

# Winston p.503, #14 (prereducer)

| Column  |   |   |   |   |   |   |   |   |   |    |          |
|---|---|---|---|---|---|---|---|---|---|----|----------|
| Row   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |          |
| 0   | 3 | 5 | 1 | 2 | 1 | 4 | 3 | 1 | 2 | 2  |          |
| 1   | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0  | $\geq 1$ |
| 2   | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | $\geq 1$ |
| 3   | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1  | $\geq 1$ |
| 4   | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0  | $\geq 1$ |
| 5   | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1  | $\geq 1$ |
| col 4 + col 5 $\geq$ col 2; c2 > c4+c5; delete column 2 |   |   |   |   |   |   |   |   |   |    |          |

| Row  | 1 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |          |
|--|---|---|---|---|---|---|---|---|----|----------|
| 0  | 3 | 1 | 2 | 1 | 4 | 3 | 1 | 2 | 2  |          |
| 1  | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0  | $\geq 1$ |
| 2  | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | $\geq 1$ |
| 3  | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1  | $\geq 1$ |
| 4  | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0  | $\geq 1$ |
| 5  | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1  | $\geq 1$ |
| col 4 + col 3 $\geq$ col 1; c1 $\geq$ c3+c4; delete column 1 |   |   |   |   |   |   |   |   |    |          |

# Winston p.503, #14 (cont'd)

| Row | 3  | 4 | 5 | 6 | 7 | 8 | 9 | 10 |          |
|-----|--|---|---|---|---|---|---|----|----------|
| 0   | 1  | 2 | 1 | 4 | 3 | 1 | 2 | 2  |          |
| 1   | 0  | 1 | 1 | 0 | 0 | 1 | 1 | 0  | $\geq 1$ |
| 2   | 1  | 0 | 0 | 0 | 0 | 0 | 0 | 0  | $\geq 1$ |
| 3   | 0  | 0 | 1 | 0 | 1 | 0 | 0 | 1  | $\geq 1$ |
| 4   | 1  | 0 | 0 | 1 | 0 | 1 | 0 | 0  | $\geq 1$ |
| 5   | 0  | 1 | 0 | 1 | 1 | 0 | 1 | 1  | $\geq 1$ |
|     |  |   |   |   |   |   |   |    |          |
|     | col 7 + col 8 $\geq$ col 6; c6 $\geq$ c7+c8; delete column 6 |   |   |   |   |   |   |    |          |

| Row | 3  | 4 | 5 | 7 | 8 | 9 | 10 |          |  |
|-----|--|---|---|---|---|---|----|----------|--|
| 0   | 1  | 2 | 1 | 3 | 1 | 2 | 2  |          |  |
| 1   | 0  | 1 | 1 | 0 | 1 | 1 | 0  | $\geq 1$ |  |
| 2   | 1  | 0 | 0 | 0 | 0 | 0 | 0  | $\geq 1$ |  |
| 3   | 0  | 0 | 1 | 1 | 0 | 0 | 1  | $\geq 1$ |  |
| 4   | 1  | 0 | 0 | 0 | 1 | 0 | 0  | $\geq 1$ |  |
| 5   | 0  | 1 | 0 | 1 | 0 | 1 | 1  | $\geq 1$ |  |
|     |  |   |   |   |   |   |    |          |  |
|     | col 4 + col 5 $\geq$ col 7; c7 $\geq$ c4+c5; delete column 7 |   |   |   |   |   |    |          |  |

# Winston p.503, #14 (cont'd)

| Row   | 3 | 4 | 5 | 8 | 9 | 10 |          |
|---|---|---|---|---|---|----|----------|
| 0   | 1 | 2 | 1 | 1 | 2 | 2  |          |
| 1   | 0 | 1 | 1 | 1 | 1 | 0  | $\geq 1$ |
| 2   | 1 | 0 | 0 | 0 | 0 | 0  | $\geq 1$ |
| 3   | 0 | 0 | 1 | 0 | 0 | 1  | $\geq 1$ |
| 4   | 1 | 0 | 0 | 1 | 0 | 0  | $\geq 1$ |
| 5   | 0 | 1 | 0 | 0 | 1 | 1  | $\geq 1$ |
| row 2 fixes x3 at 1; delete rows 2,4, col 3 |   |   |   |   |   |    |          |

| Row  | 4 | 5 | 8 | 9 | 10 |          |  |  |  |  |  |
|--|---|---|---|---|----|----------|--|--|--|--|--|
| 0  | 2 | 1 | 1 | 2 | 2  |          |  |  |  |  |  |
| 1  | 1 | 1 | 1 | 1 | 0  | $\geq 1$ |  |  |  |  |  |
| 3  | 0 | 1 | 0 | 0 | 1  | $\geq 1$ |  |  |  |  |  |
| 5  | 1 | 0 | 0 | 1 | 1  | $\geq 1$ |  |  |  |  |  |
| multiple optima; best solution is $z=4$ ; $x_3=1$ , $x_5=1$ , $x_{10}=1$ is one soln |   |   |   |   |    |          |  |  |  |  |  |

# Problem #2 (Gomory Cuts)

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- Cut from first constraint:

$$x_1 + \frac{1}{3}s_1 - \frac{1}{12}s_2 = \frac{5}{4}$$

$$x_1 + 0s_1 + \frac{1}{3}s_1 - s_2 + \frac{11}{12}s_2 = 1 + \frac{1}{4}$$

$$x_1 - s_2 - 1 = \frac{1}{4} - \frac{1}{3}s_1 - \frac{11}{12}s_2$$

$$\frac{1}{4} - \frac{1}{3}s_1 - \frac{11}{12}s_2 \leq 0, \text{ or}$$

$$\frac{1}{4} - \frac{1}{3}s_1 - \frac{11}{12}s_2 + s_3 = 0$$

- Cut from second constraint:

$$x_2 + \frac{1}{3}s_1 + \frac{1}{6}s_2 = \frac{5}{2}$$

$$x_2 + 0s_1 + \frac{1}{3}s_1 + 0s_2 + \frac{1}{6}s_2 = 2 + \frac{1}{2}$$

$$x_2 - 2 = \frac{1}{2} - \frac{1}{3}s_1 - \frac{1}{6}s_2$$

$$\frac{1}{2} - \frac{1}{3}s_1 - \frac{1}{6}s_2 \leq 0, \text{ or}$$

$$\frac{1}{2} - \frac{1}{3}s_1 - \frac{1}{6}s_2 + s_4 = 0$$