Тригонометрические уравнения 2

Иррациональные уравнения

Уравнения с модулем

Системы уравнений

Неравенства

Основные формулы тригонометрии

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$1 + \mathrm{tg}^2\, x = \frac{1}{\cos^2 x}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

http://yourtutor.info/

Начнем с… систем (nm)

$$\begin{cases} \sin \pi x = 0, \\ \sin \pi y = 0. \end{cases}$$

$$\begin{cases} x = k \\ y = n \end{cases}$$

Ясно видно

$$\begin{cases} \sin x + \cos y = 1, \\ \cos 2x - \cos 2y = 1. \end{cases}$$

$$\begin{cases} \cos x \cos y = \dfrac{3}{4}, \\ \sin x \sin y = -\dfrac{1}{4}. \end{cases}$$

$$\begin{cases} \cos(x) * \cos(y) = 3/4 \\ \sin(x) * \sin(y) = -1/4 \end{cases}$$

$$\begin{cases} \cos(x) * \cos(y) - \sin(x) * \sin(y) = 1 \\ \cos(x) * \cos(y) + \sin(x) * \sin(y) = 1/2 \end{cases}$$

$$\begin{cases} \cos(x+y) = 1 \\ \cos(x-y) = 1/2 \end{cases}$$

$$\begin{cases} x + y = 2\pi n \\ x - y = \pm\dfrac{\pi}{3} + 2\pi k \end{cases}$$

$$\begin{cases} x = \pm\dfrac{\pi}{6} + \pi(\mathrm{n}+\mathrm{k}) \\ y = \pm\dfrac{\pi}{6} + \pi(\mathrm{n}-\mathrm{k}) \end{cases}$$

$$\begin{cases} x = \pm\dfrac{\pi}{6} + \pi p \\ y = \pm\dfrac{\pi}{6} + \pi q \end{cases}$$

$$\begin{cases} \sin(x) + \cos(\mathrm{y}) = 1 \\ \cos(2\,\mathrm{x}) - \cos(2\,\mathrm{y}) = 1 \end{cases}$$

$$\begin{cases} \sin(x) + \cos(\mathrm{y}) = 1 \\ 1 - 2\sin^2(x) - 2\cos^2(\mathrm{y}) + 1 = 1 \end{cases}$$

$$\begin{cases} \sin(x) + \cos(\mathrm{y}) = 1 \\ 2\sin^2(x) - 2\cos^2(\mathrm{y}) = -1 \end{cases}$$

$$\sin(x) = a, \cos(y) = b$$

$$\begin{cases} a + b = 1 \\ 2a - 2b = -1 \end{cases}$$

$$\sin(x) + \cos(\mathrm{y}) = \cos(2\,\mathrm{x}) - \cos(2\,\mathrm{y}) = 2\sin(\mathrm{x}+\mathrm{y}) * \sin(\mathrm{y}-\mathrm{x})$$

Сведение к тригонометрической единице

$$\begin{cases} \sin y = 5\sin x, \\ 3\cos x + \cos y = 2. \end{cases}$$

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$$\begin{cases} \sin(y) = 5\sin(x) \\ (5*\sin(\mathrm{x}))^2 + (2 - 3\cos(\mathrm{x}))^2 = 2 \end{cases}$$

$$25*(1-\mathrm{t}^2) + (2-3t)^2 = 2$$

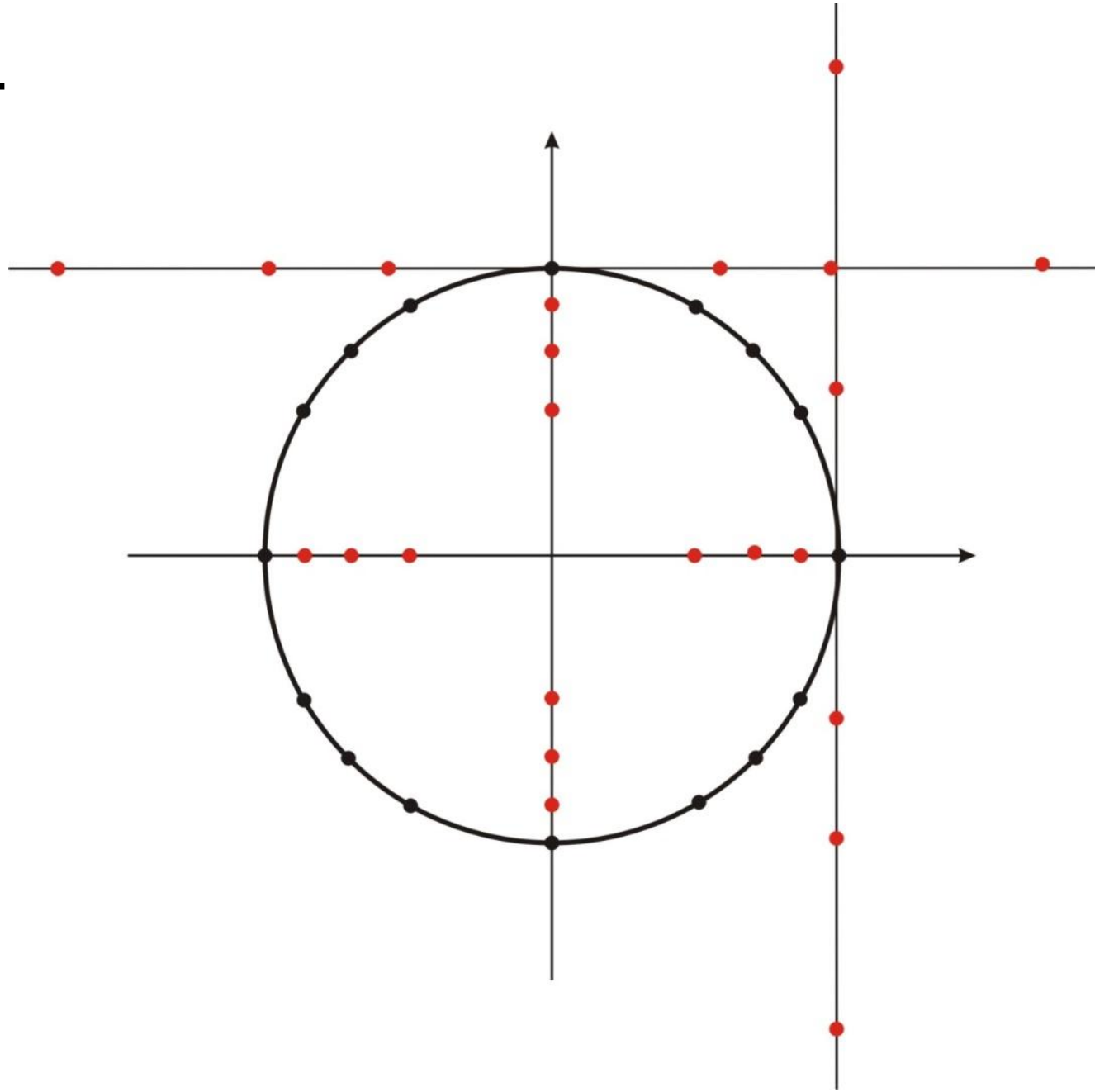
$$25 - 25\,\mathrm{t}^2 + 4 - 12t + 9\,\mathrm{t}^2 = 2$$

$$16\,\mathrm{t}^2 + 12t - 27 = 0$$

Неравенства и круг

$$\sin x < -\frac{1}{2}$$

$$\operatorname{tg} x \le 2.$$



$$\sqrt[4]{\frac{7-\cos 4x}{2}} > -2\cos x.$$

$$\left[\begin{array}{l}\begin{cases}-2\cos(x) > 0\\ \dfrac{7-\cos(4\,x)}{2} > (2\cos(x))^4\end{cases}\\ \begin{cases}-2\cos(x) < 0\\ \dfrac{7-\cos(4\,x)}{2} > 0\end{cases}\end{array}\right.$$

$$\left[\begin{array}{l}\begin{cases}-2\cos(x) > 0\\ \dfrac{7-\cos(4\,x)}{2} > (2\cos(x))^4\end{cases}\\ -2\cos(x) < 0\end{array}\right.$$

$$\frac{7-(2\cos^2(2\,x)-1)}{2} > (2\cos(x))^4$$

$$\frac{7-(2(2\cos^2(x)-1)^2-1)}{2} > (2\cos(x))^4$$

$$\cos x = t$$

$$\frac{7-(2(2t^2-1)^2-1)}{2} > (2t)^4$$

$$7-(8\,t^4-8t^2+1) > 32\,t^4$$

$$0 > 40\,t^4-8t^2-6$$

$$\begin{cases}-1 < t < 1\\ \left[\begin{array}{l}\begin{cases}-2t > 0\\ 0 > 40\,t^4-8t^2-6\end{cases}\\ -2t < 0\end{array}\right.\end{cases}$$

Проверка почему? За что?

$$\operatorname{arctg} 3x = \arccos 8x.$$

$$\cos(x)$$

$$3\,\mathrm{x} = \frac{\sin(\arccos(8\,\mathrm{x}))}{8x}$$

$$3\,\mathrm{x} = \frac{\sqrt{1-64x^2}}{8x}$$

$$\sin() = \pm\sqrt{1-\cos^2()}\,?$$

$$\left(24x^2\right)^2 = 1-64x^2$$

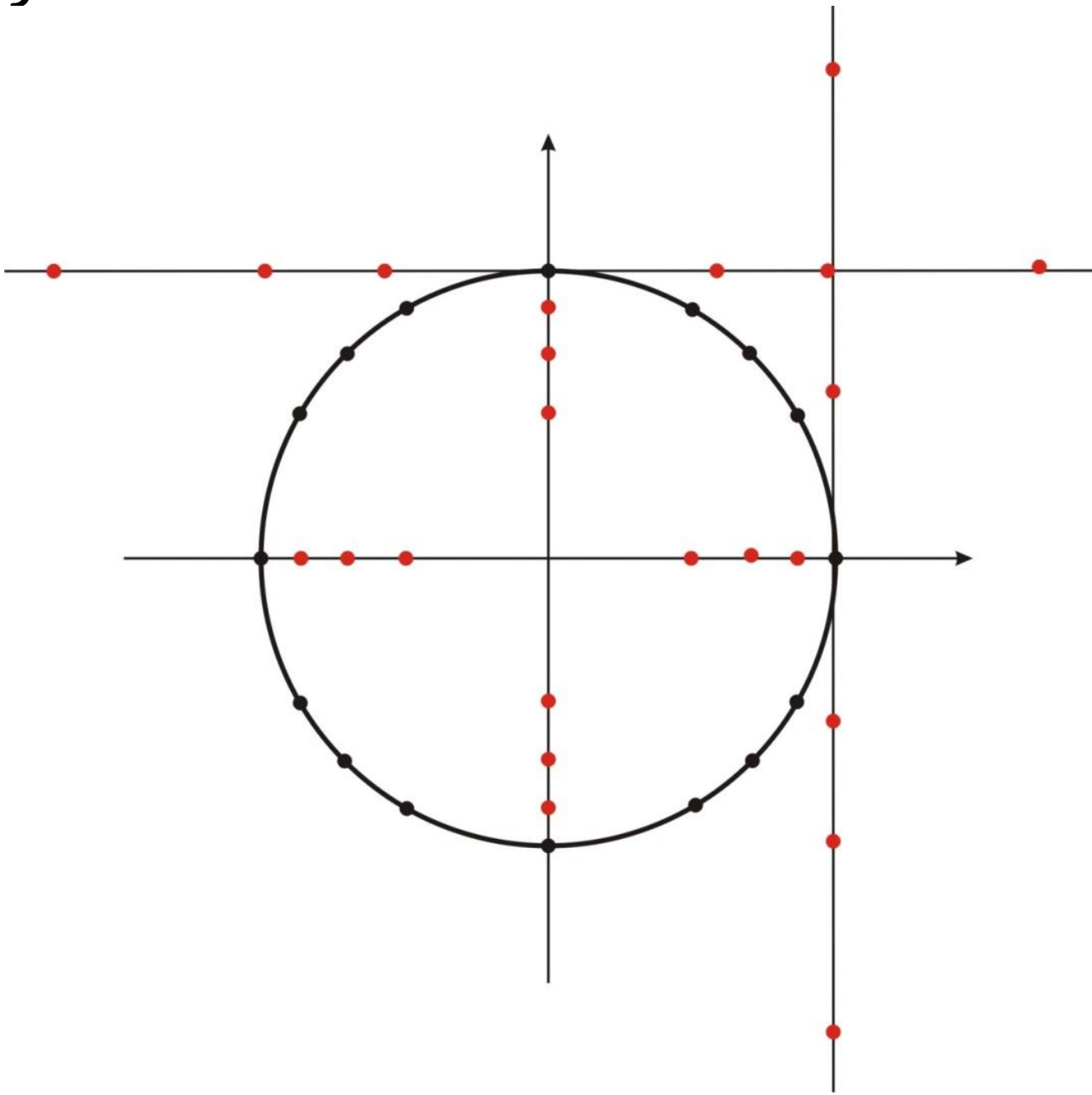
$$9(8\,\mathrm{x}^2)^2 + 8*8x^2 - 1 = 0$$

$$9t^2 + 8t - 1 = 0$$

$$\left[\begin{array}{l} \text{не подходит} \\ t = \dfrac{1}{9} \end{array}\right.$$

$$\frac{1}{9} = 8\,\mathrm{x}^2$$

$$\left[\begin{array}{l} x = \dfrac{1}{6\sqrt{2}} \\ x = \dfrac{-1}{6\sqrt{2}} \end{array}\right.$$



На счет ОДЗ…

Пример 29. При каких значениях параметра a, уравнение $(x-a)\arccos(x+3)=0$ имеет единственное решение?

$$\left[\begin{array}{l} x-a=0 \\ \arccos(x+3)=0 \end{array}\right.$$

$$\left[\begin{array}{l} x=a \\ x=-2 \end{array}\right.$$

1*сл*

~~*одно*~~ 2 *решение*

2*сл*

$a \neq -2$

$-4 <= a < -2$

2 *корня*

$x = 2$

$x = a$

~~*неуд*~~ *ОДЗ*

~~*корень*~~ 2

ОДЗ

$-1 <= x+3 <= 1$

$-4 <= x <= -2$

КВ12

$$\sin(x)+\cos(x)=\cos(2x)*(1-2*\sin(2x))$$

$$\sin(x)+\cos(x)=(\cos^2 x-\sin^2 x)*(1-4*\sin(x)*\cos(x))$$

$$(\sin(x)+\cos(x))*\{(\cos(x)-\sin(x))*[1-4*\sin(x)*\cos(x)]-1\}=0$$

$$\left[\begin{array}{l}\sin(x)+\cos(x)=0\\(\cos(x)-\sin(x))*(1-4*\sin(x)*\cos(x))-1=0\end{array}\right.$$

Формула универсальной подстановки

$$t = tg(\mathrm{x}/\,2)$$

$$\cos(\mathrm{x}) = \frac{1-t^2}{1+t^2}, \sin(\mathrm{x}) = \frac{2t}{1+t^2}$$

$$(\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2}) * (1 - 4 * \frac{1-t^2}{1+t^2} * \frac{2t}{1+t^2}) - 1 = \frac{1-2t-t^2}{1+t^2} * \frac{(1+\mathrm{t}^2) - 8\,\mathrm{t}(1-t^2)}{(1+t^2)^2} - \frac{(1+t^2)^3}{(1+t^2)^3} =$$

$$= -\frac{2\ \mathrm{t}\ (1+\mathrm{t})\ (5+\mathrm{t}\ (-12+\mathrm{t}\ (6+\mathrm{t}\ (4+\mathrm{t})))}{(1+t^2)^3}$$

(вообщерешения мож но заметить выше, остаток показать, что решений не *имеет*)