Data Features

Factor	Equation	Domain Type	Dimensions	Reference
Mean		Time	3	(Sun et al. 2010) (Bao & Intille 2004)
Variance		Time	3	(Sun et al. 2010) (Bao & Intille 2004)
Skewness		Time	3	(Godfrey et al. 2008)
Kurtosis		Time	3	(Ermes et al. 2008)
Covariance		Time	3	(Bao & Intille 2004)
Zero Crossing Rate		Time	3	(Mase 2002)
Mean Crossing Rate		Time	3	(Mase 2002) (Altini et al. 2012)
Average Resultant Acceleration		Time	1	(Kwapisz et al. 2011)
Magnitude		Time	1	(Khan et al. 2010)
Average Absolute Difference		Time	3	(Kwapisz et al. 2011)
Average Absolute Value		Time	3	(Kwapisz 2010)
Root Mean Square		Time	3	(Staudenmayer et al. 2009)
FFT Transformation	N/A	Time		
Dominant frequency		Frequency	3	(Sun et al. 2010)
Energy		Frequency	3	(Bao & Intille 2004)
Entropy		Frequency	3	(Mannini & Sabatini 2010)

Example

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- Mean
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $\overline{X} = \frac{1+3+5}{3} = 3$
  - $\overline{Y} = \frac{(-3)+1+(-1)}{3} = -1$
  - $\overline{Z} = \frac{4+1+(-5)}{3} = 0$

Example cont'd

$$var(x) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

- Variance
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

- $var(X) = \frac{(1-3)^2+(3-3)^2+(5-3)^2}{3} = \frac{8}{3}$
- $var(Y) = \frac{(-3-(-1))^2+(1-(-1))^2+(-1-(-1))^2}{3} = \frac{8}{3}$
- $var(Z) = \frac{(4-0)^2+(1-0)^2+(-5-0)^2}{3} = 14$

Example cont'd

$$M_k = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^k$$

$$\text{skewness } g_1 = \frac{\sqrt{n}M_3}{M_2^{\frac{3}{2}}}$$

- Skewness
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $skewness\ X =$
$\sqrt{3}(\frac{(1-3)^3+(3-3)^3+(5-3)^3}{3})/(\frac{(1-3)^2+(3-3)^2+(5-3)^2}{3})^{\frac{3}{2}} =$
$\sqrt{3}(\frac{(-8)+0+8}{3})/(\frac{4+0+4}{3})^{\frac{3}{2}} = 0$

Example cont'd

- $-$ $skewness\, Y =$
  $\sqrt{3}(\frac{(-3-(-1))^3+(1-(-1))^3+(-1-(-1))^3}{3})/(\frac{(-3-(-1))^2+(1-(-1))^2+(-1-(-1))^2}{3})^{\frac{3}{2}}$
  $\sqrt{3}(\frac{(-8)+8+0}{3})/(\frac{4+4+0}{3})^{\frac{3}{2}} = 0$
  $-$ $skewness\, Z = \sqrt{3}(\frac{(4-0)^3+(1-0)^3+(-5-0)^3}{3})/(\frac{(4-0)^2+(1-0)^2+(-5-0)^2}{3})^{\frac{3}{2}} =$
  $\sqrt{3}(\frac{64+1+125}{3})/(\frac{16+1+25}{3})^{\frac{3}{2}} \approx 6.28$

Example cont'd

$$\text{kurtosis } g_2 = \frac{nM_4}{M_2^2} - 3$$

- Kurtosis
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $kurtosis\ X = 3\left(\frac{(1-3)^4+(3-3)^4+(5-3)^4}{3}\right)/\left(\frac{(1-3)^2+(3-3)^2+(5-3)^2}{3}\right)^2 - 3 = 3\left(\frac{16+0+16}{3}\right)/\left(\frac{4+0+4}{3}\right)^2 - 3 = 4.5 - 3 = 1.5$

Example cont'd

- – $kurtosis\ Y =$
  $3(\frac{(-3-(-1))^4+(1-(-1))^4+(-1-(-1))^4}{3})/(\frac{(-3-(-1))^2+(1-(-1))^2+(-1-(-1))^2}{3})^2 -$
  $3\ (\frac{16+16+0}{3})/(\frac{4+4+0}{3})^2 - 3 = 4.5 - 3 = 1.5$
  – $kurtosis\ Z =$
  $3\ (\frac{(4-0)^4+(1-0)^4+(-5-0)^4}{3})/(\frac{(4-0)^2+(1-0)^2+(-5-0)^2}{3})^2 - 3 =$
  $3\ (\frac{256+1+625}{3})/(\frac{16+1+25}{3})^2 - 3 = 4.5 - 3 = 1.5$

Example cont'd

$$\mathrm{cov}(x,y) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x}) * (y_i - \bar{y})$$

- Covariance
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $cov(x,y) = \frac{(1-3)\left(-3-(-1)\right)+(3-3)\left(1-(-1)\right)+(5-3)(-1-(-1))}{2} = \frac{4+0+0}{2} = 2$
  - $cov(y,z) = \frac{\left(-3-(-1)\right)(4-0)+\left(1-(-1)\right)(1-0)+(-1-(-1))(-5-0)}{2} = \frac{-8+2}{2} = -3$
  - $cov(x,z) = \frac{(1-3)(4-0)+(3-3)(1-0)+(5-3)(-5-0)}{2} = \frac{-8+0+(-10)}{2} = -9$

Example cont'd

$$C = \{x | (x_i > 0 \wedge x_{i-1} < 0) \vee (x_i < 0 \wedge x_{i-1} > 0)\}$$
$$ZCR = count(C)$$

- ZCR
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $ZCR\ X = 0$
  - $ZCR\ Y = 2$
  - $ZCR\ Z = 1$

Cross Zero

Example cont'd

$$C = \{x | (x_i > \bar{x} \wedge x_{i-1} < \bar{x}) \vee (x_i < \bar{x} \wedge x_{i-1} > \bar{x})\}$$
$$MCR = count(C)$$

- MCR
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

$$\bar{X} = 3$$
$$\bar{Y} = -1$$
$$\bar{Z} = 0$$

  - $MCR\ X = 0$
  - $MCR\ Y = 1$
  - $MCR\ Z = 1$

Cross Mean

Example cont’d

$$\mathrm{ARA} = \overline{\sqrt{\mathrm{x_i^2} + \mathrm{y_i^2} + \mathrm{z_i^2}}} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{N}} \sqrt{\mathrm{x_i^2} + \mathrm{y_i^2} + \mathrm{z_i^2}}}{\mathrm{N}}$$

- ARA
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

- $ARA = \frac{\sqrt{1+9+16}+\sqrt{9+1+1}+\sqrt{25+1+25}}{3} = \frac{\sqrt{26}+\sqrt{11}+\sqrt{51}}{3} \approx 5.18$

Example cont'd

$$\text{magnitude} = \sqrt{x^2 + y^2 + z^2}$$

- Magnitude
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $Case\ t_1: Magnitude = \sqrt{1 + 9 + 16} \approx 5.1$
  - $Case\ t_2: Magnitude = \sqrt{9 + 1 + 1} \approx 3.32$
  - $Case\ t_3: Magnitude = \sqrt{25 + 1 + 25} \approx 7.14$

Example cont'd

- AAD

$$AAD = \frac{\sum_{i=2}^{N} |x_i - x_{i-1}|}{N}$$

  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $AAD\ X = \frac{|3-1|+|5-3|}{3} = \frac{4}{3} = 1.3\overline{3}$
  - $AAD\ Y = \frac{|1-(-3)|+|(-1)-1|}{3} = \frac{6}{3} = 2$
  - $AAD\ Z = \frac{|1-4|+|-5-1|}{3} = \frac{9}{3} = 3$

Example cont'd

$$AAV = \frac{\sum_{i=1}^{N} |x_i|}{n}$$

- AAV
  - Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

  - $AAV\ X = \frac{|1|+|3|+|5|}{3} = \frac{9}{3} = 3$
  - $AAV\ Y = \frac{|-3|+|1|+|-1|}{3} = \frac{3+1+1}{3} = \frac{5}{3} = 1.6\overline{6}$
  - $AAV\ Z = \frac{|4|+|1|+|-5|}{3} = \frac{4+1+5}{3} = \frac{10}{3} = 3.3\overline{3}$

Example cont'd

- RMS
  - Given a data set

$$RMS = \sqrt{\frac{\sum_{i=1}^{N}(x_i^2)}{N}}$$

	1	3	5
	-3	1	-1
	4	1	-5

  - $RMS\ X = \sqrt{\frac{1+9+25}{3}} = \sqrt{\frac{35}{3}} \approx 3.42$
  - $RMS\ Y = \sqrt{\frac{9+1+1}{3}} = \sqrt{\frac{11}{3}} \approx 1.91$
  - $RMS\ Z = \sqrt{\frac{16+1+25}{3}} = \sqrt{\frac{42}{3}} \approx 3.74$

Factor	1 or 3 – dimension Result
Mean	
Variance	
Skewness	
Kurtosis	
Covariance	
ZCR	
MCR	
ARA	5.18
Magnitude	
AAD	
AAV	
RMS	

	1	3	5
	-3	1	-1
	4	1	-5