Оптимальное управление динамических систем

- **Гамильтониан и принцип максимума**

Общие положения

$$H[\mathbf{x}(t),\ \mathbf{u}(t),t] = L[\mathbf{x}(t),\ \mathbf{u}(t),t] + \boldsymbol{\lambda}^T(t)\mathbf{f}[\mathbf{x}(t),\mathbf{u}(t),t]$$

(3.25)

$$\frac{\partial H}{\partial \mathbf{u}} = \mathbf{0}$$

(3.26)

$$H_{\mathbf{uu}} = \frac{\partial^2 H}{\partial \mathbf{u}^2} > \mathbf{0}$$

(3.27)

$\hat{\mathbf{x}}(t)$ $\hat{\boldsymbol{\lambda}}(t)$ $\hat{\mathbf{u}}(t)$

$$H[\hat{\mathbf{x}}(t),\ \mathbf{u}(t),\hat{\boldsymbol{\lambda}},t] \geq H[\hat{\mathbf{x}}(t),\ \hat{\mathbf{u}}(t),\hat{\boldsymbol{\lambda}},t]$$

(3.28)

$\mathbf{u}(t)$ $[\hat{\mathbf{x}}(t),\hat{\boldsymbol{\lambda}}(t)]$ $\hat{\mathbf{u}}(t)$

Уравнения Гамильтона-Якоби – Беллмана

$$\hat{\mathbf{x}}(t)$$

$$J = \varphi[\mathbf{x}(t_{\mathrm{f}}), t_{\mathrm{f}}] + \int_{t_0}^{t_{\mathrm{f}}} L[\mathbf{x}(t), \mathbf{u}(t), \mathrm{t}]\mathrm{dt} \qquad (3.29)$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \qquad (3.30)$$

$$\hat{\mathbf{u}}(t) \qquad t_0 \le t \le t_{\mathrm{f}}$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] = \varphi[\hat{\mathbf{x}}(t_{\mathrm{f}}), t_{\mathrm{f}}] + \int_{t}^{t_{\mathrm{f}}} L[\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau), \tau]\mathrm{d}\tau \qquad (3.31)$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] = \varphi[\hat{\mathbf{x}}(t_{\mathrm{f}}), t_{\mathrm{f}}] - \int_{t_{\mathrm{f}}}^{t} L[\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau), \tau]\mathrm{d}\tau \qquad (3.32)$$

Уравнения Гамильтона-Якоби – Беллмана

$$\frac{\mathrm{d}\hat{V}}{\mathrm{d}t} = -L[\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), \mathrm{t}]$$ (3.33)

$$\frac{\mathrm{d}\hat{V}}{\mathrm{d}t} = \frac{\partial \hat{V}}{\partial t} + \frac{\partial \hat{V}}{\partial \mathbf{x}}\dot{\mathbf{x}} = \frac{\partial \hat{V}}{\partial t} + \frac{\partial \hat{V}}{\partial \mathbf{x}}\mathbf{f}$$ (3.34)

$$-\frac{\partial \hat{V}}{\partial t} = L[\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), \mathrm{t}] + \frac{\partial \hat{V}}{\partial \mathbf{x}}\mathbf{f}$$ (3.35)

$$\hat{V}[\hat{\mathbf{x}}(t_{\mathrm{f}}), t_{\mathrm{f}}] = \varphi[\hat{\mathbf{x}}(t_{\mathrm{f}}), t_{\mathrm{f}}]$$ (3.36)

$\hat{\mathbf{x}}$ t $\mathbf{x}_0$

Уравнения Гамильтона-Якоби – Беллмана

$$-\frac{\partial \hat{V}}{\partial t} = H[\hat{\mathbf{x}}(t), \hat{\boldsymbol{\lambda}}(t), \hat{\mathbf{u}}(t), \mathrm{t}] \qquad (3.37)$$

$$\hat{\boldsymbol{\lambda}}^T = \frac{\partial \hat{V}}{\partial \mathbf{x}} \qquad (3.38)$$

$$\mathbf{u}(t) = \hat{\mathbf{u}}[t, \mathbf{x}(t)] \qquad (3.39)$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \qquad (3.40) \qquad \hat{V}[\hat{\mathbf{x}}(t), t]$$

$$\mathbf{x}_{\mathrm{e}} = \mathbf{0} \qquad \hat{V}[\hat{\mathbf{x}}(t), t] \qquad \mathbf{x} = \mathbf{0}$$

$$\hat{V}[\hat{\mathbf{x}}(t), t] \qquad V(\mathbf{x}, t)$$

$$V(\mathbf{0}, t) = 0 \qquad V(\mathbf{x}, t) > 0 \qquad \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{x}, t) < 0, \mathbf{x} \neq \mathbf{0} \qquad (3.41)$$

$$\|\mathbf{x}\| \to \infty, V(\mathbf{x}, t) \to \infty \qquad (3.42)$$

Линейные зависящие от времени системы с квадратичным показателем качества

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{3.43}$$

$$J = \mathbf{x}^T(t_f)\mathbf{Q}_f\mathbf{x}(t_f) + \int_{t_0}^{t_f}[\mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + 2\mathbf{x}^T(t)\mathbf{S}(t)\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t)]\mathrm{d}t \tag{3.44}$$

$$L[\mathbf{x}(t), \mathbf{u}(t), \mathrm{t}] = \mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + 2\mathbf{x}^T(t)\mathbf{S}(t)\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t) \tag{3.45}$$

$V[\mathbf{x}(t), t] \quad \mathbf{x}(t) \quad \mathbf{P}(t)$

$$V[\mathbf{x}(t), t] = \mathbf{x}^T(t)\mathbf{P}(t)\mathbf{x}(t) \tag{3.46}$$

$$\hat{V}[\hat{\mathbf{x}}(t_f), t_f] = \mathbf{x}^T(t_f)\mathbf{Q}_f\mathbf{x}(t_f) \tag{3.47}$$

$$\boldsymbol{\lambda}^T = \frac{\partial V}{\partial \mathbf{x}} = 2\mathbf{x}^T(t)\mathbf{P}(t) \tag{3.48}$$

$$\begin{aligned} H &= \mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + 2\mathbf{x}^T(t)\mathbf{S}(t)\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t) + \\ &+ 2\mathbf{x}^T(t)\mathbf{P}(t)[\mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)] \end{aligned} \tag{3.49}$$

$$H_{\mathbf{u}} = \mathbf{0}$$

$$\hat{\mathbf{x}}^T(t)\mathbf{S}(t) + \hat{\mathbf{u}}^T(t)\mathbf{R}(t) + \hat{\mathbf{x}}^T(t)\mathbf{P}(t)\mathbf{B}(t) = \mathbf{0} \qquad (3.50)$$

$$\hat{\mathbf{u}}(t) = -\mathbf{R}^{-1}(t)[\mathbf{B}^T(t)\hat{\mathbf{P}}(t) + \mathbf{S}^T(t)]\hat{\mathbf{x}}(t) \qquad (3.51)$$

$\mathbf{P}(t)$ $V[\mathbf{x}(t), t] > 0$ $\mathbf{x}(t)$

$$\hat{H} = -\frac{\partial \hat{V}}{\partial t} = -\hat{\mathbf{x}}^T \dot{\hat{\mathbf{P}}} \hat{\mathbf{x}} \qquad (3.52)$$

$H_{\mathbf{uu}} > \mathbf{0}$ $\dot{\hat{\mathbf{P}}}(t)$ $\dot{V}[\mathbf{x}(t), t] < 0, \mathbf{x}(t) \in E$

$\hat{\mathbf{P}}(t)$ $\mathbf{R}(t)$

$$\hat{\mathbf{x}}^T \dot{\hat{\mathbf{P}}} \hat{\mathbf{x}} = -\hat{\mathbf{x}}^T [(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T)^T \hat{\mathbf{P}} + \hat{\mathbf{P}}(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T) -$$
$$- \hat{\mathbf{P}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \hat{\mathbf{P}} + \mathbf{Q} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T]\hat{\mathbf{x}} \tag{3.53}$$

$$-\dot{\hat{\mathbf{P}}} = \mathbf{Q} + (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T)^T \hat{\mathbf{P}} + \hat{\mathbf{P}}(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{S}^T) - \hat{\mathbf{P}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \hat{\mathbf{P}} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T \tag{3.54}$$

$$\hat{\mathbf{P}}_{\mathrm{f}} = \mathbf{Q}_{\mathrm{f}} \tag{3.55}$$

$\hat{\mathbf{P}}(t)$ $\mathbf{Q}_{\mathrm{f}}, \mathbf{Q}(t), \mathbf{R}(t)$

Лекция

Метод АКОР

$$J(u) = \frac{1}{2} x^T(t_k)\Phi_k x(t_k) + \frac{1}{2}\int_{t_0}^{t_k}\left[x^T(t)\Phi x(t) + u^T(t)\Psi u(t)\right]dt$$

$$x^T(t)\Phi x(t) \qquad u^T(t)\Psi u(t) \qquad \frac{1}{2} x^T(t_k)\Phi_k x(t_k)$$

$$u_{onm}(t) = \Psi^{-1}B^T(t)K(t)x(t) = -D(t)x(t); D(t) = \Psi^{-1}B^T(t)K(t)$$

$$u_j = \sum_{i=1}^{n} d_{ij} x_i$$

$$\dot{K}(t) = -K(t)A(t) - A^T(t)K(t) + K(t)B(t)\Psi^{-1}B^T(t)K(t) - \Phi$$

$$\dot{x}(t) = \left[A(t) - B(t)D(t)\right]x(t) = A_o(t)x(t);\ A_o(t) = A(t) - B(t)D(t)$$