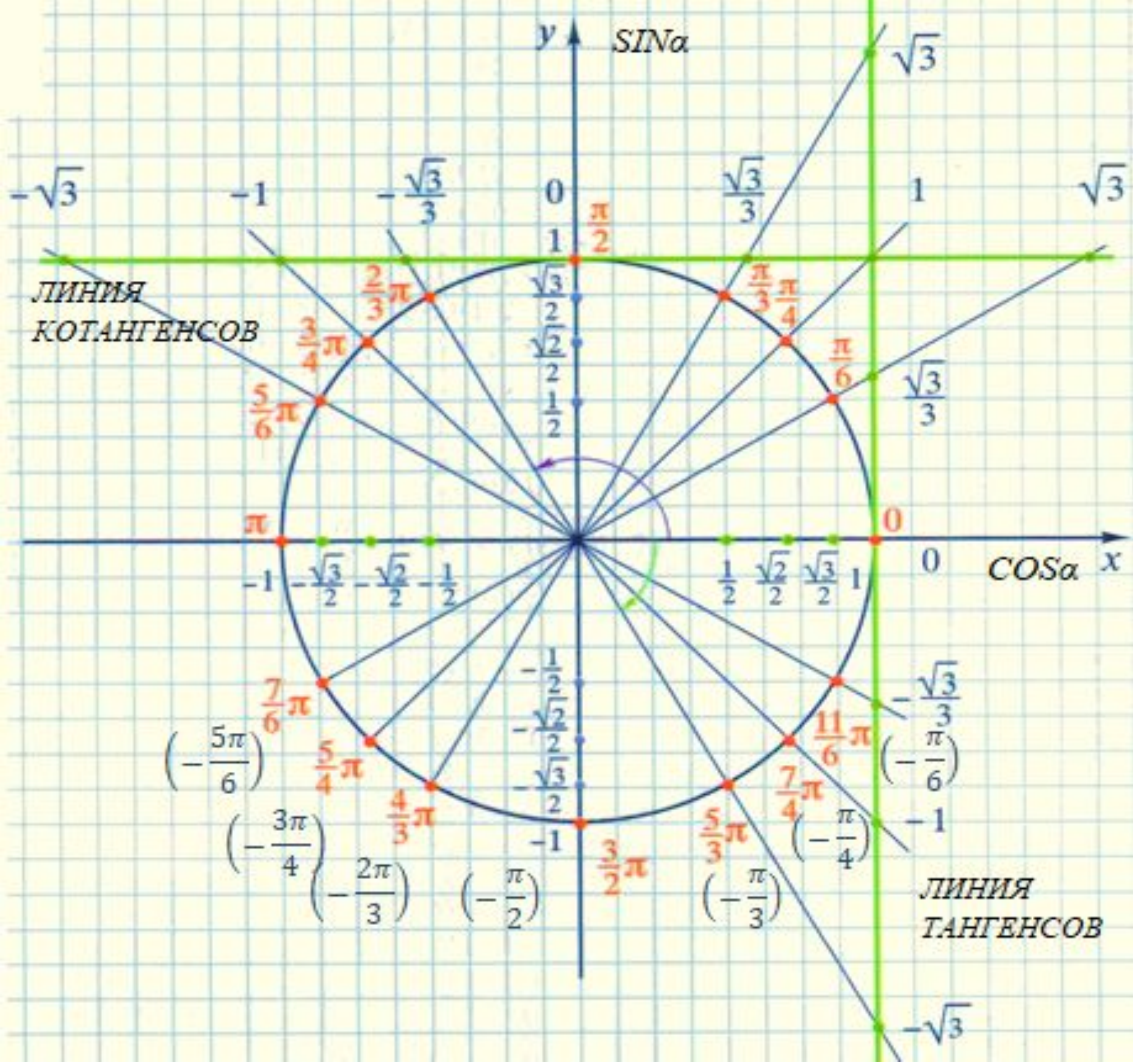
Практическое занятие

Тригонометрические уравнения



y
SINα
ЛИНИЯ КОТАНГЕНСОВ
ЛИНИЯ ТАНГЕНСОВ
COSα
x

- $$\sin\left(\frac{x}{3}+\frac{\pi}{6}\right)=\frac{1}{2}$$

$$\left[\begin{array}{l}\frac{x}{3}+\frac{\pi}{6}=\frac{\pi}{6}+2\pi n\\ \frac{x}{3}+\frac{\pi}{6}=\frac{5\pi}{6}+2\pi n\end{array}\right. => \left[\begin{array}{l}\frac{x}{3}=-\frac{\pi}{6}+\frac{\pi}{6}+2\pi n\\ \frac{x}{3}=-\frac{\pi}{6}+\frac{5\pi}{6}+2\pi n\end{array}\right.$$

$$=> \left[\begin{array}{l}\frac{x}{3}=2\pi n\\ \frac{x}{3}=\frac{2\pi}{3}+2\pi n, n\in Z\end{array}\right.$$

$$=> \left[\begin{array}{l}x=6\pi n\\ x=2\pi+6\pi n, n\in Z\end{array}\right.$$

- $$\cos\left(2x+\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$$

$$\left[\begin{array}{l} 2x+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi n \\ 2x+\frac{\pi}{4}=-\frac{3\pi}{4}+2\pi n \end{array}\right. => \left[\begin{array}{l} 2x=-\frac{\pi}{4}+\frac{3\pi}{4}+2\pi n \\ 2x=-\frac{\pi}{4}-\frac{3\pi}{4}+2\pi n \end{array}\right. =>$$

$$\left[\begin{array}{l} 2x=\frac{\pi}{2}+2\pi n \\ 2x=-\pi+2\pi n \end{array}\right. => \left[\begin{array}{l} x=\frac{\pi}{4}+\pi n \\ x=-\frac{\pi}{2}+\pi n, n \in Z \end{array}\right.$$

- $$\sqrt{3}tg\left(3x+\frac{\pi}{6}\right)=3$$
$$tg\left(3x+\frac{\pi}{6}\right)=\frac{3}{\sqrt{3}}=\sqrt{3}$$
$$3x+\frac{\pi}{6}=\frac{\pi}{3}+\pi n$$
$$3x=-\frac{\pi}{6}+\frac{\pi}{3}+\pi n$$
$$3x=\frac{\pi}{6}+\pi n$$
$$x=\frac{\pi}{18}+\frac{\pi n}{3},n\in Z$$

- $$ctg\left(\frac{x}{4}+\frac{\pi}{3}\right) = -\sqrt{3}$$

$$\frac{x}{4}+\frac{\pi}{3}=\frac{5\pi}{6}+\pi n$$

$$\frac{x}{4} = -\frac{\pi}{3}+\frac{5\pi}{6}+\pi n$$

$$\frac{x}{4}=\frac{\pi}{2}+\pi n$$

$$x = 2\pi + 4\pi n, n \in Z$$

Тригонометрические уравнения, приводящиеся к квадратным

- Определение: уравнения вида

$$asin^2x + bsinx + c = 0$$
$$acos^2x + bcosx + c = 0$$
$$atg^2x + btgx + c = 0$$
$$actg^2x + bctgx + c = 0$$

называются уравнениями, приводящимся к квадратному.

Замена $t = sinx, t \in [-1; 1]; t = cosx, t \in [-1; 1]; t = tgx; t = ctgx$

- $$2sin^2x - 3sinx + 1 = 0$$

Пусть $sinx = t, t \in [-1; 1]$

$$2t^2 - 3t + 1 = 0$$

$$D = 9 - 8 = 1$$

$$t_1 = \frac{3+1}{4} = 1 \in [-1; 1]$$

$$t_2 = \frac{3-1}{4} = \frac{1}{2} \in [-1; 1]$$

$$t_1 = 1 => sinx = 1 => x = -\frac{\pi}{2} + 2\pi n, n \in Z$$

$$t_2 = \frac{1}{2} => sinx = \frac{1}{2} => \left[\begin{array}{c} x = \frac{\pi}{6} + 2\pi n \\ x = \frac{5\pi}{6} + 2\pi n, n \in Z \end{array} \right.$$

- $$cos^2x - 4cosx + 3 = 0$$

Пусть $cosx = t, t \in [-1; 1]$

$$t^2 - 4t + 3 = 0$$

$$D = 16 - 12 = 4$$

$$t_1 = \frac{4+2}{2} = 3$$

$$t_2 = \frac{4-2}{2} = 1 \in [-1; 1]$$

$$t = 1 => cosx = 1$$

$$x = 2\pi n, n \in Z$$

- $$3tg^2x + 2tgx - 1 = 0$$

Пусть $tgx = t$,

$$3t^2 + 2t - 1 = 0$$

$$D = 4 + 12 = 16$$

$$t_1 = \frac{-2+4}{6} = \frac{1}{3}$$

$$t_2 = -1$$

$$t_1 = \frac{1}{3} => tgx = \frac{1}{3} => x_1 = arctg\frac{1}{3} + \pi n, n \in Z$$

$$t_2 = -1 => tgx = -1 => x_2 = -\frac{\pi}{4} + \pi n, n \in Z$$

- $$cos^2x + 3sinx = 3$$
$$cos^2x + sin^2x = 1$$
$$cos^2x = 1 - sin^2x$$
$$sin^2x = 1 - cos^2x$$
$$1 - sin^2x + 3sinx - 3 = 0$$
$$-sin^2x + 3sinx - 2 = 0$$
$$sin^2x - 3sinx + 2 = 0$$
Пусть $sinx = t, t \in [-1; 1] =>$
$$t^2 - 3t + 2 = 0$$
$$t_1 = 2, t_2 = 1 \in [-1; 1]$$
$$t = 1 => sinx = 1 => x = \frac{\pi}{2} + 2\pi n, n \in Z$$

- $$tgx - 2ctgx + 1 = 0$$
$$tgx \cdot ctgx = 1$$
$$tgx = \frac{1}{ctgx}; ctgx = \frac{1}{tgx}$$
$$tgx - \frac{2}{tgx} + 1 = 0 => \frac{tg^2x - 2 + tgx}{tgx} = 0$$
$$\begin{cases} tg^2x - 2 + tgx = 0 \\ tgx \neq 0 \end{cases}$$
$$tg^2x - 2 + tgx = 0$$
$$\text{Пусть } tgx = t$$
$$t^2 + t - 2 = 0$$
$$D = 1 + 8 = 9$$
$$t_1 = 1, t_2 = -2$$
$$t_1 = 1 => tgx = 1 => x_1 = \frac{\pi}{4} + \pi n$$
$$t_2 = -2 => tgx = -2 => x_2 = -arctg2 + \pi n, n \in Z$$

Задания для самостоятельного решения

- Решите уравнения:

1) $3sin^2x - 5sinx - 2 = 0$
2) $2cos^2x + 3cosx - 2 = 0$
3) $ctg^2x - 6ctgx + 5 = 0$
4) $4sinx + cos^2x = 4$
5) $5sin^2x + 6cosx - 6 = 0$
6) $2tg^2x + 3tgx - 2 = 0$