

$$\begin{aligned}
(\Delta x)^2(\Delta s)^2 &= \frac{\int x^2 f f^* dx \int s^2 F F^* ds}{\int f f^* dx \int F F^* ds} \\
&= \frac{\int x f \cdot x f^* dx \int f' f'^* dx}{4\pi^2 \left(\int f f^* dx \right)^2} \\
&\cong \frac{\left| \int (x f^* \cdot f' + x f \cdot f'^*) dx \right|^2}{16\pi^2 \left(\int f f^* dx \right)^2} \\
&= \frac{\left| \int x \frac{d}{dx} (f f^*) dx \right|^2}{16\pi^2 \left(\int f f^* dx \right)^2} \\
&= \frac{\left| \int f f^* dx \right|^2}{16\pi^2 \left(\int f f^* dx \right)^2} \\
&= \frac{1}{16\pi^2}.
\end{aligned}$$

Фильтры Габора

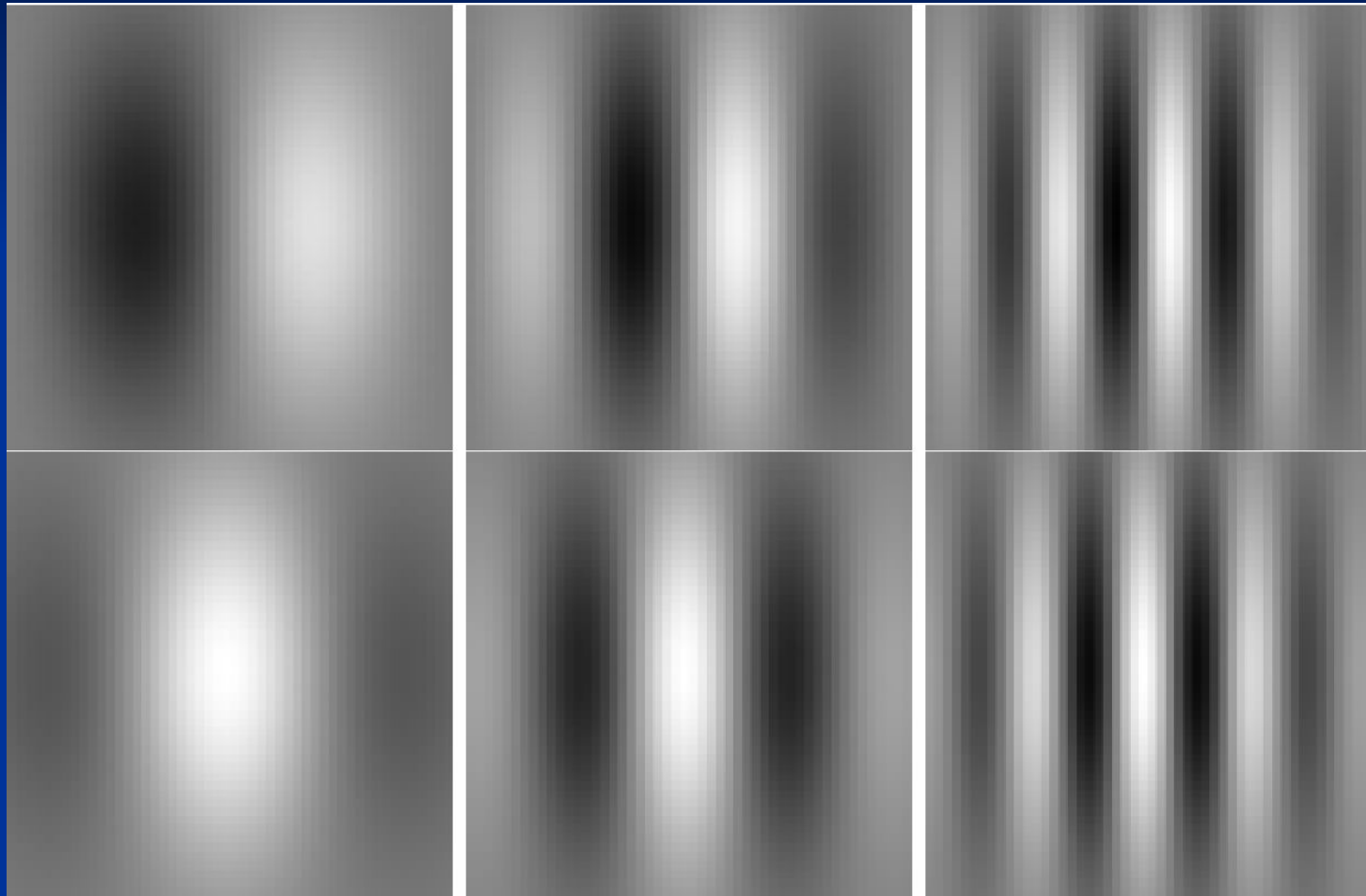
Gabor filters are formed by modulating a complex sinusoid by a Gaussian function:

$$g(x, y) = \overbrace{\frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{1}{2}\left(\frac{\tilde{x}^2}{\sigma_x^2} + \frac{\tilde{y}^2}{\sigma_y^2}\right)\right)}^{\text{gaussian envelope}} \cdot \overbrace{\exp(2\pi j\omega\tilde{x})}^{\text{complex sinusoidal}}$$

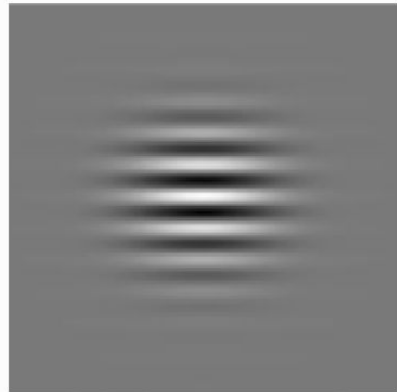
$$\text{with } \begin{cases} \tilde{x} = x \cos(\theta) + y \sin(\theta) \\ \tilde{y} = -x \sin(\theta) + y \cos(\theta) \end{cases}$$

- σ_x and σ_y control spatial extent of filter
- θ is the orientation
- ω is the radial frequency of the sinusoid

Фильтры Габора

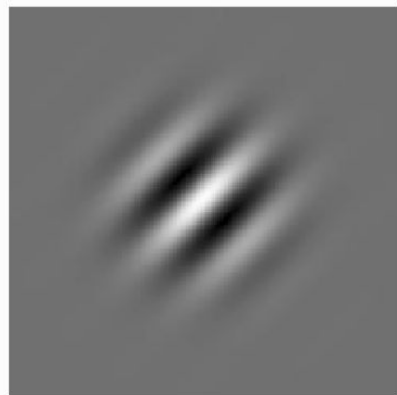
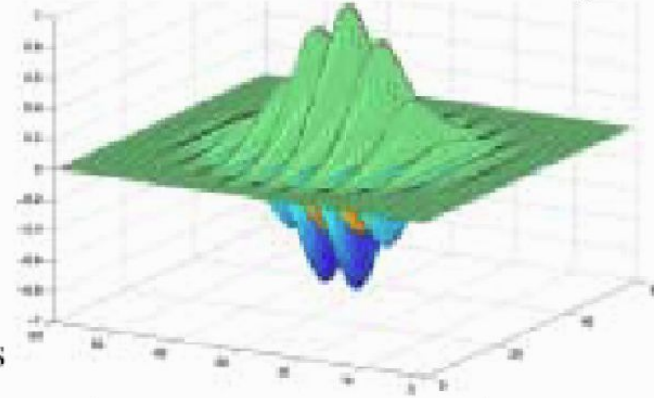


Фильтры Габора

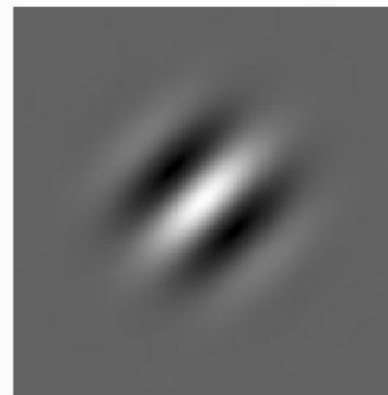


High frequency along axis

$$e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi(k_x x + k_y y))$$

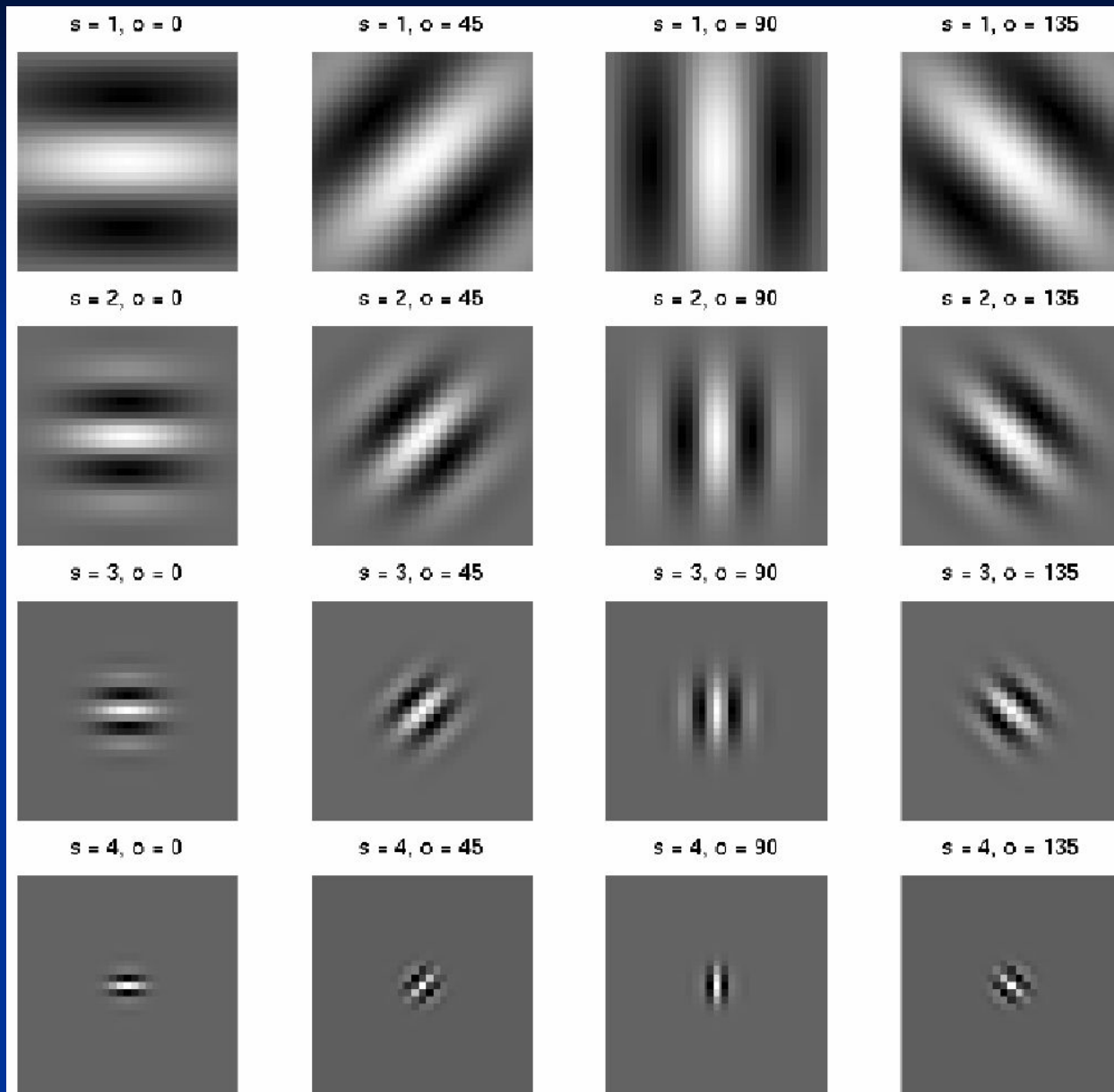


Lower frequency



Even lower frequency

Фильтры Габора



Функции Габора

$$g(x, y) = \left(\frac{1}{2\pi\sigma_x\sigma_y} \right) \exp \left[-\frac{1}{2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) + 2\pi j W x \right],$$

$$G(u, v) = \exp \left\{ -\frac{1}{2} \left[\frac{(u - W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2} \right] \right\},$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

Габоровские вейвлеты

$$g_{mn}(x, y) = a^{-m}g(x', y'), \quad a > 1, \quad m, n = \text{integer}$$

$$x' = a^{-m}(x \cos \theta + y \sin \theta) \quad \text{and}$$

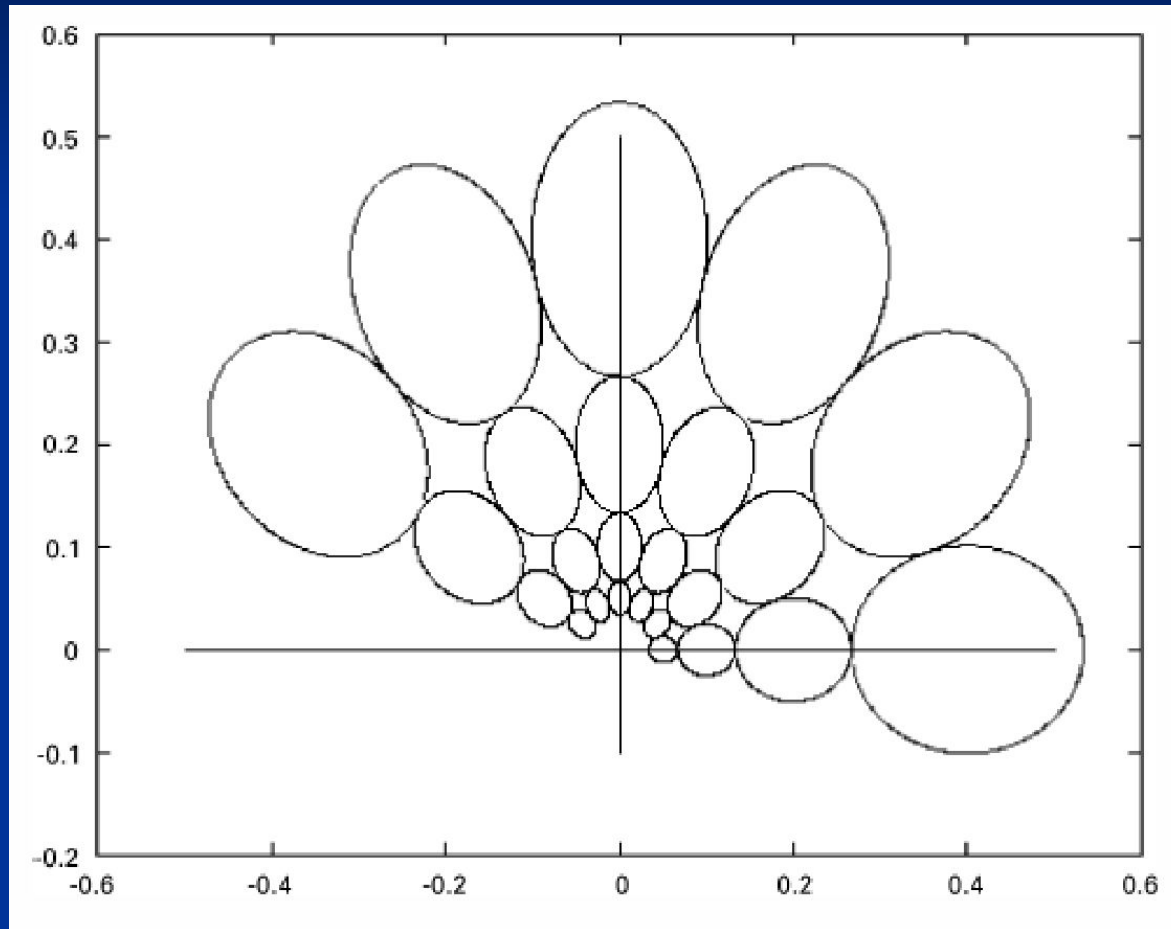
$$y' = a^{-m}(-x \sin \theta + y \cos \theta),$$

$$\theta = n\pi/K$$

K – number of orientations

Габоровские вейвлеты

$$W_{mn}(x, y) = \int I(x, y) g_{mn}^*(x - x_1, y - y_1) dx_1 dy_1$$



$$U_h = 0.04, U_1 = 0.05, K = 6 \text{ and } S = 4$$

Габоровские вейвлеты

$$a = (U_{\mathbf{h}}/U_1)^{1/(S-1)}, \quad \sigma_u = \frac{(a-1)U_{\mathbf{h}}}{(a+1)\sqrt{2 \ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right) \left[U_{\mathbf{h}} - 2 \ln 2 \left(\frac{\sigma_u^2}{U_{\mathbf{h}}} \right) \right] \\ \times \left[2 \ln 2 - \frac{(2 \ln 2)^2 \sigma_u^2}{U_{\mathbf{h}}^2} \right]^{-1/2}$$

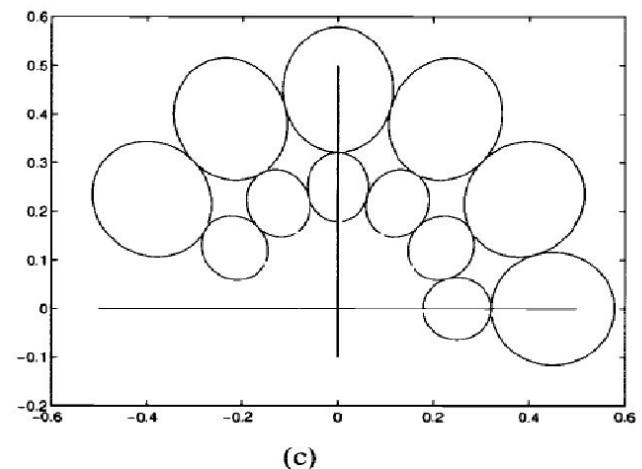
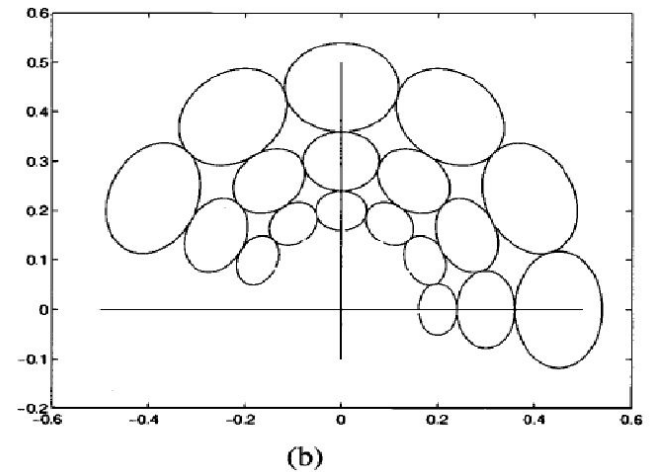
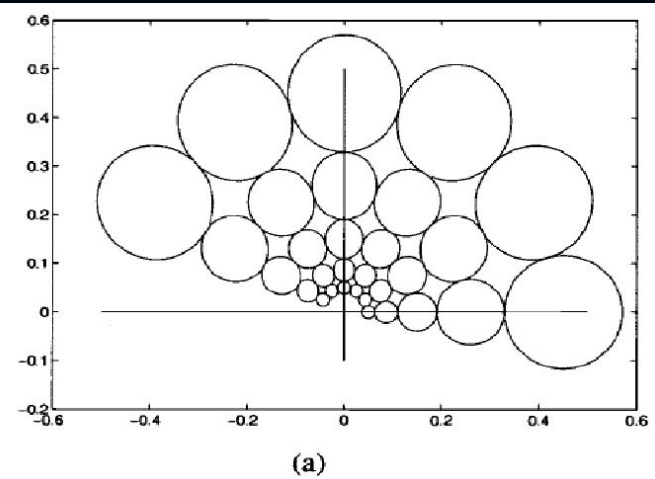
$$W = U_{\mathbf{h}} \text{ and } m = 0, 1, \dots, S - 1$$

$K=6$

$a) \quad \sigma = 5, S = 5$

$b) \quad \sigma = 1.25, S = 3$

$c) \quad \sigma = 1, S = 2$



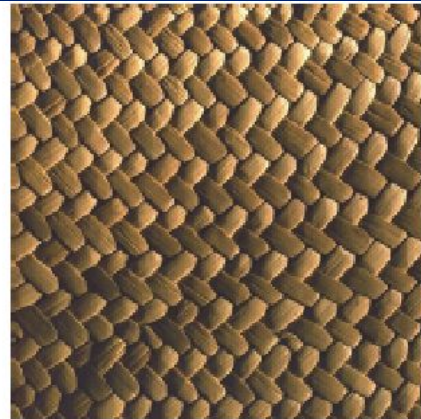
Текстуры



Bark



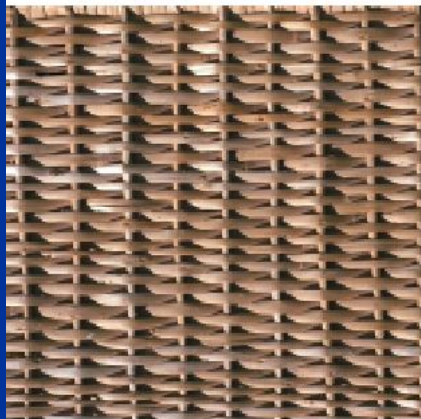
Bark



Fabric



Fabric



Fabric



Flowers



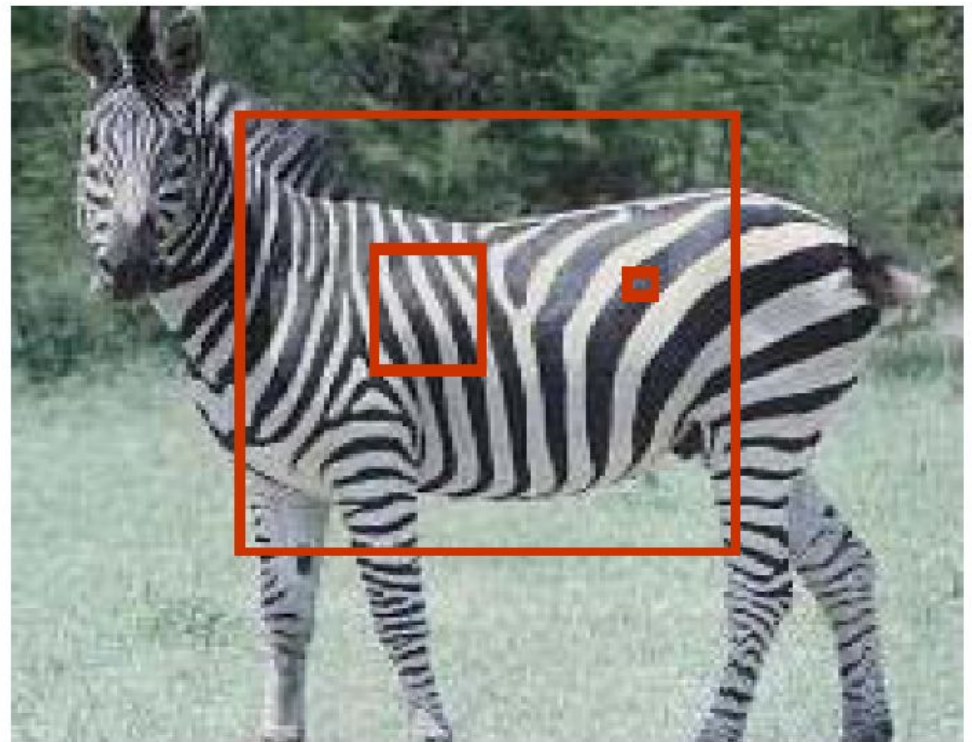
Flowers



Flowers

Текстуры

- Whether an effect is a texture or not depends on the scale at which it is viewed.



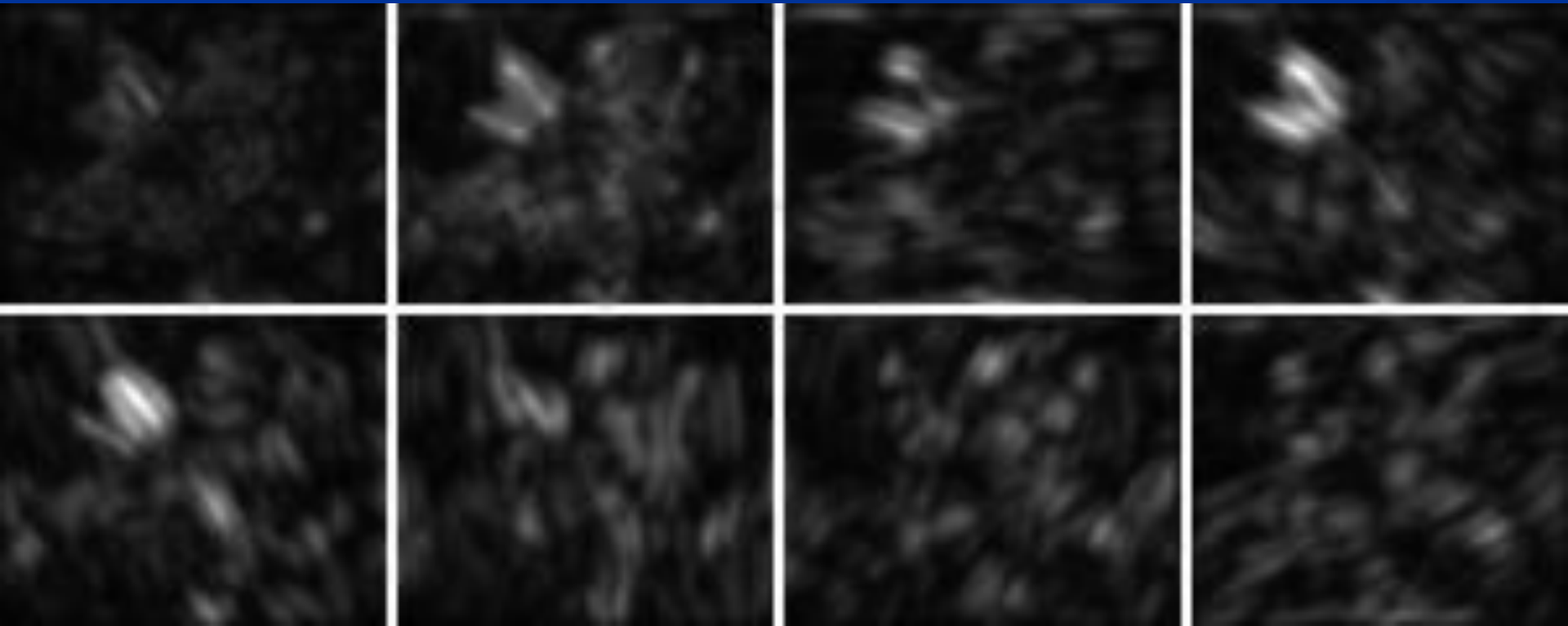
Текстуры



Банки фильтров



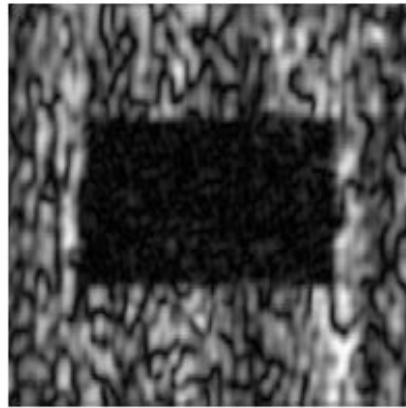
Банки фильтров



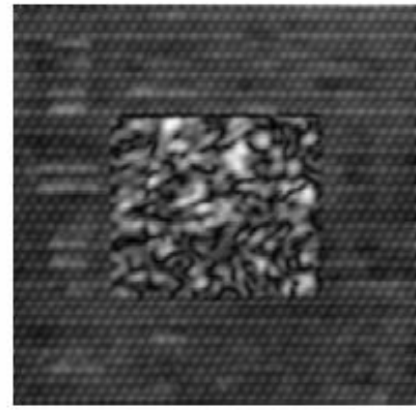
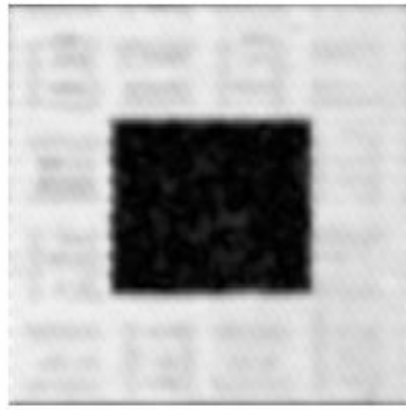
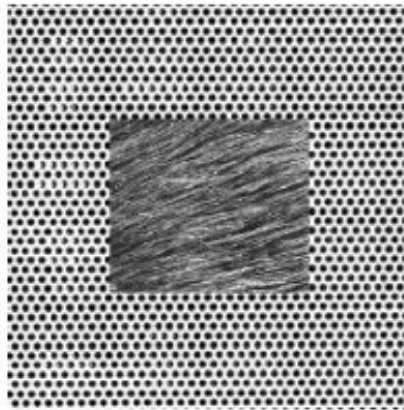
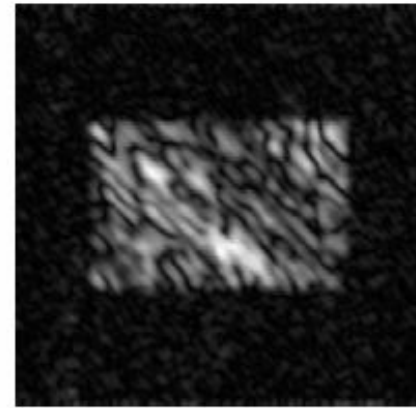
Фильтры Габора



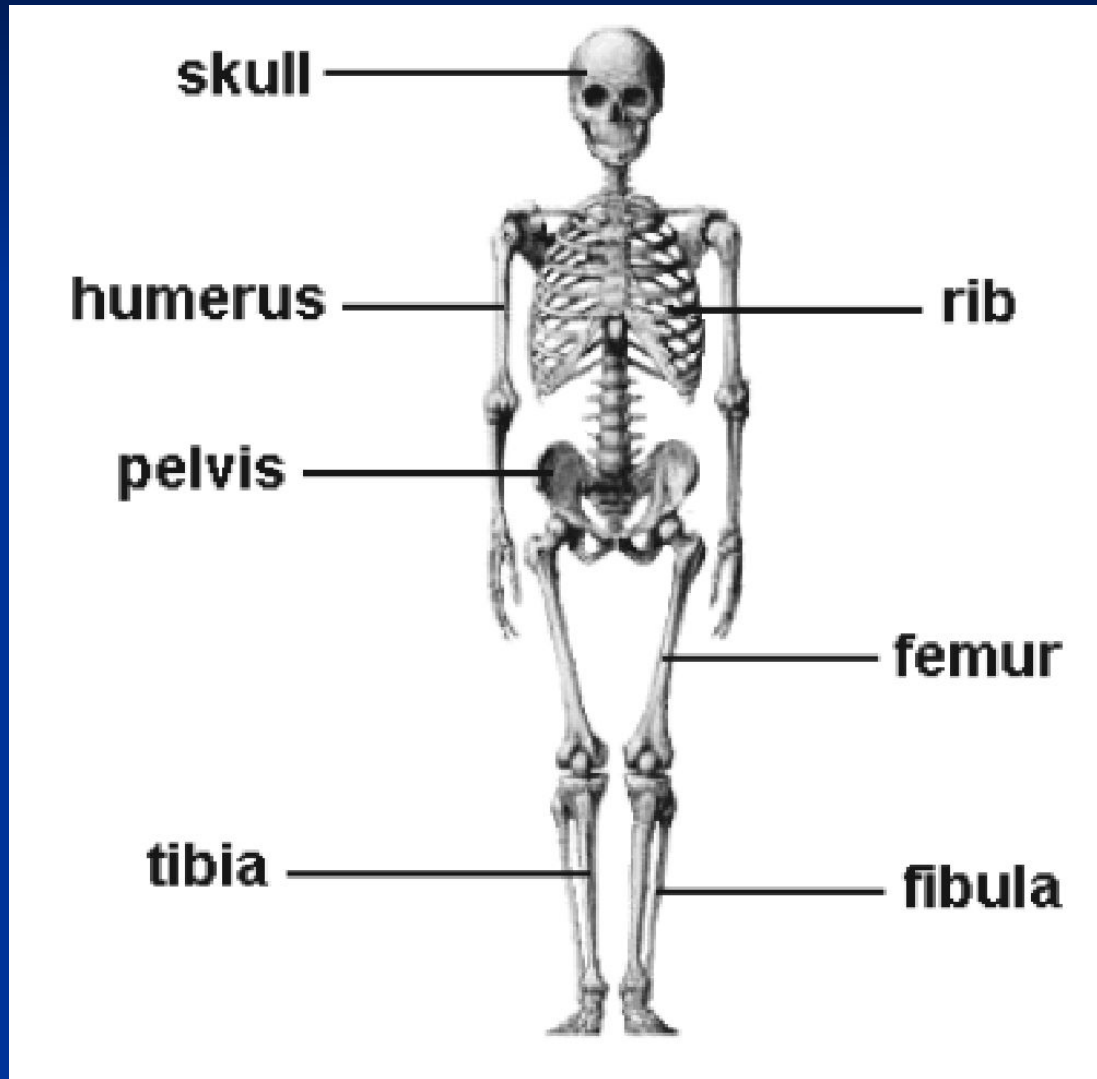
Texture image



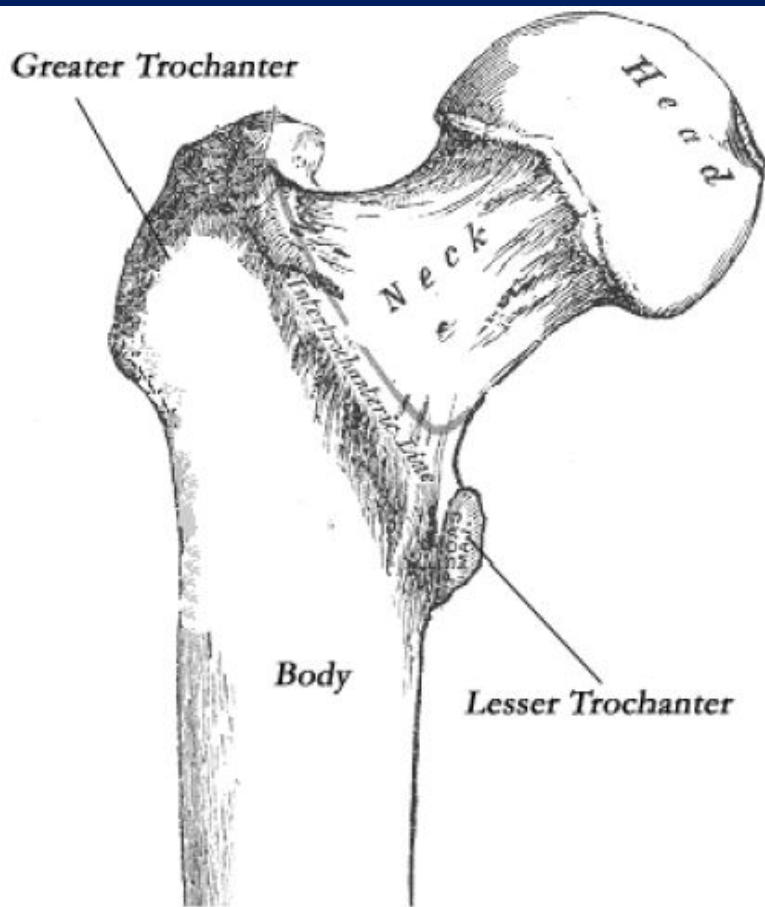
Magnitude of Gabor filter responses



Анализ бедренной кости (femur)



Шейка бедра



Шейка бедра

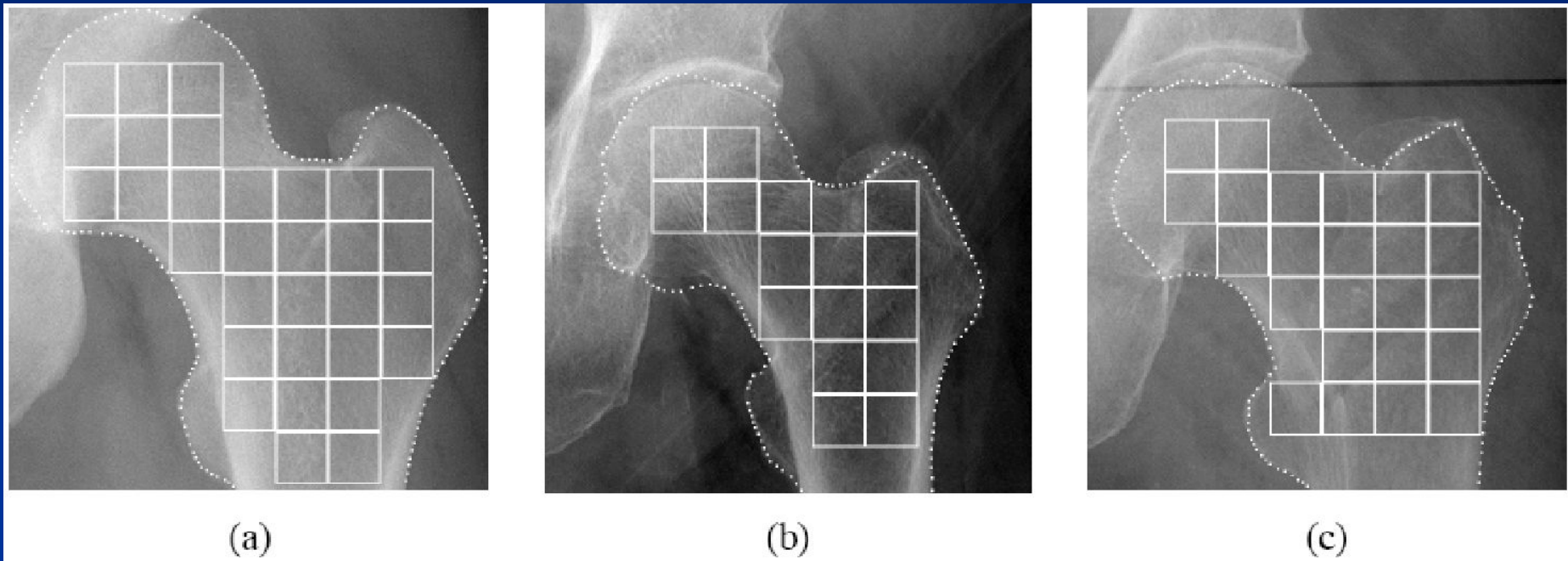


Figure 3.3. A femur is a natural structure that exhibits variations. (a) and (b) above are healthy, while (c) is fractured. Here, (a) is larger than (b). (c) has a relatively shorter neck, due to the fracture. Each grid square is a region sampled for texture orientation feature extraction

$$h(x, y) = g(x', y') \exp(2\pi jfx').$$

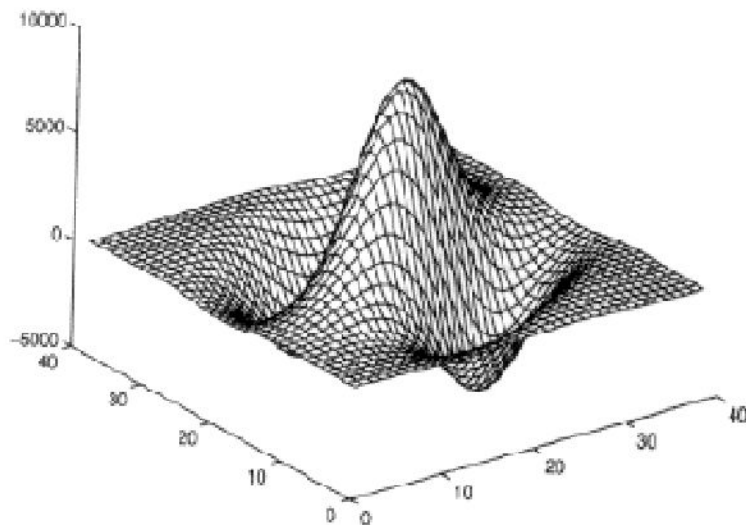
The oriented Gaussian function $g(x', y')$ is given by:

$$g(x', y') = \frac{1}{2\pi\lambda\sigma^2} \exp\left[-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right],$$

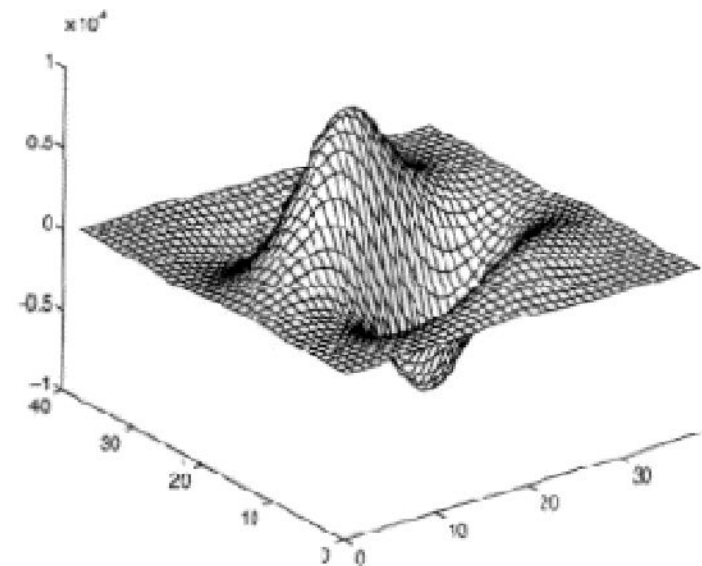
where $(x', y') = (x \cos \theta + y \sin \theta, -x \sin \theta + y \cos \theta)$ are rotated coordinates

$$h_{c,f\theta}(x, y) = g(x', y') \cos(2\pi fx'),$$

$$h_{s,f\theta}(x, y) = g(x', y') \sin(2\pi fx').$$

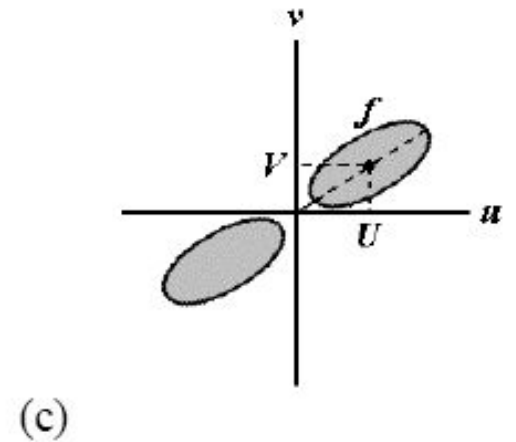
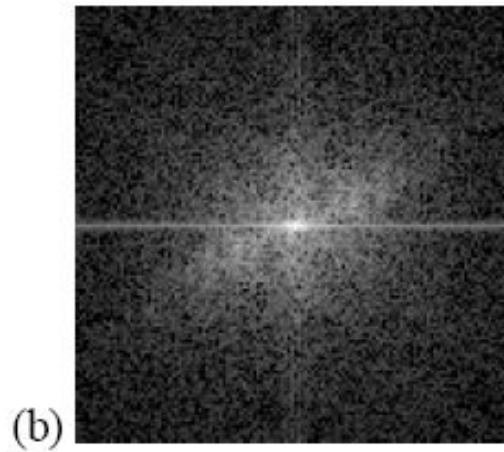
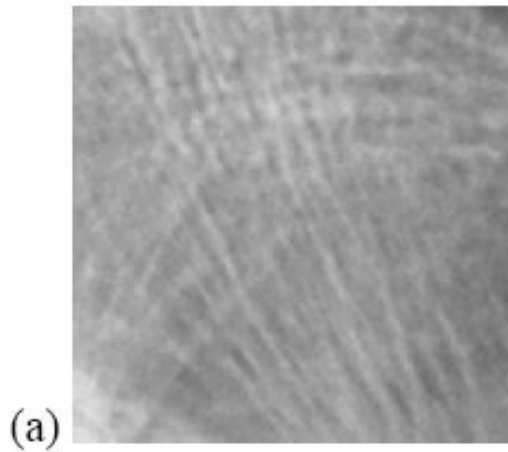


(a)



(b)

Фурье анализ текстуры кости



$$f_p = \sqrt{U^2 + V^2}$$

$$f_p \cong 0.13 \text{ cycles per pixel}$$

A Gabor filter bank of 1 frequency channel and 8 orientation channels is used to extract the orientation of the texture patterns in the femur. The centre frequency f of the Gabor filters is set to 0.13 cycles per pixel, and orientations range from 0° to 157.5° , incrementing in steps of 22.5° .

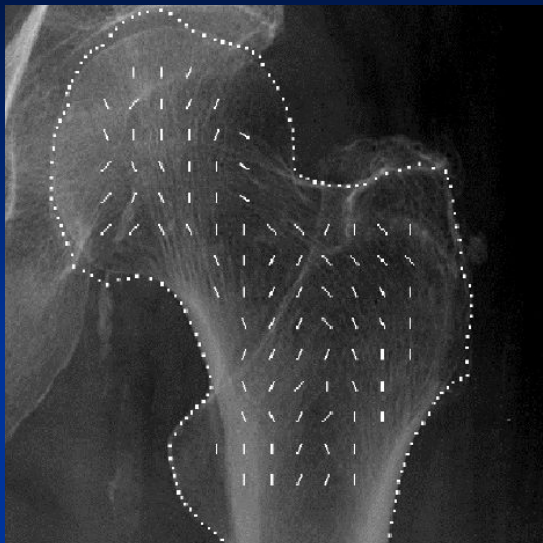
$$e_{c,f\theta}(x, y) = I(x, y) * h_{c,f\theta},$$

$$e_{s,f\theta}(x, y) = I(x, y) * h_{s,f\theta}.$$

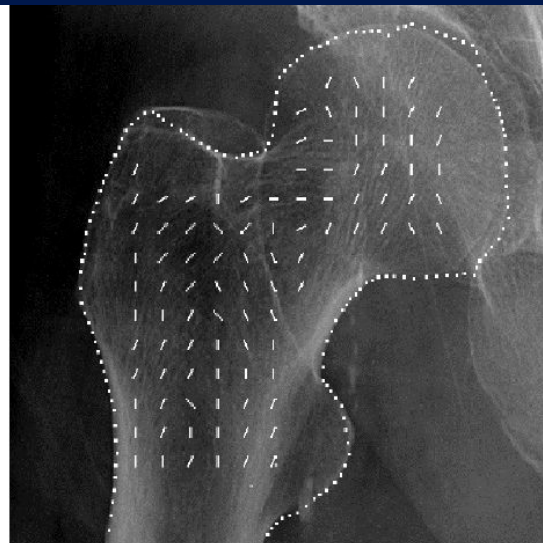
$$E_{f\theta}(x, y) = e_{c,f\theta}^2(x, y) + e_{s,f\theta}^2(x, y)$$

$$\bar{E}_{f\theta} = \frac{1}{S_x S_y} \sum_{x=1}^{S_x} \sum_{y=1}^{S_y} E_{f\theta}(x, y)$$

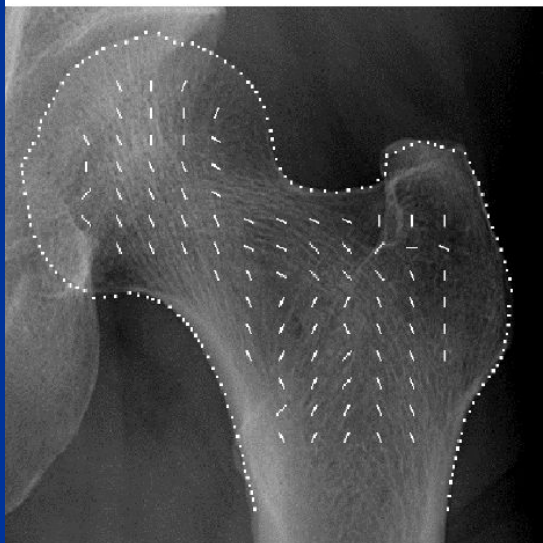
ЦЕЛЫЕ КОСТИ



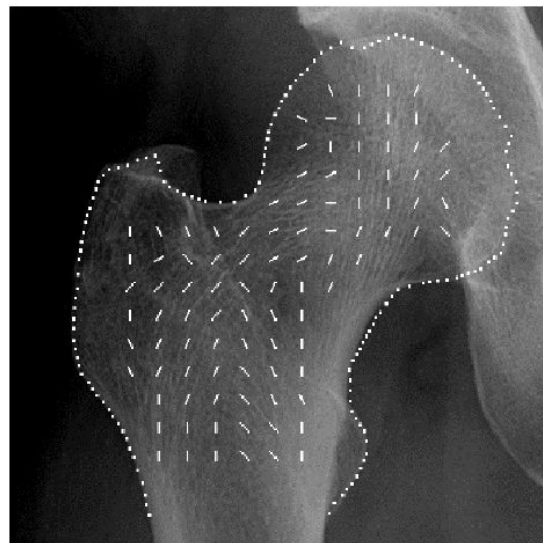
(a)



(b)

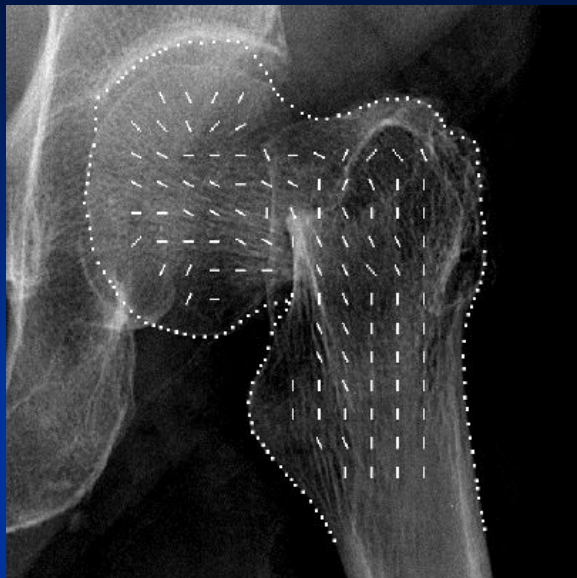


(c)

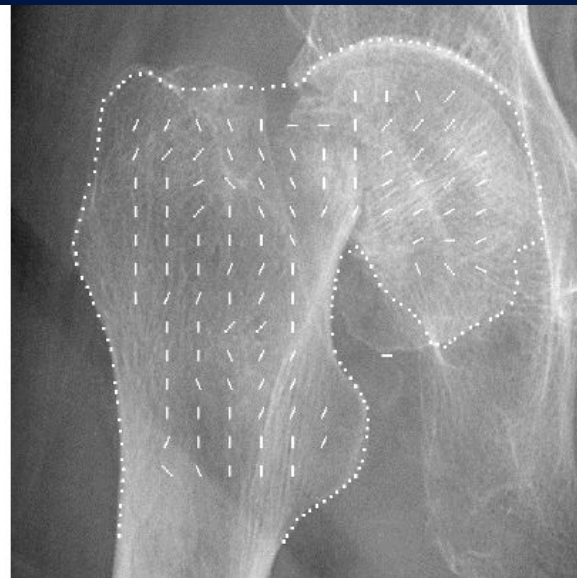


(d)

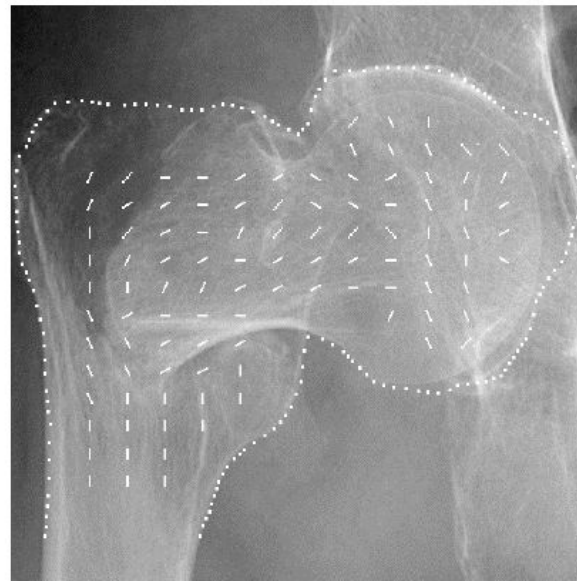
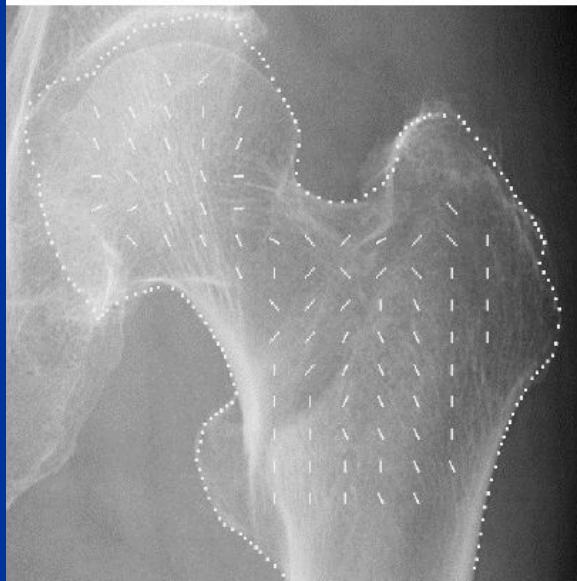
Кости с переломом



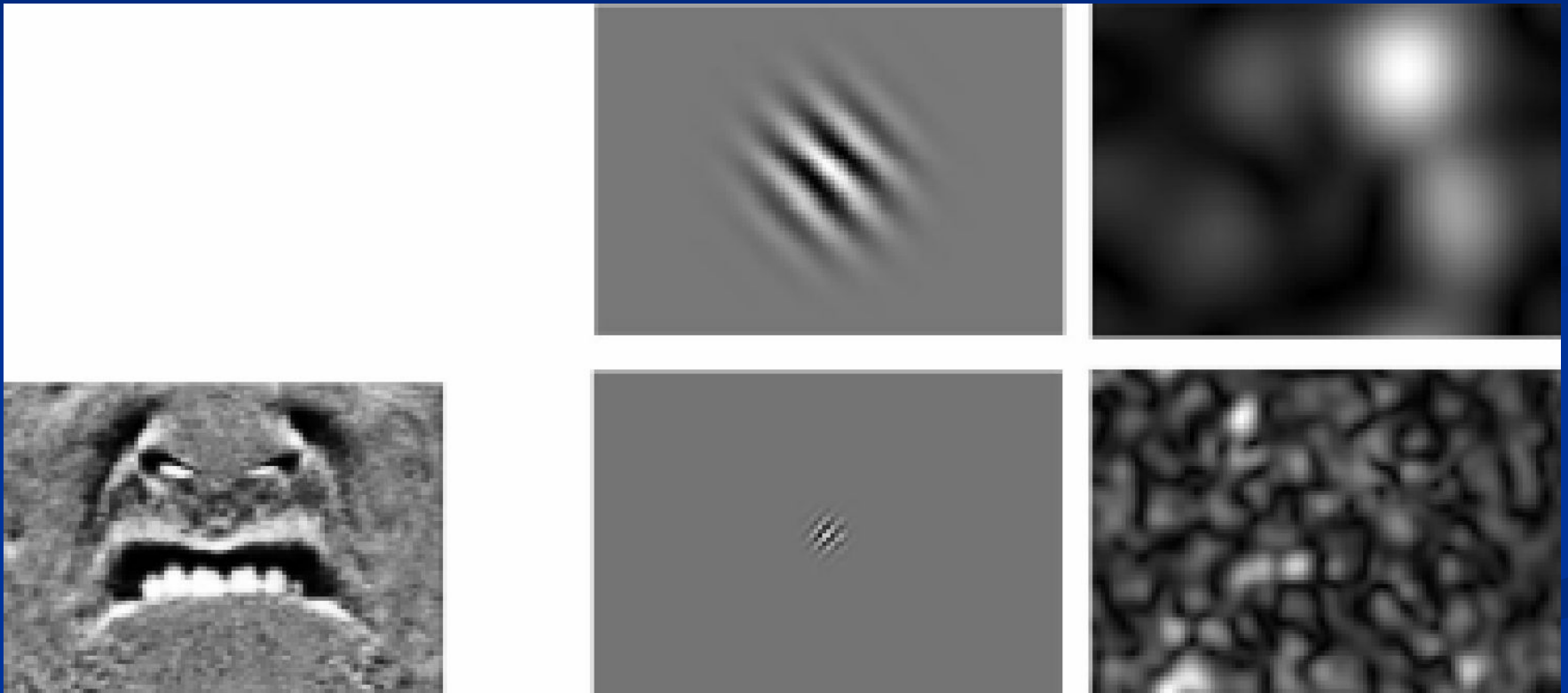
(a)



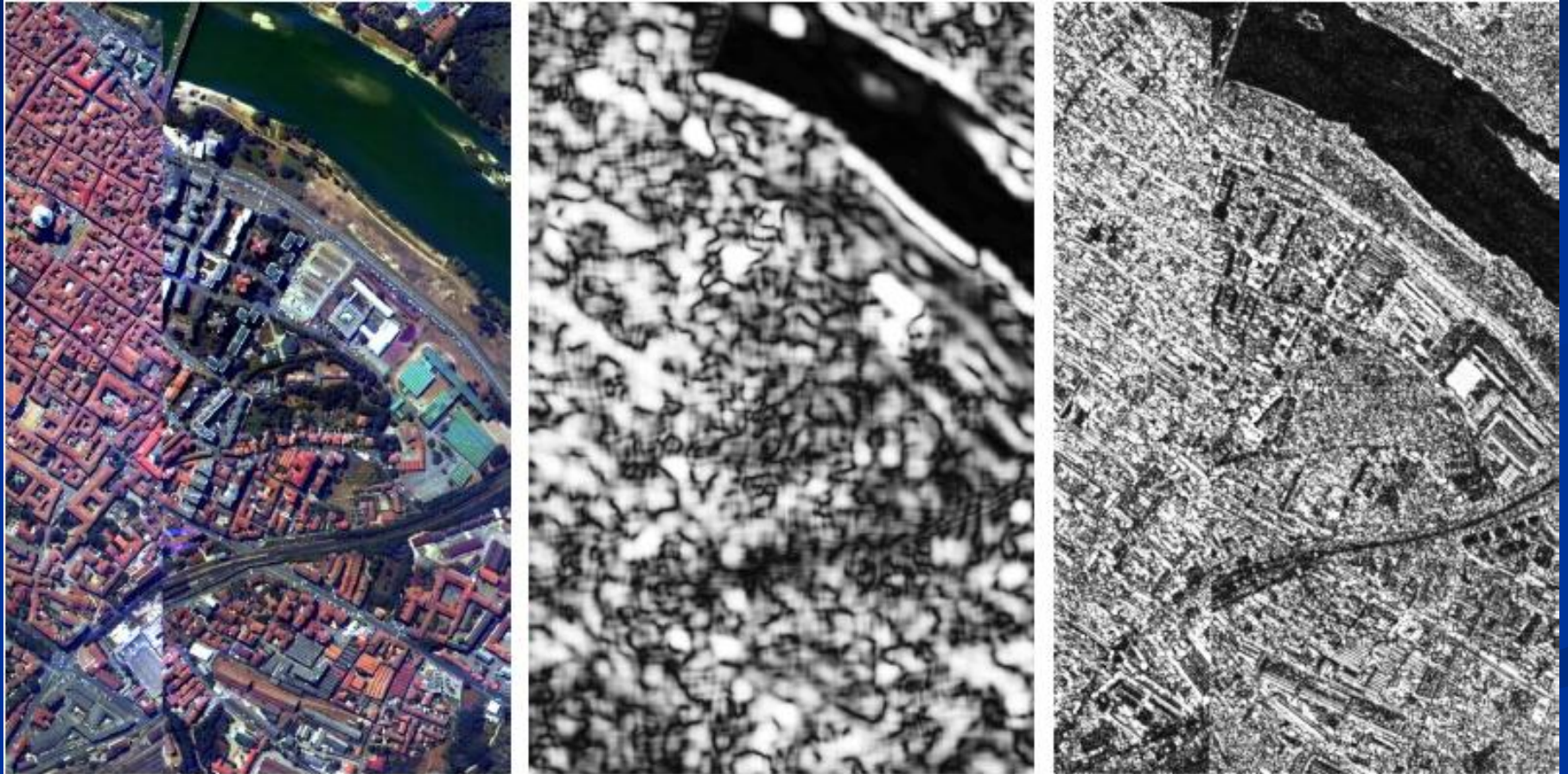
(b)



Фильтры Габора

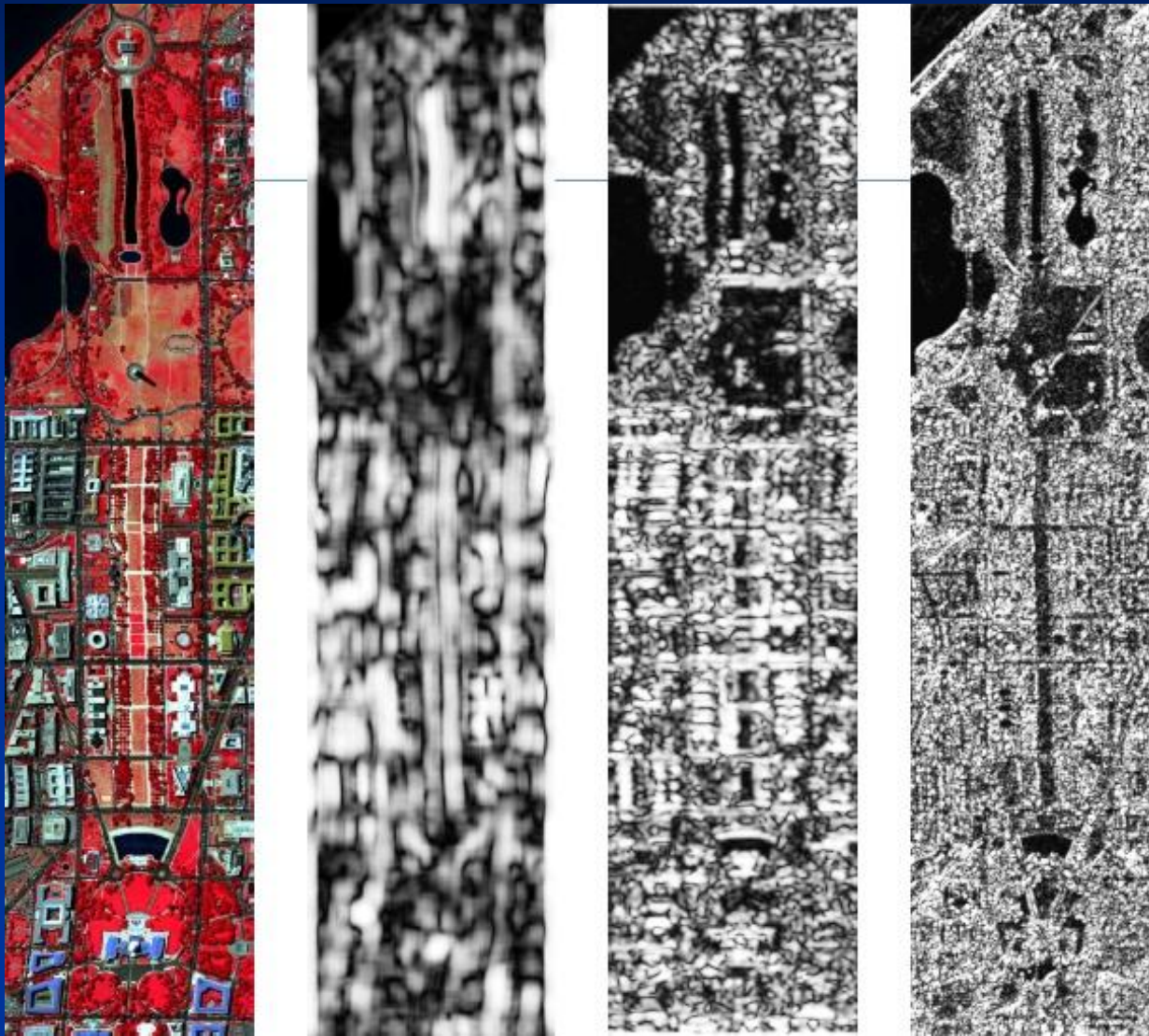


Фильтры Габора



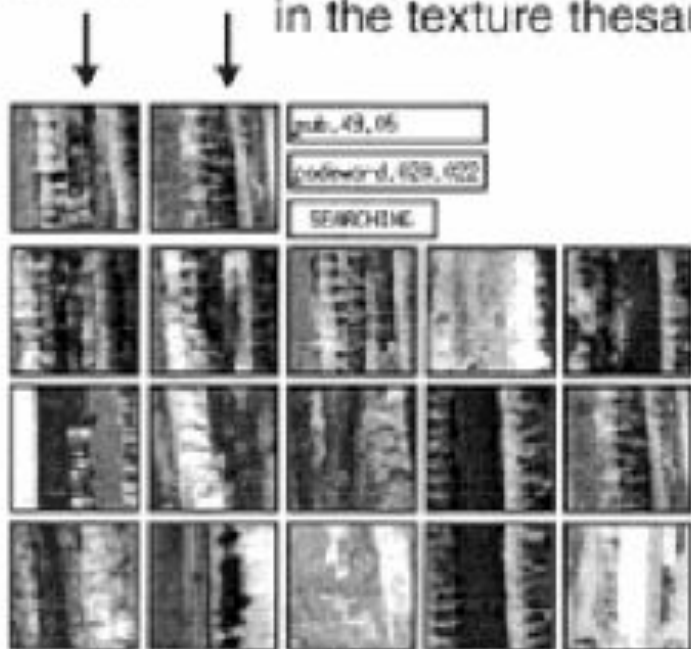
Gabor filter responses for a satellite image.

Фильтры Габора

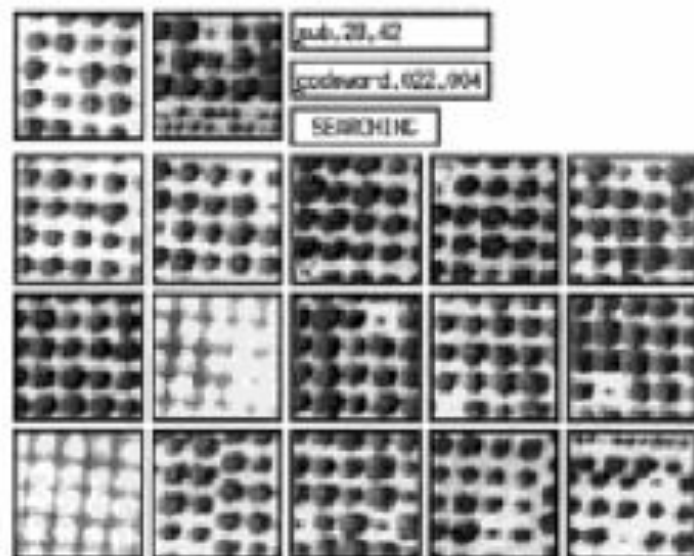


Query pattern

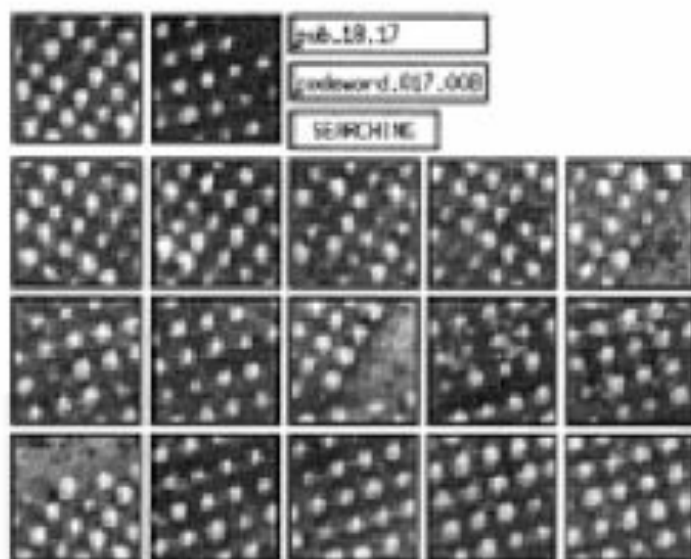
Matched codeword
in the texture thesaurus



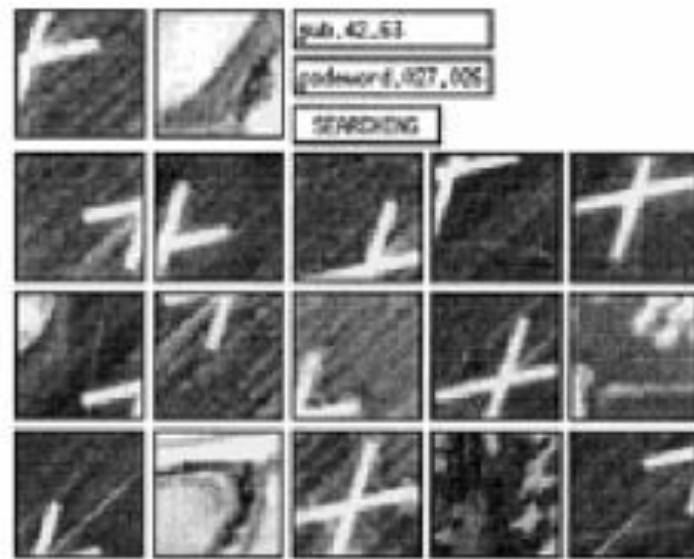
(a)



(b)



(c)



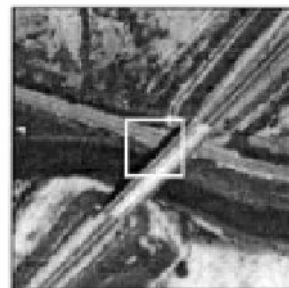
(d)

Query pattern

Codeword

Retrieval

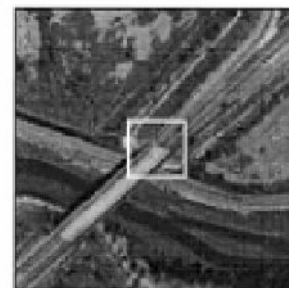
Selected image tile



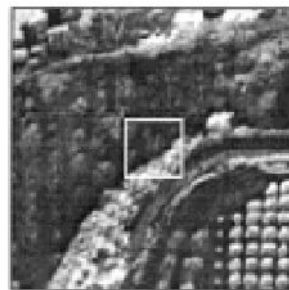
#1



#2



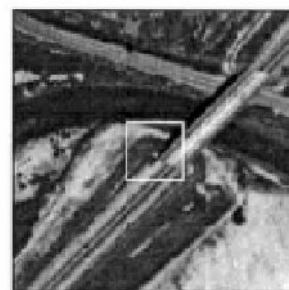
#3



#4



#5



#9

Full resolution of some retrieved tiles