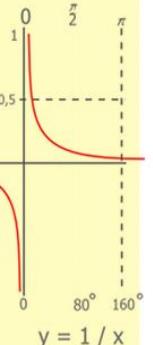
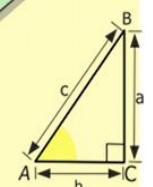
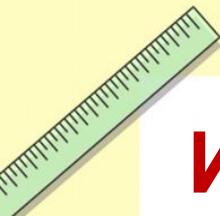
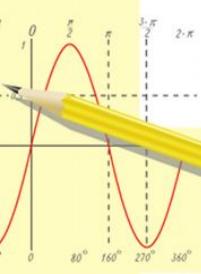


Иррациональные уравнения



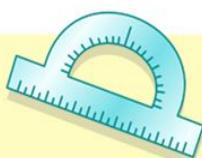
$$\begin{array}{r} \frac{1}{2500} \\ \times 42 \\ \hline + 210 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

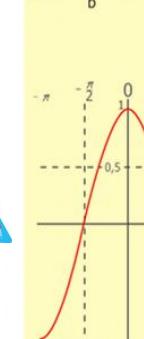
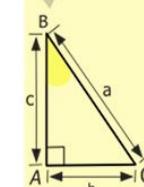
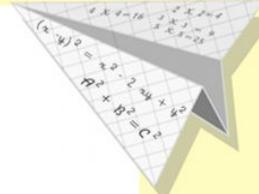
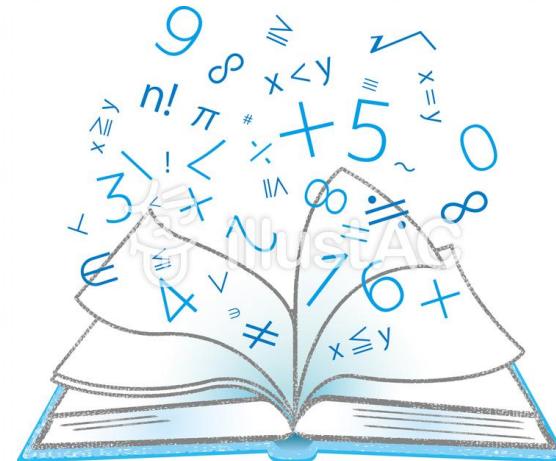


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

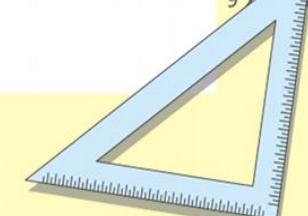
$$(x+y)(x-y) = x^2 - y^2$$

x = 70



$$y = \cos x$$

$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$



Повторение

Среди пар уравнений найдите пары
равносильных:

a) $5x + 10 = 0$ и $x + 2 = 0$;

б) $x = 5$ и $x^2 = 25$;

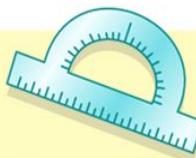
в) $\sqrt{x^2 - 2x + 1} = 3$ и $|x - 1| = 3$;

г) $\sqrt{x} = -4$ и $x^2 + 1 = 0$.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

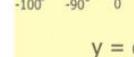
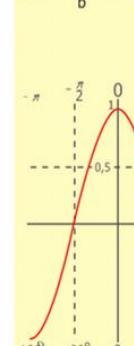
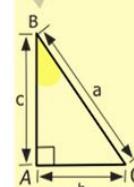
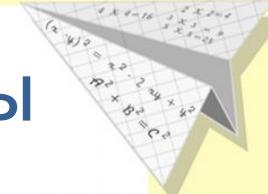


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

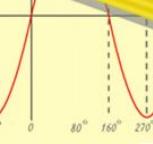
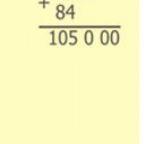
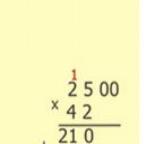
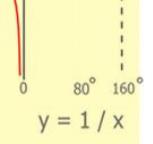
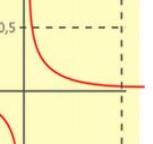
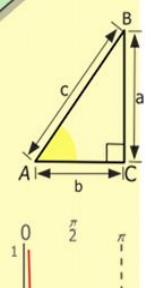
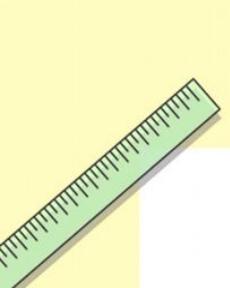
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$



$2 \times 2 = 4$
$3 \times 3 = 9$
$4 \times 4 = 16$
$5 \times 5 = 25$
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$7 \times 7 = 49$
$8 \times 8 = 64$
$9 \times 9 = 81$



Повторение

Определите, какое из двух уравнений является следствие другого:

a) $x - 5 = 0$ и $x(x - 5) = 0;$

b) $x = -3$ и $x^2 = 25;$

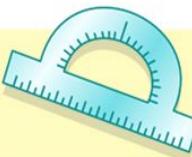
c) $\frac{x^2 - 3x}{x} = 0$ и $x^2 - 3x = 0;$

z) $\frac{x - 7}{x} = 0$ и $x - 7 = 0.$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

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$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$

Повторение

- Арифметическим квадратным корнем из числа **a** называется неотрицательное число **b**, квадрат которого равен **a**

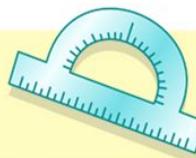
$$\sqrt{a} = b$$

, где $b \geq 0$, если $a=b^2$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

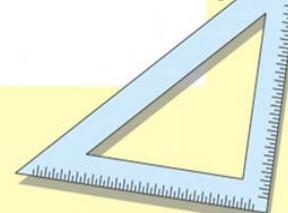


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

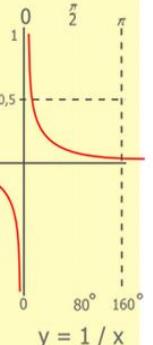
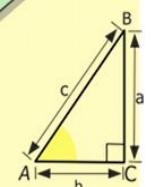
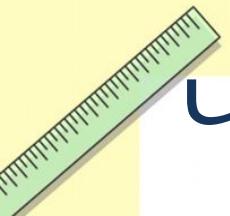
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

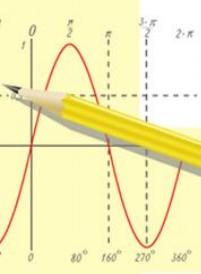
$$x = 70$$



Что общего в этих уравнениях?



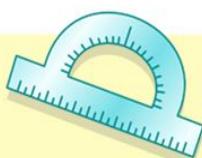
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

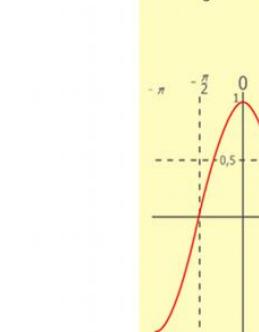
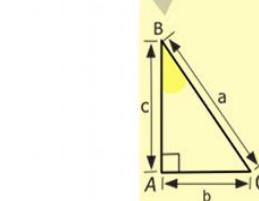
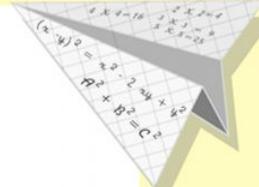


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

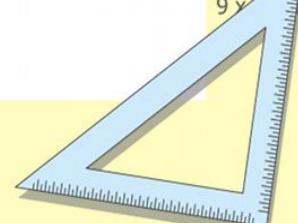
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$

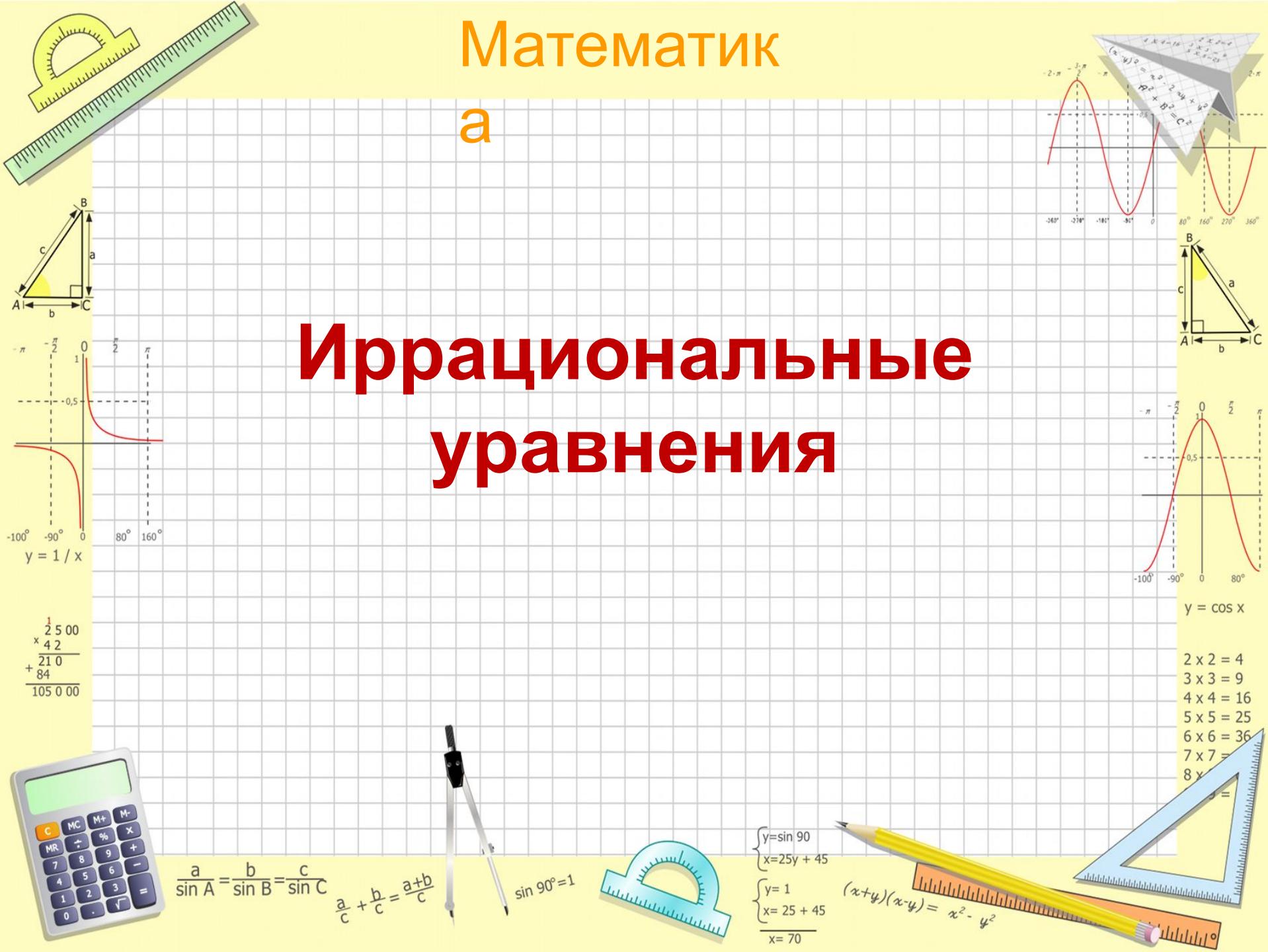


$2 \times 2 = 4$
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 $9 \times 9 = 81$



Математик а

Иrrациональные уравнения



Определение

Иrrациональными называются
уравнения, в которых переменная
содержится под знаком корня
(радикала).

Примеры:

$$\sqrt{x+12} - x = 0, \quad \sqrt[3]{x-1} = x.$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

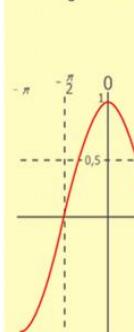
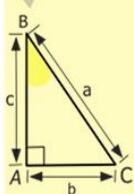
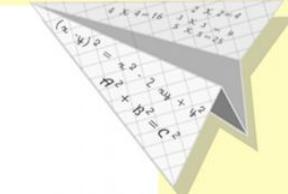


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$



$$y = \cos x$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

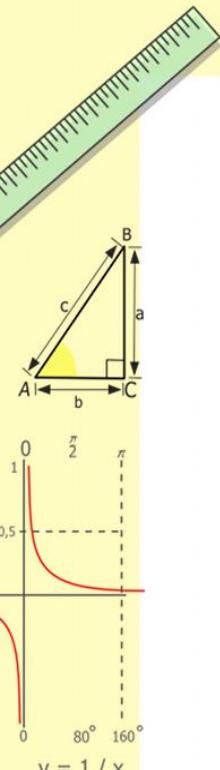
$$5 \times 5 = 25$$

$$6 \times 6 = 36$$

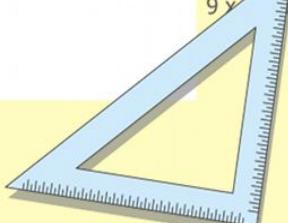
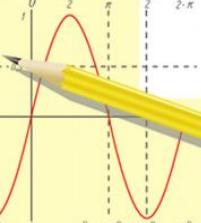
$$7 \times 7 = 49$$

$$8 \times 8 = 64$$

$$9 \times 9 = 81$$



$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



План изучения темы

Иррациональные уравнения

Определение

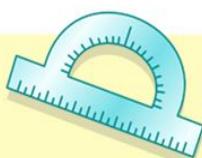
Простейшие
уравнения

Сложные
уравнения

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

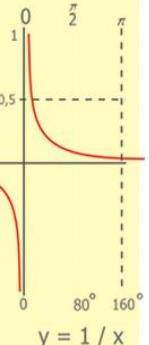
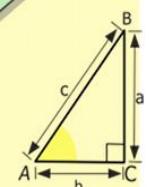
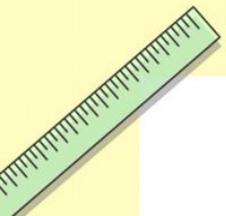


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

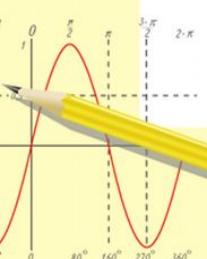
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$\frac{x=70}{(x+y)(x-y) = x^2 - y^2}$$

Какие из уравнений не являются иррациональными?



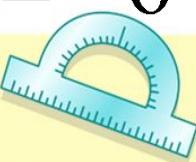
$$\begin{array}{r} \frac{1}{2500} \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

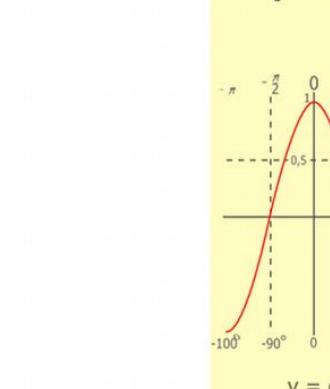
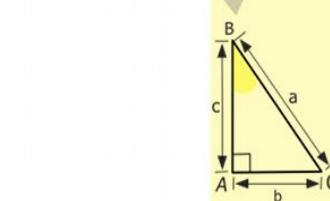
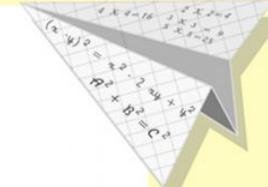


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

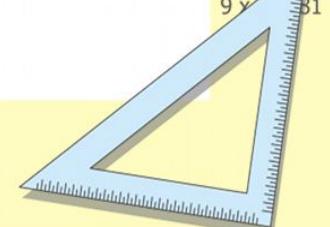
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$x = 70$



$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$



Идея решения

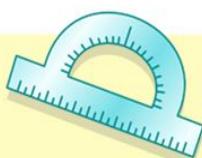
Основная идея решения иррационального уравнения состоит в сведении его к рациональному алгебраическому уравнению, которое либо равносильно исходному иррациональному уравнению, либо является его следствием.

Главный способ избавиться от корня и получить рациональное уравнение – возведение обеих частей уравнения в одну и ту же степень, которую имеет корень, содержащий неизвестное.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

x = 70



Простейшие иррациональные уравнения

$$\sqrt{f(x)} = a$$

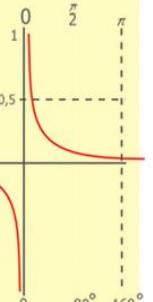
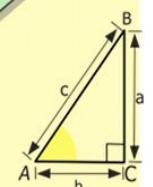
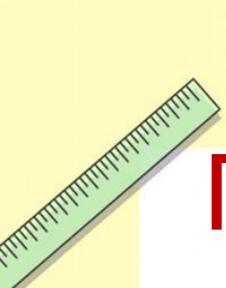
$$\sqrt{f(x)} = g(x)$$

$$\sqrt{f(x)} = \sqrt{g(x)}$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

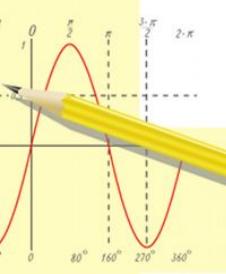
$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$



$$y = 1 / x$$

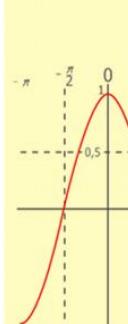
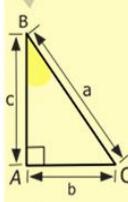
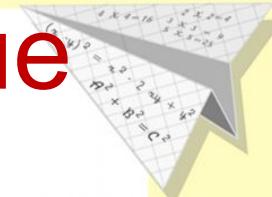
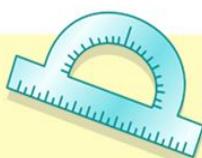
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

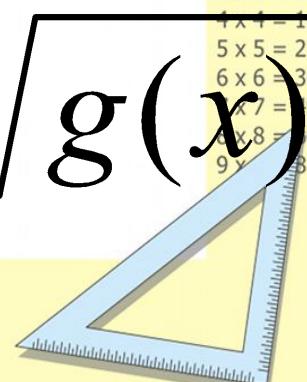
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$y = \cos x$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



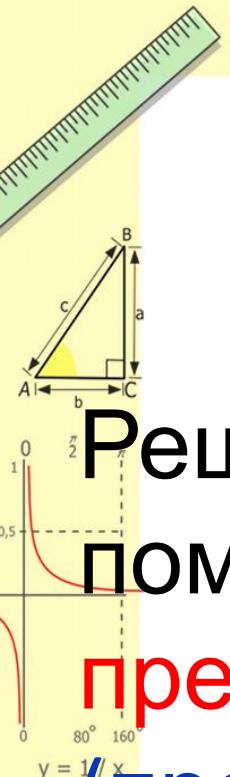
Запомни!

При возведении обеих частей уравнения

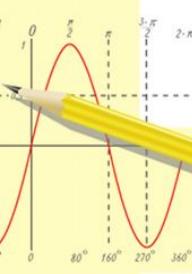
- в **четную** степень (показатель корня – **четное** число) – возможно появление постороннего корня (проверка необходима)

- в **нечетную** степень (показатель корня – **нечетное** число) – получается уравнение, равносильное исходному (**проверка не нужна**)

Запомни!



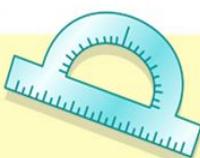
$$\begin{array}{r} \frac{1}{2500} \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

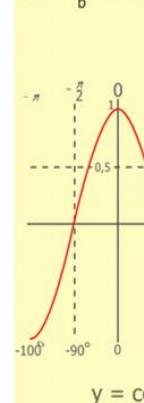
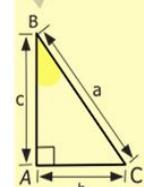
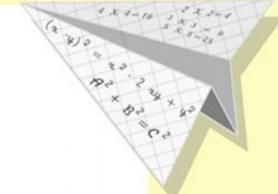


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

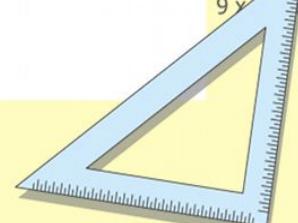
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$



$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$



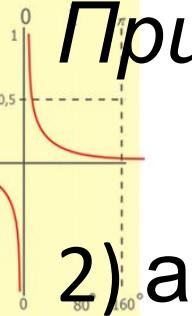
Решение уравнения

$$\sqrt{f(x)} = a$$



1) $a < 0$, то $\sqrt{f(x)} = a$ уравнение корней не имеет

Пример: $\sqrt{2x - 5} = -3$



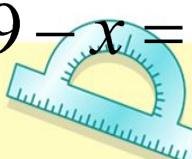
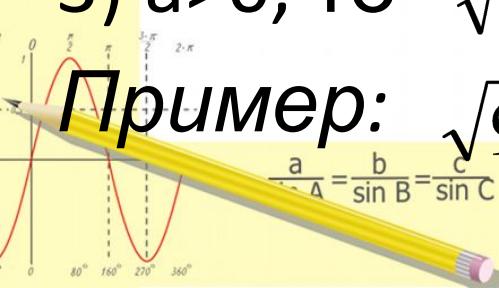
2) $a = 0$, то $\sqrt{f(x)} = 0 \Leftrightarrow f(x) = 0$

Пример: $\sqrt{x - 7} = 0 \Leftrightarrow x - 7 = 0 \Leftrightarrow x = 7$

$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

3) $a > 0$, то $\sqrt{f(x)} = a \Leftrightarrow (\sqrt{f(x)})^2 = a^2 \Leftrightarrow f(x) = a^2$

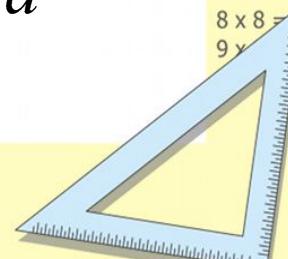
Пример: $\sqrt{9 - x} = 10 \Leftrightarrow 9 - x = 100 \Leftrightarrow x = -91$



$$\sin 90^\circ = 1$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \frac{(x+y)(x-y)}{x = 70} = x^2 - y^2 \end{cases}$$

$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$



Решение уравнения

$$\sqrt{f(x)} = g(x)$$

1 способ

$$\begin{aligned}\sqrt{-3x+3} &= x-1 \\ -3x+3 &= (x-1)^2 \\ -3x+3 &= x^2 - 2x + 1\end{aligned}$$

$$x^2 - x - 2 = 0$$

$$\begin{array}{r} \frac{1}{2}x_0 = -1 \\ \hline \frac{1}{2}x_0 + \frac{1}{2}x_0 = 2 \\ \hline x_0 = 2 \end{array}$$

проверка

при $x = -1$ $\sqrt{-3 \cdot (-1) + 3} \neq -1 - 1$

при $x = 2$ $\sqrt{-3 \cdot 2 + 3} = 2 - 1$

ответ : 2

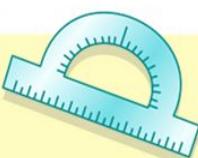
$$\sqrt{-3x+3} = x-1$$

2 способ

$$\sqrt{-3x+3} = x-1 \Leftrightarrow \begin{cases} x-1 \geq 0 \\ -3x+3 = (x-1)^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 1 \\ x_1 = -1 \text{ ответ : 2} \\ x_2 = 2 \end{cases}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases} \quad (x+y)(x-y) = x^2 - y^2$$

$$x = 70$$

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$$4^2 = 16$$

Вывод

Уравнение вида

решается:

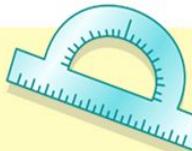
- 1) Возведением в квадрат обеих частей равенства с последующей проверкой;
- 1) Осуществляется переход к системе равносильной данному уравнению, т. е.

$$\sqrt{f(x)} = g(x) \Leftrightarrow \begin{cases} f(x) = g^2(x), \\ g(x) \geq 0. \end{cases}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

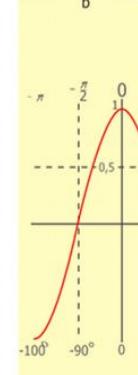
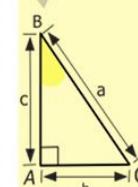
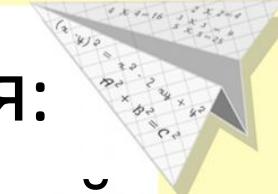


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

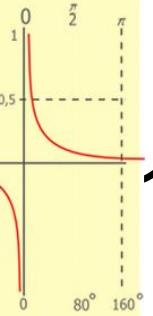
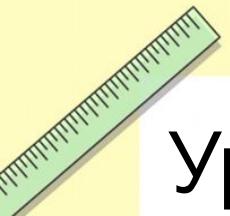
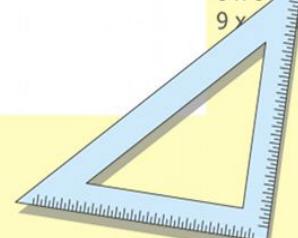
$$(x+y)(x-y) = x^2 - y^2$$

$$x = 70$$

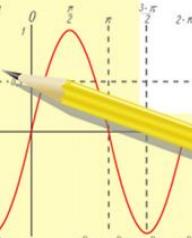


$$y = \cos x$$

$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$

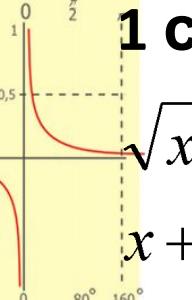
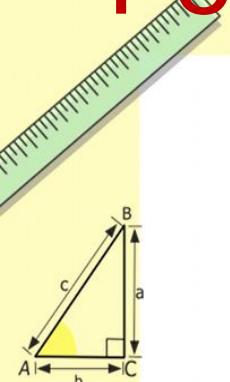


$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



Решение уравнения

$$\sqrt{f(x)} = \sqrt{g(x)}$$



1 способ

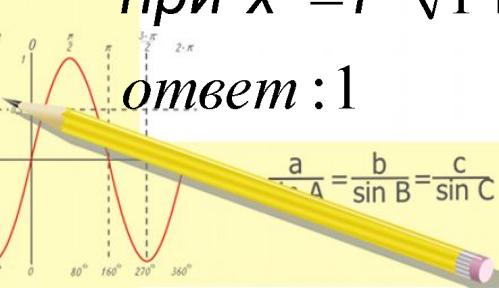
$$\begin{aligned}\sqrt{x+3} &= \sqrt{5-x} \\ x+3 &= 5-x \\ 2x &= 2\end{aligned}$$

$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

проверка

$$\text{при } x = 1 \quad \sqrt{1+3} = \sqrt{5-1}$$

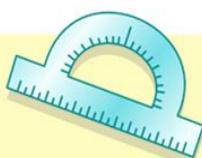
ответ : 1



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



2 способ

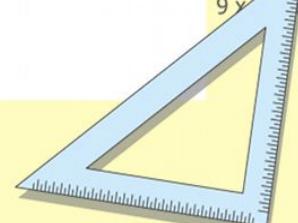
$$\begin{aligned}\sqrt{x+3} &= \sqrt{5-x} \Leftrightarrow \begin{cases} x+3 \geq 0 \\ 5-x \geq 0 \end{cases} \\ &\Leftrightarrow \begin{cases} x \geq -3 \\ x \leq 5 \end{cases} \text{ ответ : 1.} \\ &\Leftrightarrow x = 1 \quad \text{ответ : 1.}\end{aligned}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases} \quad (x+y)(x-y) = x^2 - y^2$$

$$x = 70$$



Вывод

Уравнение вида $\sqrt{f(x)} = \sqrt{g(x)}$ решается:

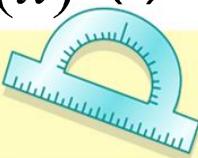
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$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\frac{y=\sin 90}{y=1} \quad \frac{x=25+45}{x=70}$$

$$(x+y)(x-y) = x^2 - y^2$$

Домашнее задание

I

$$\sqrt{x+1} = x - 5$$

$$\sqrt[3]{x^2 - 28} = 2$$

III

$$\sqrt{x-2} = x - 8$$

$$\sqrt[3]{x+12} = 4$$

II

$$\sqrt{x-6} = \sqrt{4-x}$$

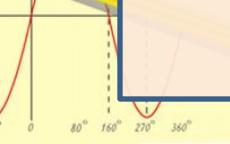
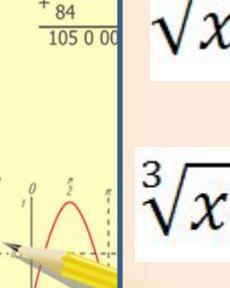
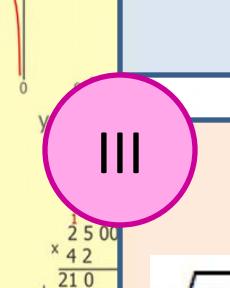
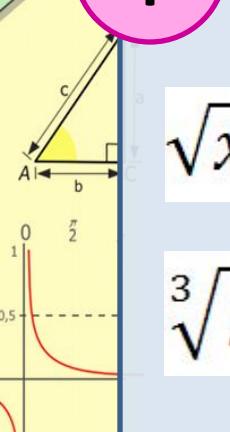
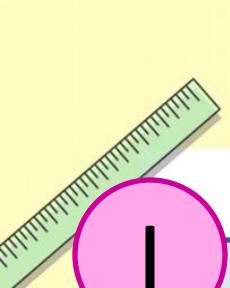
$$\sqrt[3]{x^2 - 8} = 2$$

IV

$$\sqrt{x^4 + 19} = 10$$

$$\sqrt[3]{x-1} = -1$$

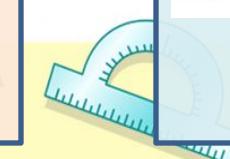
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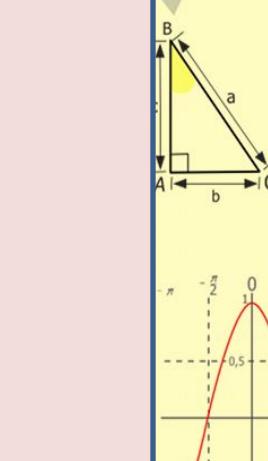
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$$\frac{a+b}{A+C} = \frac{a-b}{B-C}$$

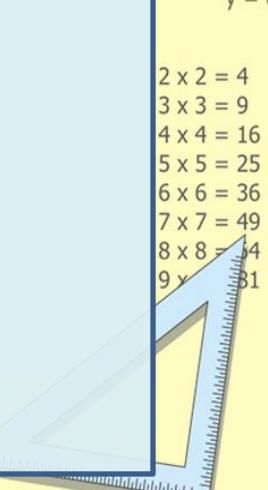
$$\sin 90^\circ = 1$$



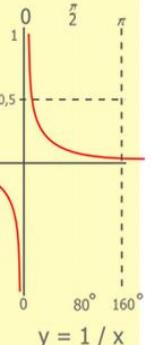
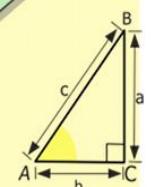
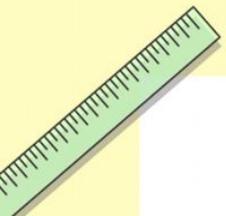
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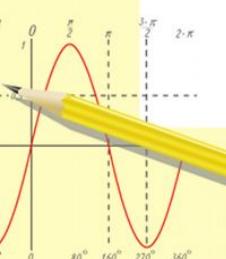
$$y = \cos x$$



Домашнее задание



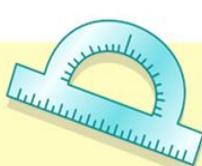
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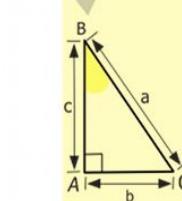
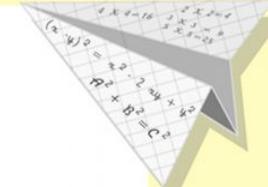


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