

# Data Features

Factor	Equation	Domain Type	Dimensions	Reference
Mean		Time	3	(Sun et al. 2010) (Bao & Intille 2004)
Variance		Time	3	(Sun et al. 2010) (Bao & Intille 2004)
Skewness		Time	3	(Godfrey et al. 2008)
Kurtosis		Time	3	(Ermes et al. 2008)
Covariance		Time	3	(Bao & Intille 2004)
Zero Crossing Rate		Time	3	(Mase 2002)
Mean Crossing Rate		Time	3	(Mase 2002) (Altini et al. 2012)
Average Resultant Acceleration		Time	1	(Kwapisz et al. 2011)
Magnitude		Time	1	(Khan et al. 2010)
Average Absolute Difference		Time	3	(Kwapisz et al. 2011)
Average Absolute Value		Time	3	(Kwapisz 2010)
Root Mean Square		Time	3	(Staudenmayer et al. 2009)
FFT Transformation	N/A	Time		
Dominant frequency		Frequency	3	(Sun et al. 2010)
Energy		Frequency	3	(Bao & Intille 2004)
Entropy		Frequency	3	(Mannini & Sabatini 2010)

# Example

- Mean

- Given a data set

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

	1	3	5
	-3	1	-1
	4	1	-5

- $\bar{X} = \frac{1+3+5}{3} = 3$

- $\bar{Y} = \frac{(-3)+1+(-1)}{3} = -1$

- $\bar{Z} = \frac{4+1+(-5)}{3} = 0$

# Example cont'd

- Variance

- Given a data set

$$\text{var}(x) = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$$

	1	3	5
	-3	1	-1
	4	1	-5

- $\text{var}(X) = \frac{(1-3)^2 + (3-3)^2 + (5-3)^2}{3} = \frac{8}{3}$

- $\text{var}(Y) = \frac{(-3-(-1))^2 + (1-(-1))^2 + (-1-(-1))^2}{3} = \frac{8}{3}$

- $\text{var}(Z) = \frac{(4-0)^2 + (1-0)^2 + (-5-0)^2}{3} = 14$

# Example cont'd

$$M_k = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^k$$

- Skewness

- Given a data set

$$\text{skewness } g_1 = \frac{\sqrt{n}M_3}{M_2^{\frac{3}{2}}}$$

	1	3	5
	-3	1	-1
	4	1	-5

- skewness  $X =$

$$\sqrt{3} \left( \frac{(1-3)^3 + (3-3)^3 + (5-3)^3}{3} \right) / \left( \frac{(1-3)^2 + (3-3)^2 + (5-3)^2}{3} \right)^{\frac{3}{2}} =$$
$$\sqrt{3} \left( \frac{(-8) + 0 + 8}{3} \right) / \left( \frac{4 + 0 + 4}{3} \right)^{\frac{3}{2}} = 0$$

# Example cont'd

- – *skewness*  $Y = \frac{\sqrt{3} \left( \frac{(-3-(-1))^3 + (1-(-1))^3 + (-1-(-1))^3}{3} \right)}{\left( \frac{(-3-(-1))^2 + (1-(-1))^2 + (-1-(-1))^2}{3} \right)^{\frac{3}{2}}} = \frac{\sqrt{3} \left( \frac{(-8)+8+0}{3} \right)}{\left( \frac{4+4+0}{3} \right)^{\frac{3}{2}}} = 0$
- *skewness*  $Z = \frac{\sqrt{3} \left( \frac{(4-0)^3 + (1-0)^3 + (-5-0)^3}{3} \right)}{\left( \frac{(4-0)^2 + (1-0)^2 + (-5-0)^2}{3} \right)^{\frac{3}{2}}} = \frac{\sqrt{3} \left( \frac{64+1+125}{3} \right)}{\left( \frac{16+1+25}{3} \right)^{\frac{3}{2}}} \approx 6.28$

# Example cont'd

- Kurtosis

$$\text{kurtosis } g_2 = \frac{nM_4}{M_2^2} - 3$$

– Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

– *kurtosis*  $X =$

$$3 \left( \frac{(1-3)^4 + (3-3)^4 + (5-3)^4}{3} \right) / \left( \frac{(1-3)^2 + (3-3)^2 + (5-3)^2}{3} \right)^2 - 3 =$$
$$3 \left( \frac{16+0+16}{3} \right) / \left( \frac{4+0+4}{3} \right)^2 - 3 = 4.5 - 3 = 1.5$$

# Example cont'd

- – *kurtosis*  $Y =$   
$$3 \left( \frac{(-3-(-1))^4 + (1-(-1))^4 + (-1-(-1))^4}{3} \right) / \left( \frac{(-3-(-1))^2 + (1-(-1))^2 + (-1-(-1))^2}{3} \right)^2 .$$
$$3 \left( \frac{16+16+0}{3} \right) / \left( \frac{4+4+0}{3} \right)^2 - 3 = 4.5 - 3 = 1.5$$
- *kurtosis*  $Z =$   
$$3 \left( \frac{(4-0)^4 + (1-0)^4 + (-5-0)^4}{3} \right) / \left( \frac{(4-0)^2 + (1-0)^2 + (-5-0)^2}{3} \right)^2 - 3 =$$
$$3 \left( \frac{256+1+625}{3} \right) / \left( \frac{16+1+25}{3} \right)^2 - 3 = 4.5 - 3 = 1.5$$

# Example cont'd

- Covariance

- Given a data set

$$\text{cov}(x,y) = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x}) * (y_i - \bar{y})$$

	1	3	5
	-3	1	-1
	4	1	-5

- $\text{cov}(x,y) = \frac{(1-3)(-3-(-1))+(3-3)(1-(-1))+(5-3)(-1-(-1))}{2} = \frac{4+0+0}{2} = 2$

- $\text{cov}(y,z) = \frac{(-3-(-1))(4-0)+(1-(-1))(1-0)+(-1-(-1))(-5-0)}{2} = \frac{-8+2}{2} = -3$

- $\text{cov}(x,z) = \frac{(1-3)(4-0)+(3-3)(1-0)+(5-3)(-5-0)}{2} = \frac{-8+0+(-10)}{2} = -9$



# Example cont'd

$$C = \{x | (x_i > 0 \wedge x_{i-1} < 0) \vee (x_i < 0 \wedge x_{i-1} > 0)\}$$
$$ZCR = \text{count}(C)$$

## • ZCR

– Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

–  $ZCR X = 0$

–  $ZCR Y = 2$

–  $ZCR Z = 1$

→ Cross Zero

# Example cont'd

$$C = \{x | (x_i > \bar{x} \wedge x_{i-1} < \bar{x}) \vee (x_i < \bar{x} \wedge x_{i-1} > \bar{x})\}$$
$$MCR = \text{count}(C)$$

## • MCR

– Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

$$\bar{X} = 3$$
$$\bar{Y} = -1$$
$$\bar{Z} = 0$$

–  $MCR X = 0$

–  $MCR Y = 1$

–  $MCR Z = 1$

→ Cross Mean

# Example cont'd

- ARA

- Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

- $ARA = \frac{\sqrt{1+9+16} + \sqrt{9+1+1} + \sqrt{25+1+25}}{3} = \frac{\sqrt{26} + \sqrt{11} + \sqrt{51}}{3} \approx 5.18$

$$ARA = \frac{\sum_{i=1}^N \sqrt{x_i^2 + y_i^2 + z_i^2}}{N}$$

# Example cont'd

$$\text{magnitude} = \sqrt{x^2 + y^2 + z^2}$$

## • Magnitude

- Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

- *Case  $t_1$ : Magnitude =  $\sqrt{1 + 9 + 16} \approx 5.1$*
- *Case  $t_2$ : Magnitude =  $\sqrt{9 + 1 + 1} \approx 3.32$*
- *Case  $t_3$ : Magnitude =  $\sqrt{25 + 1 + 25} \approx 7.14$*

# Example cont'd

- AAD

$$AAD = \frac{\sum_{i=2}^N |x_i - x_{i-1}|}{N}$$

- Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

- $AAD X = \frac{|3-1|+|5-3|}{3} = \frac{4}{3} = 1.3\bar{3}$

- $AAD Y = \frac{|1-(-3)|+|(-1)-1|}{3} = \frac{6}{3} = 2$

- $AAD Z = \frac{|1-4|+|(-5)-1|}{3} = \frac{9}{3} = 3$

# Example cont'd

- AAV

$$AAV = \frac{\sum_{i=1}^N |x_i|}{n}$$

– Given a data set

	1	3	5
	-3	1	-1
	4	1	-5

–  $AAV X = \frac{|1|+|3|+|5|}{3} = \frac{9}{3} = 3$

–  $AAV Y = \frac{|-3|+|1|+|-1|}{3} = \frac{3+1+1}{3} = \frac{5}{3} = 1.6\bar{6}$

–  $AAV Z = \frac{|4|+|1|+|-5|}{3} = \frac{4+1+5}{3} = \frac{10}{3} = 3.3\bar{3}$

# Example cont'd

- RMS

- Given a data set

$$RMS = \sqrt{\frac{\sum_{i=1}^N (x_i^2)}{N}}$$

	1	3	5
	-3	1	-1
	4	1	-5

- $RMS X = \sqrt{\frac{1+9+25}{3}} = \sqrt{\frac{35}{3}} \approx 3.42$

- $RMS Y = \sqrt{\frac{9+1+1}{3}} = \sqrt{\frac{11}{3}} \approx 1.91$

- $RMS Z = \sqrt{\frac{16+1+25}{3}} = \sqrt{\frac{42}{3}} \approx 3.74$

Factor	1 or 3 – dimension Result		
Mean			
Variance			
Skewness			
Kurtosis			
Covariance			
ZCR			
MCR			
ARA		5.18	
Magnitude			
AAD			
AAV			
RMS			

	1	3	5
	-3	1	-1
	4	1	-5