$$
\begin{aligned}
(\Delta x)^2(\Delta s)^2 &= \frac{\int x^2 f f^* \, dx \int s^2 F F^* \, ds}{\int f f^* \, dx \int F F^* \, ds} \\
&= \frac{\int x f . x f^* \, dx \int f' f'^* \, dx}{4\pi^2 \left( \int f f^* \, dx \right)^2} \\
&\geq \frac{\left| \int (x f^* . f' + x f . f'^*) \, dx \right|^2}{16\pi^2 \left( \int f f^* \, dx \right)^2} \\
&= \frac{\left| \int x \frac{d}{dx} (f f^*) \, dx \right|^2}{16\pi^2 \left( \int f f^* \, dx \right)^2} \\
&= \frac{\left| \int f f^* \, dx \right|^2}{16\pi^2 \left( \int f f^* \, dx \right)^2} \\
&= \frac{1}{16\pi^2}.
\end{aligned}
$$

# Фильтры Габора

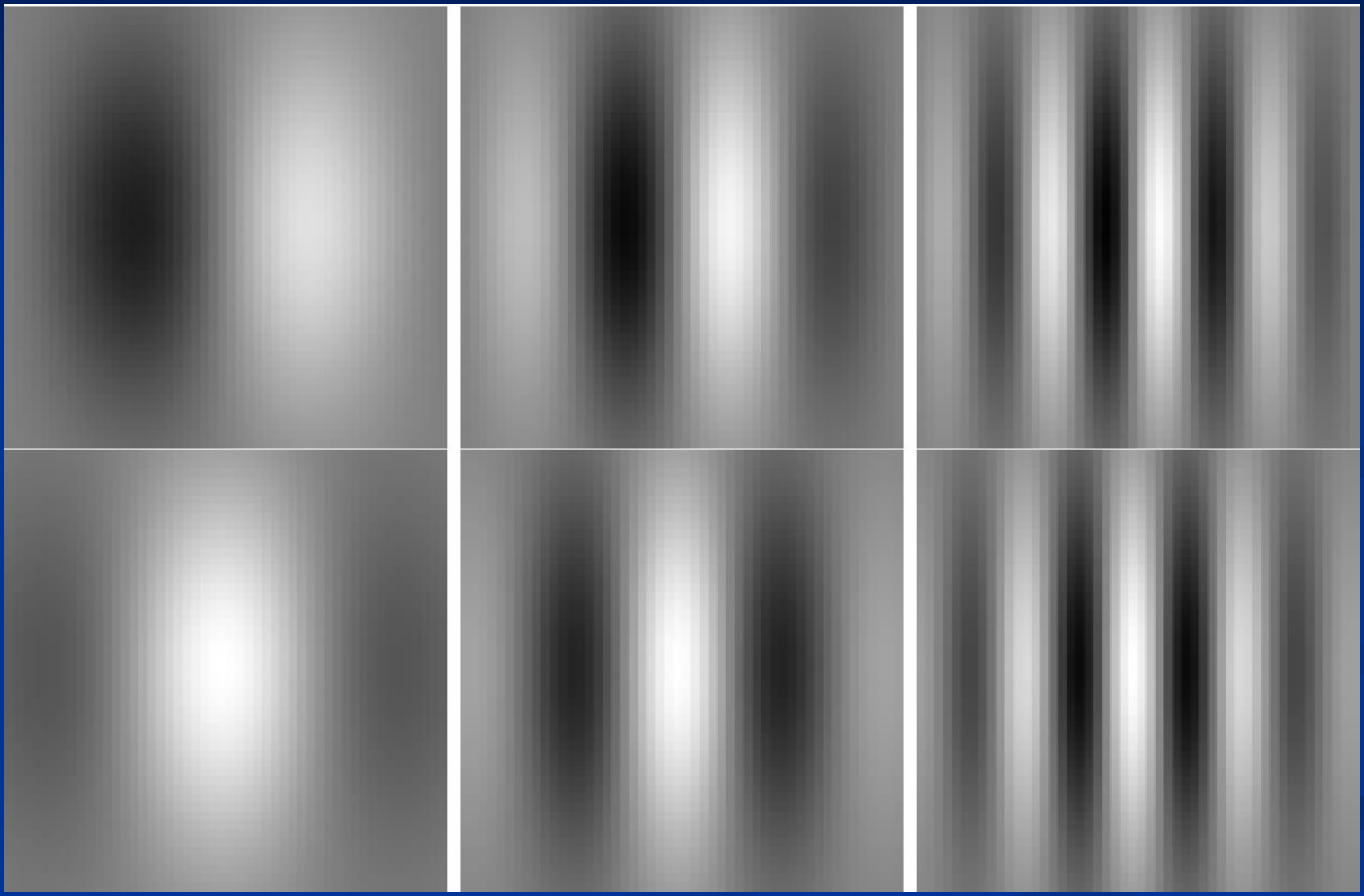
Gabor filters are formed by modulating a complex sinusoid by a Gaussian function:

$$g(x,y) = \overbrace{\frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{\tilde{x}^2}{\sigma_x^2}+\frac{\tilde{y}^2}{\sigma_y^2}\right)\right)}^{\text{gaussian envelope}} \cdot \overbrace{\exp\left(2\pi j\omega\tilde{x}\right)}^{\text{complex sinusoidal}}$$

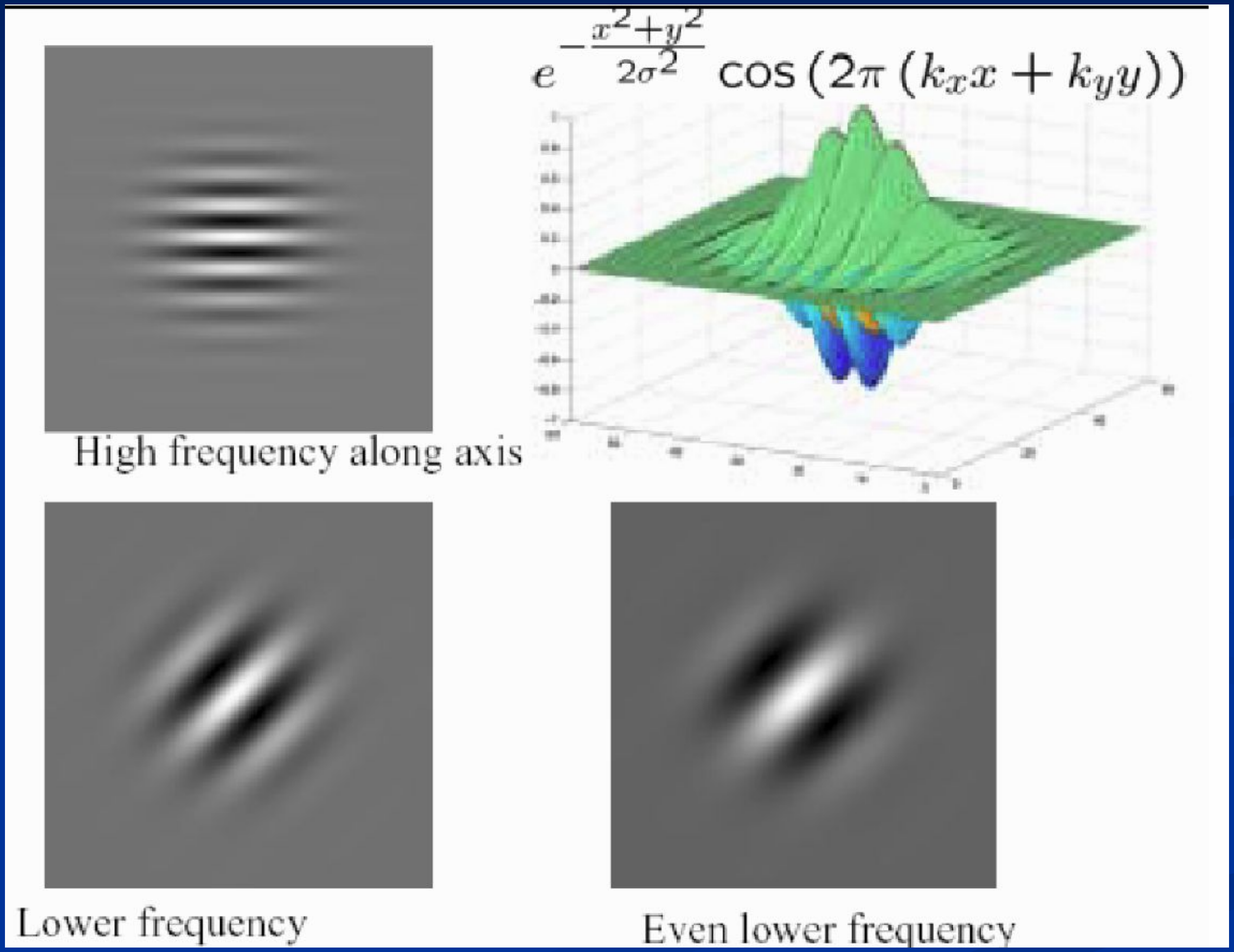
$$\text{with} \quad \begin{cases} \tilde{x} = x\cos(\theta) + y\sin(\theta) \\ \tilde{y} = -x\sin(\theta) + y\cos(\theta) \end{cases}$$

- $\sigma_x$ and $\sigma_y$ control spatial extent of filter
- $\theta$ is the orientation
- $\omega$ is the radial frequency of the sinusoid

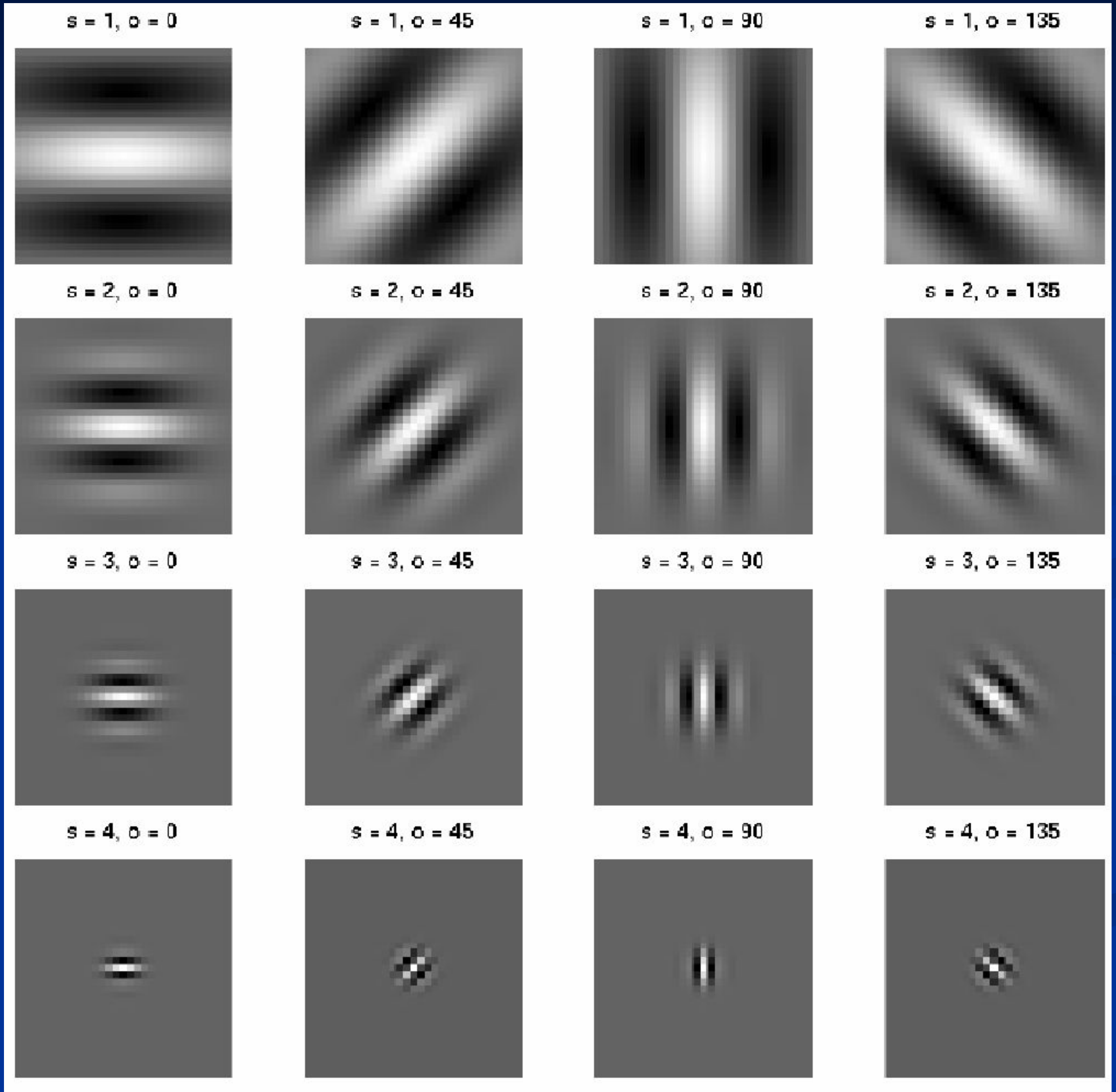
# Фильтры Габора



# Фильтры Габора



# Фильтры Габора



# Функции Габора

$$g(x, y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right) + 2\pi jWx\right],$$

$$G(u, v) = \exp\left\{-\frac{1}{2}\left[\frac{(u - W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right]\right\},$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

# Габоровские вейвлеты

$$g_{mn}(x, y) = a^{-m} g(x', y'), \quad a > 1, \quad m, n = \text{integer}$$

$$x' = a^{-m}(x \cos \theta + y \sin \theta) \quad \text{and}$$

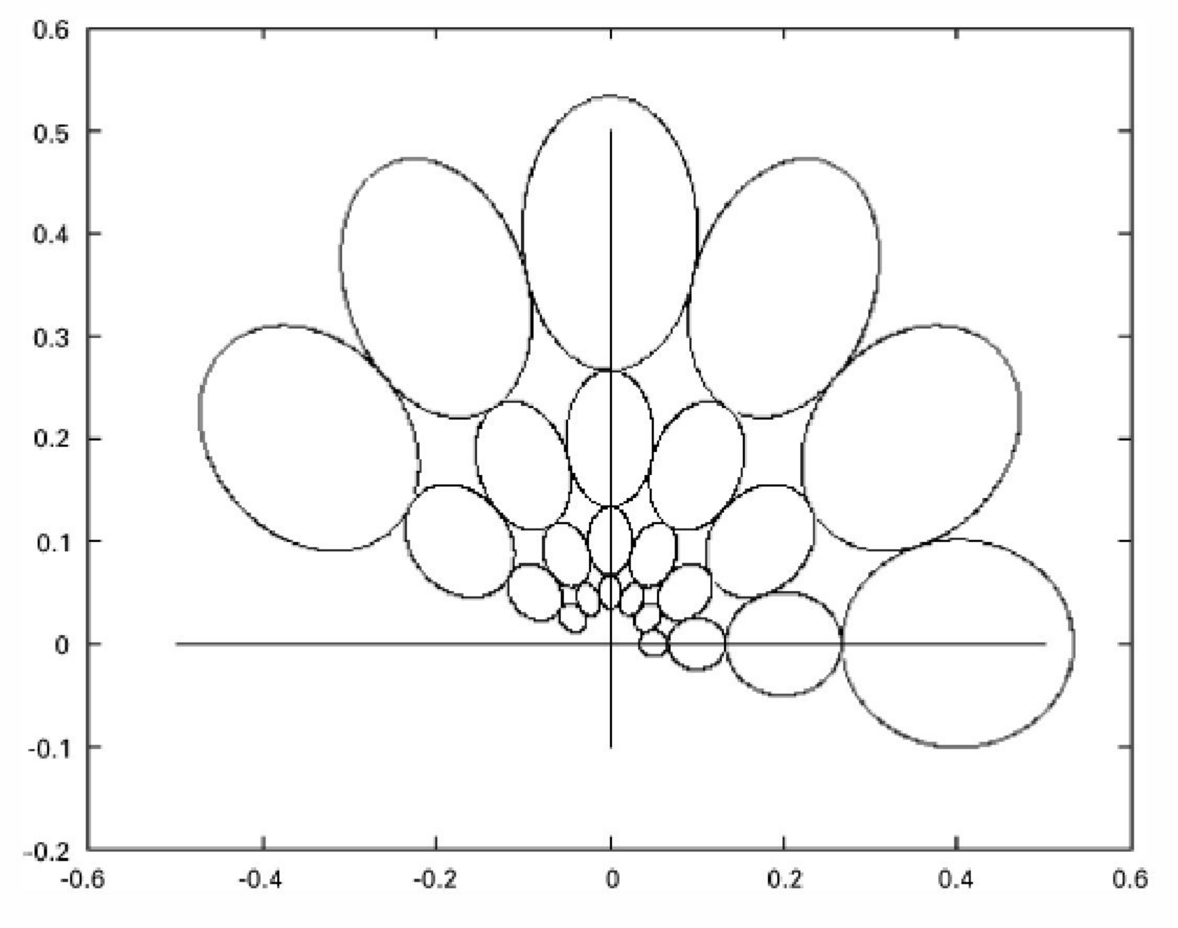
$$y' = a^{-m}(-x \sin \theta + y \cos \theta),$$

$$\theta = n\pi/K$$

$K$ – number of orientations

# Габоровские вейвлеты

$$W_{mn}(x, y) = \int I(x, y) g^{*}_{mn}(x - x_1, y - y_1)\, \mathrm{d}x_1\, \mathrm{d}y_1$$




$U_{\mathrm{h}} = 0.04,\ U_{1} = 0.05,\ K = 6$ and $S = 4$

# Габоровские вейвлеты

$$a = (U_{\mathbf{h}}/U_{\mathbf{l}})^{1/(S-1)}, \quad \sigma_u = \frac{(a-1)U_{\mathbf{h}}}{(a+1)\sqrt{2\ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right)\left[U_{\mathbf{h}} - 2\ln 2\left(\frac{\sigma_u^2}{U_{\mathbf{h}}}\right)\right] \times \left[2\ln 2 - \frac{(2\ln 2)^2\sigma_u^2}{U_{\mathbf{h}}^2}\right]^{-1/2}$$

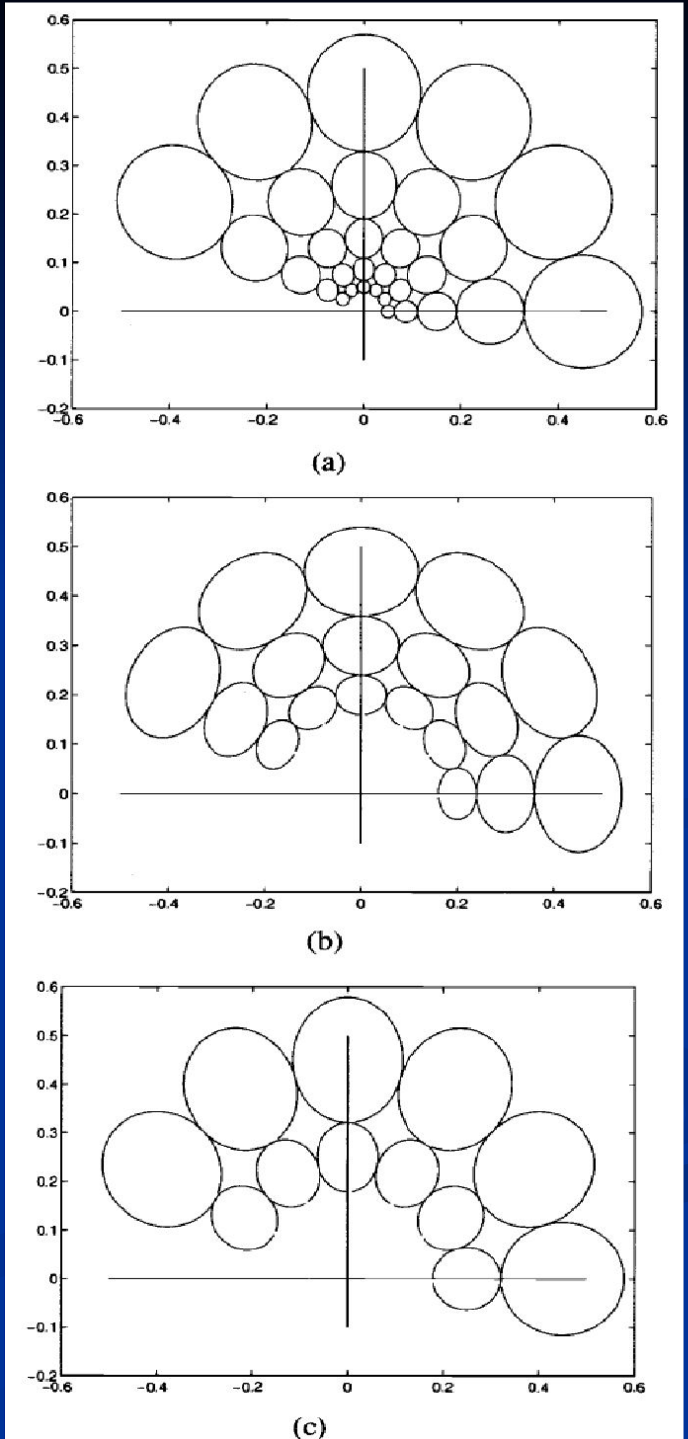
$W = U_{\mathbf{h}}$ and $m = 0, 1, \ldots, S-1$

K=6

$a)\quad \sigma = 5, S = 5$

$b)\quad \sigma = 1.25, S = 3$

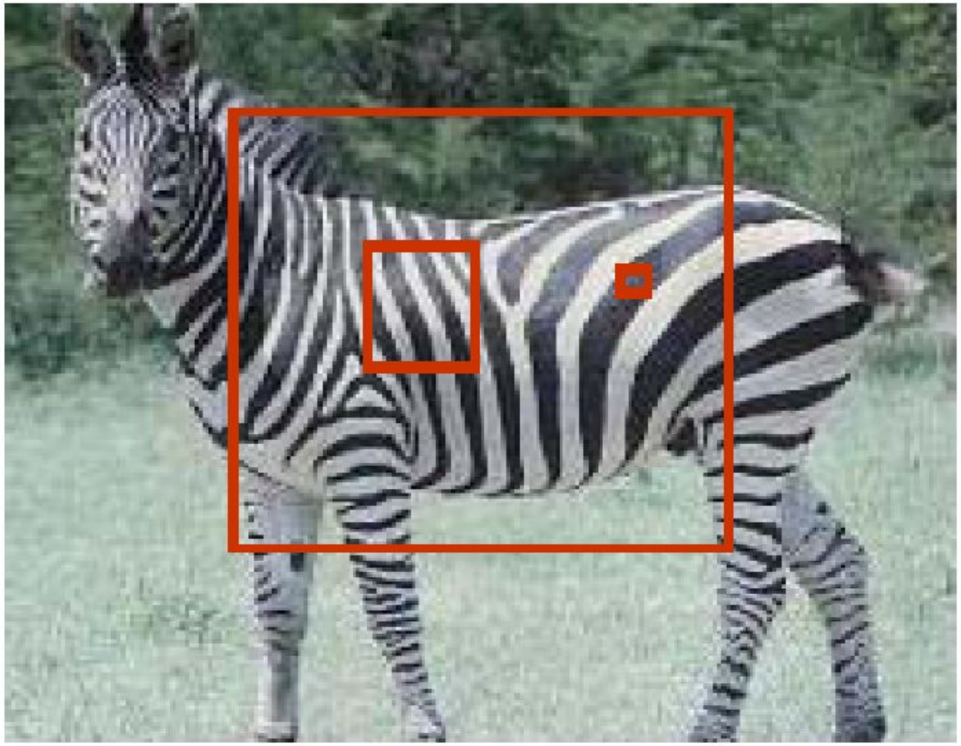
$c)\quad \sigma = 1, S = 2$



# Текстуры

# Текстуры

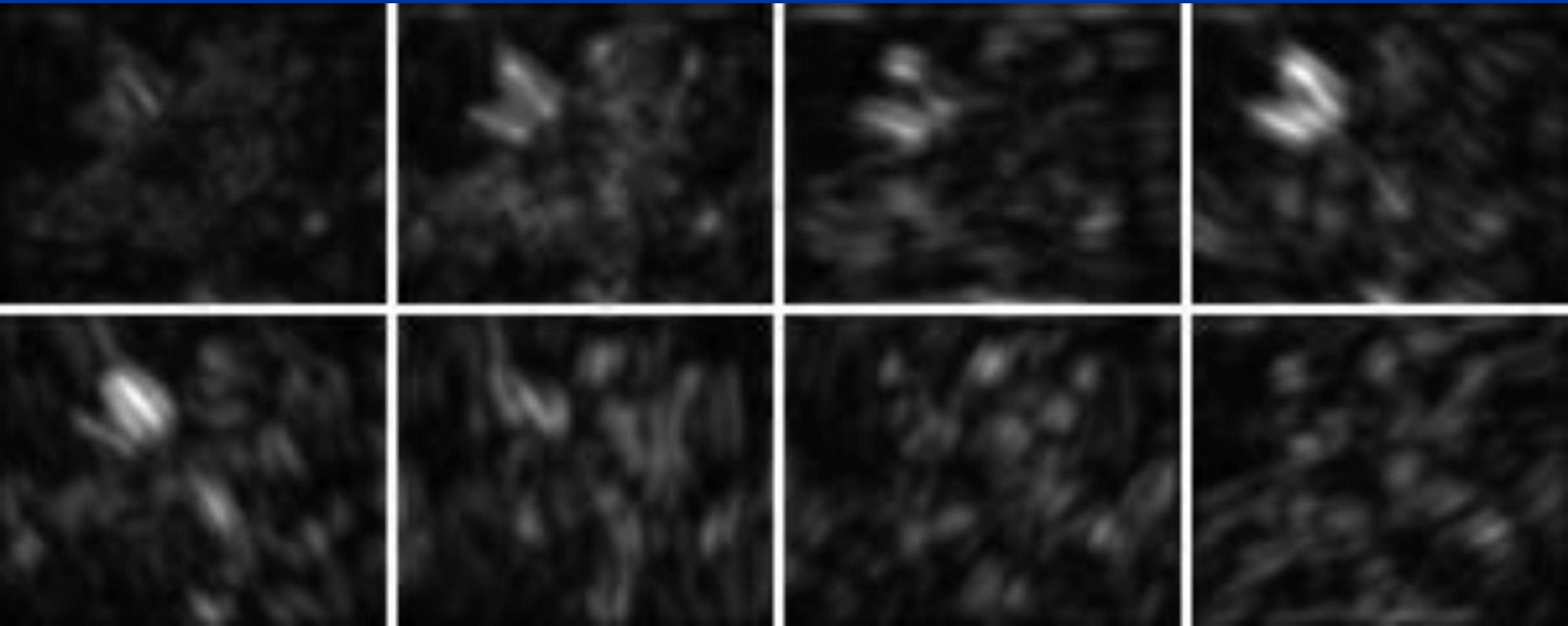
- Whether an effect is a texture or not depends on the scale at which it is viewed.



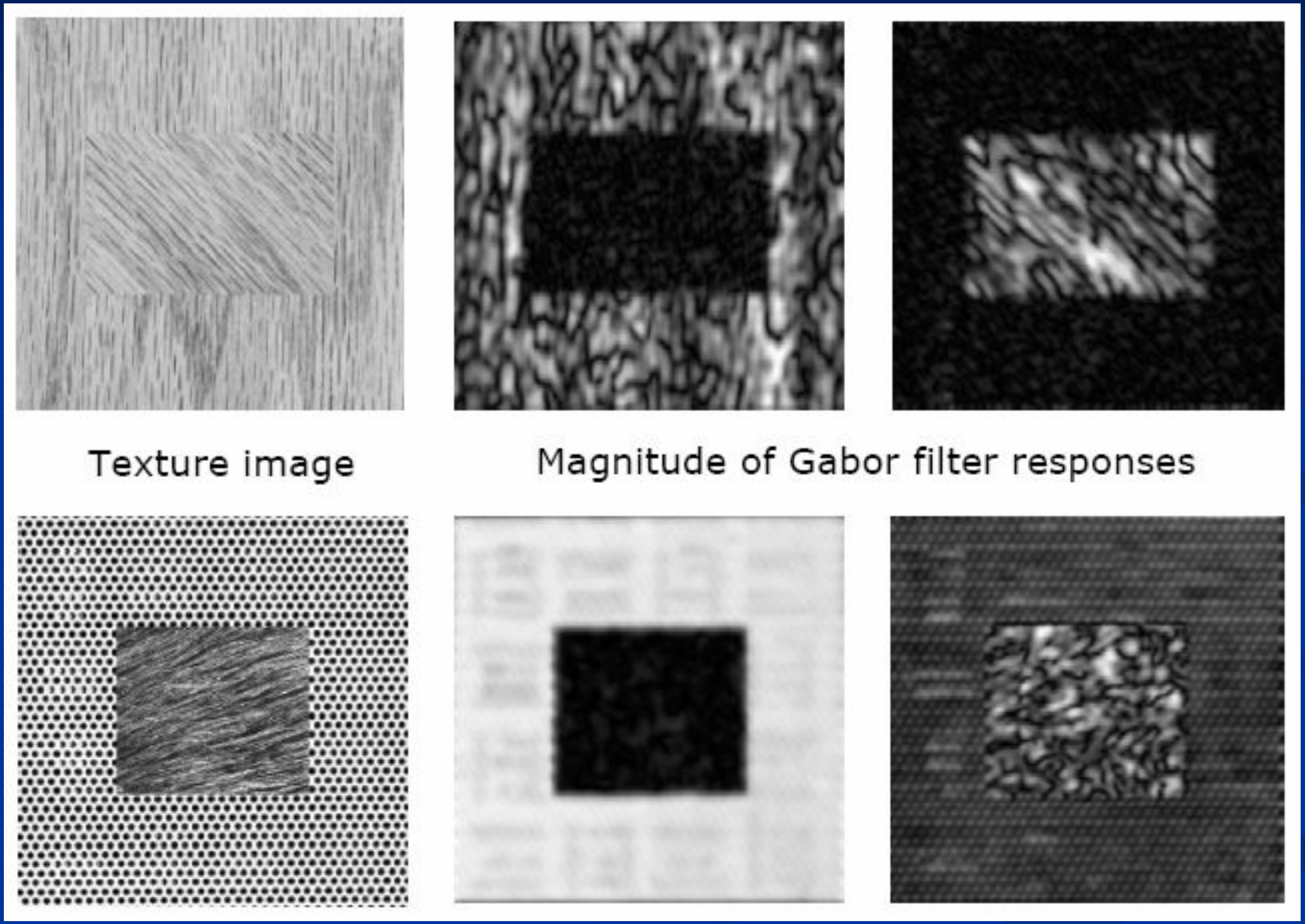
# Текстуры

# Банки фильтров

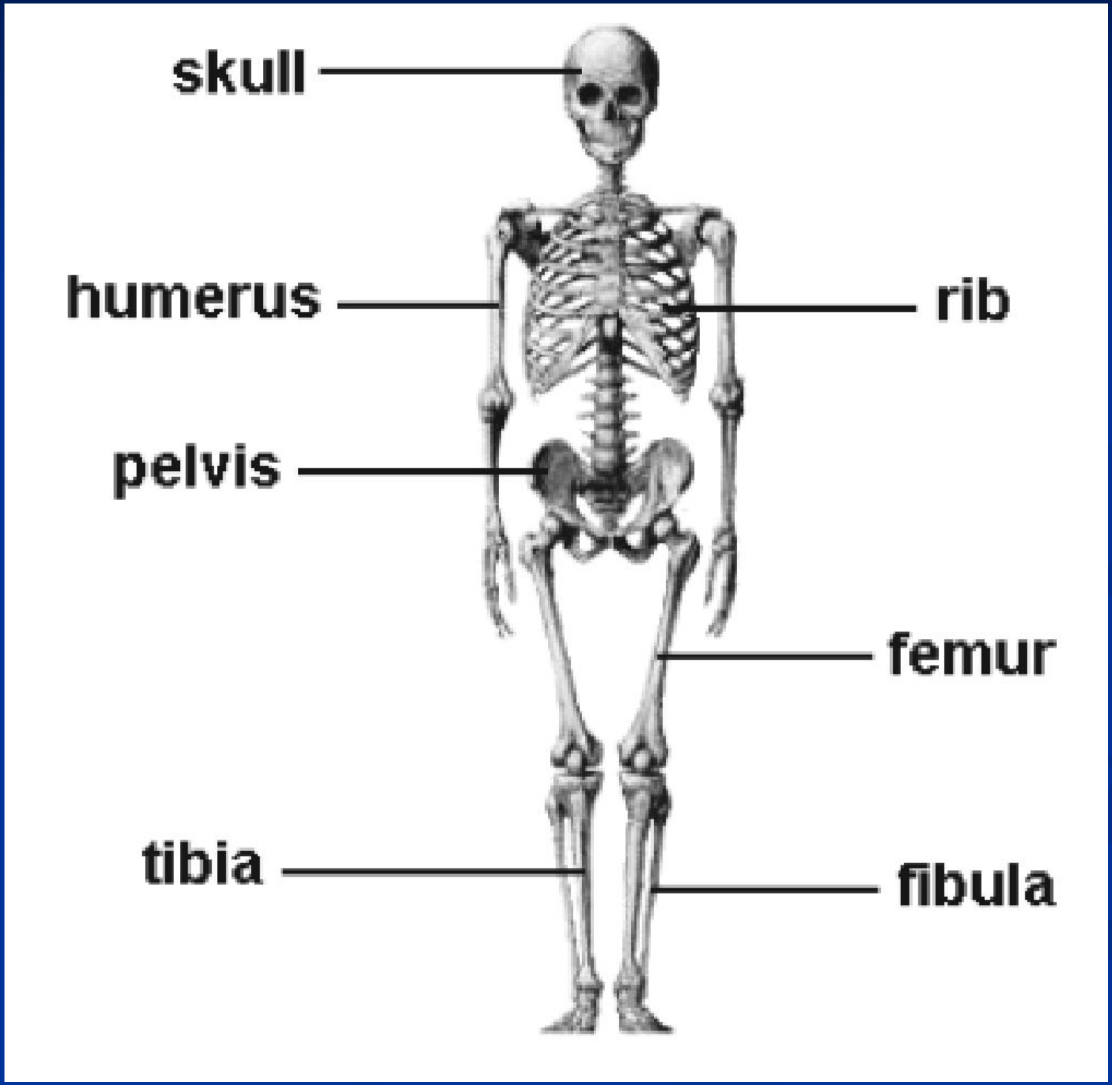
# Банки фильтров



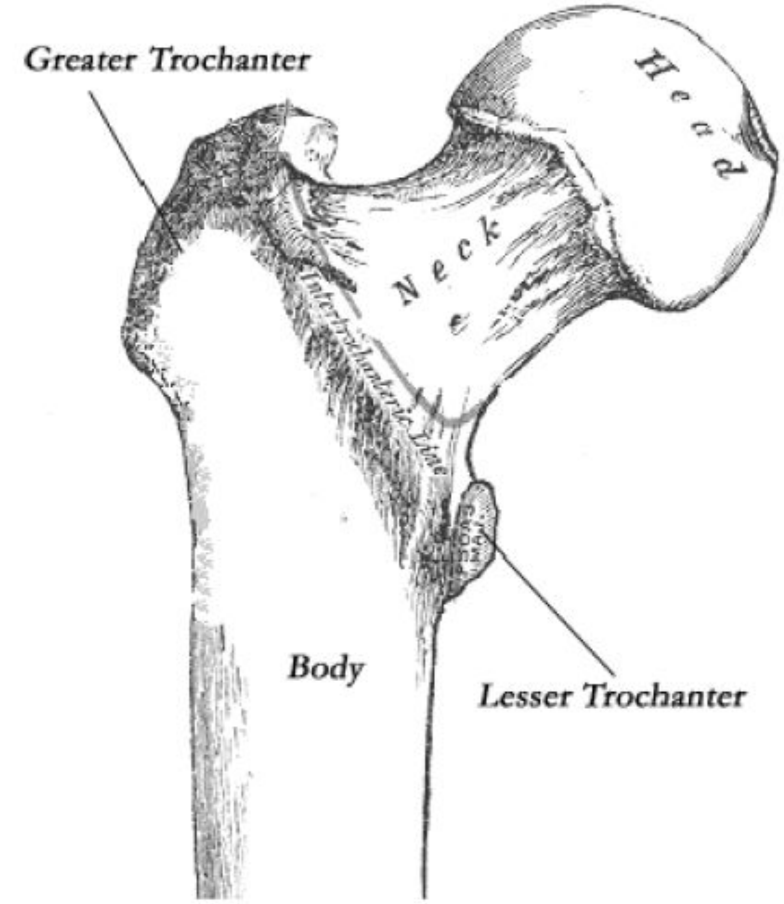
# Фильтры Габора

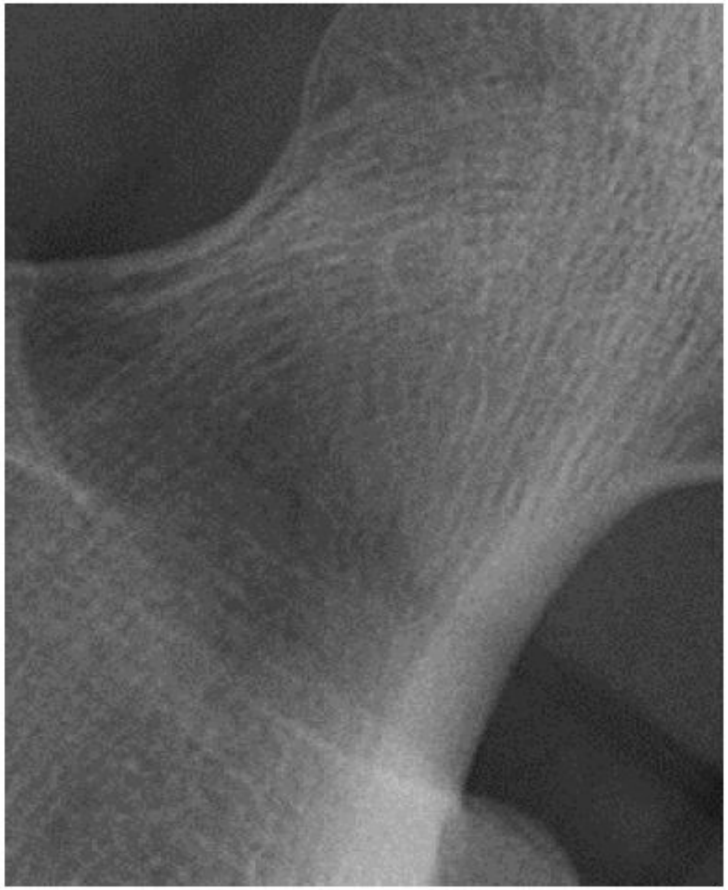


# Анализ бедренной кости (femur)



# Шейка бедра



# Шейка бедра

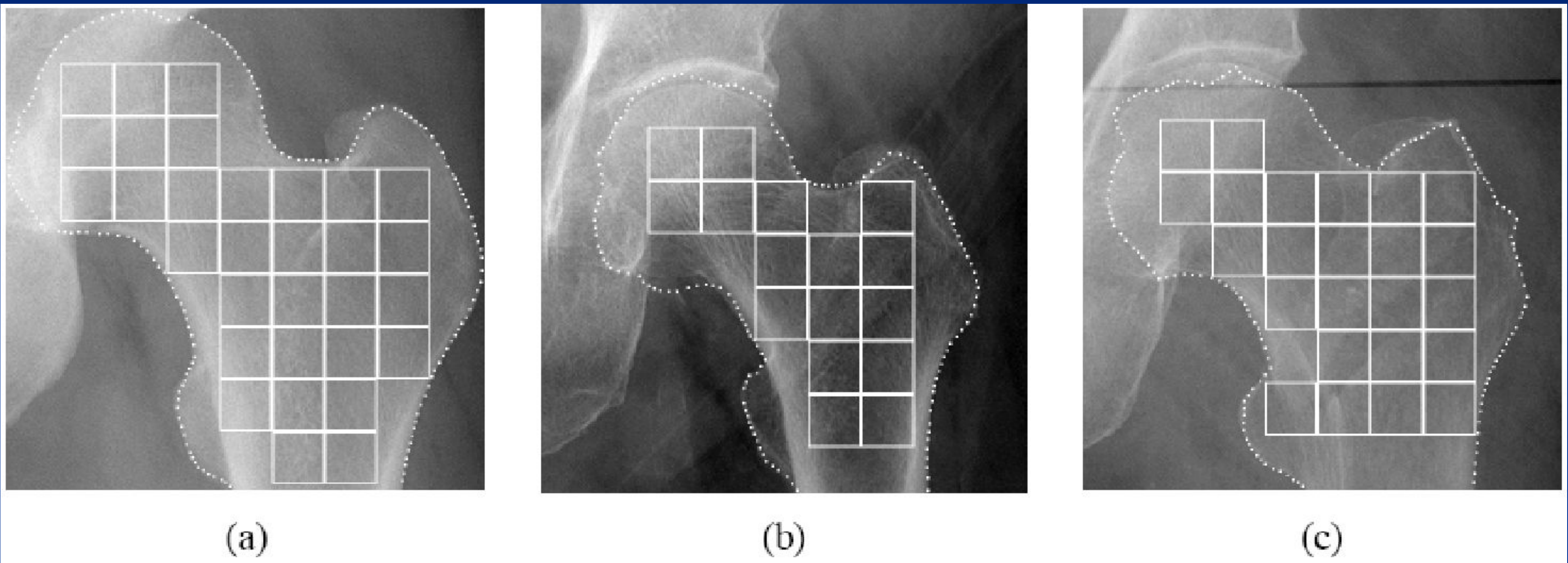



**Figure 3.3.** A femur is a natural structure that exhibits variations. (a) and (b) above are healthy, while (c) is fractured. Here, (a) is larger than (b). (c) has a relatively shorter neck, due to the fracture. Each grid square is a region sampled for texture orientation feature extraction

$$h(x, y) = g(x', y')\exp(2\pi j f x') .$$

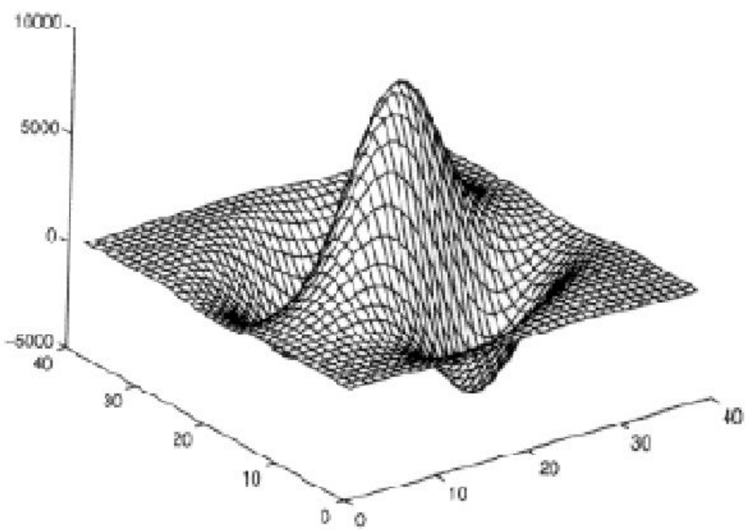
The oriented Gaussian function $g(x', y')$ is given by:

$$g(x', y') = \frac{1}{2\pi\lambda\sigma^2}\exp\left[-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right],$$

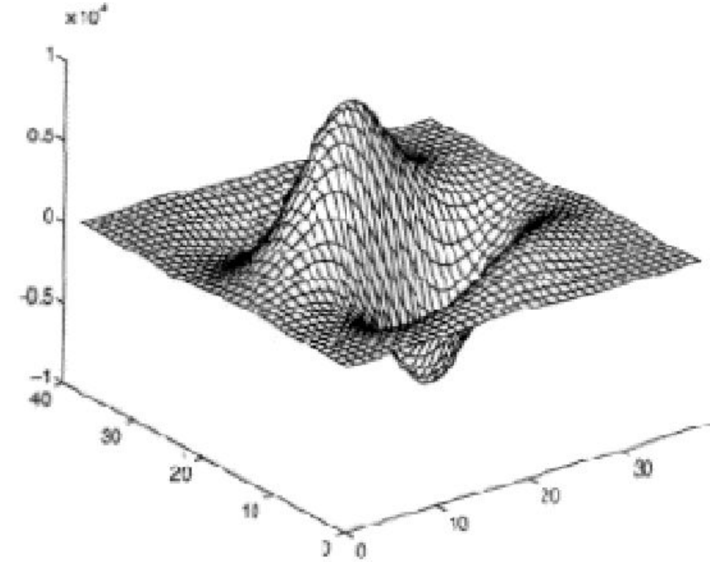
where $(x', y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ are rotated coordinates

$$h_{c,f\theta}(x, y) = g(x', y')\cos(2\pi f x') ,$$

$$h_{s,f\theta}(x, y) = g(x', y')\sin(2\pi f x') .$$

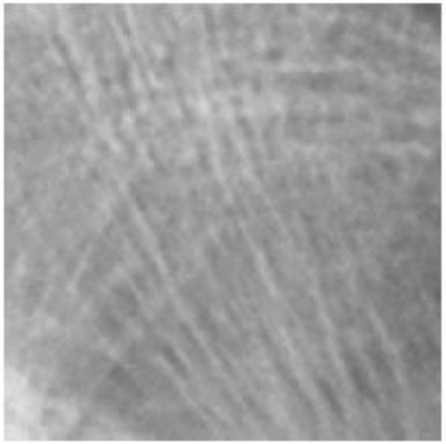


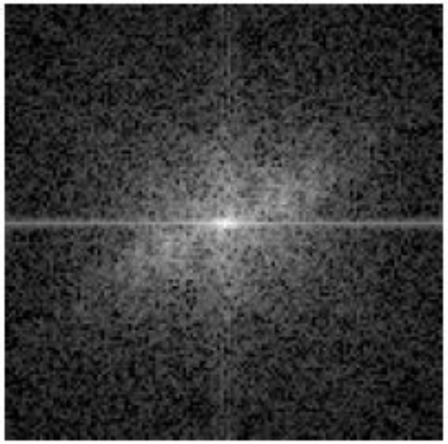

(a)

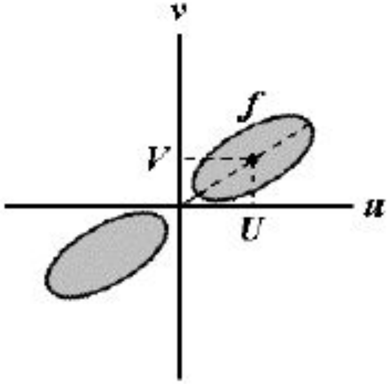



(b)

# Фурье анализ текстуры кости

(a) 

(b) 

(c) 


$$f_p = \sqrt{\overline{U}^2 + \overline{V}^2}$$

$f_p \cong$ 0.13 cycles per pixel

A Gabor filter bank of 1 frequency channel and 8 orientation channels is used to extract the orientation of the texture patterns in the femur. The centre frequency *f* of the Gabor filters is set to 0.13 cycles per pixel, and orientations range from 0° to 157.5°, incrementing in steps of 22.5°.

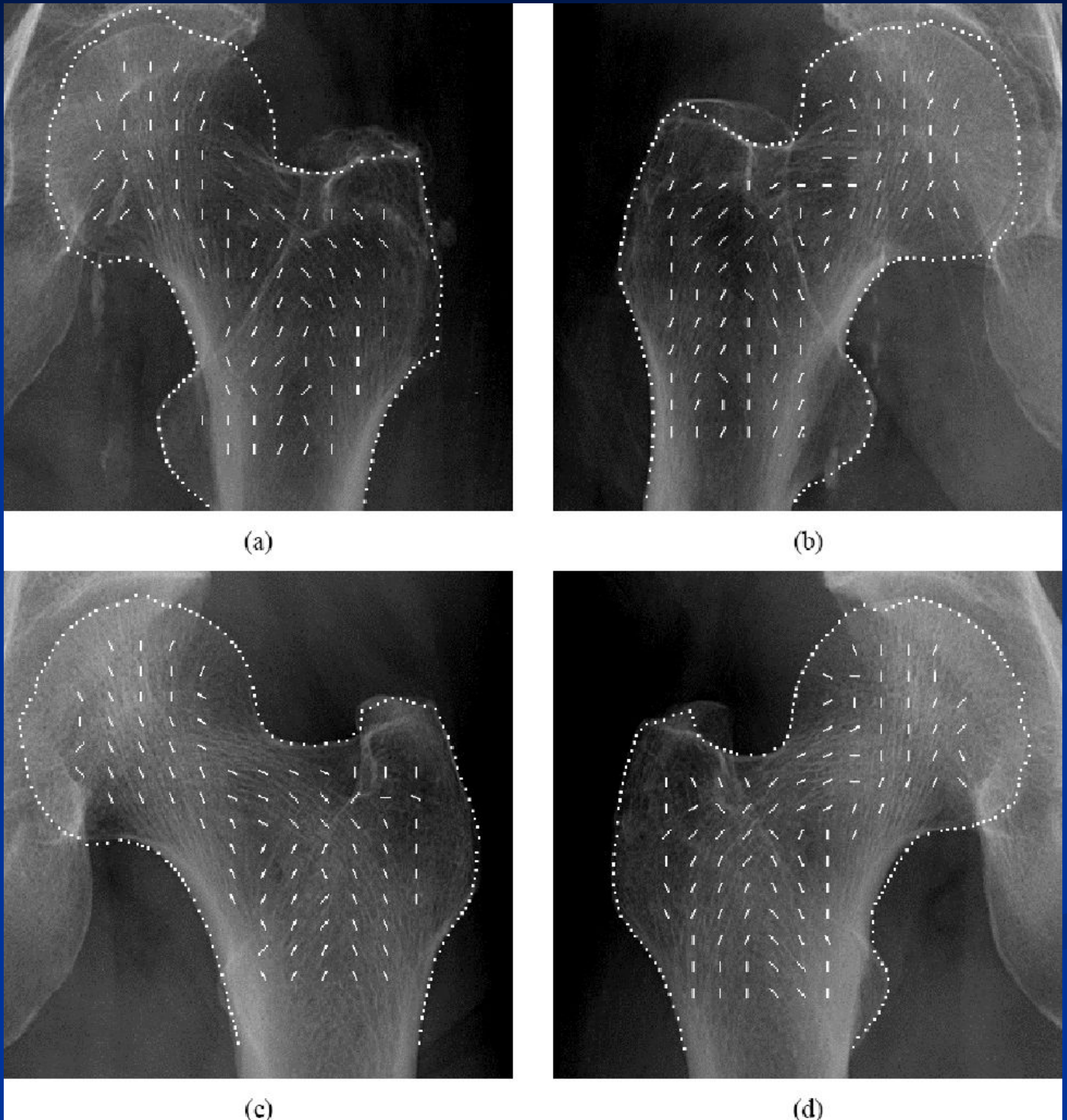
$$e_{c,f\theta}(x,y) = I(x,y) * h_{c,f\theta},$$

$$e_{s,f\theta}(x,y) = I(x,y) * h_{s,f\theta}.$$

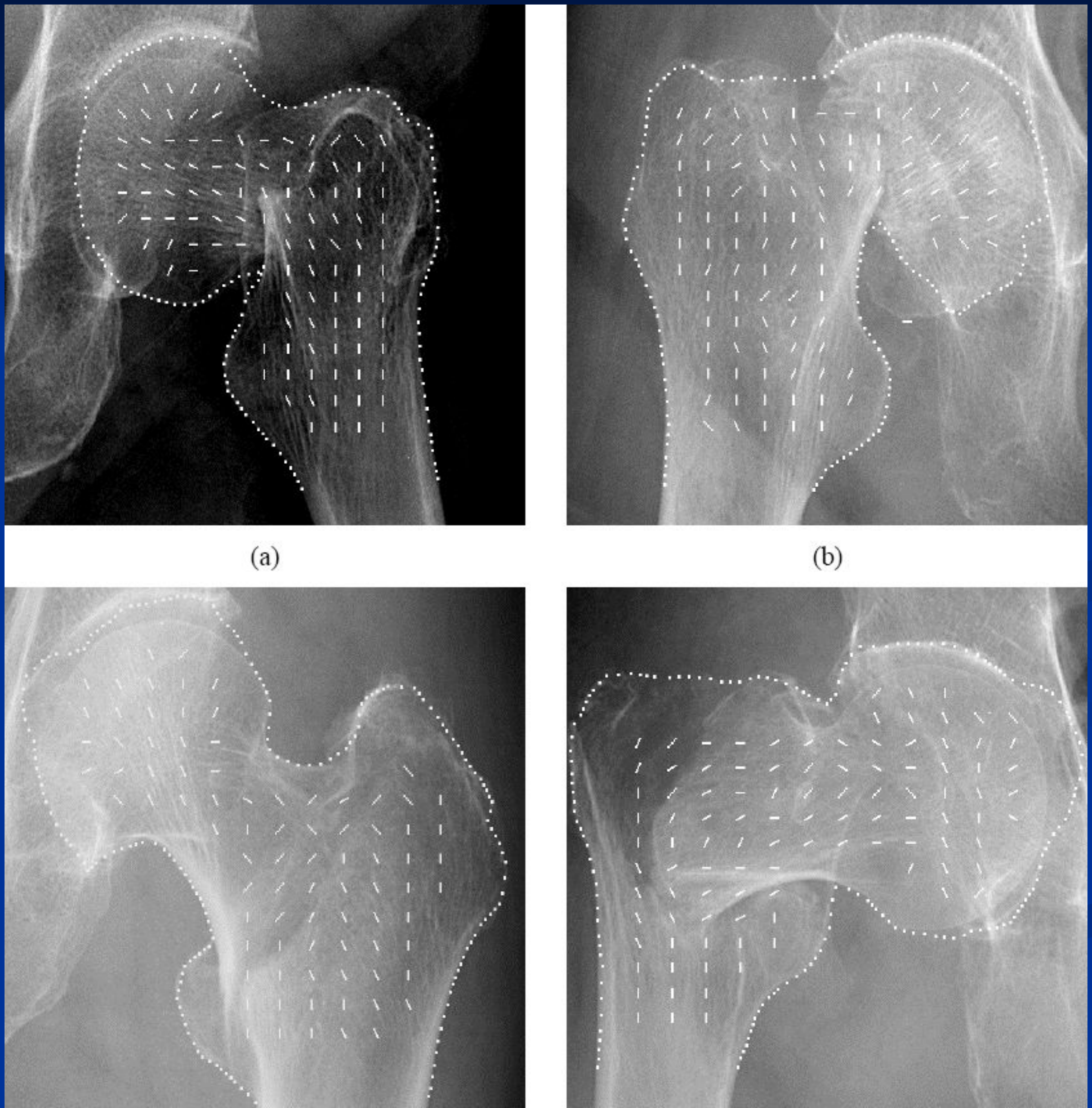
$$E_{f\theta}(x,y) = e^2_{c,f\theta}(x,y) + e^2_{s,f\theta}(x,y)$$

$$\overline{E}_{f\theta} = \frac{1}{S_x S_y} \sum_{x=1}^{S_x} \sum_{y=1}^{S_y} E_{f\theta}(x,y)$$

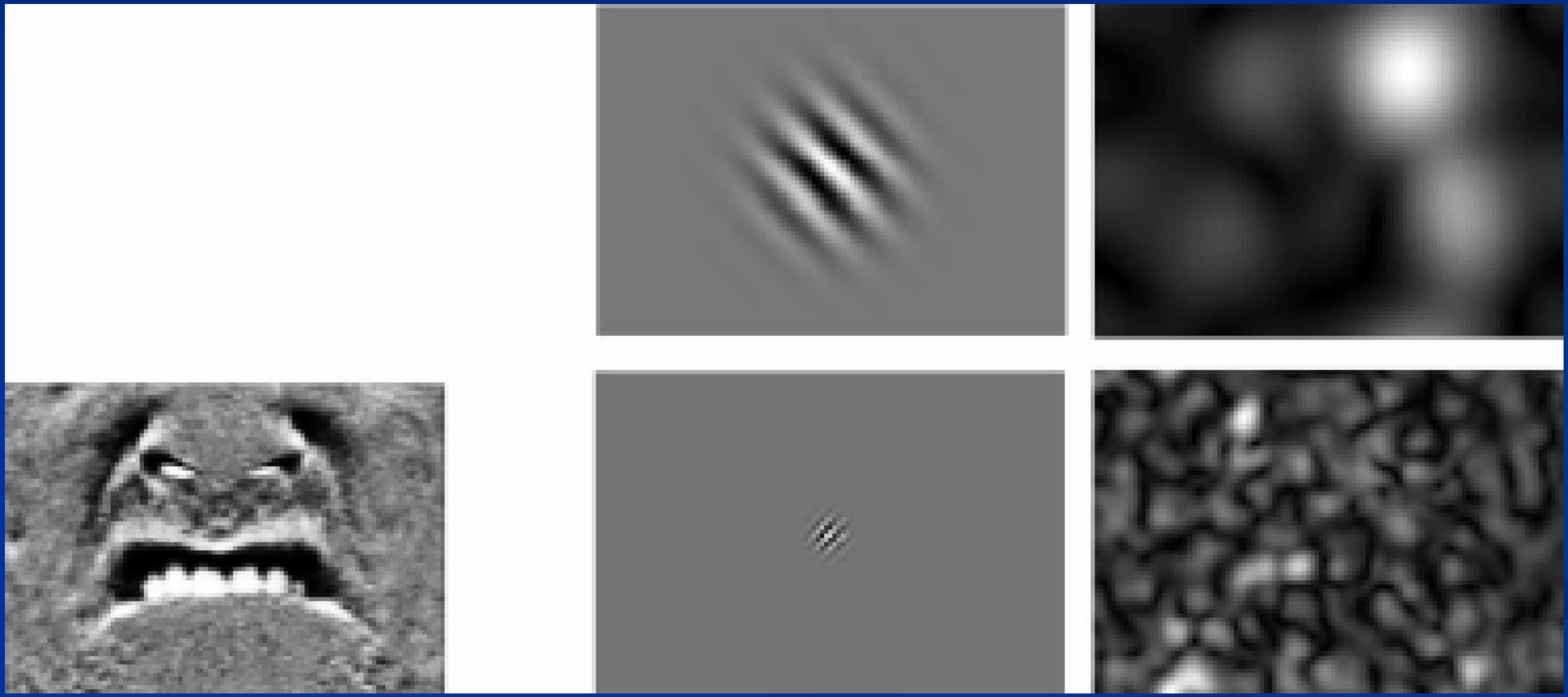
# Целые кости



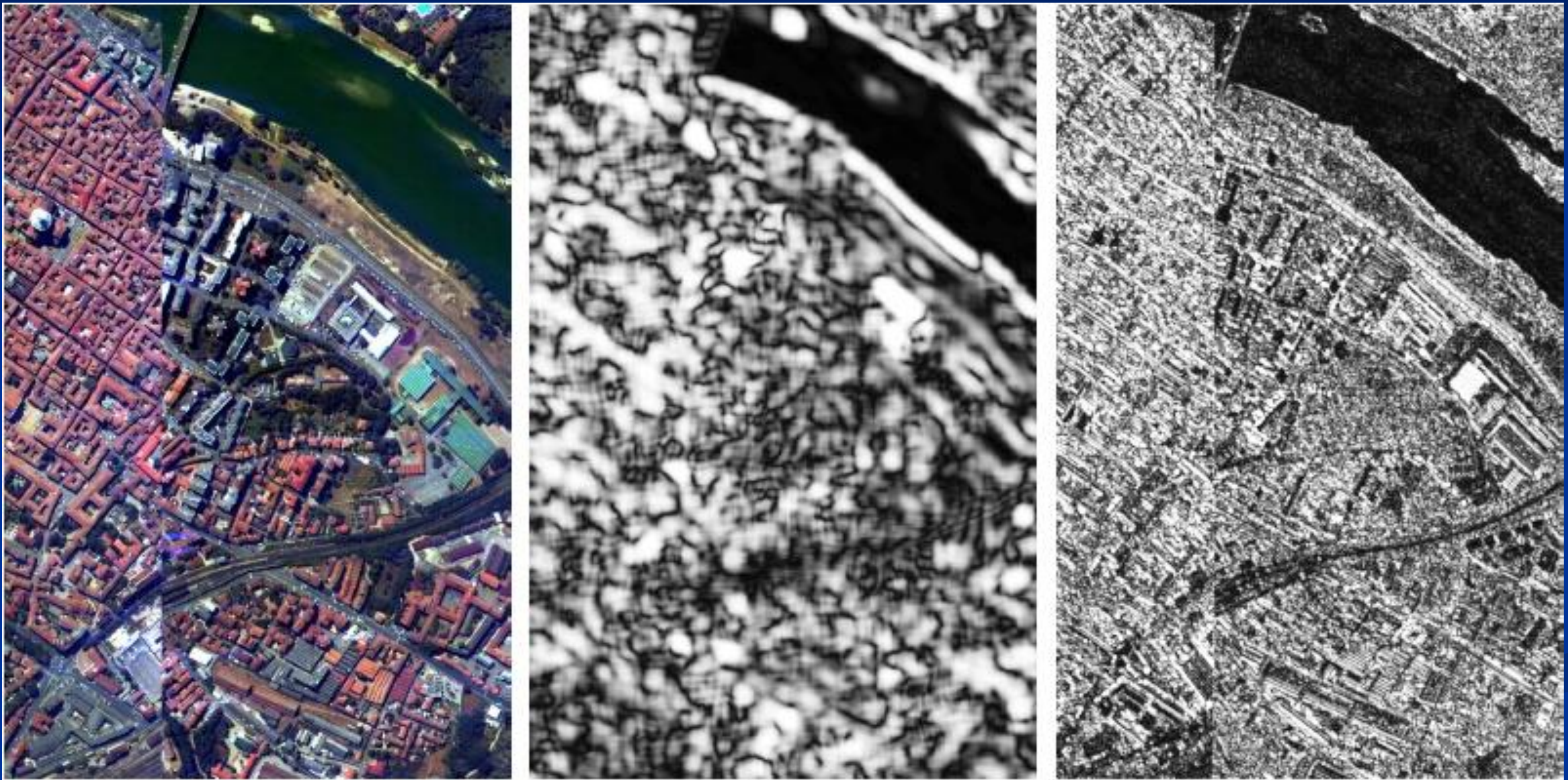
# Кости с переломом



# Фильтры Габора

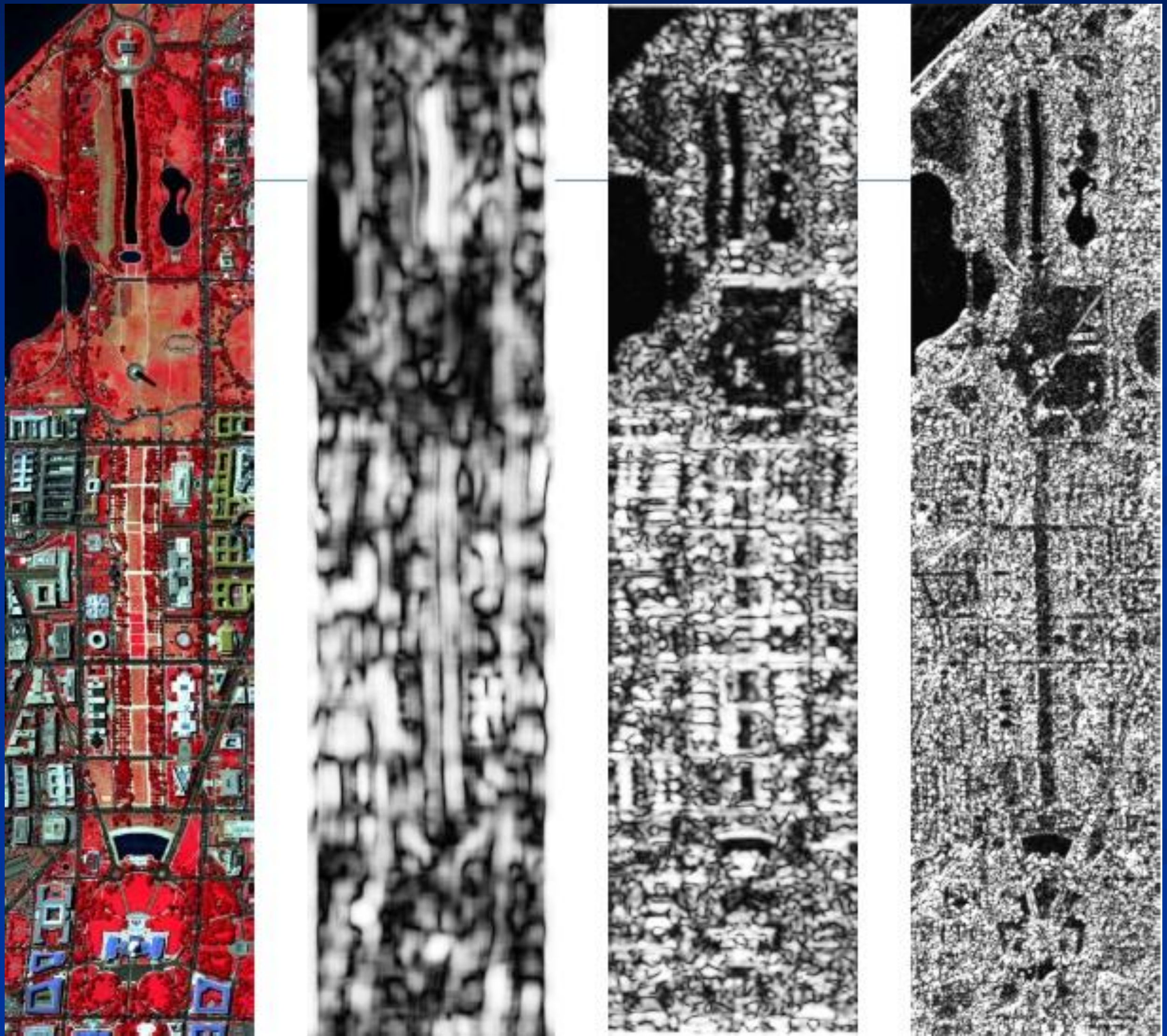


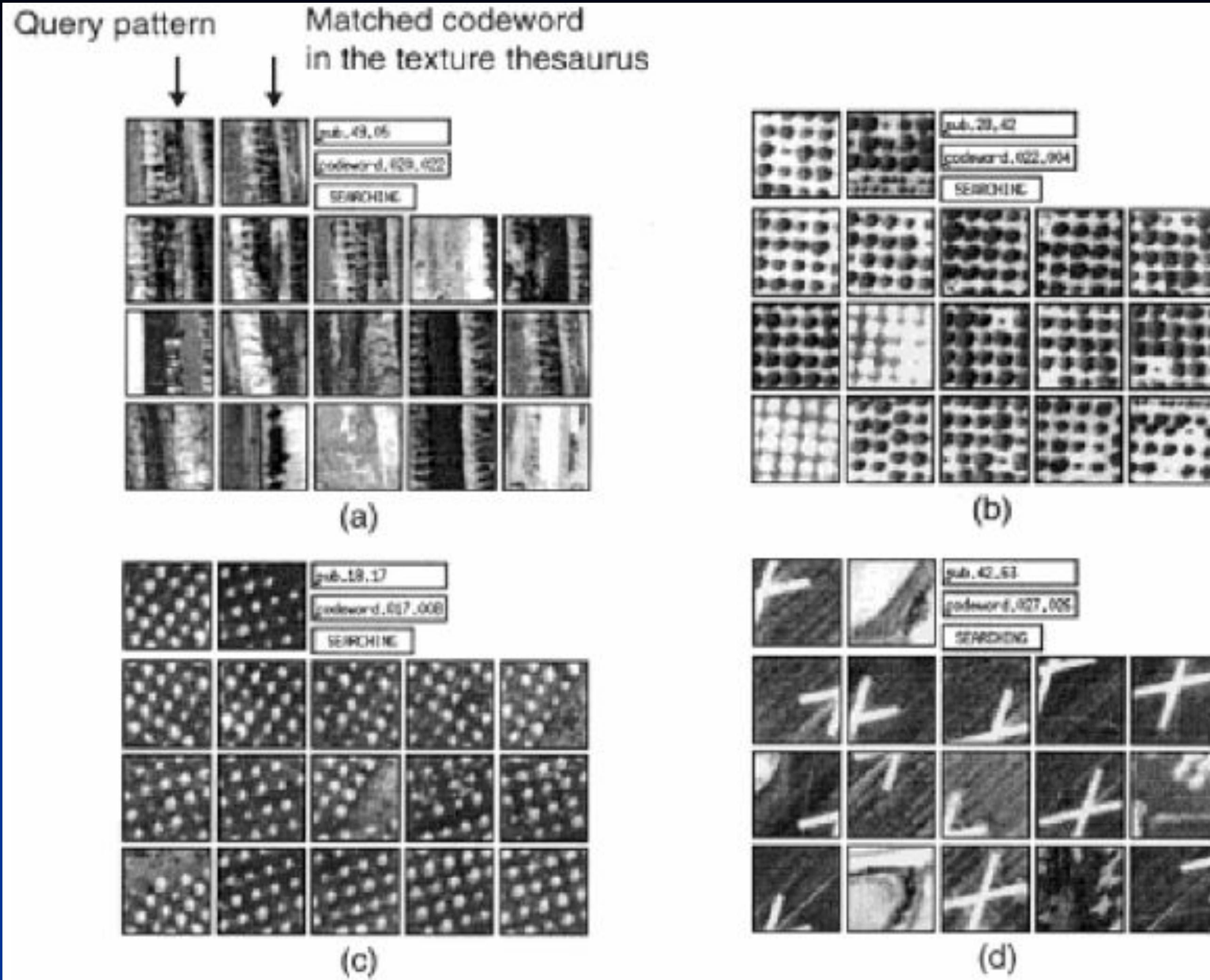
# Фильтры Габора

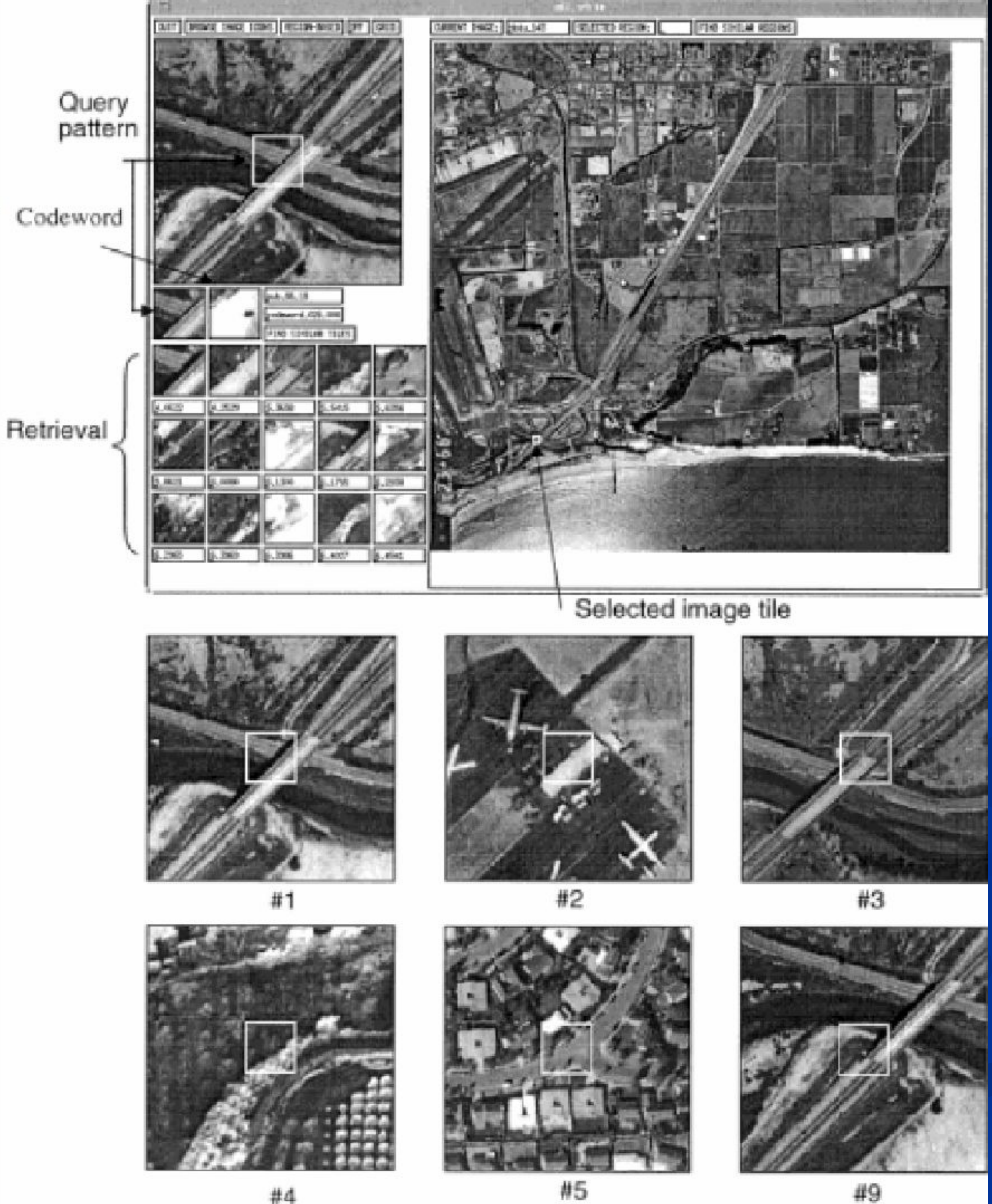


Gabor filter responses for a satellite image.

# Фильтры Габора




Query pattern
Matched codeword in the texture thesaurus
(a)
(b)
(c)
(d)




Full resolution of some retrieved tiles