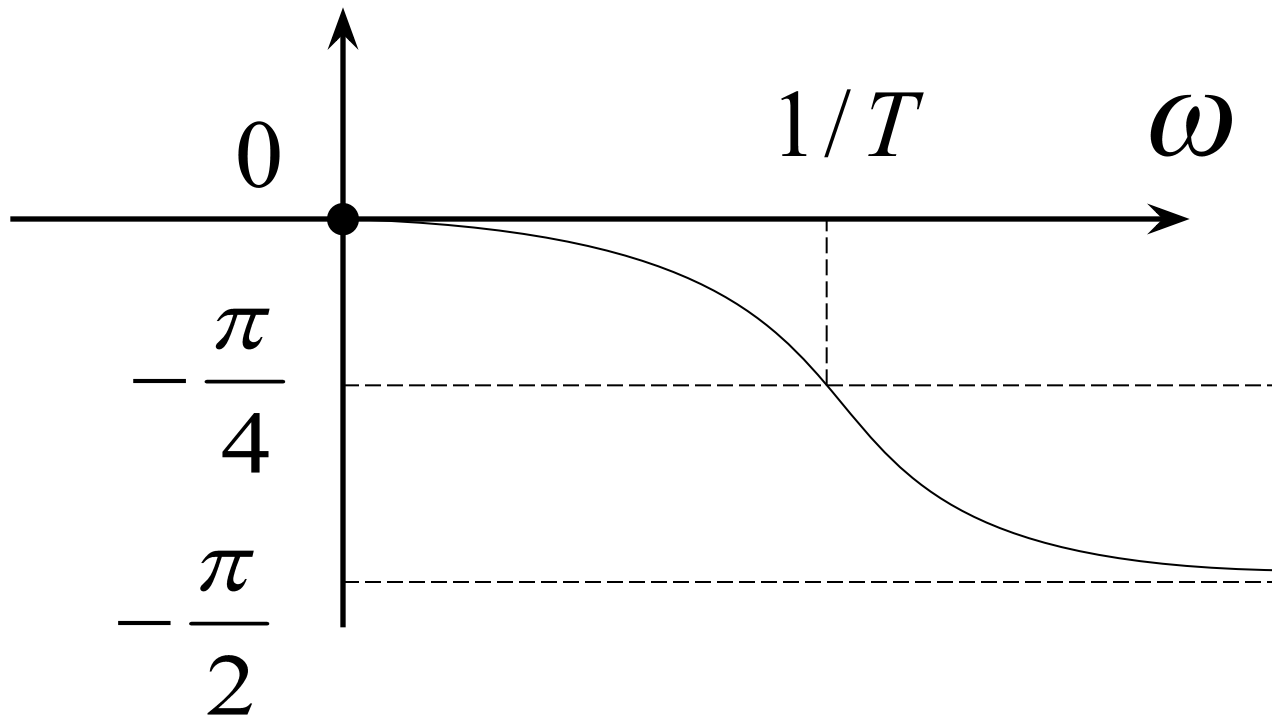
$$= \frac{k}{1+T^2\omega^2} - j \frac{kT\omega}{1+T^2\omega^2}$$



$$4.2 \quad A(\omega) = \frac{|k|}{|1 + jT\omega|} = \frac{k}{\sqrt{1 + T^2\omega^2}}$$



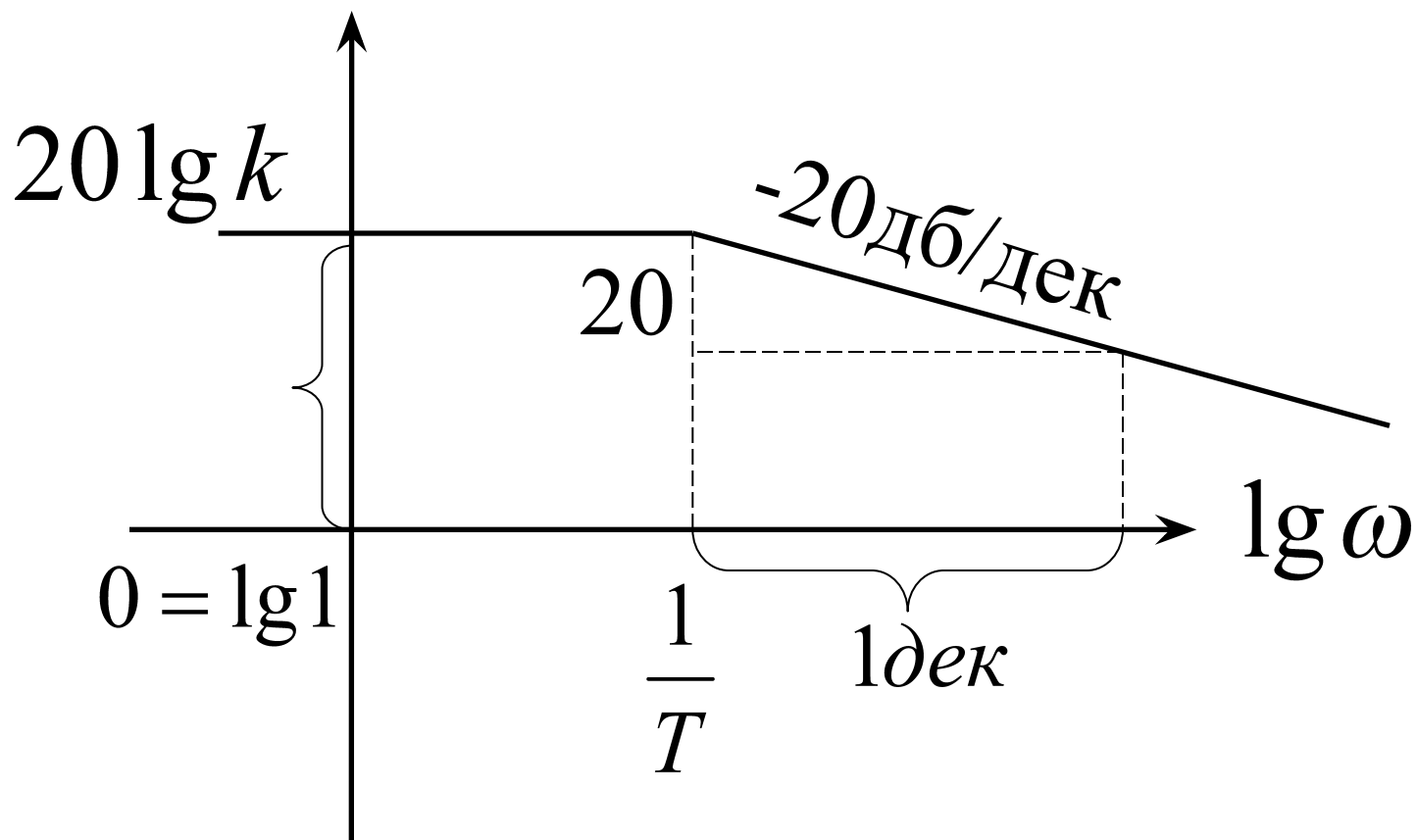
4.3  $\varphi(\omega) = \text{arctg}(-T\omega)$



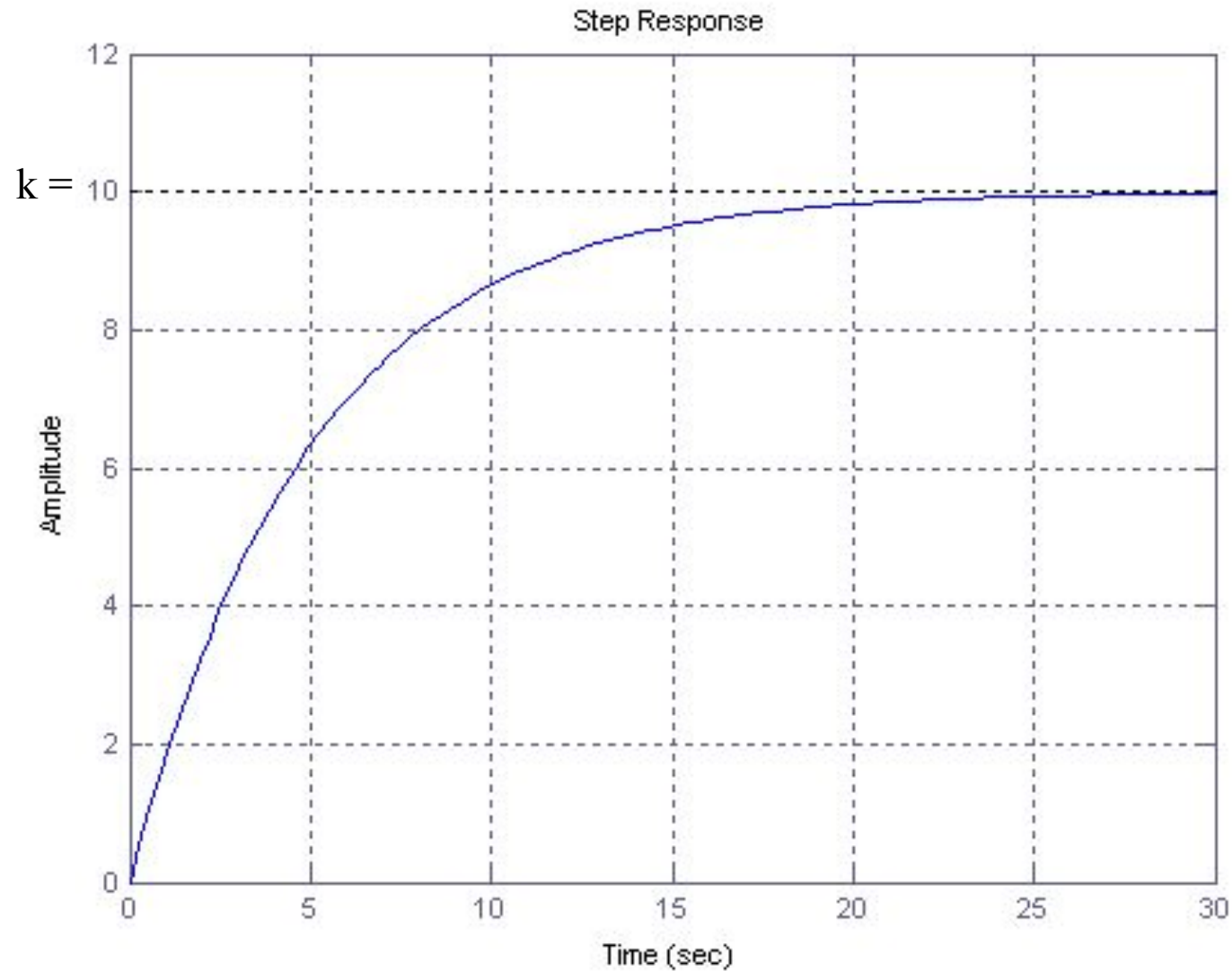
$$4.4 \quad L(\omega) = 20 \lg k - 10 \lg(1 + T^2 \omega^2)$$

$$\omega \ll \frac{1}{T} \quad L(\omega) = 20 \lg k$$

$$\omega \gg \frac{1}{T} \quad L(\omega) = 20 \lg k - 20 \lg T \omega$$



$$5. \quad h(t) = k(1 - e^{-t/T})$$





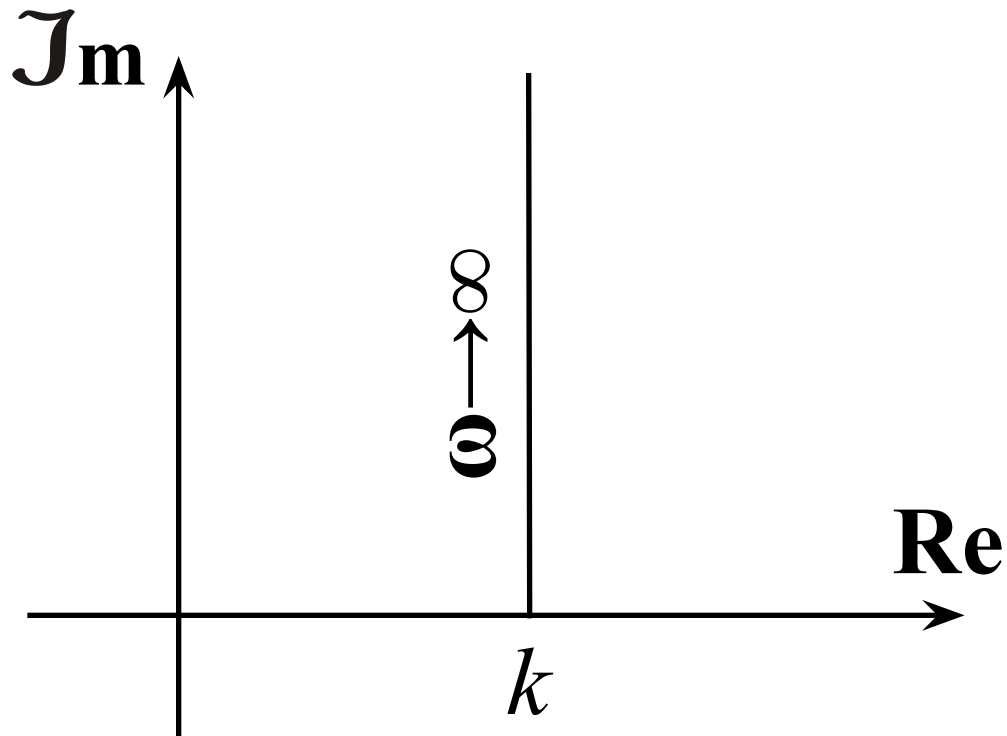
## Форсирующее звено

$$1. \quad x(t) = k\left(u + T \frac{du}{dt}\right)$$

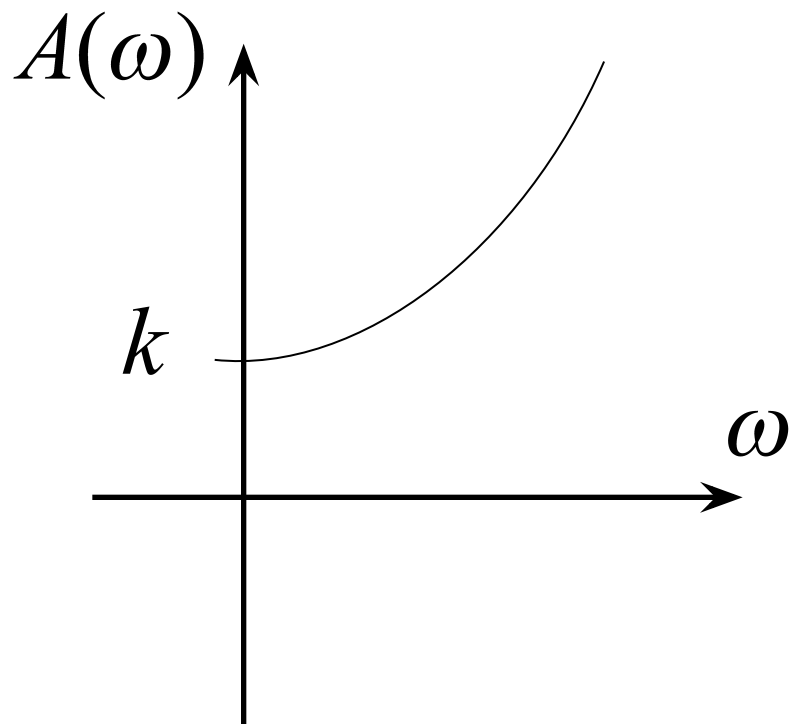
$$2. \quad X(p) = k(Tp + 1)U(p)$$

$$3. \quad W(p) = k(Tp + 1)$$

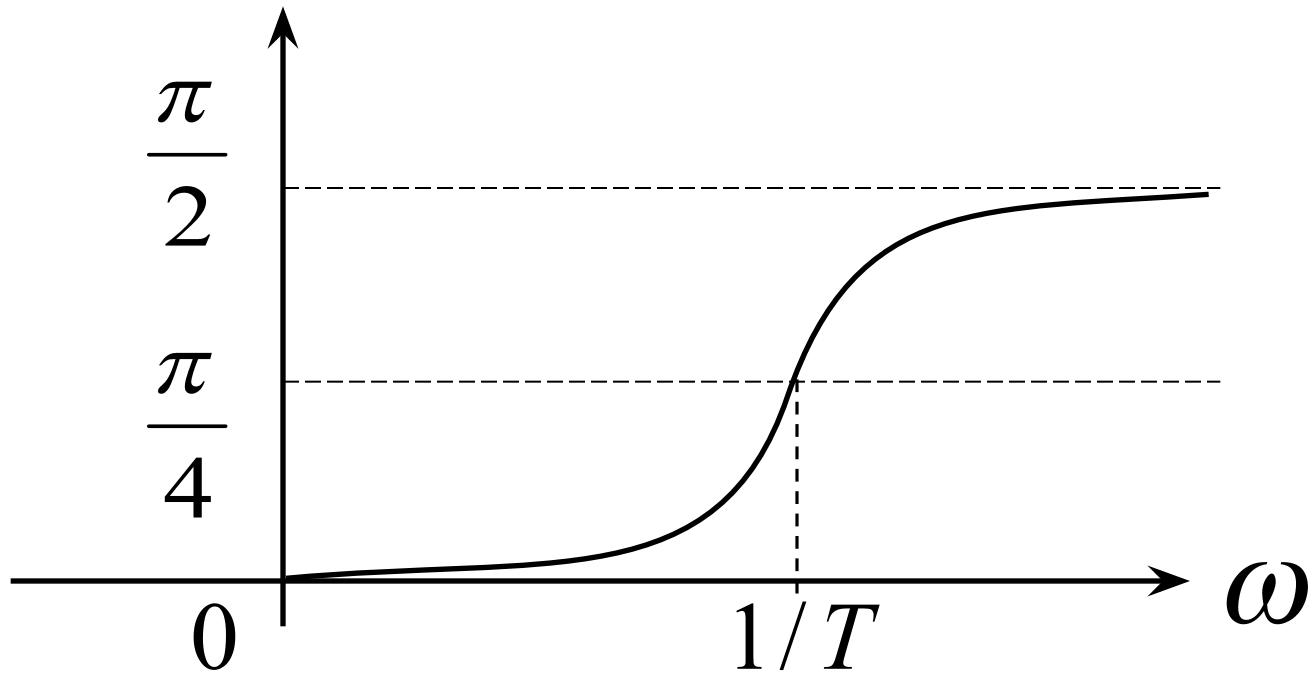
$$4.1 \quad W(j\omega) = k + jkT\omega$$



$$4.2 \quad A(\omega) = k\sqrt{1 + T^2\omega^2}$$



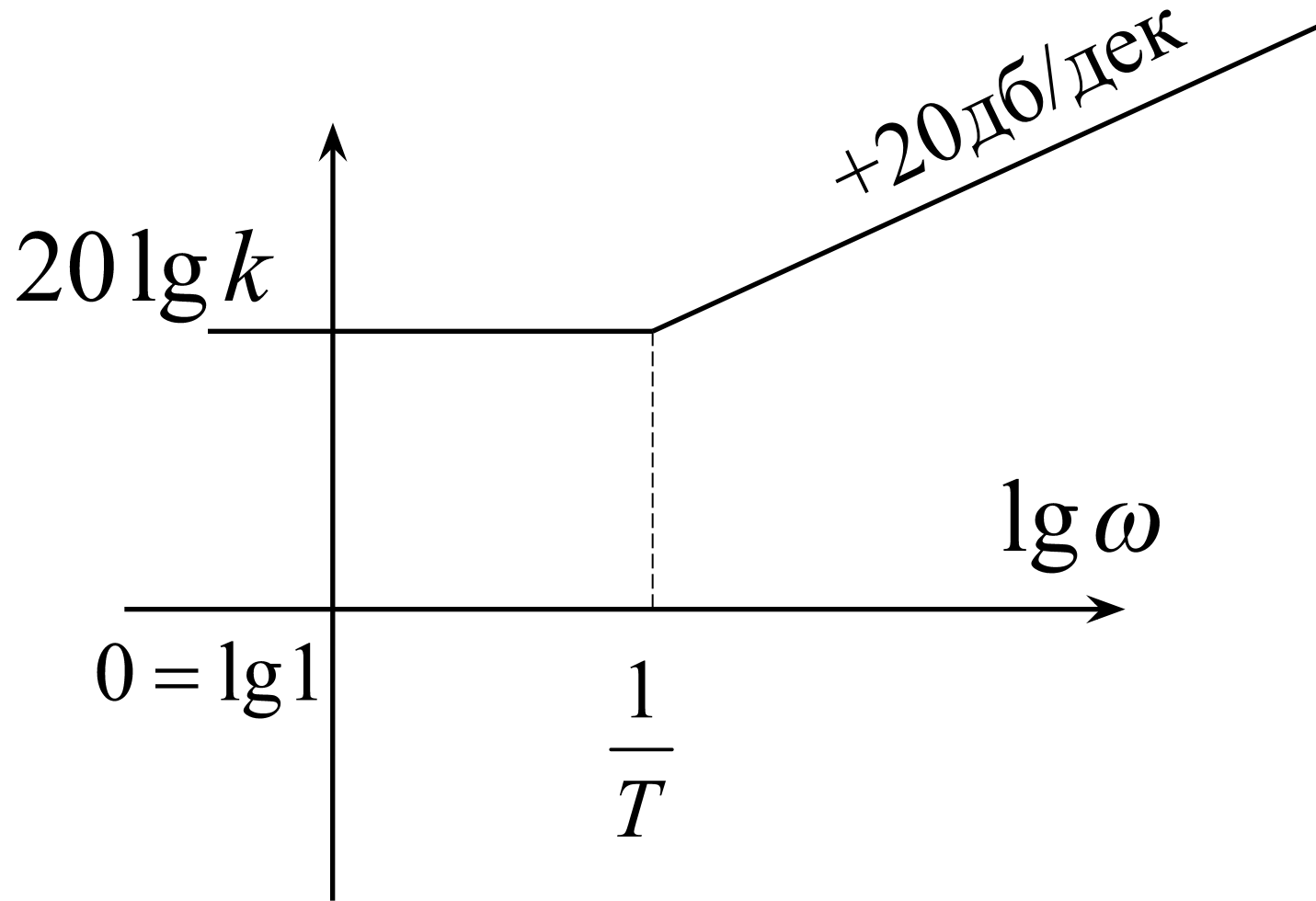
4.3  $\varphi(\omega) = \text{arctg}(T\omega)$



$$4.4 \quad L(\omega) = 20 \lg k + 10 \lg(1 + T^2 \omega^2)$$

$$\omega \ll \frac{1}{T} \quad L(\omega) = 20 \lg k$$

$$\omega \gg \frac{1}{T} \quad L(\omega) = 20 \lg k + 20 \lg(T\omega)$$



# Колебательное звено

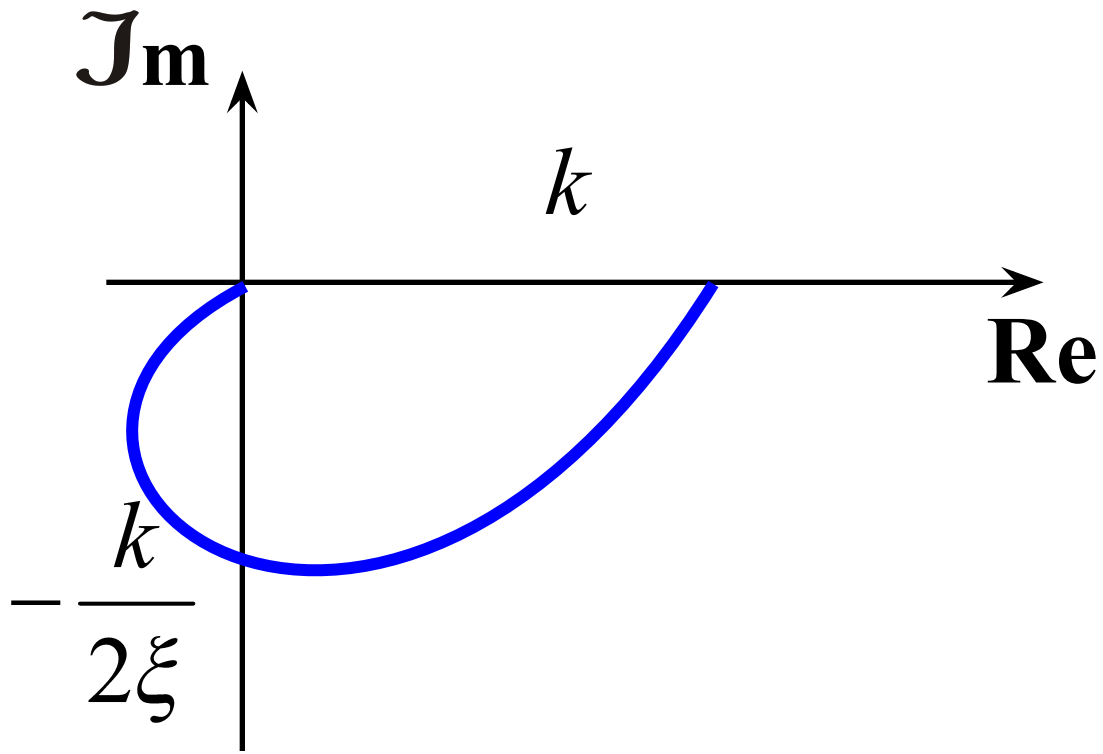
$$1. T^2 \frac{d^2 x}{dt^2} + 2T\xi \frac{dx}{dt} + x = ku$$

$$2. (T^2 p^2 + 2T\xi p + 1) \cdot X(p) = kU(p)$$

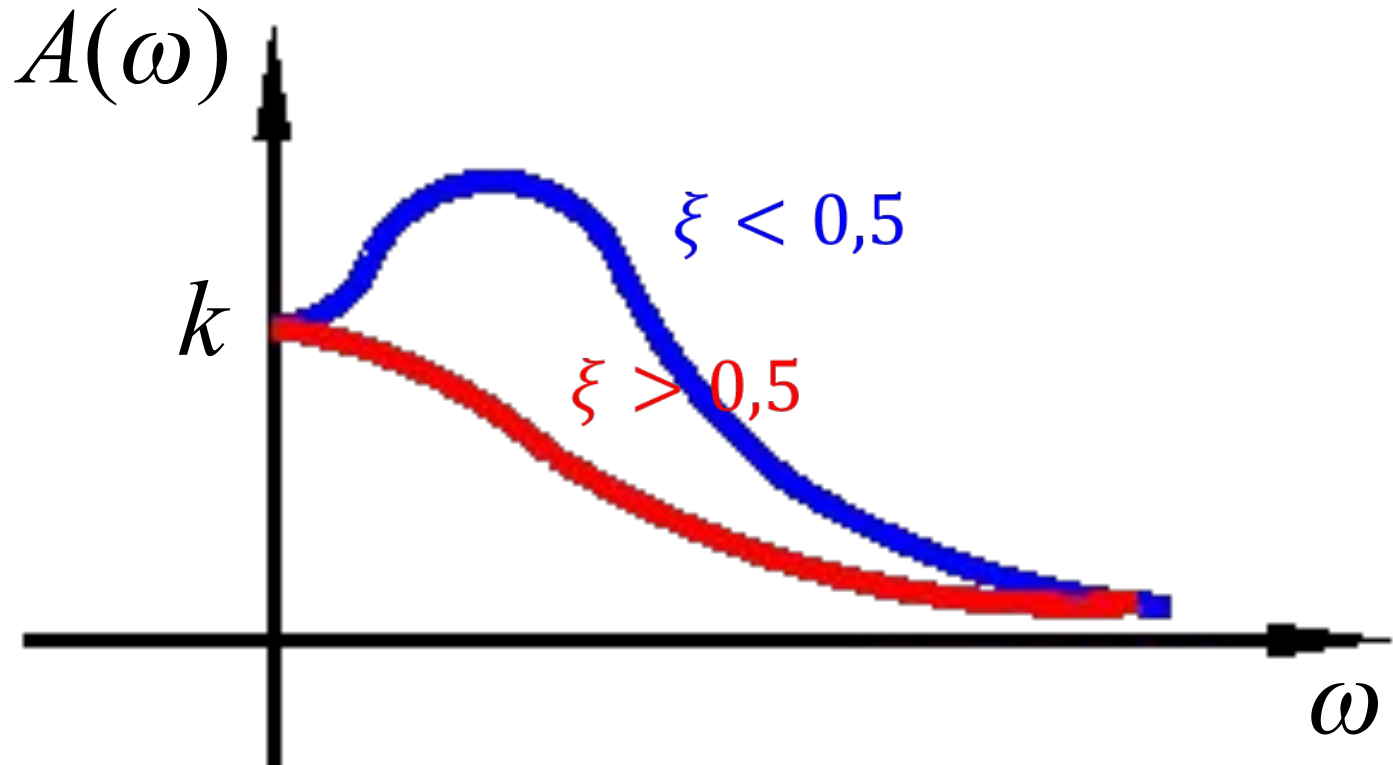
$$3. W(p) = \frac{k}{T^2 p^2 + 2T\xi p + 1}$$

$$\begin{aligned}
4.1 \quad W(j\omega) &= \frac{k}{(1 - T^2\omega^2) + j2\xi T\omega} = \\
&= \frac{k(1 - T^2\omega^2)}{(1 - T^2\omega^2)^2 + 4\xi^2 T^2\omega^2} - \\
&\quad - j \frac{2\xi T\omega k}{(1 - T^2\omega^2)^2 + 4\xi^2 T^2\omega^2}
\end{aligned}$$

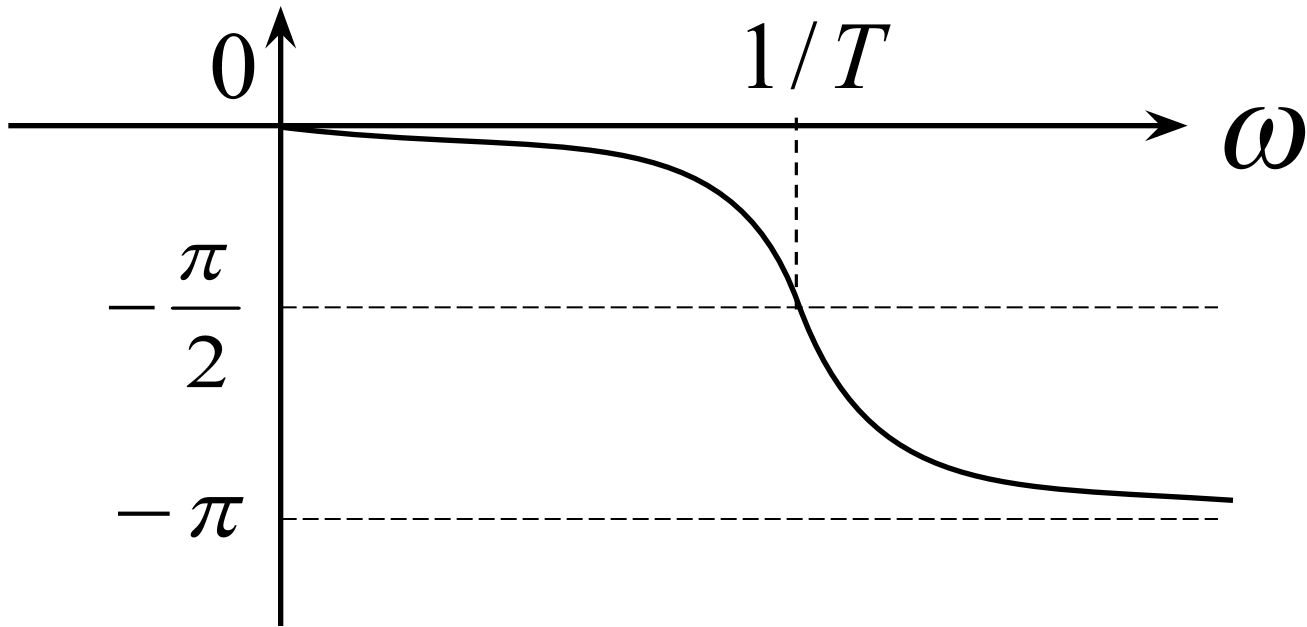




$$4.2 \quad A(\omega) = \frac{k}{\sqrt{(1 - T^2 \omega^2)^2 + 4\xi^2 T^2 \omega^2}}$$



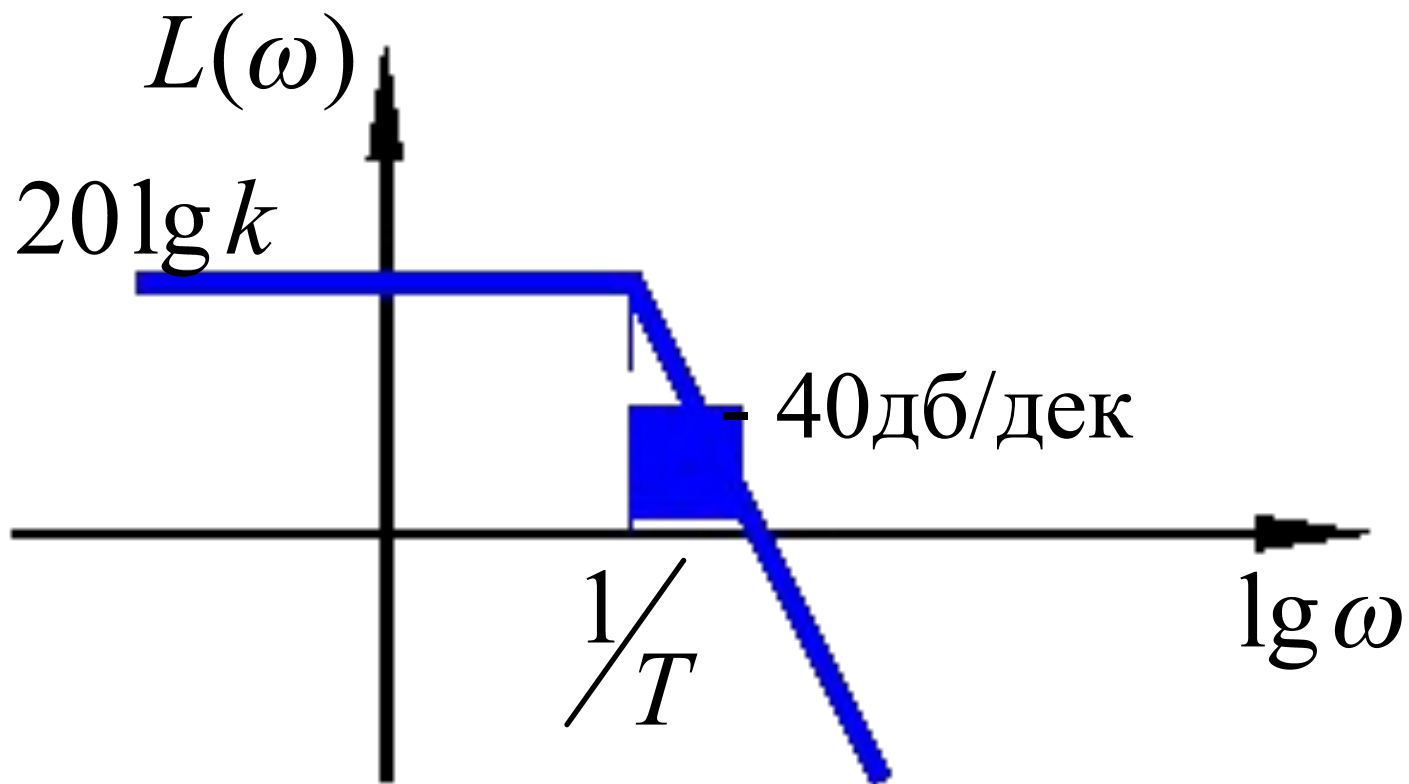
$$4.3 \varphi(\omega) = \operatorname{arctg} \left( -\frac{2\xi T\omega}{1-T^2\omega^2} \right)$$



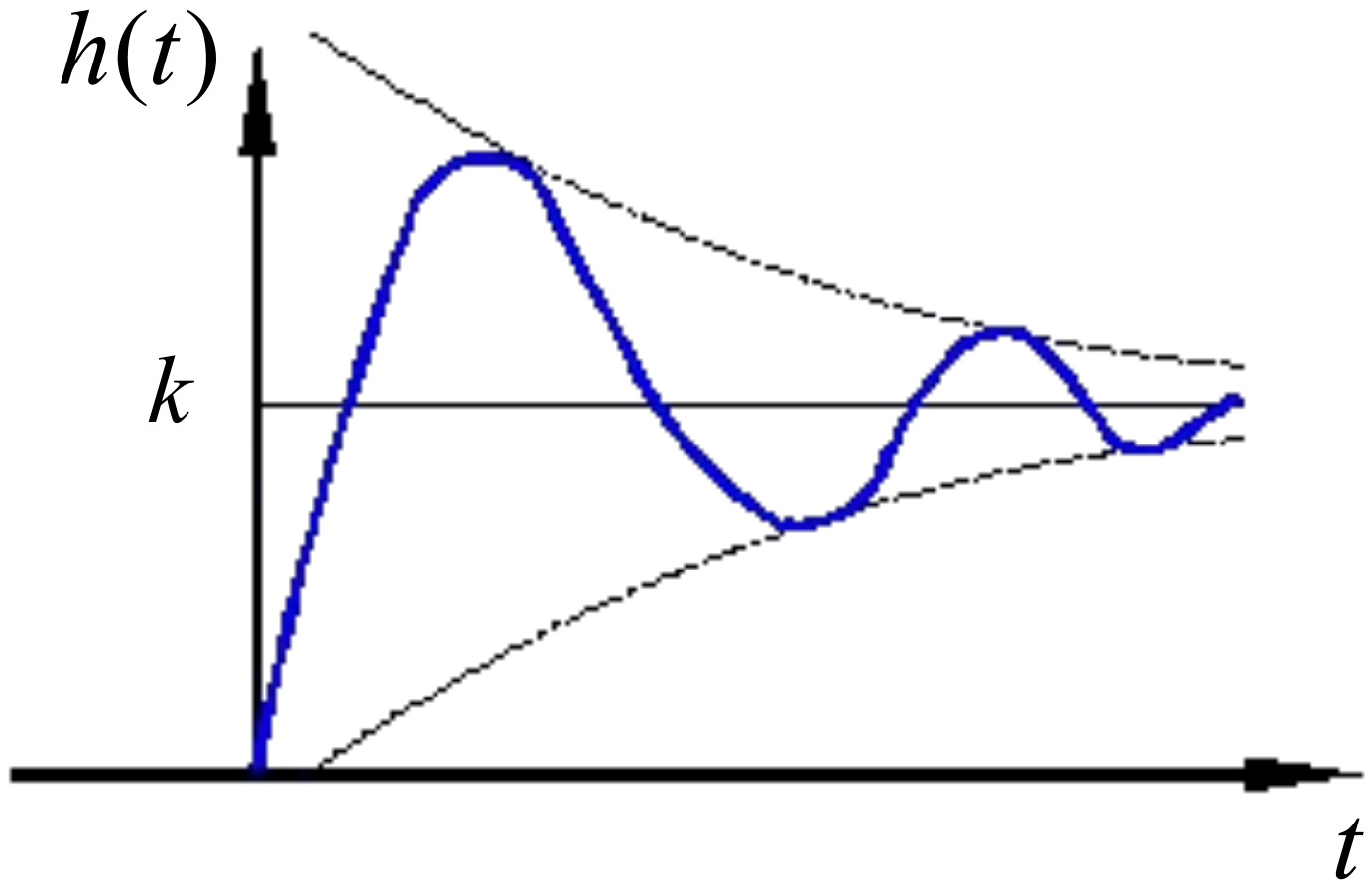
$$4.4 \quad 20 \lg k - 10 \lg \left( (1 - T^2 \omega^2)^2 + 4\xi^2 T^2 \omega^2 \right)$$

$$\omega \ll \frac{1}{T} \rightarrow L(\omega) = 20 \lg k$$

$$\omega \gg \frac{1}{T} \rightarrow L(\omega) = 20 \lg k - \underbrace{20 \lg T^2 \omega^2}_{40 \lg T \omega}$$



$$5. h(t) = k \left( 1 - e^{-\alpha t} \left( \cos \omega t + \frac{\alpha}{\omega} \sin \omega t \right) \right)$$



# Консервативное звено

$$W(p) = \frac{k}{T^2 p^2 + 1}$$

# Форсирующее второго порядка

$$W(s) = k(T^2 s^2 + 2T\xi s + 1)$$