ЗАКОНЫ РЕГУЛИРОВАНИЯ:

1. Пропорциональный (П):

1.1 $x(t) = ku(t)$

1.2 $X(p) = kU(p)$

1.3 $W(p) = k$

2. Интегральный (И):

$$2.1 \quad x(t) = k\int_0^t u(t)dt = \frac{1}{T_u}\int_0^t u(t)dt$$

$$2.2 \quad X(p) = \frac{1}{T_u p}U(p)$$

$$2.3 \quad W(p) = \frac{1}{T_u p}$$

3. Дифференциальный (Д):

$$3.1 \quad x(t) = T_д \frac{du}{dt}$$

$$3.2 \quad X(p) = T_д p \cdot U(p)$$

3.3 $$W(p) = T_Д p$$

3.4 $$W(p) = \frac{T_Д p}{\tau_б p + 1},$$

$$где \quad \tau_б = (0{,}01 \div 0{,}1) T_Д$$

4. ПИ:

4.1 $$x(t) = k(u(t) + \frac{1}{T_u}\int_0^t u(t)dt)$$

4.2 $$X(p) = k(1 + \frac{1}{T_u p})U(p)$$

4.3 $$W(p) = k(1 + \frac{1}{T_u p})$$

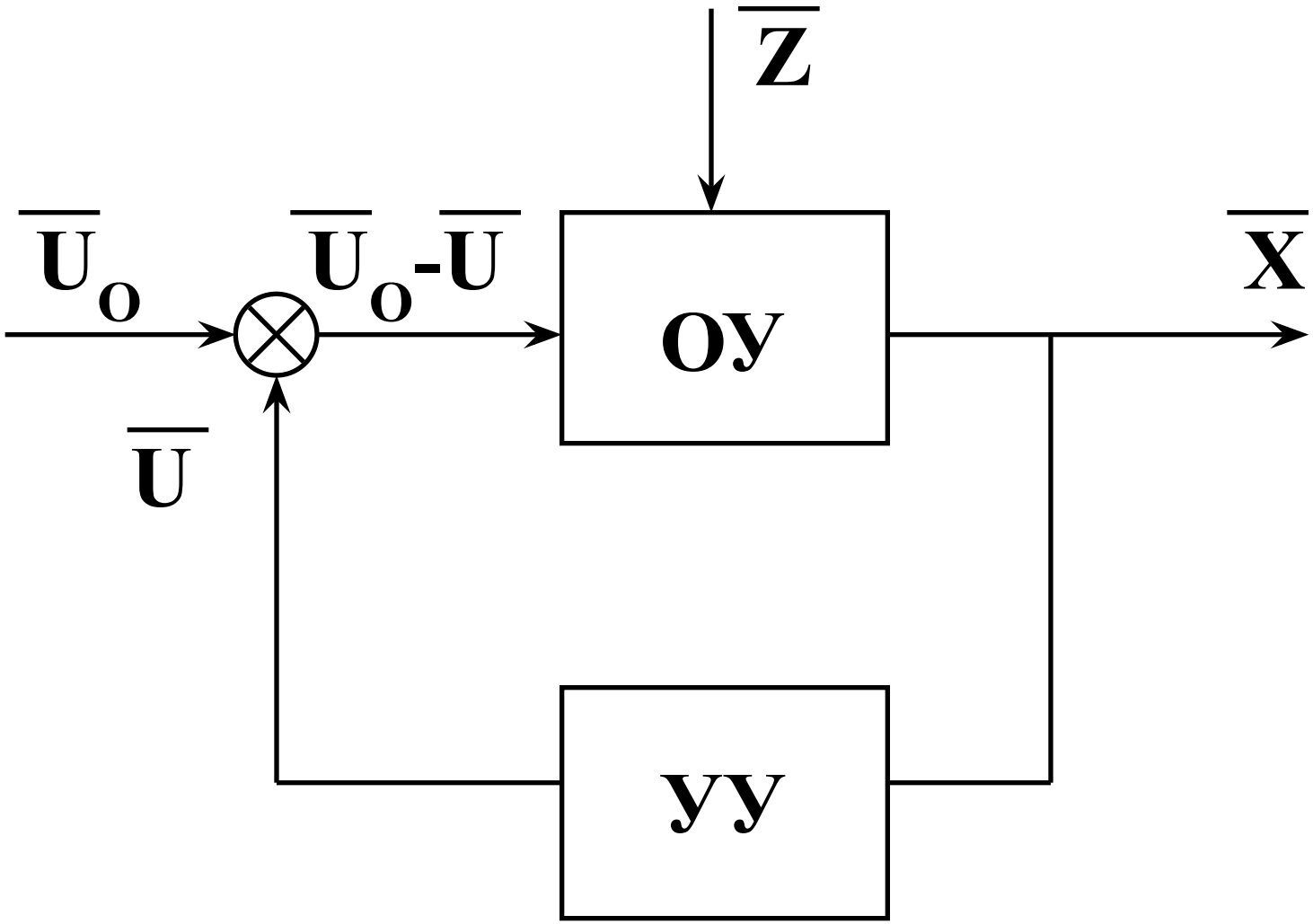
5. ПИД:

5.1 $$x(t) = k(u(t) + \frac{1}{T_u}\int_0^t u(t)dt + T_д \frac{du}{dt})$$

5.2 $$X(p) = k(1 + \frac{1}{T_u p} + T_д p) \cdot U(p)$$

5.3 $$W(p) = k(1 + \frac{1}{T_u p} + T_д p)$$

5.4 $$W(p) = k(1 + \frac{1}{T_u p} + \frac{T_д p}{\tau_б p + 1})$$


Z
U o
U o-U
ОУ
X
U
УУ

$$a_o \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_n x =$$

$$= b_o \frac{d^m u}{dt^m} + b_1 \frac{d^{m-1} u}{dt^{m-1}} + \ldots + b_m u +$$

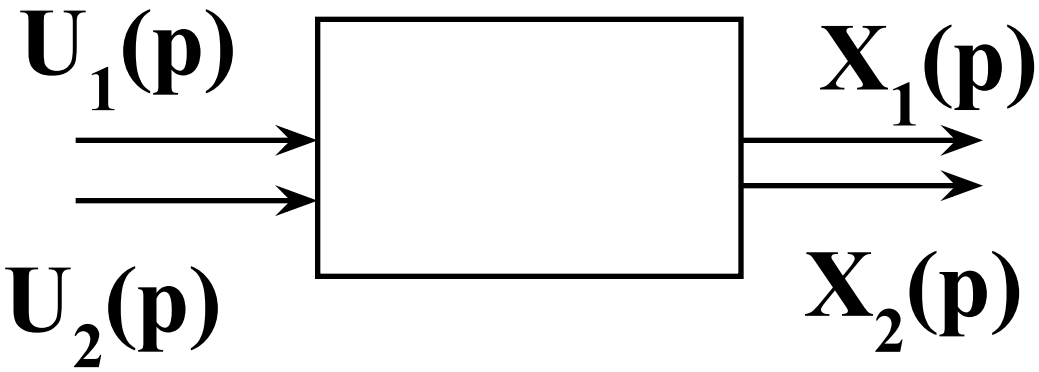
$$+ c_o \frac{d^k z}{dt^k} + c_1 \frac{d^{k-1} z}{dt^{k-1}} + \ldots + c_k z \Rightarrow$$

$$\bar{A}\bar{x}(t) = \bar{B}\bar{u}(t) + \bar{C}\bar{z}(t) \Rightarrow$$

$$A(p)X(p) = B(p)U(p) + C(p)Z(p)$$

$$X(p) = \int_0^\infty x(t)e^{-pt}dt$$

$$W(p) = \frac{X(p)}{U(p)}$$



1) $U_1(p) = 0$ $\qquad U_2(p) \to X_{12}(p)$

$\to X_{22}(p)$

$$W_{12}(p) = \frac{X_{12}(p)}{U_2(p)}; \qquad W_{22}(p) = \frac{X_{22}(p)}{U_2(p)}$$

$$2)\ U_2(p) = 0 \qquad U_1(p) \to X_{11}(p)$$
$$\to X_{21}(p)$$

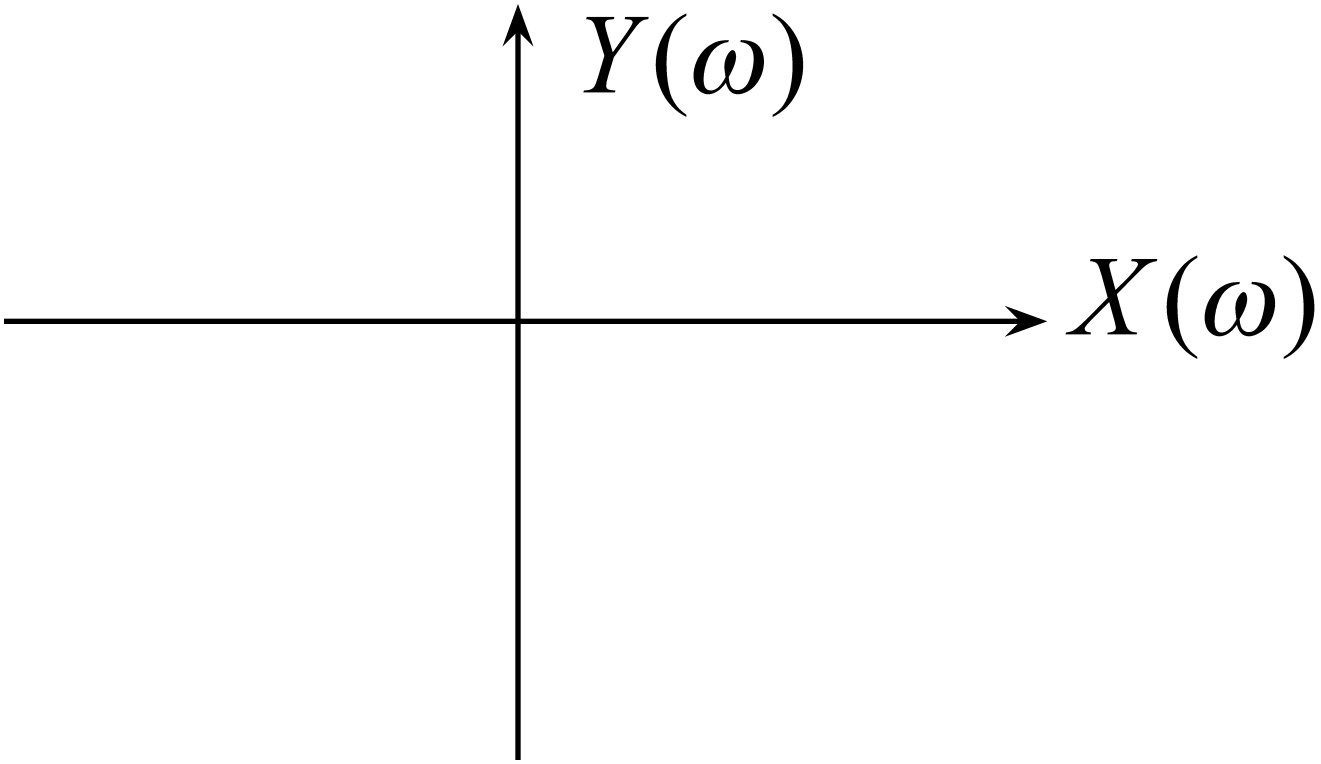
$$W_{11}(p) = \frac{X_{11}(p)}{U_1(p)}; \qquad W_{21}(p) = \frac{X_{21}(p)}{U_1(p)}$$

3) $X_i(p) = X_{1i}(p) + X_{2i}(p)$

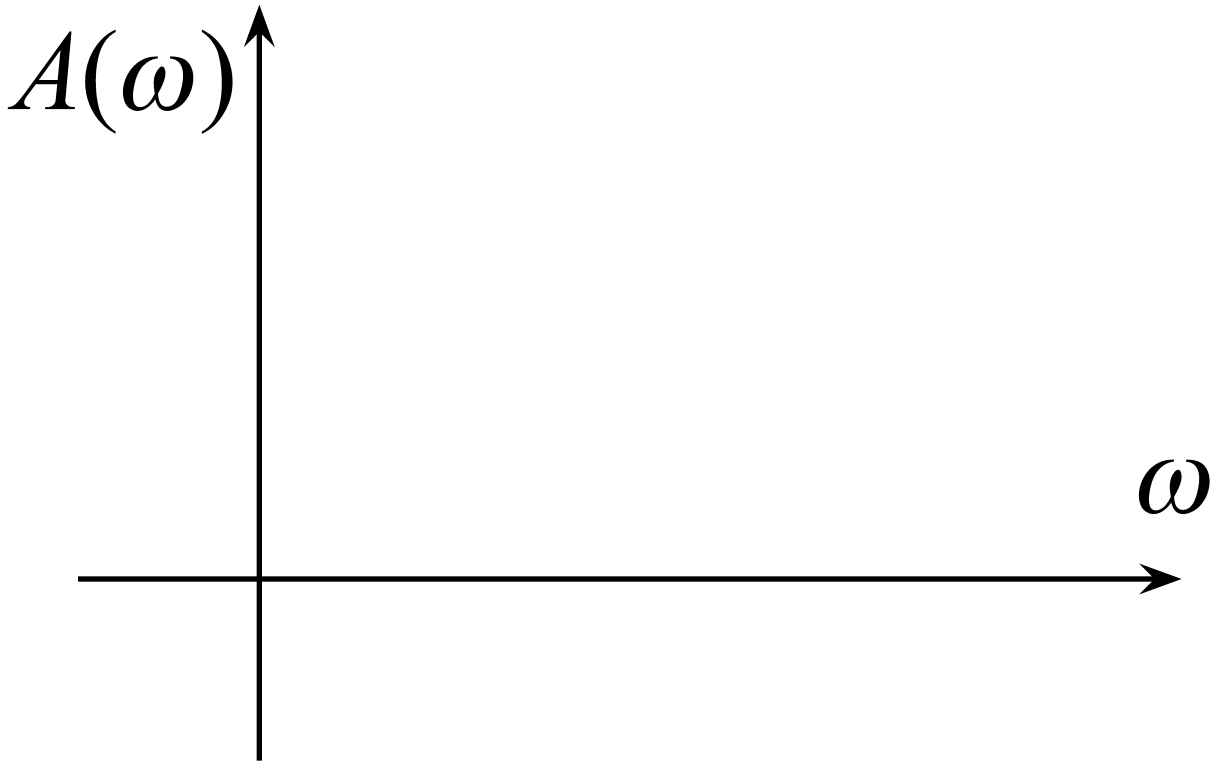
$$W_{1i}(p) = \frac{B_{1i}(p)}{A_{1i}(p)}$$

$$W_{2i}(p) = \frac{C_{2i}(p)}{A_{2i}(p)}$$

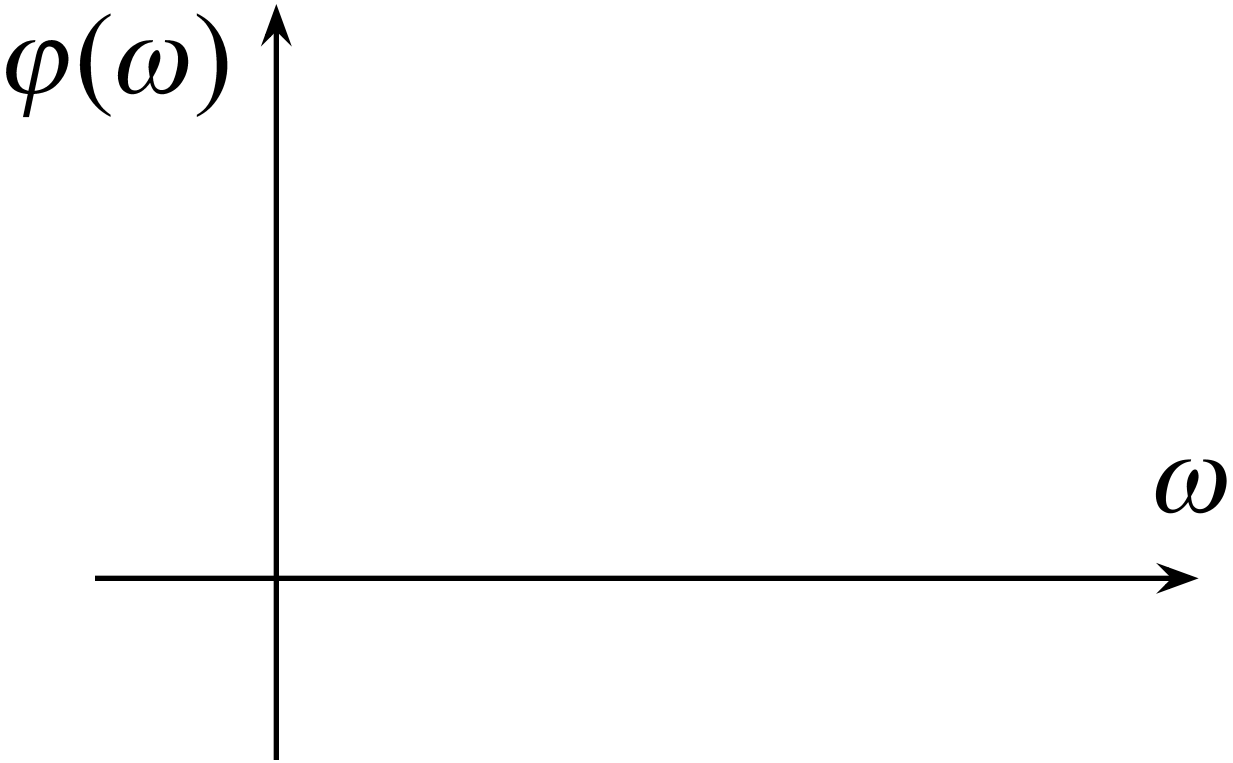
$$W(j\omega) = X(\omega) + jY(\omega)$$



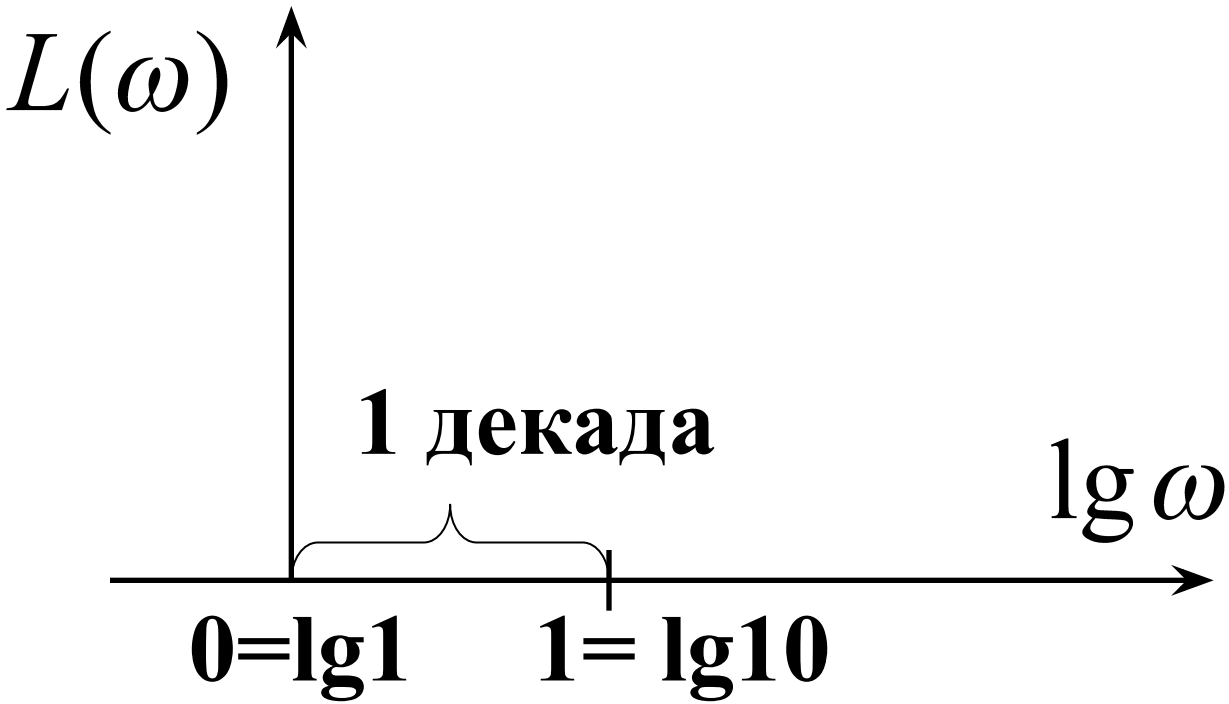
$$A(\omega) = |W(j\omega)| = \sqrt{X^2(\omega) + Y^2(\omega)}$$



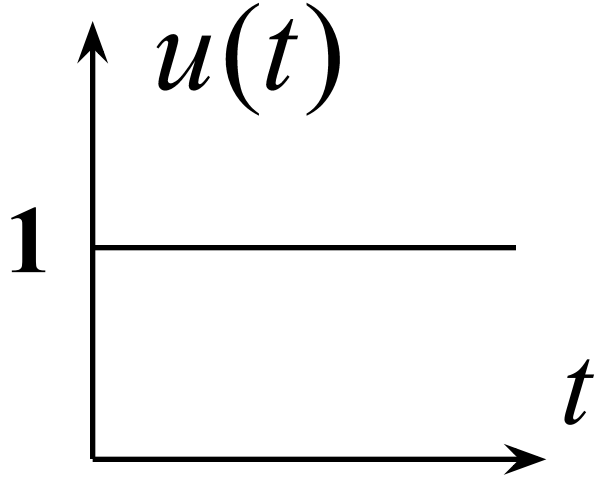
$$\varphi(\omega) = arctg \frac{Y(\omega)}{X(\omega)}$$



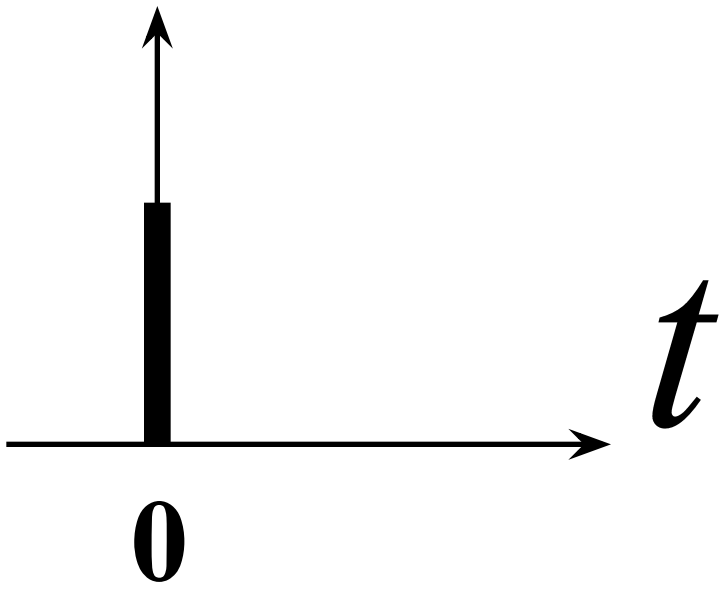
$$L(\omega) = 20 \lg A(\omega)$$

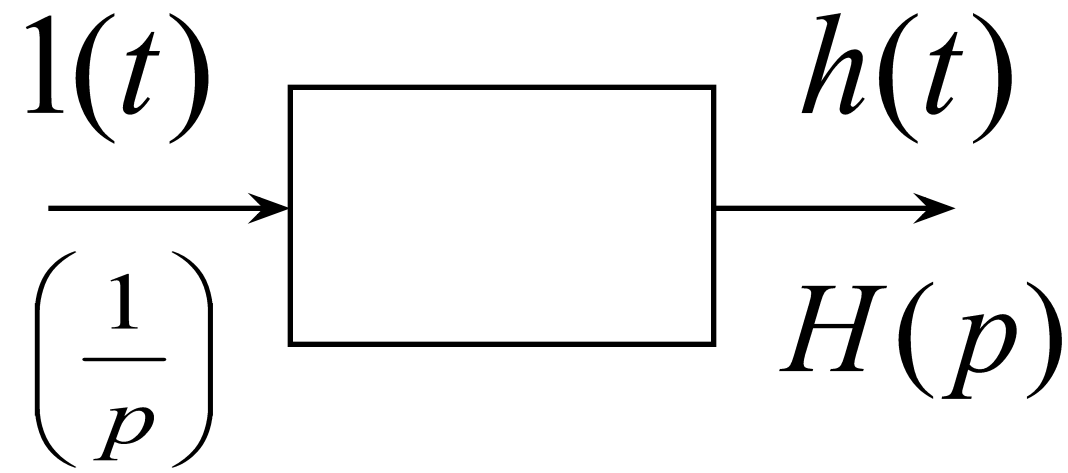


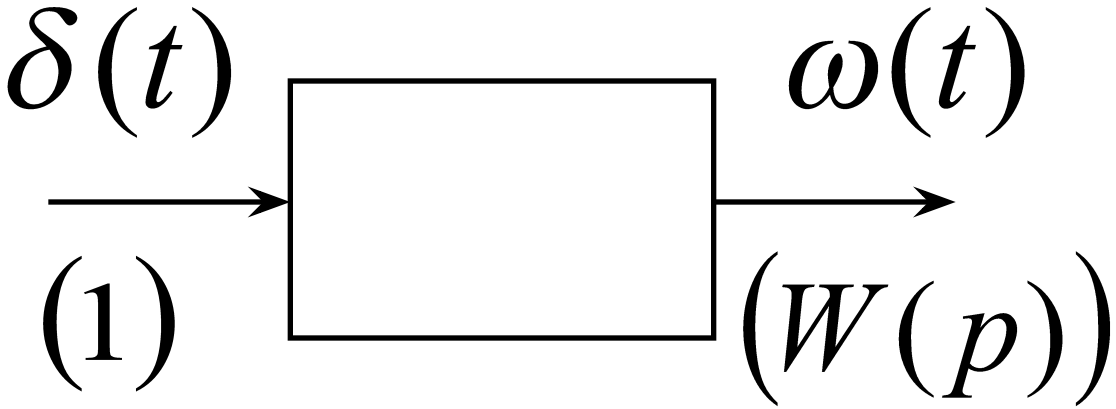
1) $u(t) = 1(t)$



2) $\delta(t) = (1(t))'$




1(t)
h(t)
1/p
H(p)


δ(t)
ω(t)
(1)
(W(p))

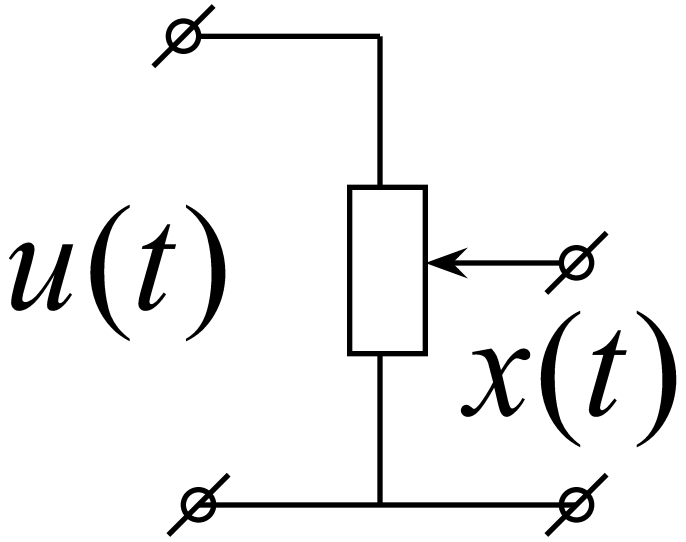
$$h(t) = L^{-1}\left(\frac{W(p)}{p}\right)$$

$$\omega(t) = L^{-1}(W(p))$$

Пропорциональное звено

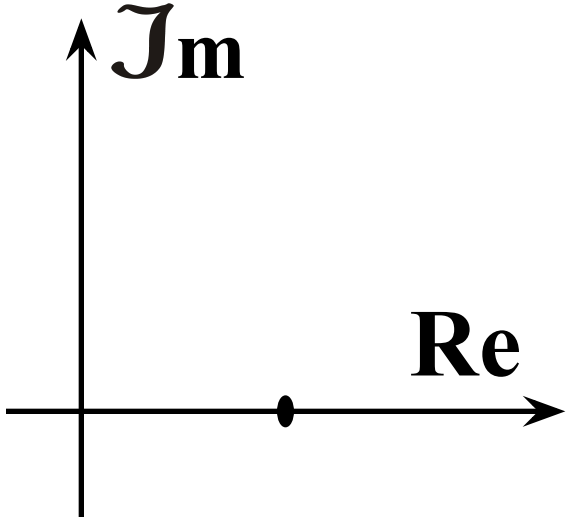
1. $x(t) = ku(t)$

2. $X(p) = kU(p)$

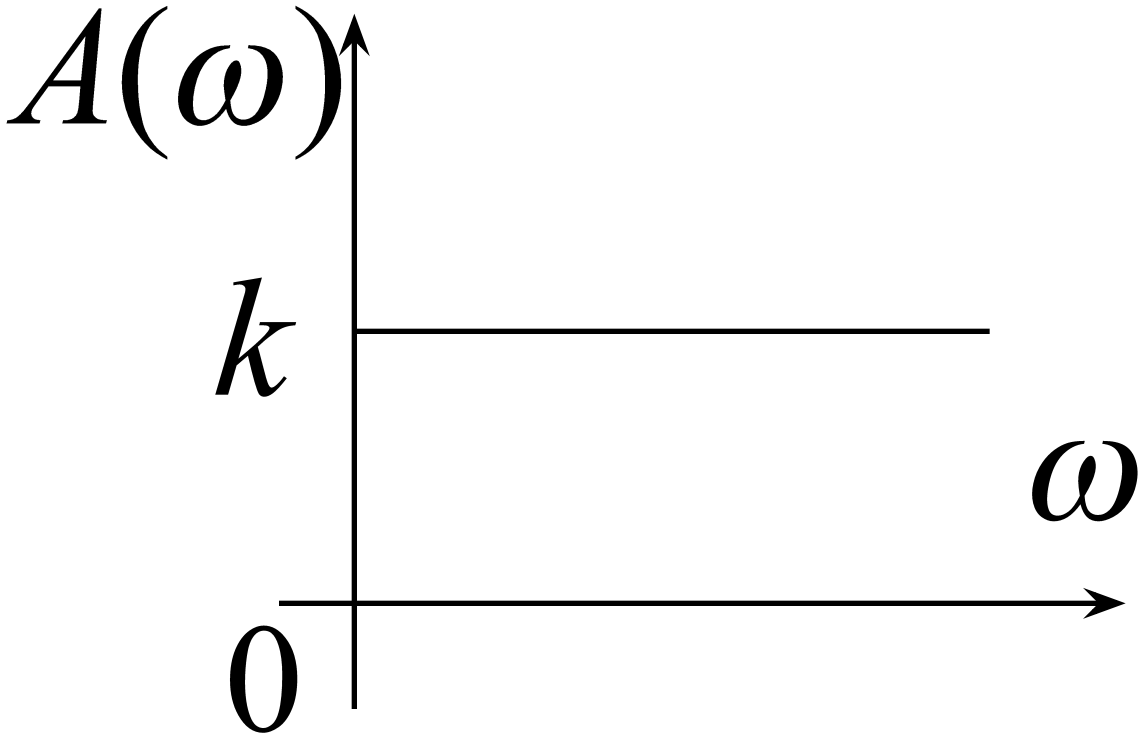


3 $W(p) = k$

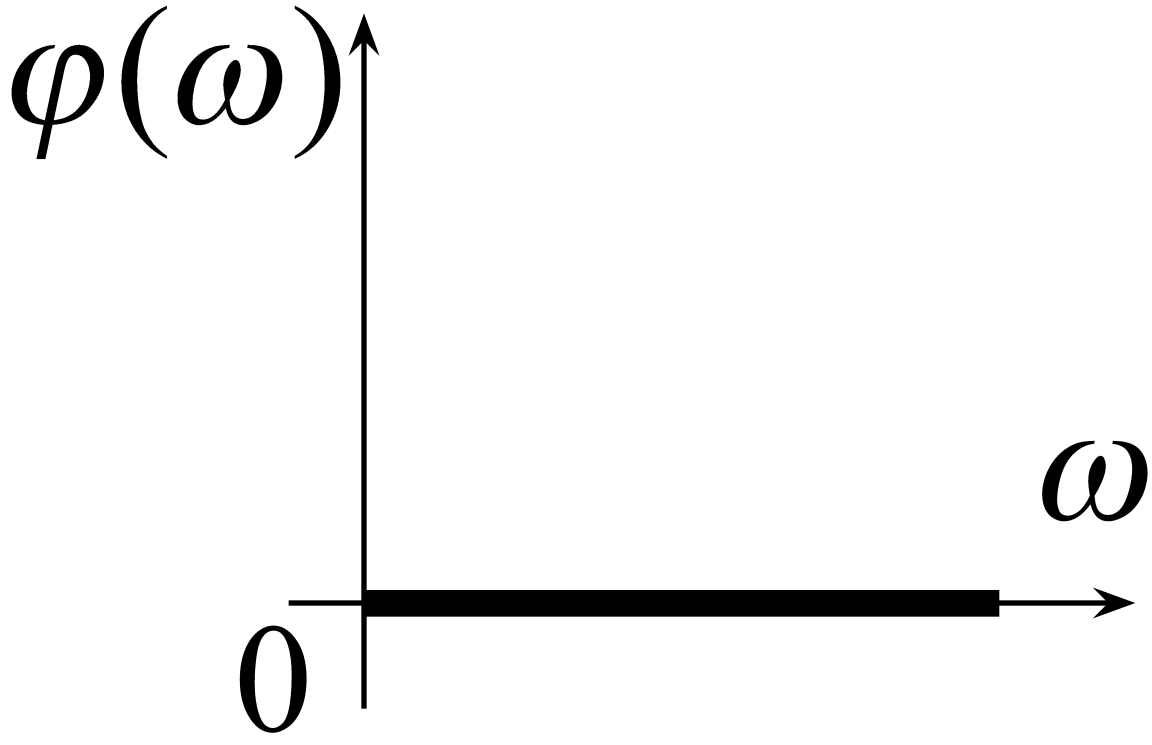
4.1 $W(j\omega) = k$



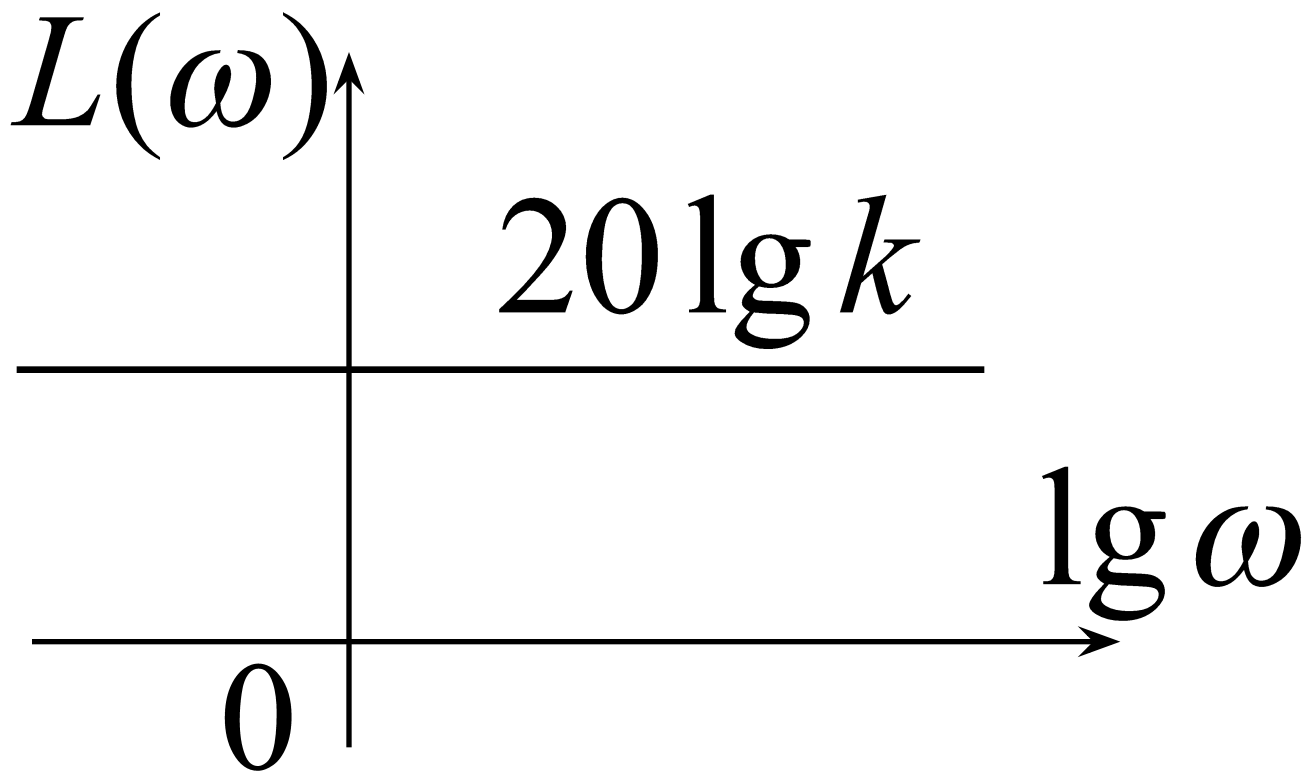
4.2 $A(\omega) = k$



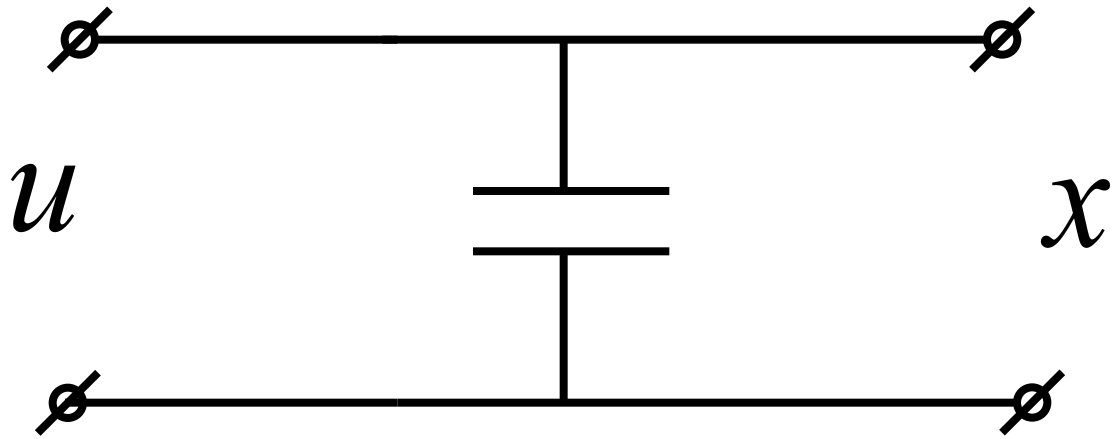
$$4.3 \quad \varphi(\omega) = arctg\left(\frac{0}{k}\right) = 0$$



4.4 $L(\omega) = 20\lg k$



Интегрирующее звено

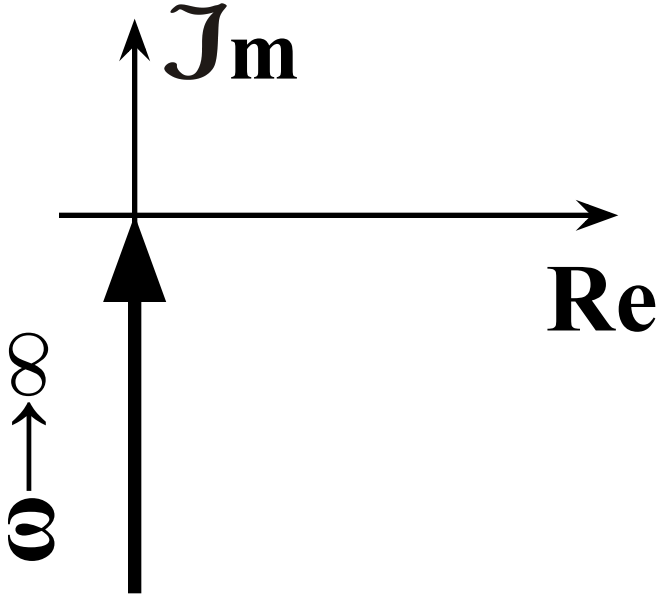


1. $X(t) = k\int_0^t u(t)dt$

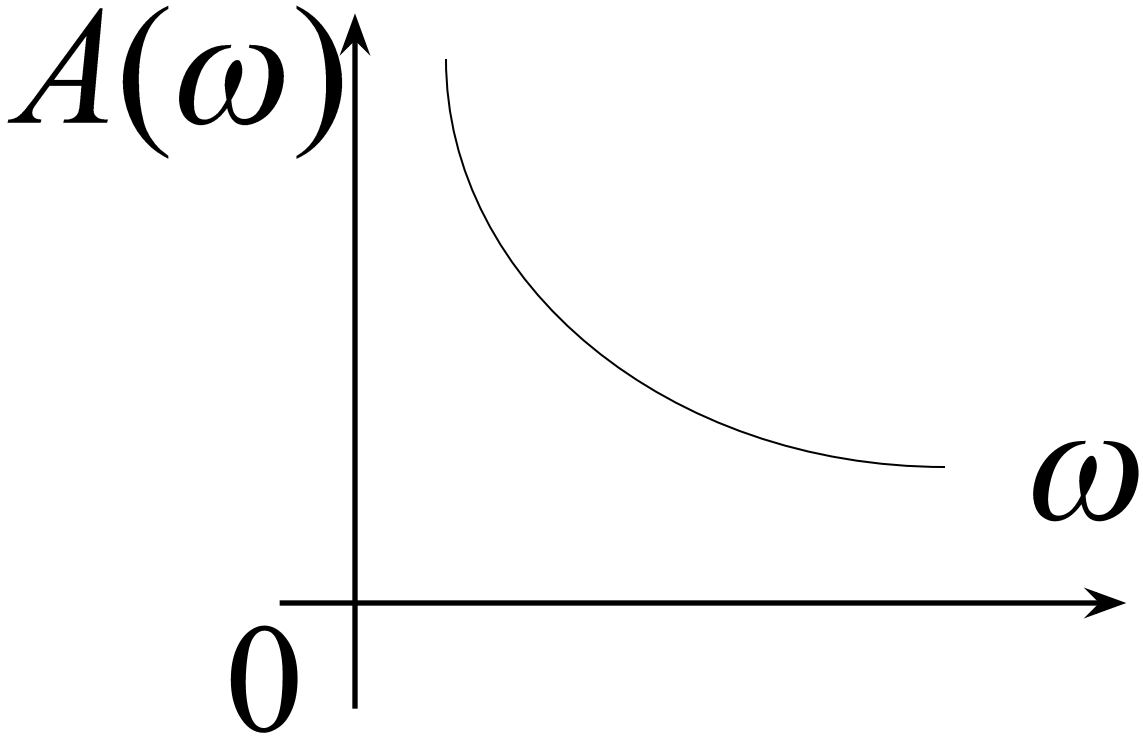
2. $X(p) = \frac{k}{p}U(p)$

3. $W(p) = \frac{k}{p}$

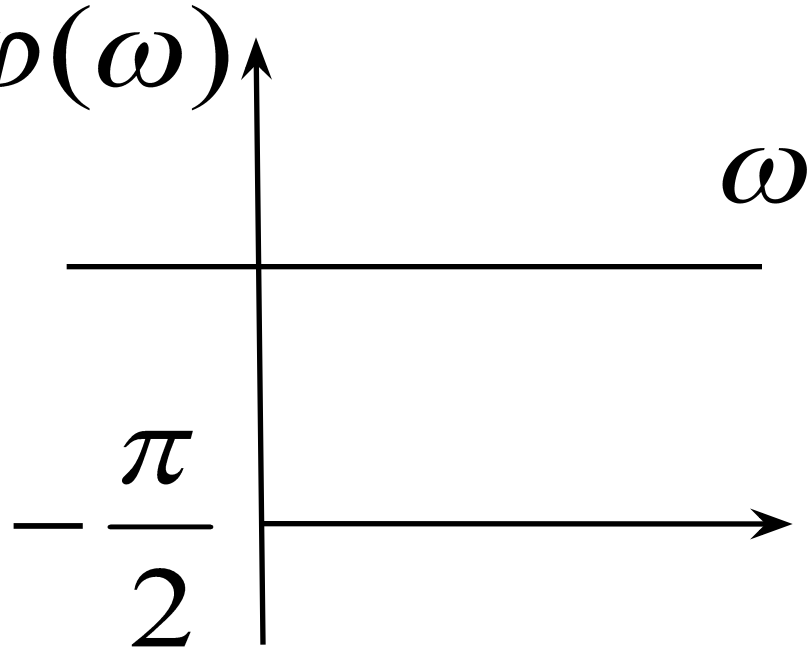
$$4.1 \quad W(j\omega) = \frac{k}{j\omega} = -j\frac{k}{\omega}$$



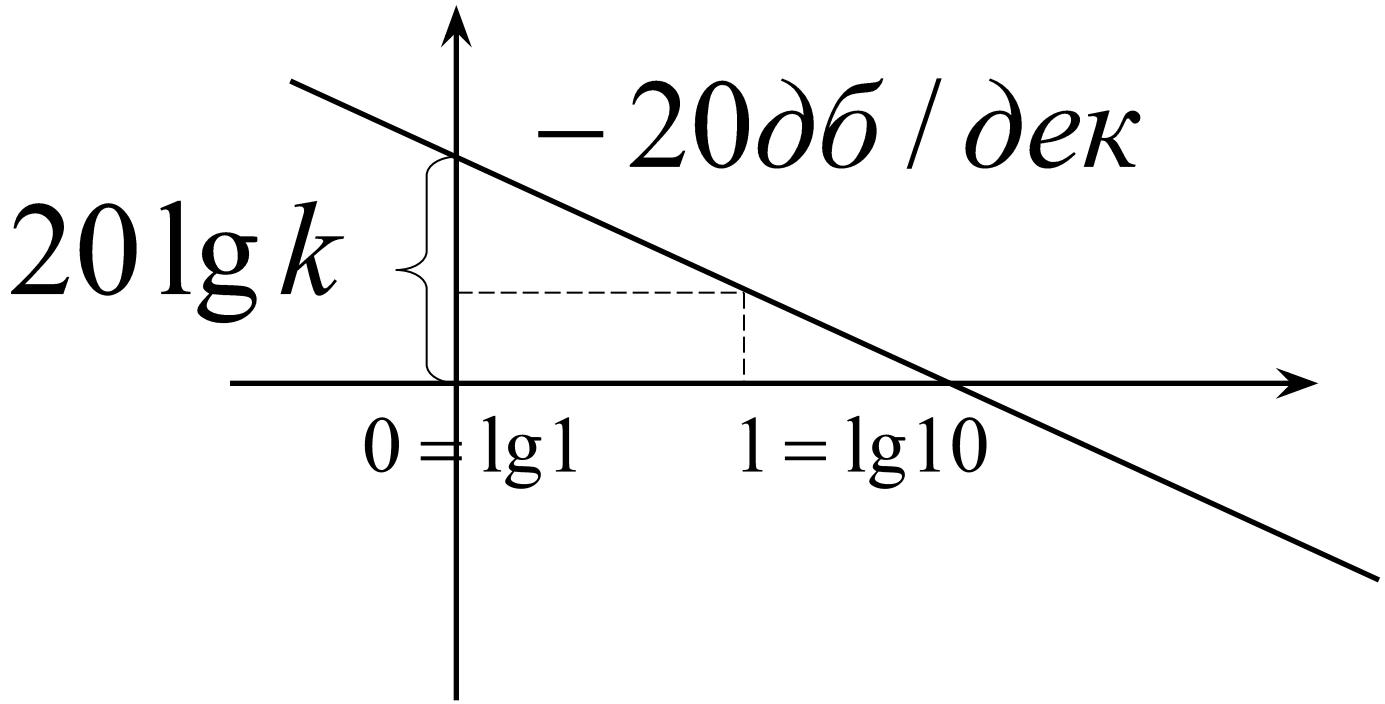
$$4.2 \quad A(\omega) = \left| -j\frac{k}{\omega} \right| = \frac{k}{\omega}$$



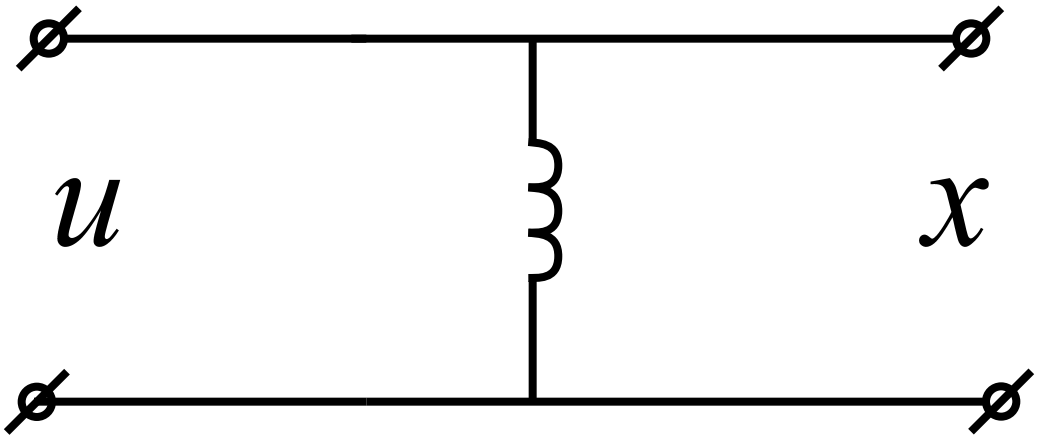
$$4.3 \quad \varphi(\omega) = arctg\left(\frac{-k/\omega}{0}\right) = -\frac{\pi}{2}$$



$$4.4 \qquad L(\omega) = 20\lg\left(\frac{k}{\omega}\right) = 20\lg k - 20\boldsymbol{lg}\omega$$



Дифференцирующее звено

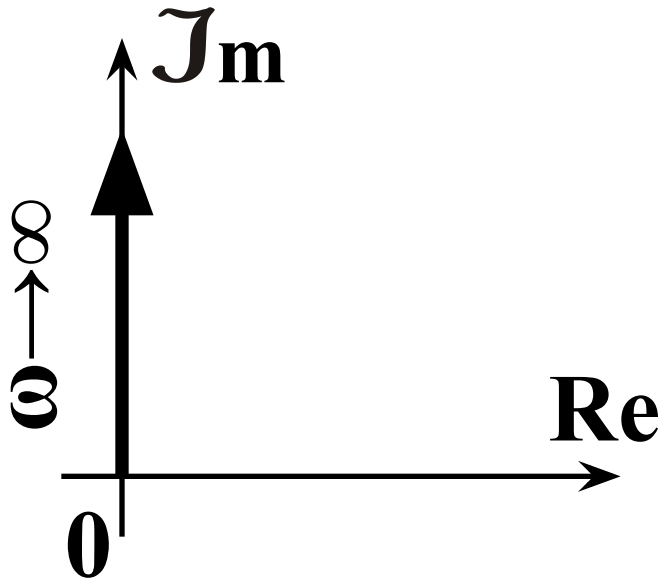


1. $x(t) = T_д \frac{du}{dt}$

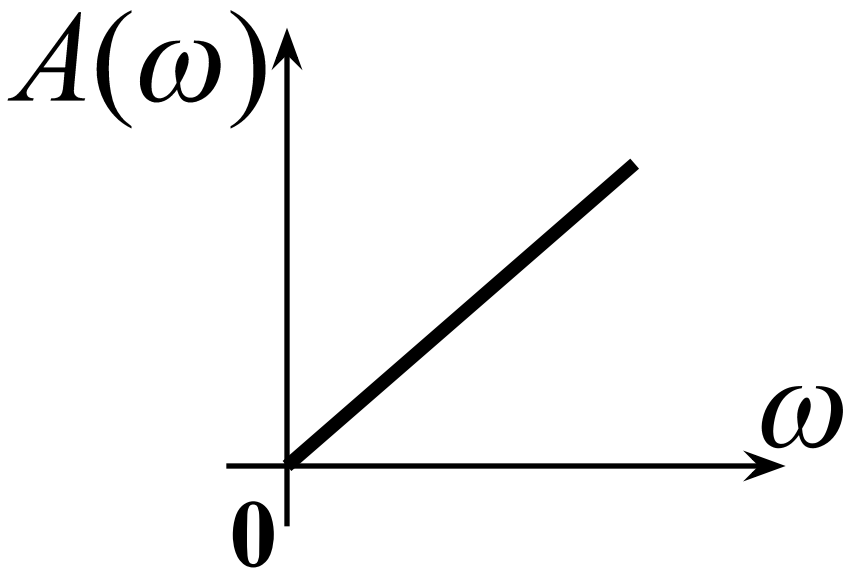
2. $X(p) = T_д p \cdot U(p)$

3. $W(p) = T_д p$

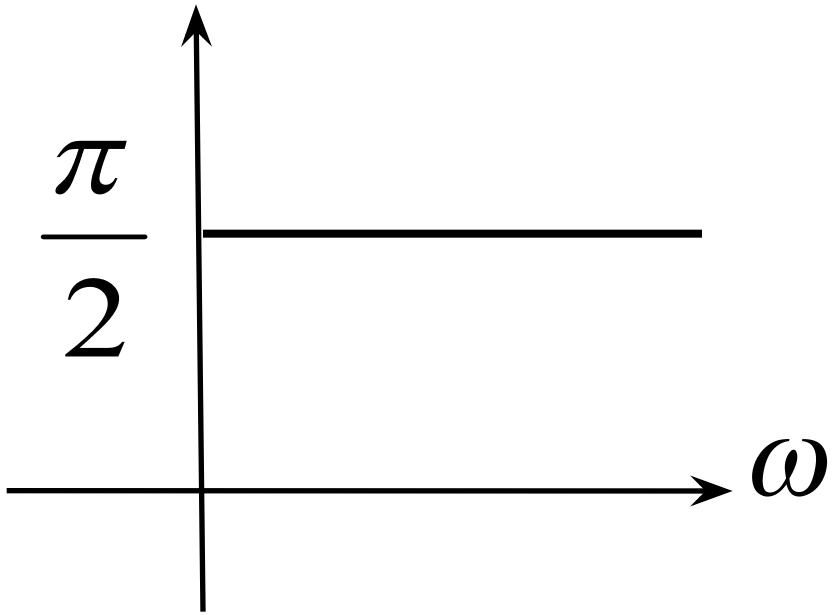
$$4.1 \quad W(j\omega) = T_{\partial}(j\omega)$$



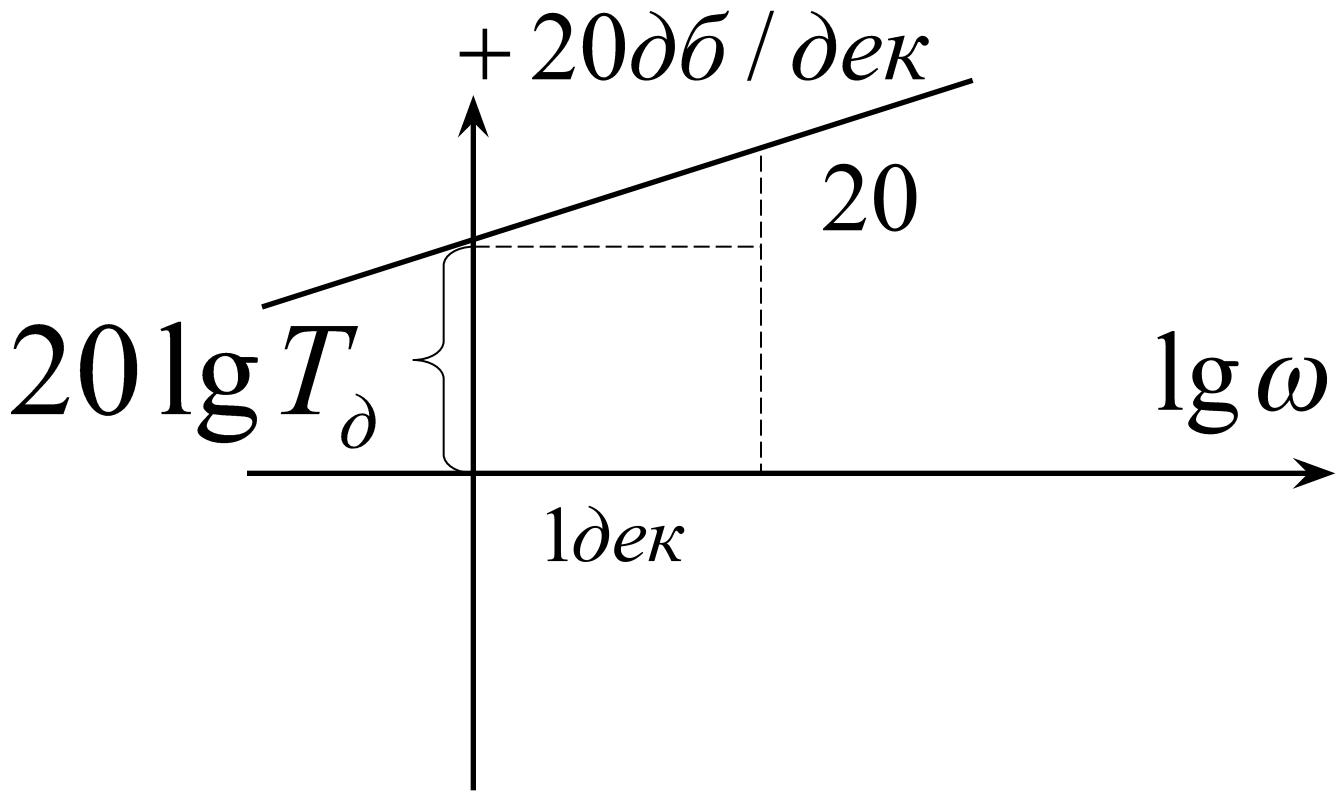
4.2 $A(\omega) = T_{\partial}\omega$



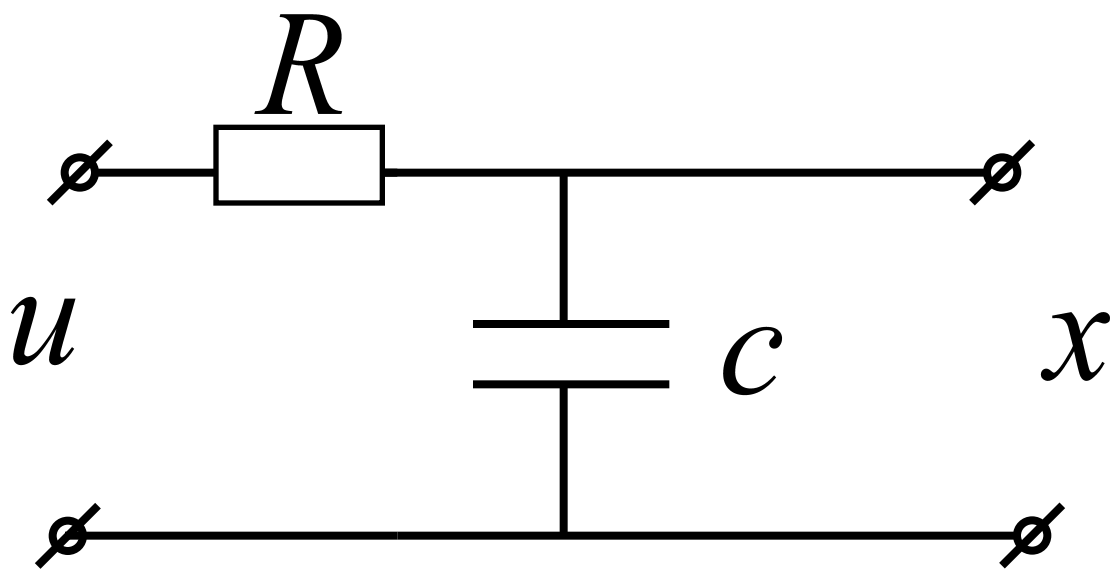
4.3 $$\phi(\omega) = arctg\left(\frac{\boldsymbol{T}\omega}{0}\right) = \frac{\pi}{2}$$



4.4 $L(\omega) = 20\lg(T_д\omega) = 20\lg T_д + 20\lg\omega$



Апериодическое звено



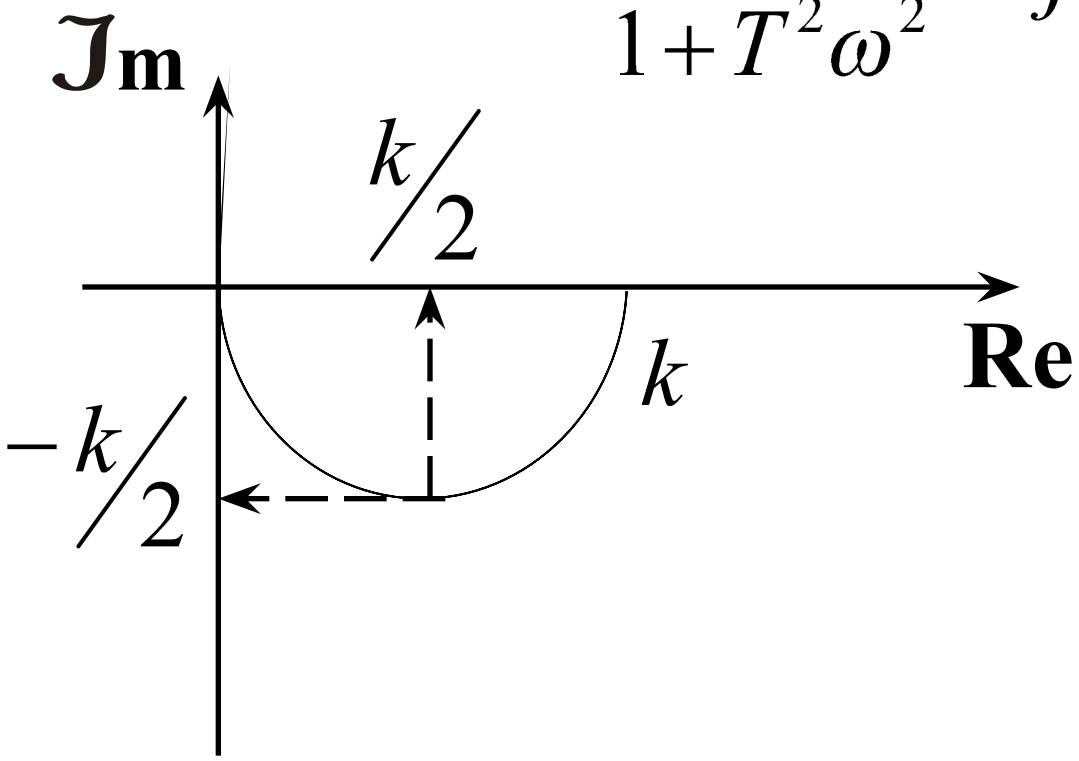
1. $$T\frac{dx}{dt}+x=ku$$

2. $$(Tp+1)X(p)=kU(p)$$

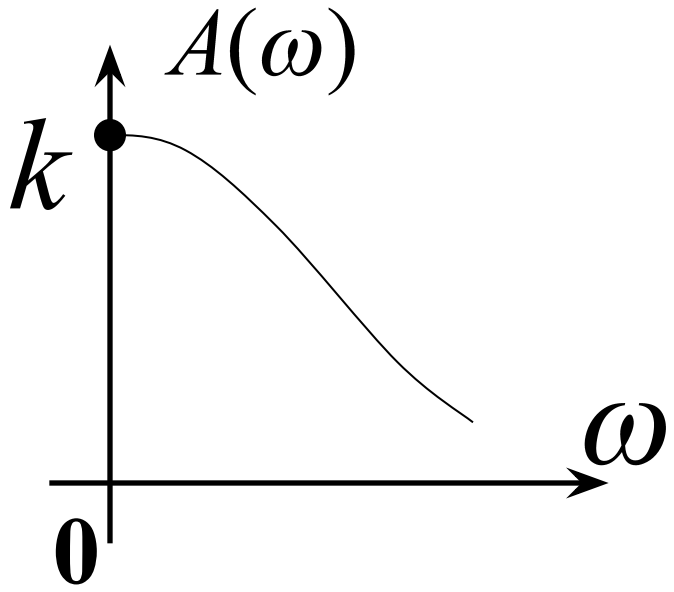
3. $$W(p)=\frac{k}{Tp+1}$$

$$4.1 \quad W(j\omega) = \frac{k}{1+T(j\omega)} = \frac{k(1-jT\omega)}{1+T^2\omega^2} =$$

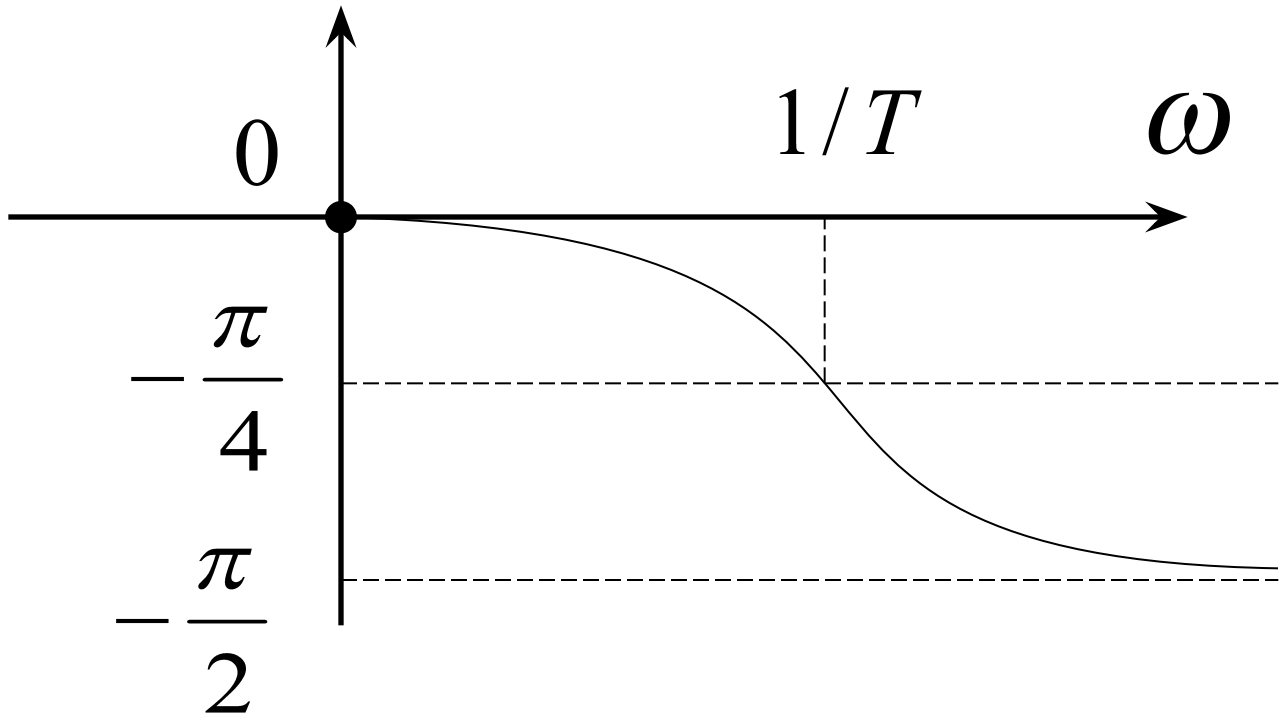
$$= \frac{k}{1+T^2\omega^2} - j\frac{kT\omega}{1+T^2\omega^2}$$



4.2 $$A(\omega) = \frac{|k|}{|1 + jT\omega|} = \frac{k}{\sqrt{1 + T^2\omega^2)}}$$



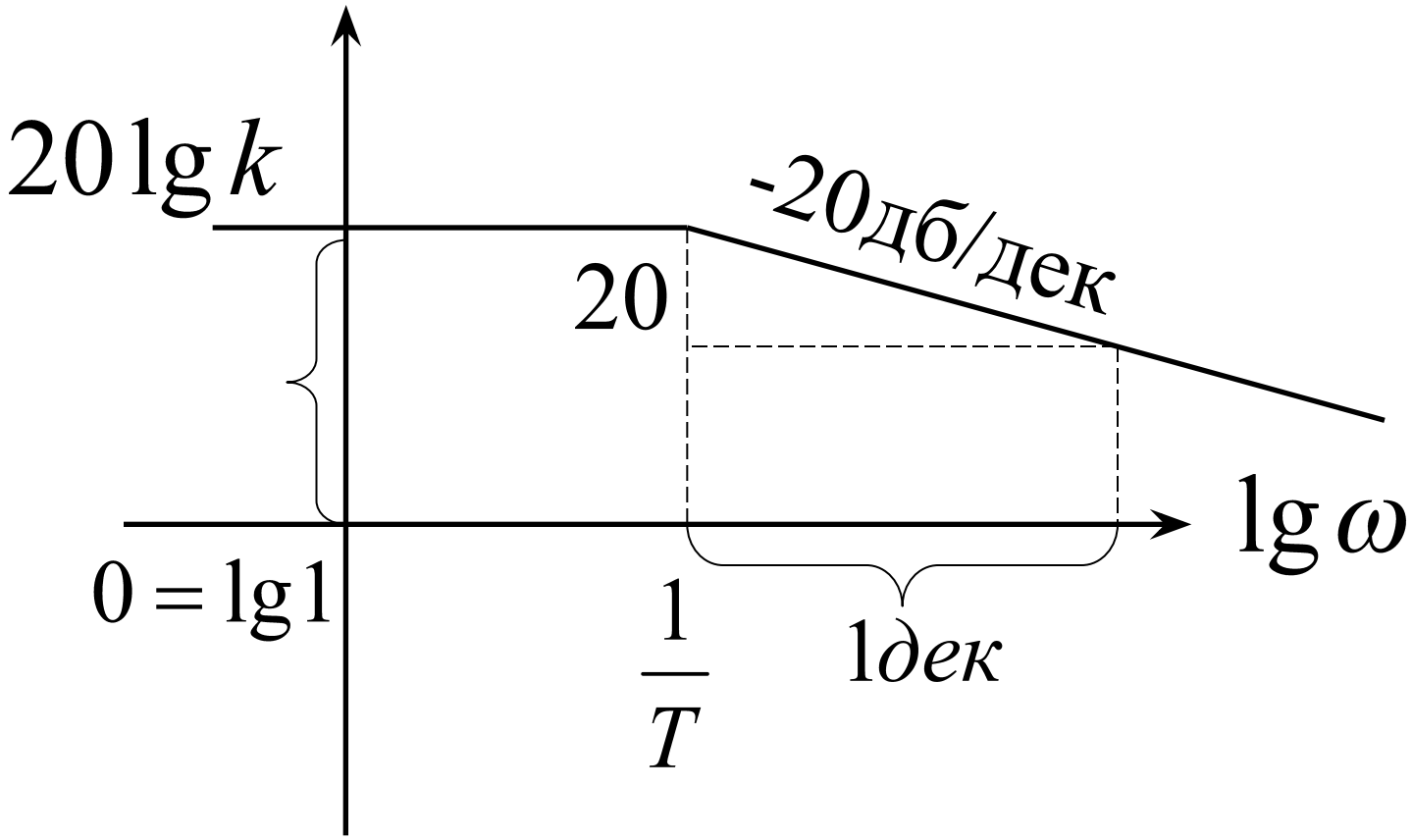
4.3 $\varphi(\omega) = arctg(-T\omega)$



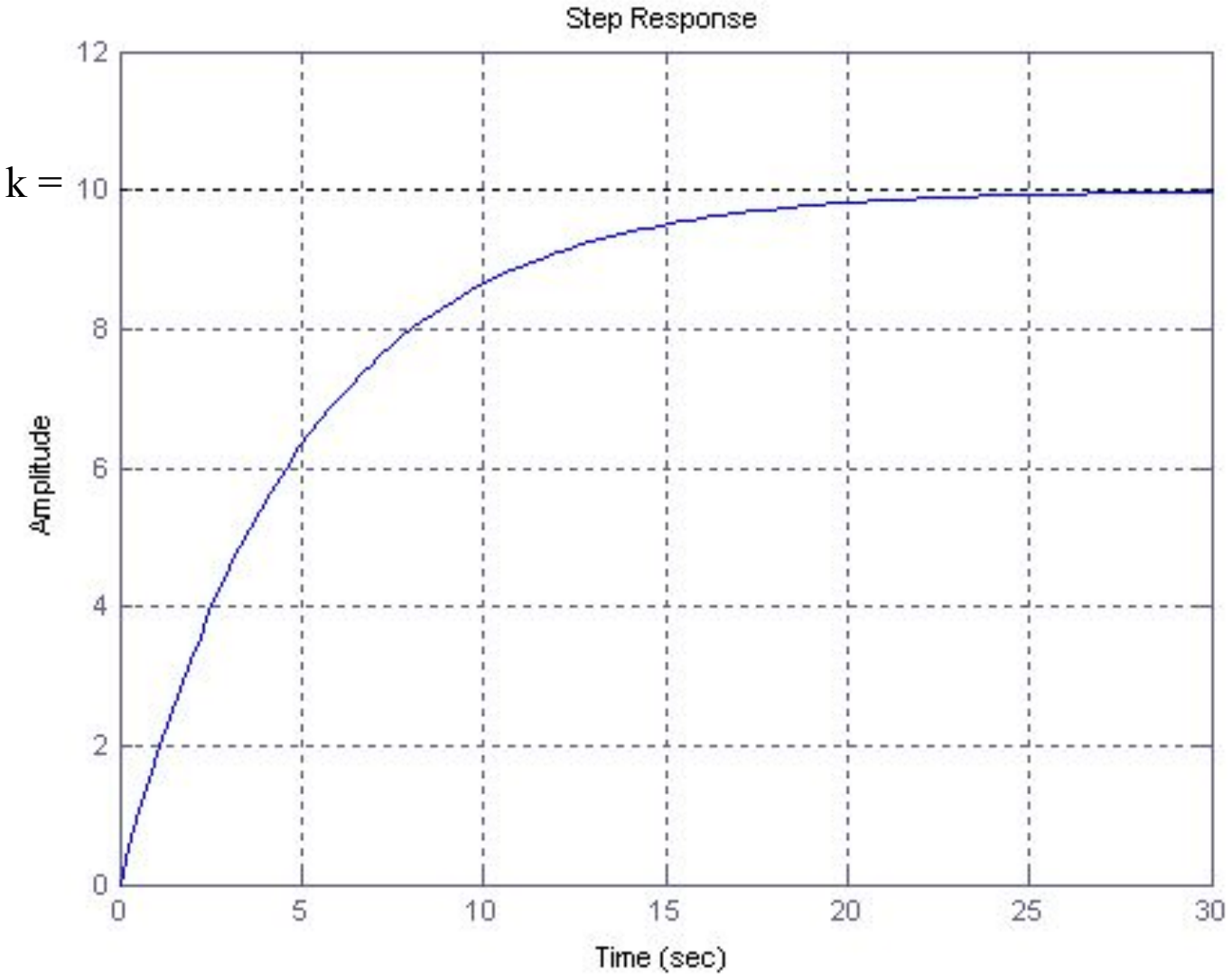
$$4.4 \qquad L(\omega) = 20\lg k - 10\lg(1 + T^2\omega^2)$$

$$\omega << \frac{1}{T} \qquad L(\omega) = 20\lg k$$

$$\omega >> \frac{1}{T} \qquad L(\omega) = 20\lg k - 20\lg \mathrm{T}\,\omega$$


20 lg k
-20дб/дек
20
lg ω
0 = lg 1
1/T
1дек

5. $h(t) = k(1 - e^{-t/T})$


Step Response
k =
Amplitude
Time (sec)
0
2
4
6
8
10
12
0
5
10
15
20
25
30

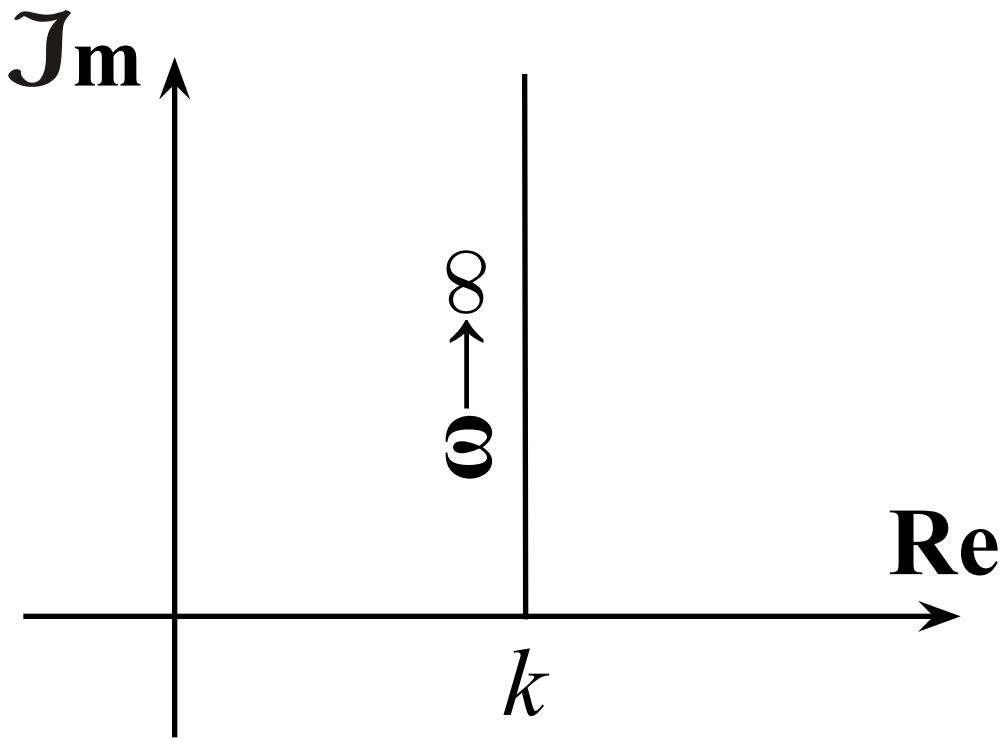
Форсирующее звено

1. $x(t) = k(u + T\frac{du}{dt})$

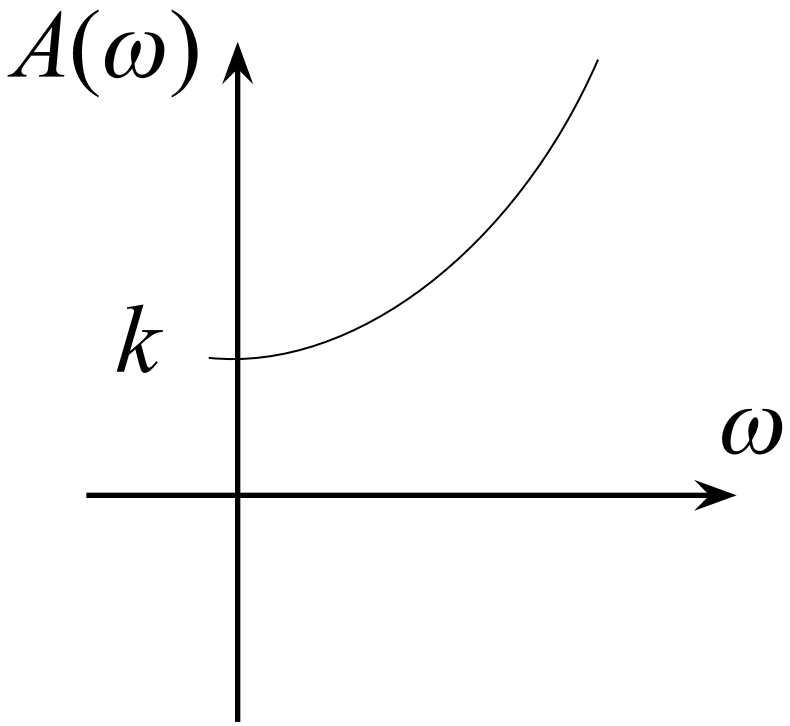
2. $X(p) = k(Tp + 1)U(p)$

3. $W(p) = k(Tp + 1)$

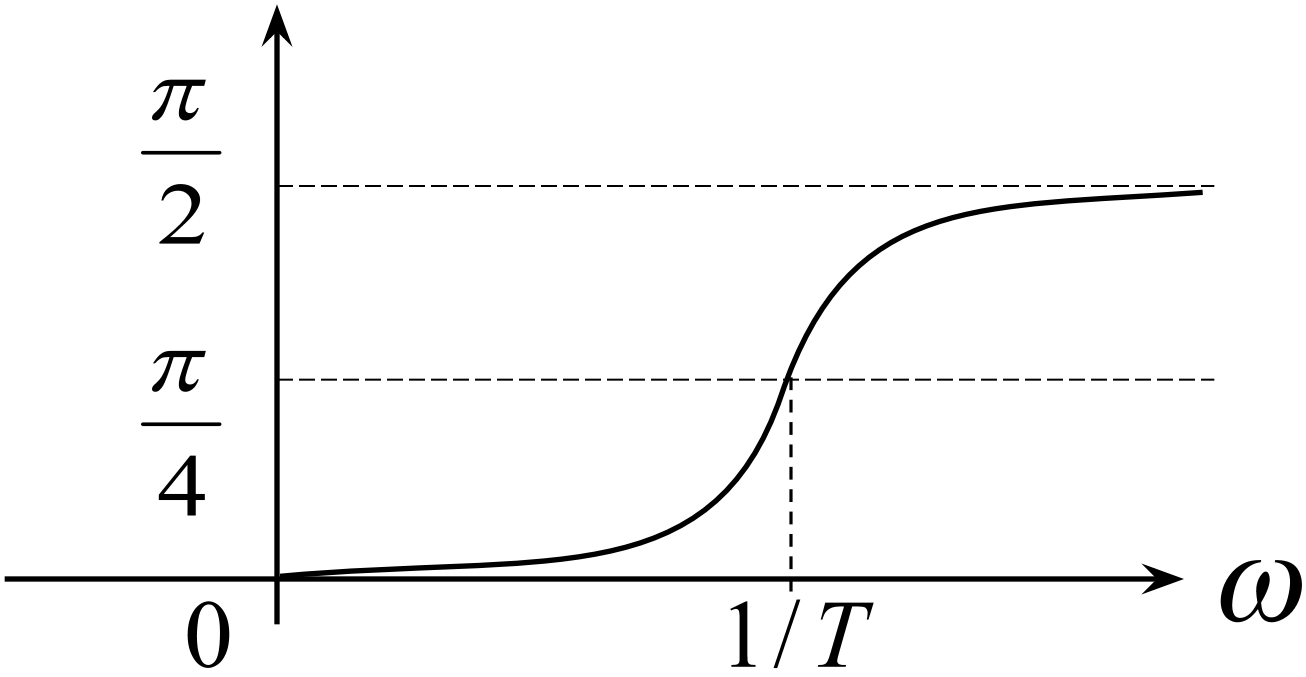
4.1 $W(j\omega) = k + jkT\omega$



4.2 $A(\omega) = k\sqrt{1+T^2\omega^2}$



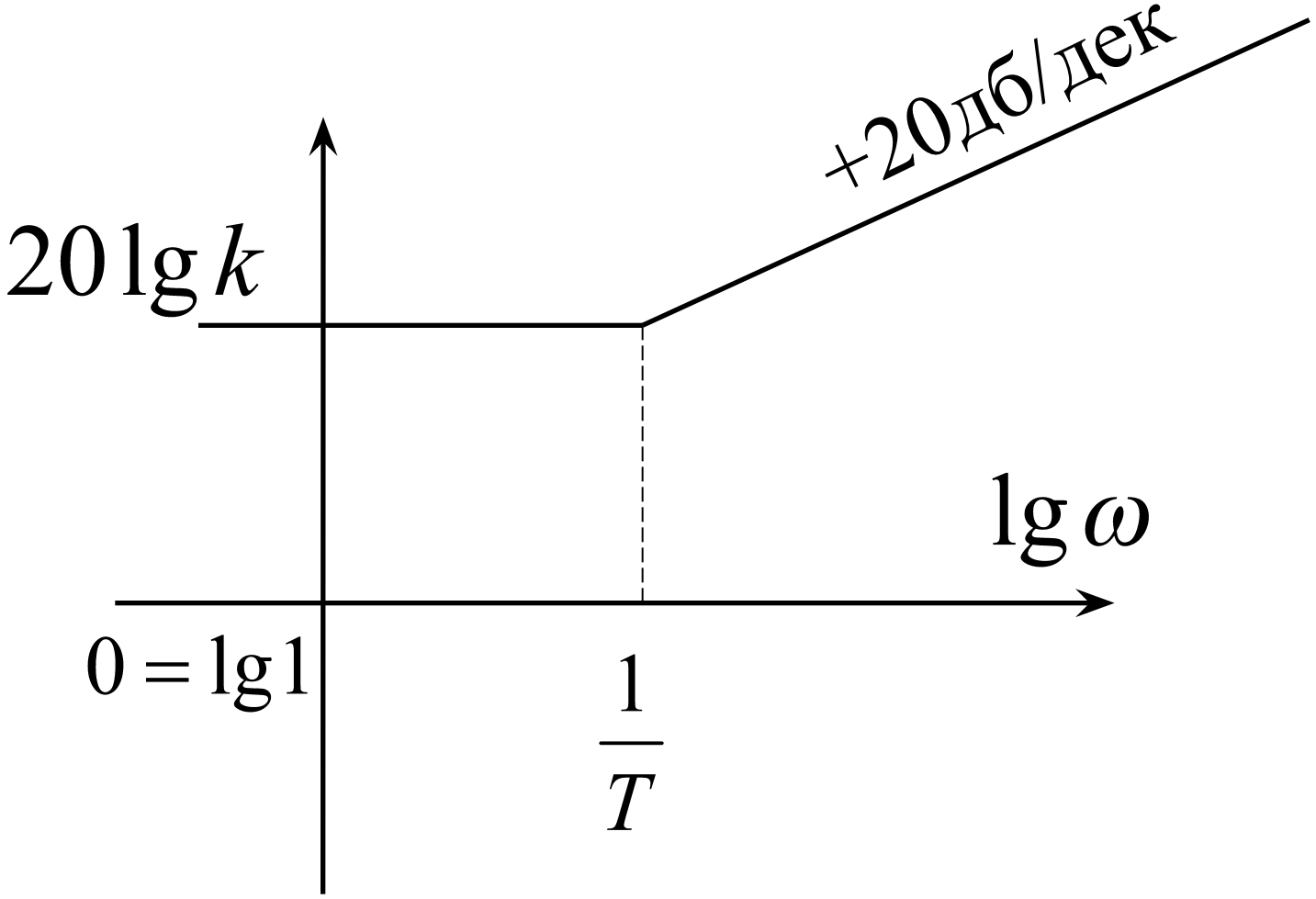
4.3 $\varphi(\omega) = arctg(T\omega)$



4.4 $L(\omega) = 20\lg k + 10\lg(1 + T^2\omega^2)$

$\omega << \frac{1}{T}$ $\quad L(\omega) = 20\lg k$

$\omega >> \frac{1}{T}$ $\quad L(\omega) = 20\lg k + 20\lg(T\omega)$


+20дб/дек
20lg k
lg ω
0 = lg1
1
T

Колебательное звено

$$1.\ T^2 \frac{d^2x}{dt^2} + 2T\xi \frac{dx}{dt} + x = ku$$

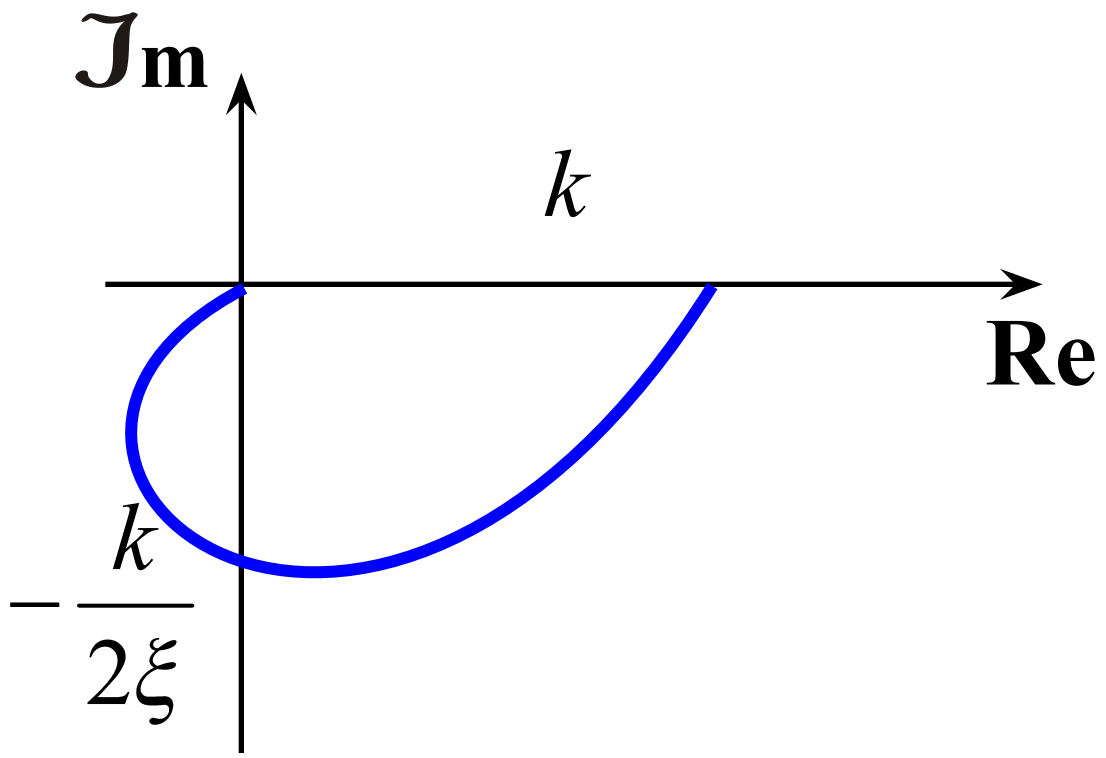
$$2.\ \left(T^2 p^2 + 2T\xi p + 1\right) \cdot X(p) = kU(p)$$

$$3.\ W(p) = \frac{k}{T^2 p^2 + 2T\xi p + 1}$$

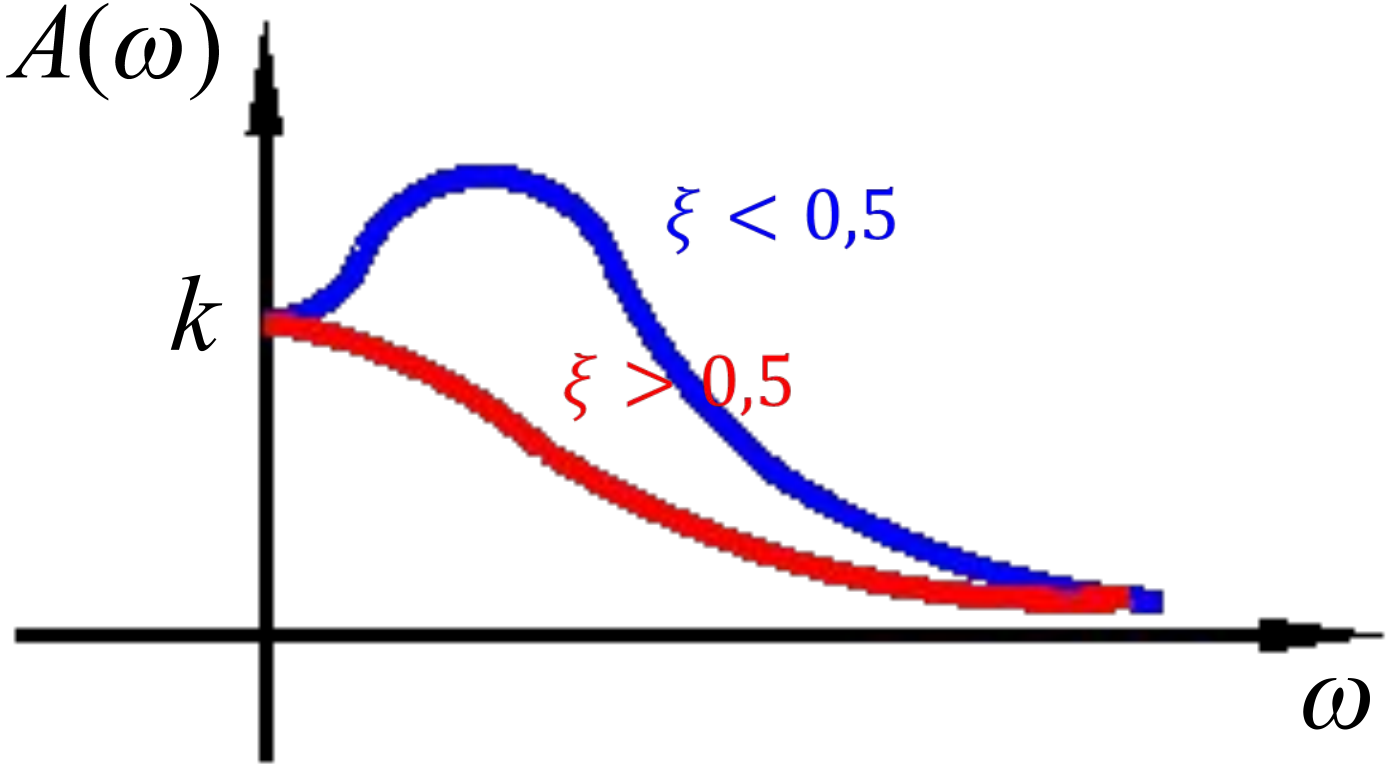
$$4.1\, W(j\omega) = \frac{k}{(1 - T^2\omega^2) + j2\xi T\omega} =$$

$$= \frac{k(1 - T^2\omega^2)}{(1 - T^2\omega^2)^2 + 4\xi^2 T^2\omega^2} -$$

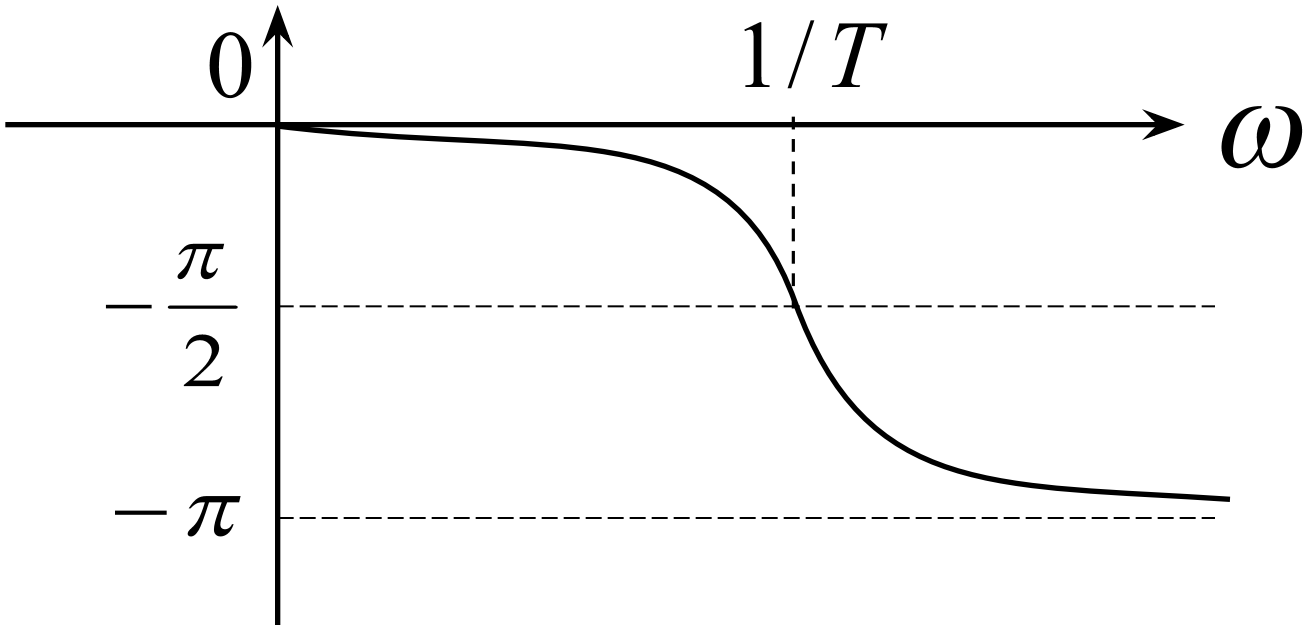
$$- j\frac{2\xi T\omega k}{(1 - T^2\omega^2)^2 + 4\xi^2 T^2\omega^2}$$


Jm
k
Re
k
2ξ

$$4.2\ A(\omega) = \frac{k}{\sqrt{(1 - T^2\omega^2)^2 + 4\xi^2 T^2 \omega^2}}$$



$$4.3\ \varphi(\omega) = arctg\left(-\frac{2\xi T\omega}{1-T^2\omega^2}\right)$$

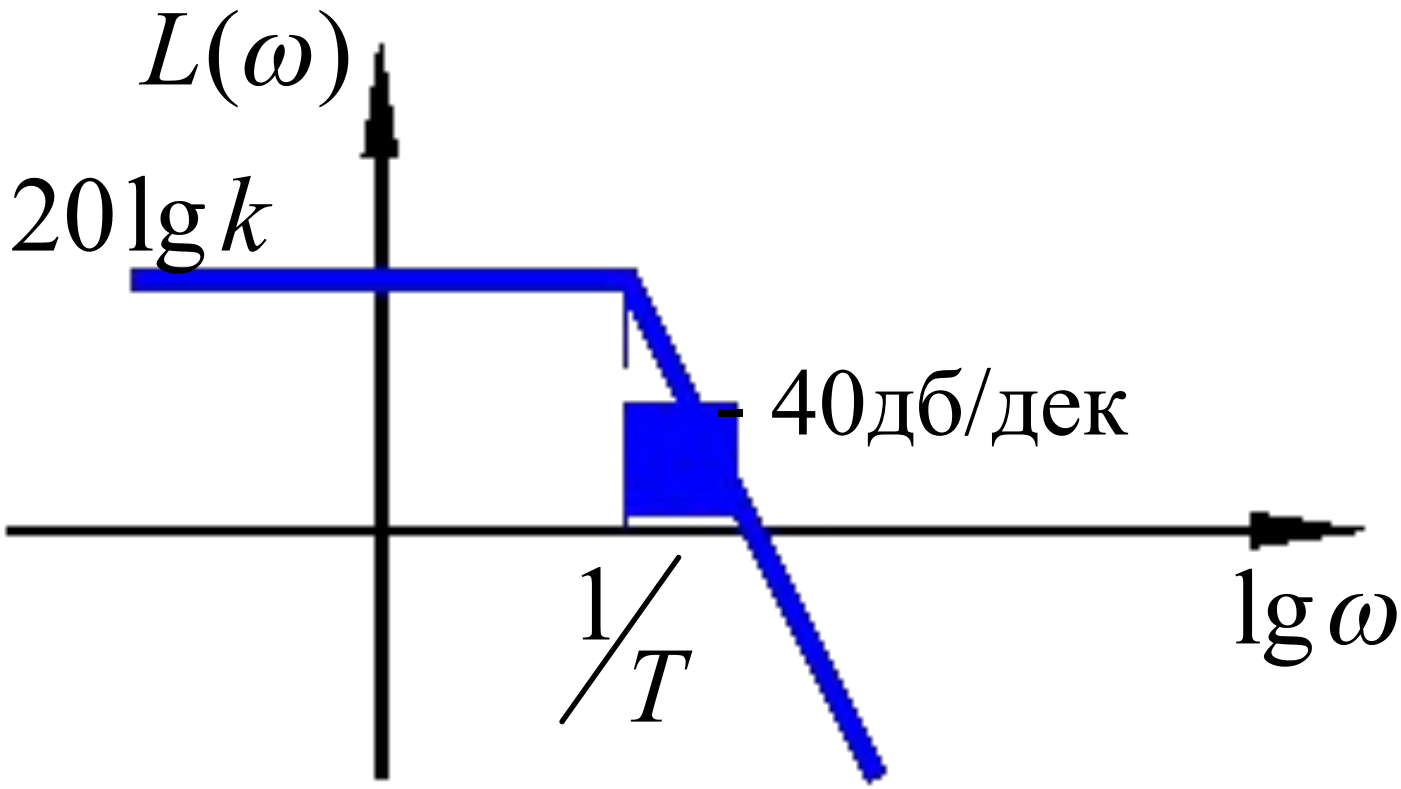


$$4.4\ 20\lg k - 10\lg\left((1-T^2\omega^2)^2 + 4\xi^2 T^2\omega^2\right)$$

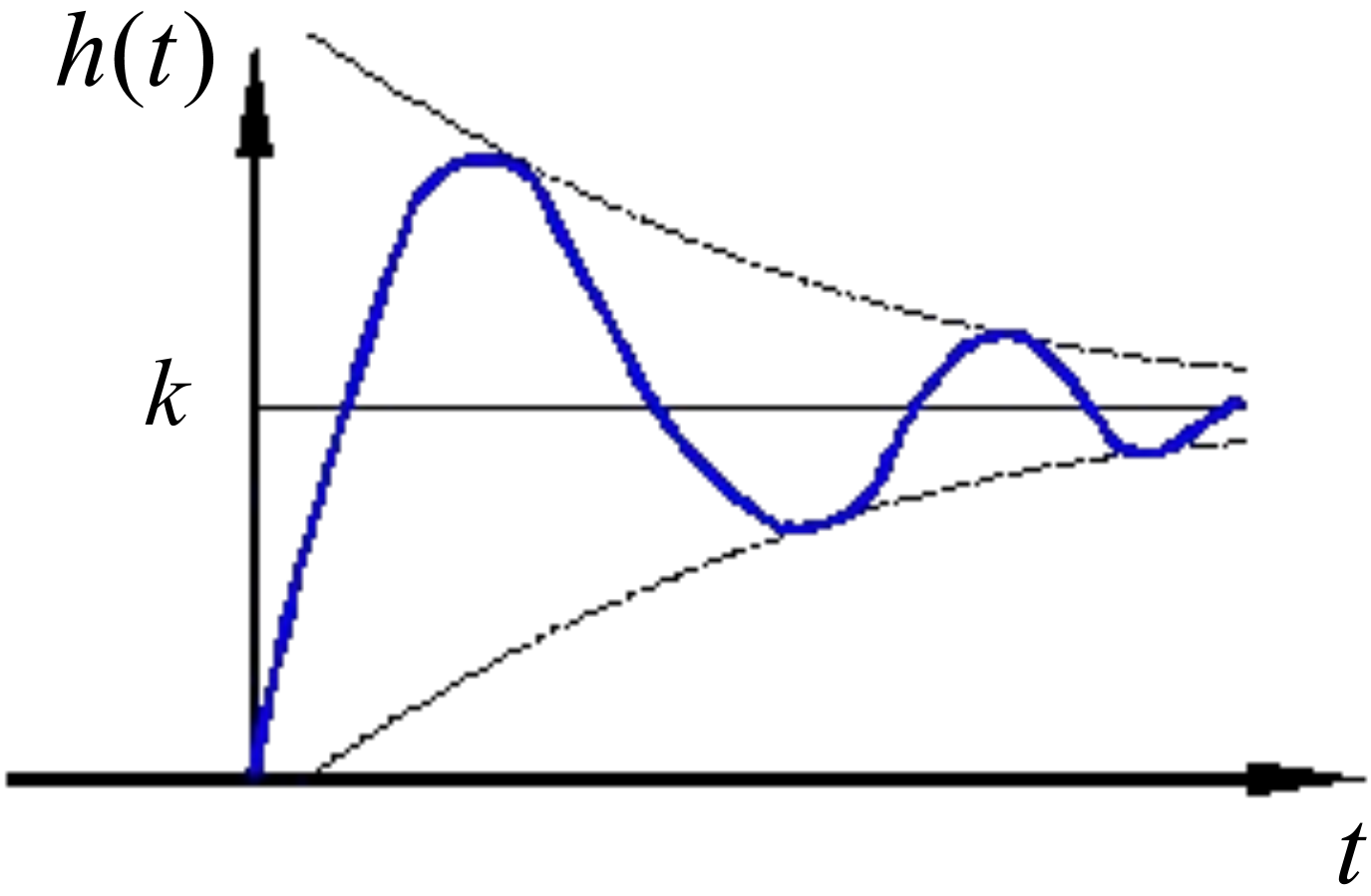
$$\omega << \frac{1}{T} \rightarrow L(\omega) = 20\lg k$$

$$\omega >> \frac{1}{T} \rightarrow L(\omega) = 20\lg k - 20\lg T^2\omega^2$$

$$40\lg T\omega$$


$L(\omega)$
$20\lg k$
40дб/дек
$1/T$
$\lg\omega$

$$5.\ h(t) = k\left(1 - e^{-\alpha t}\left(\cos\omega t + \frac{\alpha}{\omega}\sin\omega t\right)\right)$$



Консервативное звено

$$W(p) = \frac{k}{T^2 p^2 + 1}$$

Форсирующее второго порядка

$$W(s) = k(T^2 s^2 + 2T\xi s + 1)$$