

# Оптимальное управление динамических систем

- ✓ Гамильтониан и принцип максимума

# Общие положения

$$H[\mathbf{x}(t), \mathbf{u}(t), t] = L[\mathbf{x}(t), \mathbf{u}(t), t] + \boldsymbol{\lambda}^T(t) \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t), t] \quad (3.25)$$

$$\frac{\partial H}{\partial \mathbf{u}} = \mathbf{0} \quad (3.26)$$

$$H_{\mathbf{u}\mathbf{u}} = \frac{\partial^2 H}{\partial \mathbf{u}^2} > \mathbf{0} \quad (3.27)$$

$$\hat{\mathbf{x}}(t) \quad \hat{\boldsymbol{\lambda}}(t) \quad \hat{\mathbf{u}}(t)$$

$$H[\hat{\mathbf{x}}(t), \mathbf{u}(t), \hat{\boldsymbol{\lambda}}, t] \geq H[\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), \hat{\boldsymbol{\lambda}}, t] \quad (3.28)$$

$$\mathbf{u}(t) \quad [\hat{\mathbf{x}}(t), \hat{\boldsymbol{\lambda}}(t)] \quad \hat{\mathbf{u}}(t)$$

## Уравнения Гамильтона-Якоби - Беллмана

$$\hat{\mathbf{x}}(t)$$

$$J = \varphi[\mathbf{x}(t_f), t_f] + \int_{t_0}^{t_f} L[\mathbf{x}(t), \mathbf{u}(t), t] dt \quad (3.29)$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad (3.30)$$

$$\hat{\mathbf{u}}(t) \quad t_0 \leq t \leq t_f$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] = \varphi[\hat{\mathbf{x}}(t_f), t_f] + \int_t^{t_f} L[\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau), \tau] d\tau \quad (3.31)$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] = \varphi[\hat{\mathbf{x}}(t_f), t_f] - \int_{t_f}^t L[\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau), \tau] d\tau \quad (3.32)$$

## Уравнения Гамильтона-Якоби - Беллмана

$$\frac{d\hat{V}}{dt} = -L[\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), t] \quad (3.33)$$

$$\frac{d\hat{V}}{dt} = \frac{\partial \hat{V}}{\partial t} + \frac{\partial \hat{V}}{\partial \mathbf{x}} \mathbf{f} = \frac{\partial \hat{V}}{\partial t} + \frac{\partial \hat{V}}{\partial \mathbf{x}} \mathbf{f} \quad (3.34)$$

$$-\frac{\partial \hat{V}}{\partial t} = L[\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), t] + \frac{\partial \hat{V}}{\partial \mathbf{x}} \mathbf{f} \quad (3.35)$$

$$\hat{V}[\hat{\mathbf{x}}(t_f), t_f] = \varphi[\hat{\mathbf{x}}(t_f), t_f] \quad (3.36)$$

$$\hat{\mathbf{x}} \quad t \quad \mathbf{x}_0$$

## Уравнения Гамильтона-Якоби - Беллмана

$$-\frac{\partial \hat{V}}{\partial t} = H[\hat{\mathbf{x}}(t), \hat{\boldsymbol{\lambda}}(t), \hat{\mathbf{u}}(t), t] \quad (3.37)$$

$$\hat{\boldsymbol{\lambda}}^T = \frac{\partial \hat{V}}{\partial \mathbf{x}} \quad (3.38)$$

$$\mathbf{u}(t) = \hat{\mathbf{u}}[t, \mathbf{x}(t)] \quad (3.39)$$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, t) & \mathbf{x}(t_0) &= \mathbf{x}_0 & (3.40) & \hat{V}[\hat{\mathbf{x}}(t), t] \\ \mathbf{x}_e &= \mathbf{0} & \hat{V}[\hat{\mathbf{x}}(t), t] & & & \mathbf{x} = \mathbf{0} \end{aligned}$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] \quad V(\mathbf{x}, t)$$

$$V(\mathbf{0}, t) = 0 \quad V(\mathbf{x}, t) > 0 \quad \frac{d}{dt} V(\mathbf{x}, t) < 0, \mathbf{x} \neq \mathbf{0} \quad (3.41)$$

$$\|\mathbf{x}\| \rightarrow \infty, V(\mathbf{x}, t) \rightarrow \infty \quad (3.42)$$

Линейные зависимости от времени  
системы с квадратичным показателем  
качества

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad (3.43)$$

$$J = \mathbf{x}^T(t_f)\mathbf{Q}_f\mathbf{x}(t_f) + \int_{t_0}^{t_f} [\mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + 2\mathbf{x}^T(t)\mathbf{S}(t)\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t)]dt \quad (3.44)$$

$$L[\mathbf{x}(t), \mathbf{u}(t), t] = \mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + 2\mathbf{x}^T(t)\mathbf{S}(t)\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t) \quad (3.45)$$

$$V[\mathbf{x}(t), t] = \mathbf{x}^T(t) \mathbf{P}(t) \mathbf{x}(t)$$

$$V[\mathbf{x}(t), t] = \mathbf{x}^T(t) \mathbf{P}(t) \mathbf{x}(t) \quad (3.46)$$

$$\hat{V}[\hat{\mathbf{x}}(t_f), t_f] = \hat{\mathbf{x}}^T(t_f) \mathbf{Q}_f \hat{\mathbf{x}}(t_f) \quad (3.47)$$

$$\boldsymbol{\lambda}^T = \frac{\partial V}{\partial \mathbf{x}} = 2\mathbf{x}^T(t) \mathbf{P}(t) \quad (3.48)$$

$$\begin{aligned} H = & \mathbf{x}^T(t) \mathbf{Q}(t) \mathbf{x}(t) + 2\mathbf{x}^T(t) \mathbf{S}(t) \mathbf{u}(t) + \mathbf{u}^T(t) \mathbf{R}(t) \mathbf{u}(t) + \\ & + 2\mathbf{x}^T(t) \mathbf{P}(t) [\mathbf{A}(t) \mathbf{x}(t) + \mathbf{B}(t) \mathbf{u}(t)] \end{aligned} \quad (3.49)$$



$$H_u = \mathbf{0}$$

$$\hat{\mathbf{x}}^T(t)\mathbf{S}(t) + \hat{\mathbf{u}}^T(t)\mathbf{R}(t) + \hat{\mathbf{x}}^T(t)\mathbf{P}(t)\mathbf{B}(t) = \mathbf{0} \quad (3.50)$$

$$\hat{\mathbf{u}}(t) = -\mathbf{R}^{-1}(t)[\mathbf{B}^T(t)\hat{\mathbf{P}}(t) + \mathbf{S}^T(t)]\hat{\mathbf{x}}(t) \quad (3.51)$$

$$\mathbf{P}(t) \quad V[\mathbf{x}(t), t] > 0 \quad \mathbf{x}(t)$$

$$\hat{H} = -\frac{\partial \hat{V}}{\partial t} = -\hat{\mathbf{x}}^T \hat{\mathbf{P}} \hat{\mathbf{x}} \quad (3.52)$$

$$H_{uu} > \mathbf{0} \quad \hat{\mathbf{P}}(t) \quad V[\mathbf{x}(t), t] < 0, \mathbf{x}(t) \in E$$

$$\hat{\mathbf{P}}(t) \quad \mathbf{R}(t)$$

$$\begin{aligned} \hat{\mathbf{x}}^T \dot{\hat{\mathbf{P}}} \hat{\mathbf{x}} = & -\hat{\mathbf{x}}^T [(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T)^T \hat{\mathbf{P}} + \hat{\mathbf{P}}(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T) - \\ & - \hat{\mathbf{P}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \hat{\mathbf{P}} + \mathbf{Q} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T] \hat{\mathbf{x}} \end{aligned} \quad (3.53)$$

$$-\dot{\hat{\mathbf{P}}} = \mathbf{Q} + (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T)^T \hat{\mathbf{P}} + \hat{\mathbf{P}}(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T) - \hat{\mathbf{P}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \hat{\mathbf{P}} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T \quad (3.54)$$

$$\hat{\mathbf{P}}_f = \mathbf{Q}_f \quad (3.55)$$

$$\hat{\mathbf{P}}(t) \quad \mathbf{Q}_f, \mathbf{Q}(t), \mathbf{R}(t)$$

# Лекция

Метод АКОР

$$J(u) = \frac{1}{2} x^T(t_k) \Phi_k x(t_k) + \frac{1}{2} \int_{t_0}^{t_k} [x^T(t) \Phi x(t) + u^T(t) \Psi u(t)] dt$$

$$x^T(t) \Phi x(t) \quad u^T(t) \Psi u(t) \quad \frac{1}{2} x^T(t_k) \Phi_k x(t_k)$$

$$u_{opt}(t) = \Psi^{-1} B^T(t) K(t) x(t) = -D(t) x(t); D(t) = \Psi^{-1} B^T(t) K(t)$$

$$u_j = \sum_{i=1}^n d_{ij} x_i$$

$$\dot{K}(t) = -K(t)A(t) - A^T(t)K(t) + K(t)B(t)\Psi^{-1}B^T(t)K(t) - \Phi$$

$$\dot{x}(t) = [A(t) - B(t)D(t)]x(t) = A_o(t)x(t); A_o(t) = A(t) - B(t)D(t)$$