IITU

Neural Networks

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Week 7
Lecture 7

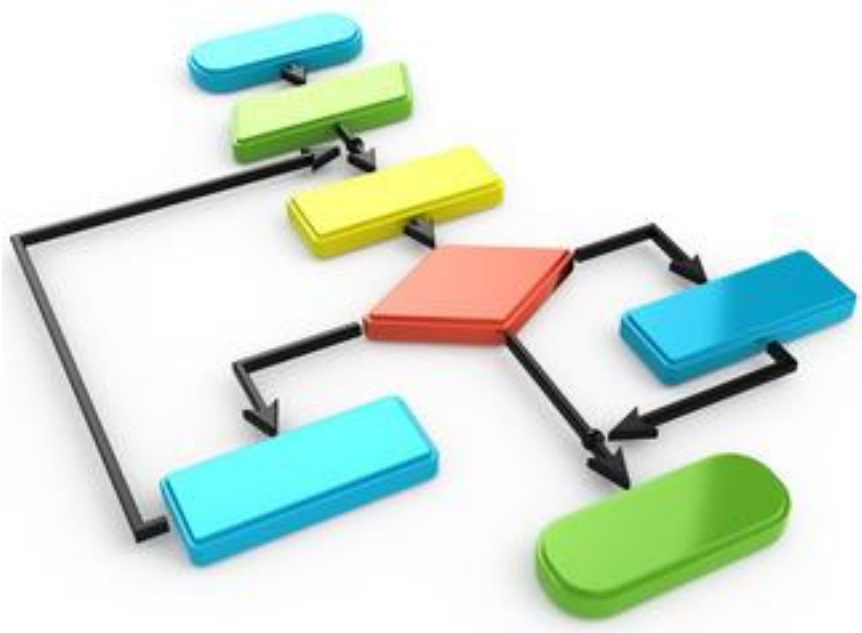
Topics

- Types of Optimization Algorithms used in Neural Networks
- Gradient descent

Have you ever wondered which optimization algorithm to use for your Neural network Model to produce slightly better and faster results by updating the Model parameters such as **Weights** and **Bias** values .

Should we use **Gradient Descent** or **Stochastic** gradient Descent?

What are Optimization Algorithms ?



Optimization algorithms helps us to ***minimize (or maximize)*** an **Objective** function (*another name for* ***Error*** *function*) **E(x)** which is simply a mathematical function dependent on the Model's internal **learnable parameters** which are used in computing the target values(**Y**) from the set of *predictors*(**X**) used in the model.

For example—we call
the **Weights(W)** and the **Bias(b)** values of the neural network as its internal learnable *parameters* which are used in computing the output values and are learned and updated in the direction of optimal solution i.e minimizing the **Loss** by the network's training process and also play a major role in the ***training*** process of the Neural Network Model .

The internal parameters of a Model play a very important role in efficiently and effectively training a Model and produce accurate results.

This is why we use various Optimization strategies and algorithms to update and calculate appropriate and optimum values of such model's parameters which influence our Model's learning process and the output of a Model.

Optimization Algorithm falls in 2 major categories

First Order Optimization Algorithms—These algorithms minimize or maximize a Loss function **E(x)** using its ***Gradient*** values with respect to the parameters. Most widely used First order optimization algorithm is **Gradient Descent.**

The First order derivative tells us whether the function is decreasing or increasing at a particular point. First order Derivative basically give us a **line** which is ***Tangential*** *to a point on its Error Surface.*

What is a Gradient of a function?

A **Gradient** is simply a vector which is a multi-variable generalization of a ***derivative***(**dy/dx**) which is the *instantaneous rate of change of* ***y with respect to x.***

The difference is that to calculate a derivative of a function which is dependent on more than one variable or multiple variables, a ***Gradient takes its place. And a gradient is calculated using Partial Derivatives .*** *Also another major difference between the* ***Gradient*** *and a* ***derivative*** *is that a* ***Gradient*** *of a function produces a* ***Vector Field****.*

A **Gradient** is represented by a ***Jacobian*** Matrix—which is simply a Matrix consisting of ***first order partial Derivatives(Gradients)***.

Hence summing up, a derivative is simply defined for a function dependent on single variables , whereas a Gradient is defined for function dependent on multiple variables.

Second Order Optimization Algorithms—Second-order methods use the **second order derivative** which is also called **Hessian** to minimize or maximize the **Loss** function.

The Hessian is a Matrix of *Second Order Partial Derivatives*. *Since the second derivative is costly to compute, the second order is not used much* .

The second order derivative tells us whether the ***first derivative*** is increasing or decreasing which hints at the function's curvature.

Second Order Derivative provide us with a **quadratic** surface which touches the curvature of the **Error Surface**.

- **Some Advantages of Second Order Optimization over First Order —**
- Although the Second Order Derivative may be a bit costly to find and calculate, but the advantage of a ***Second order Optimization*** *Technique is that is does not neglect or ignore the* ***curvature of Surface****. Secondly, in terms of Step-wise Performance they are better.*

What are the different types of Optimization Algorithms used in Neural Networks ?

- **Gradient Descent**

- **Variants of Gradient Descent:** *Batch Gradient Descent;* **Stochastic gradient descent; Mini Batch Gradient Descent**

Gradient Descent is the most important technique and the foundation of how we train and optimize ***Intelligent Systems.*** What is does is **—**

"Gradient Descent—Find the Minima , control the variance and then update the Model's parameters and finally lead us to Convergence."

$$\theta = \theta - \eta \cdot \nabla J(\theta)$$

—is the formula of the parameter updates, where '**η'** is the learning rate ,' **∇J(θ)'** is the **Gradient** of ***Loss function*-J(**θ**)** w.r.t *parameters*-'*θ'*.

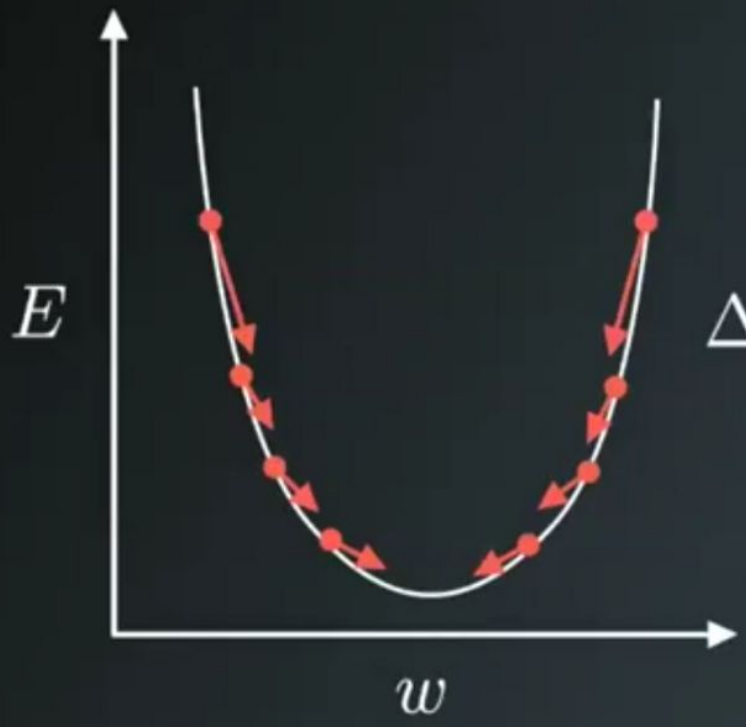
The parameter **η** is the **training rate.** This value can either set to a fixed value or found by one-dimensional optimization along the training direction at each step. An optimal value for the training rate obtained by line minimization at each successive step is generally preferable. However, there are still many software tools that only use a fixed value for the training rate.

It is the most popular Optimization algorithms used in optimizing a Neural Network. Now gradient descent is majorly used to do **Weights updates** in a Neural Network Model , i.e update and tune the Model's parameters in a direction so that we can minimize the **Loss function (**or **cost function)**.

Now we all know a Neural Network trains via a famous technique called **Backpropagation ,** in which we first propagate forward calculating the dot product of Inputs signals and their corresponding Weights and then apply a ***activation function*** to those sum of products, which transforms the input signal to an output signal and also is important to model complex Non-linear functions and introduces **Non-linearities** to the Model which enables the Model to learn almost any *arbitrary functional mappings.*

After this we propagate **backwards** in the Network carrying **Error** terms and updating **Weights** values using *Gradient Descent, in which we calculate the gradient of* ***Error(E) function*** *with respect to the* ***Weights*** *(**W**) or the parameters , and update the parameters (here* ***Weights****) in the opposite direction of the Gradient of the Loss function w.r.t to the Model's parameters.*

$$E = \frac{1}{2}(y - f(\sum w_i x_i))^2$$



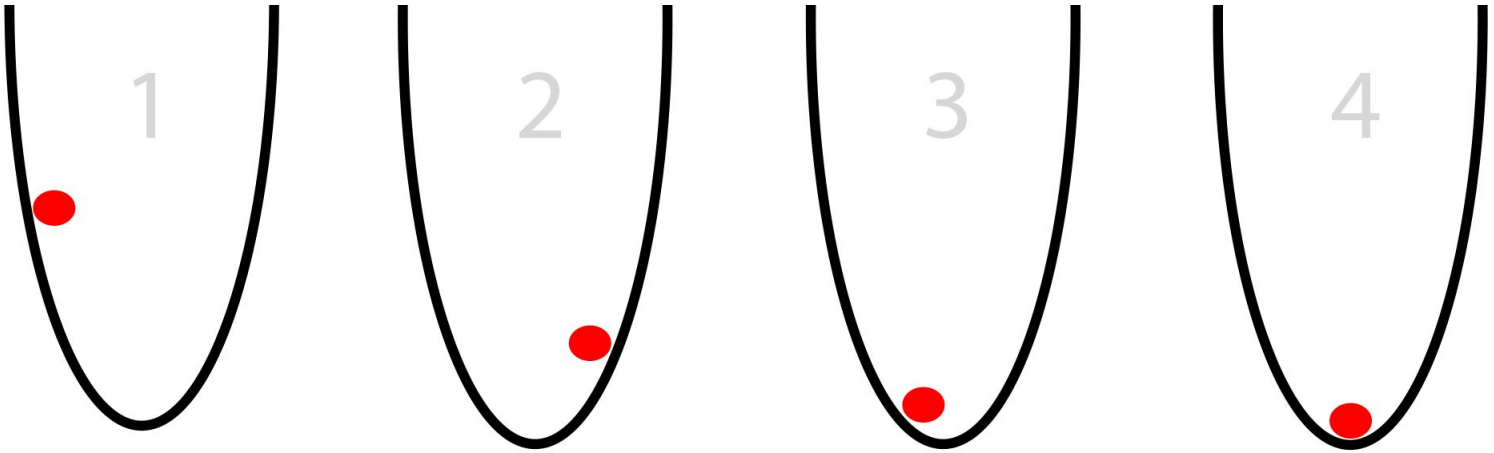
$$\Delta w = -gradient$$

*The image on above shows the process of Weight updates in the opposite direction of the Gradient Vector of Error w.r.t to the Weights of the Network. The **U-Shaped** curve is the Gradient(slope).*

*As one can notice if the Weight**(W)** values are too small or too large then we have large Errors , so want to update and optimize the weights such that it is neither too small nor too large , so we descent downwards opposite to the Gradients until we find a **local minima**.*

Gradient Descent

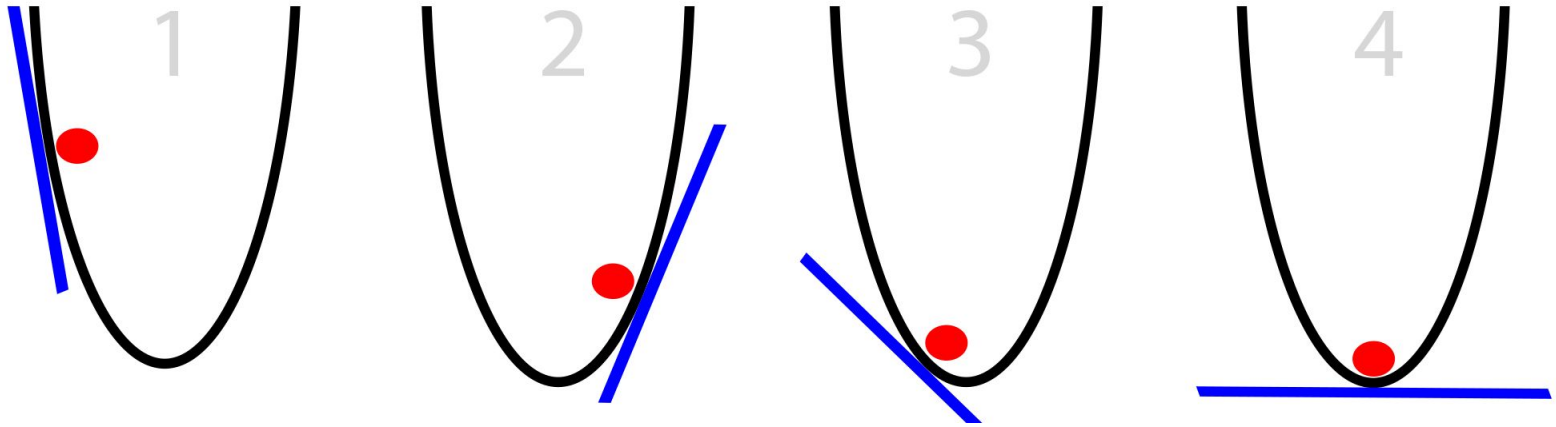
*we descent downwards opposite to the Gradients until we find a **local minima**.*



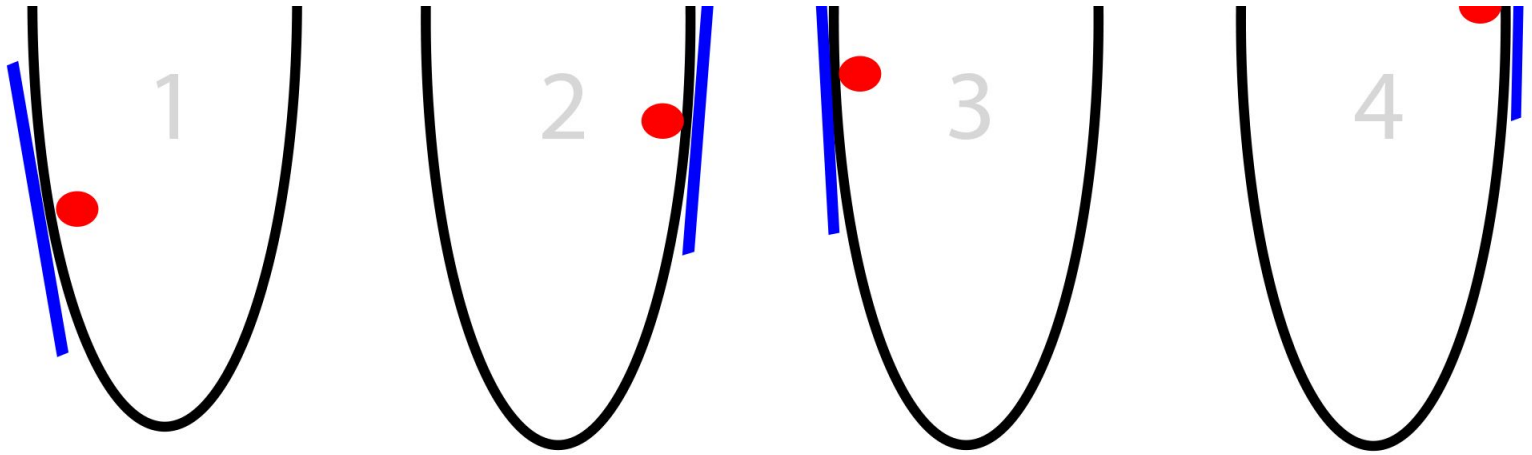
1.find slope

2. (x = x — slope)

until slope=0

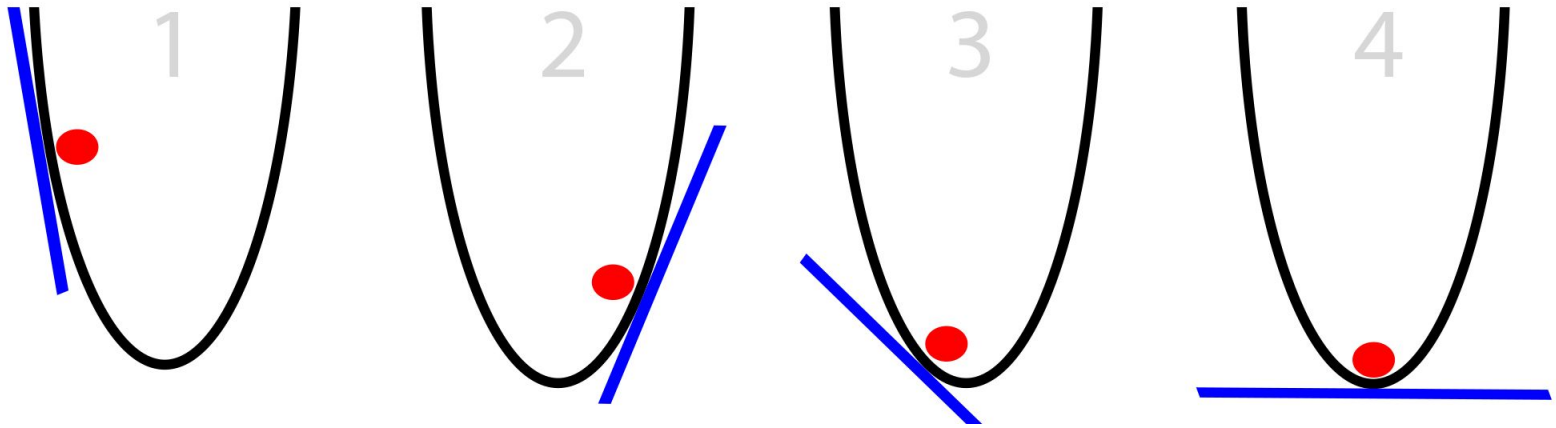


Problem

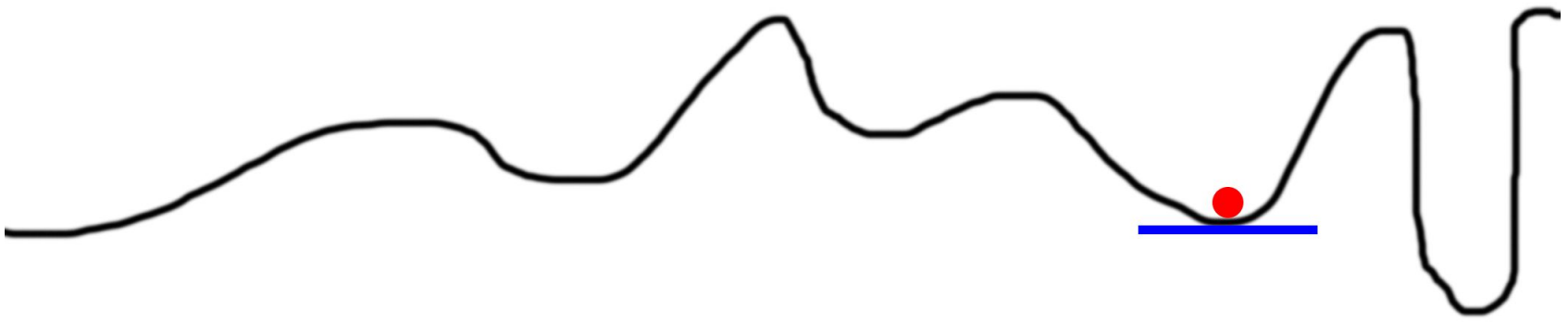


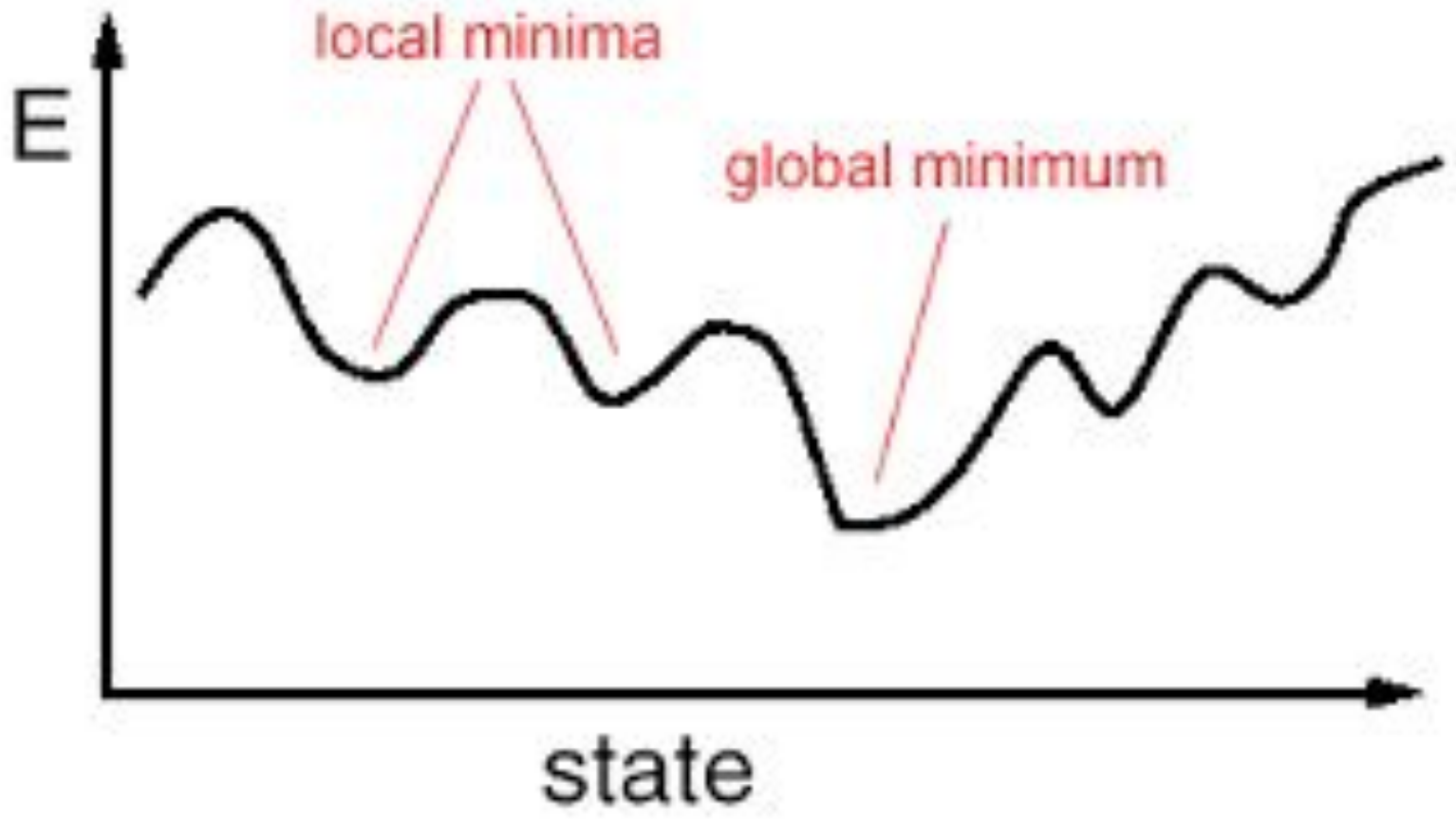
1. find slope
2. alpha = 0.1 (or any number from 0 to 1)
3. x = x — (alpha*slope)

until slope=0

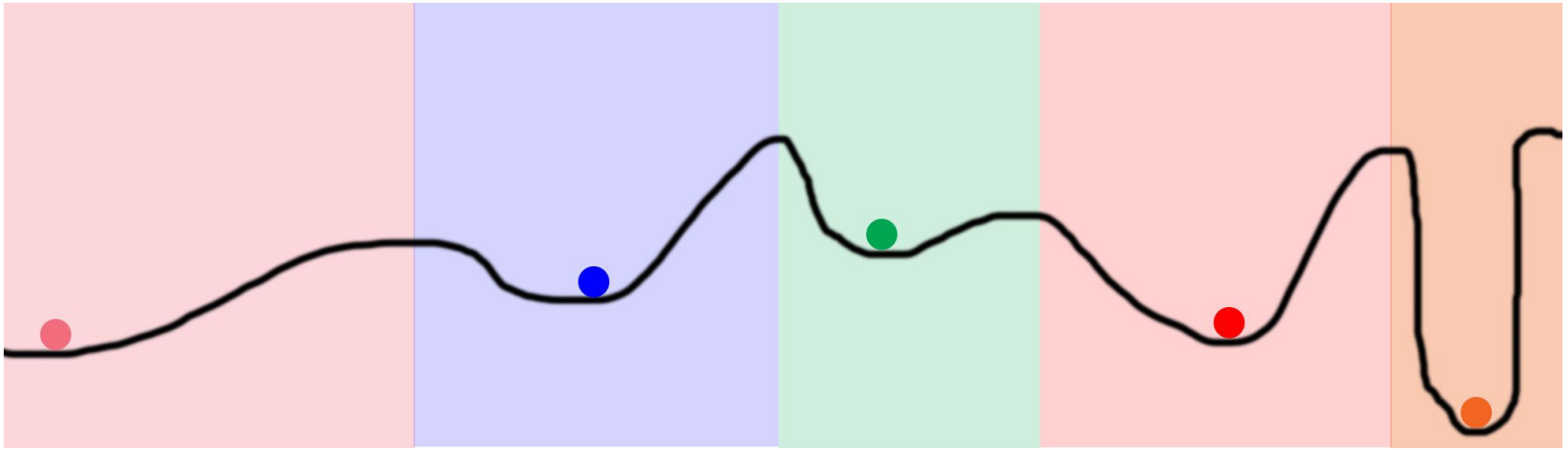


Problem

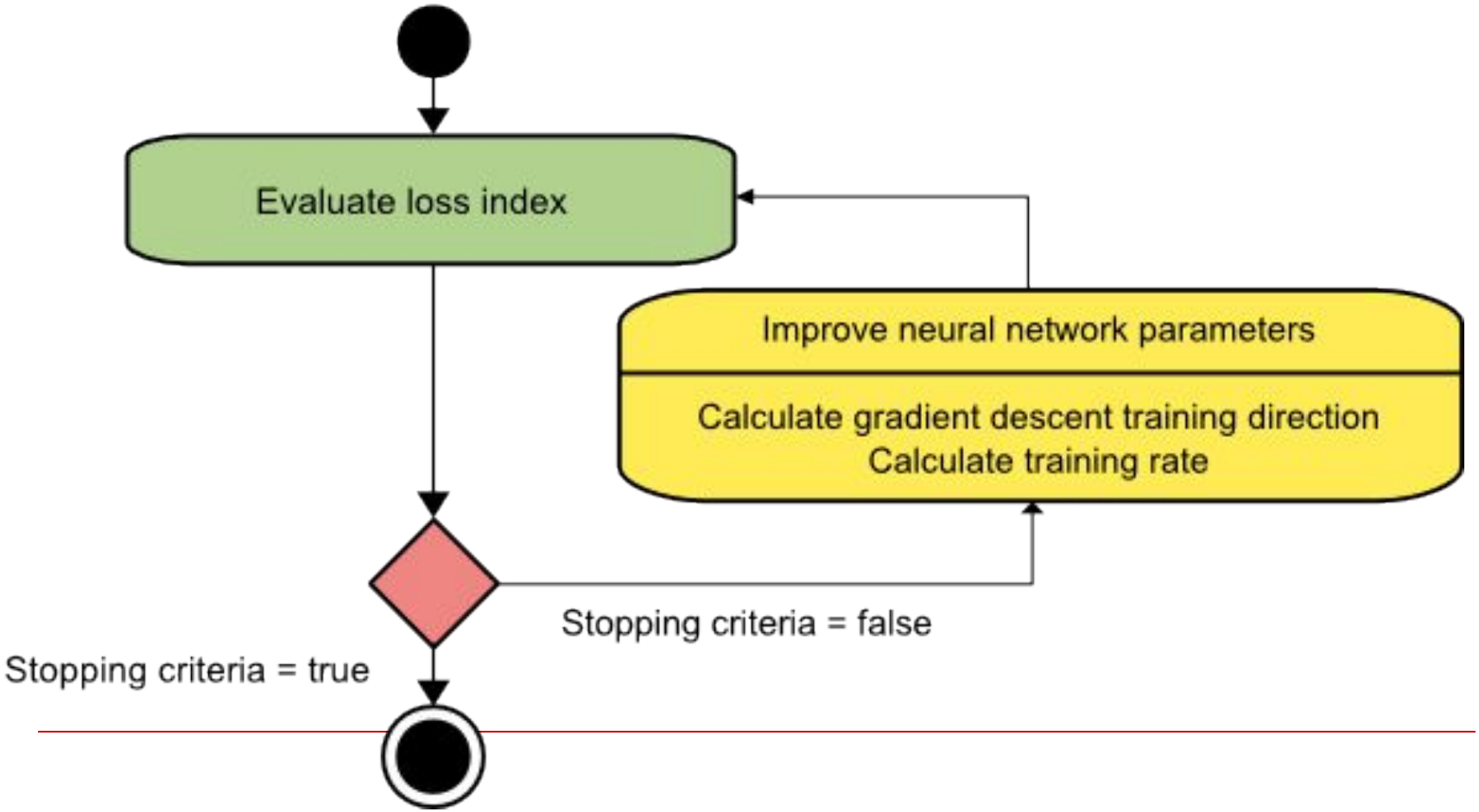



local minima
E
global minimum
state

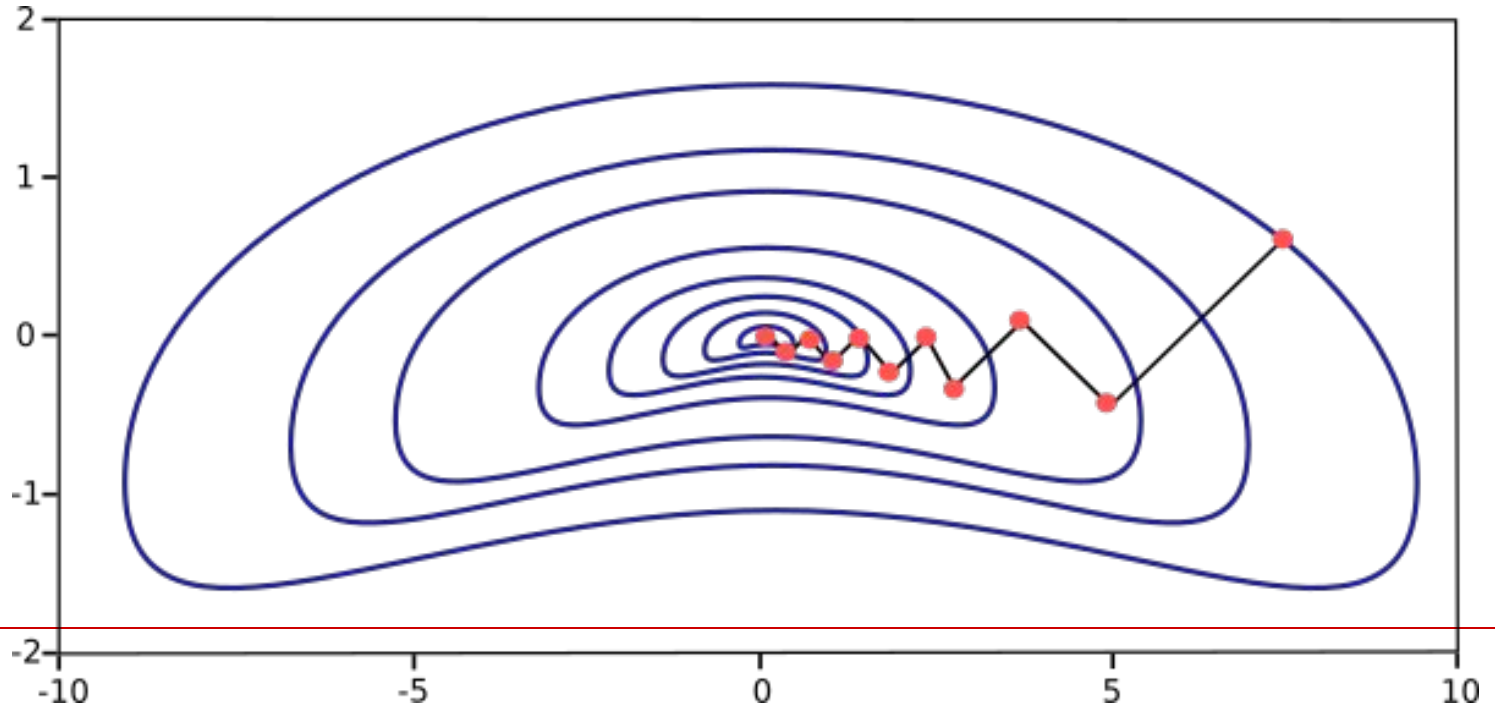
Solving the problem



The next picture is an activity diagram of the training process with gradient descent. As we can see, the parameter vector is improved in two steps: First, the gradient descent training direction is computed. Second, a suitable training rate is found.



The gradient descent training algorithm has the severe drawback of requiring many iterations for functions which have long, narrow valley structures. Indeed, the downhill gradient is the direction in which the loss function decreases most rapidly, but this does not necessarily produce the fastest convergence. The following picture illustrates this issue.



Gradient descent is the recommended algorithm when we have very big neural networks, with many thousand parameters. The reason is that this method only stores the gradient vector (size n), and it does not store the Hessian matrix (size n^2).

Optimization algorithm for Neural network Model

- Annealing
- Stochastic Gradient Descent
- AW-SGD
- Momentum
- Nesterov Momentum
- AdaGrad
- AdaDelta
- ADAM
- BFGS
- LBFGS

Thank you for your attention!